O‘ZBEKISTON RESPUBLIKASI

OLIY TA’LIM, FAN VA INNOVATSIYALAR VAZIRLIGI

RENESSANS TA’LIM UNIVERSITETI

Matematika va axborot texnologiyalari kafedrasi

ELEMENTAR MATEMATIKANING ASOSIY TUSHUNCHALARI

(1-kurs talabalari uchun uslubiy qo‘llanma)

Toshkent – 2023

Ushbu uslubiy qo‘llanma 1-kurs talabalari uchun mo‘ljalangan bo‘lib unda elementar matematikaning ba’zi bo‘limlari, jumladan haqiqiy sonlar ustida amallar, haqiqiy sonning moduli va uning asosiy xossalari, ratsional va irratsional ifodalarni ayniy shakl almashtirishlar, bir noma’lumli birinchi va ikkinchi darajali tenglamalar va tengsizliklar, modul qatnashgan tenglama va tengsizliklar, tenglamalar va tengsizliklar sistemalarini yechishning turli usullari,irratsional tenglama va tengsizliklar, ko‘rsatkichli va logarifmik tenglama va tengsizliklar, tekislikda va fazoda to`g`ri burchakli koordinatalar sistemasi,vektorlar va ular ustida amallar bajarish haqida nazariy ma’lumotlar berilib, namunaviy masalalar yechimlari ko‘rsatilib talabalar mustaqil yechishlari uchun misollar keltirilgan.

Bu qo‘llanmada keltirilgan ma’lumotlarni muvaffaqiyatli o‘zlashtirish talabalarga universitetda o‘qitiladigan matematika va unga turdosh fanlar, shuningdek matematik usullarni tadbiq qililishni talab qiladigan turli mutaxassislik fanlaridan yuqori bilim, ko‘nikma va malakalarga ega bo‘lishida zamin bo‘ladi.

Tuzuvchi:

Sh.I.Sharipova - Renessans ta’lim universiteti Matematika va axborot texnologiyalari kafedrasi katta o’qituvchisi

-----

Mundarija

1. *Natural, butun, ratsional va irratsional sonlar………………….4*
2. *Haqiqiy sonlar ustida amallar...7*
3. *Haqiqiy sonning moduli va uning asosiy xossalari…………….11*
4. *Ratsional ifodalarni ayniy shakl almashtirishlar……………….12*
5. *Bir noma'lumli birinchi va ikkinchi darajali tenglamalar.......16*
6. *Chiziqli va kvadrat tengsizliklar...20*
7. *Modul qatnashgan tenglama va tengsizliklar............................24*
8. *Tenglamalar va tengsizliklar sistemalarini yechishning turli usullari...27*
9. *Irratsional tenglama va tengsizliklar..31*
10. *Ko'rsatkichli tenglama va tengsizliklar....................................33*
11. *Logarifmlar va ularning asosiy xossalari................................38*
12. *Logarifmik tenglama va tengsizliklar.......................................41*
13. *Tekislikda to`g`ri burchakli koordinatalar sistemasi..............45*
14. *Chiziqli funksiya va uning grafigini yasash 47*
15. *Vektorlar va ular ustida amallar..49*

Natural, butun, ratsional va irratsional sonlar.

Narsalarni sanashda ishlatiladigan sonlar *natural sonlar* deyiladi. Barcha natural sonlar hosil qilgan cheksiz to'plam *N* harfi bilan belgilanadi: *N={l,* 2, ..., *n, ...}*. Natural sonlar to'plamida eng katta son (element) mavjud emas, lekin eng kichik son (element) mavjud, u *1* soni. *1* soni faqat bitta bo'luvchiga ega (1 ning o'zi). 1 dan boshqa barcha natural sonlar kamida ikkita bo'luvchiga ega (sonning o'zi va 1). 1 dan va o'zidan boshqa natural bo'luvchiga ega bo'lmagan *1* dan katta natural son *tub son* deyiladi. Masalan, 2, 3, 5, 7, 11, 13, 17, 19 sonlar 20 dan kichik bo'lgan barcha tub sonlardir. 1 dan va o'zidan boshqa natural bo'luvchiga ega bo'lgan *1* dan katta natural son *murakkab son* deyiladi. Masalan, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18 sonlar 20 dan kichik bo'lgan barcha murakkab sonlardir. Tub va murakkab sonlarga berilgan ta'riflardan 1 soni na tub, na murakkab son ekanligi ma'lum bo'ladi. Bunday xossaga ega natural son faqat *1* ning o'zidir.
Natural sonlarning ayrim xossalarini qaraymiz:
1- xossa. Har qanday $p > 1$ natural sonining 1 ga teng bo'lmagan bo'luvchilarining eng kichigi tub son bo'ladi.
2- xossa. Murakkab *p* sonining 1 dan katta eng kichik bo'luvchisi $\sqrt{p}$ dan katta bo'lmagan tub sondir.
3- xo ssa (Yevklid teoremasi). Tub sonlar cheksiz ko'pdir.
Biror *n* sonidan katta bo'lmagan tub sonlar jadvalini tuzishda *Eratosfen g'alviri* deb ataladigan oddiy usuldan foydalanadilar. Uning mohiyati bilan tanishamiz.
Ushbu:

$$1,2,3,.,.,n \qquad (1)$$

sonlarini olaylik. (1) ning 1 dan katta birinchi soni 2; u faqat 1 ga va o'ziga bo'linadi, demak, 2 tub son. (1) da 2 ni qoldirib, uning karralisi bo'lgan hamma murakkab sonlarni o'chiramiz; 2 dan keyin turuvchi o'chirilmagan son 3; u 2 ga bo'linmaydi, demak, 3 faqat 1 ga va o'ziga bo'linadi, shuning uchun u tub son. (1) da 3 ni qoldirib, unga karrali bo'lgan hamma sonlarni o'chiramiz; 3 dan keyin turuvchi o'chirilmagan birinchi son 5 dir; u na 2 ga va na 3 ga bo'linadi. Demak, 5 faqat 1 ga va o'ziga bo'linadi, shuning uchun u tub son bo'ladi va h.k.
1- miso1. 827 sonining eng kichik tub bo'luvchisini toping.
Yechish. $\sqrt{827}$ dan kichik bo'lgan tub sonlar 2, 3, 5, 7, 11, 13, 17, 19, 23 ekanligini aniqlab, 827 ni shu sonlarga bo'lib chiqamiz. 827 u sonlarning hech qaysisiga bo'linmaydi, bundan 827 ning tub son ekanligi kelib chiqadi.
2- misol. 15 va 50 sonlari orasida joylashgan tub sonlarni aniqlang.
Yechish. 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50 sonlarni olib, 2, 3, 5, 7 ga karrali sonlarning tagiga chizamiz. 17, 19, 23, 29, 31, 37, 41, 47 sonlari izlangan tub sonlardir.
Natural sonlar qatorida tub sonlar turlicha taqsimlangan. Ba'zan qo'shni tub sonlar bir-biridan 2 gagina farq qiladi, masalan, 11 va 13, 101 va 103 va hokazo. Bu sonlar *egizak*

tub sonlar deyiladi. Egizak tub sonlar to'plamining chekli yoki cheksizligi hozirgacha noma'lum.

Hisoblash mashinalari yordami bilan juda katta tub sonlar topilgan. Masalan, 25000 xonali 2^{86243}- 1 son tub sondir.

Matematikaning asosiy tushunchalaridan biri son tushunchasi hisoblanadi. Son haqidagi tushuncha qadimda paydo bo‘lib, uzoq vaqt davomida kengaytirilib va umumlashtirib borilgan. Natural sonlar to‘plami ustida faqat ikkita amal: qo‘shish va ko‘paytirish bajariladi. Agar $a \in N, b \in N$ bo‘lsa, $(a+b) \in N, ab \in N$ bo‘ladi.

Natural sonlarga 0 ni va hamma butun manfiy sonlarni qo‘shsak, sonlarning yangi to‘plami – butun sonlar to‘plami hosil bo‘ladi, uni Z bilan belgilash qabul qilingan; Z={…, -2, -1, 0, 1, 2, …}. Butun sonlar ustida qo‘shish, ko‘paytirish amallaridan tashqari ayirish amali ham bajariladi, haqiqatda agar $a \in Z, b \in Z$ bo‘lsa, $-a \in Z, -b \in Z$.Bundan $a-b=a+(-b)$ bo‘ladi. Butun sonlar hosil qilinishidan $N \in Z$ ekanligi kelib chiqadi.

Endi $\frac{p}{q}(p \in Z, \ q \in N)$ ko‘rinishdagi kasrlarni, oddiy kasr ham deyiladi, ko‘rib chiqamiz. p ixtiyoriy butun qiymatni, q ixtiyoriy natural qiymatni qabul qilganda $\frac{p}{q}$ hosil qiladigan sonlar to‘plamiga ratsional sonlar to‘p-lami deyiladi va Q bilan belgilanadi: $Q=\left\{\frac{p}{q}, \ p \in Z, \ q \in N\right\}$, Q ustida to‘rt amal: qo‘shish, ayirish, ko‘paytirish va bo‘lish bajariladi. Natural sonlar va butun sonlar ratsional sonlar to‘plamiga qism to‘plam bo‘ladi, ya’ni $N \subset Q, Z \subset Q$.

Ratsional sonlarning ba’zi xossalarini keltiramiz:

1. $\frac{a}{b}=\frac{c}{d}$.dan $a=c, b=d$ kelib chiqadi. $\frac{a}{b}=\frac{a}{b}$ hamma vaqt bajariladi.
2. $\frac{a}{b}=\frac{c}{d}$ bo‘lib $\frac{c}{d}=\frac{e}{f}$ bo‘lsa, $\frac{a}{b}=\frac{e}{f}$bo‘ladi.
3. $\frac{a}{b}$ va $n \neq 0$ bo‘lsa $\frac{a}{b}=\frac{an}{bn}$ va $\frac{a}{b}=\frac{a:n}{b:n}$ bo‘ladi.

1-ta’rif. $\frac{a}{b}$ va $\frac{b}{a}$ kasrlar o‘zaro teskari kasrlar deyiladi. Boshqacha qilib aytganda, ko‘paytmasi 1 ga teng bo‘lgan kasrlar o‘zaro teskari kasrlar deyiladi. $\frac{5}{7}, \frac{14}{10}$ o‘zaro teskari kasrlar, chunki $\frac{5}{7} \cdot \frac{14}{10}=1$ Shunga o‘xshash, $2\frac{1}{3} \cdot \frac{3}{7}=1$ bo‘lgani uchun ular o‘zaro teskari sonlardir.

2-ta’rif. Agar kasrning surati maxrajidan katta yoki teng bo‘lsa, kasr noto‘g‘ri kasr deyiladi. Bu holda suratni maxrajga bo‘lib noto‘g‘ri kasrni butun son va to‘g‘ri kasr (surat maxrajdan kichik) yig‘indisi ko‘ri-nishida tasvirlash mumkin: $\frac{27}{4}$ noto‘g‘ri kasr, suratni maxrajga bo‘lsak, 27:4=6(3 qoldiq) hosil bo‘ladi, shuning uchun $\frac{27}{4}=6+\frac{3}{4}=6\frac{3}{4}$ hosil bo‘ladi. Boshqa misol $\frac{117}{23}=5\frac{2}{23}, \frac{17}{3}=5\frac{2}{3}$

Butun va to‘g‘ri kasr yig‘indisidan iborat son aralash son deyiladi. Uni noto‘g‘ri kasrga aylantirish uchun butun maxrajga ko‘paytiriladi, ko‘paytma suratga qo‘shiladi. Hosil bo‘lgan son noto‘g‘ri kasrning surati bo‘ladi, maxraj o‘zgarmaydi.

$A\frac{a}{b}=\frac{Ab+a}{b}; \quad 2\frac{1}{3}=\frac{2x3+1}{3}=\frac{7}{3};$

3-ta’rif. Agar kasrning maxraji *10^n* dan iborat bo‘lsa, o‘nli kasr deyiladi. Bu holda suratni maxrajga bo‘lish yakunlanadi.

$\frac{3}{10}=0{,}3; \ \frac{27}{10}=2{,}7; \ \frac{97}{100}=0{,}97$

$\frac{5237}{1000}=5\frac{237}{1000}=5{,}237; \ \frac{17}{1000}=0{,}017$

Kasrning suratini maxrajga bo‘lganda, bo‘lish chekli (yakunlanadi) yoki cheksiz (yakunlanmaydi) bo‘lishi mumkin. Birinchi holatda chekli o‘nli kasr hosil bo‘ladi, ikkinchi holatda cheksiz o‘nli kasr hosil bo‘ladi. Umuman olganda, agar kasrning maxraji $b=2^n5^k$ ko‘rinishida bo‘lsa, bu kasr chekli o‘nli kasr ko‘rinishida tasvirlanadi, bu yerda n, k=0,1,2,…

Haqiqatda, $\frac{a}{b}=\frac{a}{2^n\cdot 5^k}$ bo‘lsin. Faraz qilaylik, n>k va n=k+m bo‘lsin. Kasr surat va maxrajini 5^m ga ko‘paytiramiz va $\frac{a}{b}=\frac{a}{2^n\cdot 5^k}x\frac{5^m}{5^m}=\frac{5^m a}{2^n\cdot 5^n}=\frac{5^n a}{10^n}$ ni hosil qilamiz, bu esa o‘nli kasrdir.

Agar kasr maxraji 2 va 5 dan tashqari boshqa tub bo‘luvchiga ega bo‘lsa, kasrni chekli o‘nli kasr ko‘rinishida tasvirlab bo‘lmaydi. Bu hol-da cheksiz o‘nli davriy kasr hosil bo‘ladi:

$\frac{1}{3}=0{,}333...=0,(3); \frac{7}{33}=0{,}2121...=0,(21);$

$\frac{11}{30}=0{,}36666...=0{,}3(6).$

Chekli o‘nli kasrni davri 0 yoki 9 bo‘lgan cheksiz o‘nli kasrlar ko‘rinishida yozish mumkin.

Aytilganlardan kelib chiqqan holda, ratsional sonlarga quyidagicha ta’rif berish mumkin.

4-ta’rif. Cheksiz davriy o‘nli kasrlar ratsional sonlar to‘plamiga kiradi.

5-ta’rif. Davriy bo‘lmagan cheksiz o‘nli kasrlar irratsional sonlar to‘plamini tashkil etadi. $\sqrt{2}, \ 2-\sqrt{3}, \ \pi, \ 1{,}2109327...$

Ratsional va irratsional sonlar (ya’ni cheksiz davriy va davriy bo‘l-magan o‘nli kasrlar) haqiqiy sonlar deyiladi va R bilan belgilanadi. Ta’rifdan $Q\subset R$ kelib chiqadi, bundan esa

$N \subset R, Z \subset R$ hosil bo‘ladi. Haqiqiy sonlarni sonlar o‘qida tasvirlaydigan bo‘lsak, har bir haqiqiy songa o‘qda bitta nuqta mos keladi va aksincha, sonlar o‘qidagi har bir nuqtaga faqat bitta haqiqiy son mos keladi. Demak, haqiqiy sonlar bilan sonlar o‘qidagi nuqtalar orasida o‘zaro bir qiymatli mos kelish mavjud bo‘lib, “Haqiqiy son” o‘rniga “nuqta” ni ishlatish imkonini beradi.

Misollar

1.Berilgan sonlarga teskari sonlarni yozing:

1) -5; 2) 3; 3) $\frac{2}{3}$; 4) $-\frac{5}{7}$; 5) $-2\frac{1}{2}$; 6) $4\frac{1}{5}$.

2.Quyidagi juftliklardan o‘zaro teskari sonlarni aniqlang.

1) $\frac{2}{3}$ *va* $\frac{3}{2}$ 2) $-\frac{1}{5}$ *va* 5 3) $\frac{\sqrt{5}-1}{2}$ *va* $\frac{\sqrt{5}+1}{2}$ 4) $\sqrt{2}-1$ *va* $\sqrt{2}+1$

5) $\frac{\sqrt{3}-1}{2}$ *va* $\frac{\sqrt{3}+1}{2}$ 6) $\sqrt{6}-\sqrt{5}$ *va* $\sqrt{6}+\sqrt{5}$ 7) $1\frac{4}{5}$ *va* $\frac{5}{4}$ 4) $2\frac{1}{3}$ *va* $\frac{6}{7}$

3.Quyidagi kasrlardan qaysilari chekli o‘nli kasr bo‘ladi.

1) $\frac{3}{4}$; 2) $\frac{7}{25}$; 3) $\frac{23}{20}$; 4) $\frac{37}{40}$;

5) $\frac{11}{18}$; 6) $\frac{5}{24}$; 7) $\frac{14}{75}$; 8) $\frac{3}{125}$.

4.Quyidagi kasrlarni chekli yoki cheksiz o‘nli kasr shaklida yozing.

1) $\frac{1}{2}$; 2) $\frac{1}{4}$; 3) $\frac{3}{8}$; 4) $\frac{23}{40}$.

5) $\frac{2}{3}$; 6) $\frac{5}{6}$; 7) $\frac{17}{12}$; 8) $\frac{25}{18}$.

Haqiqiy sonlar ustida amallar

Butun sonlar ustida arifmetik amallar bizga ma’lum. Ratsional kasrlar ustida bu amallar quyidagicha bajariladi:

1) qo‘shish: $A\frac{a}{b}+B\frac{c}{d}=(A+B)\frac{ad+bc}{bd}$.

Umumiy maxraj topishda agar (*b*, *d*)=1 bo‘lsa, umumiy maxraj ularning ko‘paytmasi *bd* bo‘ladi, agar (*a*, *b*)>1 bo‘lsa, umumiy maxraj *a* ga ham *b* ga ham bo‘linadigan sonlardan eng kichigi bo‘ladi.

2) ayirish: $A\frac{a}{b}-B\frac{c}{d}=(A-B)\frac{ad-bc}{bd}$

ad-*bc* ning ishorasi *A*-*B* ning ishorasiga qarama-qarshi bo‘lsa, *A*-*B* dan bitta butun olib quyidagicha yozamiz va amalni bajaramiz:

$$(A-B)\frac{ab-bc}{bd}=(A-B-1)\frac{(bd+ad)-bc}{bd}$$

Misol: $5\frac{1}{4}-2\frac{2}{3}=3\frac{3-8}{12}=2\frac{15-8}{12}=2\frac{7}{12}$

3) ko‘paytirish: Amalni bajarishdan oldin aralash sonlar noto‘g‘ri kasrlarga keltiriladi:

$$A\frac{a}{b}\cdot B\frac{c}{d}=\frac{Ab+a}{b}\cdot\frac{Bd+c}{d}=\frac{(Ab+a)(Bd+c)}{bd}$$

Qisqartirish mumkin bo‘lsa, qisqartiramiz va suratni suratga, max-rajni maxrajga ko‘paytiramiz. Noto‘g‘ri kasr hosil bo‘lsa, butun ajratamiz:

$$2\frac{1}{3}\cdot 1\frac{2}{7}=\frac{7}{3}\cdot\frac{9}{7}=3$$

$$1\frac{2}{3}\cdot 3\frac{1}{2}=\frac{5}{3}\cdot\frac{7}{2}=\frac{35}{6}=5\frac{5}{6}.$$

4) **Bo‘lish.** Aralash sonlarni noto‘g‘ri kasrlarga aylantiramiz, so‘ng bo‘lishni birinchi (bo‘linuvchini) kasrni ikkinchi (bo‘luvchi) kasrning teskarisiga ko‘paytirish bilan almashtiramiz:

$$A\frac{a}{b}:B\frac{c}{d}=\frac{Ab+a}{b}:\frac{Bd+c}{d}=\frac{Ab+a}{b}\cdot\frac{d}{Bd+c}.$$

Misol: $4\frac{1}{5}:2\frac{1}{3}=\frac{21}{5}:\frac{7}{3}=\frac{21}{5}\cdot\frac{3}{7}=\frac{3\cdot 3}{5}=\frac{9}{5}=1\frac{4}{5}$.

Irratsional ifodalar ustida amallarni ko‘rib chiqamiz.

1. $\sqrt{8}+\sqrt{18}-4\sqrt{2}=\sqrt{4x2}+\sqrt{9x2}-4\sqrt{2}=2\sqrt{2}+3\sqrt{2}-4\sqrt{2}=\sqrt{2}$ yoki

$$\sqrt{8}+\sqrt{18}-4\sqrt{2}=\sqrt{2}(\sqrt{4}+\sqrt{9}-4)=\sqrt{2}(2+3-4)=\sqrt{2}$$

2. $\frac{1}{\sqrt{4}+\sqrt{3}}+\frac{1}{\sqrt{3}}$ qo‘shishni bajarishdan oldin kasrlarning maxrajini irratsionallikdan ozod qilamiz:

$$\frac{\sqrt{4}-\sqrt{3}}{(\sqrt{4}+\sqrt{3})(\sqrt{4}-\sqrt{3})}+\frac{\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}=2-\sqrt{3}+\frac{\sqrt{3}}{3}=\frac{6-3\sqrt{3}+\sqrt{3}}{3}=\frac{6-2\sqrt{3}}{3}.$$

Umuman olganda, irratsional ifodalar ustida amallar arifmetik amal-lar qonunlariga va ildizlar ustida amallar qoidalariga muvofiq bajariladi.

Darajadan ildiz chiqarishda daraja ko‘rsatkichi ildiz ko‘rsatkichiga bo‘linadi, bo‘linma va qoldiq mos ravishda ildizdan chiqqan va ildiz osti-da qolgan asosning daraja ko‘rsatkichi bo‘ladi:

$\sqrt[3]{a^5} = a\sqrt[3]{a^2}$, chunki (5=3 $\cdot$ 1+2)

$\sqrt[4]{a^9 \cdot b^{14}} = a^2 b^3 \sqrt[4]{ab^2}$.

$\sqrt[5]{\dfrac{a^{12}}{b^{18}}} == \dfrac{a^2}{b^3}\sqrt[5]{\dfrac{a^2}{b^3}}$.

2-misol. O‘xshash ildizlarni keltiramiz:

$A\sqrt[n]{a} + B\sqrt[m]{b} + C\sqrt[n]{a} + D\sqrt[m]{b} = (A+C)\sqrt[n]{a} + (B+D)\sqrt[m]{b}$.

3-misol. Ildizlarni ko‘paytirish yoki bo‘lishda ularni umumiy ko‘rsatkichiga keltiramiz:

$\sqrt[n]{A} \cdot \sqrt[k]{B} = \sqrt[nk]{A^k} \cdot \sqrt[nk]{B^n} = \sqrt[nk]{A^k B^n}$.

$\dfrac{\sqrt[n]{A}}{\sqrt[k]{B}} = \dfrac{\sqrt[nk]{A^k}}{\sqrt[nk]{B^n}} = \sqrt[nk]{\dfrac{A^k}{B^n}}$.

1) $\sqrt{2\sqrt{2}-1} \cdot \sqrt[4]{9+4\sqrt{2}} = \sqrt[4]{(2\sqrt{2}-1)^2} \cdot \sqrt[4]{9+4\sqrt{2}} =$

$= \sqrt[4]{(9-4\sqrt{2})(9+4\sqrt{2})} = \sqrt[4]{81-32} = \sqrt[4]{49} = \sqrt{7}$.

2) $\dfrac{\sqrt[4]{324}}{\sqrt{2}} = \dfrac{\sqrt[4]{324}}{\sqrt[4]{4}} = \sqrt[4]{\dfrac{324}{4}} = \sqrt[4]{81} = \sqrt[4]{3^4} = 3$.

Ildizlarni hisoblashda murakkab kvadrat ildizni almashtirish:

$\sqrt{A \pm \sqrt{B}} = \sqrt{\dfrac{A+\sqrt{A^2-B}}{2}} \pm \sqrt{\dfrac{A-\sqrt{A^2-B}}{2}}$ formulasidan foydalanish mumkin.

$\sqrt{\dfrac{9+\sqrt{65}}{2}} + \sqrt{\dfrac{9-\sqrt{65}}{2}}$ ifoda hisoblansin.

$\sqrt{9 \pm \sqrt{65}} = \sqrt{\dfrac{9+\sqrt{81-65}}{2}} \pm \sqrt{\dfrac{9-\sqrt{81-65}}{2}} = \sqrt{\dfrac{9+4}{2}} \pm \sqrt{\dfrac{9-4}{2}}$ ni hisobga olib topamiz.

$\sqrt{\dfrac{9+\sqrt{65}}{2}} + \sqrt{\dfrac{9-\sqrt{65}}{2}} = \sqrt{\dfrac{9+4}{4}} + \sqrt{\dfrac{9-4}{4}} + \sqrt{\dfrac{9+4}{4}} - \sqrt{\dfrac{9-4}{4}} = \dfrac{\sqrt{13}}{2} + \dfrac{\sqrt{13}}{2} = \sqrt{13}$.

Misollar

Hisoblang:

1. 1) $7\dfrac{5}{8} + 2\dfrac{3}{8}$;   2) $6\dfrac{5}{7} + 3\dfrac{2}{7}$;   3) $5\dfrac{5}{6} - 2\dfrac{1}{6}$;

$4)\ 5\frac{4}{5}-3\frac{2}{5};$ $5)\ 8\frac{2}{7}-5\frac{3}{7};$ $6)\ 4\frac{1}{4}-2\frac{3}{4}.$

2. $1)\ 5\frac{1}{3}-2\frac{1}{2};$ $2)\ 4\frac{1}{7}-3\frac{1}{5};$ $3)\ 5-1\frac{5}{6};$

$4)\ 6-3\frac{1}{6};$ $5)\ 5\frac{1}{5}-3;$ $6)\ 3\frac{2}{3}-2.$

3. $1)\ 2\frac{1}{3}\cdot\frac{5}{7};$ $2)\ 3\frac{1}{2}\cdot\frac{4}{7};$

$3)\ 5\frac{1}{3}\times 1\frac{1}{8};$ $4)\ 3\frac{1}{4}\times 1\frac{3}{13}.$

4. $1)\ 6\frac{2}{3}:3\frac{3}{4};$ $2)\ 3\frac{4}{7}:1\frac{1}{14};$

$3)\ 3\frac{1}{3}:1\frac{1}{4};$ $4)\ 5\frac{1}{4}:3\frac{1}{8}.$

5. Hisoblang.

1) $2\sqrt{3\cdot 27}-6\sqrt{2\cdot 18}$ 2) $\sqrt{2^2+3\cdot 7}$ 3) $\sqrt{17^2-15^2}$

4) $3\sqrt{121}-2\sqrt{144}$ 5) $\sqrt{3^6}$ 6) $\sqrt{2^8}$

7) $\sqrt{(-5)^6}$ 8) $\sqrt{(-3)^4}$ 9) $\sqrt{(-25)^2}$

10) $\sqrt{49\cdot 25}$ 11) $\sqrt{0{,}01\cdot 169}$ 12) $\sqrt{625\cdot 9\cdot 36}$

13) $\sqrt{108\cdot 27}$ 14) $\sqrt{27\cdot 12}$ 15) $\sqrt{2}\cdot\sqrt{32}$

16) $\sqrt{3}\cdot\sqrt{7}\cdot\sqrt{21}$ 17) $\sqrt{2}\cdot\sqrt{22}\cdot\sqrt{11}$ 18) $\sqrt{\frac{1}{2}}\cdot\sqrt{\frac{2}{3}}\cdot\sqrt{3}$

19) $\sqrt{\frac{2}{5}}\cdot\sqrt{\frac{5}{7}}\cdot\sqrt{\frac{7}{8}}$ 20) $\sqrt{113^2-112^2}$ 21) $\sqrt{82^2-18^2}$

22) $\sqrt{65^2-63^2}$ 23) $\sqrt{313^2-312^2}$ 24) $\sqrt{5^4\cdot 3^2}$

25) $\sqrt{12^2\cdot 3^4}$ 26) $\sqrt{(-5)^4\cdot(0{,}1)^2}$ 27) $(\sqrt{7}+\sqrt{6})\cdot(\sqrt{7}-\sqrt{6})$

28) $(\sqrt{8}+\sqrt{2})^2$ 29) $(\sqrt{7}-\sqrt{28})^2$ 30) $3\sqrt{20}-\sqrt{5}$

31) $(5\sqrt{2}+2\sqrt{5})\cdot(5\sqrt{2}-2\sqrt{5})$ 32) $\frac{1}{3}\sqrt{18}+2\sqrt{2};$ 33) $2\sqrt{20}-2\sqrt{45}+\frac{1}{4}\sqrt{16}$

34) $\frac{1}{3}\sqrt{18}+2\sqrt{2}$ 35) $5\sqrt{8}+\frac{1}{2}\sqrt{2}-2\sqrt{18};$

36) $3\sqrt{48}-\sqrt{75}+\frac{1}{7}\sqrt{147}.$

37) $3\sqrt{45}-\sqrt{125}+\sqrt{80}$ 38) $2\sqrt{27}-\sqrt{12}$ 39) $2\sqrt{8}+0{,}5\sqrt{32}-\frac{1}{3}\sqrt{18}$

6. Hisoblang.

1) $\sqrt{\frac{9}{100}}$ 7) $\sqrt{\frac{100}{49}}$ 13) $\sqrt{3\frac{1}{16}}$

2) $\sqrt{5\frac{4}{9}}$ 8) $\sqrt{\frac{4}{9}}+\sqrt{\frac{1}{9}}$ 14) $5\sqrt{\frac{1}{25}}-3\sqrt{\frac{1}{9}}$

3) $\frac{\sqrt{27}}{\sqrt{3}}$ 9) $\frac{\sqrt{128}}{\sqrt{8}}$ 15) $\sqrt{\frac{16}{81}}-\sqrt{\frac{169}{225}}$

4) $\sqrt{\frac{64\cdot 49}{196\cdot 324}}$ 10) $\sqrt{5\frac{4}{9}\cdot 11\frac{11}{25}}$ 16) $\sqrt{\frac{9}{16}\cdot\frac{4}{81}\cdot\frac{36}{169}}$

5) $\frac{2}{\sqrt{11}-3}-\frac{7}{\sqrt{11}-2}$ 11) $\frac{3}{3+\sqrt{6}}+\frac{2}{2+\sqrt{6}}$ 17) $\frac{\sqrt{5}}{\sqrt{5}-2}-\frac{10}{\sqrt{5}}$

6) $\frac{3}{\sqrt{7}-2}-\frac{2}{\sqrt{7}+3}-2\sqrt{7}$ 12) $\frac{1}{3-\sqrt{5}}-\frac{1}{2-\sqrt{5}}+\frac{3\sqrt{5}}{4}$ 18) $\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{3}-1}$

7. Hisoblang

1) $\sqrt{5-2\sqrt{6}}-\sqrt{5+2\sqrt{6}}$; 2) $\sqrt{7-4\sqrt{3}}-\sqrt{7+4\sqrt{3}}$;

3) $\sqrt{11+6\sqrt{2}}+\sqrt{11-6\sqrt{2}}$; 4) $\sqrt{19+8\sqrt{3}}+\sqrt{19-8\sqrt{3}}$;

Haqiqiy sonning moduli va uning asosiy xossalari

Ta'rif: Haqiqiy son *a* ning absolut qiymati yoki moduli deb ($|a|$ bi-lan belgilanadi) *a* songa, agar *a≥0* bo‘lsa, va – *a* songa, agar *a*<0 bo‘lsa, aytiladi, ya'ni:

$$|a|=\begin{cases} a, & agar, a\geq 0 \text{ bo`lsa,} \\ -a, & agar, a<0 \text{ bo`lsa.}\end{cases}$$

Misol: *|3|=3, |0|=0, |-4|=4.*

Ta'rifdan har qanday haqiqiy *a* son uchun *a*≤|*a*| munosabat kelib chiqadi.

Absolut qiymatning ba'zi xossalarini ko‘rib chiqamiz:

1. |*a*+*b*|≤|*a*|+|*b*|, ya'ni ikkita haqiqiy son algebraik yig‘indisining moduli shu sonlar modullarining yig‘indisidan katta emas.

2. |*a*-*b*|≥|*a*|-|*b*|, ya'ni ayirmaning absolut qiymati kamayuvchi va ayriluvchi absolut qiymatlarining ayirmasidan kichik emas.

3. Ko‘paytmaning moduli ko‘payuvchilar modullarining ko‘paytmasiga teng, ya'ni:

$|a\cdot b\cdot ......\cdot c|=|a|\cdot|b|\cdot ....\cdot|c|$;

4. Bo‘linmaning moduli bo‘linuvchi bilan bo‘luvchi modullarining nisbatiga teng, ya’ni $|\frac{a}{b}|=\frac{|a|}{|b|}$.

Misollar

1. Sonlarni taqqoslang:

1) $|6{,}7|$ va 6;

2) $-|0{,}5|$ va $-0{,}5$;

3) $|4{,}2|$ va $4{,}2$;

4) $|-3{,}4|$ va $3{,}4$;

5) $|-3\frac{2}{3}|$ va $-3\frac{2}{3}$;

6) $|a|$ va a.

2. Harflarning berilgan qiymatlarida ifodaning qiymatini hisoblang:

1) $|a|+|2b|,\ a=-4, b=3;$

2) $|a+b|-2|b|,\ \ a=5,\ \ b=-3;$

3) $\frac{3-|3a|+2|b|}{|a|-|b|},\ \ a=1,\ \ b=3;$

4) $\frac{2+|a-b|-3|a|}{2|a|+|b|},\ \ a=-1,\ \ b=3.$

3. Hisoblang**.**

1) $|5-16|$ 2) $|25-16|$ 3) $|5-|25-29|+25|$ 4) $|5-16|+|9-4|$

5) $|5\cdot|-46-21|-16|$ 4) $|2-3\cdot|-9+36||$ 6) $|2+4\cdot|-9-36||$

Ratsional ifodalarni ayniy shakl almashtirishlar

Ratsional ifodani ayniy almashtirish deb berilgan ifodani berilganiga o‘xshamaydigan shunday yangi ifoda bilan almashtirish tushunila-diki, ikkalasining qiymatlari teng bo‘lsin.

Misol: $\frac{x^2+8x+15}{(x+5)^2}$ berilgan bo‘lsa, kasr suratini $x^2+8x+15=(x+3)(x+5)$ ko‘rinishda yozib berilgan kasrni $\frac{x+3}{x+5}$ bilan almashtiramiz. Ikkala kasrning barcha *x≠-5* dagi qiymatlari o‘zaro teng bo‘ladi.

Umumiy mavjudlik sohasida bir ratsional ifodani unga aynan teng ifoda bilan almashtirishga shu ifodani aynan almashtirish deyiladi. Bunday almashtirishlar tenglamani yechishda, teoremalar va ayniyatlarni isbotlashda, masala va misollarni yechishda ishlatiladi. Almashtirishlar kasrlarni qisqartirish, qavslarni ochish, umumiy ko‘paytuvchini qavsdan chiqarish, ifodani ko‘paytuvchilarga ajratish, o‘xshash hadlarni ixcham-lash va shu kabilardan iborat bo‘ladi. Almashtirishlarni bajarishda quyida-gilardan foydalanish tavsiya etiladi.

Agar x_1 va x_2 $ax^2+bx+c=0$ tenglamaning ildizlari bo‘lsa, u holda $ax^2+bx+c=a(x-x_1)(x-x_2)$ tenglik o‘rinli bo‘ladi.

Qisqa ko‘paytirish formulalari va ba`zi umumlashtirilganlari:

$(a\pm b)^2=a^2\pm 2ab+b^2$

$(a\pm b)^3=a^3\pm 3a^2b+3ab^2\pm b^3$

$(a+b)(a-b)=a^2-b^2$

$(a+b)(a^2-ab+b^2)=a^3+b^3$

$(a-b)(a^2+ab+b^2)=a^3-b^3$

$(a\pm b)^4=a^4\pm 4a^3b+6a^2b^2\pm 4ab^3+b^4$

$(a\pm b)^5=a^5\pm 5a^4b+10a^3b^3\pm 10a^2b^3+5ab^4\pm b^5$

$a^4-b^4=(a-b)(a^3+a^2b+ab^2+b^3)=(a-b)(a+b)(a^2+b^2)$

$a^5+b^5=(a+b)(a^4-a^3b+a^2b^2-ab^3+b^4)$

Daraja bilan amallar

$$\underbrace{a\cdot a\ldots\ldots\cdot a}_{n\ ma}=a^n$$

$a^n\cdot a^k=a^{n+k}$, $\quad \dfrac{a^n}{a^k}=a^{n-k}$, $\quad a^0=1$,

$a^{-n}=\dfrac{1}{a^n}$, $\quad \dfrac{1}{a^{-n}}=a^n$, $\quad \left(\dfrac{a}{b}\right)^{-n}=\left(\dfrac{b}{a}\right)^n$, $\quad (a^nb^k)^m=a^{nm}b^{km}$.

Misollar. 1) Kasrni qisqartiring. $\dfrac{3x+3y}{12x}\cdot\dfrac{4x^2}{x^2-y^2}$ ketma-ket amallarni bajarib, topamiz:

$$\frac{3(x+y)}{12x}\cdot\frac{4x^2}{(x+y)(x-y)}=\frac{x}{x-y}$$

2) Ifodani soddalashtiring. $\dfrac{x^3-2x^2+5x+26}{x^3-5x^2+17x-13}$

Suratdagi ko‘phadning butun ildizlari ozod had 26 ning bo‘luvchilari -26, -13, -2, -1, 1, 2, 13, 26 orasida bo‘lishi mumkin. Bu sonlarni ketma-ket $x^3-2x^2+5x+26$ ko‘phaddagi x o‘rnida qo‘yib x=-2 ildiz ekanligini aniqlaymiz. Shuning uchun (bu ko‘phadni x+2 ga bo‘lib, bo‘linma x^2-4x+1 ni topamiz):

$x^3-2x^2+5x+26=(x+2)(x^2-4x+1)$

Shunga o‘xshash x^3-5x^2+17x-13=(x-1)(x^2-4x+1)

Bularni kasrning surat va maxrajga qo‘yib, topamiz:

$$\frac{x^3-2x^2+5x+26}{x^3-5x^2+17x-13}=\frac{(x+2)(x^2-4x+1)}{(x-1)(x^2-4x+1)}=\frac{x+2}{x-1}.$$

Misollar

1\. 1) $\frac{116^8\cdot 87^4}{58^9\cdot 174^3}$ 2) $\frac{51^2\cdot 16\cdot 125}{102\cdot 5^2\cdot 153}$ **ni hisoblang.**

2\. **Quyidagi kasrlarni qisqartiring:**

1) $\frac{48a^7b^3c}{64a^8bc^4}$ 2) $\frac{a^{n+1}b^{m+2}}{a^nb^m}$ 3) $\frac{x^{m-1}y^{n-3}}{x^my^n}$

4) $\frac{12a^mb^{m-2}}{27a^{m+n}b^m}$ 5) $\frac{7x^my^{m+n}}{21x^{m-2}y^m}$ 6) $\frac{6a^mb^{n+3}}{9a^{m-2}b^{n+1}}$

7) $\frac{a^{-3}b^4}{9}\cdot\left(\frac{3}{a^{-2}b^3}\right)^{-3}$ 8) $\left(\frac{c}{10a^5b^2}\right)^{-1}\cdot(5a^3bc^2)^{-2}$ 9) $\left(\frac{x^2y^{-3}}{6z}\right)^{-3}\cdot\left(\frac{x^2y^{-2}}{9z}\right)^2$

3\. **Ifodalarni soddalashtiring.**

1) $(x-y)^2+(x+y)^2$

2) $(a+2b)^2+(2a-b)^2$

3) $3(2-a)^2+2(a-5)^2$

4) $(c-3)^2-(c+3)\cdot(3-c)$

5) $(2x+3y)\cdot(2x-3y)+(2x+3y)^2$

6) $(b-a)\cdot(-a-b)+2b^2$

7) $(z+5)\cdot(z^2-5z+25)$

8) $(2x+3y)\cdot(4x^2-6xy+9y^2)$

9) $(2a+b)^2-(a-3b)^2$

10) $(x-y)^2-(x+2y)^2$

11) $-(5+x)^2+3(1-x)^2$

12) $(a+2)^2-(a+2)\cdot(2-a)$

13) $(-b-a)\cdot(-a-b)+a^2+b^2$

14) $(2x+3y)\cdot(2x-3y)+(2x-3y)^2$

15) $(y+2)\cdot(y^2-2y+4)$

16) $(4c-5d)\cdot(16c^2+20cd+25d^2)$

4\. **Ko`paytuvchilarga ajrating.**

1) $36-x^2$

2) $81-a^4$

3) $\frac{1}{9}y^2-\frac{16}{25}x^2$

4) $25x^2-9$

5) $0{,}09y^2-0{,}49x^2$

6) x^2y^4-16

29) $1-b^2$

30) $324-16a^2$

31) $\frac{4}{9}a^2-\frac{1}{16}b^2$

32) c^2d^2-9

33) $36x^2y^2-1$

34) a^4-b^8

57) $49-y^4$

58) $256x^4-y^4$

59) $0{,}25y^2-49x^2$

60) $4a^2-9b^2$

61) $121-9x^2$

7) a^6-b^4

8) $(3y-y)^2-4y^2$

9) $(2a+b)^2-(2b+a)^2$

10) $c^2-(a-b)^2$

11) $36a^2-12a+1$

12) $x^4+2x^2y+y^2$

13) $9a^2-6a+1$

14) a^3-27

15) $8a^3-27$

16) $125a^3-64$

17) $54+2b^3$

18) $2c^3+2d^3$

19) $7-56x^3y^3$

20) $\frac{1}{8}a^2-a^5$

21) $(8a^3-27b^3)-2a\cdot(4a^2-9b^2)$

22) $(a^3+b^3)+(a+b)^2$

23) $2a^2+4ab+2b^2$

24) $(x^2+1)^2-4x^2$

25) $(x^2+2x)^2-1$

26) $(a^2+2ab+b^2)-c^2$

27) c^3-c

28) $1-a^2-2ab-b^2$

35) $(a+b)^2-c^2$

36) $(a+b)^2-(a-c)^2$

37) $(a-3b)^2-(3a+b)^2$

38) $9a^2-(a+2b)^2$

39) $100-60a+9a^2$

40) $25a^6+30a^3b+9b^2$

41) $4c^4+12c^2b^3+9b^6$

42) a^3-64

43) $27a^3-64$

44) $125-b^3$

45) $8x^3y-y^4$

46) $2cd^3-16c$

47) $54x^3-16$

48) $4a^2b+32a^5b$

49) $(64a^3-125b^3)+5b\cdot(16a^2-25b^2)$

50) $(a^3-b^3)+(a-b)^2$

51) $27a^2b^2-18ab+3$

52) $2cd^3-16c$

53) $4y^2-(y-c)^2$

54) a^2-b^2+a+b

55) $9b^2-(x^4+2x^2y+y^2)$

56) $4+(-x^2-2xy-y^2)$

62) $2a^3-2ab^2$

63) $16a^2-25b^2$

5. Ko`paytuvchilarga ajrating.

1) $6(a+b)+(a+b)^2$

2) $(a-b)+(a-b)^2$

3) $(x+y)^3-x(x+y)^2$

4) $a(a-b)^2-(b-a)^2$

5) $4(x-y)+3(x-y)^2$

6) $(a-b)^2-(b-a)^2$

7) $(a-b)^2-(a+b)(b-a)$

8) $(a+b)-(a+b)^2$

6. Ko`paytuvchilarga ajrating.

1) $(y+z)(12x^2+6x)+(y-z)(12x^2+6x)$

2) *$(6x^2-3)+7x(6x^2-3)-4y(6x^2-3)$*

3) *$18a^2-27ab+14ac-21bc$*

4) *$35ax+24xy-20ay-42x^2$*

5) *$16ab^2-5b^2c-10c^3+32ac^2$*

6) *$-28ac+35c^2-10cx+8ax$*

7) $(y-z)(12x^2-6x)+(y-z)(12x^2+6x)$

8) *$2x(8x-4y)-3y(8x-4y)-(8x-4y)$*

9) *$10x^2+10xy+5x+5y$*

10) *$48xz^2+32xy^2-15yz^2-10y^3$*

11) *$6mnk^2+15m^2k-14n^3k-35mn^2$*

12) *$-24bc-15cx+40bx+9c^2$*

7. Kasrlarni ifodalar ustida amallarni bajaring:

1) $\frac{a^2}{a+1}-a+1$

6) $\frac{7}{a+b}+\frac{8}{a-b}-\frac{16b}{a^2-b^2}$

11) $\frac{3}{a+3}+\frac{2}{3-a}-\frac{6}{a^2-9}$

2) $\frac{7}{m}-\frac{4}{m-2n}-\frac{m-n}{4n^2-m^2}$

3) $a-2+\frac{4a}{2+a}-\frac{a^3+1}{a^2+2a}$

4) $\frac{a+b}{a^2-ab+b^2}-\frac{1}{a+b}$

5) $\frac{6a}{9a^2-1}+\frac{3a+1}{3-9a}+\frac{3a-1}{6a+2}$

7) $\frac{6a}{9a^2-1}+\frac{3a+1}{3-9a}+\frac{3a-1}{6a+2}$

8) $\frac{a+1}{a^3-1}-\frac{1}{a^2+a+1}$

9) $\frac{a^2+4}{a^3+8}-\frac{1}{a+2}$

10) $\frac{m^2-3m+9}{m^3-27}-\frac{1}{m-3}$

12) $\frac{3}{4a^2-9}-\frac{8}{2a+3}-\frac{7}{3-2a}$

13) $\frac{a+b}{a}-\frac{a}{a-b}-\frac{b}{a^2-ab}$

14) $\frac{5b-1}{3b^2-3}+\frac{b+2}{2b+2}-\frac{b+1}{b-1}$

15) $x-\frac{xy}{x+y}-\frac{x^3}{x^2-y^2}$

8.Ifodani soddalashtiring:

1) $\frac{m^3-n^3}{m^2-n^2}-\frac{m^2+n^2}{m+n}-\frac{m^2n+mn^2}{m^2+n^2+2nm}$

2) $\frac{a^2-1}{an+n^2}\cdot\frac{1}{n-1}\cdot\frac{a-an^3-n^4+n}{1-a^2}$

3) $\frac{a}{a^2+b^2}-\frac{b(a-b)^2}{a^4-b^4}$

4) $\frac{2a}{a^2-4b^2}+\frac{b+3}{2b^2+6b-ab-3a}$

5) $\frac{b}{ab-2a^2}-\frac{2+2b}{b^2+b-2ab-2a}$

6) $\left[\frac{x3+y3}{xy^3}:\left(\frac{x-y}{y2}+\frac{1}{x}\right)\right]:\frac{x(x-y)2+4x^2y}{x+y}$

7) $\left[\frac{(y-x)^2}{x^2}-\frac{(x+y)^2-4xy}{x^2-xy}\right]^2\cdot\frac{x^4}{x^2y^2-y^4}$

Bir noma'lumli birinchi va ikkinchi darajali tenglamalar

Ta'rif. $ax=b$ ko'rinishdagi tenglama birinchi darajali bir noma'lumli *chiziqli tenglama* deyiladi. Agar bu tenglama $a\neq 0$, $b\in R$ bo'lsa yagona yechimga ega ; agar $a=b=0$ va $x\in R$ bo'lsa cheksiz ko'p yechimga ega; agar $a=0$, $b\neq 0$, $b\in R$ bo'lsa yechimga ega bo'lmaydi.

1-misol: *3x=12 , x= 4* yagona yechimga ega bo'ladi.

2-misol: $0\cdot x=0$ *x=1* da ham *x=2* da ham tenglikning ikki tomoni "0" ga teng bo'ladi, demak $x\in R$ da cheksiz ko'p yechimga ega bo'ladi.

3-misol: *0·x= 2 ,* bundan x ni topsak maxraji 0 bo'ladi, bunday bo'lishi mumkin emas, demak tenglama yechimga ega emas.

Ta'rif. Agar ikki tenglamadan birining ildizlari ikkinchisining ham ildizlari bo'lsa va , aksincha, ikkinchi tenglamaning ildizlari birinchi tenglamaning ham ildizlari bo'lsa yoki ikkala tenglama ham ildizlarga ega bo'lmasa, bunday ikki tenglama *teng kuchli* yoki *ekvivalent t*englamalar deyiladi.

4-misol: $4 \cdot x + 3 = 2 \cdot x + 15$ tenglama $4 \cdot x - x = x + 12$ tenglama bilan teng kuchli, ikkala tenglama ham $x=6$ ildizga ega.

3-ta'rif: Kvadrat tenglama deb $ax^2+bx+c=0$ $(a \neq 0)$ ko'rinishdagi tenglamaga aytiladi, bunda x – noma'lum son, a,b,c ixtiyoriy haqiqiy sonlar bo'lib, koeffitsiyentlar deyiladi. Ikkinchi darajali bir noma'lumli tenglama soddalashtirishdan keyin

$$ax^2+bx+c=0 \quad (1)$$

ko'rinishga keltiriladi.
Tenglamaning o'ng tomonidan to'la kvadrat ajratamiz:

$a\left(x^2+2\frac{b}{2a}x+\frac{b^2}{4a^2}-\frac{b^2}{4a^2}\right)+c=0$ yoki $a\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a}=-c$ bundan

$a\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a}-c=\frac{b^2-4ac}{4a}$ yoki $(x+\frac{b}{2a})^2=\frac{b^2-4ac}{4a^2}$ ikkala tomonidan kvadrat ildiz topamiz:

$x_{1,2}=\frac{b}{2a}\pm\sqrt{\frac{b^2-4ac}{4a^2}}=\pm\frac{\sqrt{b^2-4ac}}{2a}$ va $x_{1,2}=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$ yoki $x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ (2)

b^2-4ac kvadrat tenglamaning diskriminanti deyiladi va D bilan belgilanadi:
$D=b^2-4ac$.
1. Agar $D>0$ bo'lsa, (1) tenglama $x_1 \neq x_2$ haqiqiy ildizlarga ega bo'ladi;
2. Agar $D=0$ bo'lsa, (1) tenglama $x_1=x_2$ haqiqiy ildizlarga ega bo'ladi;
3. Agar $D<0$ bo'lsa, (1) tenglama haqiqiy ildizlarga ega emas, kompleks ildizlarga ega bo'ladi.

Misollar

1) $3x^2-5x+2=0$ ikkita haqiqiy ildizga ega. Haqiqatda:

$x_{1,2}=\frac{5\pm\sqrt{25-24}}{6}=\frac{5\pm1}{6}; \quad x_1=\frac{2}{3}; \quad x_2=1$

2) $4x^2-12x+9=0$ tenglamada $D=144-144=0$ bo'lib tenglama $(2x-3)^2=0$ ko'rinishini oladi, bundan $x_{1,2}=\frac{3}{2}$

3) $5x^2-4x+1=0$ tenglamani yechib:

$x_{1,2}=\frac{4\pm\sqrt{16-20}}{10}=\frac{4\pm2i}{10}=\frac{2\pm i}{5}; \quad x_1=\frac{2-i}{5}; \quad x_2=\frac{2+i}{5}$ kompleks ildizlarni hosil qildik.

Keltirilgan kvadrat tenglama deb

$$x^2+px+q=0$$

tenglamaga aytiladi. Buni yechish uchun (2) formuladan tashqari yana

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$$

formuladan foydalanish mumkin.

Misol: x^2-$6x+5=0$ tenglamani yechamiz.

Xususiy holda kvadrat tenglama. $x_{1,2} = 3 \pm \sqrt{9-5} = 3 \pm 2;\ x_1 = 1;\ x_2 = 5.$

Agar tenglama $ax^2+2kx+c=0$ koʻrinishda boʻlsa, ildizlarini

$$x_{1,2} = \frac{-k \pm \sqrt{k^2 - ac}}{a}$$

formula yordamida topish qulay boʻladi.

Agar x_1 va x_2 kvadrat tenglama ildizlari boʻlsa, u holda

$ax^2+bx+c=a(x-x_1)(x-x_2)$

$x^2+px+q=(x-x_1)(x-x_2)$ boʻladi.

Viyet teoremasi: Agar x_1 va x_2 keltirilgan $x^2+px+q=0$ kvadrat tenglamaning ildizlari boʻlsa,

$\begin{cases} x_1 + x_2 = -p \\ x_1 \cdot x_2 = q \end{cases}$ boʻladi.

Chala kvadrat tenglamalar

1. $ax^2+bx+c=0$ da $c=0$ boʻlsa:

$ax^2+bx=0$ boʻlib, bundan $(ax+b)x=0$ ni hosil qilamiz va $x_1=0$, $ax+b=0$; $x_2 = -\frac{b}{a}$ ni topamiz.

2. $b=0$ boʻlsa, $ax^2+c=0$ hosil boʻladi. Bundan $ax^2=-c$, $x^2 = -\frac{c}{a}$, $x_{1,2} = \pm\sqrt{-\frac{c}{a}}$ ni topamiz.

Bu holda $-\frac{a}{c} \geq 0$ boʻlganda tenglama haqiqiy ildizlarga ega boʻladi.

3. $b=c=0$ boʻlsa, $ax^2=0$, $x^2=0$, $x_{1,2}=0$ hosil boʻladi.

Misollar.

1.Tenglamalarni yeching

1) $7x - (6x+8) = -7$

2) $\frac{3x}{5} = \frac{6+x}{3}$

16) $4 + 8y + 8 = y - 9 - 8y + 6$

17) $\frac{y}{3} + \frac{y}{2} = 5$

3) $3y + 5 = 4(9 - \frac{y}{2})$

4) $8\left(11 - \frac{3}{4}z\right) = 16z - 44$

5) $0{,}18y - 7{,}4 = 0{,}05y - 5{,}71$

6) $11\frac{2}{3}x - 5\frac{1}{6} = 3\frac{3}{4} + 2\frac{3}{4}x$

7) $12\frac{3}{4} + 3\frac{3}{7}y = \frac{y}{2} - 10\frac{1}{28}$

8) $\frac{3x - 7}{4} - \frac{9x + 11}{8} = \frac{3 - x}{2}$

9) $\frac{4x - 51}{3} - \frac{17 - 3x}{4} = \frac{x + 5}{2}$

10) $(x + 2)(x - 3) = x^2 - 7$

11) $\frac{96}{7{,}2} = \frac{4x + 300}{21}$

12) $\frac{3x + 14{,}7}{20{,}4} = \frac{7{,}5}{10}$

13) $\frac{5x - 3}{4} = \frac{x}{2} + 5 - \frac{11 - 3x}{8}$

14) $8(x+2) – 5x = - 2(x + 4{,}5)$

15) $\frac{x - 4}{6} + 2 = 1{,}5x - \frac{x - 2}{12}$

18) $3(5 + \frac{x}{2}) = 4 + 2x$

19) $2\left(3 - \frac{x}{4}\right) = 6x + 4$

20) $0{,}36x - 0{,}6 = 0{,}3(0{,}4x - 1{,}2)$

21) $\frac{6x + 7}{7} = 3 - \frac{5x - 3}{8}$

22) $10 - \frac{3x - 1}{2} = \frac{6x + 3}{11}$

23) $\frac{4x - 3}{2} - \frac{5 - 2x}{3} = \frac{3x - 4}{3}$

24) $\frac{9x - 5}{2} - \frac{3 + 5x}{3} - \frac{8x - 2}{4} = 2$

25) $2(x + 3)(x - 4) = (2x - 1)(x + 2) - 27$

26) $4{,}2 : (2x - 7) = 10 : 7\frac{1}{7}$

27) $4\frac{1}{11} = 4{,}5 : (3x - 1)$

28) $\frac{2x + 1}{3} + 2 = \frac{3x - 2}{2} - \frac{x + 1}{6}$

29) $3(x+2) – 2(x+3) = 7 – 5(x+10)$

30) $6 - \frac{2x - 5}{4} = \frac{4x + 2}{4}$

2.Tenglamalarni yeching

1) $x^2 = 1$

2) $x^2 = \frac{9}{16}$

3) $x^2 = 2\frac{1}{4}$

4) $x^2 - 49 = 0$

5) $x^2 + 12 = 0$

6) $3x^2 + 5x = 0$

7) $x^2 + 6x + 9 = 0$

8) $(x - 2)(x^2 + 2x + 4) - x^2(x - 18) = 0$

9) $(x + 1)(x^2 - x + 1) - x^2(x + 4) = 0$

10) $9x^2 = 81$

11) $x^2 - 7x = 0$

12) $\frac{x^2 - 1}{3} = 5$

16) $x^2 = 9$

17) $x^2 = \frac{16}{49}$

18) $x^2 = -6$

19) $x^2 - 121 = 0$

20) $x^2 - x = 0$

21) $5x^2 - 3x = 0$

22) $x^2 + 9 = 0$

23) $4x^2 - 64 = 0$

24) $4x^2 - 169 = 0$

25) $\frac{9 - x^2}{5} = 1$

29) $x^2 = 16$

30) $x^2 = 13$

31) $x^2 = -\frac{1}{4}$

32) $\frac{x^2}{5} = 0$

33) $x^2 + 2x = 0$

34) $x^2 - 4x + 4 = 0$

35) $x^2 - 15 = 0$

36) $25 - 16x^2 = 0$

37) $3x^2 = 15$

38) $x^2 - 27 = 0$

39) $2x^2 - 16 = 0$

40) $\frac{x^2 - 5}{5} = 4$

13) $\frac{x^2-3x}{5}=0$ 26) $\frac{x^2+7x}{8}=0$ 41) $x(x-15)=3(108-5x)$

14) $\frac{x^2-3x}{x-3}=0$ 27) $\frac{x^2-9}{x-3}=0$ 42) $\frac{x^2+2x}{x+2}=0$

15) $2x^2=\frac{1}{8}$ 28) $3x^2=5\frac{1}{3}$ 43) $9x^2+1=0$

3.Ildizlari

1) 2 va 3 2) -1 va 4

3) 0 va 4 4) $\frac{1}{2}$ va $\frac{1}{3}$ bo‘lgan kvadrat tenglamalarni tuzing.

4.Tenglamalarni yeching.

1) $\frac{ax-b}{a+b}+\frac{bx+a}{a-b}=\frac{a^2+b^2}{a^2-b^2}$

2) $\frac{1}{a}+\frac{1}{a+x}+\frac{1}{a+2x}=0$

3) $\frac{2x}{x+b}-\frac{x}{b-x}=\frac{b^2}{4(x^2-b^2)}$

4) $\frac{x}{x+2}+\frac{2x}{x-2}=\frac{5}{x^2-4}$

5) $\frac{x^2+3x}{2}=\frac{x+7}{4}$; 6) $\frac{x^2-3x}{7}+x=11$;

7) $\frac{2x^2+x}{3}-\frac{2-3x}{4}=\frac{x^2-6}{6}$; 8) $\frac{x^2+x}{4}-\frac{3-7x}{20}=0,3$.

Chiziqli va kvadrat tengsizliklar

Chiziqli tengsizliklar.

Agar *x* ga bog‘liq bo‘lgan *A(x)* va *B(x)* ifodalar quyidagi munosabatlardan *A(x)>B(x), A(x)≥B(x), A(x)<B(x), A(x)≤B(x)* birini qanoatlantirsa, bir noma’lumli tengsizlik berilgan deyiladi. Bu ifodalarni ikkala tomoni ma’noga ega bo‘ladigan x ning qiymatlari to‘plami tengsizliklarning mavjudlik sohasi deyiladi. O‘zgaruvchi x ning tengsizlikni qanoatlantiradigan qiymatlar to‘plami tengsizlikning yechimi deyiladi.

2x–6≤0 bo‘lsin, bundan *2x≤6=>x≤3* bo‘lib, tengsizlikning yechimi $x\in(-\infty,3)$ bo‘ladi.

Tengsizliklarning yechimini topishda quyidagi qoidalarga rioya qilish lozim:

1. Tengsizlikning ikkala tomoniga bir xil ifodani qo‘shish yoki ayirishdan tengsizlik ishorasi o‘zgarmaydi;
2. Tengsizlikning ikkala tomonini bir xil musbat ifodaga ko‘pay-tirish yoki bo‘lishdan tengsizlik ishorasi o‘zgarmaydi;

3. Tengsizlikning ikkala tomonini bir xil manfiy ifodaga ko‘paytirsak yoki bo‘lsak, tengsizlik ishorasi teskarisiga o‘zgaradi, ya’ni $A(x)>B(x)$ bo‘lsa:

1) *A(x)+C(x)>B(x)+C(x)*

2) *C(x)>0* bo‘lsa, $A(x)\cdot C(x)>B(x)\cdot C(x)$ *va* $\frac{A(x)}{C(x)}>\frac{B(x)}{C(x)}$

3) *C(x)<0* bo‘lsa, $A(x)\cdot C(x)<B(x)\cdot C(x)$ *va* $\frac{A(x)}{C(x)}<\frac{B(x)}{C(x)}$ bo‘ladi.

Soddalashtirishdan keyin *ax>b, ax≥b, ax<b, ax≤b* ko‘rinishidan biriga keltirilishi mumkin bo‘lgan tengsizlik chiziqli (birinchi darajali) tengsizlik deyiladi.

Misol. *5x+0,7<3x-15,3* tengsizlikni yeching.

Yechish: Ayniy almashtirishlar tengsizlikni *2x<-16* ko`rinishga keltiradi. Bu tengsizlikning har ikki tomonini *2* ga bo`lib, *x<-8* ni hosil qilamiz. **Javob:** $(-\infty;-8)$.

Misol. $\frac{2x-1}{2}-3>x-\frac{x+3}{3}$ tengsizlikni yeching.

Yechish: Ikkala tomonini 6 ga ko‘paytirib *6x-3-18>6x-2x-6* ni, bundan esa *2x>15* ni hosil qilamiz. Ikkala tomonini 2 ga bo‘lib, *x>7,5* ni topamiz.

Javob: $x\in(7{,}5;\infty)$

Misollar

1. **Tengsizliklarni yeching:**

1) 4(x-2)≤2x-5; 2) 5-6(x+1)≥2x+3;

3) 3x-7<4(x+2); 4) 7-6x≥$\frac{1}{3}$(9x-1);

5) 1,5(x-4)+2,5x<x+6 6) 1,4(x+5)+1,6x>9+x;

7) $\frac{x-1}{3}-\frac{x-4}{2}\le 1$ 8) $\frac{x+4}{5}-\frac{x-1}{4}\ge 1$

9) $\frac{2x-5}{4}-\frac{3-2x}{5}<1$ 10) $\frac{x+2}{4}+x<3$

11) $3(x+2)+\frac{2}{3}x<4x+5$ 12) $\frac{5x+6}{3}+2\le 3x-\frac{x}{2}$

Kvadrat tengsizliklar

$$ax^2+bx+c>0 \qquad (ax^2+bx+c\ge 0) \qquad ax^2+bx+c<0 \qquad (ax^2+bx+c\le 0)$$

ko‘rinishidagi yoki shu ko‘rinishga keltirilishi mumkin bo‘lgan tengsizlik kvadrat tengsizlik deyiladi (bunda *x* – o‘zgaruvchi, *a, b, c* – o‘zgarmas sonlar).

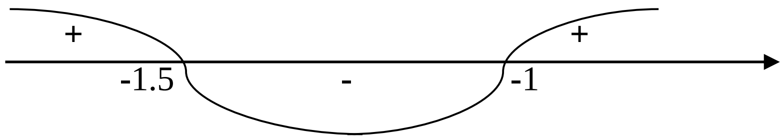
Kvadrat tengsizlikni yechishda quyidagilarga amal qilish kerak. $ax^2+bx+c<0$ kvadrat uchhadni $a(x-x_1)(x-x_2)<0$ ko‘rinishida tasvirlaymiz (x_1 va x_2) kvadrat uchhadlarning nollari).

$a(x-x_1)(x-x_2)<0$ yechimi *a>0* bo‘lganda $x\in(x_1,x_2), a<0$ bo‘lganda $x\in(-\infty,x_1)\bigcup(x_2,\infty)$ bo‘ladi, chunki ax^2+bx+c ning ishorasi *a* ning qiymatiga qarab u yoki bu oraliqning ishorasi bilan bir xil bo‘ladi. $a(x-x_1)(x-x_2)>0$ bo‘lganda, aksincha.

Agar ax^2+bx+c uchhadning diskriminanti D<0 bo‘lsa, $ax^2+bx+c>0$ tengsizlik *a>0* bo‘lganda *x* ning barcha qiymatlarida o‘rinli, *a<0* bo‘lsa, yechimga ega emas. Amalda bu qoidaning qo‘llanishini misollarda ko‘rib chiqamiz.

Misol. 1) $2x^2+5x+3>0$ tengsizlik yechilsin.

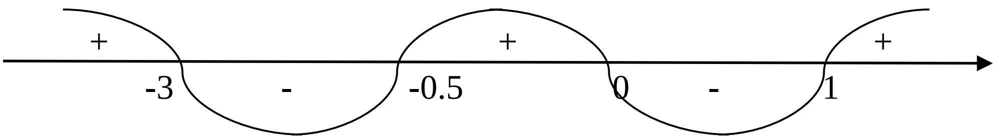
Yechish: Kvadrat uchhadning ildizlarini topib, tengsizlikni $2(x+\frac{3}{2})(x+1)>0$ ko‘rinishida yozamiz. Kvadrat uchhadning aniqlanish so-hasi $(-\infty,\infty)$ ekanligini bilgan holda, uni $x_1=-\frac{3}{2},\ x_2=-1$ nuqtalar yordamida oraliqlarga ajratamiz: $(-\infty,-\frac{3}{2}),\ (-\frac{3}{2},-1)\ va\ (-1,\infty)$. Bu oraliqlarni sonlar o‘qi-da tasvirlaymiz:



$2(x+\frac{3}{2})(x+1)>0-$ tengsizlikda ikkala qavsning ishorasi chapdagi oraliqda hamma vaqt musbat bo‘ladi, undan bitta oldingi oraliqda esa qavslarning ishorasi qarama-qarshi bo‘lib, umumiy ishora minus bo‘ladi, keyingisida musbat bo‘ladi va hokazo. Tengsizlik yechimi $x\in(-\infty;\frac{3}{2})\cup(-1,\infty)$ bo‘ladi. Bu usulda ko‘paytuvchilar (qavslar) soni ko‘p bo‘lganda ham foydala-nish mumkin.

Misol. 2) $x(x+3)(x+\frac{1}{2})(x-1)<0$ bo‘lsin.

Bu tengsizlikda chap tomondagi ifodaning nollari $-3,\ -\frac{1}{2},\ 0.1$ bo‘-ladi, shuning uchun yechim tasviri quyidagicha bo‘ladi:



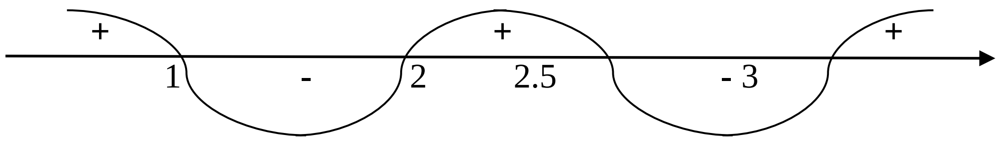
Ifoda manfiy qiymatlarni $(-3;-\frac{1}{2})$ va $(0;1)$ oraliqlarda qabul qiladi. Yechim $x \in (-3;-\frac{1}{2}) \cup (0;1)$.

Keltirilgan usuldan (u intervallar usuli deyiladi) kasr ifoda bo‘l-ganda ham foydalanish mumkin.

Misol: 3) $\frac{2x^2-7x+5}{x^2-5x+6} \geq 0$ tengsizlik yechilsin.

Yechish: Bu tengsizlikni quyidagicha yozamiz:

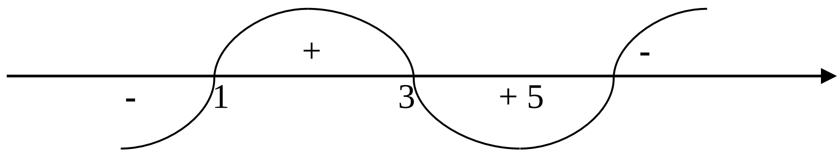
$\frac{2(x-1)(x-\frac{5}{2})}{(x-2)(x-3)} \geq 0$ va sonlar o‘qida belgilab topamiz:



Yechim: $x \in (-\infty,1] \cup (2,\frac{5}{2}] \cup (3,\infty)$.

Misol. 4) $\frac{x+7}{x-5}+\frac{3x+1}{2} \leq 0$ tengsizlik yechilsin.

Yechish: $\frac{2x+14+3x^2+x-15x-5}{2(x-5)} \leq 0, \frac{3x^2-12x+9}{2(x-5)} \leq 0, \frac{3(x-1)(x-3)}{2(x-5)} \leq 0$



Yechim: $x \in (-\infty;1] \cup [3;5)$.

Tengsizliklarni yeching

1) $(x-1)\cdot(x-3)>0$

2) $x^2+5x>0$

3) $(x-5)^2(x-25)>0$

4) $(x-5)(x+2)(x^2-4)\leq 0$

12) $(x+2)\cdot(x+5)>0$

13) $x^2-9x>0$

14) $(x+7)^2(x-49)<0$

15) $(x-8)(x-1)(x^2-1)\geq 0$

23) $(x-7)\cdot(x-10)>0$

24) $x^2+3x\leq 0$

25) $(x-4)(x^2-16)>0$

26) $x>\frac{1}{x}$

5) $\left(x^2-5x+6\right)\left(x^2-1\right)>0$

6) $\frac{x-2}{x+5}>0$

7) $\frac{(2x+1)(x+2)}{x-3}<0$

8) $\frac{(x+4)^2}{2x^2-3x+1}\geq 0$

9) $\frac{x^2-7x-8}{x^2-64}>0$

10) $\frac{x}{x-2}+\frac{3}{x}>\frac{3}{x-2}$

11) $1\leq\frac{1}{x}$

16) $\left(x^2-x+2\right)\left(x^2-7x+12\right)\leq 0$

17) $\frac{x-4}{x+3}<0$

18) $\frac{(x-3)(2x+4)}{x+1}\geq 0$

19) $\frac{x^2-x-12}{x-1}>0$

20) $\frac{x^2-16}{2x^2+5x-12}>0$

21) $\frac{x^2}{x^2+3x}+\frac{2-x}{x+3}<\frac{5-x}{x}$

22) $2>\frac{1}{2x}$

27) $2\geq\frac{1}{2x^2}$

28) $\frac{3{,}5+x}{x-7}\leq 0$

29) $\frac{x^2+2x+3}{(x-2)^2}\leq 0$

30) $\frac{x^2+3x-10}{x^2+x-2}>0$

31) $\frac{x^2+7x+10}{x^2-4}>0$

Modul qatnashgan tenglama va tengsizliklar

Modul qatnashgan tenglamalarni shartli ravishda quyidagi turlarga bo'lishimiz mumkin:

$$|f(x)|=a, a\geq 0 \qquad f(|x|)=a,\ \ |f(x)|=\varphi(x),\ \ |f(x)|=|\varphi(x)|,$$

$$|f_1(x)|+|f_2(x)|+\cdots+|f_n(x)|=a$$

Modulli tengsizliklarni yechishda quyidagi teng kuchliliklardan foydalaniladi

1. $|f(x)|<a\ \ (a>0)\ \ \Leftrightarrow\ \ -a<f(x)<a$

2. $|f(x)|>a\ \ (a>0)\ \ \Leftrightarrow\ \ f(x)<-a\ \ va\ \ f(x)>a$

3. $|f(x)|<|g(x)|\ \ \Leftrightarrow\ \ f^2(x)<g^2(x)$

Misol: 1) $|4x+5|=9$ tenglamani yeching.

Yechish: Bu tenglama quyidagi ikki holga ajraladi:

1) $\begin{cases}4x+5\geq 0\\4x+5=9\end{cases}$ 2) $\begin{cases}4x+5<0\\4x+5=-9\end{cases}$

Bularni yechamiz: 1) $\begin{cases}x\geq -\frac{5}{4}\\x=1\end{cases}$ 2) $\begin{cases}x<-\frac{5}{4}\\x=-3{,}5\end{cases}$

Ikkala yechim ham tenglamani qanoatlantiradi. **Javob:** -3,5 va 1

2) $\left|2x+4\right|-\left|x-2\right|=3$ tenglamani yeching.

Yechish: Modul ostidagi ifodalarni nolga tenglashtirib x_1=-2, x_2=2 qiymatlarni topamiz. Bu qiymatlar yordamida sonlar o‘qini qismlarga ajratamiz. Har qismda tenglamani alohida-alohida yechamiz:

I) $x \le -2; \quad -(2x+4)+(x-2)=3$

$$-2x-4+x-2=3$$

$-x=9, \qquad x_1=-9$

II) $-2 < x \le 2; \qquad 2x+4+x-2=3$

$$3x=1, \qquad x_2=\frac{1}{3}$$

III) $x>2 \qquad 2x+4-x+2=3$

$$x \ne -3$$

Javob: $x_1=-9; \qquad x_2=\frac{1}{3}$

3) $\left|2x^2-7x+3\right|=2x^2-7x+3$ tenglamani yeching.

Yechish: Ikkala tomonda bir xil ifoda turibdi. Bir tomonda modul ostida, boshqa tomonda modulsiz. Tenglik o‘rinli bo‘lishi uchun bu ifo-da manfiy bo‘lmasligi yetarli, ya’ni:

$2x^2-7x+3 \ge 0 \Rightarrow 2(x-\frac{1}{2})(x-3) \ge 0 \Rightarrow$

$x \le \frac{1}{2};\ x \ge 3$

Javob: $x \in (-\infty, \frac{1}{2}] \cup [3, \infty)$.

4) $\left|x-2\right| \le 6$ tengsizlikni yeching.

Yechish: $-6 \le x-2 \le 6 \Rightarrow -4 \le x \le 8$ **Javob:** $x \in [-4, 8]$.

5) $\left|x+3\right| > 1$ tengsizlikni yeching.

Yechish: Agar modul ostidagi ifoda manfiy bo‘lsa, *x+3<-1*, bun-dan *x<-4* bo‘lishi, agar modul ostidagi ifoda musbat bo‘lsa, *x+3>1* bo‘-lishi, bundan *x>-2* bo‘lishi lozim. Yechim: $x \in (-\infty, -4) \cup (-2, \infty)$.

6) $\left|x^2-5x+4\right| > 0$ tengsizlik $x^2-5x+4=0$ tenglamaning ildizlaridan tashqari x ning barcha qiymatlarida o‘rinli bo‘ladi, ya’ni $x \ne 1,\ x \ne 4$.

Javob: $x \in (-\infty, 1) \cup (1, 4) \cup (4, \infty)$

7) $|x^2 + 3x + 1| > 5$ tengsizlikni yeching.

Yechish: Bu tengsizlik quyidagi ikkita tengsizlikka teng kuchli.
a) $x^2 + 3x + 1 < -5$ b) $x^2 + 3x + 1 > 5$

Birinchisini yechamiz:
$x^2 + 3x + 6 < 0$ *D=9-24<0* bo‘lgani uchun $x^2 + 3x + 6$ ifoda hamma vaqt musbat bo‘ladi.
Ikkinchi tengsizlikni yechamiz: $x^2 + 3x - 4 > 0$ yoki $(x+4)(x-1) > 0$
Bundan $x \in (-\infty, -4) \cup (1, \infty)$ yechimni topamiz.

Misollar

1. Hisoblang.

1) $|5-16|$ **3)** $|5-|25-29|+25|$ **5)** $|5-16|+|9-4|$ **7)** $|25-16|$

2) $|5 \cdot |-46-21| - 16|$ **4)** $|2-3 \cdot |-9+36||$ **6)** $|2+4 \cdot |-9-36||$

2. Modulli tenglamalarni yeching.

1) $|x| = 5$ 9) $|x-1| = 2$ 17) $|x+3| = 3$

2) $|x+4| = 0$ 10) $|3-4x| = 0$ 18) $\left|\frac{2}{3}x + \frac{1}{6}\right| = \frac{1}{3}$

3) $\left|\frac{3}{4}x - \frac{1}{2}\right| = \frac{1}{4}$ 11) $|x| = 2{,}5$ 19) $|5-x| = 5$

4) $|4-5x| = 5$ 12) $|4-x| = -4+x$ 20)
$|x-2| = 2-x$

5) $|2x-5| = 5-2x$ 13) $|3x-6| = -2$ 21) $|5x-8| = -0{,}5$

6) $|x-1| = |x-2|$ 14) $|x+1| = |2x-3|$ 22) $|x+3| = |x+7|$

7) $|5x-8| = 5x-8$ 15) $|3-4x| = 3-4x$ 23) $|x+1| = x+1$

8) $|x| = 1{,}5$ 16) $|2x-3| = 0$ 24) $|-x| = 3.,4$

3. Modulli tengsizliklarni yeching.

1) $|x| < 5$ 7) $|x| \le 4$ 13) $|x| \ge 3$ 19) $|x| > 2$

2) $|x| \le 3$ 8) $|1+x| \le 0{,}3$ 14) $|2+x| < 0{,}2$ 20) $|3-x| \le \frac{3}{4}$

3) $|3x-4| \le 5$ 9) $|x-2| \ge 11$ 15) $|5-4x| \le 1$ 21) $|2-3x| \le 2$

4) $|1+x|\le 1{,}3$ 10) $|x-2|\ge 1{,}1$ 16) $|1-x|\ge \frac{1}{2}$ 22) $|3x-2|>4$

5) $|4-5x|\ge 4$ 11) $|4-5x|\ge -4$ 17) $|5-2x|<-4$ 23) $|3x-2|\ge -3$

6) $|x-1|>|x-2|$ 12) $|x+3|\le|x+7|$ 18) $|2x-6|\ge|2x+9|$

Tenglamalar va tengsizliklar sistemalarini yechishning turli usullari

Tenglamalar sistemalarini yechish

Tenglamalar sistemasini yechishda turli usullar: noma’lumlarni ketma-ket yo‘qotish, o‘rniga qo‘yish, o‘zgaruvchilarni almashtirish va boshqalar qo‘llanilishi mumkin. Bularni misolda ko‘rib chiqamiz.

1-misol $\begin{cases}2x+3y=8\\3x-y=1\end{cases}$. Sistemani o‘zgaruvchini yo‘qotish yo‘li bilan yeching.

Yechish. Birinchi tenglamani o‘zgarishsiz qoldirib, ikkinchi teng-lamani 3 ga ko‘paytiramiz va ularni qo‘shsak, hosil bo‘lgan tenglama fa-qat x ga nisbatan bo‘ladi, ya’ni:

$\begin{cases}2x+3y=8\\9x-3y=3\end{cases}$ bo‘lib, qo‘shib $11x=11$ va $x=1$ ni topamiz.

Ikkinchi tenglamada x ning o‘rniga $x=1$ ni qo‘yib, y ning qiymatini topamiz:

$3\cdot 1-y=1;$ $y=3-1=2$ Yechim; (1;2)

2-misol. $\begin{cases}x+y=7\\xy=12\end{cases}$ sistemani o‘rniga qo‘yish usuli bilan yeching.

Yechish: Birinchi tenglamadan $y=7-x$ ni topib, ikkinchi tenglamadagi y ning o‘rniga qo‘yib topamiz:

$x(7-x)=12;$ $7x-x^2-12=0,$ $x^2-7x+12=0;$

$x_{1,2}=\frac{7\pm\sqrt{49-48}}{2}=\frac{7\pm 1}{2};$ $x_1=3;$ $x_2=4.$

y=7-x da x ning o‘rniga topilgan qiymatlarni qo‘yib, $y_1=4$ *va* $y_2=3$- ni topamiz.
Yechim: *(3,4); (4,3).*

3-misol. $\begin{cases} x-y=1 \\ x^3-y^3=7 \end{cases}$ sistemani o‘rniga qo‘yish usuli bilan yeching.

Yechish: Birinchi tenglamadan x=y+1ni topib, ikkinchi tenglamaga qo‘yamiz:

$$(y+1)^3-y^3=7 \Rightarrow y^3+3y^2+3y+1-y^3-7=0 \Rightarrow y^2+y-2=0 \Rightarrow y_{1,2}=\frac{-1\pm3}{2};$$

$y_1=1$; $y_2=-2$. $x_1=2$, $x_2=-1$. Yechim: (*2;1), (-1; -2*).

4-misol. $\begin{cases} x+y+xy=11 \\ x^2y+xy^2=30 \end{cases}$ sistemani belgilab yeching.

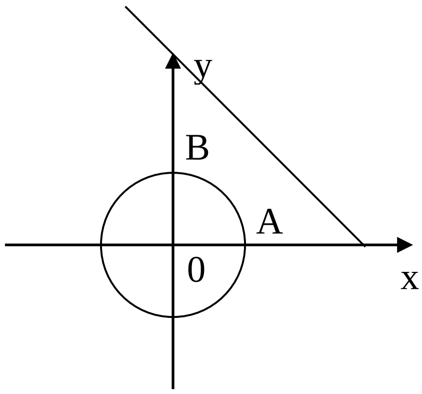
Yechish: Sistemani $\begin{cases} x+y+xy=11 \\ xy(x+y)=30 \end{cases}$ shaklda yozib, x+y=u, xy=v, deb belgilab, $\begin{cases} u+v=11 \\ uv=30 \end{cases}$ sistemani hosil qilamiz. Viyet teoremasiga ko‘ra u va v $z^2-11z+30=0$ kvadrat tenglamaning ildizlari bo‘ladi:

$$z_{1,2}=\frac{11\pm\sqrt{121-120}}{2\cdot1}=\frac{11\pm1}{2}; \ z_1=5, \ z_2=6.$$

Bundan $u_1=5$, $v_1=6$ *va* $u_2=6$, $v_2=5$ ni topamiz va ikkita sistema $\begin{cases} x+y=5 \\ xy=6 \end{cases}$ va $\begin{cases} x+y=6 \\ xy=5 \end{cases}$ ni hosil qilamiz. Bularni yechib, sistemaning yechimi (2;3), (3;2) (5; 1), (1; 5) ni hosil qilamiz.

5-misol. $\begin{cases} x+y=2 \\ x^2+y^2=4 \end{cases}$ sistemani grafik usulda yeching.

Yechish: $x+y=2$ to‘g‘ri chiziqni va $x^2+y^2=2^2$ aylanani bitta chizma-da chizamiz va ularning kesishish nuqtalari *A(2;0)* va *B(0; 2)* ni topamiz. Sistemaning yechimi (2; 0) va (0; 2) bo‘ladi.



Misollar

1.Quyidagi sistemalarni qulay usul bilan yeching:

1) $\begin{cases} x+y-5=0 \\ 2x-3y+10=0 \end{cases}$ 2) $\begin{cases} 2x-3y+7=0 \\ 3x+2y-4=0 \end{cases}$ 3) $\begin{cases} \frac{x}{2}-\frac{y}{3}+1=0 \\ \frac{x}{4}+y-\frac{13}{2}=0 \end{cases}$ 4) $\begin{cases} \frac{3x}{2}-\frac{y}{5}-2=0 \\ 2x-\frac{y}{3}-\frac{1}{3}=0 \end{cases}$

5) $\begin{cases} -x+y=6 \\ x^2-4y=-3 \end{cases}$ 6) $\begin{cases} x+y=2 \\ y^2+x=32 \end{cases}$ 7) $\begin{cases} x+2y=1 \\ x+y^2=4 \end{cases}$ 8) $\begin{cases} y-3x=7 \\ x^2+xy=2 \end{cases}$

9) $\begin{cases} xy=4 \\ x^2+y^2=17 \end{cases}$ 10) $\begin{cases} xy=3 \\ x^2+y^2=10 \end{cases}$ 11) $\begin{cases} (x^2-y^2)xy=180 \\ x^2-xy-y^2=-11 \end{cases}$

12) $\begin{cases} 3x^2-2xy+5y^2=35 \\ 5x^2-10y^2=5 \end{cases}$

Tengsizliklar sistemalarini yechish

Tengsizliklar sistemasida qatnashgan bir noma’lumli har bir teng-sizlik alohida-alohida yechilib, ularning yechimlarini umumiy qismi sis-temaning yechimi bo‘ladi. Buni misollarda ko‘ramiz.

1) $\begin{cases} 2x+1\geq 4 \\ x^2-3x+2<0 \end{cases}$ tengsizliklar sistemasini yeching.

Yechish: $\begin{cases} 2x\geq 3 \\ (x-1)(x-2)<0 \end{cases} \Rightarrow \begin{cases} x\geq 1{,}5 \\ 1<x<2 \end{cases} \Rightarrow x\in[1{,}5;2)$. **Javob:** $x\in[1{,}5;2)$.

2) $y=\frac{1}{\sqrt{3x-5}}+\frac{1}{x-5}$ funksiyaning aniqlanish sohasi topilsin.

Yechish: Birinchi kasr mavjud bo‘lishi uchun *3x-5>0* bo‘lishi, ikkinchi kasr mavjud bo‘lishi uchun $x-5\neq 0$ bo‘lishi zarur. Bularni birlashtirib $\begin{cases} 3x-5>0 \\ x-5\neq 0 \end{cases}$ sistemaga ega bo‘lamiz. Bu sistemani yechib, $\begin{cases} x>\frac{5}{3} \\ x\neq 5 \end{cases} \Rightarrow x\in(\frac{5}{3},5)\cup(5,\infty)$ yechimni topamiz.

3) $y=\sqrt{\frac{x^2+4x}{\sqrt{x^2-5x+6}}}$ funksiyaning aniqlanish sohasini toping.

Yechish: Ildiz mavjud bo‘lishi uchun, $\frac{x^2+4x}{\sqrt{x^2-5x+6}} \geq 0$ bo‘lishi zarur.

Kasr mavjud bo‘lishi uchun, maxraj noldan farqli bo‘lishi zarur, demak:

$\begin{cases} x^2-5x+6>0 \\ x^2+4x \geq 0 \end{cases}$ sistemani hosil qilamiz.

Har bir tengsizlikni alohida-alohida yechib, topamiz:

$\begin{cases} (x-2)(x-3)>0 \\ x(x+4) \geq 0 \end{cases} \Rightarrow \begin{cases} x<2 \qquad x>3 \\ x \leq -4 \qquad x \geq 0 \end{cases}$ bu yechimlarni umumiy qismi $x \in (-\infty; -4] \cup [0;2) \cup (3;\infty)$ ni topamiz. Bu sistemaning yechimi funksiyaning aniqlanish sohasini beradi.

Misollar

1.Tengsizliklar sistemalarini yeching:

1) $\begin{cases} 2x+5>1 \\ x-3<6 \end{cases}$ 2) $\begin{cases} \frac{x+1}{2}-\frac{1}{3}<\frac{x}{6} \\ 3x-\frac{1}{4}>\frac{5x-1}{2}-3 \end{cases}$

3) $\begin{cases} x^2-3x<0 \\ x^2-3x+2>0 \end{cases}$ 4) $\begin{cases} \frac{x}{3}-1<\frac{2x+1}{2}+2 \\ 2x^2-5x+2>0 \end{cases}$

2. Funksiyaning aniqlanish sohasini toring:

1) $y=\sqrt{(x+3)\sqrt{2-x}}$, 2) $y=\sqrt{(4-x)\sqrt{2x+3}}$

3. Funksiyaning aniqlanish sohasini toring:

1) $y=\frac{1}{(x-2)\sqrt{x^2-1}}$, 2) $y=\frac{x}{(x+3)\sqrt{4x+x^2}}$

4. Funksiyaning aniqlanish sohasini toring:

1) $y=\frac{1}{x-5}+\frac{1}{\sqrt{3x-2}}$, 2) $y=\frac{1}{x^2-1}+\sqrt{x^2-6x+8}$

Irratsional tenglama va tengsizliklar.

Ta'rif. Noma'lum qatnashgan ifoda ildiz belgisi ostida bo'lgan tenglamalar **irratsional tenglamalar** deyiladi.

Irratsional tenglamalar xususiy hollarda quyidagi ko'rinishlarda bo'lishi mumkin:

a) Bitta kvadrat ildiz qatnashgan irratsional tenglama.

b) Ikkita kvadrat ildiz qatnashgan tenglama.

1 – misol: $2 \cdot \sqrt{x} - 7 = 1$ irratsional tenglamani yeching.

Yechish: $2 \cdot \sqrt{x} = 8$, $\sqrt{x} = 4$; $x = 16$ **javob:** $x = 16$

2 – misol: $\sqrt{3x+7} - \sqrt{x+1} = 2$ irratsional tenglamani yeching.

Yechish: Aniqlanish sohasini qaraymiz: $\begin{cases} 3x+7 \geq 0 \\ x+1 \geq 0 \end{cases}$ => $\begin{cases} x \geq -\frac{7}{3} \\ x \geq -1 \end{cases}$ => $x \geq -1$

tenglamaning ikkala tomonini kvadratga oshiramiz:

$3 \cdot x + 7 - 2 \cdot \sqrt{(3x+7)\cdot(x+1)} + x + 1 = 4,$

$\sqrt{(3x+7)\cdot(x+1)} = 2x + 2$ ni yana kvadratga oshiramiz:

$(3x+7)\cdot(x+1) = 4x^2 + 8x + 4$ dan $x^2 - 2x - 3 = 0$ tenglamani yechib $x_1 = 3$ va $x_2 = -1$ ildizlarga ega bo'lamiz.

Agar A(x) va B(x) ifodalar ratsional ifoda va $\sqrt[2k]{A(x)} \geq 0$, $k \epsilon N$ bo'lganda quyidagi munosabat o'rinli bo'ladi: $\sqrt[2k]{A(x)} = B(x) <=> \begin{cases} A(x) = B^{2k}(x) \\ B(x) \geq 0 \end{cases}$.

3 – misol: $\sqrt{x^2+3x+1} = x - 2$ tenglamani yeching.

Yechish: tenglama ushbu sistemaga teng kuchlidir: $\begin{cases} x^2+3x+1 = (x-2)^2 \\ x - 2 \geq 0 \end{cases}$

$x^2 + 3x + 1 = (x-2)^2$ tenglama yagona $x = \frac{3}{7}$ ildizga ega, lekin $x - 2 \geq 0$ni qanoatlantirmaydi. Tenglama yechimga ega emas.

2. Irratsional tengsizliklar.

a va *b* sonlari nomanfiy bo'lgandagina $a<b$ dan $a^n < b^n$ kelib chiqadi. Shunga ko'ra *A(x)*, *B(x)* irratsional ifodali tengsizliklarni yechishda ularning ishoralari e'tiborga olinishi kerak. Umuman, $\sqrt[2k]{A(x)} < B(x) <=> \begin{cases} A(x) \geq 0 \\ B(x) > 0 \\ A(x) < B^{2k}(x) \end{cases}$ bo'ladi.

Sistemadagi 1- tengsizlik ildiz ostidagi ifodaning nomanfiyligini, ikkinchisi B(x) ning musbatligini ifodalaydi, uchinchisi $a \geq 0$, $b \geq 0$ da $a<b$ va $a^{2k} < b^{2k}$ kelib chiqadi. $\sqrt[2k]{A(x)} > B(x)$ tengsizligi *B(x)≥0*, $A(x) > B^{2k}(x)$ bo'lganda yoki *A(x)≥0* , *B(x)<0* bo'lganda o'rinli. Shunga ko'ra $\sqrt[2k]{A(x)} > B(x)$ tengsizlikni yechish uchun

$\begin{cases} A(x) > B^{2k}(x) \\ B(x) \geq 0 \end{cases}$ va $\begin{cases} A(x) \geq 0 \\ B(x) < 0 \end{cases}$ tengsizliklar sistemalarini yechish va ularning yechimlarini birlashtirish kerak.

1 – misol: $\sqrt{x^2 + 6x - 16} > x - 1$ tengsizlikni yeching.

Yechish: Berilgan tengsizlikdan ushbu tengsizliklar sistemalari hosil bo‘ladi:

$\begin{cases} x^2 + 3x + 1 > (x-1)^2 \\ x - 1 \geq 0 \end{cases}$ va $\begin{cases} x^2 + 6x - 16 \geq 0 \\ x - 1 < 0 \end{cases}$ Birinchi sistemaning yechimi $\left(2\frac{1}{8}; +\infty\right)$ dan, ikkinchi sistemaniki $(-\infty; -8)$ to‘plamdan iborat.

Javob: $(-\infty; -8) \cup \left(2\frac{1}{8}; +\infty\right)$

Agar irratsional tengsizlik $\sqrt{A(x)} + \sqrt{B(x)} < C(x)$ ko‘rinishda berilgan bo‘lsa, $A(x) \geq 0$, $B(x) \geq 0$ va $\sqrt{B(x)} < C(x)$ (yoki $\sqrt{A(x)} < C(x)$) shartlar bajarilganda berilgan tengsizlik $A(x) < (C(x) - \sqrt{B(x)}\,)^2$

(yoki $B(x) < (C(x) - \sqrt{A(x)}\,)^2$) tengsizlikka teng kuchli bo‘lib, yuqorida qaralgan turlardan biriga keladi.

2 – misol: $\sqrt{x-1} + \sqrt{x+4} < 5$ tengsizlikni yeching.

Yechish: $\begin{cases} x - 1 \geq 0 \\ x + 4 \geq 0 \\ \sqrt{x-1} < 5 \\ x + 4 < \left(5 - \sqrt{x-1}\right)^2 \end{cases}$ => $\begin{cases} x \geq 1 \\ x \geq -4 \\ x < 26 \\ x < 5 \end{cases}$ => $1 \leq x \leq 5$

Javob: $1 \leq x \leq 5$.

Tenglamani yeching:

1) $\sqrt{x} = 3$; 2) $\sqrt{x} = 7$; 3) $\sqrt{2x-1} = 0$; 4) $\sqrt{3x+2} = 0$.

1) $\sqrt{x+1} = 2$; 2) $\sqrt{x-1} = 3$;
3) $\sqrt{1-2x} = 4$; 4) $\sqrt{2x-1} = 3$.

1) $\sqrt{x+1} = \sqrt{2x-3}$; 2) $\sqrt{x-2} = \sqrt{3x-6}$;
3) $\sqrt{x^2+24} = \sqrt{11x}$; 4) $\sqrt{x^2+4x} = \sqrt{14-x}$.

1) $\sqrt{x+2} = x$; 2) $\sqrt{3x+4} = x$;
3) $\sqrt{20-x^2} = 2x$; 4) $\sqrt{0,4-x^2} = 3x$.

1) $\sqrt{x^2-x-8}=x-2$;   2) $\sqrt{x^2+x-6}=x-1$.

7. Berilgan tenglama nima uchun ildizlarga ega emasligini tushuntiring:

1) $\sqrt{x}$ = - 8 ;   2) $\sqrt{x}$ + $\sqrt{x-4}$ = - 3;

3) $\sqrt{-2-x^2}$ = 12;   4) $\sqrt{7x-x^2-63}$ = 5 .

Tenglamani yeching:

8. 1) $\sqrt{x^2-4x+9}=2x-5$;   2) $\sqrt{x^2+3x+6}$ = 3x + 8;

3) 2x = 1 + $\sqrt{x^2+5}$;   4) $x+\sqrt{13-4x}=4$.

9. 1) $\sqrt{x+12}=2+\sqrt{x}$;   2) $\sqrt{4+x}+\sqrt{x}=4$.

10. 1) $\sqrt{2x+1}+\sqrt{3x+4}=3$;   2) $\sqrt{4x-3}+\sqrt{5x+4}=4$;

3) $\sqrt{x-7}-\sqrt{x+17}=-4$;   4) $\sqrt{x+4}-\sqrt{x-1}=1$.

Tengsizliklarni yeching:

1. $\sqrt{x-4} > x-2$
2. $\sqrt{x}+\sqrt{x-2} \geq 3$
3. $\sqrt{x-2}+\sqrt{x+3} < 4$

Ko‘rsatkichli tenglama va tengsizliklar

Ko‘rsatkichli tenglamalar.

Ta’rif x noma’lum daraja ko‘rsatkichda qatnashgan tenglama ko‘rsatkichli tenglama deyiladi. $a^x = a^b$ eng sodda ko‘rsatkichli tenglama bo‘lib, bunda $a > 0$; $a \neq 1$ va x – noma’lum. Bu erda asoslari teng bo‘lgan darajalar teng bo‘lishi uchun ularning daraja ko‘rsatkichlari teng bo‘lishi yetarli, degan tasdiq o‘rinli ekanligi osongina ko‘rinadi.

Ko‘p hollarda ko‘rsatkichli tenglama $a^x = a^b$ ko‘rinishga keltiriladi. Bu yerda $a > 1$ $a \neq 1$Bu tenglama yagona yechim $x = b$ ga ega, chunki quyidagi teorema o‘rinli.

Teorema. Agar $a > 0$ va $a \neq 1$ bo‘lsa, $a^{x_1} = a^{x_2}$ tenglikdan $x_1 = x_2$ hosil bo‘ladi.

1-misol. $\sqrt{3} \cdot 3^x = 1$ tenglamani yeching.

Yechish: Tenglamani $3^{\frac{1}{2}} \cdot 3^x = 1$ yoki $3^{\frac{1}{2}+x} = 3^0$ shaklda yozamiz va $\frac{1}{2} + x = 0$ ni hosil qilib, bundan $x = -\frac{1}{2}$ ni topamiz. **Javob**: $x = -\frac{1}{2}$.

2-misol. $2^{2x} \cdot 3^x = 144$ tenglamani yeching.

Yechish: $2^{2x} = (2^2)^x = 4^x$ va $144 = 12^2$ bo‘lgani uchun, tenglamani $4^x \cdot 3^x = 12^2$ yoki $12^x = 12^2$ ko‘rinishda yozib, $x = 2$ ni topamiz. **Javob:** $x = 2$.

3-misol. $4^x = 3^{2x}$ tenglamani yeching.

Yechish: Tenglamani $4^x = 9^x$ shakldа yozamiz va $9 \neq 0$ ekanligini hisobga olib, ikkala tomonini 9^x ga bo‘lamiz:

$\frac{4^x}{9^x} = 1 \Rightarrow (\frac{4}{9})^x = \left(\frac{4}{9}\right)^0$, bundan $x = 0$ ni topamiz: **Javob:** $x = 0$

4-misol. $3^{x^2-2x-1} = 9$ tenglamani yeching.

Yechish: Bu tenglamani $3^{x^2-2x-1} = 3^2$ ko‘rinishda yozamiz. Ko‘rsat-kichlarni tenglashtirib, $x^2 – 2x - 1 = 2$ ni yoki $x^2 – 2x - 3 = 0$ ni hosil qilamiz. Buni yechib $x_1 = –1$ va $x_2 = 3$ ni topamiz. Ikkala ildiz ham tenglamani qanoatlantiradi. **Javob:** $x_1 = –1$ va $x_2 = 3$

5-misol. $2^{8-x} + 7^{3-x} = 7^{4-x} - 2^{3-x} \cdot 11$ tenglamani yeching.

Yechish: Tenglamani quyidagicha yozamiz:
$2^{8-x} - 2^{3-x} \cdot 11 = 7^{4-x} - 7^{3-x}$ bundan $2^{3-x}(2^5 - 11) = 7^{3-x}(7-1)$ yoki $2^{3-x} \cdot 21 = 7^{3-x} \cdot 6$ yoki $\left(\frac{2}{7}\right)^{3-x} = \frac{6}{21} = \frac{2}{7}$ ni hosil qilamiz va $3 - x = 1$, $x = 2$ ni topamiz. **Javob:** $x = 2$.

6-misol. $49^x - 8 \cdot 7^x + 7 = 0$ tenglamani yeching.

Yechish: Tenglamani $7^{2x} - 8 \cdot 7^x + 7 = 0$ shakldа yozib $7^x = t$ deb bel-gilaymiz va kvadrat tenglama $t^2 - 8t + 7 = 0$ ni hosil qilamiz. Uning ildizlari $t_1 = 1$ va $t_2 = 7$ bo‘ladi.
Bularni $7^x = t$ tenglikka qo‘yib, $7^x = 1 = 7^0 \Rightarrow x_1 = 0$, $7^x = 7 \Rightarrow x_2 = 1$ ni hosil qilamiz.
Javob: $x_1 = 0$, $x_2 = 1$

1 – misol. $3^x = 81$ ko‘rsatkichli tenglamani yeching. Yechish: $3^x = 3^4$ =>
J: $x = 4$
7-misol. $25^x - 6 \cdot 5^x + 5 = 0$ ko‘rsatkichli tenglamani yeching.
Yechish: $5^x = y$ deb belgilash kiritamiz, natijada kvadrat tenglamaga keltirib $y_{1,2}$ larni topamiz: $y^2 - 6 \cdot y + 5 = 0$ => y_1=1, y_2=5. Topilgan qiymatlarni o‘rniga qo‘yib, $x_{1,2}$ noma’lumlarni topamiz. $5^x = 1$ => $5^x = 5^0$ => $x_1 = 0$, $5^x = 5^1$ => **J:** $x_2 = 1$;$x_1 = 0$

8-misol. $3^{2x-1}+3^{2x}=108$

Yechish: $\frac{3^{2x}}{3}+3^{2x}=108$ => $3^{2x}\cdot\left(\frac{1}{3}+1\right)=108$ => $3^{2x}=\frac{108\cdot 3}{4}$ => $3^{2x}=3^4$ =>

2x=4 x=2 **J:** *x=2*

Ko'rsatkichli tengsizliklar.

Ko'rsatkichli tengsizliklarni yechishda ko'rsatkichli funksiya xossalaridan foydalaniladi. Ular $a^x<a^b$, $a^x>a^b(a>0\ ;\ a\neq 1)$ kabi beriladi.

Bu tengsizliklarni yechish ko'rsatkichli funksiyaning o'sish yoki kamayish xossalari, ya'ni agar $a>1$ shart bajarilsa, u holda $x_2>x_1$ bo'ladi va $a^{x_2}>a^{x_1}$ tengsizlik o'rinli bo'ladi. agar $0<a<1$ shart bajarilsa, u holda $x_2>x_1$ bo'ladi va $a^{x_2}<a^{x_1}$ tengsizlik o'rinli bo'ladi.

Ko'pgina ko'rsatkichli tengsizliklar $a^{f(x)}>a^{g(x)}\ (a>0\ ;\ a\neq 1)$ (1) ko'rinishdagi tengsizliklarni yechishga keltiriladi. (1) tengsizlikni yechish quyidagi teoremalarga asoslanadi.

Teorema. Agar *a>1* bo'lsa, u holda $a^{f(x)}>a^{g(x)}$ tengsizlik $f(x)>g(x)$ tengsizlikka teng kuchli bo'ladi.

Teorema. Agar *0<a<1* bo'lsa, u holda $a^{f(x)}>a^{g(x)}$ tengsizlik $f(x)<g(x)$ tengsizlikka teng kuchli bo'ladi.

1-misol. $2^{x^2}>4$ tengsizlikni yeching.

Yechish: Asos 2>1 bo'lgani uchun $2^{x^2}>2^2$ dan $x^2>2$ ni hosil qila-miz. Bundan esa $|x|>\sqrt{2}$ yoki $x<-\sqrt{2}$ va $x>\sqrt{2}$ hosil bo'ladi.

Javob: $x\in\left(-\infty,\sqrt{2}\right)\cup\left(\sqrt{2},\infty\right)$.

2-misol. $0{,}5^{2-3x}<42$. tengsizlikni yeching.

Yechish. Tengsizlikni $0{,}5^{2-3x}<2^2=0{,}5^{-2}$ ko'rinishida yozamiz va asos 0,5<1 ekanligini hisobga olib, $2-3x>-2$ ni hosil qilamiz. Bundan -3x>-4 yoki $x<\frac{4}{3}$ hosil bo'ladi. **Javob**: $x\in\left(-\infty,\frac{4}{3}\right)$.

3-misol. $\left(\frac{4}{3}\right)^{x+1}-\left(\frac{4}{3}\right)^{x}\geq\frac{3}{16}$ tengsizlikni yeching.

Yechish: Qavsdan $\left(\frac{4}{3}\right)^{x}$ ni chiqarib, topamiz:

$$\left(\frac{4}{3}\right)^{x}\left(\frac{4}{3}-1\right)\geq\frac{3}{16}\Rightarrow\left(\frac{4}{3}\right)^{x}\cdot\frac{1}{3}\geq\frac{3}{16}\Rightarrow\left(\frac{4}{3}\right)^{x}\geq\frac{9}{16}\Rightarrow\left(\frac{4}{3}\right)^{x}\geq\left(\frac{4}{3}\right)^{-2}$$

asos $\frac{4}{3}>1$ bo‘lgani uchun x≥-2 ni topamiz. **Yechim:** $x\in[-2,\infty)$.

4-misol. $\left(\frac{1}{36}\right)^x-5\cdot6^{-x}-6\geq0$ tengsizlikni yeching.

Yechish. Tengsizlikni $\frac{1}{6^{2x}}-\frac{5}{6^x}-6\geq0$ ko‘rinishida yozamiz va 6^x=t deb belgilab, $\frac{1}{t^2}-\frac{5}{t}-6\leq0$ ni hosil qilamiz. Ikkala tomonni $t^2=6^{6x}>0$ ga ko‘paytirib, $1-5t-6t^2\leq0$ yoki $6t2+5t-1\leq0$ ni hosil qilamiz. Bu tengsizlikni yechamiz:

$6(t+1)(t-\frac{1}{6})\leq0\Rightarrow-1\leq t\leq\frac{1}{6}$, noma’lumga o‘tsak, $-1\leq6^x\leq6^{-1}$ hosil bo‘ladi. Chap tomondagi tengsizlik barcha x uchun bajariladi, o‘ng tomon esa x≤-1 bo‘lganda bajariladi.

Javob: $x\in(-\infty,-1]$.

5-misol. $5^x-3^{x+1}>2(5^{x-1}-3^{x-2})$ tengsizlikni yeching.

Yechish: Ketma-ket quyidagicha o‘zgartirib yozamiz:

$$5^x-3\cdot3^x>2\cdot5^x\cdot5^{-1}-2\cdot3^x\cdot3^{-2}\Rightarrow5^x-\frac{2}{5}\cdot5^x>3\cdot3^x-\frac{2}{9}\cdot3^x\Rightarrow5^x\cdot\frac{3}{5}>3^x\cdot\frac{25}{9}.$$

Ikkala tomonni $\frac{5}{3}\cdot\frac{1}{3^x}$ ga ko‘paytiramiz:

$\frac{5^x}{3^x}>\frac{5}{3}*\frac{25}{9}\Rightarrow\left(\frac{5}{3}\right)^x>\left(\frac{5}{3}\right)^3\Rightarrow x>3$ hosil bo‘ladi. **Yechim:** $x\in(3,\infty)$.

6-misol. $2^{x^2+x-12}>1$ tengsizlikni yeching.

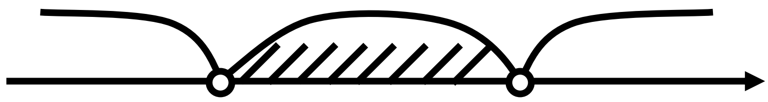
Yechish: Tengsizlikni $2^{x^2+x-12}>2^0$ ko‘rinishida yozib, $x^2+x-12>0$ ni hosil qilamiz. Bundan (x +4)(x -3)>0 va x<-4, x >3 ni topamiz. **Javob**: $x\in(-\infty,-4)\cup(3,\infty)$.

7 – misol. $5^x>125$ tengsizlikni yeching.
Yechish: $5^x>5^3$ => *x>3* **Javob:** *x>3*

8 – misol. $\left(\frac{1}{3}\right)^x>\sqrt{27}$ tensizlikni yeching.

Yechish: $\left(\frac{1}{3}\right)^x>3^{\frac{3}{2}}$ => $3^{-x}>3^{\frac{3}{2}}$ => $-x>\frac{3}{2}$ => $x<-\frac{3}{2}$ **Javob:** $x<-\frac{3}{2}$

9 – misol. $(0,(4))^{x^2-1}>(0,(6))^{x^2+6}$ tengsizlikni yeching.
Yechish: $0,(4)=\frac{4}{9}$ va $0,(6)=\frac{2}{3}$ bo‘lishini e’tiborga olib, berilgan tengsizlikning ko‘rinishini o‘zgartiramiz:$(\frac{4}{9})^{x^2-1}>(\frac{2}{3})^{x^2+6}$ => $(\frac{2}{3})^{2x^2-2}>(\frac{2}{3})^{x^2+6}$ $a=\frac{2}{3}<1$ bo‘lgani uchun $2x^2-2<x^2+6$. Bu tengsizlikni yechamiz:
$x^2<8$ => $|x|<2\sqrt{2}$ =>$-2\sqrt{2}<x<2\sqrt{2}$ x ning topilgan bu qiymatlari berilgan tengsizlikning ham yechimalri bo‘ladi.



Ko‘rsatkichli tengsizliklarni yechishning yana bir usuli $a^{2x}+a^x>b$, $a^{2x}+a^x<b$, $a^{2x}+a^x\le b$, $a^{2x}+a^x\ge b$ tengsizliklarga keltirib yechiladigan tengsizliklardir.

10 – misol. $2^{2x+2}-0{,}75\cdot 2^{x+2}>1$ tengsizlikni yeching.

Yechish: $2^{2x}\cdot 2^2-0{,}75\cdot 2^x\cdot 2^2-1<0$ => $4\cdot 2^{2x}-3\cdot 2^x-1<0$ agar $2^x=y$ deb belgilash kiritsak, $4y^2-3\cdot y-1<0$ tengsizlikni yechib, $y_1=-\frac{1}{4}$ va $y_2=1$ larni topamiz. $-\frac{1}{4}<y<1$ => $-\frac{1}{4}<2^x<1$ => $-\frac{1}{4}<2^x<2^0$ => $-\frac{1}{4}<x<0$

Javob: $-\frac{1}{4}<x<0$

Misollar

1.Tenglamalarni yeching.

1) $8^{x-1}=2$; 2) $3^{2x}=1$; 3) $0{,}5^{x^2-x}=1$;

4) $2^{2x-1}=\dfrac{1}{4}$; 5) $\left(\dfrac{1}{3}\right)^{2x+1}=9$; 6) $25^x=\sqrt{5}$;

7) $4\cdot 2^{2x}=16$; 8) $\dfrac{3^x}{27}=9$; 9) $0{,}7^{x-1}=0{,}7^{2x+3}$;

10) $\left(\dfrac{2}{9}\right)^x\left(\dfrac{9}{8}\right)^x=\dfrac{81}{256}$; 11) $\sqrt{2^x}\cdot\sqrt{3^x}=216$; 12) $5^{2x}\cdot 3+5^{2x+1}=40.$

13) $7^x+7^{x-1}=56$; 15) $2^{\frac{x+1}{2}}=3^{x+1}$;

16) $3^{x+1}+3^{x-1}=2^{x+1}+7\cdot 2^{x-1}$; 17) $3^{x+1}+3^{x-1}=7^{x+1}+5\cdot 7^x$;

18) $4^x+2^x-6=0$; 19) $100^x-11\cdot 10^x+10=0$;

2.Tengsizliklarni yeching.

1) $5^{x-1}\le\sqrt{5}$ 2) $\left(\dfrac{7}{9}\right)^{2x^2-3x}\ge\dfrac{9}{7}$ 3) $2^{x-1}+2^{x+3}>17$

4) $5^{3x+1}-5^{3x-3}\le 624$ 5) $9^x-3^x-6>0$ 6) $5^{2x+1}+4\cdot 5^x-1>0$

7) $5^{x-1} > \sqrt[3]{5}$ 8) $\left(\frac{1}{3}\right)^{x^2-3} < \left(\frac{1}{3}\right)^6$ 9) $(3)^{x^2-2} < 9$

10) $8{,}4^{\frac{x-3}{x^2+1}} < 1$ 11) $2^{x^2} \cdot 5^{x^2} < 10^{-3} \cdot \left(10^{3-x}\right)^2$

12) $\frac{4^x - 2^{x+1} + 8}{2^{1-x}} < 8^x$ 13) $7^{x-2} > 49$ 14) $3^{x-2} > 9$

15) $5^{2x} < \frac{1}{25}$ 16) $0{,}7^{x^2+2x} < 0{,}7^3$ 17) $\left(\frac{1}{3}\right)^{x^2} > \frac{1}{81}$

18) $x^2 \cdot 3^x - 3^{x+1} \le 0$ 19) $3^{8x} - 4 \cdot 3^{4x} \le -3$ 20) $\left(\sqrt{6}\right)^x \le \frac{1}{36}$

Logarifmlar va ularning asosiy xossalari

Quyidagi misollarni ko‘ramiz:

1. $2^x=4$ ni yechish uchun $2^x=2^2$ deb yozamiz va $x=2$ yechimni topamiz.

2. $2^x=5$ bo‘lsin. o‘ng tomondagi 5 ni asosi 2 bo‘lgan daraja ko‘rini-shida tasvirlash mushkul. Lekin bu tenglamaning haqiqiy ildizi mavjud-ligi bizga ma’lum. Bunday tenglamalarni yechish uchun logarifm tu-shunchasi kiritiladi.

Umuman olganda, $a^x=b$ $(a>0,\ a\neq 1,\ b>0)$ tenglamaning ildizi a asosga ko‘ra b sonning logarifmi deyiladi.

Ta’rif. b sonning a asosga ko‘ra logarifmi deb b sonni hosil qilish uchun a sonni ko‘tarish kerak bo‘ladigan daraja ko‘rsatkichiga aytiladi va $log_a b$ kabi belgilanadi. $a^x=b$ tenglamani ($x=log_a b$ bo‘lgani uchun)

$$a^{\log_a b} = b \qquad (1)$$

ko‘rinishida yozish mumkin. (1) formula asosiy logarifmik ayniyat deyi-ladi, bu yerda

$a>0$ $a\neq 1$ va $b>0$

Misollar: 1) $log_2 16$ 2) $log_5 0{,}04$ ning qiymatini toping.

Yechish: 1) $16=2^4$ bo‘lgani uchun, 16 ni hosil qilish uchun ikkini to‘rtinchi darajaga ko‘tarish kerak, demak $log_2 16=4$.

2) $0{,}04 = \frac{4}{100} = \frac{1}{25} = 5^{-2}$ ekanligi ma’lum. Shuning uchun $log_5 0,\ 04=-2$

Misollar: 3) $\log_4 x = \frac{1}{2}$, 4) $\log_x 4 = -\frac{3}{4}$ tenglamalarni qanoatlantiruv-chi x larni topamiz.

Yechish: Asosiy logarifmik ayniyatdan foydalanib:

3) $x=4^{\frac{1}{2}}=2$ 4) $x^{\log_x 4}=4$, ya'ni $x^{-\frac{3}{4}}=4,\ x=4^{-\frac{4}{3}}=\frac{1}{\sqrt[3]{256}}$ larni topamiz.

Har qanday *a>0, b>0, a≠1, b≠1, x>0, y>0* va haqiqiy istalgan *n* va *m* sonlar uchun quyidagi tengliklar bajariladi:

1. $a^{\log_a x}=x$
2. $\log_a a=1 \qquad \log_a 1=0$
3. $\log_a(xy)=\log_a x+\log_a y \qquad x,y>0$
4. $\log_a \frac{x}{y}=\log_a x-\log_a y \qquad x,y>0$
5. $\log_a x^p=p\log_a x \qquad x>0$
6. $\log_a x=\frac{\log_b x}{\log_b a}$
7. $\log_a b\cdot\log_b a=1 \qquad \log_a b=\frac{1}{\log_b a}$
8. $\log_a b=\log_{a^p}(b^p),\ \ p\neq 0 \qquad \log_{a^n}(b^m)=\frac{m}{n}\log_a b$
9. $a^{\log_b c}=c^{\log_b a}$
10. $a^{\sqrt{\log_a b}}=b^{\sqrt{\log_b a}}$

Bu tengliklar ko'rsatkichli funksiya xossalaridan kelib chiqadi. Bulardan ba'zilarini isbot qilamiz.

Logarifmik ayniyatdan foydalanib $x=a^{\log_a x}$, $\quad y=a^{\log_a y}$ ni topamiz.

Bu tengliklarni hadlab ko'paytirsak yoki bo'lsak

$xy=a^{\log_a x}*a^{\log_a y}=a^{\log_a x+\log_a y}$, $\frac{x}{y}=a^{\log_a x}:a^{\log_a y}=a^{\log_a x-\log_a y}$, hosil bo'ladi.

Bu tengliklardan logarifm ta'rifiga ko'ra 3) va 4) tengliklar kelib chiqadi.

$x=a^{\log_a x}$ ayniyatning ikkala tomonini n – darajaga oshirsak, $x^n=a^{n\log_a x}$ hosil bo'lib, bundan $\log_a x^n=n\log_a x$ ni topamiz.

Bir asosli logarifmdan boshqa asosli logarifmga o'tish formulasi 8) ni xususiy holda 9) ni isbotlash uchun quyidagicha amal qilamiz:

$$\log_a x=b\Rightarrow x=a^b$$

Hosil bo'lgan $x=a^b$ ifodaning ikkala tomonidan *b* asosga ko'ra logarifm topamiz:

$$\log_b x=\log_b a^b=b\log_b a\Rightarrow b=\frac{\log_b x}{\log_b a}$$

Chap tomonga *b* ning qiymatini qo'yib, 8) formulani hosil qilamiz. Agar bu formuladan *x=b* desak, 9) formula hosil bo'ladi.

5-misol. Agar $\log_2 5 = a$ va $\log_2 3 = b$ bo‘lsa, $\log_2 3000$ ni *a* va *b* orqali ifodalang?

Yechish: $\log_2 3000 = \log_2(3\cdot 5^3\cdot 2^3) = \log_2 3 + 3\log_2 5 + 3\log_2 2 = b + 3a + 3$

O‘nli va natural logarifmlar

1-ta’rif. Asosi *a=10* bo‘lgan logarifmlar o‘nli logarifmlar deyiladi va *lgx* orqali ifodalanadi, ya’ni $log_{10}x=lgx$

7-misol. $lg100=lg10^2=2$

8: $lg0,01=lg10^{-2}=-2$

2-ta’rif. Natural logarifm deb asosi *e* son bo‘lgan logarifmga aytiladi va *lnx* bilan belgilanadi, ya’ni $log_e x=lnx$, *e* soni irratsional son bo‘lib, *e=2,7182818284…* amalda *e≈2,7* deb qabul qilish mumkin.

O‘nli va natural logarifmlar orasida

$\lg x = \frac{1}{\ln 10}\cdot \ln x \approx 0,434294\ln x$ va

$\ln x = \frac{1}{\lg e}\cdot \lg x \approx 2,302551\lg x$ bog‘lanish mavjud. Amalda $\lg x \approx 0,4\ln x$ va $\ln x \approx 2,3\lg x$ tengliklardan foydalanish mumkin.

9-misol. *ln100*, lge^2 ni hisoblang.

Yechish: $\ln 100 \approx 2,3\cdot \lg 100 = 2,3\cdot 2 = 4,6.$

$\lg e^2 = 2\lg e \approx 2\cdot 0,4\ln e = 0,8.$

Misollar

1.Hisoblang

1) $\log_2 16$ 2) $\log_2 \frac{1}{2}$ 3) $\log_3 27$ 4) $\log_3 1$

5) $\log_{\frac{1}{2}} \frac{1}{32}$ 6) $\log_{\frac{1}{2}} \sqrt[3]{2}$ 7) $\log_{0.5} 0.125$ 8) $\log_5 625$

9) $\log_5 \frac{1}{125}$ 10) $\log_{\frac{1}{5}} 125$ 11) $\log_{\frac{1}{4}} \frac{1}{64}$ 12) $3^{\log_3 18}$

13) $\left(\frac{1}{4}\right)^{\log_{\frac{1}{4}} 6}$ 14) $3^{5\log_3 2}$ 15) $\left(\frac{1}{2}\right)^{6\log_{\frac{1}{2}} 2}$ 16) $\log_2 \sqrt[4]{2}$

17) $\log_3 \frac{1}{3\sqrt{3}}$ 18) $\left(\frac{1}{4}\right)^{-5\log_2 3}$ 19) $10^{3-\log_{10} 5}$

20) $\log_2 \log_3 81$ 21) $\log_4 \log_{16} 256 + \log_4 \sqrt{2}$

22) $\log_{10} 5 + \log_{10} 2$ 23) $\log_3 6 + \log_3 \frac{3}{2}$

24) $\log_2 15 - \log_2 \frac{15}{16}$ 25) $\log_{\frac{1}{3}} 54 - \log_{\frac{1}{3}} 2$

26) $\log_{13} \sqrt[5]{169}$ 27) $\log_2 \frac{1}{\sqrt[6]{128}}$

28) $\log_8 12 - \log_8 15 + \log_8 20$ 29) $\frac{1}{2}\log_7 36 - \log_7 14 - 3\log_7 \sqrt[3]{21}$

30) $\frac{\log_3 8}{\log_3 16}$ 31) $\frac{\log_7 8}{\log_7 15 - \log_7 30}$

32) $\frac{\log_2 24 - \frac{1}{2}\log_2 72}{\log_3 18 - \frac{1}{3}\log_3 72}$ 33) $\frac{\log_2 4 + \log_2 \sqrt{10}}{\log_2 20 + 3\log_2 2}$

2. 1) Agar $\log_4 125 = a$ bo\` lsa , $\lg 64$ ni a orqali ifodalang .

2) Agar $\log_{50} 40 = a$ bo\` lsa , $\log_5 2$ ni a orqali ifodalang .

3) Agar $\log_{98} 56 = a$ bo\` lsa , $\log_7 2$ ni a orqali ifodalang .

4) Agar $\log_{147} 63 = a$ bo\` lsa , $\log_7 3$ ni a orqali ifodalang .

Logarifmik tenglama va tengsizliklar

Logarifmik tenglamalar

Logarifmik tenglama ma’lum almashtirishlardan keyin $\log_a x = \log_a b$ (1) yoki $\log_a x = b$ (2) ko‘rinishga keltiriladi. (1) dan $x=b$ va (2) dan $x=a^b$ yechimni topamiz.

1-misol. $\log_2(x^2+5x+2)=3$ tenglamani yeching.

Yechish: Berilgan tenglama x ning $x^2+5x+2=2^3$ tenglik bajarila-digan qiymatlardagina qanoatlantiradi. Bundan $x^2+5x-6=0$ kvadrat teng-lamaga ega bo‘lib, $x_1=1,\ x_2=-6$ yechimni topamiz.

2-misol. $\log_3(2x+3)=\log_3(x+1)$ tenglamani yeching.

Yechish: Bu tenglama x ning $2x+3>0$ va $x+1>0$ shartlarni qanoat-lantiruvchi qiymatlari uchun aniqlangan. Bu tengsizliklarni yechib teng-lamaning mavjudlik sohasi $x\in(-1,\infty)$ ni aniqlaymiz. Berilgan tenglama $2x+3=x+1$ tenglamaga teng kuchlidir. Bundan $x=-2$ ni topamiz. Ammo bu ildiz tenglamaning mavjudlik sohasiga kirmaydi. Binobarin, berilgan tenglamaning ildizlari mavjud emas.

3-misol. $\log_x(x^2-3x+3)=1$ tenglamani yeching.

Yechish: bu tenglama x ning $x>0,\ x\neq 1$ (x- logarifmning asosi bo‘l-gani uchun) shartlar va $x^2-3x+3=x$ yoki $x^2-4x+3=0$ tenglik bajariladigan qiymatlardagina qanoatlantiriladi. Hosil bo‘lgan kvadrat tenglamaning ildizlari 1 va 3 bo‘lib, $x=1$ berilgan tenglamaning yechimi bo‘la olmaydi. Demak, berilgan tenglamaning ildizi faqat $x=3$.

4-misol. $\log_5 x-6\log_x 5=1$ tenglamani yeching.

Yechish: Bu tenglamaning mavjudlik sohasi $x\in(0,1)\cup(1,\infty)$ bo‘ladi. x asosli logarifmdan 5 asosli logarifmga o‘tib, $\log_5 x-\dfrac{6}{\log_5 x}-1=0$ ni, bun-dan $\log_5^2 x-\log_5 x-6=0$ ni hosil qilamiz. Bu kvadrat tenglamani noma’lum $\log_5 x$ ga nisbatan yechib, $\log_5 x_1=3$ va $\log_5 x_2=-2$ ni topamiz. Bu tengla-malardan $x_1=5^3=125$ va $x_2=5^{-2}=\dfrac{1}{25}$ larni topamiz. Bu ildizlarning ikka-lasi ham tenglamani qanoatlantiradi.

Tenglamalarni yeching.

1) $\lg(x-1)-\lg(2x-11)=\lg 2$

2) $\log_3(x^3-x)-\log_3 x=\log_3 3$

3) $\dfrac{1}{2}\lg(x^2+x-5)=\lg 5x+\lg\dfrac{1}{5x}$

4) $\dfrac{1}{2}\lg(x^2-4x-1)=\lg 8x-\lg 4x$

5) $\log_2(x-5)+\log_2(x+2)=3$

6) $\lg(x+\sqrt{3})+\lg(x-\sqrt{3})=0$

7) $\log_3(5x+3)=\log_3(7x+5)$

8) $\log_{\frac{1}{2}}(3x-1)=\log_{\frac{1}{2}}(6x+8)$

9) $\log_{\frac{1}{3}} x \log_{\frac{1}{3}}(3x-2) = \log_{\frac{1}{3}}(3x-2)$

10) $\log_{\sqrt{3}}(x-2)\log_5 x = 2\log_3(x-2)$

11) $2^{3\lg x} \cdot 5^{\lg x} = 1600$

12) $\frac{1}{5-\lg x} + \frac{2}{1+\lg x} = 1$

13) $\log_2 x + \log_x 2 = 2{,}5$

14) $\log_3 x - 6\log_x 3 = 1$

Logarifmik tengsizliklar

1. Agar $0 < a < 1$ bo`lsa,

$$\log_a f(x) > \log_a g(x) \quad \Leftrightarrow \quad \begin{cases} f(x) < g(x) \\ f(x) > 0 \\ g(x) > 0 \end{cases}$$

2. Agar $a > 1$ bo`lsa,

$$\log_a f(x) > \log_a g(x) \quad \Leftrightarrow \quad \begin{cases} f(x) > g(x) \\ f(x) > 0 \\ g(x) > 0 \end{cases}$$

Logarifmik tengsizlik lozim bo‘lgan almashtirishlar bajarilgandan keyin

$\log_a x \geq b$ $\quad (\log_a x \leq b)$ $\quad$ yoki

$\log_a x \geq \log_a b$ $\quad (\log_a x \leq \log_a b)$

ko‘rinishiga keladi.

Yechim: $\log_a x \geq b \Rightarrow \begin{cases} x \geq a^b, & \text{agar} \quad a > 1 \quad \text{bo'lsa,} \\ x \leq a^b, & \text{agar} \quad 0 < a < 1 \quad \text{bo'lsa.} \end{cases}$

$\log_a x \geq \log_a b \Rightarrow \begin{cases} x \geq b, agar\, a > 1 bo'lsa, \\ x \leq b, agar\, 0 < a < 1 bo'lsa, \end{cases}$ bo‘ladi.

6-misol. *lg(x+2)<1* tengsizlikni yeching.

Yechish: Tengsizlikning mavjudlik sohasi *x+2>0*, yechimi esa *x+2<10* bo‘lib tengsizlik yechimini topish uchun

$\begin{cases} x+2 > 0 \\ x+2 < 10 \end{cases}$ tengsizliklar sistemasiga ega bo‘lamiz,

Uni yechib $\begin{cases} x > -2 \\ x < 8 \end{cases}$ ni yoki $x \in (-2{,}8)$ ni hosil qilamiz.

Yechim: $x \in (-2, 8)$.

7-misol. $\log_{\frac{1}{3}}(2x-4) > \log_{\frac{1}{3}}(x+1)$ tengsizlikni yeching.

Yechish: Mavjudlik sohasi uchun *2x-4>0, x+1>0,* tengsizlikning bajarilishi uchun *2x-4<x+1* (asos $\frac{1}{3} < 1$ bo‘lgani uchun tengsizlik ishorasi teskarisiga o‘zgaradi) tengsizliklarga, ya’ni $\begin{cases} 2x-4>0 \\ x+1>0 \\ 2x-4<x+1 \end{cases}$ sistemaga ega bo‘lamiz. Bundan $\begin{cases} x>2 \\ x>-1 \\ x<5 \end{cases}$ ni hosil qilamiz, demak yechim $x \in (2,5)$ bo‘ladi.

8-misol. $\log_{\frac{1}{3}}(x-2) + \log_{\frac{1}{3}}(12-x) \geq -2$ tengsizlikni yeching.

Yechish: Tengsizlikning mavjudlik sohasi

$\begin{cases} x-2>0 \\ 12-x>0 \end{cases}$ yoki $\begin{cases} x>2 \\ x<12 \end{cases}$ dan iborat.

Tengsizlikni $\log_{\frac{1}{3}}(x-2)(12-x) \geq \log_{\frac{1}{3}}(\frac{1}{3})^{-2}$ teng kuchli tengsizlik bilan almashtirib,

$\begin{cases} (x-2)(12-x) \leq 3^2 \\ 2<x<12 \end{cases}$ sistemaga ega bo‘lamiz.

Sistemani yechib, $\begin{cases} x \leq 3, x \geq 11 \\ 2<x<12 \end{cases}$ ni topamiz. **Javob:** $x \in (2,3] \cup [11,12)$

Tengsizliklarni yeching.

1) $\log_3(x+2) < 3$

2) $\log_3(x+1) < -2$

3) $\log_{\frac{1}{5}}(4-3x) \geq -1$

4) $\lg x > 2 - \lg 4$

5) $\log_{\frac{1}{3}}(x-2) + \log_{\frac{1}{3}}(12-x) \geq -2$

6) $\log_5 \frac{3x-2}{x^2+1} > 0$

7) $\lg(3x-4) < \lg(2x+1)$

8) $\log_6(x^2-3x+2) \geq 1$

9) $\log_{\frac{2}{3}}(x^2-2.5x+7) < -1$

10) $\lg(x^2-8x+13) > 0$

11) $\log_2(x^2+2x) < 3$

12) $\log_{0.2} x - \log_5(x-2) < \log_{0.2} 3$

13) $\lg x - \log_{0.1}(x-1) > \log_{0.1} 0.5$

14) $\log_{0.2}^2 x - 5\log_{0.2} x < -6$

15) $\log_{0.1}^2 x + \log_{0.1} x > 4$

16) $\log_3(2-3^{-x}) < x+1-\log_3 4$

17) $\log_{\frac{x-1}{5x-6}}(\sqrt{6}-2x) < 0$

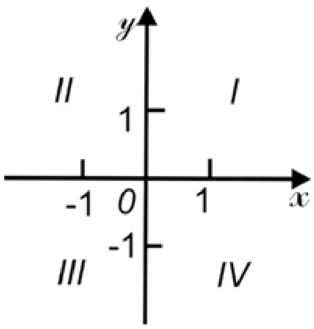
18) $\frac{1}{5-\lg x} + \frac{2}{1+\lg x} < 1$

19) $\log_{\frac{1}{5}}(3x-5) > \log_{\frac{1}{5}}(x+1)$

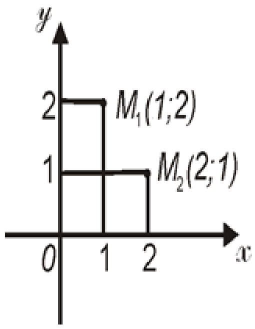
20) $\log_{x^2-3}(4x+7) > 0$

Tekislikda to`g`ri burchakli koordinatalar sistemasi

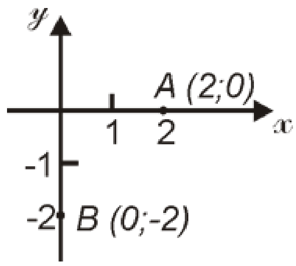
Tekislikda ikkita o`zaro perpendikular to`g`ri chiziq o`tkazamiz: biri — gorizontal, ikkinchisi — vertikal (1- rasm).



1 -rasm.



2 -rasm.



3 -rasm.

Ularning kesishish nuqtasini *O* harfi bilan belgilaymiz. Shu to`g`ri chiziqlarda yo`nalishlar tanlaymiz: gorizontal to`g`ri chiziqda chapdan o`ngga, vertikal to`g`ri chiziqda pastdan yuqoriga. Har bir to`g`ri chiziqda bir xil uzunlik birligini ajratamiz.
Gorizontal to`g`ri chiziq *Ox* bilan belgilanadi va abssissalar o`qi deyiladi;
Vertikal to`g`ri chiziq *Oy* bilan belgilanadi va ordinatalar o`qi deyiladi.
Abssissalar o`qini va ordinatalar o`qini koordinata o`qlari, ularning kesishish nuqtasini koordinatalar boshi deyiladi. Koordinatalar boshi har bir o`qdagi nol sonini tasvirlaydi.
Abssissalar o`qida musbat sonlar O nuqtadan o`ngda joylashgan nuqtalar bilan, manfiy sonlar esa O nuqtadan chapda joylashgan nuqtalar bilan tasvirlanadi. Ordinatalar o`qida musbat sonlar O nuqtadan yuqorida joylashgan nuqtalar bilan, manfiy sonlar esa O nuqtadan pastda joylashgan nuqtalar bilan tasvirlanadi.
Yo`nalishlar va uzunlik birligi tanlangan ikkita o`zaro perpendikular to`g`ri chiziq tekislikda to`g`ri burchakli koordinatalar sistemasini hosil qiladi. Koordinatalar sistemasi tanlangan tekislik koordinata tekisligi deyiladi. Koordinata o`qlari tashkil qilgan to`g`ri burchaklar koordinata burchaklari (kvadrantlar) deyiladi va 1- rasmda ko`rsatilgan tartibda raqamlanadi.

M nuqtaning abssissasi va ordinatasi M nuqtaning koordinatalari deyiladi. M(x;y) yozuvi *M* nuqta *x* abssissaga va *y* ordinataga ega ekanini bildiradi. Bu holda M nuqta *(x; y)* koordinatalarga ega deb ham aytiladi.

Masalan, $M_1(1; 2)$ nuqtaning abssissasi 1 ga, ordinatasi 2 ga teng. (2-rasm)
$M_2(2; 1)$ nuqtaning abssissasi 2 ga, ordinatasi 1 ga teng. (2-rasm)

Agar nuqta *Ox* o`qida (abssissalar o`qida) yotsa, u holda uning ordinatasi nolga ($y = 0$) teng bo`ladi. (3-rasm)
Masalan, $A(2; 0)$ nuqta $x = 2$; $y = 0$ koordinatalarga ega. (Demak nuqta abssissalar o`qida yotadi.)

Agar nuqta *Oy* o`qida (ordinatalar o`qida) yotsa, u holda uning abssissasi nolga ($x = 0$) *teng bo`ladi. (3-rasm)*
Masalan, $B(0; -2)$ nuqta $x = 0$; $y = -2$ koordinatalarga ega. (Demak nuqta ordinatalar o`qida yotadi.)

Koordinatalar boshining abssissasi va ordinatasi nolga teng: O (0; 0).
($x = 0$; $y = 0$ koordinatalarga ega.)

Ikkita A va B nuqtalar orasidagi masofa deb, $\overrightarrow{AB}$ vektor moduliga $|\overrightarrow{AB}|$ aytiladi va $\rho(A,\ B) = AB = |\overrightarrow{AB}|$ ko'rinishida yoziladi.

$$\rho(A,\ B) = |\overrightarrow{AB}| = \sqrt{\overrightarrow{AB}^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad (1)$$

Shunday qilib A va B nuqtalar orasidagi masofa yuqoridagi formula bilan hisoblanadi.

Masala. $A(-1, 0)$ va $B(2, 3)$ nuqtalar orasidagi masofani hisoblang.

Yechish. (1) formuladan topamiz.

$$\overrightarrow{AB} = |\overrightarrow{AB}| = \sqrt{(2-(-1))^2 + (3-0)^2} = \sqrt{9+9} = 3\sqrt{2}\,.$$

Masala. Uchburchak uchlarining koordinatalari to'g'ri burchakli dekart koordinatalar sistemasida *A(3, 2), B(6, 5), C(1, 10)* berilgan. Uchburchakning to'g'ri burchakli uchburchak ekanligini isbotlang.

Yechish. Uchburchak tomonlarini topamiz.

$$|\overrightarrow{AB}| = \sqrt{(6-3)^2 + (5-2)^2} = \sqrt{3^2 + 3^2} = 3\sqrt{2}$$
$$|\overrightarrow{BC}| = \sqrt{(1-6)^2 + (10-5)^2} = \sqrt{5^2 + 5^2} = 5\sqrt{2}$$
$$|\overrightarrow{AC}| = \sqrt{(1-3)^2 + (10-2)^2} = \sqrt{(-2)^2 + 9^2} = 2\sqrt{17}$$

$\overrightarrow{AB}^2 + \overrightarrow{BC}^2 = \overrightarrow{AC}^2$ ikkinchi tomondan

$\overrightarrow{AB}(3,3), \overrightarrow{BC}(-5,5)$ $\overrightarrow{AB} \cdot \overrightarrow{BC} = -15 + 15 = 0, \Rightarrow \angle B = 90^0$.

Masalalar

1. Nuqtalarni yasang:
1) A (3; 4), B (2; -5), C (-2; 5), E (-6; -2), F (3; -0,5), K (3; 0),
2) A (-1,5; 2,5), B (-2,5; 1,5), F (2; -2), M (0; 2,5). N (-3,5; 3,5)

2. Quyidagi nuqtalardan o`tuvchi to`g`ri chiziqni yasang:
1) A (3; -2) va B (-2; 2); 2) M (2; 0) va N (0; -2).

3. Oxirlarining koordinatalari: 1) A (3; 4), B (-6; 5); 2) M (0; -5), N (4; 0) bo`lgan kesmani yasang.

4. Uchlarining koordinatalari: 1) K (-2; 2), M (3; 2), N (-1; 0); 2) A (0; -1), B (0; 5), C (4; 0) bo`lgan uchburchakni yasang.

5. Uchlarining koordinatalari: A (-2; 0), B (-2; 3), C (0; 3), O (0; 0) bo`lgan to`g`ri to`rtburchakni yasang.

6. Kvadratning uchta uchi berilgan: A (1; 2), B (4; 2), C (4; 5). ABCD kvadratni yasang. D uchining koordinatalarini toping.

7. A (5; 4), B (2; -1), C (-3; 2), D (- 4; - 4) nuqtalarga *Ox* o`qiga nisbatan simmetrik bo`lgan nuqtalarni yasang va ularning koordinatalarini aniqlang.

8. A (2; - 2), B (1; 1), C (-3; 2), nuqtalarga Oy o`qiga nisbatan simmetrik bo`lgan nuqtalarni yasang va ularning koordinatalarini aniqlang.

9. A (4; - 3), B (-1; 2), C (3; -2), nuqtalarga koordinatalar boshiga nisbatan simmetrik bo`lgan nuqtalarni yasang va ularning koordinatalarini aniqlang.

Chiziqli funksiya va uning grafigini yasash

Chiziqli funksiya deb $y = kx + b$ ko`rinishidagi funksiyaga aytiladi, bu yerda $k$ va $b$ — berilgan sonlar. $b = 0$ bo`lganda, chiziqli funksiya $y = kx$ ko`rinishga ega bo`ladi va uning grafigi koordinatalar boshidan o`tuvchi to`g`ri chiziq bo`ladi. Bu dalilga asoslanib, $y = kx+ b$ chiziqli funksiyaning grafigi to`g`ri chiziq bo`lishini ko`rsatish mumkin. Ikki nuqta orqali birgina to`g`ri chiziq o`tganligi sababli, $y = kx+b$ funksiyaning grafigini yasash uchun shu grafikning ikki nuqtasini yasash yetarli, so`ngra esa shu nuqtalar orqali chizg`ich yordamida to`g`ri chiziq o`tkaziladi.

Funksiyaning grafigini yasang:
1. 1) $y = 3x$; 2) $y = 5x$; 3) $y = - 4x$; 4) $y = - 0{,}8x$.
2. 1) $y = 1{,}5x$; 2) $y = -2{,}5x$; 3) $y = - 0{,}2x$; 4) $y = 0{,}4x$.

3. 1) $y = 2\frac{1}{2}x$; 2) $y = \frac{1}{4}x$; 3) $y = 0{,}6x$; 4) $y = -\frac{5}{3}x$.

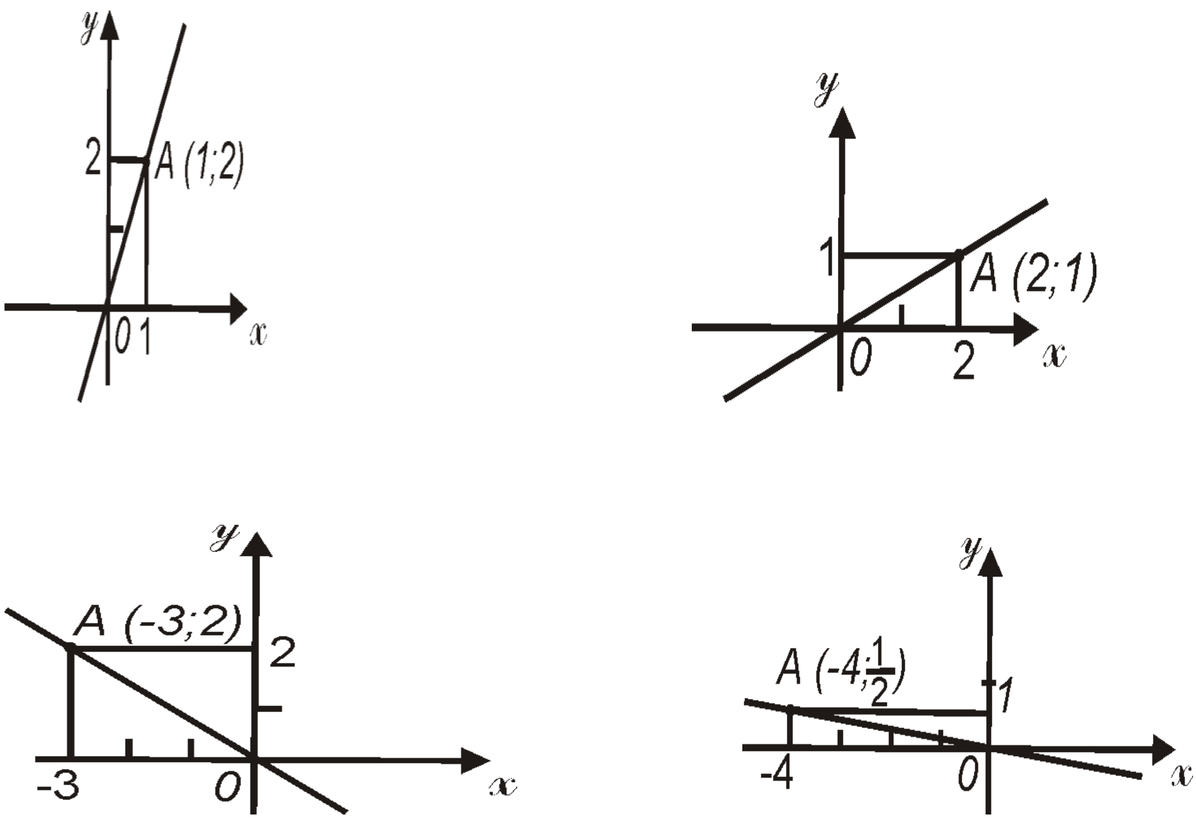
4. Funksiyaning grafigini yasang va shu grafik qaysi koordinata choraklarida joylashganligini ko`rsating:

1) $y = \frac{1}{3}x$; 2) $y = -\frac{1}{3}x$; 3) y = 4,5x; 4) y = - 4,5x.

5. Funksiyaning grafigini yasang:

1) y = 3,5x; 2) $y = -\frac{2}{5}x$.

6. Grafigi rasmdagi to`g`ri chiziq bilan tasvirlangan funksiyani formula bilan yozing:



7. Agar B nuqta *y = kx* funksiyaning grafigiga tegishli ekanligi ma`lum bo`lsa, shu funksiyaning grafigini yasang: 1) *B* (2; -3); 2) $B\left(3\frac{1}{3};-2\right)$

8. Funksiyaning grafigini yasang:

1) $y = 2x + 1$; 3) $y = -2x + 1$; 5) $y = 3x - 4$;

2) $y = 0{,}5x - 1$; 4) $y = \frac{1}{4}x - 2$; 6) $y = \frac{1}{2}x + 2$.

9. Grafikning koordinata o`qlari bilan kesishish nuqtalarining koordinatalarini toping:

1) $y = -1{,}5x + 3$; 3) $y = -2x + 4$; 5) $y = -1{,}5x - 6$;

2) $y = 0{,}8x - 0{,}6$; 4) $y = -\frac{1}{4}x + 2$; 6) $y = \frac{2}{3}x - 5$.

10. Funksiyaning grafigini uning koordinata o`qlari bilan kesishish nuqtalarini topib, yasang:

1) $y = 2x + 2;$ 3) $y = -\frac{1}{2}x - 1;$ 5) $y = 4x + 8;$

2) $y = -3x + 6;$ 4) $y = 2{,}5x + 5;$ 6) $y = -6x - 2.$

11. Funksiyaning grafigini yasang:

1) $y = 7;$ 2) $y = -3{,}5;$ 3) $y = \frac{1}{4};$ 4) $y = 0.$

12. Chiziqli funksiya $y = x + 2$ formula bilan berilgan. Shu funksiyaning grafigiga nuqtalardan qaysilari *M(0;2), A(-1,-1), C(-2.5,0.5)*

13. $y = kx + 2$ funksiyaning grafigi: $P(-7;\ -12)$ nuqtadan o`tishi ma`lum bo`lsa, k ning qiymatini toping.

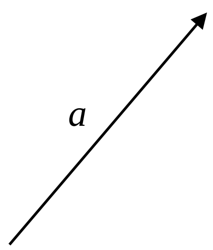
14. $y = -3x + b$ funksiyaning grafigi $N(5;\ 2)$ nuqtadan o`tishi ma'lum bo`lsa, b ning qiymatini toping.

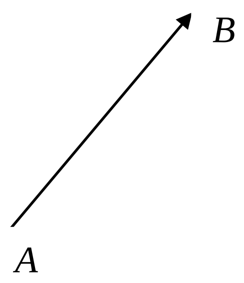
15. Agar $y = kx + 1$ funksiya grafigiga $M(2;\ -7)$ nuqta tegishli ekanligi ma'lum bo`lsa, shu funksiyaning grafigini yasang.

16) $y = kx + b$ funksiyaning grafigi $M(0;\ -1)$ ва $N(1;\ -5)$ nuqtalardan o`tadi. k va b ning qiymatini toping.

Vektorlar va ular ustida amallar

Matematika va fizika fanlarida ko'pincha ikki turli miqdor bilan ish ko'rishga to'g'ri keladi, ulardan biri faqat son qiymati bilan aniqlangani holda, ikkinchisi o'zining son qiymatidan fazodagi yo'nalishi bilan aniqlanadi. Masalan, uzunlik, hajm, temperatura, potensial energiya kabi miqdorlar faqat o'zlarining son qiymatlari bilangina aniqlanadi va bunday miqdorlar, ya'ni, faqat o'zining son qiymati bilan aniqlangan miqdorlar skalyar deyiladi.Lekin, kuch, tezlik, tezlanish kabi miqdorlarni aniqlash uchun fazodagi yo'nalishlari ham ma'lum bo'lishi lozim. Bunday miqdorlarni ya'ni o'zining son qiymatidan boshqa yana fazodagi yo'nalishi bilan aniqlanadigan miqdorlar vektorial miqdorlar yoki qisqacha vektorlar deyiladi. Demak, yo'nalishga ega bo'lgan kesma vektor deyiladi.

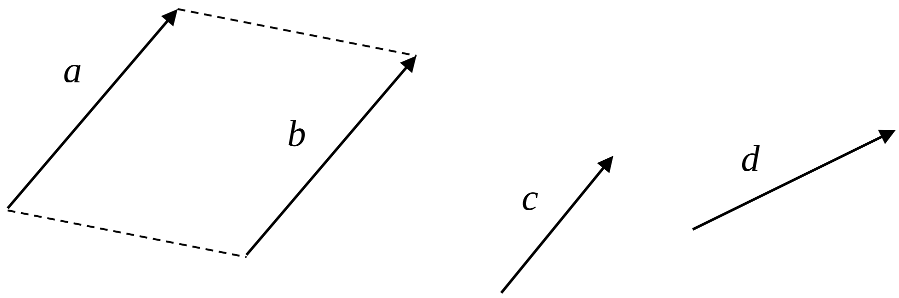




Vektor odatda strelka yordami bilan tasvir qilinadi. Strelkaning yo’nalishi vektorning yo’nalishini va uning uzunligi vektorning uzunligini ko’rsatadi.Vektorni belgilash uchun kichik lotin harflari *a,b,c…*dan foydalanamiz.Yoki, ustiga strelka qo’yilgan ikkita bosh harf bilan belgilanadi, masalan, $\overrightarrow{AB}$. Bu holda *A*-vektorning boshlang’ich nuqtasi va *B*-uning oxirgi nuqtasi yoki uchi bo’lib, vektorning yo’nalishi *A* dan *B* ga tomon yo’nalgan.

Vektorning moduli deb, shu vektorni tasvirlovchi kesmaning uzunligiga aytiladi. $\vec{a}$ vektorning moduli $|\vec{a}|$ kabi belgilanadi.

Har bir vektor o’zining son qiymati va yo’nalishi bilan aniqlangani uchun, uzunliklari teng va yo’nalishlari bir xil bo’lgan ikki vektor o’zaro teng deyiladi



Ikki vektorning tengligi to’g’risida berilgan ta’rifga qaraganda, vektorning boshlang’ich nuqtasining o’rni rol o’ynamaydi. Masalan, chizmada $\vec{a}$ va $\vec{b}$ vektorlar o’zaro teng, chunki ularning uzunliklari teng va yo’nalishlari bir xil. Bu holda algebradagi kabi

$$\vec{a}=\vec{b}$$

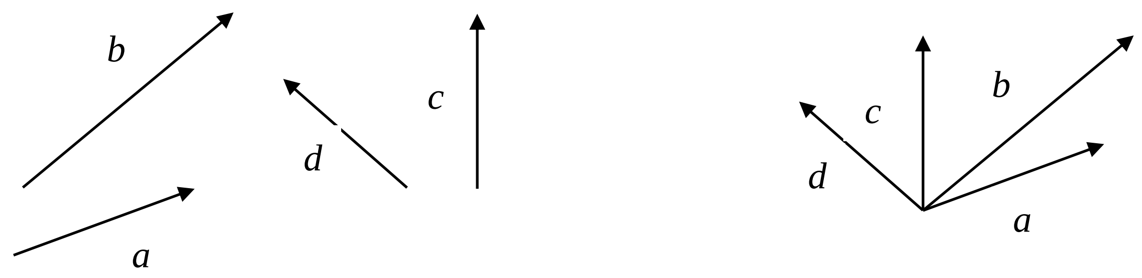
yoziladi. $\vec{b}$ va $\vec{c}$ vektorlarning yo’nalishlari bir xil bo’lsada, lekin uzunliklari teng emas, demak, $\vec{b}$ vektor $\vec{c}$ vektorga teng emas

$$\vec{b} \neq \vec{c}$$

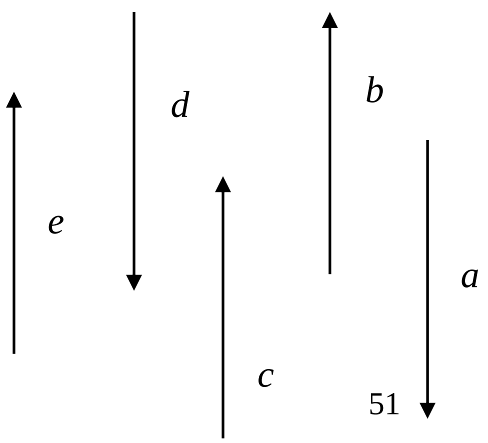
Shunga o'xshash $\vec{c}$ va $\vec{d}$ vektorlar ham o'zaro teng emas, chunki ularning uzunliklari teng bo'lsada, lekin yo'nalishlari har xil.

Kollinear vektorlar.

Faraz qilaylik, bir necha vektor, masalan, $\vec{a}$, $\vec{b}$, $\vec{c}$ va $\vec{d}$ berilgan bo'lsin. Ikki vektorning tengligi to'g'risida berilgan ta'rifga muvofiq vektorning boshlang'ich nuqtasi rol o'ynamagan edi. Bunga asoslanib, istalgan biror O nuqtada haligi $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$ vektorlarga teng bo'lgan vektorlarni yasash mumkin, yoki boshqacha qilib aytganda, berilgan vektorlarni bir boshlang'ich nuqtaga ko'chirish mumkin. Bunday yasash 1.2.3-chizmada bajarilgan, berilgan O nuqtaga $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$ vektorlarga teng qilib vektorlar yasalgan.



Agarda shuning kabi bir necha vektorni bir boshlang'ich nuqtaga ko'chirganda, ular bir to'g'ri chiziqda yotsa, bunday vektorlar kollinear vektorlar deyiladi.



Kollinear bo’lgan $\vec{a}$ va $\vec{b}$ vektorlar $\vec{a} \parallel \vec{b}$ ravishda ifoda qilinadi. Masalan, 1.2.2-chizmadagi $\vec{a}, \vec{b}, \vec{c}$ vektorlar kollinear vektorlardir. Shunga o’xshash 1.2.4-chizmadagi $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ va $\vec{e}$ vektorlar ham kollinear vektordan iborat, chunki ularni bir boshlang’ich nuqtaga keltirganda, ular bir to’g’ri chiziqda yotadi.

Vektorning koordinatalari

$A_1(x_1, y_1)$ nuqta $\vec{a}$ vektorning boshi, $A_2(x_2, y_2)$ nuqta esa uning oxiri bo’lsin. $a_1 = x_2 - x_1,\ a_2 = y_2 - y_1$ sonlarni $\vec{a}$ vektorning koordinatalari deb ataymiz. Vektorning koordinatalarini uning harfiy belgisi yoniga qo’yamiz, qaralayotgan holda $\vec{a}(a_1, a_2)$ yoki to’g’ridan-to’g’ri $(\overrightarrow{a_1}, \overrightarrow{a_2})$. Nol vektorning koordinatalari nolga teng.

Ikki nuqta orasidagi masofani shu nuqtalarning koordinatalari orqali ifodalovchi formuladan koordinatalari a_1, a_2 dan iborat vektorning moduli

$$\sqrt{{a_1}^2 + {a_2}^2}$$

ga teng degan natija chiqadi.

Teorema. Teng vektorlar mos ravishda teng koordinatalarga ega. Va aksincha, agar vektorlarning mos koordinatalari teng bo’lsa, vektorlar teng bo’ladi.

1-masala. A(1;2), B(0;1), C(-2;2), nuqtalar berilgan. Shunday $D(x; y)$ nuqtani topingki, $\overrightarrow{AB}$ va $\overrightarrow{CD}$ vektorlar teng bo’lsin.

Yechilishi: $\overrightarrow{AB}$ vektorning koordinatalari $(-1; 1)$ bo’ladi. $\overrightarrow{CD}$ vektorning koordinatalari (x+2;y-2). $\overrightarrow{AB} = \overrightarrow{CD}$ dan $x + 2 = -1, y - 2 = -1$. Bundan D nuqtaning koordinatalarini topamiz: $x = -3\ , y = 1$

Vektorlarni qo’shish

Koordinatalari a_1, a_2 va b_1, b_2 bo’lgan $\vec{a}$ va $\vec{b}$ vektorlarning yig’indisi deb, koordinatalari $a_1 + b_1$, $a_2 + b_2$ bo’lgan $\vec{c}$ vektorga aytiladi, ya’ni,

$$\vec{a}(a_1, a_2) + \vec{b}(b_1, b_2) = \vec{c}(a_1 + b_1, a_2 + b_2)$$

har qanday $\vec{a}(a_1, a_2)$, $\vec{b}(b_1, b_2), \vec{c}(c_1, c_2)$, vektorlar uchun

$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \qquad \vec{a} + (\vec{b}+\vec{c}) = (\vec{a} + \vec{b})+\vec{c}$$

tengliklar o’rinli.

Teorema. A, B, C nuqtalar qanday bo’lmasin

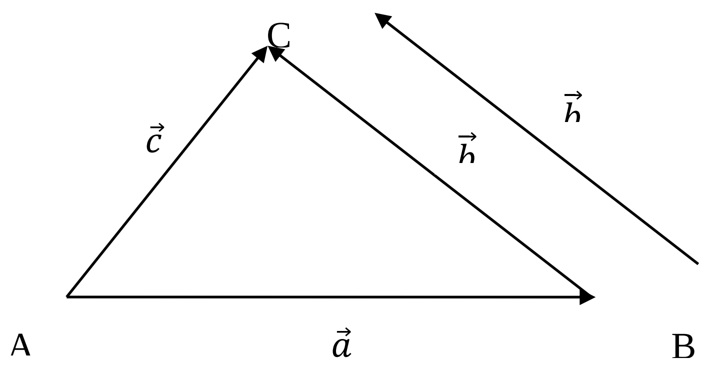
$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

vektor tenglik o’rinlidir.

Ixtiyoriy $\vec{a}$ va $\vec{b}$ vektorlar yig’indisini yasashning ushbu usulda bajariladi:

Faraz qilaylik, biror *A* nuqta oldin *A* dan *B* ga *AB* bo’yicha, so’ngra *B* dan *C* ga *BC* bo’yicha harakat qilsin. Natijada u nuqta *A* dan *C* ga keladi. SHuning uchun $\overrightarrow{AB} = \vec{a}$ va $\overrightarrow{BC} = \vec{b}$ vektorlarning yig’indisi uchun $\overrightarrow{AC} = \vec{c}$ vektorni qabul qilish tabiiydir. Buni e’tiborga olib, ikki $\vec{a}$ va $\vec{b}$ vektorlarning yig’indisi deb, quyidagicha hosil bo’lgan $\vec{c}$ vektorga aytiladi: berilgan $\overrightarrow{AB}=\vec{a}$ vektorni *B* uchidan $\overrightarrow{BC}=\vec{b}$ vektorni yasab, so’ngra birinchi $\vec{a}$ vektorning boshini ikkinchi $\vec{b}$ vektorning uchi bilan tutashtiriladi. Hosil bo’lgan $\overrightarrow{AC}=\vec{c}$ vektor−berilgan $\vec{a}$ va $\vec{b}$ vektorlarning yig’indisi bo’ladi (1.2.5-chizma). Vektorlarni qo’shish amalining ishorasi uchun odatdagi algebraik qo’shish ishorasi ishlatiladi, ya’ni

$$\vec{a} + \vec{b} = \vec{c}$$

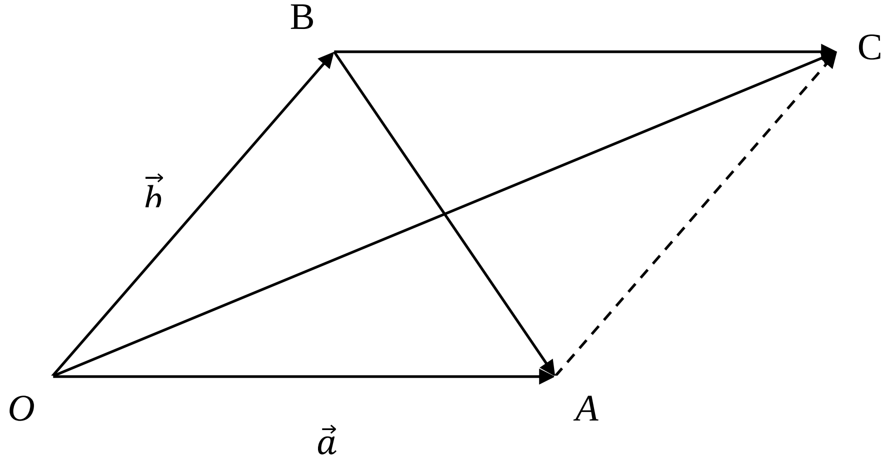


Ikkita $\vec{a}$ va $\vec{b}$ vektorlarning ayirmasi deb, shunday uchinchi $\vec{c}$ vektorga aytiladiki, $\vec{b}$ va $\vec{c}$ vektorlarning yig'indisi $\vec{a}$ ga teng bo'lsa, ya'ni $\vec{a}-\vec{b}=\vec{c},$ agar $\vec{b}+\vec{c}=\vec{a}$ bo'lsa

Agar keyingi tenglikning ikkala tomoniga $(-\vec{b})$ vektordan qo'shilsa,

$$\vec{c}=\vec{a}-\vec{b}=\vec{a}+(-\vec{b})$$

bo'ladi, ya'ni $\vec{a}$ vektordan $\vec{b}$ vektorni ayirish uchun $\vec{c}$ vektorga $(-\vec{b})$ vektorni qo'shish kerak. $\vec{a}$ va $\vec{b}$ vektorlarning ayirmasini yasash uchun bu vektorlarni biror boshlang'ich O nuqtaga ko'chirib, so'ngra $\vec{b}$ vektorning B uchidan $\vec{a}$ vektorning A uchiga $\overrightarrow{BA}$ vektor o'tkazilsa, shuning o'zi izlangan $\vec{a}-\vec{b}$ vektor bo'ladi, chunki.



$$\overrightarrow{BA}=\overrightarrow{BO}+\overrightarrow{OA}=-\vec{b}+\vec{a}=\vec{a}-\vec{b}$$

Shuning bilan, $\vec{a}$ va $\vec{b}$ vektorlarda yasalgan parallelogrammda diagonallaridan biri u vektorlarning yig'indisini va ikkinchisi ayirmasini ifoda qiladi.

2-masala: Boshi umumiy bo'lgan $\overrightarrow{AB}$ va $\overrightarrow{AC}$ vektorlar berilgan bo'lsin (1.2.5-chizma) $\overrightarrow{AC}-\overrightarrow{AB}=\overrightarrow{BC}$ ekanini isbotlang.

Yechilishi: $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$ tenglikka egamiz. Bu esa $\overrightarrow{AC}-\overrightarrow{AB}=\overrightarrow{BC}$ ekanini bildiradi.

Ikki vektorning skalyar ko'paytmasi

Ta'rif. *Ikki* $\vec{a}$ *va* $\vec{b}$ *vektorning skalyar ko'paytmasi* deb bu vektorlar uzunliklari bilan ular orasidagi burchak kosinusi ko'paytmasiga teng songa aytiladi va u $\vec{a}\vec{b}$ (yoki $\vec{a}\cdot\vec{b}$ yoki $(\vec{a},\vec{b})$) kabi belgilanadi, ya'ni

$$\vec{a}\vec{b}=|\vec{a}|\cdot|\vec{b}|\cdot\cos\varphi,$$

bu yerda $\varphi-\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak (bunda vektorlarning boshi bir nuqtaga qo'yiladi).

qqa proeksiyasining ko'paytmasiga teng.

Skalyar ko'paytmaning xossalari

1-xossa. *Ko'paytuvchilarning o'rin almashtirish* xossasi:

$$\vec{a}\vec{b}=\vec{b}\vec{a}.$$

2-xossa. *Skalyar ko'paytuvchiga nisbatan guruhlash* xossasi:

$$(\lambda\vec{a})\vec{b}=\lambda(\vec{a}\vec{b}).$$

3-xossa. *Qo'shishga nisbatan taqsimot* xossasi:

$$\vec{a}(\vec{b}+\vec{c})=\vec{a}\vec{b}+\vec{a}\vec{c}\,.$$

4-xossa. Agar $\vec{a}$ va $\overleftarrow{b}$ vektorlar perpendikular bo'lsa, u holda ularning skalyar ko'paytmasi nolga teng bo'ladi. Shunindek, teskari tasdiq o'rinli: agar $\vec{a}\vec{b}=0$ $(|\vec{a}|\neq 0,|\vec{b}|\neq 0)$ bo'lsa, u holda $\vec{a}\perp\vec{b}$ bo'ladi.

5-xossa. Vektorning skalyar kvadrati uning uzunligi kvadratiga teng, ya'ni $\vec{a}^2=|\vec{a}|^2$.

1-izoh. Agar $\vec{a}$ vektorni skalyar kvadratga oshirib, keyin kvadrat ildiz chiqarilsa, $\vec{a}$ vektorning o'zi emas, balki uning moduli hosil bo'ladi, ya'ni

$$\sqrt{\vec{a}^2}=|\vec{a}|\;(\sqrt{\vec{a}^2}\neq\vec{a}).$$

1-misol. $|\vec{a}|=4$, $|\vec{b}|=6$, $\varphi=(\vec{a}\,\hat{,}\,\vec{b})=\dfrac{\pi}{3}$ bo'lsin. $(3\vec{a}-\vec{b})\cdot(2\vec{a}+4\vec{b})$ ko'paytmani toping.

Yechish. Avval 3-xossadan foydalanib qavslarni ochamiz va keyin skalyar ko‘paytmaning ta’rifi va xossalaridan foydalanib, topamiz:

$$(3\vec{a}-\vec{b})\cdot(2\vec{a}+4\vec{b})=3\vec{a}\cdot 2\vec{a}-\vec{b}\cdot 2\vec{a}+3\vec{a}\cdot 4\vec{b}-\vec{b}\cdot 4\vec{b}=6\vec{a}^2+10\vec{a}\vec{b}-4\vec{b}^2=$$

$$=6|\vec{a}|^2+10|\vec{a}|\cdot|\vec{b}|\cos\frac{\pi}{3}-4|\vec{b}|^2=$$

$$=6\cdot 4^2+10\cdot 4\cdot 6\cdot\frac{1}{2}-4\cdot 6^2=96+120-144=72.$$

2-misol. $|\vec{a}|=4,\ |\vec{b}|=3, \varphi=(\vec{a}\,\hat{,}\,\vec{b})=\frac{2\pi}{3}$ bo‘lsin. Bu vektorlarga qurilgan parallelogramm diagonallarining uzunliklarini toping.

Yechish. $\vec{a}$ va $\vec{b}$ vektorlarga qurilgan parallelogram diagonallarini $\vec{a}+\vec{b}$ va $\vec{a}-\vec{b}$ vektorlar orqali ifodalash mumkin.

Skalyar ko‘paytmaning xossalaridan foydalanib, topamiz:

$$|\vec{a}+\vec{b}|=\sqrt{(\vec{a}+\vec{b})^2}=\sqrt{\vec{a}^2+2\vec{a}\vec{b}+\vec{b}^2}=\sqrt{|\vec{a}|^2+2|\vec{a}||\vec{b}|\cos\varphi+|\vec{b}|^2}=$$

$$=\sqrt{16+2\cdot 4\cdot 3\cdot\left(-\frac{1}{2}\right)+9}=\sqrt{13},$$

$$|\vec{a}-\vec{b}|=\sqrt{(\vec{a}-\vec{b})^2}=\sqrt{\vec{a}^2-2\vec{a}\vec{b}+\vec{b}^2}=\sqrt{|\vec{a}|^2-2|\vec{a}||\vec{b}|\cos\varphi+|\vec{b}|^2}=$$

$$=\sqrt{16+2\cdot 4\cdot 3\cdot\frac{1}{2}+9}=\sqrt{37}.$$

Koordinatalari bilan berilgan vektorlarning skalyar ko‘paytmasi

Ikkita $\vec{a}=\{a_x;a_y;a_z\}$ va $\vec{b}=\{b_x;b_y;b_z\}$ vektor berilgan bo‘lsin.

U holda bu vektorlarni skalyar ko‘paytmasi

$$\vec{a}\vec{b}=a_xb_x+a_yb_y+a_zb_z,$$

ya’ni koordinatalari bilan berilgan ikki vektorning skalyar ko‘paytmasi ularning mos koordinatalari ko‘paytmalarining yig‘indisiga teng.

3-misol. $\vec{a}=\{4;-2;3\},\ \vec{b}=\{1;-2;0\},\ \vec{c}=\{2;1;-3\}$ bo‘lsin. $(\vec{a}+3\vec{b})\cdot(\vec{a}-\vec{b}+\vec{c})$ ko‘paytmani toping.

Yechish. Avval $\vec{m}=\vec{a}+3\vec{b}$ va $\vec{n}=\vec{a}-\vec{b}+\vec{c}$ vektorlarning koordinatalarini topamiz:

$$\vec{m}=\{4+3\cdot 1;\ -2+3\cdot(-2);\ 3+3\cdot 0\}=\{7;-8;3\},$$

$$\vec{n}=\{4-1+2;\ -2+2+1;\ 3-0-3\}=\{5;1;0\}.$$

Bundan

$$\vec{m}\cdot\vec{n}=7\cdot 5+(-8)\cdot 1+3\cdot 0=27.$$

Ikki vektor orasidagi burchak.

$\vec{a}=\{a_x;a_y;a_z\}$ va $\vec{b}=\{b_x;b_y;b_z\}$ vektorlar orasidagi φ burchak kosinusini quyidagicha topamiz:

$$\cos\varphi=\frac{\vec{a}\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$$

yoki

$$\cos\varphi=\frac{a_xb_x+a_yb_y+a_zb_z}{\sqrt{a_x^2+a_y^2+a_z^2}\cdot\sqrt{b_x^2+b_y^2+b_z^2}}.$$

Ikki vektorning perpendikulyarlik sharti.

$\vec{a}\perp\vec{b}$ bo‘lsin. U holda $\cos\varphi=0$ bo‘lganidan

$$a_xb_x+a_yb_y+a_zb_z=0$$

kelib chiqadi.

4-misol. $\vec{m}=\vec{a}+2\vec{b}$ va $\vec{n}=5\vec{a}-4\vec{b}$ o‘zaro perpendikular vektorlar bo‘lsin. $\vec{a}$ va $\vec{b}$ birlik vektorlar orasidagi burchakni toping.

Yechish. $\vec{m}\perp\vec{n}$ bo‘lgani uchun $(\vec{a}+2\vec{b})\cdot(5\vec{a}-4\vec{b})=0$ bo‘ladi.

Bundan

$$5\vec{a}^2+6\vec{a}\vec{b}-8\vec{b}^2=0 \quad \text{yoki} \quad 5|\vec{a}|^2+6|\vec{a}|\cdot|\vec{b}|\cos\varphi-8|\vec{b}|^2=0.$$

$\vec{a}$ va $\vec{b}$ birlik vektorlar bo‘lgani sababli: $5+6\cos\varphi-8=0.$

U holda

$$\cos\varphi = \frac{1}{2} \text{ yoki } \varphi = \frac{\pi}{3}.$$

$$V_{ABCD} = \frac{32}{3}.$$

Masalalar

1. Tomonlari 1 ga teng bo‘lgan teng tomonli ABC uchburchak berilgan. $\overrightarrow{AB}\cdot\overrightarrow{BC}+\overrightarrow{BC}\cdot\overrightarrow{CA}+\overrightarrow{CA}\cdot\overrightarrow{AB}$ ifodaning qiymatini toping.

2. Tomonlari $BC=5,\ CA=6,\ AB=7$ ga teng bo‘lgan ABC uchburchak berilgan.
$\overrightarrow{AB}\cdot\overrightarrow{BC}$ skalyar ko‘paytmani toping.

3. Agar $|\vec{a}|=6,\ |\vec{b}|=4,\ \varphi=(\vec{a}\,\hat{,}\,\vec{b})=\frac{2\pi}{3}$ bo‘lsin. Toping:

1) $(2\vec{a}+\vec{b})^2$; 2) $(2\vec{a}-3\vec{b})\cdot(\vec{a}-2\vec{b})$.

4. $\vec{a}=\{1;-2;2\}$ va $\vec{b}=\{2;4;-5\}$ vektorlar berilgan. Toping:

1) $(3\vec{a}-2\vec{b})\cdot(\vec{a}+\vec{b})$; 2) $(\vec{a}-\vec{b})^2$.

5. Berilgan vektorlar m ning qanday qiymatlarida perpendikular bo‘ladi?

1) $\vec{a}=\{1;-2m;0\},\ \vec{b}=\{4;2;3m\}$; 2) $\vec{a}=\{m;-5;2\},\ \vec{b}=\{m-2;m;m+3\}$.

6. $\vec{e}_1,\ \vec{e}_2,\ \vec{e}_3$ birlik vektorlar uchun $\vec{e}_1+\vec{e}_2+\vec{e}_3=0$ bo‘lsa, $\vec{e}_1\vec{e}_2+\vec{e}_2\vec{e}_3+\vec{e}_3\vec{e}_1$ ni toping.

7. Tomonlari $\vec{a}=2\vec{i}+\vec{j}$ va $\vec{b}=-\vec{j}+2\vec{k}$ vektorlardan iborat bo‘lgan parllelogrammning diagonallari orasidagi burchakni toping.

8. Uchlari $A(-1;-2;4),\ B(-4;-2;0),\ C(3;-2;1)$ bo‘lgan ABC uchburchak berilgan. $\angle B$ ni toping.

9. $\vec{a}=\{3;-1;5\}$ va $\vec{b}=\{1;2;-3\}$ vektorlar berilgan. Agar $\vec{x}\cdot\vec{a}=9$, $\vec{x}\cdot\vec{b}=-4$ va $\vec{x}$ vektor Oz oqiga perpendikular bo‘lsa, $\vec{x}$ vektorning koordinatalarini toping.

10. $\vec{a}=\{2;-3;1\},\ \vec{b}=\{1;-2;3\}$ va $\vec{c}=\{1;2;-7\}$ vektorlar berilgan. Agar

$\vec{x} \perp \vec{a}$, $\vec{x} \perp \vec{b}$, $\vec{x} \cdot \vec{c} = 10$ bo‘lsa, $\vec{x}$ vektorni toping.