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Stationary stress-strain state of a three-layer viscoelastic cylindrical shell under normal loading

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Abstract. Multilayer viscoelastic cylindrical shells have found their wide application in construction, mechanical engineering, aircraft and rocket engineering. The aim of the work is to investigation the reaction of an infinitely long three-layer cylindrical shell to the action of a normal load moving along an axis with a constant to resonant velocity. The paper presents a mathematical formulation of the problem, developed solution methods and obtained numerical results for the problems of stationary deformation of an infinitely long three-layered cylindrical shell under normal loading. The equations of motion of the bearing layers satisfy the Kirchhoff-Love hypotheses. The solution methods are based on the joint application of the integral Fourier transform in the axial coordinate and the decomposition of all given and desired quantities into Fourier series in the angular coordinate. The outer and inner shells satisfy the Kirchhoff-Love hypotheses. The Lame viscoelasticity equation is used as a linear equation of the filler motion. An effective algorithm for solving the problem of osculations of a three-layer viscoelastic cylindrical shell under normal loading has been developed on a computer. Critical velocities of wave propagation in a three-layer shell under the influence of moving loads are found.

1. Introduction

The dynamics of the half-plane under the action of moving loads is considered in [1]. In [2], the stress distribution in an elastic half-plane is studied, along the boundary of which an arbitrary load with vertical and horizontal components moves at an arbitrary speed. The solution is presented in integral form, numerical results are given for a concentrated normal force moving at a constant speed. The effect of a normal pressure pulse moving at transonic speed is considered in [3]. Displacements and bending moments depending on the velocity of linear load displacement in narrow plates elastically supported along the edges are determined in [4] using the Fourier transform in geometric coordinate and Laplace in time. The dynamics of a half-plane made of a non-linearly compressible material (soil) under the influence of a monotonically decreasing profile load was studied in [5]. The effect of the inhomogeneity of the band on the dynamic response under moving loads.

The purpose of this work is to study the reaction of an infinitely long three-layer cylindrical shell to the action of a non-axisymmetric normal load moving along an axis with a constant to resonant velocity.

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The equations of motion of the bearing layers satisfy the Kirchhoff-Love hypotheses. And also the filler satisfies the Lame equations and the boundary conditions on the contacts.

2. Methods

A shell of symmetrical structure is considered, in which the mechanical characteristics and thicknesses of the bearing layers are the same. For normal loading, the contact between the shells and the filler is assumed to be sliding with a two-way nature of the connection between them.

The non - axisymmetric movement of the shell is written by the equations

$$L_{ij\kappa}\vec{U}_{0k} - \int_0^t L_{ij\kappa}R_{E\kappa}(t-\tau)\vec{U}_{0k}(\vec{r},\tau)\,d\tau = \frac{(1-\nu_{0\kappa}{}^2)}{G_{0\kappa}h_{0\kappa}}\vec{P}_\kappa + \rho_{0\kappa}\frac{(1-\nu_{0\kappa}{}^2)}{G_{0\kappa}}\frac{\partial^2\vec{U}_{0k}}{\partial t^2}\,.\,(k=1,2) \quad (1)$$

Here the index k=1- for the inner shell (cylinder), k=2 -for the outer shell, $\vec{U}_k$ - the vector of displacements of the points of the median surface of the carrier layer. For shells obeying the Kirchhoff-Love hypotheses, the displacement vector has a dimension equal to three. For shells obeying the hypotheses of Timoshenko, the dimension of the displacement vector is equal to five. The linear equation of motion of the filler of the considered mechanical system has the form:

$$\tilde{\mu}_c\nabla^2\vec{u}_c + (\tilde{\lambda}_c + \tilde{\mu}_c)graddi\vartheta\vec{u}_c = \rho_c\frac{\partial^2\vec{u}_c}{\partial t^2}, \quad (2)$$

where $\vec{u}$– displacement vector;ρ_κ- medium density; κ - sequence number of layers, $\nu_\kappa -$ the Poisson's ratio, which we consider to be a non-relaxing value [6,7],

$$\tilde{\lambda}_\kappa f(t) = \lambda_{0\kappa}\left[f(t) - \int_{-\infty}^{t} R_{\lambda\kappa}(t-\tau)f(\tau)d\tau\right];$$

$$\tilde{\mu}_\kappa f(t) = \mu_{0\kappa}\left[f(t) - \int_{-\infty}^{t} R_{\mu\kappa}(t-\tau)f(\tau)d\tau\right], \quad (3)$$

$\vec{r} = \vec{r}(x,y,z,t)$; $R_{\lambda k}(t-\tau), R_{\mu k}(t-\tau)$– the core of relaxation;$\lambda_{\kappa 0}, \mu_{\kappa 0}$– instantaneous elastic modulus; $f(t)$– an arbitrary function of time.

When performing a sliding contact between the shell and the filler, when the shell obeys the Kirchhoff-Love hypotheses, the following conditions are placed on the contact

$$r = a_k: \sigma_{rz\kappa} = \sigma_{r\theta\kappa} = 0;$$

$$u_{r\kappa} = w_k; \sigma_{rr\kappa} = \pm q_{r\kappa}$$

$$(k=1, r=a_1: \sigma_{rr1} = -q_{r1}; k=2, r=a_2: \sigma_{rr2} = q_{r2};). \quad (4)$$

In the considered problem of propagation of free damped waves, the components of the external load are zero, i.e.

$$p_{rk} = 0, p_{\theta k} = 0, p_{zk} = 0.$$

When considering the steady-state process, the Galilean transformation is applied

$$\eta = (x - ct)/H,$$

where H is some characteristic quantity in the problem under consideration, which has the dimension of length (for shells -the radius of the outer or inner carrier layer) [8,9]. Using the Galileo transformation, the problem is reduced to the joint solution of the shell and placeholder equations.

Applying the Fourier transform in a moving coordinate system by η and decomposing all functions into Fourier series by θ, we find the Fourier coefficients (transformant of normal loads), which are transmitted to the filler from the side of the skin:

$$q^0_{rk,n} = -p^0_{k,n} - \frac{2G}{1-v} k^2 f(k) \frac{u^0_{k,n}}{h}. \quad (5)$$

Introducing potential functions [10], we search for the general solution of equations (1) in the form of Fourier series and satisfying the conditions (4), we obtain a system of algebraic equations for determining the functions $A_n(\xi)\ldots S_n(\xi)$. The solution of this system can be written as

$$\{A_n + S_n\} = -\frac{a_2^0 p_n^0(\xi)}{2G_c\, det_n\|a_{ij}\|}\{A'_n + S'_n\}\ , \ (i,j = 1,\ldots,6), \quad (6)$$

$$A'_n = \frac{A_{61}}{K_{n+1}(m\xi)}\ ; \quad B'_n = \frac{A_{62}}{I_n(m\xi)}\ ;$$

$$C'_n = i\frac{a_2 A_{63}}{\xi K_{n+1}(m_s\xi)}; D'_n = -i\frac{a_2 A_{64}}{\xi I_n(m_s\xi)}; E' = \frac{A_{65}}{K_n(m_s\xi)};$$

$$S'_n = \frac{A_{66}}{I_n(m_s\xi)}.$$

Here A_{6j} algebraic additions of elements a_{6j} , and the elements of determinants $det_n\|a_{ij}\|$ are calculated according to (6).

Knowing $A_n\ldots S_n$, we obtain expressions for the terms of the expansions of the transformant of displacements and stresses in the placeholder [10]. Then the transformants of the radial components during the movement of the normal load on the shell surface are represented by the formulas

$$u_r^0(\xi,\theta,r_*) = -\frac{a_2}{G_c}\sum_{n=0}^{\infty}\frac{p^0_{r,n}(\xi)u_n(\xi,r_*)}{det_n\|a_{rj}\|}cos(n\theta); \quad (7)$$

$$u_n(\xi,r_*) = \left(\frac{n}{r_*}s_{14} - m\xi s_{13}\right)A_{61} + \left(\frac{n}{r_*}s_{16} + m\xi s_{15}\right)A_{62} +$$

$$+\left(\frac{n}{r_*}s_{18} - m\xi s_{17}\right)A_{63} - \left(\frac{n}{r_*}s_{20} + m\xi s_{19}\right)A_{64} - \frac{n}{r_*}s_{18}A_{65} + \frac{n}{r_*}s_{20}A_{66}; \quad (8)$$

$$\sigma^0_{rr}(\xi,\theta,r_*) = -\sum_{n=0}^{\infty}\frac{p^0_{r,n}(\xi)\sigma_n(\xi,r_*)}{det_n\|a_{rj}\|}cos(n\theta); \quad (9)$$

$$\sigma_n(\xi,r_*) = \left\{\left[t_1\xi^2 + \frac{n(n-1)}{r_*^2}\right]s_{14} + \frac{m\xi}{r_*}s_{13}\right\}A_{61} + \left\{\left[t_1\xi^2 + \frac{n(n-1)}{r_*^2}\right]s_{16} - \frac{m\xi}{r_*}s_{15}\right\}A_{62} +$$

$$+\left\{\left[m_s^2\xi^2 + \frac{n(n-1)}{r_*^2}\right]s_{18} + \frac{m\xi}{r_*}s_{17}\right\}A_{63} - \left\{\left[m_s^2\xi^2 + \frac{n(n-1)}{r_*^2}\right]s_{20} + \frac{m\xi}{r_*}s_{19}\right\}A_{64} -$$

$$-\frac{n}{r_*}\left[\frac{n-1}{r_*}s_{18} - m_s\xi s_{17}\right]A_{65} + \frac{n}{r_*}\left[\frac{n-1}{r^2{}_*}s_{20} + m_s\xi s_{19}\right]A_{66};$$

$$s_{13} = \frac{K_{n+1}(m\xi r_*)}{K_{n+1}(m\xi)}; s_{14} = \frac{K_n(m\xi r_*)}{K_{n+1}(m\xi)};\ s_{15} = \frac{I_{n+1}(m\xi r_*)}{I_n(m\xi)}; s_{16} = \frac{I_n(m\xi r_*)}{I_n(m\xi)};$$

s_{17}, s_{18} are obtained from s_{13}, s_{14} a s_{19}, s_{20} -из s_{15}, s_{16} replacement m on m_s. Calculations are carried out for the system l of self-balanced forces concentrated at the same distance along the circumference [11]. In this case, radial displacements and stresses in the filler are calculated using the formulas

$$\frac{2Gu_r}{p_0} = -\frac{2\gamma}{l}\sum_{n=0}^{\infty}\left[\int_0^{\infty}\frac{u_n(\xi, r_*)\,cos(\xi\eta)d\xi}{det_n\|a_{ij}\|}\right]a_n\,cos(n\theta);$$

$$\frac{\sigma_{rr}a_2}{p_0} = -\frac{2}{l}\sum_{n=0}^{\infty}\left[\int_0^{\infty}\frac{a_n(\xi, r_*)\,cos(\xi\eta)d\xi}{det_n\|a_{ij}\|}\right]a_n\,cos(n\theta). \quad (10)$$

Note the special cases of the problem.

1. For a single-layer shell of the same radius as the outer bearing layer, the formula for determining deflections has the form

$$\frac{2G\omega}{p_0} = -\frac{2(1-\nu)}{kl}\sum_{n=0}^{\infty}\left[\int_0^{\infty}\frac{cos(\xi\eta)d\xi}{t_4(n,\xi)}\right]a_n\,cos(n\theta). \quad (11)$$

2. If the inner surface of the filler is not in contact with the supporting layer (a shell with a hollow filler, on the inner surface of which there are no stresses), then the boundary conditions have the form (4) and the solution is obtained by replacing the elements of the fifth row in the determinants $det_n\|a_{ij}\|$ according to (4).

3. Results and analysis

The pre-resonant modes of motion are considered, while for obtaining numerical results in finite Fourier series (11), which were determined by numerical experiment on a computer. The calculation results for the displacements are shown in figure 1.

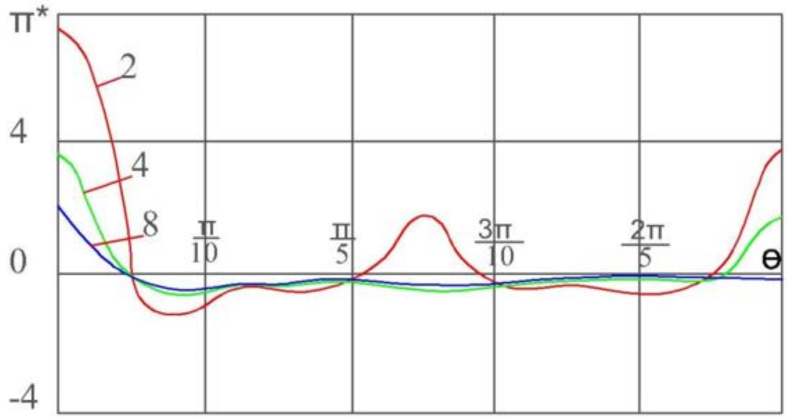

Figure 1. Change in the radial distribution of deflections of the loaded outer bearing layer depending on θ.

Figure 1 shows the distribution of dimensionless deflections of the outer bearing layer w^* by circumference in cross section $\eta = 0$ for the impact of concentrated forces. As the relaxation kernel, we take the three-parameter Koltunov - Rzhanitsyn relaxation kernel:

$$R_k(t) = Ae^{-\beta t}/t^{1-\alpha},$$

with parameters:

$$A = 0.048; \quad \beta = 0.05; \quad \alpha = 0.10.$$

The calculations were carried out at the following values of dimensionless parameters:

$$k = 0.02; \nu = \nu_c = 0.33; \gamma = 250; p^* = 12.5; k_s = 20; c_{01} = 0.05\,.$$

In the considered case, shells with a solid filler, with a circumferential distance from the place of application of the stress force, become tensile, which indicates the possibility of the carrier layer lagging behind the filler. Figure 1 shows the distribution of deflections of the loaded outer bearing layer along the circumference [12].

Figure 2 shows the distribution of the radial stress of the filler from θ. It can be seen that the voltage distribution over the filler circuit is non-monotonic. In the considered case, shells with a solid filler, with a circumferential distance from the place of application of the stress force, become tensile, which indicates the possibility of the carrier layer lagging behind the filler.

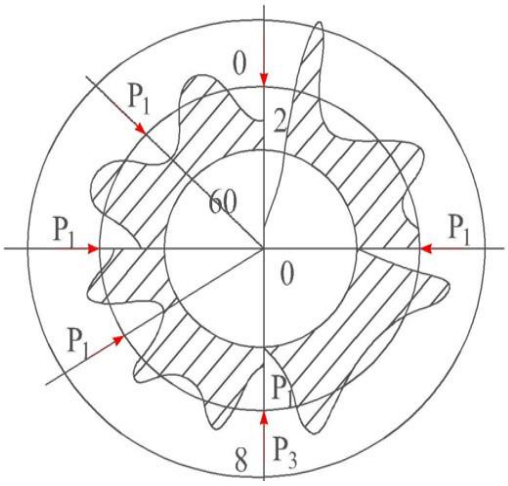

Figure 2. Distribution of the radial stress of the filler from θ.

4. Conclusions

Thus, in the considered case, coaxial shells with a solid filler, with a circumference away from the place of application of the stress force, become tensile, which indicates the possibility of the carrier layer lagging behind the filler.

An effective algorithm for solving the problem of vibrations of a three-layer viscoelastic cylindrical shell under normal loading has been developed on a computer. Critical velocities of wave propagation in a three-layer shell under the influence of moving loads are found.

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