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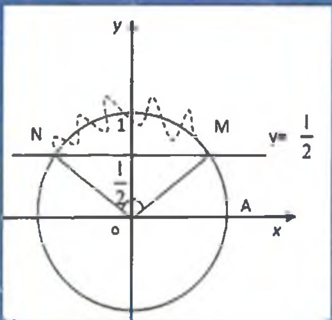
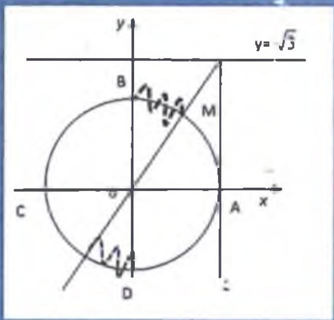
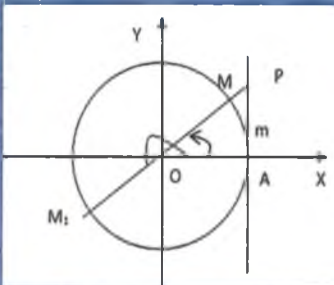
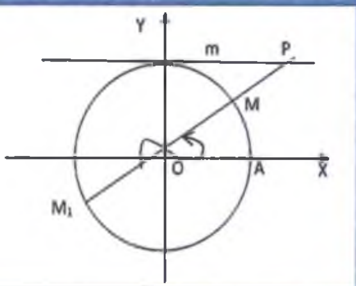
ALGEBRA. TRIGONOMETRIYA









O‘ZBEKISTON RESPUBLIKASI
OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

NIZOMIY NOMIDAGI
TOSHKENT DAVLAT PEDAGOGIKA UNIVERSITETI

Qudrat JUMANIYOZOV, Gulchehra MUXAMEDOVA

MATEMATIKADAN MISOL VA MASALALAR YECHISH METODIKASI

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TOSHKENT – 2014

Taqrizchilar: Nizomiy nomidagi TDPU fizika-matematika fakulteti matematika va uni o‘qitish metodikasi kafedrasi dotsenti, fizika-matematika fanlari nomzodi **D.Davletov;**
TAQI qoshidagi akademik litseyi matematika o‘qituvchisi fizika-matematika fanlari nomzodi, dotsent **A.Amanov**

Mazkur o‘quv qo‘llanma 5110100 matematika o‘qitish metodikasi ta’lim yo‘nalishining “matematikadan misol va masalalar yechish metodikasi” kursi dasturi bo‘yicha yozilgan bo‘lib, matematikaning arifmetika, algebra va trigonometriya bo‘limlarini qamrab olgandir. Qo‘llanmaning maqsadi talabalarning matematikadan olgan nazariy bilimlarini umumiy o‘rta ta’lim maktablari va o‘rta maxsus ta’lim muassasalari matematikasi kurslari mazmuni bilan bog‘lash, ularda misol va masalalar yechish malakasini takomillashtirish hamda rivojlantirishdan iborat.
Ushbu o‘quv qo‘llanma keng o‘quvchilar ommasiga mo‘ljallangan bo‘lib, undan oliy o‘quv yurtlari talabalari va shu bilan birgalikda akademik litsey, kasb-hunar kollejlari o‘quvchilari va o‘qituvchilari foydalanishlari mumkin.

Nizomiy nomidagi Toshkent davlat pedagogika universiteti professori, fizika-matematika fanlari nomzodi **A.S.Yunusov** ning umumiy tahriri ostida

SO‘Z BOSHI

Pedagogika oliy ta’lim muassasalarining Fizika-matematika fakultetlari matematika o‘qitish metodikasi yo‘nalishi bo‘yicha o‘qituvchi kadrlar tayyorlashga o‘tganligi sababli yangi o‘quv rejasi asosida faoliyat yurita boshladi. Yangi o‘quv rejaga «matematikadan misol va masalalar yechish metodikasi» o‘quv fani kiritilgan bo‘lib, talabalarni matematikadan masalalar yechishning umumiy va xususiy metodlari hamda masalalar yechishga o‘rgatish metodikasi bilan mufassal tanishtirish zaruriyati paydo bo‘ldi.

Mazkur kitob mavjud Davlat Ta’lim Standartida belgilangan va matematik ta’lim mazmuniga kiritilgan «matematikadan misol va masalalar yechish metodikasi» mavzularini o‘zlashtirishda talabalarga o‘quv qo‘llanma sifatida tayyorlangan. Ushbu o‘quv qo‘llanmada yuqorida ko‘rsatilgan mavzularga oid aksariyat misollar to‘liq yechimlari bilan berilgan. O‘quv qo‘llanmadan oliy o‘quv yurtiga kirish uchun mustaqil tayyorlanayotgan o‘quvchi yoshlar ham foydalanishlari mumkin.

O‘quv qo‘llanmada butun sonlar va kombinatorika, ayniy shakl almashtirishlar, ayniyatlar va tengsizliklarni isbotlash, algebraik tenglama va tengsizliklar, trigonometrik funksiyalar va ular orasidagi munosabatlar, trigonometrik tenglamalar va tengsizliklar, trigonometrik tenglamalar va tengsizliklar sistemalariga oid tushuncha va ma’lumotlar misollar yordamida yoritilgan.

O‘quv qo‘llanmaning I, III boblari Q.Jumaniyozov va G.Muxamedovalar tomonidan birgalikda, II, IV va V boblari Q.Jumaniyozov tomonidan yozilgan.

O‘quv qo‘llanmaning har bir bobi bir nechta paragraflardan tashkil topgan bo‘lib, har bir paragrafda mustaqil ishlash uchun mashqlar keltirilgan. O‘ylaymizki, ushbu o‘quv qo‘llanma pedagogika oliy ta’lim muassasalarining “matematika o‘qitish metodikasi” yo‘nalishida ta’lim olayotgan talabalarga zarur kasbiy va metodik, bilim va ko‘nikmalarning shakllanishiga hamda rivojlanishiga xizmat qiladi.

O‘quv qo‘llanmani yozishda rus va o‘zbek tillarida chop etilgan mavjud adabiyotlardan foydalanildi.

Mualliflar – o‘quv qo‘llanma haqida fikr bildirgan barcha taklif va fikr-mulohazalarni minnatdorchilik bilan qabul qiladilar.

Mualliflar.

I QISM. ALGEBRA

I BOB. BUTUN SONLAR VA KOMBINATORIKA

1-§. Qoldiqli bo'lish. Tub va murakkab sonlar. Natural sonning kanonik yoyilmasi. Arifmetikaning asosiy teoremasi

Agar berilgan n va p natural sonlar uchun $n = p \cdot q$ tenglikni qanoatlantiruvchi q natural son topilsa, u holda n soni p ga qoldiqsiz bo'linadi deyiladi. 1 dan farqli har qanday natural son kamida ikkita - 1 va shu sonning o'zidan iborat bo'lgan **natural sonlardan** iborat bo'luvchiga ega. Boshqa natural sonlardan iborat bo'luvchilarga ega bo'lmagan birdan farqli natural sonlar tub sonlar, ikkitadan ortiq natural bo'luvchilarga ega bo'lgan natural sonlar murakkab sonlar deyiladi.

1.1-teorema. Birdan farqli har qanday natural son kamida bitta tub bo'luvchiga ega.

Har qanday murakkab n sonni $n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot ... \cdot p_n^{\alpha_n}$ kanonik yoyilma shaklida yozish mumkinligi ma'lum. Bu erda $\alpha_1, \alpha_2, ..., \alpha_n$ lar $p_1, p_2, ..., p_n$ tub sonlarning n son uchun necha karrali ekanligini bildiradi. n natural sonning barcha turli natural bo'luvchilari sonini $\tau(n)$ bilan, barcha natural bo'luvchilari yig'indisini $s(n)$ bilan belgilaylik.

1.2-teorema. Agar n natural sonning kanonik yoyilmasi $n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot ... \cdot p_n^{\alpha_n}$ bo'lsa, u holda

$\tau(n) = (\alpha_1 + 1)(\alpha_2 + 1)...(\alpha_n + 1)$,

$$s(n)=\frac{p_1^{\alpha_1+1}-1}{p_1-1}\cdot\frac{p_2^{\alpha_2+1}-1}{p_2-1}\cdots\frac{p_n^{\alpha_n+1}-1}{p_n-1}$$ bo‘ladi.

1-misol. $n=360$ sonning kanonik yoyilmasi $n=2^3\cdot 3^2\cdot 5$ bo‘lib, uning turli natural bo‘luvchilari $\tau(n)=(\alpha_1+1)(\alpha_2+1)...(\alpha_n+1)=(3+1)(2+1)(1+1)=24$ ta va bu natural bo‘luvchilari yig‘indisi esa,

$$s(n)=\frac{p_1^{\alpha_1+1}-1}{p_1-1}\cdot\frac{p_2^{\alpha_2+1}-1}{p_2-1}\cdots\frac{p_n^{\alpha_n+1}-1}{p_n-1}=\frac{2^4-1}{1}\cdot\frac{3^3-1}{2}\cdot\frac{5^2-1}{4}=1170$$ ga teng bo‘ladi.

1.3-teorema. (Arifmetikaning asosiy teoremasi). Har qanday murakkab son ko‘paytuvchilar tartibigacha aniqlikda bir va faqat birgina usul bilan tub sonlar ko‘paytmasi shaklida tasvirlanishi mumkin.

Misol. $12=2\cdot 2\cdot 3=2\cdot 3\cdot 2=3\cdot 2\cdot 2$

1.4-teorema. m murakkab sonning eng kichik tub bo‘luvchisi $\sqrt{m}$ dan katta emas.

2-misol. 659 sonining eng kichik tub bo‘luvchisini toping.

Yechilishi. 659 sonini $\sqrt{659}$ sonidan kichik bo‘lgan tub sonlar $2,3,5,7,11,13,17,19,23$ ga ketma – ket bo‘lib chiqib, 659 ularning hech biriga bo‘linmasligini aniqlaymiz, demak, 659 soni tub son ekan.

Yuqoridagi misolda berilgan sonni bir nechta tub songa bo‘lishga to‘g‘ri keldi. Bu ishni osonlashtirish uchun sonlarning bo‘linish belgilaridan foydalaniladi:

1) 2 ga bo‘linish belgisi: agar sonning oxirgi raqami 2 ga qoldiqsiz bo‘linsa, u holda bu son 2 ga qoldiqsiz bo‘linadi;

2) 3 ga va 9 ga bo‘linish belgisi: agar sonning raqamlari yig‘indisi 3 ga (9 ga) qoldiqsiz bo‘linsa, u holda bu son 3 ga (9 ga) qoldiqsiz bo‘linadi;

3) 5 ga bo'linish belgisi: agar sonning oxirgi raqami 5 yoki 0 bilan tugasa, u holda bu son 5 ga bo'linadi;

4) 4 va 25 ga bo'linish belgisi: agar sonning oxirgi ikki raqami 4 ga (25 ga) bo'linsa, u holda bu son 4 *ga* (25 ga) bo'linadi (umumiy holda, 2^k va 5^k ga bo'linadigan sonlar belgisi: agar sonning oxirgi k ta raqamidan tuzilgan sonlar 2^k ga (5^k ga) bo'linsa, u holda faqat shunday sonlar 2^k ga (5^k ga) bo'linadi.

5) 7 ga bo'linish belgisi: a) berilgan sonning oxirgi raqamini o'chirishdan hosil bo'lgan sondan uning oxirgi raqamini ikkilantirib ayirilganda chiqqan son 7 ga bo'linsa, bu son 7 ga bo'linadi:

3-misol. 259 soni 7 ga bo'linadi, chunki $25-(2\cdot 9) = 7$ ga bo'linadi.

b) 9 va 11 ga bo'linish belgisi kabi qulay bo'lgan 7 ga bo'linish belgilari mavjud emas, lekin $7\cdot 11\cdot 13=1001$ ko'paytmadan foydalanish mumkin. Misol uchun, 859 516 sonining 7 ga bo'linishini tekshiraylik. Bu sonni $859\cdot 1001 - 859+516 = 859859 - 343$. 859859 soni 1001 ga (jumladan, 7 ga ham), 343 ning 7 ga bo'linishini tekshirish kifoya. $343 = 49\cdot 7$, demak 859516 ning o'zi ham 7 ga bo'linadi.

4-misol. 1) 85314507229 va363862625 sonlarining 7 ga bo'linishini tekshiring.

Yechilishi. 1) Ikki yig'indi tuzib olamiz: $85 + 507 = 592$ va $314 + 229 = 543$. Ularning ayirmasi $592 - 543 = 49 = 7\cdot 7$ Berilgan son 7 ga bo'linadi.

2) 625-862+363=126, 7 ga bo‘linishning a) belgisiga ko‘ra, $12-(2\cdot 6)=0$ esa 7 ga bo‘linadi, shuning uchun 363862625 ham 7 ga bo‘linadi.

6) 11 ga bo‘linish belgisi: agar berilgan sonning juft o‘rinda turgan raqamlari yig‘indisidan toq o‘rinda turgan raqamlari yig‘indisini yoyilganda hosil bo‘lgan ayirma 11 ga bo‘linsa yoki 0 ga teng bo‘lsa, u holda bu son 11 ga bo‘linadi.

5-misol. 1) 7896 va 2) 208912 sonlarining 11 ga bo‘linishini tekshiring.

1) $(7+9)-(8+6)=16-14=2.$ 2 soni 11 ga bo‘linmaydi, sonning o‘zi ham 11 ga bo‘linmaydi.

2)$(2+8+1)-(0+9+2)=0$, demak, berilgan son 11 ga bo‘linadi: $208912:11=18992$.

Eng katta umumiy bo‘luvchi (EKUB). Eng kichik umumiy karrali (EKUK).

m va *n* sonlarining umumiy bo‘luvchilarining eng kattasiga shu sonlarning eng katta umumiy bo‘luvchisi deyiladi va uni EKUB(m,n), D(m,n) yoki (m,n) ko‘rinishida belgilanadi.

6-misol. 168, 180 va 3024 sonlarining EKUB ini toping.

Yechilishi. Sonlarni kanonik ko‘rinishda yozib olamiz:

$$168 = 2\cdot 2\cdot 2\cdot 3\cdot 7 = 2^3\cdot 3^1\cdot 7^1,$$
$$180 = 2\cdot 2\cdot 3\cdot 3\cdot 5 = 2^2\cdot 3^2\cdot 5^1,$$
$$3024 = 2\cdot 2\cdot 2\cdot 2\cdot 3\cdot 3\cdot 3\cdot 7 = 2^4\cdot 3^3\cdot 7^1.$$

Bu yerdagi umumiy bo‘luvchilar 2 va 3 ning kichik darajalarini ko‘paytirib EKUB ni topamiz: $D(168, 180, 3024)=2^2\cdot 3^1=12$ Agar *m* va *n* sonlar uchun $D(m,n)=d=1$ bo'lsa, m va n sonlar o'zaro tub sonlar deyiladi va $(m,n)=1$ bo'ladi.

2.1-teorema. m va n sonlar uchun $m \vdots n$ bo'lsa, u holda m va n sonlarning umumiy bo'luvchilari n sonning bo'luvchilari bilan bir xil bo'ladi va $D(m,n)=n$ bo'ladi.

2.2-teorema. Agar m va $n\,(m>n)$ sonlar uchun $m=nq+r$ bo'lsa, u holda $D(m,n)=D(n,r)$ bo'ladi.

2.3-teorema. $m_1,m_2,...,m_k$ sonlar uchun

$D(m_1,m_2,...,m_{k-1})=d_{k-1}; D(d_{k-1},m_k)=d_k$ bo'lsa, u holda

$D(m_1,m_2,...,m_k)=d_k$ bo'ladi.

7-misol. 60, 45, 90 sonlarining eng katta umumiy bo'luvchisini toping.

Yechilishi. $D(60, 45, 90) = D[D(60, 45), 90)] = D(15, 90) = 15.$

2.4-teorema. m va n sonlar uchun $D(m,n)=d$ bo'lsa, u holda $D\left(\frac{m}{d},\frac{n}{d}\right)=1$ bo'ladi.

2.5-teorema. m va n sonlarining har qanday umumiy bo'luvchisi ularning eng katta umumiy bo'luvchisining ham bo'luvchisidir.

2.6-teorema. m va n sonlar uchun $D(m,n)=d$ bo'lsa, u holda $D(mk,nk)=dk$ bo'ladi.

Berilgan m va n sonlarning umumiy bo'luvchisi tushunchasi bilan birgalikda ularning umumiy bo'linuvchisi yoki karralisi tushunchasi ham matematikada muhim ahamiyatga ega. Sonlarning har biriga qoldiqsiz bo'linadigan eng kichik son shu sonlarning eng kichik umumiy karralisi (EKUK) deyiladi va uni $K(m,n)$ yoki $[m,n]$ ko'rinishda belgilaymiz.

8-misol. 168, 180 va 3024 sonlarining EKUK ini toping.

Yechilishi. Sonlarni kanonik ko'rinishda yozib olamiz:

$180 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 2^2 \cdot 3^2 \cdot 5^1,$

$3024 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 7 = 2^4 \cdot 3^3 \cdot 7^1$

$168 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 = 2^3 \cdot 3^1 \cdot 7^1,.$

Bu yerdagi barcha tub bo'luvchilarning eng katta darajalarini ko'paytirib, EKUK ni topamiz: $K(168, 180, 3024) = 2^4 \cdot 3^3 \cdot 5^1 \cdot 7^1 = 15120.$

2.7-teorema. m va n sonlarning umumiy karralisi (bo'linuvchisi) shu sonlarning eng kichik karralisiga bo'linadi.

2.8-teorema. m va n sonlarning eng kichik umumiy karralisi $\frac{m \cdot n}{D(m \cdot n)}$ ga teng.

9-misol. D (252, 120) =12 bo'lsa, K(252, 120) ni toping.

Yechilishi. $K(252,120) = \frac{252 \cdot 120}{12} = 2520.$

2.9-teorema. m va n sonlarni biror $t \neq 0$ songa bo'linsa, u holda $K(\frac{m}{t}, \frac{n}{t}) = K(m,n):t$ bo'ladi.

Berilgan sonlarning eng katta umumiy bo'luvchisi yoki eng kichik umumiy karralisini topish masalasi bevosita Evklid algoritmi tushunchasi bilan bog'liq. Berilgan m va $n\,(m > n)$ natural sonlarning EKUB ini topish uchun Evklid algoritmi yordamida quyidagicha topiladi:

$$m = nq_1 + r_1, \qquad 0 \le r_1 < n;$$
$$n = r_1 q_2 + r_2, \qquad 0 \le r_2 < r_1;$$
$$r_1 = r_2 q_3 + r_3, \qquad 0 \le r_3 < r_2;$$
$$\ldots\ldots\ldots\ldots \qquad \ldots\ldots\ldots\ldots$$
$$r_{n-2} = r_{n-1} q_n + r_n, \qquad 0 \le r_n < r_{n-1};$$

$r_{n-1} = r_n q_{n+1}, \qquad r_{n+1} = 0.$

Hosil bo'lgan oxirgi noldan farqli r_n qoldiq m va n sonlarning EKUBi bo'ladi.

10-misol. 525 va 231 sonlarining EKUB va EKUKini toping.

Yechilishi. 525 va 231 sonlarining kattasini kichigiga bo'lamiz, so'ngra 231 ni hosil bo'lgan qoldiqqa bo'lamiz, keyin birinchi qoldiqni ikkinchisiga bo'lamiz va hokazo. Noldan farqli oxirgi qoldiq berilgan sonlarning EKUBi bo'ladi:

$$
\begin{array}{r|l}
525 & 231 \\
\underline{462} & 2 \\
231 & 63 \\
\underline{189} & 3 \\
63 & 42 \\
\underline{42} & 1 \\
42 & 21 \\
\underline{42} & 2 \\
0 &
\end{array}
$$

Demak, D (525, 231) = 21 va K (525, 231) = 5775.

Mustaqil yechish uchun misollar

1. Ikkita ketma – ket keluvchi butun son kvadratlarining yig'lndisini 4 ga bo'lgandagi qoldiq 1 ga tengligini isbotlang.
2. a) m juft sonni 6 ga bo'lganda qoldiq 1 yoki 3 ga teng bo'lishi mumkinmi? b) 3 ga karrali m sonni 12 ga bo'lganda qoldiq 2 ga teng bo'lishi mumkinmi?

3. a) 3 ga karrali va 4 ga bo‘lganda 1 qoldiq qoladigan sonlarning umumiy ko‘rinishini yozing. b) 3 ga bo‘lganda 1, 5 ga bo‘lganda 3 qoldiq qoladigan barcha sonlarni toping.

4. a) m sonni 5 ga bo‘lganda 2 qoldiq, 3 ga bo‘lganda 1 qoldiq qolishi ma’lum. Bu sonni 15 ga bo‘lganda necha qoldiq qoladi? b) m sonni 5 ga bo‘lganda 1 qoldiq, 3 ga bo‘lganda 2 qoldiq qolishi ma’lum. Bu sonni 15 ga bo‘lganda necha qoldiq qoladi?

5. 12 ga bo‘lganda 11, 18 ga bo‘lganda 1 qoldiq qoladigan son mavjudmi?

6. 2 ga bo‘lganda 1, 3 ga bo‘lganda 2, ..., 9 ga bo‘lganda 8 qoldiq qoladigan eng kichik sonni toping.

7. Birinchi kishi savatdagi olmalarni sanab ko‘rib, agar 1 ta olmani yeb qo‘ysa, qolgan olmalar sonining n ga bo‘linishini aniqladi. Bitta olmani yeb, $\frac{1}{n}$ ta olmani olib yo‘lida davom etdi. Har bir keyingi kishi olmalarni sanab ko‘rib, uni n ga bo‘lganda 1 qoldiq qolishini aniqladi va 1 ta olmani yeb, $\frac{1}{n}$ ta olmani olib yo‘lida davom etaverdi. n - yo‘lovchidan so‘ng, savatda qolgan olmalar soni n ga qoldiqsiz bo‘lindi. Savatda nechta olma qoldi va dastlab savatda nechta olma bo‘lgan?

8. Berilgan uch xonali son bilan unga teskari tartibda yozilgan son orasidagi farq 9 ga bo‘linishini isbotlang.

9. Bir xil raqamdan tashkil topgan ixtiyoriy uch xonali son 37 ga bo‘linishni isbotlang.

10. Ketma – ket kelgan ixtiyoriy uchta natural sonning ko‘paytmasi 6 ga qoldiqsiz bo‘linishini isbotlang.

11. $n(n^2-1)(n^2-5n+26)$ son $\forall n \in N$ da 120 ga qoldiqsiz bo'linishini isbotlang.

12. 3^{110} ni 7 ga; 15^{256} ni 17 ga; 6^{592} ni 11 ga; $5^{1985}-9^{17}$ ni 4 ga; $13^{16}-2^{25}\cdot 5^{15}$ ni 3 ga va 37 ga; $(116+17^{17})^{21}$ ni 8 ga bo'lishdan hosil bo'ladigan qoldiqlarni toping.

13. Berilgan sonni 13 ga, 17 ga, 23 ga bo'linish belgilarini toping va 116 909 sonini bu sonlarning qaysi biriga bo'linishini tekshiring.

14. a) 78 va 700;1345 va 1355; 89769 va 89799 sonlaridan qaysi biri tub son?; b) 1973 tub sondan keyin keladigan tub sonni toping.

15. 428, 227, 825, 1529, 67323, 222333, 224433, 3082607, 138364854 sonlarini tub ko'paytuvchilarga ajrating.

16. $235^2 + 972^2$ sonni ko'paytuvchilarga ajrating.

17. 3 10 + 35 +1 sonni ko'paytuvchilarga ajrating.

18. $2^{18}+3^{18}$ ni tub ko'paytuvchilarga ajrating, uning kanonik yoyilmasini tuzing.

19. $p>15$ tub sonning kvadratini 30 ga bo'lganda qoldiqda 1 yoki 19 hosil bo'lishini isbotlang.

20. Agar p va q tub sonlar bo'lib, ular 3 dan katta bo'lsa, u holda p^2+q^2 son 24 ga karrali ekanini isbotlang.

21. n natural sonning shunday qiymatini topingki, $n+4$, $n+14$ va $n+20$ tub sonlar bo'lsin.

22. 90, 91, 92, 93, 94, 95, 96 sonlari 7 ta ketma – ket keladigan murakkab sonlardir. 9 ta ketma – ket keladigan murakkab sonlarni toping.

23. 78; 560; 459; 1208 sonlarining bo'luvchilari soni va bo'luvchilari yig'indisini toping.

24. a) bitta; b) ikkita; c) uchta bo'luvchiga ega bo'lgan sonlarning umumiy ko'rinishini yozing.

25. Quyidagi sonlarning murakkab ekanligini isbotlang.

a) $13^{25}+17^{18}+2^{31}$; b) $42^{100}+19$; c) 123....898...321; g) $2^{33}+27$; d) $2^{101}+1$; e) $2^{102}-1$ j) $2^{m}-1$, bu erda m – murakkab son; z) n^4+4n^2+3, bu yerda $n\in N$

i) $2^{20}+6^{11}+3^{22}$

26. $\overline{1234xy}$ soni 8 va 9 ga qoldiqsiz bo'linadi. x va y raqamlarni topib, $\overline{1234xy}$ son bilan $\overline{y1234x}$ sonni taqqoslang.

27. Quyidagi sonlarning EKUB va EKUKini toping

1) 1403, 1058;	2) 10140, 92274;
3) 56595, 82467;	4) 3640, 14300;
5) 36372, 147220;	6) 35574, 192423;
7) 3327449, 6314153;	8) 179370199, 4345121.

28. Quyidagi kasrlarni qisqartiring

1) $\frac{17501}{11137}$; 2) $\frac{1491}{2247}$; 3) $\frac{237419}{294817}$; 4) $\frac{1253}{406}$; 5) $\frac{438875}{747843}$; 6) $\frac{127936}{161919}$.

29. 48 ta konfet, 60 ta nok va 36 ta olmaning barchasini ishlatib, nechta bir xil sovg'a tayyorlash mumkin? Har bir sovg'ada nechtadan konfet, nok va olma bo'ladi?

30. Ishchilarga shahardan tashqariga chiqishlari uchun o'rindiqlar soni teng bo'lgan bir nechta avtobus ajratildi. 424 kishi bog'ga, 477 kishi daryo bo'yiga chiqishdi. Avtobuslardagi barcha joylar band bo'ldi va hech bir kishi joysiz qolmadi. Avtobuslar soni nechta bo'lgan va har bir avtobusda nechtadan joy bo'lgan?

31. Ahror qadamining uzunligi 75 sm, Azizaniki esa 60 sm. Qanday eng qisqa masofada ularning har ikkisi butun son marta qadam tashlaydi?

32. Velosipedning zanjirni tutib turuvchi oldingi tishli diskining (shesternasining) 44 ta tishi bor, kichik tishli diskida esa 20 ta tish bor. Oldingi tishli disk necha marta aylangandan so'ng bu disklar yana boshlang'ich vaziyatga qaytadi? Bu vaqt davomida kichik disk necha marta aylanadi?

33. Stol tennisi bo'yicha jami 145 nafar o'g'il bola va 87 nafar qiz boladan iborat, ishtirokchilar soni teng bo'lgan jamoalar qatnashdi. Barcha jamoalarda bir xil sonda o'g'il bola va bir xil sonda qiz bolalar qatnashdilar. Har bir jamoada nechtadan o'g'il bola va nechta qiz bola bo'lgan?

34. Agar a va b natural sonlar bo'lsa, EKUB (a,b) ·EKUK (a,b) ni toping.

35. a) $2n$ va $2n+2$; b) $3n$ va $6n+3$; c) $2n$ va $4n+2$; d) $30n+25$ va $20n+15$; i) n va $2n+1$; f) $10n+9$ va $n+1$; j) $3n+1$ va $10n+3$ sonlarining ($n\in N$) EKUB ini toping.

36. Agar EKUB (a,b) = 6, EKUK (a,b) = 90 bo'lsa, a va b ni toping.

37. Agar EKUK (a,b) = 60, EKUK (a,d) = 270 bo'lsa, EKUK (b,d) ni toping.

38. AC asosidagi burchagi 36^0 bo'lgan ABC teng yonli uchburchak berilgan. AB va AC kesmaga butun son marta yotqizish mumkin bo'lgan kesma mavjud emasligini isbotlang.

2-§. Matematik induksiya va uning tatbiqlari

Xususiy xulosalardan umumiy xulosalarga o'tishdan iborat mulohazalar induktiv hisoblanadi. Matematik induksiya $P(n)$ xossa yoki mulohazaning barcha natural sonlar uchun (yoki $n>d$ sonlar uchun, bu yerda d berilgan natural son) o'rinli ekanligini isbotlash usulidir. Buning uchun:

1. Bu xossaning to'g'riligini $P(1)$ (*yoki* $P(d)$) uchun tekshirib ko'rish (bu qadam induksiyaning asosi yoki bazisi deyiladi);
2. Berilgan xossa $P(k)$ uchun to'g'ri deb faraz qilib, uni $P(k+1)$ uchun to'g'riligini keltirib chiqarish (bu – induktiv qadam).

Bu ikki mulohaza bajarilgach, $P(n)$ xossani ixtiyoriy natural son uchun (yoki $\forall n>d$ natural son uchun) to'g'ri deb qabul qilish mumkin.

1-misol. 1 dan $n+1$ gacha dastlabki n ta natural sonlar kvadratlarining yig'indisi $\frac{n(n+1)(2n+1)}{6}$ ga tengligini, ya'ni

$S_n = 1^2+2^2+3^2+...+n^2 = \frac{n(n+1)(2n+1)}{6}$ $n \in N$ tenglikni isbotlang.

Isboti. 1. Agar $n=1$ bo`lsa, $S_1 = \frac{1\cdot(1+1)(2\cdot 1+1)}{6} = 1$

2. Agar $n = k$ bo`lsa, $S_k = \frac{k\cdot(k+1)(2\cdot k+1)}{6}$ ekanidan

$n=k+1$ uchun $S_{k+1} = 1^2+2^2+3^2+...+(k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$

bo`lishigini isbotlaymiz:

$$S_{k+1}=S_k+(k+1)^2=\frac{k(k+1)(2k+1)}{6}+(k+1)^2=\frac{k+1}{6}\big(k(2k+1)+6(k+1)\big)=$$

$$=\frac{k+1}{6}\left(2k^2+7k+6\right)=\frac{2(k+1)(k+2)\left(k+\frac{3}{2}\right)}{6}=\frac{(k+1)(k+2)(2k+3)}{6}$$

Matematik induksiya metodiga ko'ra, $1+2^2+3^2+...+n^2=\frac{n(n+1)(2n+1)}{6}$ tenglik o'rinli ekanligi isbotlandi.

2-misol. Ixtiyoriy natural son va $q\neq 1$ haqiqiy son uchun $1+q+q^2+...+q^n=\frac{1-q^{n+1}}{1-q}$ tenglikning o'rinli ekanini isbotlang (barchaga ma'lum bo'lgan geometrik progressiyaning hadlari yig'indisi formulasi).

Isboti. Bu tenglikni n ga nisbatan matematik induksiya yordamida isbotlaymiz:

1. Induksiyaning asosi $n = 1$ da

$$1+q=\frac{(1-q)(1+q)}{1-q}=\frac{1-q^{1+1}}{1-q}$$

tenglik o'rinli.

2. $k<n$ natural son uchun $1+q+q^2+...+q^k=\frac{1-q^{k+1}}{1-q}$ tenglikni to'g'ri deb faraz qilamiz va $n=k+1$ uchun to'g'riligini keltirib chiqaramiz:

$$1+q+q^2+...+q^k+q^{k+1}=\frac{1-q^{k+1}}{1-q}+q^{k+1}=$$

$$=\frac{1-q^{k+1}+(1-q)q^{k+1}}{1-q}=\frac{1-q^{k+1}+q^{k+1}-q^{(k+1)+1}}{1-q}=\frac{1-q^{(k+1)+1}}{1-q}$$

Demak, berilgan $1+q+...+q^n=\frac{1-q^{n+1}}{1-q}$ tenglik ixtiyoriy natural son uchun o'rinli.

3- misol. $n \geq 2$ bo'lganda $(7^n + 8^{2n-3}) \vdots 19$ ekanligini isbotlang.

Isboti. 1. $n = 2$ da $7^2 + 8 = 57$ bo'lib, $57 = 19 \cdot 3$tenglik o'rinli.

2. $n = k > 2$ da $(7^k + 8^{2k-3}) \vdots 19$ ni o'rinli deb faraz qilib, n=k+1 uchun $(7^{k+1} + 8^{2(k+1)-3}) \vdots 19$ ekanligini isbotlaymiz:

$$(7^{k+1} + 8^{2(k+1)-3}) = 7 \cdot 7^k + 8^{2k-3} \cdot 64 = 7\left(7^k + 8^{2k-3}\right) + 57 \cdot 8^{2k-3}$$

Hosil bo'lgan yig'indida har ikki qo'shiluvchining 19 ga bo'linishi ma'lum, demak, yig'indining o'zi ham 19 ga bo'linadi.

Induktiv qadamni amalga oshirish doim ham oson emas. Avvalambor, u ham berilgan teoremaning o'zi kabi cheksiz holat (k- ixtiyoriy) uchun tekshirilishi kerak. Biroq matematik induksiya metodining afzalligi shundaki, juda ko'p hollarda induksiya qadami berilgan teoremaning o'zini isbotlashga qaraganda osonroq bo'ladi.

Ko'pincha, induksiya bilan teoremani barcha n uchun emas, balki yetarlicha katta n lar uchun, ya'ni biror berilgan N sonidan katta n lar uchun isbotlashga to'g'ri keladi.

4-misol. $n \geq 2000$ bo'lganda $n^3 - 4 > 1000n^2 + 3n$ tengsizlikning to'g'riligini isbotlang.

Isboti. 1. $n = 2000$da $2000^3 - 4 > 1000 \cdot 2000^2 + 3 \cdot 2000$ tengsizlikni bevosita tekshirib uning to'g'riligiga ishonch hosil qilamiz.

2. $k > 2000$ da $k^3 - 4 > 1000k^2 + 3k$ tengsizlikni to'g'ri deb faraz qilib, $n = k + 1$ uchun $(k+1)^3 - 4 > 1000(k+1)^2 + 3(k+1)$ tengsizlikdan

$k^3 + 3k^2 + 3k + 1 - 4 > 1000k^2 + 2000k + 1000 + 3k + 3$ tengsizlikka ega bo'lamiz.

Induksiya faraziga ko'ra, $3k^2 + 3k + 1 \geq 2000k + 1003$ tengsizlikni yoza olamiz. Bu tengsizlikni isbotlash uchun yana bir bor matematik induksiya metodini qo'llaymiz.

1. $k=2000$ da yordamchi tengsizlik o‘rinli bo‘ladi.

2. Qandaydir $q>2000$ da $3q^2+3q+1\geq 2000q+1003$ tengsizlikni o‘rinli deb faraz qilib, $k=q+1$ uchun $3(q+1)^2+3(q+1)+1\geq 2000(q+1)+1003$ tengsizlikning o‘rinli ekanligini isbotlaymiz.

Ba’zi soddalashtirishlarni amalga oshirgandan so‘ng, $6q+6\geq 2000$ tengsizlikka ega bo‘lamiz, bu tengsizlik esa $q\geq 2000$ da o‘rinli. Berilgan tengsizlik isbotlandi.

5 - misol. Quyidagi tengsizlikni $n>1$ natural sonlar uchun isbotlang:

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+...+\frac{1}{\sqrt{n}}>\sqrt{n}$$

Isboti. 1. $n=2$ bo`lganda $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}>\sqrt{2}$ tengsizlik o`rinli.

2. Faraz qilaylik, berilgan tengsizlik $n=k$ uchun o`rinli, ya’ni $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+...+\frac{1}{\sqrt{k}}>\sqrt{k}$ bo`lsin, uning $n=k+1$ uchun o`rinli ekanini ko`rsatamiz:

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+...+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}>\sqrt{k+1}$$

Bu tengsizlikda induksiya faraziga ko‘ra, $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+...+\frac{1}{\sqrt{k}}$ o`rniga $\sqrt{k}$ ni qo`yib $\sqrt{k}+\frac{1}{\sqrt{k+1}}>\sqrt{k+1}$ (1) hosil qilamiz. Bu tengsizlikni o`rinli ekanini ko`rsatsak, berilgan tengsizlik isbotlangan bo`ladi.

(1) ning har ikki tomonini kvadratga ko`taramiz, u holda

$$k+\frac{1}{k+1}+\frac{2\sqrt{k}}{\sqrt{k+1}}>k+1 \quad \frac{2\sqrt{k}}{\sqrt{k+1}}>\frac{k}{k+1}$$

tengsizlik hosil bo`ladi. Bu tengsizlikni har ikkala tomonini $\sqrt{\frac{k}{k+1}}$ ga bo`lsak, $2>\sqrt{\frac{k}{k+1}}$ tengsizlik k ning $k>1$ natural qiymatlarida o`rinli, shuning uchun

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+...+\frac{1}{\sqrt{n}}>n$$

Tengsizlik n ning har qanday qiymatida o'rinli bo'ladi.

6-misol. Fibonachchi ketma-ketligi quyidagi shartlar bilan beriladi: $a_0=0,\ a_1=1,\ a_{n+1}=a_n+a_{n-1}$. $a_{n+1}\cdot a_{n+2}-a_n\cdot a_{n+3}=(-1)^n$ tenglikni isbotlang.

Isboti. Berilgan shartga ko'ra, $a_2=1, a_3=2, a_4=3$ ekani kelib chiqadi.

1. $n=1$ da $a_2\cdot a_3-a_1\cdot a_4=1\cdot 2-1\cdot 3=(-1)^1$ bo'ladi.

2. $n=k, k\in N$ da $a_{k+1}\cdot a_{k+2}-a_k\cdot a_{k+3}=(-1)^k$ tenglik o'rinli bo'lsin, u holda $n=k+1$ da

$$a_{k+2}\cdot a_{k+3}-a_{k+1}\cdot a_{k+4}=a_{k+2}\cdot a_{k+3}-a_{k+1}(a_{k+3}+a_{k+2})=$$
$$=a_{k+2}a_{k+3}-a_{k+1}a_{k+3}-a_{k+1}a_{k+2}=(a_{k+1}+a_k)a_{k+3}-a_{k+1}a_{k+3}-a_{k+1}a_{k+2}=a_{k+1}a_{k+3}+$$
$$+a_k a_{k+3}-a_{k+1}a_{k+3}-a_{k+1}a_{k+2}=-(a_{k+1}a_{k+2}-a_k a_{k+3})=-(-1)^k=(-1)^{k+1}.$$

Demak, tenglik n ning istalgan natural qiymatida o'rinli.

7-misol. n ta to'g'ri chiziq yordamida qismlarga ajratilgan tekislikni faqat oq va qora rangdan foydalanib, umumiy tomonga ega bo'lgan ixtiyoriy ikkita bo'lakni turli rangga bo'yash mumkinligini isbotlang.

Isboti. P_n bilan isbotlanishi zarur bo'lgan tasdiqni belgilaylik, u holda P_1 o'rinli ekani ravshan. Agar P_k o'rinli bo'lsa, u holda P_{k+1} ni isbotlash uchun, *k+1* ta to'g'ri chiziqning k tasini olib, hosil bo'lgan bo'laklarni talab qilingan holda bo'yab chiqiladi, so'ngra *(k+1)* – to'g'ri chiziqning bir tomonida yotgan barcha bo'laklarni qarama-qarshi rangga bo'yaladi. Natijada, tekislikning $k+1$ ta to'g'ri chiziq yordamida hosil qilingan umumiy tomonga ega bo'lgan bo'laklarining barchasi turli rangda bo'ladi.

Mustaqil yechish uchun misollar

Har qanday natural son uchun tenglikning to'g'riligini isbotlang (39-54).

39. $1+2+...+n=\frac{n(n+1)}{2}$.

40. $1+4+7+...+(3n-2)=\frac{n(3n-1)}{2}$.

41. $2+16+56+...+(3n-2)\cdot 2^n=10+(3n-5)\cdot 2^{n+1}$.

42. $5+45+325+...+(4n+1)\cdot 5^{n-1}=n\cdot 5^n$.

43. $1-2+3-...+(-1)^{n-1}n=\frac{n(-1)^{n+1}}{2}+\frac{(-1)^{n+1}+1}{4}$.

44. $1^2+4^2+7^2+...+(3n-2)^2=\frac{n(6n^2-3n-1)}{2}$.

45. $1^3+2^3+3^3+...+n^3=\left[\frac{n(n+1)}{2}\right]^2$.

46. $1^3+3^3+5^3+...+(2n-1)^3=n^2(2n^2-1)$.

47. $1\cdot 2+2\cdot 3+...+n(n+1)=\frac{n(n+1)(n+2)}{3}$.

48. $1\cdot 2\cdot 3+2\cdot 3\cdot 4+...+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$

49. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+...+\frac{1}{n(n+1)}=\frac{n}{n+1}$.

50. $\frac{1}{1\cdot 3}+\frac{7}{3\cdot 5}+\frac{17}{5\cdot 7}+...+\frac{2n^2-1}{(2n-1)(2n+1)}=\frac{n^2}{2n+1}$.

51. $\frac{1\cdot 7}{3\cdot 5}+\frac{3\cdot 9}{5\cdot 7}+\frac{5\cdot 11}{7\cdot 9}+...+\frac{(2n-1)(2n+5)}{(2n+1)(2n+3)}=\frac{n(6n+1)}{3(2n+3)}$.

52. $\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+...+\frac{1}{n(n+1)(n+2)}=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{(n+1)(n+2)}\right).$

53. $3+20+168+...+(2n+1)\cdot2^{n-1}\cdot n!=2^n\cdot(n+1)!-1.$

54. $\frac{1}{2}\cdot2!+\frac{2}{2^2}\cdot3!+\frac{3}{2^3}\cdot4!+...+\frac{n}{2^n}\cdot(n+1)!=\frac{(n+2)!}{2^n}-2.$

Berilgan ifodaning berilgan songa bo'linishini isbotlang(55–68, $n\in N$).

55. $(nq + 5n)\vdots6.$

56. $(7^n+3n-1)\vdots9.$

57. $\left(2^{5n+3}+5^n\cdot3^{n+2}\right)\vdots17$

58. $(72n- 1)\cdot24.$

59. $(13^n + 5)\vdots6$

60. $(15^n + 6)\vdots7.$

61. $\left(6^{2n}- 1\right)\vdots35$

62. $(5^n-3^n+2n)\vdots4$

63. $(5\cdot2^{3n-2}+3^{3n-1})\vdots19$

64. $(6^{2n}+19^n-2^{n+1})\vdots17$

65. $(2^{n+5}\cdot3^{4n}+5^{3n+1})\vdots37.$

66. $(11^{n+2}+12^{2n+1})\vdots133$

67. $(3^{2n+2}-8n-9)\vdots64.$

68. $(3^{2n+2}\cdot5^{2n}-3^{3n+2}\cdot2^{2n})\vdots1053$

Berilgan tengsizliklarni matematik induksiya metodi bilan isbotlang (69-86, $n\in N$)

69. $5^n>7n-3$

70. $n\geq7,\ \ 2^{n-1}>n(n+1)$

71. $3^n\geq2^n+n.$

72. $4^n\geq3^n+n^2.$

73. $n\geq2,\ \ 4^n>3^n+2^n.$

74. $n\geq10,\ \ 2^n>n^3.$

75. $n=1,2\ va\ n\geq6,\ \ 2^n>2n^2-3n+1.$

76. $\frac{1}{n+1}+\frac{1}{n+2}+...\frac{1}{3n+1}>1.$

77. $\sqrt{n}<1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{n}}<2\sqrt{n}.$

78. $\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot...\cdot\frac{2n-1}{2n}\leq\frac{1}{\sqrt{3n+1}}.$

79. Agar $a>b$ va a,b - musbat sonlar bo‘lsa, u holda $a^n>b^n$ bo‘lishini isbotlang.

80. $\{a_n\}$ ketma–ketlik rekkurent formula bilan berilgan: $a_1=3,\ a_{n+1}=7a_n+3.$ $a_n=\frac{7^n-1}{2}$ ni isbotlang.

81. $\{b_n\}$ ketma–ketlik rekkurent formula bilan berilgan: $b_1=4,\ b_{n+1}=3b_n-2.$ b_n ni n orqali ifodalang.

82. $\{s_n\}$ ketma–ketlik rekkurent formula bilan berilgan: $c_1=6,\ c_{n+1}=2c_n-n+2.$ $c_n=2^n+3n+1$ ni isbotlang.

83. $\{d_n\}$ ketma – ketlik rekkurent formula bilan berilgan: $d_1=7,\ d_2=27,\ d_{n+2}=6d_{n+1}-5d_n.d_n$ ni n orqali ifodalang.

84. Bir tekislikda yotgan va umumiy nuqtaga ega bo‘lgan n to‘g‘ri chiziq tekislikni $2n$ ta bo‘lakka bo‘lishini isbotlang.

85. Har uchtasi kesishadigan va hech qanday to‘rttasi umumiy nuqtaga ega bo‘lmagan n ta tekislik fazoni necha qismga ajratadi? $\frac{(n-1)n(n+1)}{6}+n+1.$

86. Tekislikda n ta aylana shunday chizilganki, ulardan har ikkitasi ikki nuqtada kesishadi va hech qanday uchtasi umumiy nuqtaga ega emas. Tekislik bunda nechta qismga bo‘linadi? n^2-n+2

3-§. Sanoq sistemalari: pozitsion va pozitsion bo‘lmagan sanoq sistemalari, ixtiyoriy sanoq sistemasida arifmetik amallar

Barcha mavjud tillar kabi sonlar tili ham mavjud bo‘lib, u ham o‘z alifbosiga ega. Mazkur alifbo hozir jahonda qo‘llanilayotgan 0 dan 9 gacha bo‘lgan o‘nta arab raqamlaridir, ya’ni: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Bu tilda o‘nta belgi (raqam) bo‘lganligi uchun ham, bu til o‘nlik sanoq sistemasi deb ataladi.

Bizning kundalik hayotimizda qo‘llanilayotgan o‘nlik sanoq sistemasi hozirgidek yuqori ko‘rsatkichni tez egallamagan. Turli davrlarda turli xalqlar bir–biridan keskin farqlanuvchan sanoq sistemalaridan foydalanganlar.

Masalan, 12 lik sanoq sistemasi juda keng qo‘llanilgan.Uning kelib chiqishida albatta tabiiy hisoblash vositasi – qo‘limizning ahamiyati katta. Bosh barmog‘imizdan farqli qolgan to‘rttala barmog‘imizning har biri 3 tadan, ya’ni hammasi bo‘lib 12 ta bo‘g‘indan iboratdir. Mazkur sanoq sistema izlari hanuzgacha saqlanib qolgan. Masalan, inglizlarda

Uzunlikni o‘lchash birligi: 1 fut = 12 dyum=30 sm,

pul birligi: 1 shilling = 12 pens.

Qadimgi Bobilda ancha murakkab bo‘lgan sanoq sistemasi – 60lik sanoq sistemasi qo‘llanilgan. Bu sanoq sistemasining qoldiqlari hozir ham bor. Masalan:

1 soat = 60 minut

1 minut = 60 sekund

XVI – XVII asrlargacha Amerika qit’asining katta qismini egallagan atstek va mayyalarda 20 lik sanoq sistemasi qo‘llanilgan. Bunday misollarni ko‘plab keltirish mumkin.

Biz asosan o‘nlik sanoq sistemasidan foydalanamiz. Lekin, o‘nlik sanoq sistemasidan kichik sanoq sistemalarida sonlarni belgilash uchun arab raqami belgilaridan foydalaniladi. Masalan, beshlik sanoq sistemasida 0, 1, 2, 3, 4 raqamlari, yettilik sanoq sistemasida esa 0, 1, 2, 3, 4, 5, 6 raqamlaridan foydalaniladi.

Hisoblash texnikasida va dasturlashda asosi 2, 8 va 16 ga teng bo'lgan sanoq sistemalari qo'llaniladi.

O'n ikkilik, o'n oltilik sanoq sistemalarida qanday belgilardan foydalaniladi?– degan savolga javob aniq: raqamlardan keyin lotin alifbosidagi bosh harflardan foydalaniladi.

Shunday qilib, o'n ikkilik sanoq sistemasida raqamlar 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B kabi; o'n oltilik sanoq sistemasida esa 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F kabi yoziladi.

Sanoq sistemasi bu – sonlarni o'qish va arifmetik amallarni bajarish uchun qulay ko'rinishda yozish usuli.

Qadimda hisob ishlarida ko'proq barmoqlardan foydalanilgan. Shu sababli narsalarni 5 yoki 10 tadan taqsimlashgan. Keyinchalik o'nta o'nlik maxsus nom – yuzlik, o'nta yuzlik – minglik nomini olgan va h.k. Yozuv qulay bo'lishi uchun bu muhim sonlar maxsus belgilar bilan ifodalana boshlagan. Agar hisoblashda 2 ta yuzlik, 7 ta o'nlik, yana 4 ta birlik bo'lsa, u holda yuzlikning belgisini ikki marta, o'nlik belgisini yetti marta, birlik belgisini to'rt marta takrorlashgan. Birlik, o'nlik va yuzliklarning belgisi bir–biriga o'xshash bo'lmagan. Sonlarni bunday yozganda belgilarni ixtiyoriy tartibda joylashtirish mumkin bo'lgan, chunki yozilgan sonning qiymati tartibga bog'liq emas. Bunday yozuvda belgi holatining ahamiyati bo'lmaganidan, mos sanoq sistemasi nopozitsion sistema deb ataladi. Qadimgi misrliklar, yunonlar va rimliklarning sanoq sistemasi nopozitsion edi. Nopozitsion sanoq sistemasi qo'shish va ayirish amallari uchun ozgina yarasada, ko'paytirish va bo'lish uchun butunlay yaroqsiz edi. Ishni osonlashtirish maqsadida hisob taxtalari – abaklar ishlatilar edi. Hozirgi zamon cho'tlari abakning o'zgargan ko'rinishidir.

Qadimgi bobilliklarning sanoq sistemasi dastlab nopozitsion edi, keyinchalik ular belgilarni yozish tartibida ham informatsiya borligini sezishib, undan foydalanishga o'rganishdi va pozitsion sanoq sistemasiga o'tishdi. Bunda biz hozir qo'llayotgan sistemadan (raqamning o'rni bir xonaga siljitilganda uning qiymati 10 martaga o'zgaradigan o'nli sanoq sistemadan) farqli, bobilliklarda belgi bir xonaga siljitilganda sonning qiymati 60 marta o'zgarar edi (bunday sanoq sistemasi oltmishli sistema deb ataladi). Uzoq vaqtgacha Bobilning sanoq sistemasida nol belgisi, ya'ni bo'sh qolgan xonaning belgisi yo'q edi. Odatda, sonlarning tartibi ma'lum bo'lganidan bu noqulay emas edi. Ammo keng ko'lamli matematik va astronomik jadvallar tuzish boshlanganda, ana shunday belgiga ehtiyoj tug'ildi. Bu belgi keyinchalik mixxat yozuvlarda va eramizning boshida Iskandariyada tuzilgan jadvallarda uchraydi. IX asrda nol uchun maxsus belgi paydo bo'ldi. O'nli sanoq sistemasida sonlar ustida amallar bajarish qoidasi ishlab chiqildi. Muhammad ibn Muso al-Xorazmiy tomonidan yozilgan "Hind hisobi" nomli risola tufayli o'nli sanoq sistemasi Yevropaga, keyin esa butun dunyoga tarqaldi.

Sanoq sistemasining asosi uchun nafaqat 10 va 60 ni, balki birdan katta ixtiyoriy p natural sonni olish mumkin.

Sanoq sistemalarini tashkil etilishi deyarli bir xil. Biror p soni – sanoq sistemasi asosi sifatida qabul qilinib, ixtiyoriy N soni quyidagi ko'rinishda ifodalanadi:

$$N = a_n p^n + a_{n-1} p^{n-1} + \dots + a_1 p^1 + a_0 p^0 + a_{-1} p^{-1} + \dots + a_{-m} p^{-m}$$

Ko'phad ko'rinishida ifodalangan shu sonni

$$\left(a_n a_{n-1} \ldots a_1 a_0 a_{-1} \ldots a_{-m}\right)_p$$

kabi yozish ham mumkin (n va m – sonning butun va kasr qismi xonalari (razryadlari soni).

Sonning bu kabi ifodalanishida har bir raqam qiymati o‘z o‘rniga qarab turli xil bo‘ladi. Masalan, o‘nlik sanoq sistemasida 98327 sonida 7 – raqami birlikni, 2 – o‘nlikni, 3 – yuzlikni, 8 – minglikni, 9 – o‘n minglikni ifodalaydi (bu hol faqat o‘nlik sanoq sistemasida):

$98327 = 9\times10^4 + 8\times10^3 + 3\times10^2 + 2\times10^1 + 7\times10^0$

Biror boshqa p – asosli sanoq sistemasida $a_0, a_1, a_2 \ldots$ raqamlar $a_0, a_1 p, a_2 p^2, \ldots$ qiymatlarni bildiradi.

Bunday ko‘rinishda tuzilgan sanoq sistemalari pozitsion sanoq sistemalari deyiladi.

Pozitsiyali sanoq sistemasida ***butun sonlarni*** *quyidagi* ***qonuniyat asosida hosil qilinadi:*** *keyingi son oldingi sonning o‘ngdagi oxirgi raqamini surish orqali hosil qilinadi; agar surishda biror raqam 0ga aylansa, u holda bu raqamdan chapda turgan raqam suriladi.*

Shu qonuniyatdan foydalanib, birinchi 10 ta butun sonni hosil qilamiz:

- Ikkilik sanoq sistemasida : 0, 1, 10, 11, 100, 101, 110, 111, 1000, 1001;
- Uchlik sanoq sistemasida : 0, 1, 2, 10, 11, 12, 20, 21, 22, 100;
- Beshlik sanoq sistemasida : 0, 1, 2, 3, 4, 10, 11, 12, 13, 14;
- Sakkizlik sanoq sistemasida : 0, 1, 2, 3, 4, 5, 6, 7, 10, 11.

Pozitsion sanoq sistemasi o‘zining qulayligi bilan hayotda keng qo‘llanilmoqda.

Boshqa usulda tuziladigan sanoq sistemalari ham mavjud. Ular pozitsiyaga bog‘liq bo‘lmagan sanoq sistemalari deyiladi. Masalan rim raqamlari.

Mazkur sistemada maxsus belgilar to'plami kiritilgan bo'lib, ixtiyoriy son shu belgilar ketma–ketligidan iborat bo'ladi.

Rim sanoq sistemasida

Bir (1)	–	I belgi bilan;
Besh (5)	–	V belgi bilan;
O'n (10)	–	X belgi bilan;
Ellik (50)	–	L belgi bilan;
Yuz (100)	–	C belgi bilan;
Besh yuz (500)	–	D belgi bilan;
Ming (1000)	–	M bilan belgilanadi.

Bu belgilar va ularning kombinatsiyasi yordamida turli sonlarni hosil qilinadi. Masalan, 1 dan 3 gacha – I, II, III kabi, to'rt (4) – IV , 5 – V tarzida ifodalanadi. Bu yerda 4 sonini yozish uchun 5 sonidan 1 sonini ayirib yoziladi, ya'ni I belgi V dan oldinga qo'yilsa ayirish ma'nosini, agar keyinga qo'yilsa qo'shishni anglatadi. Umumiy holda: 6 – VI, 7 – VII, 400 – CD, 600 – DC ko'rinishda ifodalanadi.

Rim sanoq sistemasida yozilgan sonlarni o'nlik sanoq sistemasiga quyidagicha o'tkazish mumkin:

$VI \to V \geq I \to 5 + 1 = 6$

$IV \to (I \geq V)? \to 5 - 1 = 4$

$XIX \to X + (I \geq X)? \to 10 + (10-1) = 19$

$XCIX \to (X \geq C)? + (I \geq X)? \to (100-10) + (10-1) = 99$

$MCMLXIII \to M+(C \geq M)?+L+X+I+I+I \to 1000+(1000-100)+50+1+1+1 = 1963.$

Demak, bu sistemada har bir belgining ma'nosi va qiymati uning turgan pozitsiyasiga bog'liq emas. Shuning uchun rim raqamlarini hayotda keng qo'llash imkoniyati bo'lmagan. Ammo ularni kitoblar bobini qo'yishda, soatlarni yozuvida va boshqalarda qo'llab turamiz.

1-misol. Qaysi sanoq sistemasida 21+24 = 100 bo'ladi?

Yechilishi. x – qidirilayotgan sanoq sistemasini asosi bo'lsin. U holda $100_x = 1\cdot x^2 + 0\cdot x^1 + 0\cdot x^0$, $21_x = 2\cdot x^1 + 1\cdot x^0$, $24_x = 2\cdot x^1 + 4\cdot x^0$ bo'ladi. Demak, $x^2 = 2x + 2x + 5$ yoki $x^2 - 4x - 5 = 0$ bo'ladi. Bu tenglamaning musbat yechimi x=5 bo'ladi. Demak, sonlar beshlik sanoq sistemasida berilgan ekan.

Ikkilik sanoq sistemasida 2 ta raqam: 0 va 1 mavjud. O'nlik, sakkizlik sanoq sistemasidagi sonlar ikkilik sanoq sistemasida quyidagicha ifodalanadi:

O'nlik s/s. son	0	1	2	3	4	5	6	7	8	9	10	11
Sakkizlik s/s. son	0	1	2	3	4	5	6	7	10	11	12	13
Ikkilik s/s. son	0	1	10	11	100	101	110	111	1000	1001	1010	1011

Ikkilik sanoq sistemasidagi sonlar ustida turli arifmetik amallar bajarishga oid misollar ko'raylik:

2-misol. 10011+ 11001

Yechilishi.

$$\begin{array}{r} + \; 10011 \\ 11001 \\ \hline 101100 \end{array}$$

Javob: 101100

3-misol. 1101101,001+1000101,001

Yechilishi.

$$\begin{array}{r} + \; 1101101{,}001 \\ 1000101{,}001 \\ \hline 10110010{,}010 \end{array}$$

Javob: 10110010,010

4-misol. 101010 – 10011

Yechilishi.

$$\begin{array}{r} - \; 101010 \\ 10011 \\ \hline 10111 \end{array}$$

Javob: 10111

5-misol. 110011,01 – 10111,101

Yechilishi.

$$\begin{array}{r} - \; 110011{,}010 \\ 10111{,}101 \\ \hline 11011{,}101 \end{array}$$

Javob: 11011,101

6-misol. 110011 × 101

Yechilishi.

$$\begin{array}{r} 110011 \\ \times \; 101 \\ \hline + \; 110011 \\ 110011 \\ \hline 11111111 \end{array}$$

Javob: 11111111

7-misol. 101,11 × 11,01

Yechilishi.

$$\begin{array}{r} 101{,}11 \\ \times \; 11{,}01 \\ \hline 10111 \\ + \; 10111 \\ 10111 \\ \hline 10010{,}1011 \end{array}$$

Javob: 10010,1011

8-misol. 1000010010: 110101   **9-misol.** 1000000,10011:110,101

Yechilishi:

```
_1000010'010 | 110101
 110101      |--------
-------------| 1010
  _110101
   110101
  -------
        0
```

Javob: 1010

Yechilishi:

```
_ 100000010'011 | 11010100
   11010100     |----------
----------------| 1001,11
  _ 101110 011
     11010 100
  -------------
   _ 10011 1110
      1101 0100
  -------------
    _ 110 10100
      110 10100
  -------------
              0
```

Javob: 1001,11

10-misol. $a = 6467_8$, $b = 101_3$ sonlarni asosi g = 5 bo'lgan sanoq sistemasida yozing va bu sonlarning kattasini kichigiga bo'ling.

Yechilishi. $5 = 12_3$ bo'lgani uchun quyidagilarni yoza olamiz.

```
1)_6467₈    |5
   5        _1244   |5
 _14         12    _ 207    |5
  12        _ 44     17    _ 33    |5
 _26          43    _17     31     5    |5
  24           1     17      2     5    1
 _27                  0            0
  24
   3
```

$a = 6467_8 = 102013_5$.

```
2) 101₃    |12₃
   101       2

     0
```

$$b = 101_3 = 20_5$$

$$
\begin{array}{l|l}
1)102013_5 & 20_5 \\
\underline{\ 40} & 2323_5 \\
\ \ 120 & \\
\underline{110} & \\
101 & \\
\underline{40} & \\
113 & \\
\underline{110} & \\
\ \ 3 &
\end{array}
\qquad \Rightarrow 102013_5 = 20_5 \cdot 2323_5 + 3
$$

Javob: $a = 102013_5$; $b = 20_5$,

$102013_5 = 20_5 \cdot 2323_5 + 3$

11-misol. Berilgan sistematik sonlarni surati va maxraji o'nlik sanoq sistemasida yozilgan oddiy kasr ko'rinishida ifodalang: $a)2,3_4; b)0,04_5; c)2,012_3$.

Yechilishi. $a)\ 2,3_4 = 2 + \frac{3}{4} = \frac{11}{4}$;

$b)\ 0,04_5 = 0 + 0/5 + 4/5^2 = 4/25$;

$c)\ 2,012_3 = 2 + 0/3 + 1/3^2 + 1/3^3 = (54 + 0 + 3 + 2)/27 = 59/27$.

12-misol. Berilgan sistematik sonlarni surati va maxraji shu sanoq sistemasida yozilgan oddiy kasr ko'rinishida ifodalang: $a)0,04_6; b)2,3_4; c)2,012_3$.

Yechilishi. $a)0,04_6 = \left(\frac{4}{100}\right)_6 = \left(\frac{1}{13}\right)_6$;

$b)2,3_4 = \left(\frac{23}{10}\right)_4$; $c)2,012_3 = \left(\frac{2012}{1000}\right)_3$.

13-misol. Berilgan sistematik sonlarni surati va mahraji shu sanoq sistemasida yozilgan oddiy kasr ko'rinishida ifodalang: $a)0,0(2)_4; b)0,1(4)_7; c)0,(23)_6$.

Yechilishi. a) $0,0(2)_4=\left(\frac{2}{3\cdot 10}\right)_4=\left(\frac{1}{3\cdot 2}\right)_4=\left(\frac{1}{12}\right)_4$

b) $0,1(4)_7=\left(\frac{14-1}{6\cdot 10}\right)_7=\left(\frac{13}{6\cdot 10}\right)_7=\left(\frac{5}{30}\right)_7$

c) $0,(23)_6=\left(\frac{23}{55}\right)_6=\left(\frac{3}{11}\right)_6$.

14-misol. Berilgan oddiy kasrlarni shu sanoq sistemasida sistematik sonlar ko'rinishida ifodalang: $a)\frac{137}{40};b)\left(\frac{17}{40}\right)_8;c)\left(\frac{10}{8}\right)_{12}$

Yechilishi.

a)137	|40	*b*) 17_8	|40	*c*)10_{12}	|8
120	3,425	170	$0,36_8$	8	$1,6_{12}$
170		140		40	
160		300		40	
100		300		0	
80		0			
200					
200					
0					

$\frac{137}{40}=3,425$

$\left(\frac{17}{40}\right)_8=0,36_8$

$\left(\frac{10}{8}\right)_{12}=1,6_{12}$

15–misol. Berilgan oddiy kasrlarni shu sanoq sistemasidagi sistematik sonlarga yoying: $a)\left(\frac{3}{5}\right)_{11};b)\left(\frac{11}{12}\right)_4;c)\left(\frac{1}{2}\right)_3$.

Yechilishi. $a)\left(\frac{3}{5}\right)_{11}$ kasrni chekli sistematik songa yoyib bo'lmaydi, chunki 5 soni sanoq sistemasining asosi bo'lgan 11 sonini tashkil etuvchi ko‘paytuvchilarga tegishli emas. $(5,11) = 1$ bo‘lgani uchun berilgan son sof davriy kasr bo‘ladi:

$$\begin{array}{l|l} 3_{11} & 5 \\ 30 & 0,66... \\ \underline{28} & \\ \;\;30 & \end{array} \qquad ya'ni \left(\frac{3}{5}\right)_{11} = 0,(6)_{11}$$

$b)$ $12_4 = 3_4 \cdot 2_4$, $(3,4) = 1$, $(2,4) = 2$ bo‘lgani uchun berilgan kasr aralash davriy kasr ko‘rinishida bo‘ladi:

$$\begin{array}{l|l} 11_4 & 12_4 \\ 110 & 0.311... \\ \underline{102} & \\ 20 & \\ \underline{12} & \\ 20 & \end{array} \qquad ya'ni \left(\frac{11}{12}\right)_4 = 0,3(1)_4$$

$c)$ $(2,3) = 1$ bo‘lgani uchun berilgan kasr sof davriy sistematik kasr bo‘ladi:

$$\begin{array}{l|l} 1_3 & 2_3 \\ 10 & 0,11... \\ \underline{\;2} & \\ \;\;10 & \end{array} \qquad ya'ni \left(\frac{1}{2}\right)_3 = 0,(1)_3$$

Mustaqil yechish uchun misollar

Quyidagi ikkilik sanoq sistemasida berilgan sonlar ustida arifmetik amallarni bajaring:

87. 101+111 **88.** 1101+110 **89.** 1111+1011

90. 11,011+101,01

91. 10,101+11,111

92. 110,01+11,0101

93. 111,10+111

94. 1010−110

95. 1100−11

96. 1011−101,11

97. 11011,11−101,01

98. 1111−10,11

99. 1101,101−1001,01

100 10010,01−111,1

101 110001−11,01

102 10000−100,11

103 101·11

104 110·101

105 111·11

106 1011·11,01

107 1111,01·101

108 101,11·1,101

109 11010,11·10,01

110 111·11,101

111 100101·101,011

112 11110 : 101

113 1011010 : 1111

114 1111 : 101

115 11010,01 : 101,01

116 1000,1111 : 10,11

117 1101,1 : 1100

118 10111,101 : 0,11

119 111000001:11111

120 1111110 : 1,11

Sonlarni taqqoslang:

12 1101+11 va 1111 +10

12 1001,11+101,01va 01,01−101,11

12 11101−11 va 111+11

12 1110,01+101 va 10010,01

12 1111+110001 va 11110011−11001

12 1101·1101 va 1011·1011

12 1101,011−11,01 va 1011,001

12 11100111:11 va 1010111:11

12 111111:11 va 10101·11

13 10101 va 1110+111

Tenglamalarni ikkilik sanoq sistemasida yeching:

131. $x+ 1001 = 1000$

132. $(101x-100)/10 = (x+10)/100$

133. $1101\cdot(x+1101) = x-10101$

134. $x^{10}+100\cdot x+100=0$

135. $(10\cdot x-11)\cdot 101= 101101$

136. $x-(111-x)\cdot 11=101x$

137. $(1111\cdot x-11):11 = 101$

138. $x^{10}-10\cdot x+1=0$

139. $10\cdot x^{10}+101\cdot x+1=0$

140. $x^{10}+101\cdot x+10=0$

141. $x^{10}-110001=0$

142. $x^{10}-111\cdot x+1100=0$

Ikkilik sanoq sistemasida $S_n = a \cdot n + b$ ketma–ketlikning birinchi beshta hadini yozing:

143. $a = 10$; $b = 11$

144. $a = 110$; $b = 101$

145. $a = 10,1$; $b = 1,11$

146. $a = 101$; $b = 11$

147. $a = 10$; $b = 101$

148. $a = 11,1$; $b = 11$

149. $a = 10,1$; $b = 101$

150. $a = 1,01$; $b = 111$

To'g'ri burchakli uchburchakning asosi va balandligi ikkilik sanoq sistemasida berilgan. Uchburchak yuzini ikkilik sanoq sistemasida toping:

151. a = 101; h = 11

152. a = 101; h = 110

153. a = 101; h = 101

154. a = 10; h = 101

155. a = 111; h = 111

156. a = 11; h = 11

157. a = 101; h = 101

158. a = 101; h = 111

159. Ikkilik sanoq sistemasida: velosipedchining tezligi $v = 10000$ km / soat bo'lsa, 11; 110; 1111 soatdan keyin qancha yo'l bosib o'tishini toping.

160. Sonlar ikkilik sanoq sistemasida qaraladi: qayiq daryo oqimi bo'ylab 1001 km/soat tezlik bilan 10 soat, oqim bo'ylab 101 km/soat tezlik bilan 11 soat suzganda bosib o'tgan yo'lini toping.

a, b sonlarni asosi g bo'lgan sanoq sistemasida yozing va bu sonlarning kattasini kichigiga bo'ling.

161. $a = 18536$ $b = 430$ $g = 7$

162. $a = 101_2$ $b = 14320_5$ $g = 3$

163. $a = 1653_7$ $b = 201$ $g = 4$

164. $a = 15$ $b = 3571_8$ $g = 11$

165. $a = 132_4$ $b = 443_5$ $g = 2$

166. $a = 201_3$ $b = 6514_7$ $g = 5$

167. $a = 136$ $b = 2632$ $g = 7$

168. $a = 111_2$ $b = 3546_7$ $g = 4$

169. $a=121_3$ $b=4731_9$ $g=8$

170. $a=121_3$ $b=5378$ $g=12$

171. $a=3745_9$ $b=40_5$ $g=6$

172. $a=132_4$ $b=1643_7$ $g=5$

173. $a=4321_5$ $b=13$ $g=8$

174. $a=7356_8$ $b=24_4$ $g=5$

175. $a=101_2$ $b=3542_6$ $g=3$

176. $a=201_3$ $b=13765_8$ $g=4$

Berilgan sistematik sonlarni surati va maxraji o'nlik sanoq sistemasida yozilgan oddiy kasr ko'rinishida ifodalang:

177. $2{,}114_8$.

178. $5{,}442_7$.

179. $74{,}13_8$.

180. $35{,}13_7$.

181. $4{,}521_8$.

182. $0{,}6467_8$.

183. $2{,}224_6$.

184. $7{,}742_9$.

185. $3{,}201_5$.

186. $4{,}234_6$.

Berilgan sistematik sonlarni surati va maxraji shu sanoq sistemasida yozilgan oddiy kasr ko'rinishida ifodalang:

187. 2,1148

188. 5,4427

189. 74,138

190. 35,137

191. 4,5218

192. 0,64678

193. 2,2246

194. 7,7429

195. 3,2015

196. 4,2346

197. 1,1(6)

198. 0,3(2)4

199. 3,1(42)5

200. 4,2(3)5

201. 32,14(2)5

202. 5,01(3)6

203. 2,1(2)7

204. 2,10(3)6

205. 1,1(2)3

206. 0,7(4)8

Berilgan oddiy kasrlarni shu sanoq sistemasidagi sistematik sonlarga yoying:

207. $\left(\frac{112}{100}\right)_3$

208. $\left(\frac{311}{1000}\right)_5$

209. $\frac{43}{80}$

210. $\left(\frac{1}{122}\right)_4$

211. $\left(\frac{151}{30}\right)_6$

212. $\left(\frac{31}{120}\right)_6$

213. $\left(\frac{27}{30}\right)_9$

214. $\left(\frac{17}{40}\right)_9$

215. $\left(\frac{103}{10}\right)_7$

216. $\frac{13}{20}$

217. $\left(\frac{101}{20}\right)_3$

218. $\left(\frac{64}{30}\right)_7$

219. $\left(\frac{331}{40}\right)_5$

220. $\left(\frac{1}{3}\right)_6$

221. $\left(\frac{23}{33}\right)_4$

222. $\left(\frac{24}{5}\right)_6$

223. $\left(\frac{2012}{1000}\right)_3$

224. $\left(\frac{12}{37}\right)_{14}$

225. $\left(\frac{125}{6}\right)_8$

226. $\left(\frac{204}{11}\right)_9$

4-§. Kombinatorika elementlari. Nyuton binomi va uning tatbiqlari

Qo'shish qoidasi. Agar a elementni m usulda, b elementni (a elementning tanlanishiga bog'liq bo'lmagan) n usulda tanlash mumkin bo'lsa, "a yoki b" tanlanmani m+n usulda hosil qilish mumkin.

Ko'paytirish qoidasi. Agar a elementni m usulda, b elementni (a elementning tanlanishiga bog'liq bo'lmagan) n usulda tanlash mumkin bo'lsa, "a va b" tanlanmani $m\cdot n$ usulda hosil qilish mumkin.

Dirixle prinsipi (qutilar prinsipi). $nk + 1$ ta yoki undan ko'p sondagi predmetni n ta qutiga joylaganda qaysidir qutida, albatta, k + 1 tadan kam bo'lmagan predmet bo'ladi.

Misol. Omborda 41–,42– va 43 – o'lchamdagi 200 tadan etik bo'lib, bu 600 ta etikning 300 tasi o'ng va 300 tasi chap oyoqniki. Ulardan kamida 100 juft yaroqli etik olish mumkinligini isbotlang.

Yechilishi. k – o'lchamdagi o'ng va chap oyoq etiklarining soni mos ravishda N(k,l) N(k,r) bo'lsin. Masala shartiga ko'ra,

$$N(k,l) + N(k,r) = 200\ (k = 41, 42, 43);$$
$$N(41, l) + N(42, l) + N(43, l) = 300;$$
$$N(41, r) + N(42, r) + N(43, r) = 300.$$

Har bir o'lchamdagi chap (o'ng) etik uchun o'ng (chap) etiklar sonining kam bo'lishi mumkin emas. Quyidagilarni o'rinli deylik:

$$N(41, l) \le N(41, r),\ N(42, l) \le N(42, r),\ N(43, l) \ge N(43, r)$$

U holda yaroqli juftliklar soni

$$N(41, l) + N(42, l) + N(43, r) = 300 - N(43, l) + N(43, r) \ge 100$$

O'rinlashtirishlar. O'rin almashtirish. Guruhlash.

Ta'rif. Berilgan n ta elementli to'plam elementlaridan k tadan olib tuzilgan va elementlari yoki elementlarining tartibi bilan farq qiluvchi turli xil guruhlar o'rinlashtirishlar deyiladi.

Ta'rif. Berilgan n ta elementdan k tadan olib tuzilgan o'rinlashtirishlarda kamida bitta element bir va undan ortiq, lekin k tadan ortiq bo'lmagan marta qatnashsa, u holda bunday o'rinlashtirish takrorlanuvchi o'rinlashtirish deyiladi va B_n^k orqali belgilanadi.

Teorema. n ta elementdan k tadan olib tuzilgan barcha takrorlanuvchi o'rinlashtirishlar soni $B_n^k = n^k$ ga teng.

Ta'rif. Berilgan n ta elementdan k tadan olib tuzilgan o'rinlashtirishlarda har bir element bir martadan qatnashib, guruhlar bir-biridan elementlari yoki elementlarining tartibi bilan farq qilsa, u holda bunday o'rinlashtirishlar takrorlanmaydigan o'rinlashtirishlar va A_n^k orqali belgilanadi.

Teorema. n ta elementdan k tadan olib tuzilgan barcha takrorlanmaydigan o'rinlashtirish soni $A_n^k = n(n-1)(n-2)...(n-k+1)$ yoki $A_n^k = \frac{n!}{(n-k)!}$ ga teng.

Misol. 1,3,5,7 raqamlaridan nechta turli ikki xonali son hosil qilish mumkin?

Yechilishi. 1,3,5,7 raqamlar soni 4 ta, ulardan ikkitadan olib tuzilgan barcha takrorlanmaydigan o'rinlashtirish soni $A_n^k = \frac{4!}{(4-2)!} = \frac{4!}{2!} = \frac{1\cdot2\cdot3\cdot4}{1\cdot2} = 12$ ga teng. Takrorlanuvchi o'rinlashtirishlar soni esa $B_n^k = n^k = 4^2 = 16$ ga teng. Demak, agar 1,3,5,7 raqamlarning har

O‘rinlashtirishlar. O‘rin almashtirish. Guruhlash.

Ta’rif. Berilgan n ta elementli to‘plam elementlaridan k tadan olib tuzilgan va elementlari yoki elementlarining tartibi bilan farq qiluvchi turli xil guruhlar o‘rinlashtirishlar deyiladi.

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Ta'rif. Berilgan n ta elementli to'plam elementlaridan n tadan olib tuzilgan va faqat elementlarining tartibi bilan farq qiladigan o'rinlashtirishlar o'rin almashtirishlar deyiladi va P_n orqali belgilanadi.

Teorema. n ta elementdan n tadan olib tuzilgan barcha takrorlanmaydigan o'rin almashtirishlar soni $P_n = n!$ bo'ladi.

Misol. 7 kishi bir qator bo'lib necha xil usulda turishi mumkin?

Yechilishi. $P_n = n! = 7! = 1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7 = 5040.$

Ta'rif. Berilgan n ta elementdan k tadan olib tuzilgan hamda o'zining tartibi va elementlari bilan farq qiladigan takrorlanmaydigan o'rinlashtirishlar guruhlash deyiladi va C_n^k orqali belgilanadi (bu yerda $n > k$).

Teorema. n ta elementdan k tadan olib tuzilgan barcha takrorlanmaydigan guruhlashlar soni $C_n^k = \frac{A_n^k}{P_k} = \frac{n(n-1)(n-2)\ldots(n-k+1)}{k!}$ ga teng.

Misol. a) Bitta sihatgohga mo'ljallangan uchta yo'llanmani necha xil usulda to'rt kishiga berish mumkin?

Yechilishi. Bu masalani yechish uchun to'rtta elementni uchtadan guruhlash formulasidan foydalanamiz, ya'ni $C_n^k = \frac{1\cdot 3\cdot 2}{1\cdot 2\cdot 3} = 4.$

b) Turli uchta sihatgohga mo'ljallangan uchta yo'llanmani necha xil usulda to'rt kishiga berish mumkin?

Yechilishi. Bu masalani yechish uchun to'rtta elementni uchtadan takrorlanmaydigan o'rinlashtirish formulasidan foydalanamiz, ya'ni

$A_4^3 = 4(4-1)(4-2)(4-3) = 24.$

Nyuton binomi. $(x+y)^n = C_n^0 x^n + C_n^1 x^{(n-1)} y + C_n^2 x^{(n-2)} y^2 + \ldots + C_n^n y^n$ tenglikdagi $C_n^k \, (k=\overline{0,n})$ koeffitsientlar binomial koeffitsientlar deyiladi, ular x + y binomni darajaga ko‘tarish natijasida hosil qilinadi.

Binomial koeffitsientlarni Paskal uchburchagi deb ataluvchi jadval yordamida tasvirlash qulay:

C_0^0	1
$C_1^0 \quad C_1^1$	1 1
$C_2^0 \quad C_2^1 \quad C_2^2$	1 2 1
$C_3^0 \quad C_3^1 \quad C_3^2 \quad C_3^3$	1 3 3 3
$C_4^0 \quad C_4^1 \quad C_4^2 \quad C_4^3 \quad C_4^4$	1 4 6 4 1
... ...	... ...

Nyuton binomi formulasining umumiy hadi $T_{k+1} = C_n^k x^{n-k} y^k$ formula bilan hisoblanadi.

1-misol. $(1+x^2-x^3)^9$ ning yoyilmasidagi x^8 had oldidagi koffitsientni toping.

Yechilishi.
$$(1+x^2-x^3)^9 = 1 + C_9^1\left(x^2-x^3\right) + C_9^2(x^2-x^3)^2 + C_9^3(x^2-x^3)^3 +$$
$$+ C_9^4(x^2-x^3)^4 + C_9^5(x^2-x^3)^5 + \ldots + (x^2-x^3)^9$$

ga egamiz. Bu tenglikning o‘ng qismidagi ifodada x^8 faqat $C_9^3(x^2-x^3)^3$, $C_9^4(x^2-x^3)^4$ qo‘shiluvchilarda mavjud. Bundan foydalanib, x^8 had oldidagi koffitsientni topamiz, bu koeffitsient $3C_9^3 + C_9^4$ ga teng.

2-misol. $\left(x\sqrt{x}+\frac{1}{x^4}\right)^n$ yoyilmadagi uchinchi hadning binomial koeffitsienti ikkinchi had koeffitsientidan 44 ga katta. Tarkibida x bo'lmagan hadni toping.

Yechilishi: Masala shartiga ko'ra,

$$C_n^2=C_n^1+44, \text{ yoki } \frac{n(n-1)}{2}=n+44$$

Bu tenglamani yechib, n = 11 ni hosil qilamiz. $\left(x\sqrt{x}+\frac{1}{x^4}\right)^{11}$ yoyilmasining umumiy hadi formulasini $C_{11}^m x^{\frac{3}{2}(11-m)-4m}$ ko'rinishda yozish mumkin. Bu yerdan, $\frac{3}{2}(11-m)-4m=0$ yoki m = 3 ga ega bo'lamiz. Demak, izlanayotgan had C_{11}^3 ga teng.

Mustaqil yechish uchun misollar

227. a) Bir davlatda A, B va C shaharlar bo'lib, A shahardan B shaharga 6 ta yo'l, B shahardan C shaharga 4 ta yo'l olib boradi. A shahardan C shaharga necha xil usulda borish mumkin?

b) Bu davlatda yana bir D shahar qurishdi va ikkita A dan D ga boruvchi va ikkita D dan C ga boruvchi yangi yo'l yotqizildi. Endi A shahardan C shaharga necha xil usulda borish mumkin?

228. Ettita raqamdan iborat barcha telefon nomerlari soni nechta (nomer noldan boshlanishi mumkin emas)?

229. Avtomashinaning davlat raqami ingliz alifbosi (26 ta harf) ning uchta harfidan va uchta raqamdan iborat. Davlat raqami turlicha bo'lgan barcha avtomashinalar soni nechta?

230. Maktabda har bir o‘g‘il bola 32 ta qiz bolani taniydi, har bir qiz bola 29 ta o‘g‘il bolani taniydi. Bu maktabda qiz bola ko‘pmi yoki o‘g‘il bola ko‘pmi? Necha marta ko‘p?

231. a) 5 ga bo‘linuvchi nechta olti xonali son mavjud?

b) Tarkibida kamida bitta juft raqam bo‘lgan nechta olti xonali son mavjud?

c) Tarkibida kamida ikkitasi bir xil raqam bo‘lgan o‘n xonali sonlar sonini toping.

d) Yetti xonali sonlar orasida tarkibida bir raqami bo‘lganlari ko‘pmi yoki qolgan yetti xonali sonlar ko‘pmi?

232. a) O‘ng va chapdan o‘qilganda bir xil bo‘lgan besh xonali sonlar nechta (masalan, 54345, 17071 kabi sonlar)?

b) Raqamlari yig‘indisi juft bo‘lgan o‘n xonali sonlar nechta?

c) Yozuvida faqat toq sonlar bo‘lgan to‘rt xonali sonlar soni nechta?

233. Qiymati turlicha bo‘lgan 7 ta tangani uchta cho‘ntakka necha xil usulda joylash mumkin?

234. Qopchada 20 tasi qizil, 20 tasi ko‘k, 20 tasi sariq va qolganlari oq va qora rangda bo‘lgan 70 ta shar bor. Sharlarni ko‘rmasdan turib, olingan sharlarning kamida 10 tasi bir xil rangda bo‘lishi uchun eng kam miqdordagi nechta shar olish kerak?

235. Aylanma stol atrofida yarmidan ko‘prog‘i erkaklar bo‘lgan 100 kishi o‘tiribdi. Qaysidir ikki erkak kishi bir – biriga qarama – qarshi o‘tirishini isbotlang.

236. Turnirda bir nechta futbol jamoasi bir – birlari bilan turnir o‘tkazishmoqda. Bu turnirning ixtiyoriy momentida bu paytgacha o‘ynagan o‘yinlari soni teng bo‘lgan ikki jamoa mavjudligini isbotlang.

237. 2000 dan katta bo'lmagan 1002 ta son berilgan. Bu sonlardan ikkitasining yig'indisi uchinchisiga teng bo'ladigan uchta sonni ajratib olish mumkinligini isbotlang. Bu mulohaza 1002 sonini 1001 bilan almashtirilsa ham o'rinli bo'ladimi?

238. Passajir poyezdi 17 ta vagondan iborat. Agar har bir vagonga bittadan kuzatuvchi biriktirilsa, 17 ta kuzatuvchini necha xil usul bilan vagonlarga biriktirish mumkin?

239. 1,...,7 raqamlaridan o'rin almashtirishlar yordamida hosil qilinadigan barcha yetti xonali sonlar yig'indisini toping.

240. Agar tanlanayotgan lavozimga to'qqiz kishi talabgor bo'lsa, to'rt kishini to'rtta turli lavozimga necha xil usulda tanlash mumkin?

241. Ikkita matematik va o'nta iqtisodchidan sakkiz kishidan iborat tekshiruv guruhini tuzish kerak. Agar bu guruhda kamida bitta matematik bo'lishi shart bo'lsa, bunday tekshiruv guruhini necha xil usul bilan tuzish mumkin?

242. Birinchi talabada 7 ta turli kitob, ikkinchi talabada 9 ta turli kitob bor. Ular 5 ta kitobni necha xil usulda bir – birlari bilan almashtira oladilar.

243. Berilgan ifodalarning yoyilmasida nechta ratsional qo'shiluvchi bor?

a) $(\sqrt{2}+\sqrt[4]{3})^{100}$; b) $(\sqrt{2}+\sqrt[3]{3})^{300}$

244. Ixtiyoriy a natural son uchun shunday n natural son topiladiki, $n+1,\ n^n+1,\ n^{n^n}+1,\ldots$ sonlarning barchasi a ga bo'linadi. Isbotlang.

245. Qavariq a) o'nburchak; b) k burchak (k>3) ning nechta diagonali bor?

246. Har bir keyingi raqami oldingisidan kichik bo'lgan nechta olti xonali son mavjud?

247. $x_1+x_2+x_3=1000$ tenglama a) natural sonlar to‘plamida; b) manfiymas butun sonlar to‘plamida nechta yechimga ega?

248. Nima uchun $11^2=121, 11^3=1331$ tengliklar Paskal uchburchagi satrlarini eslatadi? 11^4 nimaga teng?

249. n ning qanday qiymatlarida $(x+y)^n$ Nyuton binomidagi barcha koeffitsientlar toq bo‘ladi?

250. Hisoblang:

$a) C_5^0+2C_5^1+2^2C_5^2+...+2^5C_5^5;$

$b) C_n^0-C_n^1+...+(-1)^nC_n^n;$

$c) C_n^0+C_n^1+...+C_n^n.$

251. Ayniyatlarni isbotlang:

$a) C_r^mC_m^k=C_r^kC_{(r-k)}^{(m-k)};$

$b) C_{(n+1)}^{(m+1)}=C_n^m+C_n^{(m+1)};$

$c) C_2n^n=(C_n^0)^2+(C_n^1)^2+...+(C_n^n)^2;$

$d) C_{(n+m)}^k=C_n^0C_m^k+C_n^1C_m^{(k-1)}+...+C_n^kC_m^0$

$e) C_n^k=C_{(n-1)}^{(k-1)}+C_{(n-2)}^{(k-1)}+...+C_{(k-1)}^{(k-1)}.$

252. $(x+y)^n$ Nyuton binomi yoyilmasining ikkinchi hadi 240 ga, uchinchi hadi 720 ga, to‘rtinchisi 1080 ga teng bo‘ldi. x, y va n larni toping.

253. Quyidagi tenglikdan m va n larni toping: $C_{n+1}^{m+1} : C_{n+1}^{m} : C_{n+1}^{m-1}=5:5:3$

254. $(1+\sqrt{3})^{100}$ formulaning Nyuton binomi bo‘yicha yoyilmasidagi eng katta qo‘shiluvchini toping.

255. $(1+x+\frac{6}{x})^{10}$ formulaning Nyuton binomi bo‘yicha yoyilmasidagi tarkibida x bo‘lmagan hadni toping.

256. Tenglamalarni yeching:

$a)(x+1)!/x!=6;$

$b)C_x^{(x-2)}=28;$

$c)C_{(x-3)}^{(x-5)}=x^2-3x-9;$

$d)C_x^2+C_{(x-1)}^2=x+1;$

5-§. Butun koeffitsientli aniqmas tenglamalar

Berilgan kasrni zanjirli kasrga aylantirish tushunchasi bizga algebra va sonlar nazariyasi fanidan ma'lumdir.

1-misol. $\frac{539}{103}$ sonni zanjir kasrga aylantiring.

Yechilishi: Buning uchun kasr suratini uning maxrajiga bo'lamiz, ya'ni

539|103
515|5
103|24
96|4
24|7
21|3
7 |3
6|2
3|1
3|3
0

Demak , $\frac{539}{103}=5+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3}}}}=[5;4,3,2,3].$

Agar α sonni zanjirli kasrga yoyganda $[a_0, a_1, a_2, \ldots]$ hosil bo‘lib $\frac{P_k}{Q_k}$,

$\frac{P_k}{Q_k}$

Qo‘shni yaqinlashuvchi kasr bo‘lsa, u holda $\left|\alpha - \frac{P_k}{Q_k}\right| \leq \frac{1}{Q_k Q_{k+1}}$

munosabat o‘rinli ekanligini ko‘rsatish mumkin.

Ma’lumki, zanjirli kasrng shartidan

$$\frac{P_0}{Q_0} = \alpha_0, \frac{P_1}{Q_1} = \frac{Q_0 Q_1 + 1}{Q_1}, \ldots, \frac{P_k}{Q_k} = \alpha$$

Munosabatlar aniqlangandir.

Misol. $\frac{2517}{773} = [3;3,1,9,2,2,1,2]$

$$\frac{P_0}{Q_0} = \frac{\alpha_0}{1} = 3; \frac{P_1}{Q_1} = \frac{10}{3}; \ldots$$

K	0	1	2	3	4	5	6	7
a_0	3	3	1	9	2	2	1	2
P_k	3	10	13	127	267	661	928	2517
Q_k	1	3	4	38	82	203	285	773

$$\left|\frac{2517}{773} - \frac{127}{39}\right| < \frac{1}{39 \cdot 82} = \frac{1}{3498}$$

Ekanligini hisobga olsak, u holda bo‘lishini ko‘rish mumkin.

Agar berilgan α sonni zanjirli kasrga yoyganda

$$\alpha = [\alpha_0, \alpha_1, \alpha_2, \ldots] = [\alpha_0; \alpha_1, (\alpha_2, \alpha_3) \ldots]$$

natija olinsa bu natijada a_2 va a_3 larning takrorlanishini ko'ramiz.

2-misol. $142x+82y=6$ tenglamani butun yechimlarini toping.

Yechilishi. $(142,82)=2;6:2$ bundan tenglama yechimga ega ekanligini ko'rishimiz mumkin.

Bundan $71x+41y=3$ natijani hosil qilamiz, so'ngra $\frac{71}{41}=[1;1,2,1,2,1,2]$.

Endi barcha yaqinlashuvchi kasrlarni tuzamiz:

$$\frac{P_0}{Q_0}=1,\frac{P_1}{Q_1}=2,\frac{P_2}{Q_2}=\frac{5}{3},\frac{P_3}{Q_3}=\frac{7}{4},\frac{P_4}{Q_4}=\frac{19}{11},\frac{P_5}{Q_5}=\frac{26}{15},\frac{P_6}{Q_6}=\frac{71}{41},$$

Yaqinlashuvchi kasrning

$P_{k-1}Q_k-P_kQ_{k-1}=(-1)^k$ xossasiga ko'ra

$26\cdot41=71\cdot15=(-1)^k$ yoki $71\cdot(-15)+41\cdot26=1$ ni hosil qilamiz,

So'ngra ikkala tomonini 3 ga ko'paytirib $71\cdot(-45)+41\cdot78=3$

ko'ra $x_0=-45$, $y_0=78$ xususiy yechimlarini hosil qilamiz, umumiy yechim esa

$x=-45+41t \qquad x=-4+41t$

$y=78-71t \qquad y=7-71t$, bu yerda $t\epsilon Z$.

3-misol. Yuk tashuvchi tashkilotdan 53t yukni bir qatnovda tashib berishni iltimos qilishdi . Bu tashkilot yukni tashish uchun yuk ko'tarish quvvati 3,5t va 4,5t li avtomashinalardan ajratdi . Tashkilot har bir mashinadan nechtadan ajratgan?

Yechilishi. Yuk tashuvchi tashkilot mashinalarning 3,5 t lisidan x ta 4,5 t lisidan y ta ajratgan bo'lsin , u holda $3{,}5x+4{,}5y=53$ tenglama hosil bo'ladi.

$35x+45y=530$ yoki $7x+9y=106$

$\frac{7}{9}=[0;1,2,3]$

K	0	1	2	3
a_k	0	1	3	2
P_k	0	1	3	7
Q_k	1	1	4	9

Jadvaldan ko'rinib turibdiki, $39-47=-1 \Rightarrow 47-39=1 \Rightarrow$

$$\begin{cases} 4\cdot106+9t\geq0 \\ (-3)\cdot106-7t\geq0 \end{cases} \quad 7\cdot(4\cdot106)+9\cdot((-3)\cdot106)=106$$

$x_0=4\cdot106, y_0=-3\cdot106, x=4\cdot106+9t, \ y=(-3)\cdot106-7t, \ t\in Z$ Endi yechimlardan musbatini ajratamiz

$$\begin{cases} 4\cdot106+9t\geq0 \\ (-3)\cdot106-7t\geq0 \end{cases} \begin{cases} t\geq-47\frac{1}{9} \\ t\leq-45\frac{3}{7} \end{cases}, \ t\in Z$$

ekanligini hisobga olsak $t_1=-46, t_2=-47$ bo'lib t_1 uchun $x_1=10, y_1=4$ t_2 uchun esa $x_2=1, \ y_2=11$ hosil bo'ladi. Demak 1- hol uchun 3,5 tdan 10 ta, 4,5 t ligidan esa 4 ta, ikkinchi hol uchun 1 ta va 11 ta ajratilgan.

Mustaqil yechish uchun misollar

Kasrni zanjir kasrga yoying

257. $\frac{323}{17}$;

258. $\frac{135}{279}$;

259. $-\frac{187}{63}$;

260. $\frac{30}{37}$;

261. $-\frac{12}{5}$

262. $\frac{127}{52}$;

263. 1.23;

264. $\frac{71}{41}$

Zanjirli kasrga ko‘ra sonning o‘zini toping

265. $[2;1,3,4,1,2]$

266. $[0;1,1,6,8]$

267. $[0;1,4,3,2]$;

268. $[0;3,1,2,7]$;

269. $[-1;1,2,4,5]$;

270. $[0;1,4,3,2]$;

271. $[0;1,4,3,2]$;

Quyidagi tenglamalarni butun sonlar to‘plamida yeching

272. $143x+169y=5$

273. $237x+44y=1$

274. $275x+145y=10$

275. $3x+8y=5$

276. $2x+5y=7$

277. $5x+28y=59$

278. $12x+7y=41$

279. $12x-7y=29$

280. $8x+3y=63$

281. $7x-19y=23$

282. $9x - 22y = 10$

283. $122x + 129y = 2$

284. $26x + 34y = 13$

285. $258x - 172y = 56$

286. $70x + 33y = 1$

287. $45x - 37y = 25$

288. $60x - 91y = 2$

289. 440 kg donni tashish uchun 60 kg va 80 kg li qoplar mavjud. Shu donni tashish uchun har bir xil qopdan nechtadan olingan?

290. Kinoteatrga tushish uchun 14900 so'mga 300 va 500 so'mlik biletlardan sotib olindi. Har bir xil biletdan nechtadan sotib olingan?

II BOB. AYNIY SHAKL ALMASHTIRISHLAR. AYNIYATLAR VA TENGSIZLIKLARNI ISBOTLASH

1-§. Ratsional ifodalarni ayniy shakl almashtirish

Birorta to‘plamda (oraliqda) analitik ifodani unga aynan teng bo‘lgan boshqa ifodaga almashtirish, shu to‘plamda (oraliqda) berilgan ifodani ayniy almashtirish deyiladi. Biror $(a;b)$ oraliqda berilgan ikkita $f(x)$ va $\phi(x)$ ifodaning shu oraliqning har bir nuqtasidagi qiymatlari teng bo‘lsa, bunday holda ifodalar bu to‘plamda (oraliqda) aynan teng deyiladi, ya’ni

$f(x)\equiv\phi(x),\ x\in(a,\ b)$

Masalan, $f(x)=1$ va $\phi(x)=\cos^2 x+\sin^2 x$ ifodalar $(-\infty;+\infty)$ oraliqda aynan tengdir, chunki istalgan x uchun $\cos^2 x+\sin^2 x=1$ tenglik o‘rinli.

Ta’rif. Ratsional ifoda deb, ratsional sonlar maydonida aniqlangan $x,y,z,...$o‘zgaruvchilar va shu sohadan olingan $a,b,c,...$ sonlar ustida qo‘shish, ayirish,ko‘paytirish, bo‘lish (nolga bo‘lishdan tashqari) amallari bilan bog‘langan ifodaga aytiladi. Agar $P(x,y,z,...)$ ratsional ifoda $Q(x,y,z,...)$ va$G(x,y,z,...)$ ifodalarning bo‘linmasidan iborat bo‘lsa, u holda $P(x,y,z,...)$ ifoda kasr-ratsional ifoda deyiladi. Ifodalarni ayniy almashtirishda ularning aniqlanish sohasi o‘zgaradi. Ba’zi hollarda ifodalarni ayniy almashtirishda ularning aniqlanish sohasi kengayadi.

Masalan, $x^2-5x+6+\sqrt{x}-\sqrt{x}$ ifodani $x\geq0$ aniqlangan bo‘lsa, soddalashtirgandan so‘ng, x^2-5x+6 ifoda $x\in R$ da aniqlangan.

Berilgan va olingan ifodalar $(0;\infty)$ to'plamda aynan teng bo'ladi. Ko'phadlarni ko'paytuvchilarga ajratishda turli usullardan foydalanish mumkin.

1-misol. Ifodani soddalashtiring.

$$f(x,y)=\frac{2x^2+xy-y^2}{x+y}$$

Kasrni suratini quyidagi ko'rinishda ko'paytuvchilarga ajratamiz. Masalan, $xy=2xy-xy$ ni qo'yamiz, u holda

$$2x^2+xy-y^2=2x^2+2xy-xy-y^2=$$
$$=2x(x+y)-y(x+y)=(x+y)(2x-y)$$

Demak, $f(x,y)=\frac{(x+y)(2x-y)}{x+y}=2x-y \qquad x\neq -y$

2-misol. Ifodani soddalashtiring.

$$f(x)=\frac{x^4-10x^2+169}{x^2+6x+13}$$

Yechilishi. Ifodani suratini ko'paytuvchilarga ajratamiz.

$x^4+169=\left(x^2\right)^2+13^2$ yig'indini to'la kvadratga keltirilsa,

$$f(x)=\left(x^4+10x^2+169\right)=26x^2-10x^2=\left(x^2+13\right)^2-(6x)^2=\left(x^2-6x+13\right)\left(x^2+6x+13\right)$$

hosil bo'ladi.

Demak, $f(x)=\frac{\left(x^2-6x+13\right)\left(x^2+6x+13\right)}{x^2+6x+13}=x^2-6x+13$

$x^2+16x+13=x^2+6x+9+4=\left(x+3\right)^2+41$ ifoda $x\in R$ da musbat bo'ladi.

3-misol. Ifodani soddalashtiring.

$$f(x)=\left(\frac{1}{x^2+3x+2}+\frac{2x}{x^2+4x+3}+\frac{1}{x^2+5x+6}\right)^2\cdot\frac{(x-3)^2+12x}{2}$$

Yechilishi. Kasrga umumiy maxraj tanlaydigan bo‘lsak, bundan $x^2+3x+2=(x+1)(x+2)$, $x^2+4x+3=(x+1)(x+3)$, $x^2+5x+6=(x+2)(x+3)$ larni hisobga olsak, $(x+1)(x+2)(x+3)$ umumiy maxrajga kelamiz.

$$f(x)=\left(\frac{x+3+2x(x+2)+x+1}{(x+1)(x+2)(x+3)}\right)^2\cdot\frac{x^2-6x+9+12x}{2}=\left(\frac{2x^2+6x+4}{(x+1)(x+2)(x+3)}\right)^2\times$$

$$\times\frac{x^2+6x+9}{2}=4\cdot\left(\frac{x^2+3x+2}{(x^2+3x+2)(x+3)}\right)^2\cdot\frac{(x+3)^2}{2}=2$$

Demak, $f(x)=2$, agar $x\neq-1$, $x\neq-2$, $x\neq-3$

4-misol. Ifodani soddalashtiring.

$$f(x,y,z)=\frac{x^2}{(x-y)(x-z)}+\frac{y^2}{(y-z)(y-x)}+\frac{z^2}{(z-x)(z-y)}$$

Yechilishi. Kasrni umumiy maxrajga keltiramiz va $y-z$ ni $y-z=(x-z)-(x-y)$ ga almashtiramiz.

$$\frac{x^2}{(x-y)(x-z)}+\frac{y^2}{(y-z)(y-x)}+\frac{z^2}{(z-x)(z-y)}=$$

$$=\frac{x^2}{(x-y)(x-z)}-\frac{y^2}{(x-y)(y-z)}+\frac{z^2}{(x-z)(y-z)}=$$

$$=\frac{x^2(y-z)-y^2(x-z)+z^2(x-y)}{(x-y)(x-z)(y-z)}=$$

$$=\frac{x^2(x-z)-x^2(x-y)+y^2(x-z)+z^2(x-y)}{(x-y)(x-z)(y-z)}=$$

$$=\frac{(x-z)(x^2-y^2)-(x-y)(x^2-z^2)}{(x-y)(x-z)(y-z)}=$$

$$=\frac{(x-z)(x-y)(x+y-x-z)}{(x-y)(x-z)(y-z)}=\frac{(x-z)(x-y)(y-z)}{(x-y)(x-z)(y-z)}=1$$

5-misol. Ifodani soddalashtiring.

$$f(x,y,z)=\frac{y-z}{(x-y)(x-z)}+\frac{z-x}{(y-z)(y-x)}+\frac{x-y}{(z-x)(z-y)}$$

Yechilishi. $\frac{y-z}{(x-y)(x-z)}=\frac{1}{x-y}-\frac{1}{x-z}$ ga asosan,

$$\frac{1}{x-y}-\frac{1}{x-z}+\frac{1}{y-z}-\frac{1}{y-x}+\frac{1}{z-x}-\frac{1}{z-y}=$$

$$=\frac{2}{x-y}+\frac{2}{x-z}+\frac{2}{y-z}\quad x\neq y,\quad y\neq z,\quad z\neq x$$

Mustaqil yechish uchun misollar

Ifodalarni soddalashtiring:

1. $\dfrac{\frac{1}{a}-\frac{1}{b+c}}{\frac{1}{a}+\frac{1}{b+c}}\cdot\left(1+\frac{b^2+c^2-a^2}{2bc}\right):\frac{a-b-c}{abc}$ Bu yerda $a=0{,}02$, $b=11{,}05$, $c=1{,}07$

2. $\dfrac{\frac{a-b}{2a-b}-\frac{a^2+b^2-c}{2a^2+ab+b^2}}{\left(4b^4+4ab^2+a^2\right):\left(2b^2+a\right)}\cdot\left(b^2+b+ab+a\right)$ $\quad a>0,\ b>0,\ b\neq 2a$

3. $\dfrac{3a^2+2ax-x^2}{(3x+a)\cdot(a+x)}-2+10\cdot\dfrac{ax-3x^2}{a^2-9x^2}$

4. $\left(\left(\frac{x^2}{y^2}+\frac{1}{x}\right):\left(\frac{x}{y^2}-\frac{1}{y}+\frac{1}{x}\right)\right):\dfrac{(x-y)^2+4xy}{1+\frac{y}{x}}$ $\quad x\neq 0, y\neq 0, x\neq -y$.

5. $\left(\frac{3}{2x-y}-\frac{2}{2x+y}-\frac{1}{2x-5y}\right):\frac{y^2}{4x^2-y^2}$

6. $\left(x^2+2x-\frac{11x-2}{3x+1}\right):\left(x+1-\frac{2x^2+x+2}{3x+1}\right)$ $\quad x=7{,}(3)$

7. $\left(2-x+4x^2+\frac{5x^2-6x+3}{x-1}\right):\left(2x+1+\frac{2x}{x-1}\right)$ $\quad x\neq -1\ x\neq\frac{1}{2}\ x\neq 1$

8. $\dfrac{x^4-x^3-x+1}{x^2-5x^2+7x-3}\cdot|x-3|$

9. $\dfrac{x^3+x^2-2x}{x|x+2|-x^2+4}$

10. $\dfrac{a\cdot|a-3|}{\left(a^2-a-6\right)\cdot|a|}$

11. $\dfrac{a^3-2a^2+5a+26}{a^3-5a^2+17a-13}$

12. $\dfrac{|a-1|+|a|+a}{3a^2-4a+1}$

13. $\dfrac{x^4-x^2-2x-1}{x^3-2x^2+1}:\dfrac{x^4+2x^3-x-3}{1+\dfrac{4}{x}+\dfrac{4}{x^2}}$

14. $\dfrac{|y^2-1|+y^2}{2y^2-1}-\dfrac{|y-1|}{y-1}$

15. $\dfrac{|a^3-1|+|a+1|}{a^3+a}$

16. $|y^2-1|+y|y+1|$

17. $\left(\dfrac{|x-1|}{x-1}\cdot x^2-2x\cdot\dfrac{|x+1|}{x+1}+2x-4\right):|x-2|$

18. $\dfrac{n^4-9n^3+12n^2+9n-13}{n^4-10n^3+22n^2-13n}$

19. $\dfrac{||x|-1|\cdot|x|}{x^2-1}$

20. $\dfrac{2x^2-xy-3y^2}{2x^2+5xy+3y^2}$

21. $$\frac{\left(x^3-1-\frac{7-x^3}{3+x^3}\right)\cdot\frac{4}{x^5+3x^2}}{\frac{6x^6-24}{x^9+6x^6+9x^3}\cdot\frac{2x}{3x^3+6}}$$

22. $$\frac{p^3+4p^2+10p+12}{p^3+p^2+2p+16}\cdot\frac{p^2-3p^2+8p}{p^2+2p+6}$$

23. $$\frac{x^3+y^3+z^3-3xyz}{(x-y)^2+(y-z)^2+(z-x)^2}$$

24. $$\frac{x^2+y^2+z^2+2xy+2yz+2zx}{x^2-y^2-z^2-2yz}$$

25. $$\frac{2x^2-xy-3y^2}{2x^2+3y^2-5xy}$$

26. $$\frac{a^3}{(a-b)(a-c)}+\frac{b^3}{(b-c)(b-a)}+\frac{c^3}{(c-a)(c-b)}$$

27. $$\frac{x^4-(x-1)^2}{\left(x^2+1\right)^2-x^2}+\frac{x^2-\left(x^2-1\right)^2}{x^2(x+1)^2-1}+\frac{x^2(x-1)^2-1}{x^4-(x+1)^2}$$

28. $$\left(\frac{x^2}{y^2}-2+\frac{y^2}{x^2}\right)\cdot\frac{x^4y^4}{xy+y^2}\cdot\frac{\frac{x}{y}-1+\frac{y}{x}}{x^3-2x^2y+xy^2}$$

29. $$\frac{x^2-x+1}{x^2+x+1}+\frac{2x(x-1)^2}{x^4+x^2+1}+\frac{2x^2\left(x^2-1\right)^2}{x^8+x^4+1}$$

30. $$\frac{a+b}{ax+by}+\frac{a-b}{ax-by}+\frac{2\left(a^2x+b^2y\right)}{a^2x^2+b^2y^2}-\frac{4\left(a^4x^3-b^4y^3\right)}{a^4x^4-b^4y^4}$$

31. $$\frac{b-c}{b+c}+\frac{c-a}{c+a}+\frac{a-b}{a+b}+\frac{(b-a)(c-a)(a-b)}{(b+a)(c+a)(a+b)}$$

Ayniyatni isbotlang

32. $$\frac{b-c}{(a-b)(a-c)}+\frac{c-a}{(b-c)(b-a)}+\frac{a-b}{(c-a)(c-b)}=\frac{2}{a-b}+\frac{2}{b-c}+\frac{2}{c-a}$$

33. $$\frac{a^2(d-b)(d-c)}{(a-b)(d-c)}+\frac{b^2(d-c)(d-a)}{(b-c)(b-a)}+\frac{c^2(d-a)(d-b)}{(c-a)(c-b)}=d^2$$

34. Agar $(a-b)^2+(b-c)^2+(c-a)^2=(a+b-2c)^2+(b+c-2a)^2+(c+a-2b)^2$ bo'lsa, u holda $a=b=c$ ni isbotlang.

35. Agar $a+b+c=0$ bo'lsa, u holda $\frac{a^5+b^5+c^5}{7}=\frac{a^3+b^3+c^3}{5}\cdot\frac{a^2+b^2+c^2}{2}$ ni isbotlang.

36. Agar $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{a+b+c}$ bo'lsa, u holda $\frac{1}{a^n}+\frac{1}{b^n}+\frac{1}{c^n}=\frac{1}{a^n+b^n+c^n}$ isbotlang.

2-§. Irratsional ifodalarni ayniy shakl almashtirishlar

Matematikada ko'p uchraydigan amallardan biri ildiz chiqarish amalidir. Agar berilgan algebraik ifodalarda to'rt arifmetik amaldan tashqari ildiz chiqarish amali ham qatnashsa, bunday ifodalar irratsional ifodalar deyiladi. Arifmetik ildizning ta'rifini keltiramiz.

Ta'rif. $a\geq 0$ sonning n-darajali arifmetik ildizi deb $(n\in N)$, n-darajasi a ga teng bo'lgan $b\geq 0$ songa aytiladi va $b=\sqrt[n]{a}$ orqali belgilanadi. Shartga ko'ra $\left(\sqrt[n]{a}\right)^n=a,\quad a\geq 0$

Teorema. Har qanday manfiy bo'lmagan haqiqiy sonning n-darajali arifmetik ildizi yagona manfiy bo'lmagan haqiqiy sondir.

Masalan, $\sqrt{16}=4$, bu yerda arifmetik ildiz 4 ga teng.

$\sqrt{25}=5,$ bu yerda arifmetik ildiz 5 ga teng.

Ta'rifdan quyidagi xulosaga kelish mumkin:

$$\sqrt[n]{a^n}=\begin{cases}|a|, & \text{agar } n \text{ - juft son bo'lsa,}\\ a, & \text{agar } n \text{ - toq bo'lsa, } n\neq 1\end{cases}$$

Masalan, $\sqrt{a^n}=|a|,\ \sqrt[3]{a}=a$

Irratsional ifodalar quyidagi xossalarga ega:

$1^0.\ \sqrt[n]{ab}=\sqrt[n]{a}\cdot\sqrt[n]{b}$, agar $a\geq 0,\ b\geq 0$ bo'lsa.

$2^0.\ \sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, agar $a\geq 0,\ b>0$ bo'lsa.

$3^0.\ \left(\sqrt[n]{a^m}\right)^k=\sqrt[n]{a^{km}}$

$4^0.\ \sqrt[n]{\sqrt[k]{a^m}}=\sqrt[nk]{a^m}$

$$5^0.\ \sqrt[mn]{a^{mk}}=\begin{cases}\sqrt[n]{\left|a^k\right|}, & \text{agar m juft bo'lsa}\\ \sqrt[n]{a^k}, & \text{agar m toq bo'lsa}\end{cases}$$

1-misol. $\frac{3}{1+\sqrt{2}-\sqrt{3}}$ ifodani maxrajini irratsionallikdan qutqaring.

Yechilishi. $1+\sqrt{2}-\sqrt{3}$ ni qo'shmasi $1+\sqrt{2}+\sqrt{3}$ ga kasrni surat va maxrajini ko'paytiramiz.

$$\frac{3\left(1+\sqrt{2}+\sqrt{3}\right)}{\left(1+\sqrt{2}-\sqrt{3}\right)\left(1+\sqrt{2}+\sqrt{3}\right)}=\frac{3\left(1+\sqrt{2}+\sqrt{3}\right)}{\left(1+\sqrt{2}\right)^2-\sqrt{3}^2}=\frac{3\left(1+\sqrt{2}+\sqrt{3}\right)}{2\sqrt{2}};$$

Endi $\sqrt{2}$ dan qutulishimiz kerak.

$$\frac{3\sqrt{2}\left(1+\sqrt{2}+\sqrt{3}\right)}{2\sqrt{2}\cdot\sqrt{2}}=\frac{3\left(\sqrt{2}+2+\sqrt{6}\right)}{4}$$

2-misol. $\sqrt{5}-\sqrt{3-\sqrt{29-12\sqrt{5}}}$ ifodani soddalashtiring.

Yechilishi. Avvalambor kvadrat ildizlar ostidagi ifodalarning musbat ekanini, ya'ni ildizlar istalgan R da ma'noga ega ekanini ko'rsatishimiz kerak.

$$3-\sqrt{29-12\sqrt{5}}>0 \Rightarrow 3>\sqrt{29-12\sqrt{5}}$$

Buning uchun $29-12\sqrt{5}>0$ ekanini ko'rsatish kerak. R da ayrim almashtirishlar bajarib quyidagiga kelamiz.

$$29-2\cdot3\cdot2\cdot\sqrt{5}=20-2\sqrt{20}+9=\left(\sqrt{20}-3\right)^2>0$$

$$3-\left(\sqrt{20}-3\right)=6-\sqrt{20}$$

$$\sqrt{5}-\sqrt{6-\sqrt{20}}=\sqrt{5}-\sqrt{\left(\sqrt{5}-1\right)^2}=\sqrt{5}-\sqrt{5}+1=1$$

3-misol. $A=\left(\sqrt{32}+\sqrt{45}-\sqrt{98}\right)\left(\sqrt{72}-\sqrt{500}-\sqrt{8}\right)$ Ifodani soddalashtiring.

Yechilishi. Oldin har bir radikalni ildiz xossasidan foydalanib soddalashtiramiz.

$\sqrt{32}=\sqrt{16\cdot2}=4\sqrt{2},\quad \sqrt{45}=\sqrt{9\cdot5}=3\sqrt{5},\quad \sqrt{98}=\sqrt{49\cdot2}=7\sqrt{2},$

$\sqrt{72}=\sqrt{36\cdot2}=6\sqrt{2},\quad \sqrt{500}=\sqrt{100\cdot5}=10\sqrt{5},\quad \sqrt{8}=\sqrt{4\cdot2}=2\sqrt{2},$

Bulardan

$$A=\left(4\sqrt{2}+3\sqrt{5}-7\sqrt{2}\right)\left(6\sqrt{2}-10\sqrt{5}-2\sqrt{2}\right)=\left(3\sqrt{5}-3\sqrt{2}\right)\left(4\sqrt{2}-10\sqrt{5}\right)$$

Qavslarni ochib chiqsak,

$$A=12\sqrt{10}-24-150+30\sqrt{10}=42\sqrt{10}-174=6\left(7\sqrt{10}-29\right)$$

4-misol. $\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{x-1}}+\frac{\sqrt[3]{x-1}}{\sqrt[3]{x+1}}-2\right)^{-\frac{1}{2}}$ $x=\frac{a^3+b^3}{a^3-b^3}$, $ab>0$ ifodani soddalashtiring.

Yechilishi.

$$\sqrt[3]{x+1}=\sqrt[3]{\frac{2a^3}{a^3-b^3}},\quad \sqrt[3]{x-1}=\sqrt[3]{\frac{2b^3}{a^3-b^3}}$$

$$\begin{cases} x+1 \\ x-1 \end{cases} \Rightarrow \begin{cases} a^3(a^3-b^3)>0 \\ b^3(a^3-b^3)>0 \end{cases} \; \square \; \begin{cases} a>0 \\ a>b \\ b>0 \end{cases} \vee \begin{cases} a<0 \\ a<b \\ b<0 \end{cases}$$

$$\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{x-1}}+\frac{\sqrt[3]{x-1}}{\sqrt[3]{x+1}}-2\right)^{-\frac{1}{2}}=\left(\frac{\left(\sqrt[3]{x+1}-\sqrt[3]{x-1}\right)^2}{\sqrt[3]{x-1}\cdot\sqrt[3]{x+1}}\right)^{-\frac{1}{2}}=\frac{\sqrt{\sqrt[3]{x+1}\cdot\sqrt[3]{x-1}}}{\sqrt[3]{x-1}-\sqrt[3]{x-1}}=$$

$$=\frac{\sqrt{\sqrt[3]{\frac{2a^3}{a^3-b^3}}\cdot\sqrt[3]{\frac{2b^3}{a^3-b^3}}}}{\sqrt[3]{\frac{2a^3}{a^3-b^3}}\cdot\sqrt[3]{\frac{2b^3}{a^3-b^3}}}=A$$

1-hol. $a>0$, b>0, a>b

$$A=\frac{\sqrt[3]{2}\cdot\sqrt{ab}}{\sqrt[3]{2}(a-b)}=\frac{\sqrt{ab}}{a-b}$$

2-hol.

$$\frac{\sqrt{\sqrt[3]{\frac{-2a^3}{b^3-a^3}}\cdot\sqrt[3]{\frac{-2b^3}{b^3-a^3}}}}{\sqrt[3]{\frac{-2a^3}{b^3-a^3}}-\sqrt[3]{\frac{-2b^3}{b^3-a^3}}}=\frac{\sqrt[3]{2}\cdot\sqrt{(-a)(-b)}}{\sqrt[3]{2}(-a+b)}=\frac{\sqrt{ab}}{b-a}.$$

Javob:

1) $\frac{\sqrt{ab}}{b-a}$ agar a>0, b>0, a>b.

2) $\frac{\sqrt{ab}}{b-a}$ agar a<0, b<0, a<b bo'lsa.

5-misol. Ifodani soddalashtiring.

$$A=\left(\frac{1}{\sqrt{a}+\sqrt{a+1}}+\frac{1}{\sqrt{a}-\sqrt{a-1}}\right)\div\left(1+\sqrt{\frac{a+1}{a-1}}\right)$$

Yechilishi. Oldin birinchi qavsni, keyin ikkinchi qavs maxrajini irratsionallikdan qutqaramiz.

1) $\frac{1}{\sqrt{a}+\sqrt{a+1}}=\frac{\sqrt{a}-\sqrt{a+1}}{\left(\sqrt{a}+\sqrt{a+1}\right)\left(\sqrt{a}-\sqrt{a}+1\right)}=\frac{\sqrt{a}-\sqrt{a+1}}{a-(a+1)}=\sqrt{a+1}-\sqrt{a}$

2) $\frac{1}{\sqrt{a}-\sqrt{a-1}}=\frac{\sqrt{a}+\sqrt{a-1}}{\left(\sqrt{a}-\sqrt{a-1}\right)\left(\sqrt{a}+\sqrt{a}-1\right)}=\frac{\sqrt{a}+\sqrt{a-1}}{a-(a-1)}=\sqrt{a}+\sqrt{a-1}$

3) $\left(\sqrt{a+1}-\sqrt{a}\right)+\left(\sqrt{a}+\sqrt{a-1}\right)=\sqrt{a+1}+\sqrt{a-1}$

4) $1+\sqrt{\frac{a+1}{a-1}}=\frac{\sqrt{a-1}+\sqrt{a+1}}{\sqrt{a-1}}$

Javob: agar $a>1$ bo‘lsa, $A=\sqrt{a-1}$.

6-misol. Kasr maxrajini irratsionallikdan qutqaring.

$$A=\frac{3}{1+\sqrt{2}-\sqrt{3}}$$

Yechilishi. Avvalambor, kasr maxrajini $\left(1+\sqrt{2}\right)-\sqrt{3}$ ning qo‘shmasi $\left(1+\sqrt{2}\right)+\sqrt{3}$ ga kasr surat va maxrajini ko‘paytiramiz.

$$A=\frac{3\left(1+\sqrt{2}+\sqrt{3}\right)}{\left(1+\sqrt{2}-\sqrt{3}\right)\left(1+\sqrt{2}+\sqrt{3}\right)}=\frac{3\left(1+\sqrt{2}+\sqrt{3}\right)}{\left(1+\sqrt{2}\right)^2-\sqrt{3}^2}=\frac{3\left(1+\sqrt{2}+\sqrt{3}\right)}{2\sqrt{2}}$$

Endi $\sqrt{2}$ ga ko‘paytirib bo‘lamiz.

7-misol. $A=\frac{\sqrt{2}}{\sqrt[4]{3}-\sqrt[4]{2}}$ kasr maxrajini irratsionallikdan qutqaring.

Yechilishi. $a^4-b^4=(a-b)\left(a^3+a^2b+ab^2+b^3\right)$dan foydalansak va $\sqrt[4]{3}=a$ $\sqrt[4]{2}=b$ desak,

$$\frac{\sqrt{2}\left(\sqrt[4]{3^3}+\sqrt[4]{3^2\cdot 2}+\sqrt[4]{3\cdot 2^2}+\sqrt[4]{2^3}\right)}{\left(\sqrt[4]{3}-\sqrt[4]{2}\right)\left(\sqrt[4]{3^3}+\sqrt[4]{3^2\cdot 2}+\sqrt[4]{3\cdot 2^2}+\sqrt[4]{2^3}\right)}=\frac{\sqrt{2}\left(\sqrt[4]{27}+\sqrt[4]{18}+\sqrt[4]{12}+\sqrt[4]{8}\right)}{\left(\sqrt[4]{3}\right)^4-\left(\sqrt[4]{2}\right)^4}=$$
$$=\sqrt[4]{108}+\sqrt[4]{72}+\sqrt[4]{48}+\sqrt[4]{32}$$

Mustaqil yechish uchun misollar

Kasrni maxrajini irratsionallikdan qutqaring

37. $\dfrac{2\sqrt{30}}{\sqrt{5}+\sqrt{6}+\sqrt{7}}$

38. $\dfrac{1}{\sqrt[4]{3}+\sqrt[4]{9}+\sqrt[4]{27}+3}$

39. $\dfrac{\sqrt{2\sqrt{3}+\sqrt{2}}}{\sqrt{2\sqrt{3}-\sqrt{2}}}$

40. $\dfrac{1}{\sqrt[3]{15}-\sqrt[3]{7}}$

41. $\dfrac{2+\sqrt{6}}{2\sqrt{2}+2\sqrt{3}-\sqrt{6}-2}$

42. $\dfrac{1}{\sqrt[3]{4}+\sqrt[3]{6}+\sqrt[3]{9}}$

Tengliklar to‘g‘riligini isbotlang

43. $\sqrt[3]{20+14\sqrt{2}}+\sqrt[3]{20-14\sqrt{2}}=4$

44. $\sqrt[3]{\dfrac{\sqrt{5}+2}{\sqrt{5}}}+\sqrt[3]{\dfrac{\sqrt{5}-2}{\sqrt{5}}}=\dfrac{1}{\sqrt[3]{5}}$

45. $\dfrac{\sqrt{9+\sqrt{65}}}{\sqrt{\sqrt{19}-\sqrt{3}}}=\dfrac{\sqrt{\sqrt{19}+\sqrt{3}}}{\sqrt{9-\sqrt{65}}}$

46. $\dfrac{1+\sqrt{3}}{2\sqrt[3]{2}}=\dfrac{2+\sqrt{3}}{\sqrt[3]{9-\sqrt{65}}}$

47. $\sqrt[3]{38+17\sqrt{5}}=\sqrt{9+4\sqrt{5}}$

48. $\dfrac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2+\sqrt{3}}}+\dfrac{2-\sqrt{3}}{\sqrt{2}-\sqrt{2-\sqrt{3}}}=2$

Ifodalarni soddalashtiring:

49. $\left(\frac{\sqrt[4]{ab}-\sqrt{ab}}{1-\sqrt{ab}}+\frac{1-\sqrt[4]{ab}}{\sqrt[4]{ab}}\right):\frac{\sqrt[4]{ab}}{1+\sqrt[4]{a^3b^3}}-\frac{1-\sqrt[4]{ab}-\sqrt{ab}}{\sqrt{ab}}$

50. $\left(\frac{\sqrt{1+a}}{\sqrt{1+a}-\sqrt{1-a}}-\frac{1-a}{\sqrt{1-a^2}-1+a}\right)\left(\sqrt{\frac{1}{a^2}-1}-\frac{1}{a}\right)$

51. $\left(\frac{a-2b}{\sqrt[3]{a^2}-\sqrt[3]{4b^2}}+\frac{\sqrt[3]{2a^2b}+\sqrt[3]{4ab^2}}{\sqrt[3]{a^2}+\sqrt[3]{4b^2}+\sqrt[3]{16ab}}\right):\frac{a\sqrt[3]{a}+b\sqrt[3]{2b}+b\sqrt[3]{a}+a\sqrt[3]{2b}}{a-b}$

52. $\left(\frac{4b^2+2ab}{\sqrt{4a^2b^2-8ab^3}}-\frac{8b\sqrt{b}}{\sqrt{4a^2b-8ab^2}}\right)\left(\frac{1}{2ab}-a^{-2}\right)^{-\frac{1}{2}}\cdot\sqrt{\frac{2a}{b}}$

53. $\frac{8-a}{\sqrt[3]{a}+2}:\left(2+\frac{\sqrt[3]{a^2}}{\sqrt[3]{a}+2}\right)+\left(\sqrt[3]{a}+\frac{2\sqrt[3]{a}}{\sqrt[3]{a}-2}-\right)\cdot\frac{\sqrt[3]{a^2}-4}{\sqrt[3]{a^2}+2\sqrt[3]{a}}$

54. $\left(2\sqrt{x^4-a^2x^2}-\frac{2a^2}{\sqrt{1-a^2x^{-2}}}\right)\cdot\frac{\left(x^2a^{-2}-4+4a^2x^{-2}\right)^{-\frac{1}{2}}}{2ax\left(x^2-a^2\right)^{-\frac{1}{2}}}$

55. $\sqrt{\frac{2a}{(1+a)\sqrt[3]{1+a}}}\cdot\sqrt[3]{\frac{4+\frac{8}{a}+\frac{4}{a^2}}{\sqrt{2}}}$

56. $\sqrt{\frac{\sqrt{2}}{a}+\frac{a}{\sqrt{2}}+2}-\frac{a^2\sqrt[4]{2}-2\sqrt{a}}{a\sqrt{2a}-\sqrt[4]{8a^4}}$

57. $\frac{\sqrt{(2p+1)^3}+\sqrt{(2p-1)^3}}{\sqrt{4p+2\sqrt{4p^2-1}}},\qquad p\geq\frac{1}{2}$

58. $\left(\left(\frac{x^2}{y^2}+\frac{1}{x}\right):\left(\frac{x}{y^2}-\frac{1}{y}+\frac{1}{x}\right)\right):\frac{(x-y)^2+4xy}{1+\frac{y}{x}},\quad x\neq0,\ y\neq0,\ x\neq-y$

59. $\left(\frac{a\sqrt{a}+b\sqrt{b}}{\sqrt{a}+\sqrt{b}}-\sqrt{ab}\right)\left(\frac{\sqrt{a}+\sqrt{b}}{a-b}\right)^2$

60. $\dfrac{\dfrac{x+y}{\sqrt{x}-\sqrt{y}}-\dfrac{x-y}{\sqrt{x}+\sqrt{y}}}{\dfrac{\sqrt{x}-\sqrt{y}}{x+y}-\dfrac{\sqrt{x}+\sqrt{y}}{x-y}}\cdot\dfrac{y-\sqrt{xy}+x}{2\sqrt{xy}}$

61. $\dfrac{\sqrt{4x+4+\frac{1}{x}}}{\sqrt{x}\left|2x^2-x-1\right|}$

62. $\sqrt{\dfrac{x-9}{x+3\sqrt{x}+9}:\dfrac{\sqrt{x}+3}{\sqrt[3]{x^2}-27}}-\sqrt{x}$

63. $\dfrac{x^4+x^2+x\sqrt{2}+2}{x^2-x\sqrt{2}+2}-x\sqrt{2}$

64. $\dfrac{\left(\sqrt{x}+\sqrt{2}\right)^2-\sqrt{2x}}{x^2+x-\sqrt{2x}+2}$, $\quad x>0$

65. $\sqrt{\dfrac{\left(x^2-3\right)^2+12x^3}{x^2}}+\sqrt{(x+2)^2-8x}$

66. $\dfrac{(x-1)\sqrt{(x-1)^2+4x}}{x^2+1+2|x|}$

67. $\sqrt{1-2\sqrt{m-m^2}}$

68. $\sqrt{\left(\dfrac{x^2-4}{2x}\right)^2+4}+\sqrt{1+\dfrac{4}{x^2}+\dfrac{4}{x}}$

3-§. Ko‘rsatkichli va logarifmik ifodalarni ayniy shakl almashtirishlar

Ta’rif. y=a^x (a>0,a≠ 1) ko‘rinishdagi funksiya ko‘rsatkichli funksiya deyiladi.

Ko‘rsatkichli funksiya quyidagi xossalarga ega:

1. Agar $a^x, a^y, x, y \in R, a>0$ bo‘lsa , $a^x \cdot a^y = a^{x+y}$ bo‘ladi.

2. Agar $a^x, a^y, x, y \in R, a>0$ bo‘lsa, $a^x : a^y = a^{x-y}$ bo‘ladi.

3. Agar $a^x, x \in R, a>0$ bo‘lsa, $\exists y \in R$ uchun $(a^x)^y = a^{xy}$ bo‘ladi.

4. Agar $\forall x \in R, a^x, b^x, a>0, b>0$ bo‘lsa $(a \cdot b)^x = a^x \cdot b^x$ bo‘ladi.

Ta’rif. b sonning a asosga ko‘ra logarifmi deb, b sonni hosil qilish uchun a sonni ko‘tarish kerak bo‘lgan daraja ko‘rsatkichiga aytiladi.

Logarifimning asosiy xossalarini keltirib o‘tamiz:

1. Agar $ab>0$ bo‘lsa, $\log_c(ab) = \log_c a + \log_c b$ bo‘ladi.

2. Agar $ab>0$ bo‘lsa, $\log_c(\frac{a}{b}) = \log_c a - \log_c b$ bo‘ladi.

Agar ba’zi hollarda $a>0, b>0$ bo‘lsa u holda $|a| = a, |b| = b$ bo‘ladi.

Undan esa $\log_c(ab) = \log_c a + \log_c b$ va $\log_c\left(\frac{a}{b}\right) = \log_c a - \log_c b$

3. Agar $a>0, n \in R$ bo‘lsa, u holda $\log_c a^n = n\log_c a$ bo‘ladi.

Agar $a \neq n$, $n = 2m(m \neq \pm1, \pm2, \ldots)$ bo‘lsa, u holda $\log_c a^n = n\log_c |a|$ bo‘ladi.

4 Agar $a>o, b>o, b \neq 1$ bo‘lsa u holda $\log_c a = \frac{\log_b a}{\log_b c}$ bo‘ladi.(Bu formulani yangi asosga almashtirish (o‘tkazish) formulasi deyiladi).

Bunda agar $a = b$ bo‘lsa

$\log_c a = \frac{1}{\log_a c}$ bo‘ladi.

1. Agar $a>0, n \in R$ bo‘lsa, u holda $\log_c a = \log_{c^n} a^n$ bo‘ladi.

2. Agar $c>0, b>0, a \neq 1, m, n, k \in R$ bo‘lsa, u holda $\log^k_{c^n} b^m = \left(\frac{m}{n}\right)^k \log^k_c b$ bo‘ladi.

3. Agar $c>0, b>0, c\neq 1, b\neq 1$, bo‘lsa, u holda $c^{\log_c^2 b}=b^{\log_c b}$ bo‘ladi.

Bu xossalar yordamida ko‘rsatkichli va logarifmik ifodalarning ayniy shakl almashtirishlariga doir misollar keltiramiz.

1-misol. $49^{1-\frac{1}{4}\log_7 25}$ ni hisoblang.

Yechilishi: $\left(7^2\right)^{1-\frac{1}{4}\log_7 25}=7^{2-\frac{1}{2}\log_7 25}$

Bunda daraja ko‘rsatkichida ayniy shakl almashtirishlar bajarib, Quyidagi ko‘rinishga keltiramiz:

$$2-\frac{1}{2}\log_7 25=2-\frac{1}{2}\log_7 5^2=2-\log_7 5=\log_7 49-\log_7 5=\log_7\frac{49}{5}$$

Shunday qilib, $7^{2-\frac{1}{2}\log_7 25}=7^{\log_7\frac{49}{5}}$ (1) xossaga asosan, $7^{\log_7\frac{49}{5}}=\frac{49}{5}=9{,}8$

ga teng.

2-misol. Agar $\log_3 12=a$ bo‘lsa $\log_3 18$ ni hisoblang.

Yechilishi: $\log_3 18=\log_3 9+\log_3 2=2+\log_3 2$ dan $\log_3 2=x$ belgilashni kiritamiz.

$\log_3 18=2+x$. Endi $\log_3 12=\log_3(3\cdot 2^2)=\log_3 3+\log_3 2^2=1+2\log_3 2$ ga ega bo‘lamiz. Bundan $\log_3 12=1+2x$.

Shartga ko‘ra, $\log_3 12=a$ ga teng. Demak, $1+2x=a$, $x=\frac{a-1}{2}$.

Shunday qilib, $\log_3 18=2+x=2+\frac{a-1}{2}=\frac{a+3}{2}$.

3-misol. Agar $\log_6 30=a, \log_{15} 24=b$ bo‘lsa, $\log_{12} 60$ ni hisoblang.

Yechilishi.

$$\log_{12}60=\frac{\log_2 60}{\log_2 12}=\frac{\log_2(3\cdot 4\cdot 5)}{\log_2(4\cdot 3)}=\frac{\log_2 4+\log_2 3+\log_2 5}{\log_2 4+\log_2 3}=\frac{2+\log_2 3+\log_2 5}{2+\log_2 3}.$$

$\log_2 3 = x$, $\log_2 5 = y$ belgilash kiritsak, $\log_{12} 60 = \frac{2+x+y}{2+x}$,

$$a = \log_6 30 = \frac{\log_2 30}{\log_2 6} = \frac{\log_2(2\cdot3\cdot5)}{\log_2(2\cdot3)} = \frac{1+x+y}{1+x}$$

$$b = \log_{15} 24 = \frac{\log_2 24}{\log_2 15} = \frac{\log_2(8\cdot3)}{\log_2(3\cdot5)} = \frac{3+x}{x+y}$$

$$\begin{cases} \frac{1+x+y}{1+x} = a \\ \frac{x+3}{x+y} = b \end{cases}$$ bu sistemani yechib x va y larni topamiz.

$x = \frac{b+3-ab}{ab-1}$, $y = \frac{2a-b-2+ab}{ab-1}$ u holda, $\log_{12} 60 = \frac{2ab+2a-1}{ab+b+1}$.

4-misol. Agar $a^2+b^2=7ab$, bo‘lsa $\lg\frac{a+b}{3} = \frac{1}{2}(\lg a + \lg b)$ ni isbotlang.

Isboti: Tenglikni ikkala tomonini 2 ga ko‘paytiramiz.

$$2\lg\frac{a+b}{3} = \lg a + \lg b$$

$$\lg\left(\frac{a+b}{3}\right)^2 = \lg a + \lg b.$$

$a^2+b^2=7ab$ da tenglikni ikkala qismiga 2ab ni qo‘shamiz.

$$a^2+b^2+2ab=7ab+2ab$$

$$(a+b)^2=9ab$$

$$\lg\frac{(a+b)^2}{9} = \lg\frac{9ab}{9} = \lg(ab) = \lg a + \lg b.$$

Mustaqil yechish uchun misollar

69. $\log_6 2 = a$ ga ko‘ra, $\log_3 6$ ni hisoblang.

70. $\lg 64 = a$ ga ko‘ra, $\lg\sqrt[3]{25}$ ni hisoblang.

71. $\lg 122{,}5 = a$ va $\lg 7 = b$ ga ko‘ra, $\lg 5$ ni hisoblang.

72. $\log_5 4 = a$ va $\log_5 3 = b$ ga ko‘ra, $\log_{25} 12$ ni hisoblang.

73. $\log_4 125 = a$ ga ko‘ra $\lg 64$ ni hisoblang.

74. $m^2 = a^2 - b^2$ deb, $\log_{a+b} m + \log_{a-b} m - 2\log_{a+b} m \log_{a-b} m$ ni soddalashtiring.

75. $\left(\left(\log_b^4 a + \log_a^4 b\right)^{\frac{1}{2}} + 2\right)^{\frac{1}{2}} - \log_b a - \log_a b$ ni soddalashtiring.

76. $a \cdot n > 0,\ b \cdot n > 0, b \neq 1$ bo'lsa, $\log_{bn} an = \dfrac{\log_b a + \log_b n}{1 + \log_b n}$ ni isbotlang.

77. $a > o,\ b > o\ a \neq 1, b \neq 1$ bo'lsa, $\dfrac{1 - \log_a^3 b}{\left(\log_a b + \log_b a + 1\right)\log_a \frac{a}{b}}$ ni soddalashtiring.

78. $\log_a N \cdot \log_b N + \log_b N \cdot \log_c N + \log_c N \cdot \log_a N = \dfrac{\log_a N \cdot \log_b N \cdot \log_c N}{\log_{abc} N}$ ni isbotlang.

79. $\log_8 \log_4 \log_2 16$ ni hisoblang.

80. $\lg \lg \sqrt{\sqrt[5]{10}}$ ni hisoblang.

81. $\left(\dfrac{16}{25}\right)^{\log_{\frac{125}{64}} 3}$ ni hisoblang.

82. $\left(\dfrac{8}{27}\right)^{\log_{\frac{81}{16}} 5}$ ni hisoblang.

83. $36^{\log_6 5} + 10^{1-\lg 2} - 3^{\log_9 36}$ ni hisoblang.

84. $\log_3 2 \cdot \log_4 3 \cdot \log_5 4 \log_6 5 \log_7 6 \log_8 7$ ni hisoblang.

85. $\lg\left(7 - \log_2 \log_3 \sqrt{\sqrt[4]{3}}\right)$ ni hisoblang.

86. $3^{\sqrt{\log_3 2}} - 2^{\sqrt{\log_2 3}}$ ni hisoblang.

87. $\log_2 5 = a$ ga ko'ra $\log_{100} 40$ ni hisoblang.

88. $\log_{12} 27 = a$ ga ko'ra $\log_6 16$ ni hisoblang.

89. $\log_3 20=a$ va $\log_3 15=b$ ga ko‘ra $\log_2 360$ ni hisoblang.

90. $\log_{12} 5=a$ va $\log_{12} 11=b$ ga ko‘ra $\log_{275} 60$ ni hisoblang.

91. $\lg_{ab} k=\dfrac{\log_a k\log_b k}{\log_a k+\log_b k}$ ni isbotlang.

92. $\lg_{ab} ak=\dfrac{\log_b a+\log_b k}{1+\log_b k}$ ni isbotlang.

93. $\dfrac{1}{(\log_{a_1} b)^{-1}+(\log_{a_2} b)^{-1}+...+(\log_{a_n} b)^{-1}}=\log_{a_1a_2...a_n} b$ ni isbotlang.

94. Agar $a^2+4b^2=12ab$ bo‘lsa, $\log\dfrac{a+2b}{4}=\dfrac{1}{2}(\lg a+\lg b)$ ni isbotlang.

95. $(\log_a b+\log_b a+2)\cdot(\log_a b-\log_{ab} b)\log_b a-1$ ni soddalashtiring.

96. $0.2\left(2a^{\log_2 b}+3b^{\log_{\sqrt{2}}\sqrt{a}}\right)$ ni soddalashtiring.

97. $\sqrt{1+2^{\frac{\lg a}{\lg\sqrt{2}}}-a^{1+\frac{1}{\log_4 a^2}}-1}$ ni soddalashtiring.

98. $2\log_a^{\frac{1}{2}} b\left[\left(\log_a\sqrt[4]{ab}+\log_b\sqrt[4]{ab}\right)^{\frac{1}{2}}-\left(\log_a\sqrt[4]{\frac{b}{a}}+\log_b\sqrt[4]{\frac{a}{b}}\right)^{\frac{1}{2}}\right]$, $(a>1)$ ni soddalashtiring.

99. Agar $m^2=a^2-b^2$ ekanligi ma’lum bo‘lsa, $\log_{a+b} m+\log_{a-b} m-2\log_{a+b} m\log_{a-b} m$ ifodani soddalashtiring.

100. $\beta=10^{\frac{1}{1-\lg\alpha}}$ va $\gamma=10^{\frac{1}{1-\lg\beta}}$ ekanligi ma’lum bo‘lsa, α *ning* γ ga bog‘lanishini toping.

101. Agar $b^2=ac$ bo‘lsa, $\dfrac{\log_a x-\log_b x}{\log_b x-\log_c x}=\dfrac{\log_a x}{\log_c x}$ ni isbotlang.

102. Agar $\frac{x(y+z-x)}{\lg x}=\frac{y(z+x-y)}{\lg y}=\frac{z(x+y-z)}{\lg z}$ bo‘lsa,

$x^y y^x = y^z z^y = z^x x^z$ ni isbotlang.

4-§. Sonli tengsizliklar va ularning xossalari. Tengsizliklarni isbotlash

Ushbu $f(x)>g(x), f(x)<g(x), f(x)\geq g(x), f(x)\leq g(x)$

ko‘rinishdagi tengsizliklar bir noma’lumli tengsizliklar deyiladi.

Tengsizliklar ustida ham xuddi tenglamalar singari ayniy shakl almashtirishlar bajarish mumkin, ya’ni, birorta $\phi(x)$ funktsiya $f(x)$ va $g(x)$ funksiyalarning ham aniqlanish sohasida aniqlangan bo‘lsa, u holda

1) $f(x)>g(x)$ va $g(x)+\phi(x)>f(x)+\phi(x)$

2) a) agar $\phi(x)>0$ bo‘lsa,

$f(x)>g(x)$ va $f(x)\cdot\phi(x)>g(x)\cdot\phi(x)$ yoki $\frac{f(x)}{\phi(x)}>\frac{g(x)}{\phi(x)}$

b) agar $\phi(x)<0$ bo‘lsa,

$f(x)>g(x)$ va $f(x)\cdot\phi(x)<g(x)\cdot\phi(x)$ yoki $\frac{f(x)}{\phi(x)}<\frac{g(x)}{\phi(x)}$

1) agar $f(x)\geq 0$, $g(x)\geq 0$ bo‘lsa, $(f(x))^n \geq (g(x))^n$

1-misol. Tengsizlikni yeching.

$\frac{(x-1)(x+2)}{(x-4)(x-5)}>0$ $f(x)=\frac{(x-1)(x+2)}{(x-4)(x-5)}$

Yechilishi. $f(x)$ ni noldan katta qiymatlarini topamiz. Funksiyalarning nollarini topamiz.

$x_1=-2, x_2=1, x_3=4, x_4=5$

$f(x)>0$, Javob: $(-\infty;-2)\cup(1;4)\cup(5;+\infty)$

Tengsizliklarni isbotlash

Tengsizliklarni isbotlashda albatta bir nechta usullardan foydalanish mumkin.

1) Tengsizliklarni ta'rifi yordamida isbotlash.

2) Tengsizliklarni sintez metodi yordamida isbotlash.

3) Tengsizliklarni teskari metod yordamida isbotlash.

4) Tengsizliklarni matematik induksiya metodi yordamida isbotlash.

1-misol. Agar $ab>0$ bo'lsa, $\frac{a}{b}+\frac{b}{a}\geq 2$ ekanligini isbotlang.

Isboti. 1) Buni "har qanday musbat sonning o'ziga teskari son bilan yig'indisi 2 dan kichik emas" degan jumladan foydalanib isbot qilish mumkin.

$$\frac{a}{b}+\frac{1}{\frac{a}{b}}\geq 2$$

$$\frac{a^2+b^2-2ab}{ab}\geq 0$$

$$\frac{(a-b)^2}{ab}\geq 0$$

$(a-b)^2\geq 0.$ $a,b\in R$ da o'rinli.

Faqat $a=b$ holda tenglik bajariladi.

2-misol. Agar $a\geq 0$, $b\geq 0$ bo'lsa, u holda

$a^2+b^2+c^2\geq ab+ac+bc$ tengsizlikni isbotlang.

Isboti. $\frac{a^2}{2}+\frac{a^2}{2}+\frac{b^2}{2}+\frac{b^2}{2}+\frac{c^2}{2}+\frac{c^2}{2}\geq ab+ac+bc$

1) $\frac{a^2}{2}+\frac{b^2}{2}\geq ab$

2) $\frac{a^2}{2}+\frac{c^2}{2}\geq ac$

3) $\frac{b^2}{2}+\frac{c^2}{2}\geq bc$

Tengsizliklarni qo'shamiz.

$$\frac{a^2}{2}+\frac{b^2}{2}+\frac{a^2}{2}+\frac{c^2}{2}+\frac{b^2}{2}+\frac{c^2}{2}\geq ab+ac+bc\Rightarrow$$

$$a^2+b^2+c^2\geq ab+ac+bc$$

3-misol. Agar $n\geq 2$ bo'lsa,

$\frac{1}{n+1}+\frac{1}{n+2}+...+\frac{1}{2n}>\frac{13}{14}$ u holda

tengsizlikni isbotlang.

Isboti. Matematik induksiya metodi yordamida isbot qilinadi.

Mustaqil yechish uchun misollar

103. Agar $a>0$, b>0 bo'lsa, $\frac{2\sqrt{ab}}{\sqrt{a}+\sqrt{b}}\leq\sqrt[4]{ab}$ ni isbotlang.

104. Agar $a>0$, b>0, c>0 bo'lsa, $\frac{bc}{a}+\frac{ac}{b}+\frac{ab}{c}\geq(a+b+c)\geq(a+b+c)$ ni isbotlang.

105. Agar $a>0$, b>0 bo'lsa, $\sqrt{a}+\sqrt{b}\leq\sqrt{\frac{a^2}{b}}+\sqrt{\frac{b^2}{a}}$ ni isbotlang.

106. Agar $a>0$, b>0, c>0 bo'lsa, $\frac{2a}{b+c}+\frac{2b}{c+a}+\frac{2c}{a+b}\geq 3$ ni isbotlang.

107. Ixtiyoriy x haqiqiy son uchun $x^8+x^6-4x^4+x^2+1\geq 0$ ni isbotlang.

108. Agar $\frac{1}{a}+\frac{1}{c}=\frac{2}{b}$ va ac>0 bo'lsa, $\frac{a+b}{2a-b}+\frac{c+b}{2c-b}\geq 4$ ni isbotlang.

109. Ixtiyoriy a va b uchun $a^2(1+b^4)+b^2(1+a^4)\leq(1+a^4)(1+b^4)$ ni isbotlang.

110. Ixtiyoriy x haqiqiy son uchun $1\le\frac{2x^2+6x+6}{x^2+4x+5}\le3$ ni isbotlang.

111. $a\ge0,\ b\ge0,\ c\ge0$ uchun $a^3+b^3+c^3\ge a^2\sqrt{bc}+b^2\sqrt{ac}+c^2\sqrt{ab}$

112. Agar $a\ge0,\ b\ge0,\ c\ge0$ bo'lsa, $(a+b+c)(a^2+b^2+c^2)\ge9abc$ ni isbotlang.

113. $(a+b)(b+c)(a+c)\ge8abc;\quad a\ge b,\quad b\ge c,\quad c\ge0.$

114. $a(1+b)+b(1+c)+c(1+a)\ge6\sqrt{abc};\ a\ge0,\ b\ge0,\ c\ge0$

115. $\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdots\frac{99}{100}<\frac{1}{12}$

116. $\left(1+\frac{1}{n+1}\right)^{n+1}>\left(1+\frac{1}{n}\right)^{n};\quad n\in N$

117. $\frac{a^2}{1+a^4}\le\frac{1}{2}$

118. Agar $a+b\ge0,\ a\ne0,\ b\ne0$ u holda $\frac{a}{b^2}+\frac{b}{a^2}\ge\frac{1}{a}+\frac{1}{b}$

119. $\frac{a+b+c+d}{4}\le\sqrt{\frac{a^2+b^2+c^2+d^2}{4}}$

120. $|a+b|\ge|a|-|b|$

121. Agar $ab\ge0$ bo'lsa $(a+b)^4\ge a^4+b^4$

122. Agar $a\ge0,\ b\ge0,\ c\ge0,\ d\ge0$ u holda $a^4+b^4+c^4+d^4\ge4abcd$

123. Agar $a\ge0,\ b\ge0$ u holda $\frac{a+b}{2}\le\sqrt{\frac{a^2+b^2}{2}}$

124. Agar $a>0,\ b>0$ u holda $\frac{2\sqrt{ab}}{\sqrt{a}+\sqrt{b}}\le\sqrt[4]{ab}$

125. $\log_2 3+\log_3 2>2$

126. Agar $a>0,\ b>0,\ c>0$ bo'lsa $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\ge\frac{3}{2}$

127. Agar $a>0$, $b>0$, c>0, d>0 bo'lsa

128. $\sqrt{(a+b)(c+d)}+\sqrt{(a+c)(b+d)}+\sqrt{(a+d)(b+c)}\geq$
$\geq\sqrt{ab}+\sqrt{ac}+\sqrt{ad}+\sqrt{bc}+\sqrt{bd}+\sqrt{cd}$

129. $a+b\geq1$ bo'lsa, $x^4+y^4\geq\frac{1}{8}$ ekanligini isbotlang?

130. $31^{11}<17^{14}$ Qay biri katta?

131. $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)>9$

132. $a^3+b^3+c^3>3abc$

133. Agar a-ixtiyoriy haqiqiy son bo'lsa, u holda $\frac{a^2+a+2}{\sqrt{a^2+a+1}}\geq2$

134. Agar $a\geq0$ va $b\geq0$ bo'lsa, u holda $ab(a+b)\leq a^3+b^3$

III BOB. ALGEBRAIK TENGLAMA VA TENGSIZLIKLAR

1-§. Ratsional tenglama. Teng kuchli tenglamalar

$f(x)=g(x)$ (1) tenglama berilgan bo‘lsin. (1) tenglamaning aniqlanish sohasi D sonli to‘plamdan iborat bo‘lsa, $D=M_1\cap M_2$ bo‘ladi. Bu yerda M_1 va M_2 mos ravishda $f(x)$ va $g(x)$ funksiyalarning aniqlanish sohalaridan iborat bo‘lgan sonli to‘plamlardir. (1) tenglama D sohada ba’zi bir ayniy almashtirishlardan umumiy maxrajga keltirish, qavslarni ochib chiqish, hadlarni tenglamaning bir qismidan ikkinchi qismiga olib o‘tish, o‘xshash hadlarni ixchamlashtirish va h.k. keyin $f_1(x)=g_1(x)$ (2) ko‘rinishni qabul qilish mumkin.

Ta’rif. Agar (1) va (2) tenglamalarning ikkalasi ham bir xil yechimlarga ega bo‘lsa, ya’ni yechimlar to‘plami ustma-ust tushsa, bunday holda (1) va (2) tenglamalar teng kuchli tenglamalar deyiladi.

Masalan,

$\frac{2x^2+2x+3}{x+3}=\frac{3x^2+2x-1}{x+3}$ va $2x^2+2x+3=3x^2+2x-1$

$a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+...+a_1x+a_0=0$ ko‘rinishdagi tenglama yuqori darajali (butun ratsional) tenglama deyiladi.

$\frac{P(x)}{Q(x)}=0$ ko‘rinishdagi tenglama kasr-ratsional tenglama deyiladi. Bu yerda $P(x)$ va $Q(x)$ - ko‘phadlar.

Ratsional tenglamalarni yechishda asosan quyidagi metodlardan foydalaniladi:

1) Ko'paytuvchilarga ajratish usuli.

2) Yangi o'zgaruvchilar kiritish usuli.

1-misol. Tenglamani yeching.

$x^3+2x^2+3x+6=0$

Yechilishi. Tenglamani chap tomonini ko'paytuvchilarga ajratamiz.

$x^2(x+2)+3(x+2)=0$ bundan $(x+2)(x^2+3)=0$. Bu tenglama $x+2=0$ yoki $x^2+3=0$ ga kelamiz. Birinchi tenglamadan $x_1=-2$ ga ega bo'lamiz. $x^2+3=0$ $x\in R$ da yechimga ega emas. Javob: $x=-2$

2-misol. $x^4+4x^3-10x^2-28x-15=0$ tenglamani yeching.

Yechilishi. To'la kvadratdan foydalanib tenglamani chap qismini ko'paytuvchilarga ajratamiz:

$x^4+4x^3+4x^2-14x^2-28x-15=0$

$(x^2+2x)^2-14(x^2+2x)-15=0$

Agar $x^2+2x=y$ deb belgilash kiritsak, y ga nisbatan $y^2-14y-15=0$ tenglama hosil bo'ladi. Tenglamani yechib $y_1=-1$, $y_2=15$ ildizlarga ega bo'lamiz.

$y_1=-1$ bo'lganda $x^2+2x=-1$, $(x+1)^2=0$. Ildizi $x_{1,2}=-1$

$y_2=15$ bo'lganda $x^2-2x-15=0$. Ildizi $x_3=3$ $x_4=-5$

Javob: $x_1=x_2=-1$ $x_3=3$ $x_4=-5$

3-misol. $4x^3-10x^2+14x-5=0$ tenglamani yeching.

Yechilishi. Bu tenglamani yechishda sal boshqacharoq usul tanlaymiz, ya'ni tenglamani shunday songa ko'paytiramizki, natijada x^3 oldidagi koeffitsient biror butun sonni kubi bo'lsin. Buning uchun 2 koeffitsientga ko'paytirish kifoya. Tenglamani har ikkala tomonini 2 ga ko'paytiramiz.

$8x^3-20x^2+28x-10=0$

$(2x)^3-5\cdot(2x)^2+14\cdot(2x)-10=0$

$t=2x$ almashtirish kiritilsa, quyidagi ko'rinishga keladi.

$t^3-5t^2+14t-10=0$

Gorner sxemasidan foydalanib, tenglama yagona $t_1=1$ yechimga ega ekanligiga kelamiz. $x=\frac{t}{2}\Rightarrow x=\frac{1}{2}$

4-misol. $\left(x^2+x+4\right)^2+8x\left(x^2+x+4\right)+15x^2=0$

Yechilishi. $y=x^2+x+4$ belgilash kiritsak, tenglama $y^2+8xy+15x^2=0$ ko'rinishga keladi. Tenglamani har bir hadini x^2 ga bo'lsak, $\left(\frac{y}{x}\right)^2+8\cdot\left(\frac{y}{x}\right)+15=0$ ga ega bo'lamiz.

$\frac{y}{x}=t$ ikkinchi belgilash kiritamiz.

$t^2+8t+15=0 \quad t_1=-3, \quad t_2=-5$

$\frac{y}{x}=-3 \quad y_1=-3x \quad \frac{y}{x}=-5 \quad y_2=-5x$

Shunday qilib, tenglama $x^2+x+4=-3x$ va $x^2+x+4=-5x$ ko'rinishga keladi.

Bulardan, $x_{1,2}=-2 \quad x_{3,4}=-3\pm\sqrt{5}$.

Mustaqil yechish uchun misollar

Quyidagi tenglamalar teng kuchlimi?

1. $x+3=0$ va $(x+3)\left(x^2+2\right)=0 \quad x\in R$.

2. $x^2+\frac{1}{x}-2x=\frac{1}{x}$ va $x^2=2x$ $x\in R$.

3. $\frac{x^2-4}{x-2}=1$ va $x-2=0$ $x\in R$.

4. $x^2+1=\sqrt{x}$ va $x^2+1+\sqrt{1-x}=\sqrt{x}+\sqrt{1-x}$.

5. $x^3+x=0$ va $\frac{x^3+x}{x}=0$.

6. $\frac{2x^2+2x+3}{x+1}=\frac{3x^2+2x-1}{x+1}$ va $2x^2+2x+3=3x^2+2x-1$.

7. $\sqrt{x}+2=\sqrt{2x}+1$ va $(\sqrt{x}+2)^2=(\sqrt{2x}+1)^2$.

8. $\frac{x^2-4}{x-2}=-4$ va $x+2=-4$ $x\in R$.

9. $2\sqrt{x}-7x^2=2\left(\frac{x}{2}+\sqrt{x}\right)$ va $2\sqrt{x}-7x^2=2x+2\sqrt{x}$.

10. $2\sqrt{x}-7x^2=2x+2\sqrt{x}$ va $-7x^2=2\sqrt{x}$.

11. $f(x)=\phi(x)$ va $|f(x)|^2=|\phi(x)|^2$ $x\in R$.

12. $x^2-1=0$ va $\sqrt{x^2-1}=0$ $x\in R$.

Tenglamalarni yeching

13. $x^4-1=0$.

14. $x^6-64=0$.

15. $x^4-8x+63=0$.

16. $(x+1)(x+3)(x+5)(x+7)=9$.

17. $(x^2+x+1)(x^2+x+2)=12$.

18. $3\left(x+\frac{1}{x^2}\right)-7\left(1+\frac{1}{x}\right)=0$.

19. $\frac{x^2-x}{x^2-x+1}-\frac{x^2-x+2}{x^2-x-2}=1$.

20. $x^3-x^2-\dfrac{8}{x^3-x^2}=2$.

21. $(x+3)^4+(x+5)^4=16$.

22. $(x-4{,}5)^4+(x+-5{,}5)^4=1$.

23. $10x^3-3x^2-2x+1=0$.

24. $4x^3-3x-1=0$.

25. $2x^4-21x^3+74x^2-105x+50=0$.

26. $x^5+4x^4-6x^3-24x^2-27x-108=0$.

27. $x^4+2x^3-2x^2+6x-15=0$.

28. $2x^3-3x^2+6x+4=0$.

29. $\dfrac{5}{x-1}-\dfrac{2}{x+1}=\dfrac{x}{2(x-3)}+\dfrac{(x-1)^2}{x^2-2x-3}$.

30. $\dfrac{24}{x^2+2x}=\dfrac{12}{x^2+x}+x^2+x$.

31. $x^2+x+x^{-1}+x^{-2}=4$.

32. $16x^4+8x^3-7x^2+2x+1=0$.

33. $\dfrac{x^2-6x-9}{x}=\dfrac{x^2-4x-9}{x^2-6x-9}$.

34. $\dfrac{3}{1+x+x^2}=3-x-x^2$.

35. $\dfrac{1}{x^2-3x+3}+\dfrac{2}{x^2-3x+4}=\dfrac{6}{x^2-3x+5}$.

36. $\dfrac{x^2+2x+2}{x+1}+\dfrac{x^2+8x+20}{x+4}=\dfrac{x^2+4x+6}{x+2}+\dfrac{x^2+6x+12}{x+3}$.

37. $\dfrac{3}{2x^2-8}=\dfrac{4-x}{x^4+2x^3+8x-16}-\dfrac{x}{x^3-8}$.

38. $\dfrac{1}{x+1}+\dfrac{x-1}{2(x+1)}=\dfrac{9}{2(4-x)}$.

39. $\frac{5}{2x}-\frac{x-1}{x(x+1)}-\frac{x+1}{x(x-1)}=0$.

40. $\frac{3}{x(x-3)}=\frac{3}{(x+2)(x-1)}-\frac{1}{x(x-1)^2}$.

41. $\dfrac{\frac{1+x}{1-x}-\frac{1-x}{1+x}}{\frac{1+x}{1-x}-1}=\frac{3}{14-x}$.

42. $\dfrac{1}{x+\dfrac{1}{1+\frac{x+2}{x-2}}}=\frac{12}{12x-7}$.

43. $\frac{3}{x(x-3)}=\frac{3}{(x+2)(x-1)}-\frac{1}{x(x-1)^2}$.

44. $\frac{1}{3-x}-\frac{1}{x+1}=\frac{x}{2(x-3)}+\frac{(x-1)^2}{x^2-2x-3}$.

45. $\frac{2x}{x^2-4x+2}+\frac{3x}{x^2+x+2}+\frac{5}{4}=0$.

46. $\frac{1}{6x^2-7x+2}+\frac{1}{12x^2-17x+6}=8x^2-6x+1$.

47. $\left(\frac{x}{x-1}\right)^2+\left(\frac{x}{x+1}\right)^2=\frac{45}{16}$.

48. $\frac{24}{x^2-2x}=\frac{12}{x^2-x}+x^2-x$.

49. $\left(x^3+\frac{1}{x^3}\right)=6\left(x+\frac{1}{x}\right)$.

50. $\frac{x^2}{3}+\frac{48}{x^2}=10\left(\frac{x}{3}-\frac{4}{x}\right)$.

51. $x^2+\frac{25x^2}{(x+5)^2}=11$.

52. $\frac{2x+1}{4x^2+3x+8}+\frac{3x}{14x^2-6x+8}=\frac{1}{6}$.

2-§. Qaytma va yuqori darajali tenglamalar. Kasr-ratsional tenglamalar

$$a_1x^n+a_2x^{n-1}+a_3x^{n-2}+...+a_{n-2}x^2+a_{n-1}x+a_n=0$$

ko'rinishdagi butun algebraik tenglama **qaytma tenglama** deyiladi.

Bu ko'rinishdagi tenglamalarda boshidan va oxiridan bir xil uzoqlikda joylashgan koeffitsientlar teng bo'ladi. Qaytma tenglamalarni $n=2k$ va $n=2k+1$ bo'lgan holatlarda qaraymiz. Buni misollarda keltirib o'tamiz.

1-misol. $21x^6+82x^5+103x^4+164x^3+103x^2+82x+21=0$ tenglamani yeching.

Yechilishi. Tenglamani x^3 ga bo'lamiz.

$$21x^3+82x^2+103x+164+103\cdot\frac{1}{x}+82\cdot\frac{1}{x^2}+21\cdot\frac{1}{x^3}=0$$

$$21\left(x^3+\frac{1}{x^3}\right)+82\left(x^2+\frac{1}{x^2}\right)+103\left(x+\frac{1}{x}\right)+164=0$$

$x+\frac{1}{x}=t$ belgilash kiritsak, $x^2+\frac{1}{x^2}=t^2-2,\quad x^3+\frac{1}{x^3}=t^3-3t$

ga ega bo'lamiz.

$$21t^3+82t^2+40t=0\Rightarrow t\left(21t^2+82t+40\right)=0$$

Bundan, $t_1=0$ va $21t^2+82t+40=0$

Tenglamani ildizlari $t_1=0,\ t_2=-\frac{4}{3},\ t_3=-\frac{10}{3}$

Agar: 1) $t_1=0$ bo'lsa, $x+\frac{1}{x}=0\Rightarrow x^2+1=0$

Tenglamaga ega bo'lamiz. $x_1=-i,\ x_2=i$

2) $t_2=-\frac{4}{3}$ bo'lsa, $7x^2+4x+7=0$ tenglamaga ega bo'lamiz. Uning ildizlari

$$x_{3,4}=\frac{-2\pm3\sqrt{5}i}{7}.$$

3) $t_3=-\frac{10}{3}$ bo'lsa, $3x^2+10x+3=0$ tenglamaga ega bo'lamiz. Uning ildizlari: $x_5=-\frac{1}{3}$, $x_6=-3$

Javob: $x_1=-i,\ x_2=i,\ x_3=\frac{-2-3\sqrt{5}i}{7},\ x_4=\frac{-3+3\sqrt{5}i}{7},\ x_5=-\frac{1}{3},\ x_6=-3$

2-misol. $x^5+x^4-3x^3+3x^2-4x-1=0$

Yechilishi. Bu tenglamaning daraja ko'rsatkichi toq son bo'lgani uchun, bitta ildizi $x_1=1$ ga teng, ya'ni

$$(x-1)(x^4+5x^3+2x^2+5x+1)=0$$

$$x_1=1,\ x^4+5x^3+2x^2+5x+1=0$$

Bu tenglamani x^2 ga bo'lamiz.

$$x^2+5x+2+\frac{5}{x}+\frac{1}{x^2}=0$$

$$\left(x^2+\frac{1}{x^2}\right)+5\left(x+\frac{1}{x}\right)+2=0$$

$x+\frac{1}{x}=t$ belgilash kiritamiz.

U holda $x^2+\frac{1}{x^2}=t^2-2$ ga teng bo'ladi.

Belgilashlarni o'rniga qo'yib, $t^2+5t=0$ ga ega bo'lamiz.

Bundan, $t_1=0,\ t_2=-5$.

Agar: 1) $t_1=0$ bo'lsa, $x^2+1=0 \Rightarrow x_{2,3}=\pm i$

2) $t_1=-5$ bo'lsa, $x^2+5x+1=0$ bo'lib, yechimi $x_{4,5}=\frac{-5\pm\sqrt{21}}{2}$ bo'ladi.

J: $x_1=1,\ x_2=-i,\ x_3=i\ x_4=\frac{-5-\sqrt{21}}{2},\ x_5=\frac{-5+\sqrt{21}}{2}$

Mustaqil yechish uchun misollar

Tenglamalarni yeching

53. $x^4 - 1 = 0$

54. $x^3 - 8x^2 + 40 = 0$

55. $x^3 - 4x^2 + x + 6 = 0$

56. $x^3 - 7x - 6 = 0$

57. $2x^3 - 3x^2 + 6x + 4 = 0$

58. $2x^4 - 5x^3 - x^2 + 3x + 1 = 0$

59. $2x^4 - 5x^3 + 5x^2 - 2 = 0$

60. $4x^4 - 7x^2 - 5x - 1 = 0$

61. $6x^4 - 13x^3 - 27x^2 + 40x - 12 = 0$

62. $x^5 - 6x^4 + 9x^3 - 6x^2 + 8x = 0$

63. $x^5 - 4x^4 - 6x^3 - 24x^2 - 27x - 108 = 0$

64. $2x^4 - 4x^3 + 13x^2 - 6x + 15 = 0$

65. $x^8 - 15x^4 - 16 = 0$

66. $(x^2 - 5x + 7)^2 - (x-2)(x-3) = 1$

67. $\frac{x^2+1}{x} + \frac{x}{x^2+1} = 2,9$

68. $(x+1)(x+2)(x+3)(x+4) = 120$

69. $(x-4,5)^4 + (x-5,5)^4 = 1$

70. $(x+3)^4 + (x+5)^4 = 16$

71. $6x^3 - 13x^2 + 9x - 2 = 0$

72. $4x^3 + 6x^2 + 5x + 69 = 0$

73. $2\left(x^2 + \frac{1}{x^2}\right) - 7\left(x + \frac{1}{x}\right) + 9 = 0$

74. $4x^2+12x+\frac{12}{x}+\frac{4}{x^2}=47$

75. $\frac{x^2}{3}+\frac{48}{x^2}=5\left(\frac{x}{3}+\frac{4}{x}\right)$

76. $x^4-2x^3-x^2-2x+1=0$

77. $x^4+2x^3-7x^2-4x+4=0$

78. $16x^4+8x^3-7x^2-2x+1=0$

79. $30x^4-17x^3-228x^2+17x+30=0$

80. $\frac{(3+x)(2+x)(1+x)}{(3-x)(2-x)(1-x)}=-35$

81. $\frac{x-2}{x-1}+\frac{x+2}{x+1}=\frac{x-4}{x-3}+\frac{x+4}{x+3}-\frac{28}{15}$

82. $\frac{3}{1+x+x^2}=3-x-x^2$

83. $\frac{1}{x^2-3x+3}+\frac{2}{x^2-3x+4}=\frac{6}{x^2-3x+5}$

84. $\frac{12x+1}{6x-2}-\frac{9x-5}{3x+1}=\frac{108x-36x^2}{4(9x^2-1)}$

85. $\frac{x+4}{2x^3-8x+6}+\frac{x-3}{8-2x^2}=\frac{x+6}{x^3+3x^2-x+3}$

86. $\frac{4(x+3)}{2x^3+x^2-8x-4}-\frac{5}{2x^2-3x-2}=1$

87. $\frac{2x-2}{x^2-36}-\frac{x-2}{x^2-6x}=\frac{x-1}{x^2+6x}$

88. $\frac{242}{48-10x-2x^2}+\frac{x^2+8x}{x^2-3x}+\frac{x+2}{x+8}=1$

3-§. Modul qatnashgan tenglamalar

Modul tushunchasi matematikaning muhim tushunchalaridan biri hisoblanadi.

Ta'rif. a sonining moduli deb, agar u son nomanfiy bo'lsa, a sonning o'ziga agar u son manfiy bo'lsa, $-a$ soniga teng songa aytiladi.

Ta'rifga ko'ra,

$$|a|=\begin{cases} a, & agar\ a\geq 0\quad bo'lsa, \\ -a, & agar\ a<0\quad bo'lsa, \end{cases}$$

O'zgaruvchisi modul ostida qatnashgan tenglamalarni yechish uchun quyidagi metodlardan foydalaniladi.

1) Ta'rifdan foydalanib yechiladi.

2) Tenglamani ikkala tomonini kvadratga oshiriladi.

3) Oraliqlarga ajratib yechiladi.

1-misol. $|4x-1|=3$ tenglamani yeching.

Yechilishi: 1-usul. $|4x-1|=3$ Ta'rifga ko'ra,

$$\begin{cases} 4x-1\geq 0 \\ 4x-1=3 \end{cases} \qquad \begin{cases} 4x-1<0 \\ 4x-1=-3 \end{cases}$$

Birinchi sistemadan $x_1=1$ yechimga keltiramiz.

Ikkinchi sistemadan $x_2=-\frac{1}{2}$

2-usul. Tenglikni ikkala tomonini kvadratga oshiramiz.

$$|4x-1|^2=3^2$$

$$16x^2-8x-1=9$$

$$16x^2-8x-10=0$$

$$8x^2-4x-5=0$$

Bundan $x_1=1,\ x_2=-\frac{1}{2}$ javobga kelamiz.

2-misol. $|3x-4|=4-x$ tenglamani yeching.

Yechilishi. Bu tenglamani yechish uchun ta'rifdan foydalamiz. $4-x<0$ bo'lganda tenglama yechimga ega emas, chunki $|3x-4|\geq 0$ bo'lishi kerak. $4-x\geq 0$ bo'lganda tenglamani ikkala tomoni nomanfiy. Shuning uchun kvadratga oshiramiz.

$$\begin{cases} 4-x\geq 0 \\ (3x-4)^2=(4-x)^2 \end{cases} \Rightarrow \begin{cases} x\leq 4 \\ x_1=0,\ x_2=2 \end{cases}$$

Bu ildizlar aniqlanish sohasiga tegishli bo'lganligi uchun

javob: 0 va 2.

3-misol. $|3-x|-|x+2|=5$ (1).

Yechilishi. Bu tenglamani yechish uchun 3-usuldan foydalanamiz. $3-x=0$ va $x+2=0$ nuqtalarni son o'qiga joylashtiramiz. U holda son o'qi $(-\infty;-2)\cup[-2;3]\cup(3;+\infty)$ oraliqlarga ajraladi.

(1)tenglamani har bir oraliqda yechib quyidagicha natija olamiz.

$$\begin{cases} x<-2 \\ 3-x+x+2=5 \end{cases} \vee \begin{cases} -2\leq x\leq 3 \\ 3-x-x-2=5 \end{cases} \vee \begin{cases} x>3 \\ -3+x-x-2=5 \end{cases}$$

yoki $\begin{cases} x<-2 \\ 5=5 \end{cases} \vee \begin{cases} -2\leq x\leq 3 \\ x=-2 \end{cases} \vee \begin{cases} x>3 \\ -5\neq 5 \end{cases}$

Javob: $(-\infty;-2]$

4-misol. $x-|2x+3|=3x-1$ tenglamani yeching.

Yechilishi: Bu tenglamani yechish uchun ta'rifdan «ichma-ich» joylashgan modul sifatida foydalanamiz, ya'ni

$$\begin{cases} x-|2x+3|\ge 0 \\ x-|2x+3|=3x-1 \end{cases} \vee \begin{cases} x-|2x+3|<0 \\ x-|2x+3|=1-3x \end{cases}$$

yoki

$$\begin{cases} |2x+3|\le x \\ |2x+3|=1-2x \end{cases} \vee \begin{cases} |2x+3|>x \\ |2x+3|=4x-1 \end{cases}$$

Birinchi tenglamalar sistemasi yechimga ega emas. Ikkinchi tenglamalar sistemasini yechamiz.

$$\begin{cases} \begin{cases} 2x+3\ge 0 \\ x>0 \end{cases} \\ 2x+3=4x-1 \end{cases} \vee \begin{cases} \begin{cases} 2x+3<0 \\ x>0 \end{cases} \\ 2x+3=1-4x \end{cases}$$

$$\begin{cases} x>0 \\ 2x=4 \end{cases} \quad x=2$$

Javob: 2.

Mustaqil yechish uchun misollar

89. $|x-2|=3$

90. $|x^2-1|=-|x|+1$

91. $|2x-x^2-1|=2x-x^2-1$

92. $\left|\frac{1}{2}x^2-2x+\frac{3}{2}\right|+\left|\frac{1}{2}x^2-3x+4\right|=\frac{3}{4}$

93. $\frac{x^2-1+|x+1|}{|x|\cdot(x-2)}=2$

94. $\frac{|x^2-1|}{x-2}=x$

95. $(1+x)\cdot|x+2|+x|x-3|=6x+2$

96. $|x^2-9|+|x^2-4|=5$

97. $|9-4x|-|4-6x|=|2x+5|$

98. $\dfrac{|x^2-3x|+5}{x^2+|x+3|}=1$

99. $4x^2=|1-8x^2|$

100. $|x|=2x+1$

101. $\dfrac{7x+4}{5}-x=\dfrac{|3x-5|}{2}$

102. $|4x-1|-|2x-3|+|x-2|=0$

103. $|x-1|-|x+2|-|2x-5|+|3-x|=-3$

104. $||x-3|-1|-2|=2$

105. $|2-|1-|x|||=1$

106. $|x^2-4|=x^2-4$

107. $|2x-x^2-1|=2x-x^2-1$

108. $\left|\dfrac{1}{2}x^2-2x+\dfrac{3}{2}\right|+\left|\dfrac{1}{2}x^2-3x+4\right|=\dfrac{3}{4}$

109. $|7-4x|=4x-7$

110. $|x-7|=|x+9|$

111. $|3x+2|=|2x-3|$

112. $|x^2-2|=x^2+1$

113. $2x^2-|x|=15$

114. $x^2+2x-3|x+1|=-3$

115. $|x-1|-|x-2|-1=0$

116. $|x+1|-|x-2|+|3x+6|-5=0$

117. $|x+1|-|x|+3|x-1|=2$

118. $|x^2-9|+|x-2|-5=0$

119. $|x^2-4|+|x^2-9|=5$

120. $|2|x-1|+3x-4|=x-2$

121. $|-5x-3|2x-3|+2|=x+11$

122. $\dfrac{|x^2-x|+1}{|x+1|-x^2}=1$

123. $\dfrac{x+|x^2-3x+2|}{1+|x^2-x|}=1$

124. $||6x|-|6x-3||=3$

125. $||3-2x|-1|=|x-1|$

126. $|x|-|2x+3|=x-1$

127. $\dfrac{|x^2-1|}{x-2}=x$

128. $\dfrac{x^2-1+|x+1|}{(x-2)|x|}-2=0$

4-§. Modul qatnashgan tengsizliklar

O‘zgaruvchi modul belgisi ostida qatnashgan tengsizliklarni yechishda haqiqiy son modulining geometrik ma’nosidan foydalanib yechiladi. a sonining moduli son to‘g‘ri chizig‘ida a nuqtadan koordinata boshigacha bo'lgan masofani bildiradi. $|x_2-x_1|$ ayirmaning moduli esa koordinatalari x_2 ва x_1 nuqtalar orasidagi masofani bildiradi. Bunga quyidagi misollarda yana ham aniqlik kiritamiz.

1-misol. $|x-1|<3$ tengsizlikni yeching.

Yechilishi: I-usul. Ta'rifdan foydalanib yechamiz:

$$\begin{cases} x-1<3 \\ x-1\ge 0 \end{cases} \vee \begin{cases} -(x-1)<3 \\ x-1<0 \end{cases} \qquad \begin{cases} x<4 \\ x\ge 1 \end{cases} \vee \begin{cases} x>-2 \\ x<1 \end{cases}$$

$1\le x<4 \quad \vee \quad -2<x<1 \Rightarrow \quad (-2;4)$ javobga kelamiz.

II-usul. Tengsizlikni ikkala qismini kvadratga oshirish usuli bilan yechamiz. (Buni o'quvchiga havola etamiz).

2-misol. $|2x-3|\le|3x-2|$ tengsizlikni yeching.

Yechilishi. Bu tengsizlikni yechish uchun (1) misoldagi 2-usuldan foydalanamiz.

Tengsizlikni ikkala qismini kvadratga oshiramiz. (Chunki, $(2x-3)$ va $(3x-2)$lar modul ostida qatnashyapti. Moduldan har doim musbat son chiqadi).

$(2x-3)^2\le(3x-2)^2$

$4x^2-12x+9\le 9x^2+12x+4$

$5x^2\ge 5$

$x^2-1\ge 0$

$(x-1)(x+1)\ge 0$

Bundan esa, $(-\infty;-1)\cup(1;+\infty)$javobga kelamiz.

3-misol. $|x^2-3x+2|\le 2x-x^2$ tengsizlikni yeching.

Yechilishi. Bu tengsizlikni modul ta'rifidan foydalanib yechamiz. Tengsizlik quyidagi tengsizliklar sistemalari birlashmalariga teng kuchli, ya'ni:

$$\begin{cases} x^2-3x+2\geq 0 \\ x^2-3x+2\leq 2x-x^2 \end{cases} \vee \begin{cases} x^2-3x+2<0 \\ -(x^2-3x+2)\leq 2x-x^2 \end{cases}$$

Bu tengsizliklar sistemasini yechib, quyidagilarga kelamiz:

$$\begin{cases} (x-1)(x-2)\geq 0 \\ \left(x-\frac{1}{2}\right)(x-2)\leq 0 \end{cases} \vee \begin{cases} (x-1)(x-2)<0 \\ x-2\leq 0 \end{cases}$$

$$\begin{cases} x\leq 1;\ x\geq 2 \\ \frac{1}{2}\leq x\leq 2 \end{cases} \vee \begin{cases} 1<x<2 \\ x\geq 2 \end{cases}$$

Bundan javoblarni umumlashtirib, $x\in\left[\frac{1}{2};2\right]$ ga ega bo'lamiz.

4-misol. $|2x+6|+|x-4|>10$ tengsizlikni yeching.

Yechilishi. Tengsizlikni yechish uchun modul belgisi ostida qatnashgan ifodalarni son to'g'ri chizig'ida nolga aylantiruvchi nuqtalarini topamiz, -3 va 4. Bu nuqtalar son to'g'ri chizig'ini quyidagi oraliqlarga ajratadi:

$(-\infty;3)$, $[-3;4]$ va $[4;+\infty)$

Bu oraliqlarda tengsizlik quyidagi uchta tengsizliklar sistemasi birlashmalariga keltiriladi.

$$\begin{cases} x\leq -3 \\ -(2x+6)-(x-4)>10 \end{cases} \vee \begin{cases} -3\leq x\leq 4 \\ (2x+6)-(x-4)>10 \end{cases} \vee \begin{cases} x\geq 4 \\ 2x+6+x-4>10 \end{cases}$$ Birinchi

$$\Rightarrow \begin{cases} x\leq -3 \\ -3x>12 \end{cases} \vee \begin{cases} -3x\leq x\leq 4 \\ x>0 \end{cases} \vee \begin{cases} x\geq 4 \\ 3x>8 \end{cases}$$

sistemadan $x<-4$, Ikkinchidan $0<x\leq 4$ va uchinchidan $x\geq 4$ ga ega bo'lamiz. Javoblarni birlashtirib, $(-\infty;-4)\cup(0;+\infty)$ javobga kelamiz.

Mustaqil yechish uchun misollar

129. $|3-2x|\ge|4x-9|$

130. $|x-2|+|3-x|>2+x$

131. $|x-6|\le|x^2-5x+9|$

132. $\frac{2x-5}{|x-3|}>-1$

133. $\frac{|x-2|}{x^2-5x+6}\ge 3$

134. $\left|\frac{x^2-3x+2}{x^2+3x+2}\right|-1\ge 0$

135. $\frac{4x-1}{|x-1|}\ge|x+1|$

136. $\frac{|x^2-2x|+4}{x^2+|x+2|}\ge 1$

137. $x^2-\left|1-\frac{2}{x^2}\right|\le 0$

138. $|2x-6|+|x-1|-3<0$

139. $|x+1|+|x+2|+|x-4|>9$

140. $|x+2|-|x+1|+|x|-|x-1|+|x-2|>2{,}5$

141. $|x^2-6x+8|<5x-x^2$

142. $|x^2-3x+2|>3x-x^2-2$

143. $\left|\frac{x^2-5x+4}{x^2-4}\right|\le 1$

144. $\frac{x^2-|x|-6}{x-2} \geq 2x$

145. $|x+5|>11$

146. $|2x-4|\leq 1$

147. $\left|\frac{3}{2x-7}\right| < \left|-\frac{6}{x+4}\right|$

148. $|x+8|<3x-1$

149. $2-x\leq|4-3x|$

150. $|5x^2-2x+1|-1<0$

151. $|-2x^2+3x+5|-2>0$

152. $\frac{|2x-3|}{|x^2-1|}\geq 2$

153. $x^2+2|x|\leq 3$

154. $|x^2+x+10|\leq 3x^2+7x+2$

155. $|3x-1|+|2x-3|-|x+5|<2$

156. $|x-6|-|x^2-5x+9|>0$

157. $\frac{|x+1|}{|x-2|-2}<1$

158. $|2x-|3-x|-2|\leq 4$

159. $||x-3|+1|\geq 2$

160. $\left|\frac{x^2-2x+1}{x^2-4x+4}\right|+\left|\frac{x-1}{x-2}-12\right|<0$

5-§. Tenglamalar sistemasi. Tenglamalar sistemasini yechishning elementar usullari

Tenglamalar sistemasi deb,

$$\begin{cases} a_1x_1+b_1x_2+...+c_1x_n=d_1 \\ a_2x_1+b_2x_2+...+c_2x_n=d_2 \\ -------------- \\ a_nx_1+b_nx_2+...+c_nx_n=d_2 \end{cases}$$

ko'rinishdagi sistemaga aytiladi. Bu yerda $x_1, x_2, ..., x_n$ lar o'zgaruvchilar, $a_1...,a_n$, $b_1...b_n,...$, $c_1,c_2,..c_n$ lar koeffitsientlar, $d_1,d_2,...,d_n$ lar ozod hadlar deyiladi.

Sistemani turli usullarda yechish mumkin.

1) o'rniga qo'yish usuli;

2) qo'shish usuli;

3) taqqoslash usuli;

4) grafik usuli.

1-misol. Tenglamalar sistemasini yeching

$$\begin{cases} xy-6=\frac{y^3}{x} \\ xy+24=\frac{x^3}{y} \end{cases} \qquad (1)$$

Yechilishi. (1) sistemadagi tenglamalarni ko'paytirib,

$$\begin{cases} xy-6=\frac{y^3}{x} \\ (xy-6)(xy+24)=\frac{x^3y^3}{xy} \end{cases} \qquad (2)$$

(2) tenglamalar sistemasini soddalashtirib quyidagi ko'rinishga keltiramiz.

$$\begin{cases} xy - 6 = \dfrac{y^3}{x} \\ xy = 8 \end{cases} \qquad (3)$$

Tenglamalarni ayirib, quyidagi sistemaga kelamiz.

$$\begin{cases} xy = 8 \\ \dfrac{y^3}{x} = 2 \end{cases} \Rightarrow \begin{cases} xy = 8 \\ y^4 = 16 \end{cases}$$

Bundan $y_{1,2} = \pm 2$, $x_{1,2} = \pm 4$

Javob: $\{(4;2);(-4;-2)\}$

2-misol. Tenglamalar sistemasini yeching.

$$\begin{cases} x^3 + y^3 = 19 \\ x^2y + xy^2 = -6 \end{cases}$$

Yechilishi. Sistemani yechish uchun ikkinchi tenglamani uchinchiga ko'paytirib, tenglamalarni qo'shib yuboramiz.

$$\begin{cases} x^3 + y^3 = 19 \\ 3x^2y + 3xy^2 = -18 \end{cases}$$

$$x^3 + 3x^2y + 3xy^2 + y^3 = 1$$

$$(x + y)^3 = 1$$

$$x + y = 1$$

$$\begin{cases} x + y = 1 \\ xy(x + y) = -6 \end{cases} \Rightarrow \begin{cases} x + y = 1 \\ xy = -6 \end{cases}$$

Bu sistemani yechib,

$x_1 = -2$, $y_1 = 3$

$x_2 = 3$, $y_2 = -2$ javobga keltiramiz.

3-misol. Tenglamalar sistemasini yeching.

$$\begin{cases} \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=3 \\ \frac{1}{xy}+\frac{1}{yz}+\frac{1}{xz}=3 \\ \frac{1}{xyz}=1 \end{cases}$$

Yechilishi. Tenglamalar sistemasini yechish uchun, birinchi va ikkinchi tenglamalarni umumiy maxrajga keltiramiz.

$$\begin{cases} \frac{xy+yz+xz}{xyz}=3 \\ \frac{x+y+z}{xyz}=3 \\ xyz=1 \end{cases} \Rightarrow \begin{cases} xy+yz+xz=3 \\ x+y+z=3 \\ xyz=1 \end{cases} \Rightarrow$$

Bu esa birorta $(t-1)^3=0$ ga keladi, ya'ni $t=1$ mos ravishda $x=y=z=1$.

Javob: $x=1, y=1, z=1$.

4-misol. Tenglamalar sistemasini yeching.

$$\begin{cases} x^3y+xy^3=10 \\ xy+x^2+y^2=7 \end{cases} \quad (1)$$

Yechilishi. Birinchi tenglamadan xy ko'paytuvchini qavsdan tashqariga chiqaramiz. $xy(x^2+y^2)=10$

$$\begin{cases} xy(x^2+y^2)=10 \\ xy+(x^2+y^2)=7 \end{cases}$$

$xy=a, \quad x^2+y^2=b$ deb belgilash kiritamiz.

$\begin{cases} ab=10 \\ a+b=7 \end{cases}$ bu esa Viet teoremasiga ko‘ra birorta $t^2-7t+10=0$ tenglamani ildizlariga keladi. Bundan mos ravishda $t_1=2;\ t_2=5$, ya’ni $a_1=2;\ b_1=5$ yoki $a_2=5;\ b_2=2$

Demak, $\begin{cases} xy=2 \\ x^2+y^2=5 \end{cases}$ (2) va $\begin{cases} xy=5 \\ x^2+y^2=2 \end{cases}$ (3)

(2) sistemani yechib, $(2;1)$, $(1;2)$, $(-2;-1)$, $(-1;-2)$ ga kelamiz.

(3) sistema R da yechimga ega emas.

Mustaqil yechish uchun misollar

Tenglamalar sistemasini yeching

161. $\begin{cases} \dfrac{x^2}{y}+\dfrac{y^2}{x}=12 \\ \dfrac{1}{x}+\dfrac{1}{y}=\dfrac{1}{3} \end{cases}$.

162. $\begin{cases} x^3-y^3=26 \\ x^4-y^3=20(x+y). \end{cases}$

163. $\begin{cases} xy+24=\dfrac{x^3}{y} \\ xy-6=\dfrac{y^3}{x}. \end{cases}$

164. $\begin{cases} x^3+4y=y^3+16x \\ \dfrac{1+y^2}{1+x^2}=5 \end{cases}$

165. $\begin{cases} x^2+y^2=7+xy \\ x^3+y^3=6xy-1 \end{cases}$

166. $\begin{cases} x^4-x^2y^2+y^4=601 \\ x^2-xy+y^2=21 \end{cases}$.

167. $\begin{cases} x^3+y^3=9y \\ 3x^2y=4(x+y) \end{cases}$.

168. $\begin{cases} 2y^2-4xy+3x^2=17 \\ y^2-x^2=16 \end{cases}$.

169. $\begin{cases} y^2-x^2=4x+4 \\ x^2++3xy=4 \end{cases}$.

170. $\begin{cases} x^3y+xy^3=\frac{10}{9}(x+y)^2 \\ x^4y+xy^4=\frac{2}{3}(x+y)^3 \end{cases}$.

171. $\begin{cases} x^4+y^4=82 \\ xy=3 \end{cases}$.

172. $\begin{cases} x^4-y^4=15 \\ x^3y-xy^3=6 \end{cases}$.

173. $\begin{cases} x^2y^3=16 \\ x^3y^2=2 \end{cases}$.

174. $\begin{cases} x+yz=8 \\ y+zx=2 \\ z+yx=2 \end{cases}$.

175. $\begin{cases} x^2+y^4=20 \\ x^4+y^2=20 \end{cases}$.

176. $\begin{cases} x^2+y^2=34 \\ x+xy+y=23 \end{cases}$.

177. $\begin{cases} \frac{x^3}{y}+xy=40 \\ \frac{x^3}{y}+xy=10 \end{cases}$.

178. $\begin{cases} x+y+z=2 \\ x^2+y^2+z^2=6 \\ x^3+y^3+z^3=8 \end{cases}$.

179. $\begin{cases} 10(x^4+y^4)=-17(x^3y+xy^3) \\ x^2+y^2=5 \end{cases}$

180. $\begin{cases} x^3+y^3=2 \\ 2xy^2-x^2y=1. \end{cases}$ (butun yechimlarini izlash bilan cheklaning)

181. $\begin{cases} x^2+3=2xy \\ 6x^2-11y^2=10 \end{cases}$.

182. $\begin{cases} \frac{1}{x}-\frac{1}{y}=36 \\ xy^2-x^2y=324 \end{cases}$.

183. $\begin{cases} (x^2+1)(y^2+1)=10 \\ (x+y)(xy-1)=3 \end{cases}$.

184. $\begin{cases} x^3-y^3=26 \\ x^4-y^4=20(x+y) \end{cases}$.

185. $\begin{cases} x+y+xy=19 \\ xy(x+y)=84 \end{cases}$.

186. $\begin{cases} xy(x+y)=20 \\ \frac{1}{x}+\frac{1}{y}=\frac{5}{4} \end{cases}$.

187. $\begin{cases} x^2+y^2=5z \\ x+y=3z \\ x^3+y^3=9z \end{cases}$.

188. $\begin{cases} x^2+y^2+z^2=1 \\ x+y+z=1 \\ x^3+y^3+z^3=1 \end{cases}$.

189. $\begin{cases} x+yx+y=1 \\ y+yz+z=2 \\ z+zx+x=3 \end{cases}$.

6-§. Kasr ratsional va yuqori darajali tengsizliklar

O'zgaruvchisi ildiz ostida qatnashgan ifodalar irratsional ifodalar deyiladi.

Arifmetik ildiz ta'rifmi eslasak,

Ta'rif. $a\geq 0$ sonning n-darajali arifmetik ildizi deb $(n\in N)$, n-darajasi a ga teng bo'lgan $b\geq 0$ songa aytiladi va $b=\sqrt[n]{R}$ ko'rinishda belgilanadi. $(\sqrt[n]{a})^n=a$

$a>0$, k∈Zva $n\in N, n\neq 1$ bo'lsa $\sqrt[n]{a^k}$ soni a ning $r=\frac{k}{n}$ ratsional ko'rsatkichli darajasi deyiladi, ya'ni, $a^r=a^{\frac{k}{n}}=\sqrt[n]{a^k}$

Yuqoridagilarga asosan, $\sqrt{36}=6$ to'g'ri, $\sqrt{36}=-6$, yoki $\sqrt{36}=\pm 6$ noto'g'ri ekanligini ko'rish mumkin.

Ta'rifga ko'ra:

$$\sqrt[n]{a^n}=\begin{cases} |a| - \text{agar } n \text{ juft son bo'lsa} \\ a - \text{agar } n \text{ toq son bo'lsa, } n\neq 1 \end{cases}$$

Masalan: $\sqrt{a^2}=|a|$, $\sqrt[3]{a^3}=a$.

Xossalari:

1^0. $\sqrt[n]{ab}=\sqrt[n]{a}\cdot\sqrt[n]{b}$, agar $a\geq 0$, b≥0 bo'lsa.

2^0. $\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, agar $a\geq 0$, b>0 bo'lsa.

3^0. $(\sqrt[n]{a^m})^k=\sqrt[n]{a^{km}}$

4^0. $\sqrt[n]{\sqrt[k]{a^m}}=\sqrt[nk]{a^m}$

5^0. $\sqrt[mn]{a^{mk}}=\begin{cases}\sqrt[n]{|a^k|}, & \text{agar m juft bo'lsa}\\ \sqrt[n]{a^k}, & \text{agar m toq bo'lsa}\end{cases}$

Misol. $\frac{3}{1+\sqrt{2}-\sqrt{3}}$ ifodani maxrajini irratsionallikdan qutqaring.

Yechilishi. $1+\sqrt{2}-\sqrt{3}$ ni qo'shmasi $1+\sqrt{2}+\sqrt{3}$ ga kasrni surat va maxrajini ko'paytiramiz.

$$\frac{3\left(1+\sqrt{2}+\sqrt{3}\right)}{\left(1+\sqrt{2}-\sqrt{3}\right)\left(1+\sqrt{2}+\sqrt{3}\right)}=\frac{3\left(1+\sqrt{2}+\sqrt{3}\right)}{\left(1+\sqrt{2}\right)^2-\sqrt{3}^2}=\frac{3\left(1+\sqrt{2}+\sqrt{3}\right)}{2\sqrt{2}};$$

Endi $\sqrt{2}$ dan qutulishimiz kerak.

$$\frac{3\sqrt{2}\left(1+\sqrt{2}+\sqrt{3}\right)}{2\sqrt{2}\cdot\sqrt{2}}=\frac{3\left(\sqrt{2}+2+\sqrt{6}\right)}{4}\qquad \text{Javob: } \frac{3\left(\sqrt{2}+2+\sqrt{6}\right)}{4}$$

Misol. $\sqrt{5}-\sqrt{3-\sqrt{29-12\sqrt{5}}}$ ifodani soddalashtiring.

Yechilishi. Avvalambor kvadrat ildizlar ostidagi ifodalarning musbat ekanini, ya'ni ildizlar istalgan R da ma'noga ega ekanini ko'rsatishimiz kerak.

$$3-\sqrt{29-12\sqrt{5}}>0\Rightarrow 3>\sqrt{29-12\sqrt{5}}$$

Buning uchun $29-12\sqrt{5}>0$ ekanini ko'rsatish kerak. R da ayrim almashtirishlar bajarib quyidagiga kelamiz.

$$29-2\cdot3\cdot2\cdot\sqrt{5}=20-2\cdot3\sqrt{20}+9=\left(\sqrt{20}-3\right)^2>0$$

$$3-\left(\sqrt{20}-3\right)=6-\sqrt{20}$$

$$\sqrt{5}-\sqrt{6-\sqrt{20}}=\sqrt{5}-\sqrt{\left(\sqrt{5}-1\right)^2}=\sqrt{5}-\sqrt{5}+1=1$$

Mustaqil yechish uchun misollar

Tengsizlikni yeching

190. $\frac{4x-17}{x-4}+\frac{10x-13}{2x-3}\le\frac{8x-30}{2x-7}+\frac{5x-4}{x-1}$.

191. $2x^2+2x+1-\frac{15}{x^2+2x+1}<0$.

192. $(x^2-2x)(2x-2)-\frac{9(2x-2)}{x^2-2x}<0$.

193. $\frac{1}{1+2x}-\frac{2}{2+3x}+\frac{3}{3+4x}<\frac{4}{5x+4}$.

194. $\frac{1}{x+1}-\frac{2}{x^2-x+1}\le\frac{1-2x}{x^3+1}$.

195. $\frac{2-x}{x^3+x^2}\ge\frac{1-2x}{x^3-3x^2}$.

196. $\frac{10(5-x)}{3(x-4)}-\frac{11(6-x)}{3(x-4)}\ge\frac{5(6-x)}{x-2}$.

197. $\frac{3}{6x^2-x-12}<\frac{25x-47}{10x-15}-\frac{3}{3x+4}$.

198. $\frac{x^2(x+1)}{(x^2+x+1)^2}>\frac{2}{9}$.

199. $\frac{x^3-2x^2-5x+6}{x-2}>0$.

200. $\frac{3}{x+1}+\frac{7}{x+2}<\frac{6}{x-1}$.

201. $\frac{x}{x^2+7x+12}<\frac{x}{x^2+3x+2}$.

202. $\frac{2x}{4x^2+3x+8}+\frac{3x}{4x^2+6x+8}>\frac{1}{6}$.

203. $\frac{x}{3}-\frac{4}{x}>\frac{4}{3}$.

204. $x^2+\frac{4x^2}{(x+2)^2}<5$.

205. $x^4+8x^3+12x^2\geq 0$.

206. $(x-1)(x^2-3x+8)<0$.

207. $(x-1)(x^2-1)(x^3-1)(x^4-1)\leq 0$.

208. $(16-x^2)(x^2+4)(x^2+x+1)(x^2-3x)\leq 0$.

209. $(2x^2-x-5)(x^2-9)(x^2-3x)\leq 0$.

210. $\frac{x^2-4x-2}{9-x^2}<0$.

211. $\frac{x^4-2x^2-8}{x^2+x-1}<0$.

212. $\frac{3x-2}{2x-3}<3$.

213. $\frac{1}{x+1}+\frac{2}{x+3}>\frac{3}{x+2}$.

214. $2x^2-5x-12<0$.

215. $4x^2+2x+5<0$.

216. $3(x-3)^2(4-x)^3(2-3x-2x^2)^5(x^2+x+3)<0$.

217. $x^3-6x^2+5x+12\leq 0$.

218. $x^4+6x^3+11x^2+6x>0$.

219. $32x^4-48x^3-10x^2+21x+5\leq 0$.

220. $\frac{(x+3)^2(5x+2)^3}{(2-x)^5(1-3x)}\geq 0$.

221. $\frac{(1-4x^2)(x+2)^2}{x^3(x+1)(x-2)^5}<0$.

222. $\dfrac{\left(5x^2+8x-13\right)^3\left(x-\sqrt{2}\right)^2(1-x)}{\left(-3x^2+2x-5\right)^5(x+3)7(x-2)^4}\geq 0$.

223. $\dfrac{4}{3x-1}<\dfrac{3}{x+2}$.

224. $\dfrac{x}{x-1}-\dfrac{2}{x+1}>\dfrac{8}{x^2-1}$.

225. $\dfrac{(x-2)(x-3)}{(x-4)(x-5)}\geq 1$.

226. $\dfrac{x^2(x+1)}{\left(x^2+x+1\right)^2}>\dfrac{2}{9}$.

7-§. Irratsional tenglama va tenglamalar sistemasi

Ta'rif. O'zgaruvchisi ildiz belgisi ostida qatnashgan tenglama irratsional tenglama deyiladi.

Irratsional tenglamalarni yechishni ba'zi metodlarini eslatib o'tamiz.

a) darajaga ko'tarish usuli bilan yechiladigan tenglamalar;

b) yangi o'zgaruvchi kiritish usuli bilan yechiladigan tenglamalar;

c) sun'iy usul bilan yechiladigan tenglamalar.

Albatta, darajaga ko'tarish usulida chet ildizlar paydo bo'lishi mumkin. Bunday hollarda tenglamaning aniqlanish sohasiga e'tibor berish kerak, yoki bo'lmasa ildizlarni tenglamani o'rniga qo'yib tekshirib ko'rish kerak.

1-misol. $\sqrt{-x^2+x+6}=2x-7$ tenglamani yeching.

Yechilishi. Tenglamani ikkala tomonini ham kvadratga oshiramiz.

$-x^2+x+6=4x^2-28x+49$

$5x^2-29x+43=0$

javob: $\varnothing$

R da yechimga ega emas.

2-misol. $\sqrt{7+\sqrt[3]{x^2+7}}=3$ tenglamani yeching.

Yechilishi. Tenglamani ikkala tomonini ham kvadratga oshiramiz.

$$7+\sqrt[3]{x^2+7}=9$$
$$\sqrt[3]{x^2+7}=2$$

endi kubga oshiramiz.

$$x^2+7=8$$
$$x^2=1$$
$$x_1=1$$
$$x_2=-1$$

Javob: $\{-1;1\}$

Mustaqil yechish uchun misollar

Tenglamalarni yeching

227. $\sqrt{3x+4}+\sqrt{x-4}=2\sqrt{x}$.

228. $1+\sqrt{1+x\sqrt{x^2-24}}=x$.

229. $\sqrt[3]{24+\sqrt{x}}-\sqrt[3]{5+\sqrt{x}}=1$.

230. $\sqrt[5]{\frac{5-x}{x+3}}-\sqrt[5]{\frac{x+3}{5-x}}=2$.

231. $x^2-4x-6=\sqrt{2x^2-8x+12}$.

232. $\sqrt{\frac{20+x}{x}}-\sqrt{\frac{20-x}{x}}=\sqrt{6}$.

233. $\sqrt{x+8+2\sqrt{x+7}}+\sqrt{x+1-\sqrt{x+7}}=4$.

234. $\sqrt{x^2+x+4}+\sqrt{x^2+x+1}=\sqrt{2x^2+2x+9}$.

235. $\sqrt[3]{x+7}-\sqrt{x+3}=0$.

236. $\sqrt[4]{x+8}-\sqrt[4]{x-8}=2$.

237. $\sqrt[3]{x+5}-\sqrt[3]{x+6}=\sqrt[3]{2x+1}$.

238. $\frac{1}{\sqrt{x}+\sqrt[3]{x}}+\frac{1}{\sqrt{x}-\sqrt[3]{x}}=\frac{1}{3}$.

239. $\frac{x^2}{\sqrt{2x+15}}+\sqrt{2x+15}=2x$.

240. $x^3+x+\sqrt{x^3+x-2}=12$.

241. $\sqrt{4-2x}+\sqrt{2+x}=2\sqrt{2}$.

242. $\sqrt{x^2-3x+3}+\sqrt{x^2-3x+6}=3$.

243. $x^2+\sqrt{x^2+20}=22$.

244. $\sqrt{x\sqrt[5]{x}}+\sqrt[5]{x\sqrt{x}}=56$.

245. $\sqrt{x^2-3x}\cdot\sqrt{2x-4}=x-1$.

246. $\sqrt{x^2-4x+3}+\sqrt{-x^2+3x-2}=\sqrt{x^2-x}$.

247. $x+\sqrt{17-x^2}+x\sqrt{17-x^2}=9$.

248. $\sqrt{x}+\sqrt{2-x}+\sqrt{2x-x^2}=\sqrt{2}$.

249. $4\left(\sqrt{1+x}-1\right)\left(\sqrt{1-x}+1\right)=x$.

250. $\frac{\sqrt{x+4}+\sqrt{x-4}}{2}=x+\sqrt{x^2-16}-6$.

251. $\sqrt{x^2+9}+\sqrt{x^2-9}=\sqrt{7}+5$.

252. $\sqrt{x+5}+\sqrt{x+3}=\sqrt{2x+7}$.

253. $\sqrt{x+5-4\sqrt{x+1}}+\sqrt{x+2-2\sqrt{x+1}}=1$.

254. $\sqrt{10-x^2}+\sqrt{x^2+3}=5$.

255. $\sqrt[3]{9-\sqrt{x+1}}+\sqrt[3]{7+\sqrt{x+1}}=4$.

256. $\sqrt{9x^2+2x-3}=3x-2$.

257. $(x+2)(x-5)+3\sqrt{x(x-3)}=0.$

258. $\sqrt{x+2\sqrt{x-1}}-\sqrt{x-2\sqrt{x-1}}=2.$

259. $\sqrt[3]{x-2}-\sqrt[3]{x+3}=\sqrt[3]{2x+1}.$

260. $\sqrt{x^2+3x-3}=2x-3.$

261. $\sqrt{1-x}=\sqrt{6-x}-\sqrt{-5-2x}.$

262. $\sqrt[3]{x+1}+\sqrt[3]{x-1}=\sqrt[3]{5x}.$

263. $\sqrt{\sqrt{6x^2+1}-2x}=1-x.$

Tenglamalar sistemalarini yeching

264. $\begin{cases}\sqrt[3]{x-y}=\sqrt{x-y}\\ \sqrt[3]{x+4}=\sqrt{x+y-4}\end{cases}.$

265. $\begin{cases}\sqrt{\dfrac{y+1}{x-y}}+2\sqrt{\dfrac{x-y}{y+1}}=3\\ x+xy+y=7\end{cases}.$

266. $\begin{cases}(x-y)\sqrt{y}=\dfrac{\sqrt{x}}{2}\\ (x+y)\sqrt{x}=3\sqrt{y}.\end{cases}$

267. $\begin{cases}x\sqrt{x}+y\sqrt{y}=341\\ x\sqrt{y}+y\sqrt{x}=330.\end{cases}$

268. $\begin{cases}\sqrt{2x-y+11}-\sqrt{3x+y-9}=13\\ \sqrt[4]{2x-y+11}-\sqrt[4]{3x+y-9}=2\end{cases}.$

269. $\begin{cases}x\sqrt{y}+y\sqrt{x}=6\\ x^2y+y^2x=20\end{cases}.$

270. $\begin{cases}x^2+x\sqrt[3]{xy^2}=80\\ y^2+y\sqrt[3]{x^2y}=5\end{cases}.$

271. $\begin{cases} \sqrt{x+\sqrt{y}}+\sqrt{x-\sqrt{y}}=2 \\ \sqrt{y+\sqrt{x}}-\sqrt{y-\sqrt{x}}=1. \end{cases}$

272. $\begin{cases} \sqrt{x-4}+\sqrt{y}+\sqrt{z+4}=6 \\ 2\sqrt{x-4}-\sqrt{y}-4\sqrt{z+4}=-12 \\ x+y+z=14 \end{cases}$

273. $\begin{cases} \sqrt[3]{x}+\sqrt[3]{y}=4 \\ x+y=28 \end{cases}$

274. $\begin{cases} \sqrt[4]{x}-\sqrt[4]{y}=1 \\ \sqrt{x}+\sqrt{y}=5. \end{cases}$

275. $\begin{cases} \sqrt{\dfrac{20y}{x}}=\sqrt{x+y}+\sqrt{x-y} \\ \sqrt{\dfrac{16x}{5y}}=\sqrt{x+y}-\sqrt{x-y}. \end{cases}$

276. $\begin{cases} \sqrt[3]{x+y}+\sqrt[3]{y+z}=3 \\ \sqrt[3]{y+z}+\sqrt[3]{z+x}=1 \\ \sqrt[3]{z+x}+\sqrt[3]{x+y}=0. \end{cases}$

8-§. Irratsional tengsizliklar

Irratsional tengsizliklarni yechishda xuddi irratsional tenglamalarni yechishdagi kabi tengsizlikni ikkala qismini bir xil darajaga ko‘tarish, (yoki bir xil darajani pasaytirish) yangi o‘zgaruvchi kiritish va h.k.lar qo‘llaniladi.

Albatta irratsional tengsizliklarni yechishda aniqlanish sohasiga e’tibor berish kerak.

1-misol. $\sqrt{x^2-3x-10}<8-x$ tengsizlikni yeching. (1)

Yechilishi. Avvalambor aniqlanish sohasini qaraylik. Tengsizlikni chap qismi $x^2-3x-10\geq 0$ bo'lishi kerak. Tengsizlikni o'ng qismi esa $8-x>0$ va $8-x<0$ birinchi holatda tengsizlikni ikkala qismini kvadratga oshiramiz. Ikkinchi holatda esa tengsizlikni chap qismi nomanfiy, o'ng qismi esa manfiy. Bunday holatda yechimga ega emas. Shunday qilib (1) tengsizlik aniqlanish sohasini e'tiborga olib quyidagi ko'rinishda yozamiz.

$$\begin{cases} x^2-3x-10\geq 0 \\ 8-x>0 \\ x^2-3x-10<(8-x)^2 \end{cases} \Rightarrow \begin{cases} x\leq -2 \vee x\geq 5 \\ x<8 \\ x<5\frac{9}{13} \end{cases} \Rightarrow \begin{cases} x\leq -2 \\ x<8 \\ x<5\frac{9}{13} \end{cases} \vee \begin{cases} x\geq 5 \\ x<8 \\ x<5\frac{9}{13} \end{cases} \Rightarrow$$

$x\leq 2 \vee 5\leq x<5\frac{9}{13}$ Javob: $(-\infty;-2]\cup\left[5;5\frac{9}{13}\right)$

Mustaqil yechish uchun misollar

Tengsizliklarni yeching

277. $\sqrt{x^2-5x+6}<2x-3$.

278. $\sqrt{x^2-7x-8}>x-8$.

279. $\sqrt{x+2}+\sqrt{2x-10}<\sqrt{x+12}$.

280. $\sqrt{x+\sqrt{x}}-\sqrt{x-\sqrt{x}}>\frac{3}{2}\sqrt{\frac{x}{x+\sqrt{x}}}$.

281. $\sqrt[3]{x^2-x}>-\sqrt[3]{2}x$.

282. $\sqrt{\frac{x+3}{4-x}}>x-8$.

283. $\frac{\sqrt{17-15x-2x^2}}{x+3}>0$.

284. $2\sqrt{x+1}-\sqrt{2x-10}\geq 2\sqrt{x-3}$.

285. $x^2+\sqrt{x^2+11}<31$.

286. $\sqrt{\frac{2x-1}{x+2}}-\sqrt{\frac{x+2}{2x-1}}\ge\frac{7}{12}$.

287. $\sqrt[3]{x+5}+2>\sqrt[3]{x-3}$.

288. $\sqrt{3x^2+5x+7}-\sqrt{3x^2+5x+2}>1$.

289. $\frac{2}{2+\sqrt{4-x^2}}+\frac{1}{2-\sqrt{4-x^2}}\ge\frac{1}{x}$.

290. $\frac{\sqrt{x^2-16}}{\sqrt{x-3}}+\sqrt{x-3}>\frac{5}{\sqrt{x-3}}$.

291. $\sqrt{x+1}+1<4x^2+\sqrt{3x}$.

292. $\frac{x-7}{\sqrt{4x^2-19x+12}}<0$.

293. $\sqrt{x^2-x-12}<x$.

294. $\frac{\sqrt{17-15x-2x^2}}{x+3}>0$.

295. $\sqrt{3x-x^2}<4-x$.

296. $\frac{4}{\sqrt{2-x}}+\sqrt{2-x}<2$.

297. $\sqrt{x^2-4x}>x-3$.

298. $\sqrt{\frac{1}{x^2}-\frac{3}{4}}<\frac{1}{x}-\frac{1}{2}$.

299. $\frac{6x}{x-2}-\sqrt{\frac{12x}{x-2}}-\sqrt[4]{\frac{12x}{x-2}}>0$.

300. $\sqrt{5x-4}+\sqrt{3x+1}<3$.

301. $\sqrt{x^4-2x^2+1}>1-x$.

302. $\sqrt{1-x}-\sqrt{x}>\frac{1}{\sqrt{3}}$.

303. $\left(x-\frac{1}{3}\right)^2-\frac{25}{9}>\frac{3x^2+\frac{4}{9}}{2\left(x-\frac{1}{3}\right)+\sqrt{x\left(x-\frac{8}{3}\right)}}$.

304. $\sqrt{x-\frac{1}{x}}-\sqrt{1-\frac{1}{x}}>\frac{x-1}{x}$.

305. $\sqrt{2x+4}-2\sqrt{2-x}>\frac{12x-8}{\sqrt{9x^2+16}}$.

306. $x+\frac{x}{\sqrt{x^2-1}}>\frac{35}{12}$.

307. $\sqrt{2x+1}<\frac{2(x+1)}{2-x}$.

9-§. Ko'rsatkichli va logarifmik tenglamalar

Ko'rsatkichli tenglamalar

Ko'rsatkichli tenglamalarda noma'lum daraja ko'rsatkichida ishtirok etadi. Ko'rsatkichli tenglamalarni yechishda asosan quyidagi ikkita usuldan foydalaniladi.

1. $a^{f(x)}=a^{g(x)}$ tenglamadan $f(x)=g(x)$ tenglama ko'rinishiga keltirish.

2. Yangi o'zgaruvchi kiritish usuli.

Teorema. Agar $a>0, a\neq 1$ bo'lsa, $a^{f(x)}=a^{g(x)}$ va $f(x)=g(x)$ tenglamalar teng kuchlidir.

1-misol. $4^{5x^2-46}=4^{2(x^2+1)}$ tenglamani yeching.

Yechilishi. Tenglamani yechish uchun 1-usuldan foydalanamiz, ya'ni tenglamani bir ko'rinishdan unga teng kuchli bo'lgan ikkinchi ko'rinishga o'tkazamiz, ya'ni, $4^{5x^2-46}=4^{2(x^2+1)}\Rightarrow 5x^2-46=2(x^2+1)$ bundan **javob:** $x_1=-4, x_2=4$

2-misol. $4^x+2^{x+1}-24=0$ tenglamani yeching.

Yechilishi. Quyidagicha almashtirishlar kiritgandan keyin, ya'ni $4^x=(2^2)^x=(2^x)^2$ $2^{x+1}=2\cdot 2^x$ dan so'ng tenglama $(2^x)^2+2\cdot 2^x-24=0$ ko'rinishga keladi. Yangi o'zgaruvchi kiritib, $2^x=t$, $t^2+2t-24=0$ kvadrat tenglamaga kelamiz.

$2^x=4\Rightarrow x=2$

$2^x=-6$

Tenglama yechimga ega emas. Javob: $x=2$

3-misol. $2^{2x+2\sqrt{x^2-2}}-5\cdot 2^{x+\sqrt{x^2-2}-1}=6$

Yechilishi. Berilgan tenglamani quyidagicha yozib olamiz.

$$2^{2\left(x+\sqrt{x^2-2}\right)}-\frac{5}{2}\cdot 2^{x+\sqrt{x^2-2}}=6$$

$2^{x+\sqrt{x^2-2}}=t$ o'zgarish kiritib $t^2-\frac{5}{2}t-6=0$ kvadrat tenglamaga keltiramiz. Bundan $t_1=4, t_2=-\frac{3}{2}$ larga kelamiz. Natijada $2^{x+\sqrt{x^2-2}}=4$, $2^{x+\sqrt{x^2-2}}=-\frac{3}{2}$ larga

ega bo'lamiz. Birinchi tenglamadan $x+\sqrt{x^2-2}=2$ va $x=\frac{3}{2}$ ga keltiramiz. Ikkinchi tenglama yechimga ega emas.

Logarifmik tenglamalar

$\log_a x=b$ $a>0, a\neq 1$ tenglamani qaraymiz. Bu tenglama eng sodda logarifmik tenglama deyiladi. $x=a^b$ son qaralayotgan tenglamani ildizi deyiladi. Logarifmik tenglamalarni yechishda asosan logarifmning quyidagi xossalaridan foydalaniladi.

1^0. $\log_a f(x)+\log_a g(x)=\log_a(f(x)\cdot g(x))$, $(a>0,\ a\geq 1)$.

2^0. $\log_a f(x)-\log_a g(x)=\log_a \frac{f(x)}{g(x)}$ $(a>0,\ a\geq 1)$.

3^0. $\log_a(f(x))^n=\begin{cases} n\log_a f(x), & \text{agar } n\in R,\ n\neq 2k,\ k\in Z \\ n\log_a|f(x)|, & \text{agar } n=2k \end{cases}$ bo'lsa

$(a>0, a\neq 1)$

Agar tenglamalarni yechishda logarifmning asoslari har xil bo'lib qolsa, u holda quyidagi bir asosdan ikkinchi asosga o'tish formulasidan foydalaniladi.

4^0. $\log_a f(x)=\frac{\log_c f(x)}{\log_c a}$ bunda $c>0, c\neq 1$

Logarifmik tenglamani yechishda asosan uchta metoddan foydalanamiz.

1) potensirlash;

2) yangi o'zgaruvchi kiritish;

3) logarifmlash;

Misollar ko'rib o'tamiz.

1-misol. Tenglamani yeching.

$\log_7(7-2x)=\log_7(x^2+3x+1)$

Yechilishi. Bu tenglamadan potensirlash yordamida $7-2x=x^2+3x+1$ tenglamaga kelamiz. Bundan, $x^2+5x-6=0$. Bu tenglamani yechib $x_1=-6, x_2=1$ ildizlarni topamiz. Albatta tekshirib ko'ramiz.

1) $\log_7(7-2(-6))=\log_7((-6)^2+3(-6)+1)$

$\log_7 19=\log_7 19$

2) $\log_7(7-2)=\log_7(1+3\cdot 1+1)$

$\log_7 5=\log_7 5$

Demak, ildizlarning har ikkalasi ham tenglamaning yechimi bo'ladi.

2-misol. Tenglamani yeching.

$$\frac{\lg\sqrt{x+7}-\lg 2}{\lg 8-\lg(x-5)}=-1$$

Yechilishi. Bu tenglamani yechish uchun logarifmning ikkinchi xossasidan foydalanamiz.

$$\lg\sqrt{x+7}-\lg 2=\lg(x-5)-\lg 8$$
$$\lg\sqrt{x+7}+\lg 8=\lg(x-5)+\lg 2$$
$$\lg 8\sqrt{x+7}=\lg 2(x-5)$$

Endi potensirlaymiz.

$$8\sqrt{x+7}=2(x-5)$$
$$4\sqrt{x+7}=x-5$$
$$16(x+7)=x^2-10x+25$$
$$x^2-26x-87=0$$
$$(x-29)(x+3)=0$$
$$x_1=29,\quad x_2=-3$$

Tekshirish:

1) $\lg\left(\sqrt{-3+7}\right)-\lg 2=\lg(-3-5)-\lg 8$ logarifm ostida manfiy son bo'lishi mumkin emas. $x_1=-3$ chet ildiz.

2) $\lg\sqrt{29+7}-\lg 2=\lg 24-\lg 8$

$\lg 3=\lg 3$ demak, $x=29$ ildiz bo'ladi.

3-misol. Tenglamani yeching.

$$\lg^2 x+\lg x+1=\frac{7}{\lg\frac{x}{10}} \qquad (1)$$

Yechilishi. $\lg\frac{x}{10}=\lg x-\lg 10=\lg x-1$ ko'rinishda yozib olamiz. Bundan (1) tenglama $\lg^2 x+\lg x+1=\frac{7}{\lg x-1}$ ko'rinishga keladi. $\lg x=t$ deb o'zgaruvchi kiritamiz. $t^2+t+1=\frac{7}{t-1}$

Bu tenglamani yechib $t=2$ ildiz olamiz.

$\lg x=2\Rightarrow x=100$

Tekshirishlar natijasiga ko'ra $x=100$ (1) tenglamaning yechimi bo'ladi.

4-misol. $\lg^2 x^3-\lg(0{,}1x^{10})=0$

Yechilishi.

$(\lg x^3)^2-\lg x^{10}-\lg 0{,}1=0$

$9\lg^2 x-10\lg|x|+1=0$

Agar tenglamaning aniqlanish sohasiga e'tibor beradigan bo'lsak, u holda tenglamani $x>0$ holini qaraymiz.

$9\lg^2 x-10\lg x+1=0$ bundan $\lg x=1$, $\lg x=\frac{1}{9}$ $x_1=10$ $x_2=\sqrt[9]{10}$. Tekshirib ko'radigan bo'lsak, topilgan ildizlarining har ikkalasi ham tenglamaning ildizlari bo'ladi.

Mustaqil yechish uchun misollar

Tenglamalarni yeching

308. $3\cdot 4^x+2\cdot 25^x=5\cdot 10^x$.

309. $4^{x-1}-3^{x-\frac{3}{2}}=3^{x-\frac{1}{2}}-2^{2x-3}$.

310. $2^{\sqrt{x}}=16\sqrt{\left(\frac{1}{4}\right)^{5\frac{1}{4}-\frac{x}{4}}}$.

311. $10^{(x+1)(3x+4)}-2\cdot 10^{(x+1)(x+2)}=10^{1-x-x^2}$.

312. $32^{\frac{1}{x^2-1}}\cdot 4^{\frac{1}{x}}-8^{\frac{1}{x-1}}=0$.

313. $2^{\sqrt{x+1}}\cdot\sqrt{2^{\sqrt{6}}}=4^{\sqrt{x+1}}$.

314. $4^x\cdot 3^{2x}-2\cdot 6^{3x-1}+4^{2x-1}\cdot 3^{4x-2}=0$.

315. $3^{x-1}-\dfrac{15}{3^{x-1}}+3^{x-2}-\dfrac{23}{3^{x-2}}=0$.

316. $\sqrt{2^x\cdot\sqrt[3]{4^x\cdot\sqrt[x]{0.125}}}=4\sqrt[3]{2}$.

317. $100^{\frac{1}{2}}+25^{\frac{1}{2}}=4.25\cdot 50^{\frac{1}{2}}$.

318. $9^x+6^x=2^{2x+1}$.

319. $2^{x-1}+2^{x-2}+2^{x-4}=6,5+3,25++1,625+\dots$

320. $3^{2x-+4}+45\cdot 6^x-9\cdot 2^{2x+2}=0$.

321. $\left(2+\sqrt{3}\right)^{x^2-2x+1}+\left(2-\sqrt{3}\right)^{x^2-2x-1}=\dfrac{101}{10\left(2-\sqrt{3}\right)}$.

322. $2^{x^2}\cdot 5^{x^2}=0{,}001\cdot\left(10^{3-x}\right)^2$.

323. $0{,}6^x\cdot\left(\dfrac{25}{9}\right)^{x^2-12}=\left(\dfrac{27}{125}\right)^3$.

324. $3^x+3^{x+1}+3^{x+2}=4^x+4^{x+1}+4^{x+2}$.

325. $\left(\sqrt[3]{5}\right)^x+\left(\sqrt[3]{5}\right)^{2x}+20=0$.

326. $\left(2+\sqrt{3}\right)^x+\left(2-\sqrt{3}\right)^x-4=0$.

327. $5^{2x}-7^x-35\cdot 5^{2x}+35\cdot 7^x=0$.

328. $\left(2^x+3^x\right)\left(3\cdot 2^x+3^x\right)=8\cdot 6^x$.

329. $\left(3x-4\right)^{2x^2+2}=\left(3x-4\right)^{5x}$.

330. $3^{x-2}+3^{x+2}=3^x$.

331. $\sqrt{x}\left(9^{\sqrt{x^2-3}}-3^{\sqrt{x^2-3}}\right)=3^{2\sqrt{x^2-3}-1}-3^{\sqrt{x^2-3}-1}+6\sqrt{x}-18.$

332. $2^{x^2-6x-\frac{5}{2}}=16\sqrt{2}.$

333. $2^{x+3}+3^{x^2-3}=3^{x^2+1}-2^{x-1}.$

334. $5^x\cdot 8^{\frac{x}{x+1}}=100.$

335. $3^x\cdot 4^x=5^x.$

336. $\left(\frac{57}{37}\right)^{1+x}+\left(\frac{57}{37}\right)^3=10.$

Logarifmik tenglamalar

337. $2x(1-\lg 5)=\lg\left(4^x+2x-6\right).$

338. $\lg\left(x^2-1\right)=\lg(x-1)^2+\lg|2-x|.$

339. $2\log_2 x-\log_{\sqrt{2}} x+\log_{\frac{1}{2}} x=9.$

340. $\log_{2x}\frac{2}{x}\cdot\log_2^2 x+\log_2^4 x=1,\qquad x>1.$

341. $\log_4\log_2 x+\log_2\log_4 x=2.$

342. $x^2\log_x 27\cdot\log_9 x=x+4.$

343. $\log_{\sqrt{3}} x+\log_{\sqrt[4]{3}} x+\ldots+\log_{\sqrt[10]{3}} x=36.$

344. $\log_{4x}\sqrt{x}+7\log_{16x} x^3-\log_{\frac{x}{2}} x^2=0.$

345. $x(-1+\lg 5)=\lg\left(2^x+1\right)-\lg 6.$

346. $\dfrac{\lg 8-\lg(x-5)}{\lg\sqrt{x+7}-\lg 2}=-1.$

347. $2^{\log_2 x^2}\cdot 5^{\log_3 x}=400.$

348. $\log_2\left(25^{x+3}-1\right)=2+\log_2\left(5^{x+3}+1\right).$

349. $\log_3 x\cdot\log_9 x\cdot\log_{27} x\cdot\log_{81} x=\frac{2}{3}$.

350. $\log_{10} x+\log_{\sqrt{10}} x+\log_{\sqrt[3]{10}} x...+\log_{\sqrt[10]{10}} x=5{,}5$.

351. $\frac{10x^{2\lg^2 x}}{x^3}=\frac{x^{3\lg x}}{10}$.

352. $\frac{\log_2\left(x^3+3x^2+2x-1\right)}{\log_2\left(x^3+2x^2-3x-5\right)}=\log_{2x} x+\log_{2x} 2$.

353. $\log_4 x+\log_x 2-\log_4\sqrt{x}=5{,}5$.

354. $\log_{\sqrt{x}}(x+12)=8\log_{x+12} x$.

355. $2\lg x^2-\lg^2(-x)=4$.

356. $\log_4\log_2\log_3(2x-1)=\frac{1}{2}$.

357. $\left|\log_{\sqrt{3}} x-2\right|-\left|\log_3 x-2\right|=2$.

358. $\log_{x+\frac{1}{8}} 2=\log_x 4$.

359. $(0{,}4)^{\lg^2 x+1}=(6{,}25)^{2-\lg x^3}$.

360. $\left(\sqrt{x}\right)^{\log_{x^2}\left(x^2-1\right)}=5$.

361. $x^{(\log_3 x)^3-3\log_3 x}=3^{-3\log_{2\sqrt{2}}4+8}$.

362. $x^2\log_2\frac{3+x}{10}-x^2\log_{\frac{1}{2}}(2+3x)=x^2-4+2\log_{\sqrt{2}}\frac{3x^2+11x+6}{10}$.

10-§. Ko'rsatkichli va logarifmik tenglamalar sistemalari

Ko'rsatkichli va logarifmik tenglamalar sistemalarini yechish uchun ularni xossalariga tayanamiz. Biz bular bilan oldingi bandda tanishganimiz uchun misollar keltiramiz.

1-misol. Tenglamalar sistemasini yeching: $\begin{cases} 25^{2x}+25^{2y}=30 \\ 25^{x+y}=5\sqrt{5} \end{cases}$

Yechilishi. $25^x=t,\ 25^y=z$ ya'ni o'zgaruvchilar kiritamiz. Bundan $\begin{cases} t^2+z^2=30 \\ t\cdot z=5\sqrt{5} \end{cases}$ tenglamalar sistemasiga kelamiz. Sistema to'rtta ildizga ega, ya'ni

$t_{1,2}=\pm 5 \quad t_{3,4}=\pm\sqrt{5}$

$z_{1,2}=\pm\sqrt{5} \quad z_{3,4}=\pm 5$

Lekin, shunga e'tibor berish kerakki, $25^x>0,\ 25^y>0$. Shuning uchun ham, faqat $\begin{cases} 25^x=5 \\ 25^y=\sqrt{5} \end{cases} \vee \begin{cases} 25^y=\sqrt{5} \\ 25^y=5 \end{cases}$ sistemalarni qaraymiz.

Bundan, $x_1=\frac{1}{2}\quad y_1=\frac{1}{4},\quad x_2=\frac{1}{4}\quad y_2=\frac{1}{2}$

Javob: $\left\{\left(\frac{1}{2};\frac{1}{4}\right);\left(\frac{1}{4};\frac{1}{2}\right)\right\}$.

2-misol. Tenglamalar sistemasini yeching: $\begin{cases} x^{\sqrt{y}}=y \\ y^{\sqrt{y}}=x^4 \end{cases}$

Yechilishi. $\begin{cases} x^{\sqrt{y}}=y \\ y^{\sqrt{y}}=x^4 \end{cases}$ bu turdagi tenglamalar sistemasini yechish uchun sistemadagi har ikki tenglamani o'nli asosga ko'ra logarifmlaymiz.

$\begin{cases} \lg x^{\sqrt{y}} = y \\ \lg y^{\sqrt{y}} = x^4 \end{cases} \Rightarrow \begin{cases} \sqrt{y}\lg x = \lg y \\ \sqrt{y}\lg y = 4\lg x \end{cases} \Rightarrow$ endi o'rniga qo'yish usuliga ko'ra

$\sqrt{y} \cdot \sqrt{y \cdot \lg x} = 4\lg x$.

Natijada $x=1 \vee y=4$ ga ega bo'lamiz. Bundan $x_1=1, y_1=1, x_2=-2, y_2=4, x_3=2, y_3=4$ javobga kelamiz.

3-misol. Tenglamalar sistemasini yeching.

$$\begin{cases} x^{y^2-5y+6} = 4 \\ x^{2y^2-9y+6} = 64 \end{cases}$$

Yechilishi. Sistemadagi har ikkala tenglamani ikki asosga ko'ra logarifmlaymiz va quyidagi sistemaga keltiramiz.

$$\begin{cases} \log_2 x^{y^2-5y+6} = \log_2 4 \\ \log_2 x^{2y^2-9y+6} = \log_2 64 \end{cases} \Rightarrow \begin{cases} (y^2-5y+6)\log_2 x = 2 \\ (2y^2-9y+6)\log_2 x = 6 \end{cases}$$

$\log_2 x = \dfrac{2}{y^2-5y+6}$ bundan, $\dfrac{2y^2-9y+6}{y^2-5y+6} = 3$. Bu tenglamani yechib, $y=3$ javobga kelamiz. Lekin, $2y^2-9y+6 \neq 0$ bo'lishi kerak edi. $(y-3)(y-2) \neq 0$.

Javob $\emptyset$ (bo'sh to'plam). Sistema yechimga ega emas.

Mustaqil yechish uchun misollar

Tenglamalar sistemasini yeching

363. $\begin{cases} 2^x \cdot 3^y = 6 \\ 3^x \cdot 4^y = 12 \end{cases}$.

364. $\begin{cases} 3^{2x} - 2^y = 725 \\ 4^x - 2^{\frac{1}{2}y} = 25 \end{cases}$.

365. $\begin{cases} 8^x = 10y \\ 2^x = 5y \end{cases}$.

366. $\begin{cases} 3^x - 2^{2y} = 77 \\ 3^{\frac{x}{2}} - 2^y = 7 \end{cases}$.

367. $\begin{cases} x^{y+1} = 27 \\ x^{2y-5} = \frac{1}{3} \end{cases}$.

368. $\begin{cases} x^{y-2} = 4 \\ x^{2y-3} = 64 \end{cases}$.

369. $\begin{cases} x^{x+y} = y^{12} \\ y^{x+y} = x^3 \end{cases}$.

370. $\begin{cases} x^{\sqrt{y}} = y \\ y^{\sqrt{y}} = x^4 \end{cases}$.

371. $\begin{cases} 3^y \cdot 9^x = 81 \\ \lg(x+y)^2 - \lg x = 2\lg 3 \end{cases}$.

372. $\begin{cases} \log_{xy}(x-y) = 1 \\ \log_{xy}(x+y) = 0 \end{cases}$.

373. $\begin{cases} \lg x + \lg y = \lg 2 \\ x^2 + y^2 = 5 \end{cases}$.

374. $\begin{cases} \log_y x - \log_x y = \frac{8}{3} \\ xy = 16 \end{cases}$.

375. $\begin{cases} \lg(x^2 + y^2) - 1 = \lg 13 \\ \lg(x+y) - \lg(x-y) = 3\lg 2 \end{cases}$.

376. $\begin{cases} 5(\log_y x + \log_x y) = 26 \\ xy = 64 \end{cases}$.

377. $\begin{cases} 2^x \cdot 4^y = 32 \\ \lg(x-y)^2 = \lg 4 \end{cases}$.

378. $\begin{cases} 10^{2-\lg(x-y)} = 25 \\ \lg(x-y) + \lg(x+y) = 1 + \lg 4 \end{cases}$

379. $\begin{cases} 2^{\frac{x-y}{2}} - (\sqrt[4]{2})^{x-y} = 12 \\ 3^{\lg(2y-x)} = 1 \end{cases}$.

380. $\begin{cases} 3^x \cdot 2^y = 576 \\ \log_{\sqrt{2}}(x-y) = 4 \end{cases}$.

381. $\begin{cases} \log_5 x + 3^{\log_3 x} = 7 \\ x^y = 5^{12} \end{cases}$.

382. $\begin{cases} 3\left(2\log_{y^2} x - \log_{\frac{1}{x}} y\right) = 10 \\ xy = 81 \end{cases}$.

383. $\begin{cases} \log_2(x+y) + 2\log_3(x-y) = 5 \\ 2^x - 5 \cdot 2^{0.5(x+y-1)} + 2^{y+1} = 0 \end{cases}$.

384. $\begin{cases} \log_2(10 - 2^y) = 4 - y \\ \log_2 \frac{x+3y-1}{3y-x} = \log_2(x-1) - \log_2(\end{cases}$

385. $\begin{cases} \lg x\cdot \lg(x+y)=\lg y\cdot \lg(x-y) \\ \lg y\cdot \lg(x+y)=\lg x\cdot \lg(x-y) \end{cases}$ 386. $\begin{cases} 4^{x+y}=27+9^{x-y} \\ 8^{x+y}-21\cdot 2^{x+y}=27^{x-y}+7\cdot 3^{x-y+1} \end{cases}$.

387. $\begin{cases} x\cdot 2^{x+1}-2\cdot 2^{y}=-3y\cdot 4^{x+y} \\ 2x\cdot 2^{2x+y}+3\cdot 8^{x+y}=1 \end{cases}$. 388. $\begin{cases} a^{x}b^{y}=ab \\ xy=1 \end{cases}$.

389. $\begin{cases} 3^{x}\cdot 2^{y}=576 \\ \log_{\sqrt{2}}(y-x)=4 \end{cases}$. 390. $\begin{cases} \left(\frac{3}{2}\right)^{x-y}-\left(\frac{2}{3}\right)^{x-y}=\frac{65}{36} \\ xy-x+y=118 \end{cases}$.

391. $\begin{cases} x^{\lg y} \\ xy=40 \end{cases}$. 392. $\begin{cases} x^{y}=\frac{1}{10\sqrt{10}} \\ \lg x : y=-6 \end{cases}$.

393. $\begin{cases} y^{\frac{x}{y}}=x \\ y^{3}=x^{2} \end{cases}$.

11-§. Ko'rsatkichli va logarifmik tengsizliklar

1. Ko'rsatkichli tengsizliklar

$a^{f(x)}>a^{g(x)}\ (a>0, a\neq 1)$ ko'rinishdagi ko'rsatkichli tengsizlik quyidagi ikkita teoremaga asoslangan.

1-teorema. Agar $a>1$ bo'lsa, $a^{f(x)}>a^{g(x)}$ tengsizlik $f(x)>g(x)$ ga teng kuchli.

2-teorema. Agar $0<a<1$ bo'lsa, $a^{f(x)}>a^{g(x)}$ tengsizlik $f(x)<g(x)$ ga teng kuchli.

1-misol. Tengsizlikni yeching.

$3\cdot 7^{x}+5-2\cdot 7^{-x}<0$

Yechilishi. $7^x = t$ deb belgilash kiritamiz. $t > 0$ ekani ravshan.

$3t+5-2\frac{1}{t}<0$ tengsizlikni ikkala tarafini t ga ko‘paytiramiz. ($t>0$ bo‘lgani uchun tengsizlik ishorasi o‘zgarmaydi).

$3t^2+5t-2<0$

$3\left(t-\frac{1}{3}\right)(t+2)<0$

$$\begin{cases} -2<t<\frac{1}{3} \\ t>0 \end{cases} \Rightarrow 0<t<\frac{1}{3} \Rightarrow \begin{cases} t>0 \\ t<\frac{1}{3} \end{cases}$$

$$\begin{cases} 7^x>0 \\ 7^x<\frac{1}{3} \end{cases} \Rightarrow \begin{cases} 7^x>0 \\ x<\log_7\frac{1}{3} \end{cases} \Rightarrow x\in\left(-\infty;\log_7\frac{1}{3}\right)$$

Javob: $(-\infty;-\log_7 3)$

2-misol. Tengsizlikni yeching.

$4\left(3^{2x}-3^x\right)>\frac{5}{3^{x-2}}+3^{x+1}$

Yechilishi. Tengsizlikni almashtirishlar yordamida quyidagicha yozamiz.

$4\cdot 3^{2x}-4\cdot 3^x>\frac{45}{3^x}+3\cdot 3^x$

$t=3^x$ o‘zgartirish kiritganimizdan so‘ng tengsizlik quyidagi ko‘rinishni oladi:

$4t^2-4t>\frac{45}{t}+3t$

Tengsizlikni ikkala qismini ($t>0$) ga ko‘paytiramiz.

$4t^3-4t^2>45+3t^2 \Rightarrow 4t^3-7t^2-45>0$ ko‘phadni ko‘paytuvchilarga ajratamiz.

$(t-3)\left(4t^2+5t+15\right)>0$ bundan $t>3$ ekanligi kelib chiqadi. Shunday qilib,

$3^x>3 \Rightarrow x>1$

Javob: $x\in(1;+\infty)$.

2. Logarifmik tengsizliklar

$\log_a f(x) > \log_a g(x)\ (a>0, a\neq 1)$ tengsizlikni yechish uchun albatta logarifmning asosi va aniqlanish sohasiga e'tibor berish kerak.

agar $a>0$ bo'lsa,

$$\begin{cases} f(x)>0 \\ g(x)>0 \\ f(x)<g(x) \end{cases}$$ bo'ladi.

3-misol. Tengsizlikni yeching.

$$\log_2((x+2)(x-3))+\log_{\frac{1}{2}}(x-3)<-\log_{\frac{1}{\sqrt{2}}}3$$

Yechilishi. Logarifm xossalaridan foydalansak tengsizlik, $\log_2(x+2)+\log_2(x-3)-\log_2(x-3)<2\log_2 3$ ko'rinishni oladi. Bundan, $x<7$ ekanligi kelib chiqadi. Aniqlanish sohasiga e'tibor bersak, $3<x<7$

Javob: (3;7).

4-misol. Tengsizlikni yeching.

$$\log_{x-2}(2x-3)>\log_{x-2}(24-6x)$$

Yechilishi. Logarifm asosida o'zgaruvchi qatnashganligi uchun 2 holni qarab o'tamiz.

1-hol. $$\begin{cases} x-2>1 \\ 2x-3>0 \\ 24-6x>0 \\ 2x-3>24-6x \end{cases} \Rightarrow \begin{cases} x>3 \\ x>\frac{3}{2} \\ x<4 \\ x>\frac{27}{8} \end{cases} \Rightarrow \frac{27}{8}<x<4$$

2-hol.

$$\begin{cases} 0<x-2<1 \\ 2x-3>0 \\ 24-6x>0 \\ 2x-3<24-6x \end{cases} \Rightarrow \begin{cases} 2<x<3 \\ x>\dfrac{3}{2} \\ x<4 \\ x>\dfrac{27}{8} \end{cases} \Rightarrow 2<x<3$$

Javob: ikkita holni birlashmasi olinadi. $x\in(2;3)\cup\left(\frac{27}{8};4\right)$

Mustaqil yechish uchun misollar

Tengsizliklarni yeching

394. $\left(\frac{1}{3}\right)^{2-\frac{x-3}{x+2}}<\left(\frac{1}{3}\right)^{\frac{x-3}{x+2}}.$

395. $\left(\frac{1}{2}\right)^{2\sqrt{x}}+2>3\cdot\left(\frac{1}{2}\right)^{\sqrt{x}}.$

396. $x^2\cdot 3^x+9>9x^2+3^x.$

397. $2^{x+2}-2^{x+1}+2^{x-1}-2^{x-2}\le 9.$

398. $2^{x^2}<-x^2+20.$

399. $2^{-x^2}<2^{-x}.$

400. $3^{\sqrt{1-x}}+3^{\sqrt{2-x}}+3^{\sqrt{6-2x}}>13.$

401. $\left(4x^2+2x+1\right)^{x^2-x}>1.$

402. $4^x>3\cdot 2^{\sqrt{x}+x}+4^{\sqrt{x}+1}.$

403. $9\cdot 4^{-\frac{1}{x}}+5\cdot 6^{\frac{1}{x}}<4\cdot 9^{-\frac{1}{x}}.$

404. $\left(\frac{1}{2}\right)^{2x+3} - \left(\frac{1}{3}\right)^{x+\frac{1}{2}} > \left(\frac{1}{3}\right)^{x+\frac{3}{2}} - \left(\frac{1}{4}\right)^{x+1}$.

405. $\frac{1}{5-\left(\frac{1}{3}\right)^{x}} + \frac{2}{1+\left(\frac{1}{3}\right)^{x}} < 1$.

406. $4^{x^2} + 3^{\sqrt{x}+1} + x \cdot 3^{\sqrt{x}} < 2x^2 \cdot 3^{\sqrt{x}} + 2x + 6$.

407. $5^{2x+1} + 6^{x+1} > 30 + 5^{x} \cdot 30^{x}$.

408. $2x^{\log_{\frac{1}{2}} x} - x^{-\log_{\frac{1}{2}} x} < -1$.

409. $\left(\frac{2}{7}\right)^{2x} \cdot \left(\frac{147}{20}\right)^{x} < \left(\frac{81}{625}\right)^{x}$.

410. $3^{\frac{1}{x}} + 3^{\frac{1}{x}+3} > 84$.

411. $\left(\frac{3}{7}\right)^{\frac{x^2-2x}{x^2}} \geq 1$.

412. $x^{2} \cdot 3^{x} - 3^{x+1} \leq 0$.

413. $3^{\sqrt{x}} + 3^{\sqrt{x}-1} - 3^{\sqrt{x}-2} < 11$.

414. $0{,}5^{x} \leq 0{,}25^{x^2}$.

415. $\sqrt{9^{x} - 3^{x+2}} > 3^{x} - 9$.

416. $\left(\frac{3}{5}\right)^{13x^2} \leq \left(\frac{3}{5}\right)^{x^4+36} < \left(\frac{3}{5}\right)^{12x^2}$.

417. $25 \cdot 2^{x} - 10^{x} + 5^{x} > 25$.

418. $\left|2^{4x^2-1}\right| \leq 3$.

419. $\left(x^2 + x + 1\right)^{\frac{x+5}{x+2}} \geq \left(x^2 + x + 1\right)^{3}$.

420. $\sqrt{2}^{\,x-3+1} < 64$.

421. $4^x - 2^{2(x-1)} + 8^{\frac{2}{3}(x-2)} > 52.$

422. $0{,}02^{1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots+(-1)^n\frac{1}{2^n}\ldots} < \sqrt[3]{0{,}02^{3x^2+5x}} < 1.$

423. $2^{2+x} - \dfrac{2^x}{2} > 15.$

424. $36^x - 2\cdot 18^x - 8\cdot 9^x > 0.$

425. $|x-3|^{2x^2-7x} > 1.$

426. $0{,}3^{2+4+6+\ldots+2x} > 0{,}3^{72}.$

427. $\sqrt{2(5^x+24)} - \sqrt{5^x-7} \geq \sqrt{5^x+7}.$

Logarifmik tengsizliklar

428. $\log_{\frac{1}{4}}(2-x) > \log_{\frac{1}{4}}\dfrac{2}{x+1}.$

429. $\dfrac{\log_{0,3}(x+1)}{\log_{0,3}100 - \log_{0,3}9} < 1.$

430. $\log_{0,2}^2(x-1) > 4.$

431. $2{,}25^{\log_2(x^2-3x-10)} > \left(\dfrac{2}{3}\right)^{\log_{\frac{1}{2}}(x^2+4x+4)}.$

432. $\log_x \dfrac{x+1}{x-1} > 1.$

433. $\log_x(x^3+1)\cdot\log_{x+1}x > 2.$

434. $\dfrac{1-\log_4 x}{1+\log_4 x} > 1.$

435. $\log_{\sqrt{2}}(5^x-1)\cdot\log_{\sqrt{2}}\dfrac{2\sqrt{2}}{5^x-1} > 2.$

436. $\log_{x^2-3}729 > 3.$

437. $\dfrac{3x^2-16x+21}{\log_{0,3}\left(x^2+4\right)}<0$.

438. $\log_{1,5}\dfrac{2x-8}{x-2}<0$.

439. $\log_x\left(\log_9\left(3^x-9\right)\right)<1$.

440. $\log_{|x-1|}0,5>0,5$.

441. $\dfrac{\log_{0,3}|x-2|}{x^2-4}<0$.

442. $\log_3\log_4\dfrac{4x-1}{x+1}-\log_{\frac{1}{3}}\log_{\frac{1}{4}}\dfrac{x+1}{4x-1}<0$.

443. $\log_{\frac{1}{5}}x+\log_4 x>1$.

444. $\log_{2x}\left(x^2-5x+6\right)<1$.

445. $\log_3\log_{x^2}\log_{x^2}x^4<0$.

446. $\log_{\frac{1}{2}}\dfrac{x+7}{2x+3}<\log_{\frac{1}{2}}(5-x)$.

447. $\log_{x-1}(x-1)>2$.

448. $x^{2-2\log_2 x\cdot\log_2^2 x}>\dfrac{1}{x}$.

449. $\log_x\dfrac{4x-2}{3}>1$.

450. $\dfrac{\log_{\frac{1}{2}}x-1}{\log_{\frac{1}{3}}x+2}<\dfrac{\log_{\frac{1}{3}}x-3}{\log_{\frac{1}{2}}x+4}$.

451. $\log_2\dfrac{1}{x^2}\leq 2$.

452. $\log_{x^2}(x+2)<1$.

453. $\log_5\sqrt{3x+4}\cdot\log_x 5\geq 1$.

454. $\lg\left(5^x+x-20\right)>x-x\lg2.$

455. $\left(\frac{1}{3}\right)^{\log_{\frac{1}{9}}\left(x^2-\frac{10}{3}x+1\right)}\le1.$

12-§. Parametr qatnashgan tenglama va tengsizliklar

Parametr qatnashgan masalalarning o'ziga xosligi shundaki, bunday masalalarda berilgan noma'lumlar bilan birga son qiymati aniq ko'rsatilmagan parametrlar qatnashib, ularni biror to'plamda berilgan ma'lum miqdorlar deb qarashga to'g'ri keladi. Bunda parametrning qiymati masalani yechish jarayoniga va yechimning ko'rinishiga mantiqiy va texnik jihatdan katta ta'sir ko'rsatadi. Parametrning aniq qiymatlarida masalaning javoblari bir-biridan keskin farq qilishi mumkin.

1-misol. a parametrning qanday qiymatlarida $ax^2-2(a+1)x+a-3=0$ tenglamaning barcha ildizlari manfiy bo'ladi?

Yechilishi. $a=0$ da tenglama bitta $x=-\frac{3}{2}$ ildizga ega va u masala shartini qanoatlantiradi.

$a\neq0$ bo'lgan holni ko'rib chiqamiz. Berilgan tenglamaning ikkala ildizi ham manfiy bo'lishi uchun quyidagi

$$\begin{cases} D\ge0 \\ x_1\cdot x_2>0 \\ x_1+x_2<0 \end{cases}$$

shartlarning bajarilishi zarur va yetarli. Viyet teoremasini tatbiq etib, bu shartlarni quyidagi ko'rinishda yozish mumkin:

$$\begin{cases} \frac{D}{4}=(a+1)^2-a(a-3)\geq 0 \\ x_1\cdot x_2=\frac{a-3}{a}>0 \\ x_1+x_2=\frac{2(a+1)}{a}<0 \end{cases}$$

Bu sistemani yechib, $a\in\left[-\frac{1}{5};0\right)$ ga ega bo'lamiz. Ikkala holni birlashtirib masalaning javobini $a\in\left[-\frac{1}{5};0\right]$ deb olamiz.

Javob: $a\in\left[-\frac{1}{5};0\right]$.

2-misol. p parametrning qanday qiymatlarida $2x^2+(p-10)x+6=0$ tenglamaning ildizlari nisbati 12 ga teng bo'ladi?

Yechilishi. Berilgan tenglama $D\geq 0$ bo'lganda haqiqiy ildizlarga ega bo'ladi. Masala shartiga ko'ra, $x_2=12x_1$. Viyet teoremasiga ko'ra, quyidagi sistemani tuzamiz:

$$\begin{cases} D=(p-10)^2-48\geq 0 \\ x_2=12x_1 \\ x_1+x_2=-\frac{p-10}{2} \\ x_1\cdot x_2=\frac{6}{2}=3 \end{cases} \Rightarrow \begin{cases} p^2-20p+52\geq 0 \\ x_2=12x_1 \\ 13x_1=\frac{10-p}{2} \\ 12x_1^2=3 \end{cases}$$

Bu sistemaning yechimlari $p=-3$, $p=23$ bo'ladi.

Javob: $p=-3$, $p=23$.

3-misol. a parametrning qiymatlariga nisbatan $x^3+2ax^2-(a+1)^2x-2a(a+1)^2=0$ tenglamaning eng kichik ildizini toping.

Yechilishi. Berilgan tenglamani $(x+2a)(x-a-1)(x+a+1)=0$ ko'rinishda yozib olamiz. Bu tenglamaning ildizlari:

$$x_1=-a-1,\ x_2=a+1,\ x_3=-2a.$$

a parametrning x_1 eng kichik ildiz bo‘ladigan qiymatlarini topamiz. Quyidagi sistemani yozamiz

$$\begin{cases}-a-1<a+1\\ -a-1<-2a\end{cases}\Rightarrow -1<a<1.$$

Agar $x_2$eng kichik ildiz bo‘ladi deb faraz qilsak, u holda

$$\begin{cases}a+1<-a-1\\ a+1<-2a\end{cases}\Rightarrow a>1,$$

va nihoyat, x_3 eng kichik ildiz bo‘ladigan bo‘lsa,

$$\begin{cases}-2a<a+1\\ -2a<-a-1\end{cases}\Rightarrow a>1$$

larga ega bo‘lamiz. Agar $a=-1$ bo‘lsa, $x_1=x_2$ ildizlar teng bo‘ladi, agar $a=1$ bo‘lsa, $x_1=x_3$ ildizlar teng bo‘ladi.

Javob: Agar $a\le -1$ bo‘lsa, $x=a+1$; agar $-1<a<1$ bo‘lsa, $x=-a-1$; agar $a\ge 1$ bo‘lsa, $x_1=-2a$.

4-misol. $\frac{x^2-2x+3^a}{x^2-4}<0$ tengsizlikni yeching.

Yechilishi. a parametrning ixtiyoriy qiymatida berilgan tengsizlik suratining diskriminanti musbat emas: $D=4(1-3^a)\le 0$.

Agar $a=0$ bo‘lsa, u holda $D=0$ va tengsizlik

$$\frac{(x-1)^2}{(x-2)(x+2)}<0$$

ko‘rinishga keladi. Bu tengsizlikni intervallar metodi bilan yechib, $x\in(-2;1)\cup(1;2)$ ga ega bo‘lamiz. Agar $a\ne 0$ bo‘lsa, u holda $|a|>0$ bo‘lib, $3^a>1$ va diskriminant $D<0$. Bundan, ixtiyoriy x uchun $x^2-2x+3^a>0$

tengsizlikning o‘rinli ekani kelib chiqadi. Shuning uchun berilgan tengsizlikning yechimi $x\in(-2;2)$ bo‘ladi.

Javob. Agar $a\neq 0$ bo‘lsa $x\in(-2;2)$, agar $a=0$ bo‘lsa, $x\in(-2;1)\cup(1;2)$.

Mustaqil yechish uchun misollar

456. a parametrning qanday qiymatlarida $(a^2-3a+2)x^2-(a^2-5a+4)x+a^2-a=0$ tenglama ikkitadan ortiq ildizga ega bo‘ladi?

457. a parametrning qanday qiymatlarida $ax^2-3x+5-a=0$ tenglamaning ildizlari musbat bo‘ladi?

458. $x^2+(a-9)x+a-1=0$ tenglamaning ildizlari turlicha va 1 dan katta bo‘ladigan a parametrning barcha qiymatlarini toping.

459. a parametrning $x^2+(a-7)x+a+8=0$ tenglamaning ildizlari turlicha va 2 dan katta bo‘ladigan barcha qiymatlarini toping.

460. b parametrning qanday qiymatlarida $x^2-3x+2b+3=0$ tenglama $5x_1+3x_2=23$ shartni qanoatlantiradi?

461. Agar $x^2-4x+p=0$ tenglamaning ildizlari kvadratlarining yig‘indisi 14 ga teng bo‘lsa, p parametrning qiymatini toping.

462. a parametrning qanday qiymatlarida $4x^2-16x+a^2-2=0$ tenglamaning ildizlari orasidagi farq 3 ga teng bo‘ladi?

463. p parametrning qanday qiymatlarida $4x^2-(3+2p)x+2=0$ tenglamaning ildizlari bir - biridan 8 marta katta bo‘ladi?

464. c parametrning qanday musbat qiymatida $8x^2-6x+9c^2=0$ tenglamaning ildizlaridan biri ikkinchisining kvadratiga teng bo‘ladi?

465. a parametrning qiymatiga bog‘liq ravishda $|x^2+ax|=2a$ tenglamaning ildizlarini toping.

466. a parametrning qanday qiymatlarida $x^2+ax+8=0$ va $x^2+x+a=0$ tenglamalar umumiy ildizga ega bo‘ladi?

467. a parametrning qanday qiymatlarida $ax^2-(a^3+2a^2+1)x+a(a+2)=0$ tenglamaning ildizlari $[0;1]$ kesmaga tegishli bo‘ladi?

468. a parametrning qanday qiymatlarida $x^4+(a-6)x^2+(a+1)^2=0$ tenglama 4 ta haqiqiy ildizlarga ega bo‘ladi. Bu ildizlar arifmetik progressiya tashkil etadigan a ning barcha qiymatlarini toping. Bu progressiyani a ning butun qiymati uchun yozing.

469. a parametrning qanday qiymatlarida $x^5+(a-4)x^3+(a+3)^2x=0$ tenglama 5 ta haqiqiy ildizlarga ega bo‘ladi. Bu ildizlar arifmetik progressiya tashkil etadigan a ning barcha qiymatlarini toping. Bu progressiyani a ning butun qiymati uchun yozing.

470. a parametrning qanday qiymatlarida $x^3+3x^2-6x+a=0$ tenglama geometrik progressiya tashkil etuvchi uchta turli ildizlarga ega bo‘ladi? Bu ildizlarni toping.

471. a parametrning qanday qiymatlarida berilgan tenglamalar umumiy ildizga ega bo‘ladi? Bu ildizlarni toping.

$x^3+ax+1=0$ va $x^4+ax^2+1=0$.

472. $(1-2a)x^2-6ax-1=0$ va $ax^2-x+1=0$ $ax^2-x+1=0$

473. a parametrning qanday qiymatlarida $\sqrt{x+a}=x+1$ tenglama yagona ildizga ega bo‘ladi? Bu ildizlarni toping.

Tenglamalarni yeching

474. $\sqrt{4x+a}=2x-1.$

475. $x+\sqrt{a+\sqrt{x}}=a.$

476. $\sqrt{x+a}+\sqrt{x}=a.$

477. $\sqrt{x}-\sqrt{a-x}=2.$

478. $\sqrt{x+1}-\sqrt{a-x}=1.$

479. $\sqrt[3]{x+a+63}-\sqrt[3]{x+a-1}=4.$

480. $\sqrt{2x+a}+\sqrt{x-a}=2.$

481. $\frac{a+5}{\sqrt{x+9}}=1.$

482. $a-x=\sqrt{x^2-1}.$

483. $x=a-\sqrt{a^2-x\sqrt{x^2+a^2}}.$

484. $\sqrt[4]{\frac{a-x}{b+x}}+\sqrt[4]{\frac{b+x}{a-x}}=2.$

485. $4^x-(2a+1)2^x-a^2+a=0.$

486. $9^{\lg(x-a)-\lg 2}=3^{\lg(x-1)}.$

487. $\log_a x^2+2\log_a(x+2)=1.$

488. $\sqrt{1+\log_x\sqrt{a^3}}\log_a x+1=0.$

489. a parametrning $x^2-2(a+4)x+18a\leq 0$ tengsizlik yechimga ega bo‘ladigan va bu yechimlar $x^2-7|x|-8\leq 0$ tengsizlikni qanoatlantiradigan barcha qiymatlarini toping.

490. a parametrning $x^2-2(a-2)x+4a-11\leq 0$ tengsizlik yechimga ega bo‘ladigan va bu yechimlar $x^2-|x|-6\leq 0$ tengsizlikni qanoatlantiradigan barcha qiymatlarini toping.

491. a parametrning qanday qiymatlarida $\frac{a-1}{x+4}=\frac{2x+3}{x^2-x-20}$ tenglama $x\le 2$ tengsizlikni qanoatlantiruvchi ildizga ega bo'ladi?

Tengsizlikni yeching

492. $\frac{x^2+4x}{3^a x^2-2x+1}>0.$

493. $\frac{2ax+3}{5x-4a}<4.$

494. $\frac{3ax+4}{3a+9}<\frac{x}{a+3}+\frac{3a-5}{3a-9}.$

495. $\sqrt{5x^2+a^2}\le -3x.$

496. $2x+\sqrt{a^2-x^2}>0.$

497. $2\sqrt{x+a}>x+1.$

498. $\sqrt{1-x^2}\ge 2x+a.$

499. $x+\sqrt{10x}\ge a-4+\sqrt{a+9x-4}.$

500. $x+\sqrt{8x}\ge 5-a+\sqrt{5+7x-a}.$

501. $a^{x+2}+6a^{x+1}+12a^x+8a^{x-1}-\frac{4}{a}>a+4.$

502. $a^2 4^{2x+1}-5a4^x+1>0.$

503. $a^2-24^{x+1}-a2^{x+1}>0.$

504. $25^{x+1}-45^{x+1}<a^2+4a.$

505. $9^{x+1}-3^{x+1}\ge a^2+a.$

506. $x^{\log_a x}>a.$

507. $\log_a(x-2)+\log_a x<1.$

508. $\log_{x+2}(x^2-2x+a)\ge 2.$

13-§. Parametr qatnashgan tenglama va tengsizliklar sistemasi

1-misol. $\begin{cases} x-y=a(1+xy) \\ 2+x+y+xy=0 \end{cases}$ sistema bitta yechimga ega bo'ladigan a parametrning qiymatlarini va parametrning shu qiymatlarida sistemaning yechimlarini toping.

Yechilishi. Bevosita tekshirib ko'rish yo'li bilan $x=-1$ sistemaning yechimi emasligiga ishonch hosil qilamiz. U holda sistemaning ikkinchi tenglamasidan $y=\frac{-x-2}{x+1}$ tenglikni hosil qilamiz. Birinchi tenglamadagi y o'rniga uning x ga nisbatan ifodasini qo'yib, quyidagi tenglamani hosil qilamiz:

$$x^2(a+1)+x(a+2)+2-a=0 \qquad (1)$$

Bu tenglama yagona yechimga ega bo'lishi uchun uning chiziqli bo'lishi, ya'ni $a+1=0$ bo'lishi kerak. $a=-1$ bo'lganda sistemaning yechimi $x=-3;\ y=-\frac{1}{2}$ bo'ladi. Bundan tashqari kvadrat tenglama diskriminanti nolga teng bo'lganda ham yagona yechimga ega bo'ladi, ya'ni $D=(a+2)^2-4(a+1)(2-a)=0.$ Bundan, $a=\pm\frac{2}{\sqrt{5}}$ parametrning bu qiymatlarida sistemaning yechimlarini topamiz:

$x=-\frac{1\pm\sqrt{5}}{2\pm\sqrt{5}};\ y=-3\pm\sqrt{5}.$ Va, nihoyat, x ga nisbatan (1) kvadrat tenglama ikkita yechimga ega bo'lib, lekin yechimlaridan bittasi sistema uchun yaroqli bo'lmasa (ya'ni $x=-1$) ham berilgan sistema yagona yechimga ega bo'ladi, (1) ga $x=-1$ ni qo'yib, $a=1$ ga ega bo'lamiz. $a=1$ dagi ikkinchi ildiz $x=-\frac{1}{2}$ va unga mos $y=-3$ ni topamiz.

Javob. Agar $a=-1$ bo‘lsa, $x=-3; y=-\frac{1}{2}$; agar $a=1$ bo‘lsa, $x=-\frac{1}{2}$ $y=-3$ agar $a=\frac{2}{\sqrt{5}}$ bo‘lsa, $x=-\frac{1+\sqrt{5}}{2+\sqrt{5}}; y=-3-\sqrt{5}$; agar $a=-\frac{2}{\sqrt{5}}$ bo‘lsa, $x=-\frac{1-\sqrt{5}}{2-\sqrt{5}}; y=-3+\sqrt{5}$;

2-misol. $\begin{cases} y=x^2-2x \\ x^2+y^2+a^2=2x+2ay \end{cases}$ sistema a parametrning qanday qiymatlarida yechimga ega bo‘ladi?

Yechilishi. Berilgan sistemani quyidagi ko‘rinishda yozamiz:

$$\begin{cases} (x-1)^2=y+1 \\ (x-1)^2+(y-a)^2=1 \end{cases}$$

Bundan,

$$\begin{cases} (y-a)^2+y+1=1 \\ y+1\geq 0 \end{cases} \Rightarrow \begin{cases} y^2+(1-2a)y+a^{10}=1 \\ y\geq -1 \end{cases}$$

Kvadrat uchhadning geometrik ma’nosiga ko‘ra, berilgan sistema quyidagi tengsizliklar sistemalari bir vaqtda hamjoyli bo‘lganda yechimga ega bo‘ladi:

$$\left[\begin{array}{l} \begin{cases} D=1-4a\geq 0 \\ y=a-\frac{1}{2}>-1 \end{cases} \\ \begin{cases} D=1-4a\geq 0 \\ y=a-\frac{1}{2}\leq -1 \\ f(-1)=a^2+2a\leq 0 \end{cases} \end{array}\right.$$

Bu tengsizliklar sistemasini yechib, $\left[\begin{array}{l} -\frac{1}{2}<a\leq\frac{1}{4} \\ -2\leq a\leq -\frac{1}{2} \end{array}\right.$ ni hosil qilamiz.

Javob. $-2 \le a \le \frac{1}{4}$.

3-misol. $\begin{cases} \sqrt{a+x}-\sqrt{y+b}=1 \\ \sqrt{y+a}-\sqrt{x+b}=1 \end{cases}$ tenglamalar sistemasini yeching.

Yechilishi. Sistemadagi ikkala tenglamani kvadratga ko‘tarsak, quyidagi sistemani hosil qilamiz:

$$\begin{cases} x+a+y+b-1=2\sqrt{(a+x)(y+b)} \\ y+a+x+b-1=2\sqrt{(a+y)(x+b)} \end{cases}$$

Bundan,

$(a+x)(y+b)=(a+y)(x+b) \Rightarrow (a-b)(x-y)=0$ $a=b$ bo‘lgan holni qaraymiz. Bu holda dastlabki sistema

$$\begin{cases} \sqrt{a+x}-\sqrt{y+a}=1 \\ \sqrt{y+a}-\sqrt{x+a}=1 \end{cases}$$

ko‘rinishda bo‘lib, bu sistemaning yechimga ega emasligi ravshan.

Endi $x=y$ bo‘lgan holni qaraymiz. Bunday holda sistema o‘rniga quyidagi tenglamaga ega bo‘lamiz:

$$\sqrt{a+x}=1+\sqrt{x+b}$$

Bu tenglamaning chap va o‘ng qismlari manfiy emas, shuning uchun uning ikkala tomonini kvadratga ko‘tarib, teng kuchli

$$\sqrt{x+b}=\frac{a-b-1}{2}$$

tenglamani hosil qilamiz. Oxirgi tenglama $a-b \ge 1$ bo‘lgandagina o‘rinli, va bu holda

$$x=\frac{(a-b)^2-2(a+b)+1}{4}.$$

Javob. Agar $a-b<1$ bo‘lsa, yechim mavjud emas; agar $a-b\geq 1$ bo‘lsa,

$$x=y=\frac{(a-b)^2-2(a+b)+1}{4}.$$

Mustaqil yechish uchun misollar

Berilgan tenglamalar (tengsizliklar) sistemasi a parametrning qanday qiymatlarida yagona yechimga ega bo‘ladi?

509. $\begin{cases} 3x+(a-1)y=a+1 \\ (a+1)x+y=3 \end{cases}.$

510. $\begin{cases} y(ax-1)=2|x+1|+2xy \\ xy+1=x-y \end{cases}.$

511. $\begin{cases} x^2-y+1=0 \\ x^2-y^2+(a+1)x+(a-1)y+a=0 \end{cases}.$

512. $\begin{cases} x=a+\sqrt{y} \\ y^2-x^2-2x+4y+3=0 \end{cases}.$

513. $\begin{cases} \sqrt{x}-y=a \\ y\sqrt{x}=1-a \end{cases}.$

514. $\begin{cases} 3y+2+xy=0 \\ x(y+1-a)+y(2a-3)+a+3=0 \end{cases}.$

515. $\begin{cases} y\geq x^2+a \\ x\geq y^2+a \end{cases}.$

516. $\begin{cases} x^2+4x+3-a\leq 0 \\ x^2-2x+6a-3\leq 0 \end{cases}.$

517. $\begin{cases} x+y=a \\ x^4+y^4=a^4 \end{cases}.$

518. $\begin{cases} x-y=8a^2 \\ \sqrt{x}+\sqrt{y}=4a \end{cases}$.

519. $\begin{cases} \sqrt{x}-\sqrt{y}=1 \\ \sqrt{x}+\sqrt{y}=a \end{cases}$.

520. $\begin{cases} x^2-(3a+1)x+2a^2+2a<0 \\ x+a^2=0 \end{cases}$.

521. a ning qanday qiymatlarida tenglamalar sistemasi ikkita yechimga ega bo'ladi? $\begin{cases} |x^2+7x+6|+x^2-5x+6-12|x|=0 \\ x^2-2(a+2)x+a(a+4)=0 \end{cases}$.

Berilgan tenglamalar (tengsizliklar) sistemasi a parametrning qanday qiymatlarida yagona yechimga ega bo'ladi? Bu yechimlarni toping.

522. $\begin{cases} y-x-yx+3=0 \\ a(x+y)=xy-3 \end{cases}$ parametr qatnashgan tenglamalar (tengsizliklar) sistemasini yeching.

523. $\begin{cases} ax>-1 \\ x+a>0 \end{cases}$.

524. $\begin{cases} x^2-(a+1)x+a<0 \\ x^2-(a+3)x<0 \end{cases}$.

525. $\begin{cases} \sqrt{a+x}-\sqrt{y+b}=1 \\ \sqrt{y+a}-\sqrt{x+b}=1 \end{cases}$.

526. a parametrning $\begin{cases} ax+3y=a^2+1 \\ (3a+14)x+(a+8)y=5a^2+5 \end{cases}$ tenglamalar sistemasi yechimga ega bo'lmaydigan qiymatlarini toping.

527. a parametrning qanday qiymatlarida $\begin{cases} 3^{2x+y}+3^{x+3y}=3 \\ 3^{y}+3^{-3x-3y}=3^{a-2x} \end{cases}$ tenglamalar sistemasi yechimga ega bo'ladi?

528. a parametrning qanday qiymatlarida ixtiyoriy b uchun $\begin{cases} bx+y=ac^2 \\ x+by=ac+1 \end{cases}$ tenglamalar sistemasi yechimga ega bo'ladigan kamida bitta c topiladi?

529. M to'plam koordinatalari quyidagi tengsizliklar sistemasini qanoatlantiruvchi nuqtalar to'plamidan iborat bo'lsa, a parametrning qanday qiymatlarida M to'plam Ox o'qning $[-2;-1]$ kesmasini o'zida saqlaydi.

$$\begin{cases} x^2+(a+4)x+4a \le y \\ 3x+y-(2a+4) \le 0 \end{cases}$$

II. QISM. TRIGONOMETRIYA

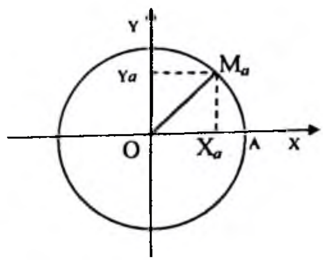
IV BOB. TRIGONOMETRIK FUNKSIYALAR VA ULAR ORASIDAGI MUNOSABATLAR

1-§. Haqiqiy argumentli trigonometrik funksiyalar va ularning xossalari

Tekislikda to‘g‘ri burchakli Oxy koordinatalar sistemasini qaraymiz. Markazi koordinatalar boshida va radiusi birga teng bo‘lgan aylana birlik aylana deyiladi va uning tenglamasi $x^2+y^2=1$ ko‘rinishda bo‘ladi. Koordinata boshi O ni istalgan burchakning uchi qilib, yarim musbat absissa o‘qini qo‘zg‘almas nur OA uchun qabul qilamiz. Bu OA radius boshlang‘ich radius, F nuqta esa boshlang‘ich nuqta deyiladi.

Bu koordinatalar sistemasida a haqiqiy son berilgan bo‘lsin. a haqiqiy songa

koordinatali aylananing koordinatasi a ga teng bo‘lgan $M(a)$ nuqtasini mos qo‘yamiz. (1-chizma).



1-chizma

Ta’rif. $M(a)$ nuqtaning x_a absissasi a sonining kosinusi, y_a ordinatasi esa a sonining sinusi deyiladi va cosa, sina kabi belgilanadi.

Ta’rif. a sonning tangensi deb, shu son sinusining uning kosinusiga nisbatiga aytiladi, ya’ni

$tga=\frac{\sin a}{\cos a}(\cos a\neq 0)$.

Ta'rif. a sonning kotangensi deb, shu son kosinusining uning sinusiga nisbatiga aytiladi.

Trigonometrik funksiyalarning asosiy xossalari

1. Aniqlanish sohasi

A) $D(\sin)=(-\infty;+\infty)$

B) $D(cos)=(-\infty;+\infty)$

C) $D(tg)=R$, faqat $\frac{\pi}{2}+\pi n, n\in Z$, sonlardan boshqa qiymatlarda aniqlangan, chunki bu qiymatlarda kosinus nolga teng. Demak, tangens mavjud emas.

D) $D(tg)=R$ faqat $n\pi, n\in Z$ sonlardan boshqa qiymatlarda kosinus nolga teng, demak kotangens mavjud emas. Albatta bular funksiyalarning ta'rifidan kelib chiqadi.

2. Har qanday burchak sinusi va kosinusining absolyut qiymati 1 dan katta bo'la olmaydi, ya'ni $\sin\alpha$ va $cos\alpha$ funksiyalarning o'zgarish sohasi $[-1;1]$ dan iborat bo'ladi,

$-1\leq\sin x\leq 1,\ |\sin x|\leq 1$

$-1\leq cos x\leq 1,\ |cos x|\leq 1$

Tangens va kotangenslar qiymatlar to'plami esa barcha haqiqiy sonlar to'plamidan iborat bo'ladi.

$E(tg)=R=(-\infty;+\infty)$

$E(ctg)=R=(-\infty;+\infty)$

Mustaqil yechish uchun misollar

1. Agar $\sin\alpha-\cos\alpha=p$ bo'lsa, $\sin 2\alpha$ ni hisoblang.

2. Agar $\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}=1,4$ bo'lsa, $\sin\alpha$ ni hisoblang.

3. Agar $\sin\alpha-\cos\alpha=\frac{1}{2}$ bo'lsa, $\sin^4\alpha+\cos^4\alpha$ ni hisoblang.

4. Agar $0<\alpha<\frac{\pi}{2}$ va $0<\beta<\frac{\pi}{2}$ bo'lib, $ctg\ \alpha=\frac{3}{4}$, $ctg\ \beta=\frac{1}{7}$ bo'lsa, $\alpha+\beta$ ni toping.

5. Agar $90^\circ<\alpha<180^\circ$ bo'lib, $\cos(\alpha-90^\circ)=0,2$ bo'lsa, $tg2\alpha$ ni toping.

6. Agar $tg\frac{\alpha}{2}=4$ bo'lsa, $\frac{6\sin\alpha-7\cos\alpha+1}{8\sin\alpha+9\cos\alpha-1}$ ni hisoblang.

7. Agar $tg\left(\frac{3\pi}{2}+\alpha\right)=\frac{3}{4}$ bo'lsa, $tg\left(\frac{5\pi}{4}+\alpha\right)+tg\left(\frac{5\pi}{4}-x\right)$ ni hisoblang.

8. Agar $\sin\alpha-\cos\alpha=n$ bo'lsa, $\sin^3\alpha-\cos^3\alpha$ ni hisoblang.

9. Agar $tg\alpha=3$ bo'lsa, $\frac{2\sin2\alpha-3\cos2\alpha}{4\sin2\alpha+5\cos2\alpha}$ ni hisoblang.

10. Agar α,β,γ lar biror uchburchakning ichki burchaklari bo'lsa, $\sin3\alpha+\sin3\beta+\sin3\gamma=-4\cos\frac{3}{2}\beta\cos\frac{3}{2}\gamma$ ekanligini isbotlang.

Ifodani soddalashtiring

11. $\frac{ctg\frac{\alpha}{2}+tg\frac{\alpha}{2}}{ctg\frac{\alpha}{2}-tg\frac{\alpha}{2}}$.

12. $\frac{\sqrt{2}-\sin\alpha-\cos\alpha}{\sin\alpha-\cos\alpha}$.

13. $\sin^2\left(\frac{\pi}{8}+\frac{\alpha}{2}\right)-\sin^2\left(\frac{\pi}{8}-\frac{\alpha}{2}\right)$.

14. $\frac{\sin160^\circ\cdot\cos70^\circ-\cos200^\circ\cdot\sin70^\circ-\cos235^\circ\cdot\sin215^\circ}{tg55^\circ\cdot ctg215^\circ}$.

15. $\frac{\cos 5\alpha+\cos 6\alpha+\cos 7\alpha}{\sin 5\alpha+\sin 6\alpha+\sin 7\alpha}$.

16. $\left(\sin x+\frac{1}{\sin x}\right)^2+\left(\cos+\frac{1}{\cos x}\right)^2-tg^2x-ctg^2x$.

17. $\left(\sqrt{\frac{1-\cos x}{1+\cos x}}+\sqrt{\frac{1+\cos x}{1-\cos x}}\right)\cdot\sin x$.

18. $\cos\alpha(1+\sin\alpha+tg\alpha)(1-\sin\alpha+tg\alpha)$.

19. $\sin^2(\alpha+2\beta)-\sin^2(\alpha-2\beta)-1$.

20. $\frac{1-tg(\pi-2\alpha)tg\alpha}{tg\left(\frac{3}{2}\pi-\alpha\right)+tg\alpha}$.

21. $\frac{\cos^2\alpha-ctg^2\alpha+1}{\sin^2\alpha+tg^2\alpha}$.

22. $\sin^2\left(\frac{9\pi}{8}+\alpha\right)-\sin^2\left(\frac{17\pi}{8}-\alpha\right)$.

23. $\frac{tg2\alpha}{tg4\alpha+tg2\alpha}$.

24. $\frac{\cos 3\alpha+\cos 4\alpha+\cos 5\alpha}{\sin 3\alpha+\sin 4\alpha+\sin 5\alpha}$.

25. $\frac{\cos\left(4\alpha-\frac{9\pi}{2}\right)}{ctg\left(\frac{5\pi}{4}+2\alpha\right)\left(1-\cos\frac{5\pi}{2}+4\alpha\right)}$.

26. $\frac{1-2\cos^2 2\alpha}{2tg\left(2\alpha-\frac{\pi}{4}\right)\cdot\sin^2\left(\frac{\pi}{4}+2\alpha\right)}$.

27. $\frac{\sin(80^\circ+4\alpha)}{4\sin(20^\circ+\alpha)\sin(70^\circ-\alpha)}$.

28. $\dfrac{tg^2\left(2\alpha-\frac{\pi}{4}\right)-1}{tg^2\left(2\alpha-\frac{5\pi}{4}\right)+1}$.

29. $\left(tg225^\circ-tg555^\circ\right)\left(tg795^\circ+tg195^\circ\right)$.

30. $\sqrt{\operatorname{cosec}^2\left(\alpha-\frac{3\pi}{2}\right)+\sec^2\left(\alpha+\frac{3\pi}{2}\right)}$.

31. $\dfrac{(\sin 8\alpha+\sin 9\alpha+\sin 10\alpha+\sin 11\alpha)(\cos 8\alpha-\cos 9\alpha-\cos 10\alpha+\cos 11\alpha)}{(\sin 8\alpha-\sin 9\alpha-\sin 10\alpha+\sin 11\alpha)(\cos 8\alpha+\cos 9\alpha+\cos 10\alpha+\cos 11\alpha)}$.

2-§. Argumentning ba'zi qiymatlarida trigonometrik funksiyalarning qiymatlari

Birlik aylanada ichki chizilgan muntazam n burchak berilgan bo'lsin. Agar shu n burchakning tomoni a_n ma'lum bo'lsa, $\frac{\pi}{n}\left(=\frac{180^0}{n}\right)$ burchakning trigonometrik qiymatlarini hisoblash mumkin. Muntazam n burchakning tomonini teng ikkiga bo'luvchi radiusni boshlang'ich tomoni sifatida qabul qilaylik. U holda $\frac{\pi}{n}$ burchakning sinusi $\frac{a_n}{2}$ ga, kosinusi esa apofema $l_n=\sqrt{1-\frac{a_n^2}{2}}$ ga teng bo'ladi, ya'ni $\sin\frac{\pi}{n}=\frac{a_n}{2},\cos\frac{\pi}{n}=\sqrt{1-\frac{a_n^2}{2}}$

Geometriyadan ma'lum bo'lgan ichki chizilgan muntazam ko'pburchakning tomoni bilan radiusi orasidagi bog'lanish formulalariga asosan:

1) $n=3$ da $a_3=\sqrt{3}$, $l_3=\frac{1}{2}$ bo'lib, $\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2},\cos\frac{\pi}{3}=\frac{1}{2},ctg\frac{\pi}{3}=\frac{1}{\sqrt{3}}$ bo'ladi;

2-chizma

2) $n=4$ da $a_4=\sqrt{2}$, $l_4=\frac{\sqrt{3}}{2}$ bo‘lib,

$\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$, $\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$, $tg\frac{\pi}{4}=1$, $ctg\frac{\pi}{4}=1$ bo‘ladi.

3) $n=6$ da $a_6=1$, bo‘lib, $\sin\frac{\pi}{6}=\frac{1}{2}, \cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}, tg\frac{\pi}{6}=\frac{1}{\sqrt{3}}, ctg\frac{\pi}{6}=\sqrt{3}$

bo‘ladi.

4) $n=8$ da $a_8=\sqrt{2-\sqrt{2}}, l_8=\frac{\sqrt{2+\sqrt{2}}}{2}$ bo‘lib,

$\sin\frac{\pi}{8}=\sin 22{,}5^0=\frac{\sqrt{2-\sqrt{2}}}{2}, \cos\frac{\pi}{8}=\frac{\sqrt{2+\sqrt{2}}}{2}, tg\frac{\pi}{8}=\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}=\sqrt{2}-1$

bo‘ladi. Bulardan tashqari, trigonometrik funksiyalarning ta’riflariga asosan, yana quyidagilarni yozish mumkin:

$\sin 0=0, \cos 0=1, tg0=0, ctg0$ - mavjud emas, $ctg\frac{\pi}{2}=0$.

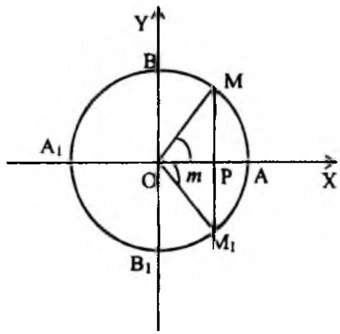
3-§. Trigonometrik funktsiyaning berilgan qiymatiga ko‘ra burchagini yasash

$y=f(x)$ trigonometrik funksiyalarning biri bo‘lsin, m-berilgan son. Shunday burchaklar to‘plamini topish kerakki, bunda $f(\alpha)=m$ tenglik o‘rinli bo‘lsin, boshqacha aytganda $f(\alpha)=m$ ko‘rinishdagi eng sodda trigonometrik tenglama yechilsin. Bular $\cos\alpha = m$, $\sin\alpha = m$, $tg\alpha = m$, $ctg\alpha = m$ ko'rinishga ega.

1. $\cos\alpha = m$. Kosinusi m ga teng bo'lgan burchakni yasash uchun absissalar o'qida $P(m,o)$ nuqtani yasaymiz. Quyidagi uch hol bo'lishi mumkin:

$|m| < 1,\ |m| = 1,\ |m| > 1$

1^0. $|m| < 1$ bo'lsin. Bu holda R nuqta AA_1 kesmaning (kosinuslar kesmasi) ichki nuqtasi bo'ladi. R nuqtadan o'tuvchi va Oy o'qiga parallel bo'lgan to'g'ri chiziq birlik aylanani M va M_1 nuqtalarda kesib o'tadi. OM va OM_1 radiuslar absissalar o'qiga simmetrik bo'lgan va qarama-qarshi yo'nalgan AOM va AOM_1 burchaklarni tashkil etadi. (3-chizma)



3-chizma

Kosinusi m ga teng bo'lgan barcha burchaklarning so'nggi tomoni OM yoki OM_1 dan iborat bo'ladi. $m = 0$ bo'lganda AOB va AOB_1 burchaklar hosil bo'lib, bunda $\angle AOB = \frac{\pi}{2}$ ва $\angle AOB_1 = \frac{\pi}{2}$.

2^0. $|m| < 1$ bo'lsin. U holda $m = 1$ yoki $m = -1$ bo'lib;

a) $m < 1$ da P nuqta A nuqta bilan ustma-ust tushadi va burchak 0 ga teng bo'ladi;

b) $m = -1$ da P nuqta A_1 nuqta bilan ustma-ust tushib, burchak π ga teng bo'ladi.

3^0. $|m|>1$ bo‘lsin. Bu holda $P\notin[A_1A]$ bo‘lib, P nuqtadan Oy o‘qqa o‘tkazilgan parallel to‘g’ri chiziq birlik aylanani kesmaydi.

Shuning uchun bu holda burchak mavjud emas. Shunday qilib, $\cos\alpha=m$ tenglama:

$|m|<1$ da ikkita yechimga ega;

$m=1$ va $m=-1$ da bittadan yechimga ega. $|m|>1$ yechimga ega emas.

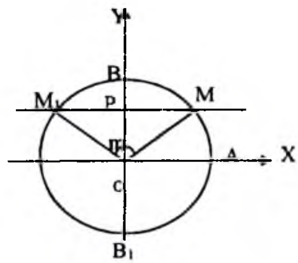
2. $\sin\alpha=m$. Sinusi m ga teng bo‘lgan burchakni yasash uchun ordinatalar o‘qida $P(O,m)$ nuqtani yasaymiz. Quyidagi uch hol bo‘lishi mumkin:

$|m|<1$, $|m|=1$, $|m|>1$.

1^0. $|m|<1$ bo‘lsin. Bu holda P nuqta BB_1 kesmaning (sinuslar kesmasi) ichki nuqtasi bo‘ladi. P nuqtadan o‘tuvchi va OX o‘qiga parallel bo‘lgan to‘g’ri chiziq birlik aylanani M va M_1 nuqtalarda kesib o‘tadi. OM va OM_1 radiuslar ordinatalar o‘qiga

simmetrik va o‘zaro π ga qadar to‘ldiruvchi bo‘lgan AOM va AOM_1 burchaklarni tashkil etadi. (4-chizma).

Sinusi $m$ga teng bo‘lgan barcha burchaklarning so‘nggi tomoni yoki OM, yoki OM_1 dan iborat bo‘ladi. $m=0$ bo‘lganda M nuqta A nuqta bilan, M_1 nuqta A_1 nuqta bilan ustma-ust tushadi va O yoki π ga teng burchaklarni hosil qiladi.



4-chizma

2^0. $|m|=1$ bo‘lsin. U holda $m=1$ yoki $m=-1$ bo‘lib:

a) $m=1$ da P nuqta B nuqta bilan ustma-ust tushadi va $\frac{\pi}{2}$ burchak hosil bo'ladi;

b) $m=-1$ da P nuqta B_1 nuqta bilan ustma-ust tushadi va $\frac{3\pi}{2}$ burchak hosil bo'ladi.

3^0. $|m|>1$ bo'lsin. Bu holda $P\notin[BB_1]$ bo'lib, P nuqtadan OX o'qiga parallel o'tgan to'g'ri chiziq birlik aylanani kesmaydi. Shuning uchun bu holda burchak mavjud emas.

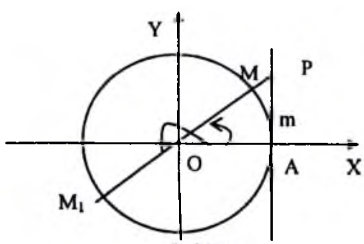
Shunday qilib, $\sin\alpha=m$ tenglama:

$|m|<1$ da ikkita yechimga ega. $m=1$ va $m=-1$ da bittadan yechimga ega.

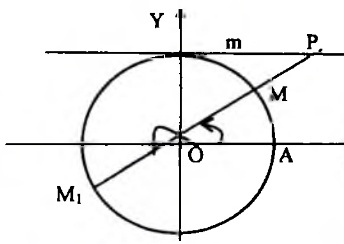
$|m|>1$ da yechimga ega emas

3. $tg\alpha=m$. Tangensi $m$ga teng bo'lgan burchakni yasash uchun tangenslar o'qida $P(1;m)$ nuqtani yasaymiz. P nuqtani aylana markazi bilan tutashtiruvchi to'g'ri chiziq aylanani M va M_1 nuqtalarda kesib o'tadi. OM va OM_1 radiuslar izlangan burchaklarning so'nggi tomonlari bo'lib, bu burchaklar bir-biri bilan π ga qadar farq qiladi.

Shunday qilib, $tg\alpha=m$ tenglama m ning ixtiyoriy haqiqiy qiymatlari uchun har doim yechimga ega. (5-chizma).



5-chizma



6-chizma

4.

$ctg\alpha = m$. Kotangensi $m$ga teng bo'lgan burchakni yasash uchun kotangenslar o'qida $P(m;1)$nuqtani yasaymiz. P nuqtani aylana markazi bilan tutashtiruvchi to'g'ri chiziq aylanani M va M_1 nuqtalarda kesib o'tadi. OM va OM_1 radiuslar izlangan burchaklarning so'nggi tomonlari bo'lib, bu burchaklar bir-biri bilan π ga qadar farq qiladi.

Shunday qilib, $ctg\alpha = m$ tenglama m ning ixtiyoriy haqiqiy qiymatlari uchun har doim yechimga ega. (6-chizma).

4-§. Bir xil argument trigonometrik funksiyalari orasidagi munosabatlar

Teorema: α argumentning ixtiyoriy qiymatlarida $\sin^2\alpha + \cos^2\alpha = 1$ ayniyat o'rinli.

Isboti. Koordinatalari $x = \cos\alpha$, $y = \sin\alpha$ bo'lgan nuqta birlik aylanada joylashgan bo'lib, $x^2 + y^2 = 1$ edi. Bundan

$$\cos^2\alpha + \sin^2\alpha = 1 \qquad (1)$$

Teorema isbot bo'ldi.

Tangens va kotangens ta'rifiga ko'ra

$$tg\alpha = \frac{\sin\alpha}{\cos\alpha} \qquad (2)$$

$$ctg\alpha = \frac{\cos\alpha}{\sin\alpha} \qquad (3)$$

edi. (2) va (3) larni ko'paytirib, $tg\alpha \cdot ctg\alpha = 1$ (4)

ga ega bo'lamiz.

(1) Ayniyatni galma-galdan $\sin^2\alpha(\sin\alpha \neq 0)$ va $\cos^2\alpha\,(\cos\alpha \neq 0)$ larga hadma-had bo'lib:

$$tg^2\alpha+1=\frac{1}{\cos^2\alpha}=\sec^2\alpha \qquad (5)$$

va

$ctg^2\alpha+1=\frac{1}{\sin^2\alpha}=\operatorname{cosec}^2\alpha$ (6) larga ega bo‘lamiz. Bu formulalar α argumentning qabul qilishi mumkin bo‘lgan barcha qiymatlarida ayniyatdir.

Bu formulalarni bilish biz uchun bitta trigonometrik funksiyani boshqalari orqali ifodalash va aksincha, trigonometrik funksiyani boshqalari orqali ifodalash imkonini beradi.

Jumladan:

1. (1), (2), (3) lardan

$$\cos\alpha=\pm\sqrt{1-\sin^2\alpha},\quad tg\alpha=\frac{\sin\alpha}{\pm\sqrt{1-\sin^2\alpha}},\quad ctg\alpha=\frac{\pm\sqrt{1-\sin^2\alpha}}{\sin\alpha} \qquad (7)$$

yoki

$$\sin\alpha=\pm\sqrt{1-\cos^2\alpha},\quad tg\alpha=\frac{\pm\sqrt{1-\cos^2\alpha}}{\cos\alpha},\quad ctg\alpha=\frac{\cos\alpha}{\pm\sqrt{1-\cos^2\alpha}} \qquad (8)$$

2. (4), (5), (6) lardan

$$\cos\alpha=\frac{1}{\pm\sqrt{1+tg^2\alpha}},\quad \sin\alpha=\frac{tg\alpha}{\pm\sqrt{1+tg^2\alpha}},\quad ctg\alpha=\frac{1}{tg\alpha} \qquad (9)$$

yoki

$$\cos\alpha=\frac{ctg\alpha}{\pm\sqrt{1+ctg^2\alpha}},\quad \sin\alpha=\frac{1}{\pm\sqrt{1+ctg^2\alpha}},\quad tg\alpha=\frac{1}{ctg\alpha} \qquad (10)$$

Bu formulalardagi “$\pm$” ishora umumiy holda masalaning ikkita yechimga ega ekanligini bildiradi. Haqiqatan ham trigonometrik funksiyaning berilgan qiymatiga ko‘ra ikkita burchak yasash mumkin edi. Shuning uchun ishoraning tanlanishi α burchakning qaysi chorakda joylashishiga bog'liq. (7)-(10) formulalarda α ning qaysi chorakda joylashishiga qarab trigonometrik funksiyaning ishorasi aniqlanadi, shunga mos kelgan ishora o'ng tomonda ildiz oldida qoldiriladi.

1. Trigonometrik funksiyalar qiymatlarining ishoralari

Chorak	$\sin x$	$\cos x$	tgx	$ctgx$	$\sec x$	$\operatorname{cosec} x$
I	+	+	+	+	+	+
II	+	-	-	-	-	+
III	-	-	+	+	-	-
IY	-	+	-	-	+	-

2. Trigonometrik funksiyalarning ayrim burchaklardagi qiymatlari jadvali

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
tgx	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	-	0	-	0
$ctgx$	-	0	1	$\frac{\sqrt{3}}{3}$	0	-	0	-

3. Keltirish formulalari

Keltirish formulalari deb, $\frac{\pi}{2}\pm x$, $\pi\pm x$, $\frac{3\pi}{2}\pm x$, $2\pi\pm x$ argumentlarning trigonometrik funktsiyalarini x argumentning trigonometrik funktsiyalari orqali ifodalovchi formulalarga aytiladi. Bular quyidagi jadvalda keltirilgan:

x	α	$\frac{\pi}{2}-\alpha$	$\frac{\pi}{2}+\alpha$	$\pi-\alpha$	$\pi+\alpha$	$\frac{3\pi}{2}-\alpha$	$\frac{3\pi}{2}+\alpha$	$2\pi-\alpha$
$\sin x$	$\sin\alpha$	$\cos\alpha$	$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$
$\cos x$	$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$	$\sin\alpha$	$\cos\alpha$
tgx	$tg\alpha$	$ctg\alpha$	$-ctg\alpha$	$-tg\alpha$	$tg\alpha$	$ctg\alpha$	$-ctg\alpha$	$-tg\alpha$
$ctgx$	$ctg\alpha$	$tg\alpha$	$-tg\alpha$	$-ctg\alpha$	$ctg\alpha$	$tg\alpha$	$-tg\alpha$	$-ctg\alpha$
$\sec x$	$\sec\alpha$	$cosec\alpha$	$-cosec\alpha$	$-\sec\alpha$	$-\sec\alpha$	$-cosec\alpha$	$cosec\alpha$	$\sec\alpha$
$cosec x$	$cosec\alpha$	$\sec\alpha$	$\sec\alpha$	$cosec\alpha$	$-cosec\alpha$	$-\sec\alpha$	$-\sec\alpha$	$-cosec\alpha$

4. Trigonometrik funksiyalar yig'indilarini ko'paytmaga almashtirish formulalari

$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha-\cos\beta=2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}$$

$$tg\alpha\pm tg\beta=\frac{\sin(\alpha\pm\beta)}{\cos\alpha\cdot\cos\beta}\quad\left(\alpha\neq\frac{\pi}{2}+\pi n,\quad \beta\neq\frac{\pi}{2}+\pi n\right)$$

$$ctg\alpha\pm ctg\beta=\frac{\sin(\beta\pm\alpha)}{\sin\alpha\cdot\sin\beta}\quad(\alpha\neq\pi k,\quad \beta\neq\pi k)$$

Ko'paytmani yig'indiga almashtiramiz.

$$\sin\alpha\cdot\cos\beta=\frac{1}{2}(\sin(\alpha-\beta)+\sin(\alpha+\beta))$$

$$\cos\alpha\cdot\cos\beta=\frac{1}{2}\left(\cos(\alpha-\beta)+\cos(\alpha+\beta)\right)$$

$$\sin\alpha\cdot\sin\beta=\frac{1}{2}\left(\cos(\alpha-\beta)-\cos(\alpha+\beta)\right)$$

1-misol. $f(x)=\dfrac{\sin^3(x-270^\circ)\cos(360^\circ-x)}{tg^3(x-90^\circ)\cos(270^\circ-x)}$ Ifodani soddalashtiring.

Yechilishi. $y=cosx$, juft funksiya, $y=\sin x$, $y=tgx$ toq funksiya, shularga ko'ra:

$$f(x)=\frac{\left(-\sin(270^\circ-x)\right)^3\cdot\cos(360^\circ-x)}{\left(-tg(x-90^\circ)\right)^3\cdot\cos^3(270^\circ-x)};$$

Keltirish formulalariga ko'ra: $f(x)=\dfrac{(\cos x)^3\cdot\cos x}{(-ctgx)^3\cdot(\sin x)^3}=\cos x$. Shunday qilib, agar $\sin x\neq0$, $\cos x\neq0$ va $x\neq\frac{\pi n}{2}$, u holda $f(x)=\cos x$.

2-misol. Agar $tg\alpha=-\dfrac{3}{4}$ va $\frac{\pi}{2}<\alpha<\pi$ bo'lsa, $\sin\alpha$, $\cos\alpha$, $\text{tg}\alpha$ ni toping.

Yechilishi.

$\cos^2\alpha=\dfrac{1}{1+tg^2\alpha}=\dfrac{16}{25}$ bundan,

$\cos\alpha=\frac{4}{5}$ yoki $\cos\alpha=-\frac{4}{5}$ .Bu yerda $\frac{\pi}{2}<\alpha<\pi$, ya'ni II-chorakda bo'lganligi uchun, $\cos\alpha=-\frac{4}{5}$.

$tg\alpha=\dfrac{\sin\alpha}{\cos\alpha}$dan, $\sin\alpha=\text{tg}\alpha\cdot\cos\alpha$ bundan $\sin\alpha=\frac{3}{5}$ $ctg\alpha=\dfrac{1}{tg\alpha}=-\dfrac{4}{3}$

shunday qilib, $\sin\alpha=\frac{3}{5}$, $\cos\alpha=-\frac{4}{5}$, $\cos\alpha=-\frac{4}{3}$.

3-misol.

Tenglikni to‘g‘riligini tekshiring.

$\sin 47^\circ + \sin 61^\circ - \sin 11^\circ - \sin 25^\circ = \cos 7^\circ$

Yechilishi.

Trigonometrik funksiyalarni yig’indisini ko‘paytmaga almashtirish formulalariga ko‘ra:

$$(\sin 47^\circ + \sin 61^\circ) - (\sin 11^\circ - \sin 25^\circ) = 2\sin 54^\circ \cdot \cos 7^\circ - 2\sin 18^\circ \cdot \cos 7^\circ =$$
$$= 2\cos 7^\circ (\sin 54^\circ - \sin 18^\circ) = 2\cos 7^\circ \cdot \sin 18^\circ \cdot \cos 36^\circ.$$

Hosil bo‘lgan ifodani $\cos 18^\circ$ ga ham ko‘paytirib, $\sin 36^\circ = 2\sin 18^\circ \cdot \cos 18^\circ$ ni e’tiborga olsak:

$$2\cos 7^\circ \cdot \frac{\sin 36^\circ \cdot \cos 36}{\cos 18^\circ} = \cos 7^\circ \cdot \frac{\sin 72^\circ}{\cos 18^\circ} = \cos 7^\circ \cdot \frac{\cos 18^\circ}{\cos 18^\circ} = \cos 7^\circ.$$

Demak, berilgan tenglik to‘g‘ri ekan.

4-misol. Agar $\cos\alpha = \frac{4}{3}$ bo‘lsa, $16\sin\frac{\alpha}{2} \cdot \sin\frac{3\alpha}{2}$ ni hisoblang.

Yechilishi. Trigonometrik funksiyalarni ko‘paytmasini yig‘indiga almashtirish, hamda darajasini pasaytirish formulalariga ko‘ra:

$$16\sin\frac{\alpha}{2} \cdot \sin\frac{3\alpha}{2} = 16 \cdot \frac{\cos\left(\frac{\alpha}{2} - \frac{3\alpha}{2}\right) - \cos\left(\frac{\alpha}{2} + \frac{3\alpha}{2}\right)}{2} =$$
$$= 8(\cos\alpha - \cos 2\alpha) = 8(\cos\alpha - 2\cos^2\alpha + 1) =$$
$$= 8\left(\frac{3}{4} - 2\left(\frac{3}{4}\right)^2 + 1\right) = 5$$

5-§. Trigonometrik ayniyatlar

Trigonometrik funksiyalar qatnashgan ifodalar xossalari va ularning o‘zaro bog‘liqligini yana o‘rganishning muhim bosqichidir. Trigonometrik ifodalarni ayniy shakl almashtirishga doir misollarda argumentning qabul qilishi mumkin bo'lgan qiymatlari to'plamida berilgan deb qaraladi. Zarur bo'lgan holda alohida aniqlanish sohasiga murojaat qilamiz.

Ta'rif. Tenglikning tarkibiga kiruvchi o'zgaruvchilarning istalgan qiymatlarida to'g'ri bo'ladigan tenglikka ayniyat deyiladi.

Albatta, trigonometrik ayniyatlarni isbotlashda tenglikda qatnashayotgan argument qabul qilishi mumkin bo'lgan qiymatlar to'plami hisobga olinib, shu to'plamda qaralayotgan ayniyat hisoblanadi. Ayrim trigonometrik ayniyatlarni isbotlashni ko'rib o'tamiz.

1-misol. Ayniyatni isbotlang.

$$\frac{tg^2 2\alpha - tg^2\alpha}{1 - tg^2 2\alpha \cdot tg^2\alpha} = tg3\alpha \cdot tg\alpha$$

Isboti. Ayniyatni isbotlash uchun chap tomonidan o'ng tomonini keltirib chiqaramiz, ya'ni

$$\frac{tg^2 2\alpha - tg^2\alpha}{1 - tg^2 2\alpha \cdot tg^2\alpha} = \frac{(tg2\alpha - tg\alpha)(tg2\alpha + tg\alpha)}{(1 - tg2\alpha \cdot tg\alpha)(1 + tg2\alpha \cdot tg\alpha)} = \frac{tg2\alpha + tg\alpha}{1 - tg2\alpha \cdot tg\alpha} \cdot \frac{tg2\alpha - tg\alpha}{1 + tg2\alpha \cdot tg\alpha} =$$
$$= tg(2\alpha + \alpha) \cdot tg(2\alpha - \alpha) = tg3\alpha \cdot tg\alpha$$

2-misol. Ayniyatni isbotlang.

$$\cos^2 x + \cos^2\left(\frac{2\pi}{3} + x\right) + \cos^2\left(\frac{2\pi}{3} - x\right) = \frac{2}{3}$$

Isboti.

$$\cos^2 x + \cos^2\left(\frac{2\pi}{3} + x\right) + \cos^2\left(\frac{2\pi}{3} - x\right) = \frac{1}{2}(1 + \cos x) + \frac{1}{2}\left(1 + \cos\left(\frac{4\pi}{3} + 2x\right)\right) + \frac{1}{2}\left(1 + \cos\left(\frac{4\pi}{3} - 2x\right)\right) =$$
$$= \frac{3}{2} + \frac{1}{2}\cos 2x + \cos\frac{4\pi}{3} \cdot \cos 2x = \frac{3}{2} + \frac{1}{2}\cos 2x - \frac{1}{2}\cos 2x = \frac{3}{2}$$

3-misol. Agar $\alpha + \beta + \gamma = \pi$ bo'lsa, $\sin\alpha + \sin\beta + \sin\gamma = 4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$ ni isbotlang.

Isboti. $\alpha+\beta+\gamma=\pi$ bundan

$$\gamma=\pi-(\alpha+\beta)$$

$$\sin\gamma=\sin(\pi-(\alpha+\beta))=\sin(\alpha+\beta)$$

$$\sin\alpha+\sin\beta+\sin\gamma=\sin\alpha+\sin\beta+\sin(\alpha+\beta)=2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}+2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha+\beta}{2}=$$

$$=2\sin\frac{\alpha+\beta}{2}\left(\cos\frac{\alpha-\beta}{2}+\cos\frac{\alpha+\beta}{2}\right)=2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha}{2}\cos\frac{\beta}{2}=4\sin\left(\frac{\pi}{2}-\frac{\gamma}{2}\right)\cos\frac{\alpha}{2}\cos\frac{\beta}{2}=$$

$$=4\cos\frac{\alpha}{2}\cdot\cos\frac{\beta}{2}\cdot\cos\frac{\gamma}{2}$$

Mustaqil yechish uchun misollar

Ayniyatlarni isbotlang

32. $\dfrac{\sin(\beta-\gamma)}{\cos\beta\cos\gamma}+\dfrac{\sin(\gamma-\alpha)}{\cos\gamma\cos\alpha}+\dfrac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}=0.$

33. $3(\sin^4\alpha+\cos^4\alpha)-2(\sin^6\alpha+\cos^6\alpha)=1.$

34. $\dfrac{\cos 2\alpha}{1+\sin 2\alpha}=\dfrac{1-tg\alpha}{1+tg\alpha}.$

35. $4\sin\left(\alpha+\dfrac{\pi}{3}\right)\sin\left(\alpha-\dfrac{\pi}{3}\right)=-3+4\sin^2\alpha.$

36. $tg(\alpha+35^\circ)tg(25^\circ-\alpha)=\dfrac{2\cos(10^\circ+2\alpha)-1}{2\cos(10^\circ+2\alpha)+1}.$

37. $9\cos 15\alpha+3\cos 7\alpha+3\cos 19\alpha+9\cos 11\alpha=24\cos^3 2\alpha\cos 13\alpha.$

38. $\sqrt{\dfrac{1+\sin\alpha}{1-\sin\alpha}}-\sqrt{\dfrac{1-\sin\alpha}{1+\sin\alpha}}=\begin{cases} agar\ -\dfrac{\pi}{2}+2\pi n<\alpha<\dfrac{\pi}{2}+2\pi n,\ bo'lsa\ 2tg\alpha \\ agar\ \dfrac{\pi}{2}+2\pi n<\alpha<\dfrac{3\pi}{2}+2\pi n,\ bo'lsa\ -2tg\alpha \end{cases}.$

39. $\sin\alpha+\sin\beta+\sin\gamma-\sin(\alpha+\beta+\gamma)=4\sin\dfrac{\alpha+\beta}{2}\sin\dfrac{\beta+\gamma}{2}\sin\dfrac{\gamma+\alpha}{2}.$

40. $tg2\alpha tg(30^\circ-\alpha)+tg2\alpha tg(60^\circ-\alpha)+tg(60^\circ-\alpha)tg(30^\circ-\alpha)=1.$

41. Agar $\alpha+\beta+\gamma=\pi$ bo'lsa, $tg\alpha+tg\beta+tg\gamma=tg\alpha\cdot tg\beta\cdot tg\gamma.$

42. $\cos\alpha\cdot cosec\alpha+\sin\alpha\cdot\sec\alpha=2ctg2\alpha$.

43. $(\sin\alpha+\cos\alpha)^2-(\sin\alpha-\cos\alpha)^2=2\sin\alpha$.

44. $3-4\sin^2\alpha=4\sin\left(\frac{\pi}{3}+\alpha\right)\sin\left(\frac{\pi}{3}-\alpha\right)$.

45. $\sin\alpha+\sin3\alpha+\sin5\alpha+\sin7\alpha=4\cos\alpha\cos2\alpha\sin4\alpha$.

46. $\sin^2\alpha+\sin^2\left(\frac{2\pi}{3}+\alpha\right)+\sin^2\left(\frac{2\pi}{3}-\alpha\right)=\frac{3}{2}$.

47. $\frac{\sin^2\alpha}{\sin^2\beta}+tg^2\beta\cdot\cos^2\alpha=\sin^2\alpha+tg^2\beta$.

48. $tg\left(\frac{\pi}{4}-\alpha\right)+ctg\left(\frac{\pi}{4}-\alpha\right)=2\sec2\alpha$.

49. $\cos\alpha\, tg(\pi+\alpha)tg\left(\frac{3\pi}{2}-\alpha\right)cosec\left(\frac{\pi}{2}-\alpha\right)=1$.

50. $\sin6\alpha\cos^3 2\alpha+\cos6\alpha\cdot\sin^2 2\alpha=\frac{3}{4}\sin8\alpha$.

51. $\cos\frac{\alpha}{2}\cdot\cos\frac{\alpha}{4}\cdots\cos\frac{\alpha}{2^n}=\frac{\sin\alpha}{2^n\cdot\sin\frac{\alpha}{2^n}}$, $\quad n\in N$.

52. $ctg\left(\alpha-\frac{3\pi}{4}\right)(1+\sin2\alpha)=\cos2\alpha$.

53. $\sin2\alpha\cos\left(\frac{\pi}{6}+\frac{\alpha}{2}\right)\cos\left(\frac{\pi}{6}-\frac{\alpha}{2}\right)=\cos\alpha\cos\frac{\alpha}{2}$.

54. $\sin^6\alpha+\cos^6\alpha=\frac{5}{8}+\frac{3}{8}\cos4\alpha$.

55. $tg2\alpha\cdot tg\left(\frac{\pi}{6}-\alpha\right)+tg2\alpha\cdot tg\left(\frac{\pi}{3}-\alpha\right)+tg\left(\frac{\pi}{6}-\alpha\right)tg\left(\frac{\pi}{3}-\alpha\right)=1$.

56. $ctg\alpha-tg\alpha-2tg2\alpha-4tg4\alpha=8ctg8\alpha$.

57. $\sin\alpha+\sin\left(\alpha+\frac{14}{3}\pi\right)+\sin\left(\alpha-\frac{8}{3}\pi\right)=0$.

58. $\frac{\sin4\alpha}{1+\cos4\alpha}\cdot\frac{\cos2\alpha}{1+\cos2\alpha}=ctg\left(\frac{3}{2}\pi-\alpha\right)$.

59. $\sin^2\left(\frac{14}{3}\pi+\alpha\right)+\sin\left(\alpha-\frac{8}{3}\pi\right)=0$.

60. $\sin^2(45^\circ+\alpha)\cdot\sin^2(30^\circ-\alpha)-\sin 15^\circ\cos(15^\circ+2\alpha)=\sin 2\alpha$.

61. $tg4\alpha+\sec 4\alpha=\frac{\cos 2\alpha+\sin 2\alpha}{\cos 2\alpha-\sin 2\alpha}$.

62. $4\cos\left(\frac{\pi}{6}-\alpha\right)\sin\left(\frac{\pi}{3}-\alpha\right)=\frac{\sin 3\alpha}{\sin\alpha}$.

63. $$\frac{\sin\left(\frac{5}{2}\pi+\frac{\alpha}{2}\right)\left(1+tg^2\left(\frac{3\alpha}{4}-\frac{\pi}{2}\right)\right)}{\sec\frac{\alpha}{4}\left(tg^2\left(\frac{3\pi}{2}-\frac{\alpha}{4}\right)-tg^2\left(\frac{3\alpha}{4}-\frac{7\pi}{2}\right)\right)}=\frac{1}{8}.$$

64. $1+tg^{-1}\alpha+\frac{1}{\sin\alpha}=\frac{\sqrt{2}\sin\frac{\alpha}{2}}{\sin\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)}$.

65. $1-\cos(2\alpha-\pi)+\cos(4\alpha-2\pi)=4\cos 2\alpha\cdot\cos\left(\frac{\pi}{6}+\alpha\right)\cos\left(\frac{\pi}{6}-\alpha\right)$.

66. $3+4\sin\left(4\alpha+\frac{3}{2}\pi\right)+\sin\left(8\alpha+\frac{5}{2}\pi\right)=8\sin^4 2\alpha$.

67. $\frac{\sqrt{ctg\alpha}+\sqrt{tg\alpha}}{\sqrt{ctg\alpha}-\sqrt{tg\alpha}}=ctg\left(\frac{\pi}{4}-\alpha\right)$.

68. $\frac{\sin 6\alpha+\cos 7\alpha+\sin 8\alpha+\sin 9\alpha}{\cos 6\alpha+\cos 7\alpha+\cos 8\alpha+\cos 9\alpha}=tg\frac{15}{2}\alpha$.

69. $ctg\alpha-tg\alpha-2tg2\alpha=4ctg2\alpha$.

70. $tg6\alpha-tg4\alpha-tg2\alpha=tg6\alpha\cdot tg4\alpha\cdot tg2\alpha$.

71. $tg4\alpha-\sec 4\alpha=\frac{\sin 2\alpha-\cos 2\alpha}{\sin 2\alpha+\cos 2\alpha}$.

72. $16\sin^5\alpha-20\sin^3\alpha+5\sin\alpha=\sec\alpha$.

73. $ctg(270^\circ-2\alpha)+ctg(210^\circ-2\alpha)+ctg(150^\circ-2\alpha)=3tg6\alpha$.

74. $8\cos^4\alpha+4\cos^3\alpha-8\cos^2\alpha-3\cos\alpha+1=-2\cos\frac{7\alpha}{2}\cos\frac{\alpha}{2}$.

75. $\cos^2\alpha+\cos^2 2\alpha+...+\cos^2 n\alpha=\frac{\cos(n+1)\alpha\cdot\sin\alpha}{2\sin\alpha}+\frac{n}{2}$.

76. $\frac{3-4\cos 2\alpha+\cos 4\alpha}{3+4\cos 2\alpha+\cos 4\alpha}=tg^4\alpha$.

77. $8\cos^4\alpha-4\cos^3\alpha-8\cos^2\alpha+3\cos\alpha+1=-2\sin\frac{7\alpha}{2}\sin\frac{\alpha}{2}$.

6-§. Qo'shish teoremalari va ularning natijalari

Teorema. α va β argumentlarning ixtiyoriy qiymatlarida:

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta, \quad (1)$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta, \quad (2)$$

Munosabatlar o'rinli.

Isboti. M va N α va β burchaklarning qiymatlarini birlik aylanada ifodalovchi nuqtalar bo'lsin (1-chizma). U holda M va N nuqtalarning koordinatalari quyidagicha bo'ladi:

$$x_M=\cos\alpha, \qquad y_M=\sin\alpha;$$
$$x_N=\cos\beta, \qquad y_N=\sin\beta.$$

M va N nuqtalar orasidagi masofani hisoblaylik:

$$MN^2=(x_M-x_N)^2+(y_M-y_N)^2=(\cos\alpha-\cos\beta)^2+$$
$$+(\sin\alpha-\sin\beta)^2=2(1-\cos\alpha\cos\beta-\sin\alpha\sin\beta) \quad (*)$$

Endi boshlang'ich nuqtasi N va oxirgi nuqtasi M bo'lgan yoyni qaraylik. Bu yoy $\alpha-\beta$ son bilan o'lchanadi. Shu yoyni $A(1;0)$ nuqtadan boshlab qo'yaylik. Bunda B uning oxirgi uchi bo'lsin. U holda yasashga ko'ra

$\cup MN=\cup AB$ bo‘lib, bularni tortib turuvchi vatarlar ham teng bo‘ladi, ya’ni $AB=MN$.

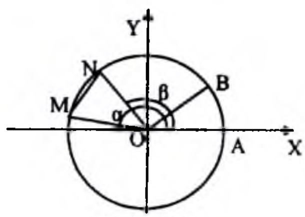
AB masofani hisoblash uchun A va B nuqtalarning koordinatalarini aniqlaylik: $x_A=1;$ $y_A=0;$ $x=\cos(\alpha-\beta),$ $y_B=\sin(\alpha-\beta)$. Demak,

$$AB^2=\left(\cos(\alpha-\beta)-1\right)^2+\sin^2(\alpha-\beta)=2\left(1-\cos(\alpha-\beta)\right)\ (**)$$

(*) va (**) larni o‘zaro tenglashtirib,

$$2\left(1-\cos\alpha\cos\beta-\sin\alpha\sin\beta\right)=2\left(1-\cos(\alpha-\beta)\right)$$
$$1-\cos\alpha\cos\beta-\sin\alpha\sin\beta=1-\cos(\alpha-\beta)$$
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

(2) formulaga ega bo‘lamiz.



1-chizma.

(2) formulada β ni $-\beta$ ga almashtirib,

$$\cos(\alpha+\beta)=\cos(\alpha-(-\beta))=\cos\alpha\cos(-\beta)+$$
$$+\sin\alpha\sin(-\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

ga ega bo‘lamiz. Bu yerda sinus va kosinuslarning juft va toqligidan foydalandik.

Natija. Agar α va β argumentlarning yig‘indisi $\frac{\pi}{2}$ ga teng bo‘lsa, u holda birining kosinusi ikkinchisining sinusiga va aksincha, teng bo‘ladi, ya’ni $\alpha+\beta=\frac{\pi}{2}$ bo‘lsa, u holda $\cos\alpha=\sin\beta$ yoki $\cos\beta=\sin\alpha$. Haqiqatan, $\cos\alpha=\cos\left(\frac{\pi}{2}-\beta\right)$ (2) formulaga binoan:

$$\cos\alpha=\cos\left(\frac{\pi}{2}-\beta\right)=\cos\frac{\pi}{2}\cos\beta+\sin\frac{\pi}{2}\sin\beta=\sin\beta$$

Ikkinchisi ham shunday isbotlanadi.

Teorema. α va β argumentlarning ixtiyoriy qiymatlarida quyidagi munosabatlar o‘rinli.

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta, \qquad (3)$$

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta, \qquad (4)$$

Teorema. $tg\alpha$ va $tg\beta$, $tg(\alpha+\beta)$ lar ma'noga ega bo'ladigan barcha α va β lar uchun:

$$tg(\alpha+\beta)=\frac{tg\alpha+tg\beta}{1-tg\alpha\cdot tg\beta} \qquad \left(\alpha\neq\frac{\pi}{2}+\pi n, \quad \beta\neq\frac{\pi}{2}+\pi k, \quad \alpha\pm\beta\neq\frac{\pi}{2}+\pi m\right).$$

munosabatlar o'rinli.

Natija. $tg(\alpha-\beta)$, $tg\alpha$, $tg\beta$ lar ma'noga ega bo'ladigan barcha α va β lar uchun β ni $-\beta$ ga almashtirib:

$$tg(\alpha-\beta)=\frac{tg\alpha-tg\beta}{1+tg\alpha\cdot tg\beta} \qquad (\alpha\neq\frac{\pi}{2}+\pi n, \quad \beta\neq\frac{\pi}{2}+\pi k, \quad \alpha\pm\beta\neq\frac{\pi}{2}+\pi m).$$

munosabatlar keltirilib chiqariladi.

Teorema. $ctg\alpha$, $ctg\beta$, $ctg(\alpha\pm\beta)$ lar ma'noga ega bo'ladigan barcha α va β lar uchun:

$$ctg(\alpha\pm\beta)=\frac{ctg\alpha\cdot ctg\beta\mp1}{ctg\alpha\pm ctg\beta} \qquad (\alpha\neq\pi n, \quad \beta\neq\pi k, \quad \alpha\pm\beta\neq\pi m).$$

formulalar o'rinli.

7-§. Trigonometrik funksiyani ko'paytmasini yig'indiga va aksincha, almashtirish formulalari

Teorema. α va β larning ixtiyoriy qiymatlarida:

$$\cos\alpha\cos\beta=\frac{1}{2}\left(\cos(\alpha+\beta)+\cos(\alpha-\beta)\right) \qquad (6)$$

$$\sin\alpha\sin\beta=\frac{1}{2}\left(\cos(\alpha+\beta)-\cos(\alpha-\beta)\right) \qquad (7)$$

$$\sin\alpha\cos\beta=\frac{1}{2}\left(\sin(\alpha+\beta)+\sin(\alpha-\beta)\right) \qquad (8)$$

ayniyatlar o'rinli.

Isboti. Oldingi mavzuda qaralgan (1), (2) formulalarni hadma-had qo'shib, ya'ni

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$\cos\alpha\cos\beta=\frac{1}{2}(\cos(\alpha+\beta)+\cos(\alpha-\beta)$ (6) formulaga ega bo'lamiz, hadma-had ayirib esa, $\sin\alpha\sin\beta=\frac{1}{2}(\cos(\alpha+\beta)-\cos(\alpha-\beta)$ (7) formulaga ega bo'lamiz.

Xuddi shu singari (3), (4) formulalarni hadma-had qo'shib,

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta,$$

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta,$$

$\sin\alpha\cos\beta=\frac{1}{2}(\sin(\alpha+\beta)+\sin(\alpha-\beta)$ (8) formulaga ega bo'lamiz.

Teorema. α va β ning barcha qiymatlarida:

$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}; \qquad (9)$$

$$\cos\alpha+\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}; \qquad (10)$$

$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}; \qquad (11)$$

$$\sin\alpha-\sin\beta=2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \qquad (12)$$

α va β larning qabul qilishi mumkin bo'lgan barcha qiymatlarida:

$\alpha,\beta\neq\frac{\pi}{2}+k\pi,\quad k\in Z$ **lar uchun:**

$$tg\alpha+tg\beta=\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} \qquad (13)$$

$$tg\alpha - tg\beta = \frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta} \qquad (13)$$

$\alpha, \beta \neq k\pi, \quad k \in Z$ lar uchun:

$$ctg\alpha + ctg\beta = \frac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta} \qquad (14)$$

$$ctg\alpha - ctg\beta = \frac{\sin(\alpha-\beta)}{\sin\alpha\sin\beta} \qquad (15)$$

Yig'indini ko'paytmaga keltirishga doir misollar

78. a) $\sin 80^0 + \sin 30^0$, б) $\sin 80^0 - \sin 30^0$.

79. a) $\cos 75^0 + \cos 48^0$, б) $\cos 75^0 - \cos 30^0$.

80. a) $\sin 5 - \sin 3$, б) $\cos 5x - \cos 3x$.

81. a) $\sin 2x + \cos 2x$, б) $\cos(3+x) - \cos(3-x)$.

82. a) $\frac{\sin 24^0 + \sin 6^0}{\cos 24^0 + \cos 6^0}$, б) $\frac{\cos 5^0 - \cos 25^0}{\sin 5^0 + \sin 25^0}$.

83. a) $tg110^0 - tg20^0$, б) $\frac{tg75^0 + tg25^0}{tg75^0 - tg25^0}$.

84. a) $ctgx + tgx$, б) $ctg(x+45^0) + tg(x+45^0)$.

85. a) $\sin^2 50^0 - \sin^2 30^0$, б) $\sin^2 70^0 - \cos^2 50^0$

86. a) $\sin^2\frac{\alpha}{2} - \sin^2\frac{\beta}{2}$, б) $1 \pm tg\alpha$.

87. a) $cosec\alpha - ctg\alpha$, б) $tg^2\alpha - tg^2\beta$.

88. a) $\sin 10^0 + \sin 50^0 - \sin 70^0$, б) $\cos 20^0 + \cos 100^0 + \cos 140^0$.

89. a) $\sin 47^0 + \sin 61^0 - \sin 11^0 - \sin 25^0$, б) $\cos 47^0 - \cos 61^0 - \cos 11^0 + \cos 25^0$

90. $\sin 87^0 - \sin 59^0 - \sin 93^0 + \sin 61^0$.

91. $tg30^0 + tg40^0 + tg50^0 + tg60^0$.

92. a) $\frac{1}{2} + \cos\alpha$, б) $\sqrt{3} + tg\alpha$.

93. a) $\sqrt{3}-2\sin\alpha$, б) $3+\sqrt{3}tg\alpha$.

94. a) $3-4\sin^2\alpha$, б) $3-tg^2\alpha$.

95. a) $\sin\alpha+tg\alpha$, б) $tg\alpha-\sin\alpha$.

96. a) $1-\sin\alpha+\cos\alpha$, б) $1-\sin\alpha-\cos\alpha$.

97. a) $1-\cos(\frac{\pi}{4}-\alpha)\sin(\frac{\pi}{4}+\alpha)$, б) $1+\cos\alpha+\cos\beta+\cos(\alpha+\beta)$.

8-§. Darajani pasaytirishning umumiy formulalari

Teorema. $\cos^n\alpha$ va $\sin^m\alpha$, shuningdek, bu darajalarning har qanday ko'paytmasi trigonometrik ko'rinishda tasvirlanishi mumkin $(n\in N)$.

$\cos^n\alpha$, $\sin^m\alpha$, $\cos^n\alpha\cdot\sin^m\alpha$ darajalarni trigonometrik ko'phadga almashtirish formulalari darajani pasaytirish formulalari deb ataladi.

Umumiy holda, $\cos^n\alpha$ va $\sin^m\alpha$ darajalarni trigonometrik ko'phadga almashtirish uchun Muavr formulasidan foydalanish qulay.

Faraz qilaylik, $u=\cos\alpha+i\sin\alpha$, $v=\cos\alpha-i\sin\alpha$

U holda $\cos\alpha=\frac{u+v}{2}$, $\sin\alpha=\frac{u-v}{2}$, $u\cdot v=1$

Muavr formulasiga ko'ra:

$u^n=\cos n\alpha+i\sin n\alpha$, $v^n=\cos n\alpha-i\sin n\alpha$

$\cos n\alpha=\frac{1}{2}\left(u^n+v^n\right)$, $\sin n\alpha=\frac{1}{2}\left(u^n-v^n\right)$

Bulardan:

$$\cos^n\alpha=\left(\frac{u+v}{2}\right)^n=\frac{1}{2^n}\left(u^n+C_n^1u^{n-1}v+C_n^2u^{n-2}v^2+...+C_n^2u^2v^{n-2}+C_n^1uv^{n-1}+v^n\right)=$$

$$=\frac{1}{2^n}\left(\left(u^n+v^n\right)++C_n^1uv\left(u^{n-2}+v^{n-2}\right)+C_n^2u^2v^2\left(u^{n-1}+v^{n-1}\right)+...\right)=$$

$$=\frac{1}{2^{n-1}}\left(\cos n\alpha+C_n^1\cos\left(n-2\right)\alpha+C_n^2\cos\left(n-4\right)\alpha+...\right)$$

Shunga o‘xshash,

$$\sin^n\alpha=\frac{1}{2^n i^n}(v-u)^n=\frac{1}{2^n i^n}\left(u^n-C_n^1u^{n-1}v+C_n^2u^{n-2}v^2-...\right)$$

Bu yerda, agar $n=2k$ bo‘lsa,

$$\sin^{2k}\alpha=\frac{(-1)^k}{2^{2k}}\left(\left(u^{2k}+v^{2k}\right)-C_{2k}^1uv\left(u^{2k-2}+v^{2k-2}\right)+...\right)=$$

$$=\frac{(-1)^k}{2^{2k}}\left(\cos 2k\alpha-C_{2k}^1\cos(2k-2)\alpha+C_{2k}^2\cos(2k-4)\alpha-...\right)$$

$n=2k+1$ bo‘lganda:

$$\sin^{2k+1}\alpha=\frac{(-1)^k}{2^{2k+1}i}\left(\left(u^{2k+1}-v^{2k+1}\right)-C_{2k+1}^1uv\left(u^{2k-1}-v^{2k-1}\right)+C_{2k+1}^1u^2v^2\left(u^{2k-1}-v^{2k-1}\right)-...\right)=$$

$$=\frac{(-1)^k}{2^{2k}}\left(\sin(2k+1)\alpha-C_{2k+1}^1\sin(2k-1)\alpha+C_{2k+1}^2\sin(2k-3)\alpha+...\right)$$

Ikkilangan burchakning trigonometrik funksiyalari

Yuqorida keltirilib chiqarilgan $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$, $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$ va $tg(\alpha+\beta)=\dfrac{tg\alpha+tg\beta}{1-tg\alpha tg\beta}$ formulalarda $\beta=\alpha$ deb olsak, u holda $\sin(\alpha+\alpha)=\sin\alpha\cos\alpha+\cos\alpha\sin\alpha=2\cos\alpha\sin\alpha$;

$\cos(\alpha+\alpha)=\cos\alpha\cos\alpha-\sin\alpha\sin\alpha=\cos^2\alpha\text{-}\sin^2\alpha$;

$$tg(\alpha+\alpha)=\frac{tg\alpha+tg\alpha}{1-tg\alpha tg\alpha}=\frac{2tg\alpha}{1-tg^2\alpha};\quad \left(\alpha\neq\frac{\pi}{4}+\frac{\pi n}{2},\quad \alpha\neq\frac{\pi}{2}+\pi k,\right),$$

$$ctg2\alpha=\frac{1-tg^2\alpha}{2tg\alpha}=\frac{ctg^2\alpha-1}{2ctg\alpha};\quad \left(\alpha\neq\frac{\pi n}{2}\right),$$

$$ctg2\alpha=\frac{1}{tg2\alpha}=\frac{1-tg^2\alpha}{2tg\alpha}=\frac{1}{2}(ctg\alpha-tg\alpha);$$

$\sec 2\alpha=\dfrac{1}{\cos 2\alpha}=\dfrac{1+tg^2\alpha}{1-tg^2\alpha}$; $\cos ec2\alpha=\dfrac{1}{\sin 2\alpha}=\dfrac{1+tg^2\alpha}{2tg\alpha}$ larga ega bo‘lamiz.

Demak,

$\sin 2\alpha = 2\cos\alpha\sin\alpha$	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$$tg2\alpha = \frac{2tg\alpha}{1-tg^2\alpha}$$

$ces2\alpha = \frac{1+tg^2\alpha}{1-tg^2\alpha}$	$\csc 2\alpha = \frac{1+tg^2\alpha}{2tg\alpha}$

fomulalari hosil bo‘ladi. Bular ikkilangan burchaklar trigonometrik funksiyalarini burchakning o‘zini trigonometrik funksiyalari orqali ifoda etadi. Shularga o‘xshash:

$\sin 3\alpha = \sin(2\alpha+\alpha) = \cos 2\alpha \sin\alpha + \cos\alpha \sin 2\alpha = 3\sin\alpha - 4\sin^3\alpha$;

$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$;

$tg3\alpha = tg(\alpha+2\alpha) = \frac{tg\alpha + tg2\alpha}{1-tg\alpha tg2\alpha}$.

9-§. Ratsionallashtiruvchi almashtirish va yordamchi burchak kiritish

Ta’rif. Agar $\cos\alpha$ va $\sin\alpha$ ga nisbatan ratsional bo‘lgan $R(\cos\alpha, \sin\alpha)$ funksiya oraliq argument deb ataluvchi $t = f(\alpha)$ ga nisbatan murakkab funksiya ko‘rinishida tasvirlangan bo‘lsa, ya’ni $R(\cos\alpha, \sin\alpha) = R_1(t)$ va $R_1(t)$ - ratsional funksiya bo‘lsa, u holda $t = f(\alpha)$ oraliq argument yordamida kiritilgan almashtirish **ratsionallashtiruvchi almashtirish** deyiladi.

Teorema. $\cos\alpha$ va $\sin\alpha$ ga nisbatan ratsional bo‘lgan har qanday $R(\cos\alpha, \sin\alpha)$ funksiya uchun $t = tg\frac{\alpha}{2}$ almashtirish **ratsionallashtiruvchi almashtirish** bo‘ladi.

Isboti. $\cos\alpha = \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2} = \frac{\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}} = \frac{1 - tg^2\frac{\alpha}{2}}{1 + tg^2\frac{\alpha}{2}}$;

$$\sin\alpha = \frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}} = \frac{2tg\frac{\alpha}{2}}{1 + tg^2\frac{\alpha}{2}}$$

$t = tg\frac{\alpha}{2}$ almashtirishdan so‘ng $\cos\alpha$ va $\sin\alpha$ lar t argumentning ratsional funksiyalari ko‘rinishida tasvirlanadi, ya’ni:

$$\cos\alpha = \frac{1 - t^2}{1 + t^2}, \ \sin\alpha = \frac{2t}{1 + t^2}. \ (*)$$

Bu formulalar α ning qabul qilishi mumkin bo‘lgan barcha $\left(\alpha \neq (2k+1)\pi\right)$ qiymatlarida ma’noga ega.

$R(\cos\alpha, \sin\alpha)$ ifodada $\cos\alpha$ va $\sin\alpha$ ni (*) orqali almashtirib, t argumentga nisbatan ifodaga ega bo‘lamiz.

Teorema. Agar a va b sonlardan hech bo‘lmaganda bittasi noldan farqli bo‘lsa, u holda $0 \leq \phi \leq 2\pi$ $\left(yoki \ -\pi < \phi \leq \pi\right)$ oraliqda

$$\begin{cases} a = r\cos\phi \\ b = r\sin\phi \\ r = \sqrt{a^2 + b^2} \end{cases}$$

sistemani qanoatlantiruvchi φ ning qiymati yagona bo‘ladi.

Misol. $a\sin\alpha + b\cos\alpha$ yig‘indini ko‘paytmaga almashtiring. Bu erda a va b – istalgan haqiqiy sonlar $a \cdot b \neq 0$.

Yechilishi. Yuqoridagi teoremaga asosan,

$$a\sin\alpha + b\cos\alpha = r\left(\sin\alpha\cos\phi + \cos\alpha\sin\phi\right) = r\sin\left(\alpha + \phi\right)$$

Bu yerda $r=\sqrt{a^2+b^2}$ va $\phi = arctg\frac{b}{a}$.

Mustaqil yechish uchun misollar

Quyidagi tengliklarni isbotlang

98. a) $\frac{\sin\alpha-\sin\beta}{\cos\alpha-\cos\beta}=-ctg\frac{\alpha+\beta}{2}$, б) $\frac{\sin\alpha+\sin\beta}{\cos\alpha+\cos\beta}=tg\frac{\alpha+\beta}{2}$.

99. a) $\frac{\sin\alpha+\sin\beta}{\cos\alpha-\cos\beta}=ctg\frac{-\alpha+\beta}{2}$, б) $\frac{\sin\alpha-\sin\beta}{\cos\alpha+\cos\beta}=tg\frac{\alpha-\beta}{2}$.

100. a) $\frac{\sin\alpha+\sin 3\alpha}{\cos\alpha+\cos 3\alpha}=tg2\alpha$, б) $\frac{\sin\frac{2}{3}\alpha+\sin\frac{4}{3}\beta}{\cos\frac{2}{3}\alpha+\cos\frac{4}{3}\beta}=tg\alpha$.

101. a) $\frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\sin(\alpha+\beta)-\sin(\alpha-\beta)}=\frac{tg\alpha}{tg\beta}$, б) $\frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos(\alpha-\beta)-\cos(\alpha+\beta)}=ctg\alpha\cdot ctg\beta$.

102. a) $\frac{tg\alpha+tg\beta}{tg\alpha-tg\beta}-\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}=0$, б) $1+ctg\alpha=\frac{\sin(45^0+\alpha)}{\sin 45^0\sin\alpha}$.

103. a) $1-tg^2\alpha=\frac{\cos 2\alpha}{\cos^2\alpha}$, б) $(\sin\alpha+\sin\beta)^2+(\cos\alpha+\cos\beta)^2=4\cos^2\frac{\alpha-\beta}{2}$.

104. a) $ctg^2\alpha-ctg^2\beta=\frac{\sin(\alpha+\beta)\cdot\sin(\beta-\alpha)}{\sin^2\alpha\cdot\sin^2\beta}$, б) $tg^2(45^0-\alpha)=\frac{1-\sin 2\alpha}{1+\sin 2\alpha}$.

105. a) $\frac{tg^2\alpha+tg^2\beta}{tg^2\alpha-tg^2\beta}=\frac{\sin^2\alpha\cos^2\beta+\cos^2\alpha\sin^2\beta}{\sin(\alpha+\beta)\sin(\alpha-\beta)}$, б) $\frac{1-\sin\alpha}{1+\sin\alpha}=tg^2(\frac{\pi}{4}-\frac{\alpha}{2})$.

106. a) $\frac{1+ctg\alpha}{1-ctg\alpha}=ctg(\alpha-45^0)$, б) $tg67^0 30'-tg22^0 30'=2$.

107. a) $1+2\cos\alpha=4\cos(30^0+\frac{\alpha}{2})\cos(30^0-\frac{\alpha}{2})$, б) $\frac{\cos 3\alpha-\cos\alpha}{2\sin\alpha\sin 2\alpha}=-1$.

108. a) $1-4\sin^2\alpha=4\sin(30^0+\alpha)\sin(30^0-\alpha)$, б) $\frac{2\sin\alpha-\sin 2\alpha}{2\sin\alpha+\sin 2\alpha}=tg^2\frac{\alpha}{2}$.

109. a) $\frac{1+\sin 2\alpha}{1-\sin 2\alpha}=ctg^2(\frac{\pi}{4}-\frac{\alpha}{2})$, б) $\frac{\sin^2\alpha-\sin^2\beta}{\sin\alpha\cos\beta-\sin\beta\cos\alpha}=\sin(\alpha+\beta)$.

110. a) $\frac{\sin\alpha-2\sin2\alpha+\sin3\alpha}{\cos\alpha-2\cos2\alpha+\cos3\alpha}=tg2\alpha$, б) $\frac{tg^2\alpha-\sin^2\alpha}{ctg^2\alpha-\cos^2\alpha}=tg^6\alpha$.

111. a) $\frac{\sin\alpha+\sin3\alpha+\sin5\alpha}{\cos\alpha+\cos3\alpha+\cos5\alpha}=tg3\alpha$, б) $\frac{tg(45^0+\alpha)+tg(-45^0+\alpha)}{ctg(\alpha+45^0)+ctg(45^0-\alpha)}=\sin2\alpha$.

112. a) $\sec\alpha+tg\alpha=tg(\frac{\pi}{4}+\frac{\alpha}{2})$,

б) $\sin^2(\alpha+\beta)-\cos^2\alpha-\cos^2\beta=2\cos\alpha\cos\beta\cos(\alpha+\beta)$.

113. a) $\sec^2\alpha-\cos^2\alpha-2\sin^2\alpha=\sin^2\alpha\cdot tg^2\alpha$,

б) $\frac{1+\cos\alpha+\cos2\alpha+\cos3\alpha}{\cos\alpha+2\cos^2\alpha-1}=2\cos\alpha$.

114. a) $\frac{\sin^2 2\alpha-4\sin^2\alpha}{\sin^2 2\alpha-4+4\sin^2\alpha}=tg^4\alpha$,

б) $\cos^2\frac{\alpha}{2}(1+\sec\frac{\alpha}{2}+tg\frac{\alpha}{2})(1-\sec\frac{\alpha}{2}+tg\frac{\alpha}{2})=\sin\alpha$.

115. a) $\cos^2\alpha-\sin^2\beta=\cos(\alpha-\beta)\cdot\cos(\alpha+\beta)$,

б) $1-\cos^2\alpha-\cos^2\beta=\cos(-\alpha+\beta)\cdot\cos(\alpha+\beta)$.

116. a) $tg\frac{x}{2}-\cos x-\sin x=-2\sqrt{2}\sin\frac{x}{2}\cdot ctgx\cdot\cos\left(\frac{x}{2}-45^0\right)$,

б) $\frac{\sin x-\cos x}{tgx+1}=\cos x\cdot tg\left(x-45^0\right)$.

117. a) $\frac{1-\sin^6\alpha-\cos^6\alpha}{1-\sin^4\alpha-\cos^4\alpha}=\frac{3}{2}$, б) $tg10^0tg20^0+tg60^0tg20^0+tg10^0tg60^0=1$

10-§. Ba'zi trigonometrik yig'indi va ko'paytmalarni hisoblash

Argumentlari arifmetik progressiya tashkil qiluvchi kosinuslar va sinuslar yig'indilarini hisoblash.

$$\cos\alpha+\cos(a+h)+\cos(a+2h)+...+\cos(a+nh)=\sum_{k=0}^{n}\cos(a+kh).\quad(*)$$

Bu yig'indilarini hisoblash uchun quyidagi ayniyatlardan foydalanamiz.

$$2\sin\frac{h}{2}\cos\alpha=\sin\left(a+\frac{h}{2}\right)-\sin\left(a-\frac{h}{2}\right),$$

$$2\sin\frac{h}{2}\cos(\alpha+h)=\sin\left(a+\frac{3h}{2}\right)-\sin\left(a+\frac{h}{2}\right),$$

....

$$2\sin\frac{h}{2}\cos(\alpha+nh)=\sin\left(a+\frac{2n+1}{2}h\right)-\sin\left(a+\frac{2n+1}{2}h\right).$$

Bularni hadma-had qo'shib, so'ng $\sin\frac{h}{2}$ ga bo'lsak:

$$\sum_{k=0}^{n}\cos(a+kh)=\frac{1}{2\sin\frac{h}{2}}\left(\sin\left(a+\frac{2n+1}{2}h\right)-\sin\left(a-\frac{h}{2}\right)\right).$$

Yoki bundan:

$$\sum_{k=0}^{n}\cos(a+kh)=\frac{\cos\left(a+\frac{n}{2}\right)\sin\frac{n+1}{2}h}{\sin\frac{h}{2}}. \qquad (1)$$

Agar bu formulada a ni $\frac{\pi}{2}-a$ ga va h ni –h ga almashtirsak:

$$\sum_{k=0}^{n}\sin(a+kh)=\frac{\sin\left(a+\frac{n}{2}\right)\sin\frac{n+1}{2}h}{\sin\frac{h}{2}} \qquad (2)$$

(*) formulada $a=0$ desak,

$$1+\cos h+\cos 2h+...+\cos nh=\frac{\cos\frac{n}{2}h\sin\frac{n+1}{2}h}{\sin\frac{h}{2}}, \qquad (3)$$

$$\sin h+\sin 2h+...+\sin nh=\frac{\sin\frac{n}{2}h\sin\frac{n+1}{2}h}{\sin\frac{h}{2}}, \qquad (4)$$

(4) ni (3) ga hadma-had bo‘lsak:

$$\frac{\sin h+\sin 2h+...+\sin nh}{1+\cos h+\cos 2h+...+\cos nh}=tg\frac{n}{2}h. \qquad (5)$$

(1) va (2) formulalarda $a=x$, $h=2x$ va n ni n-1 ga almashtirsak:

$$\sin x+\sin 2x+...+\sin(2n-1)x=\sum_{k=1}^{n}\sin(2k-1)x=$$

$$=\frac{\sin(x+(n-1)x)\sin nx}{\sin x}=\frac{\sin^2 nx}{\sin x} \qquad (6)$$

$$\cos x+\cos 3x+...+\cos(2n-1)x=\sum_{k=1}^{n}\cos(2k-1)x=$$

$$=\frac{\cos nx\sin nx}{\sin x}=\frac{\sin 2nx}{2\sin x} \qquad (7).$$

Endi shu mavzularga doir misollar ko‘rib chiqamiz.

1-misol. Tenglikni to‘g‘riligini tekshiring.

$$\sin 47^0+\sin 61^0-\sin 11^0-\sin 25^0=\cos 7^0$$

Yechilishi. Qo‘shish formulalaridan foydalanib, quyidagi natijaga ega bo‘lamiz.

$$(\sin 47^0+\sin 61^0)-(\sin 11^0+\sin 25^0)=2\sin 54^0\cos 7^0-2\sin 18^0\cos 7^0=$$

$$=2\cos 7^0\cdot(\sin 54^0-\sin 18^0)=2\cos 7^0\cdot 2\sin 18^0\cos 36^0$$

Hosil bo‘lgan ifodani $\cos 18^0$ga ham ko‘paytirib, ham bo‘lamiz, hamda $2\sin 18^0\cos 18^0=\cos 36^0$ekanligidan va $\sin 72^0=\cos 18^0$ dan foydalanib,

$$2\cos 7^0\cdot\frac{\sin 36^0\cos 36^0}{\cos 18^0}=\cos 7^0\cdot\frac{\sin 72^0}{\cos 18^0}=\cos 7^0\cdot\frac{\cos 18^0}{\cos 18^0}=\cos 7^0.$$

Demak, berilgan tenglik to‘g‘ri ekan.

2-misol. Ayniyatni isbotlang.

$$4\sin\alpha\sin\left(60^0-\alpha\right)\sin\left(60^0+\alpha\right)=\sin3\alpha$$

Isboti. Ayniyatning chap tomoniga mos ravishda (6), (7), (8) formulalarni qo'llaymiz.

$$4\sin\alpha\sin\left(60^0-\alpha\right)\sin\left(60^0+\alpha\right)=4\sin\alpha\cdot\frac{1}{2}\times$$

$$\times\left(\cos\left(60^0-\alpha-60^0-\alpha\right)-\cos\left(60^0-\alpha+60^0+\alpha\right)\right)=2\sin\left(\cos\left(-2\alpha\right)-\cos110^0\right)=$$

$$=2\sin\left(\cos2\alpha+\frac{1}{2}\right)=2\sin\alpha\cos2\alpha+\sin\alpha=2\cdot\frac{1}{2}\left(\sin\left(\alpha-2\alpha\right)+\sin\left(\alpha+2\alpha\right)\right)+$$

$$+\sin\alpha=-\sin\alpha+\sin3\alpha+\sin\alpha=\sin3\alpha$$

Shunday qilib, ayniyat α ning ixtiyoriy haqiqiy qiymatlarida bajariladi.

3-misol. Ayniyatni isbotlang.

$$\frac{\cos^2\alpha}{ct\alpha-tg\alpha}=\frac{1}{4}\sin4\alpha$$

Isboti. Tenglikni chap qismida quyidagi almashtirishlar bajaramiz.

$$\frac{\cos^2 2\alpha}{ctg\alpha-tg\alpha}=\frac{\cos^2 2\alpha}{\frac{\cos\alpha}{\sin\alpha}-\frac{\sin\alpha}{\cos\alpha}}=\frac{\cos^2 2\alpha}{\frac{\cos^2\alpha-\sin^2\alpha}{\sin\alpha\cdot\cos\alpha}}=\frac{\cos^2 2\alpha\sin\alpha\cdot\cos\alpha}{\cos^2\alpha-\sin^2\alpha}$$

Ikkilangan burchakning trigonometrik funksiyalari formulalariga ko'ra, ya'ni

$\frac{1}{2}\sin2\alpha=2\cos\alpha\sin\alpha$, $\cos2\alpha=\cos^2\alpha\text{-}\sin^2\alpha$ bo'ladi.

Shunga ko'ra,

$$\frac{\cos^2 2\alpha\sin\alpha\cdot\cos\alpha}{\cos^2\alpha-\sin^2\alpha}=\frac{\cos^2 2\alpha\cdot\sin2\alpha}{2\cos2\alpha}=\frac{1}{2}\sin2\alpha\cdot\cos2\alpha=\frac{1}{4}\sin4\alpha$$

Demak ayniyat isbotlandi.

4-misol. Ayniyatni isbotlang.

$$\cos^2 x+\cos^2\left(\frac{2\pi}{3}+x\right)+\cos^2\left(\frac{2\pi}{3}-x\right)=\frac{3}{2}$$

Isboti. Bu ayniyatni isbotlash uchun trigonometrik ifodalarni darajasini pasaytirish formulasidan foydalanamiz, ya'ni $\cos^2 x=\frac{1+\cos 2x}{2}$ ekanligidan foydalanamiz.

$$\cos^2 x+\cos^2\left(\frac{2\pi}{3}+x\right)+\cos^2\left(\frac{2\pi}{3}-x\right)=\frac{1+\cos 2x}{2}+\frac{1+\cos\left(\frac{4\pi}{3}+2x\right)}{2}+$$

$$+\frac{1+\cos\left(\frac{4\pi}{3}-2x\right)}{2}=\frac{3}{2}+\frac{\cos 2x}{2}+\frac{\cos\left(\frac{4\pi}{3}+2x\right)+\cos\left(\frac{4\pi}{3}-2x\right)}{2}=$$

$$=\frac{3}{2}+\frac{\cos 2x}{2}+\cos\frac{4\pi}{3}\cdot\cos 2x=\frac{3}{2}+\frac{\cos 2x}{2}-\frac{\cos 2x}{2}=\frac{3}{2}$$

Demak, ayniyat isbotlandi.

Mustaqil yechish uchun misollar

118. Agar $\sin\alpha=-\frac{12}{13}$ va $\frac{3\pi}{2}<\alpha<2\pi$ bo'lsa, $\cos\left(\frac{\pi}{3}-\alpha\right)$ ni hisoblang.

119. Agar $\sin\alpha=\frac{1}{\sqrt{5}}$, $\sin\beta=\frac{1}{\sqrt{10}}$ va $0<\alpha<\frac{\pi}{2}$, $0<\beta<\frac{\pi}{2}$ bo'lsa, $\cos(\alpha+\beta)$ ni hisoblang.

120. Hisoblang. a) $tg^2\alpha+ctg^2\alpha$ b) $tg^3\alpha+ctg^3\alpha$ v) agar $tg\alpha+ctg\alpha=3$ bo'lsa, $tg\alpha-ctg\alpha$ ni hisoblang.

121. Agar $\cos\alpha=0{,}8$ va $0<\alpha<\frac{\pi}{2}$ bo'lsa, $\sin\frac{\alpha}{2}$, $\cos\frac{\alpha}{2}$, $tg\frac{\alpha}{2}$ ni hisoblang.

122. Agar $tg\alpha=3\frac{3}{7}$ va $\pi<\alpha<\frac{3\pi}{2}$ bo'lsa, $\sin\frac{\alpha}{2}$, $\cos\frac{\alpha}{2}$, $tg\frac{\alpha}{2}$ ni hisoblang.

123. Agar $\sin\alpha+\cos\alpha=\frac{\sqrt{7}}{2}$ va $0<\alpha<\frac{\pi}{2}$ bo'lsa, $tg\frac{\alpha}{2}$ ni hisoblang.

124. Agar $\sin\alpha+\cos\alpha=t$ bo'lsa, quyidagi ifodalarni t parameter orqali ifodalang. *a*) $\sin\alpha\cdot\cos\alpha$ b) $\sin\alpha-\cos\alpha$ v) $\sin^4\alpha+\cos^4\alpha$.

125. Agar $tg\alpha+ctg\alpha=t$ bo'lsa, a) $tg^2\alpha+ctg^2\alpha$ b) $tg^3\alpha+ctg^3\alpha$ ni hisoblang.

Hisoblang

126. $\sin^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\sin^2\frac{5\pi}{8}+\cos^2\frac{7\pi}{8}$.

127. $tg255^0-tg195^0$.

128. agar $ctg\alpha=\frac{2}{3}$ bo'lsa, $\cos(\frac{7\pi}{4}+2\alpha)$ ni toping.

129. agar $tg\alpha=0{,}2$ bo'lsa, $\frac{2}{3+4\cos2\alpha}$ ni toping,

130. agar $tg\alpha=-2$ bo'lsa, $1+5\sin2\alpha-3\sec2\alpha$ ni toping.

131. $\sin^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{4}$.

132. $\cos\frac{3\pi}{8}\cdot\cos\frac{6\pi}{8}$.

133. $\frac{\cos67^0\cdot\cos7^0-\cos83^0\cdot\cos27^0}{\cos128^0\cdot\cos68^0-\cos38^0\cdot\cos22^0}-tg164^0$.

134. Agar $\sin\alpha+\sin\beta=-\frac{21}{65},\cos\alpha+\cos\beta=-\frac{27}{65}$ va $\frac{5\pi}{2}<\alpha<3\pi$, $-\frac{\pi}{2}<\alpha<0$ bo'lsa, $\sin\frac{\alpha+\beta}{2}$ va $\cos\frac{\alpha+\beta}{2}$ ni hisoblang.

135. Agar $tg\alpha=3$ bo'lsa, $\frac{2\sin\alpha-3\cos2\alpha}{4\sin2\alpha+5\cos2\alpha}$ ni toping.

Biror uchburchakning ichki burchaklari α, β va γ ekanligini bilgan holda quyidagi tengliklarni isbot qiling.

136. $\sin2\alpha+\sin2\beta+\sin2\gamma=4\sin\alpha\sin\beta\sin\gamma$.

137. $\sin 4\alpha + \sin 4\beta + \sin 4\gamma = -4\sin 2\alpha \sin 2\beta \sin 2\gamma$.

138. $tg\frac{\alpha}{2}tg\frac{\beta}{2} + tg\frac{\beta}{2}tg\frac{\gamma}{2} + tg\frac{\alpha}{2}tg\frac{\gamma}{2} = 1$.

139. $\sin^2\alpha + \sin^2\beta + \sin^2\gamma - 2\cos\alpha\cos\beta\cos\gamma = 2$.

140. $\sin 2n\alpha + \sin 2n\beta + \sin 2n\gamma = (-1)^{n+1}\sin n\alpha \sin n\beta \sin n\gamma$ (bu erda n butun son).

Trigonometrik tengsizliklarni isbotlang

141. Agar $\alpha < 0 < \frac{\pi}{2}$ bo'lsa, u holda $\sin\alpha < \alpha < tg\alpha$ isbotlang.

142. Agar $\alpha + \beta + \gamma = \pi$ va $\alpha, \beta, \gamma > 0$ bo'lsa, u holda $\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2} \leq \frac{1}{8}$ isbotlang.

143. Agar $\alpha \neq \frac{\pi n}{2}$, $n \in z$ bo'lsa, u holda $\frac{\sin\alpha + tg\alpha}{\cos\alpha + ctg\alpha} \geq 0$.

144. Agar $\alpha + \beta = \frac{\pi}{3}$ va $\alpha > 0$, $\beta > 0$ u holda $tg\alpha \cdot tg\beta \leq 1$ isbotlang.

145. Agar $0 < \alpha < \beta = \frac{\pi}{2}$ bo'lsa, u holda $\left(1 + \frac{1}{\sin\alpha}\right)\left(1 + \frac{1}{\cos\alpha}\right) \geq 3 + 2\sqrt{2}$ ni isbotlang.

146. $4\sin 3x + 5 \geq 4\cos 2x + 5\sin x$ ni isbotlang.

147. $0 < \alpha < \beta = \frac{\pi}{2}$ bo'lsa, u holda $\cos\alpha + \alpha\sin\alpha > 1$ ni isbotlang.

148. Agar $0 \leq \alpha < \frac{\pi}{2}$ bo'lsa, u holda $\sin(\cos\alpha) < \cos(\sin\alpha)$ ni isbotlang.

Tengsizliklarni isbotlang

149. Agar $0 < \alpha \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ bo'lsa, u holda $\sqrt{\cos\alpha} < \sqrt{2}\cos\frac{\alpha}{2}$.

150. Agar $\alpha \in \left[0;\frac{\pi}{2}\right]$ va $\beta \in \left[0;\frac{\pi}{2}\right]$ bo'lsa, u holda

a) $\sin\frac{\alpha+\beta}{2} \geq \frac{\sin\alpha + \sin\beta}{2}$.

b) $\cos\frac{\alpha+\beta}{2} \geq \frac{\cos\alpha + \cos\beta}{2}$ ekanligini isbotlang.

151. $\sin^4\alpha + \cos^4\alpha \geq \frac{1}{2}$.

152. $\cos^7\alpha - 8\cos^7\alpha + 7 \geq 0$.

153. $-\sqrt{2} \leq \sin\alpha + \cos\alpha \leq \sqrt{2}$.

154. $\sin 2\alpha + \cos 2\alpha \cos 4\alpha \cos 8\alpha \cos 16\alpha \leq \frac{1}{16}$.

155. $\cos(\alpha+\beta)\cos(\alpha-\beta) \leq \cos^2\alpha$.

156. Agar $\alpha \in \left[0;\frac{\pi}{2}\right]$ u holda $\cos\alpha + 2\sin\alpha > 1$ ekanligini isbotlang.

11-§. Teskari trigonometrik funksiyalar va ularning asosiy xossalari, grafigi

Teskari trigonometrik funksiyalar ta'riflariga to'xtalib o'tamiz.

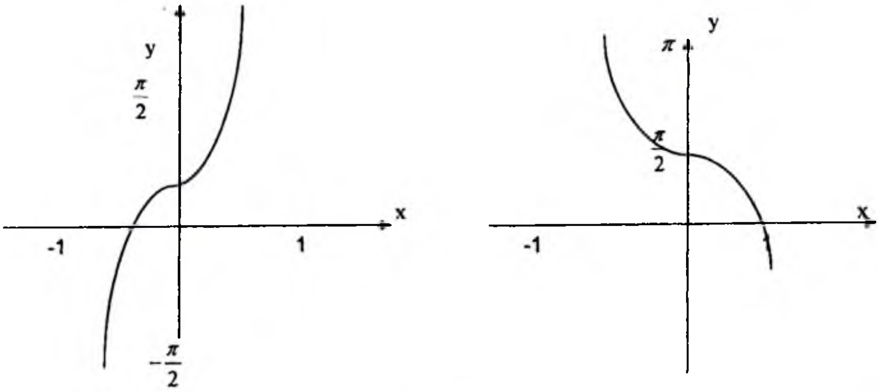
1. $y = \arcsin x$ funksiya $[-1;1]$ kesmada aniqlangan, teskari funksiya $x = \sin y$ esa , $y \in \left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ da aniqlangan.

Ta'rif. Berilgan x sonning arksinusi deb, $y \in \left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ da aniqlangan va sinusi x ga teng bo'lgan $y = \arcsin x$ funksiyaga aytiladi. (1-chizma).

Har qanday $x \in [-1;1]$ uchun, $-\frac{\pi}{2} \leq \arcsin x \leq \frac{\pi}{2}$

$$\sin(\arcsin x) = x$$

2. $y = \arccos x$ funksiya $[-1;1]$ kesmada aniqlangan. $x = \cos y$ teskari



1-chizma.

funktsiya esa $y \in [0;\pi]$ da aniqlangan.

Ta'rif. Berilgan x sonning arkkosinusi deb, kosinusi x ga teng va $0 \le y \le \pi$ da aniqlangan $y = \arccos x$ funksiyaga aytiladi. (1-chizma)

Har qanday $x \in [-1;1]$ uchun $0 \le \arccos x \le \pi$, $\cos(\arccos x) = x$

3. $y = \operatorname{arctg} x$ funksiya $(-\infty;+\infty)$ da aniqlangan $x = tgy$ funksiya esa, $y \in \left(-\frac{\pi}{2};\frac{\pi}{2}\right)$

Ta'rif. $x \in (-\infty;+\infty)$ sonning arktangensi deb, tangensi x ga teng va $-\frac{\pi}{2} < y < \frac{\pi}{2}$ da aniqlangan $y = \operatorname{arctg} x$ funksiyaga aytiladi. Har qanday x uchun:

$-\frac{\pi}{2} < \operatorname{arctg} x < \frac{\pi}{2}$ bo'ladi.

$tg(\operatorname{arctg} x) = x$

4. $y=\mathrm{arc}tgx$ funksiya $(-\infty;+\infty)$ da aniqlangan $x=ctgy$ funksiya esa $y\in(0;\pi)$ da aniqlangan.

Ta'rif. x sonning arkkotangensi deb, kotangensi x ga teng va $0<y<\pi$ da aniqlangan $y=\mathrm{arc}ctgx$ funksiyaga aytiladi. Har qanday x uchun :

$$0<arcctgx<\pi$$
$$ctg(arcctgx)=x$$

bo'ladi.

Teskari trigonometrik funksiyalar orasidagi ba'zi bir ayniyatlarni eslatib o'tamiz.

1) $\arcsin(-x)=-\arcsin x \quad (-1\le x\le 1)$

2) $\arccos(-x)=\pi-\arccos x \quad (-1\le x\le 1)$

3) $\arcsin x+\arccos x=\frac{\pi}{2} \quad (-1\le x\le 1)$

4) $arctg(-x)=-arctgx$

5) $arcctg(-x)=\pi-arcctgx$

6) $arctgx+arcctgx=\frac{\pi}{2}$

1-misol. $\sin(\mathrm{arc}tgx)$ ifodani soddalashtiring.

Yechilishi. $\mathrm{arc}tgx=y$ deb belgilash kiritamiz.

$-\frac{\pi}{2}<y<\frac{\pi}{2}$ va $tgy=x$ bo'ladi. $1+tg^2y=\frac{1}{\cos^2 y}$ dan $\cos y$ ni topamiz.

$-\frac{\pi}{2}<y<\frac{\pi}{2}$ oraliqda kosinus faqat musbat qiymatlar qabul qiladi, bundan

$\cos y=\frac{1}{\sqrt{1+tg^2y}}$ endi $\sin y$ ni topamiz. $\sin y=tgy\cdot\cos y=\frac{tgy}{\sqrt{1+tg^2y}}$. Bundan

$\sin(arctgx)=\frac{x}{\sqrt{1+x^2}}$.

2-misol. Agar, $-1<y<1$ bo'lsa, $arc\sin x=arctg\frac{x}{\sqrt{1-x^2}}$ ni isbotlang.

Isboti. Tenglikning ikkala tarafini ham tangenslaymiz:

$$tg(arc\sin x)=tg\left(arctg\frac{x}{\sqrt{1-x^2}}\right)$$

1) $tg(arc\sin x)=\dfrac{\sin(arc\sin x)}{\cos(arc\sin x)}=\dfrac{x}{\sqrt{1-x^2}}$

2) $tg\left(arctg\dfrac{x}{\sqrt{1-x^2}}\right)=\dfrac{x}{\sqrt{1-x^2}}$

Bundan ko'rinadiki, tenglikning ikkala qismini tangensi ham teng.

Demak, $-1<y<1$ da $arc\sin x$ ham $arctg\dfrac{x}{\sqrt{1-x^2}}$ ham $\left(-\dfrac{\pi}{2};\dfrac{\pi}{2}\right)$ intervalda yotadi.

Mustaqil yechish uchun misollar

157. $cos\left(arctg\sqrt{2}+\arcsin\dfrac{\sqrt{3}}{2}\right)$.

158. $tg\left(\arcsin\left(-\dfrac{12}{13}\right)+\arcsin\dfrac{3}{5}\right)$.

159. $\sin\left(\dfrac{1}{2}\arcsin\alpha\right)\cos\left(\dfrac{1}{2}\arcsin\alpha\right)$, $|a|\leq 1$.

160. $tg\left(5\mathrm{arc}tg\dfrac{\sqrt{3}}{3}-\dfrac{1}{2}\arcsin\dfrac{\sqrt{3}}{2}\right)$.

161. a) $\arccos\left(\cos\dfrac{\pi}{4}\right)$ b) $\arccos\left(-\cos\dfrac{3\pi}{4}\right)$.

162. $ctg\left(\dfrac{1}{2}\mathrm{arc}cos\left(-\dfrac{4}{7}\right)\right)$.

163. $\sin\left(2\left(\arcsin\dfrac{\sqrt{5}}{3}-\arccos\dfrac{\sqrt{5}}{3}\right)\right)$.

164. $\cos\left(\arccos\frac{5}{13}-\arcsin\frac{3}{5}\right).$

165. $\sin\left(arctg\frac{8}{15}-\arcsin\frac{8}{17}\right).$

166. $\cos\left(2arctg\left(-\frac{1}{3}\right)\right).$

Tenglik to'g'riligini tekshiring

167. $arctg\frac{2}{3}+arctg\frac{1}{5}=\frac{\pi}{4}.$

168. $\arcsin\frac{4}{5}-\arccos\frac{1}{\sqrt{5}}=arctg\frac{1}{2}.$

169. $arctg\frac{1}{3}+arctg\frac{1}{4}+arctg\frac{2}{9}=\frac{\pi}{4}.$

170. $\arcsin\frac{4}{5}+\arcsin\frac{5}{13}+\arcsin\frac{16}{65}=\frac{\pi}{2}.$

171. $\arcsin\frac{5}{13}+\arcsin\frac{12}{13}=\frac{\pi}{2}.$

172. $\arcsin\frac{3}{5}-arctg\frac{3}{5}=arctg\frac{27}{11}.$

173. $\arccos\frac{1}{2}+\arccos\frac{1}{7}=\arccos\left(-\frac{11}{14}\right).$

174. $\arccos\frac{\sqrt{2}}{2}+arctg\frac{\sqrt{2}}{2}=arctg\left(\sqrt{2}+1\right)^2.$

175. $2arctg\sqrt{\frac{1-x}{1+x}}=\arccos x,\quad -1<x\leq 1.$

176. $\arcsin(x-1)+2arctg\frac{\sqrt{2x-x^2}}{x}=\frac{\pi}{2},\quad 0<x<2.$

Ayniyatlarni isbotlang

177. $2\arccos\sqrt{\frac{1+x}{2}}=\arccos x,\ -1\leq x\leq 1.$

178. $2arctgx+\arcsin\frac{2x}{1+x^2}=\pi,\ x>1.$

179. $\mathrm{arc}tgx + arctgy = \mathrm{arc}tg\frac{x+y}{1-xy}$.

180. $\sin(\arcsin x) = x \quad |x| \le 1$.

181. $\cos(\arcsin x) = \sqrt{1-x^2}, \quad |x| \le 1$.

182. $tg(\arcsin x) = \frac{x}{\sqrt{1-x^2}}, \ |x| < 1$.

183. $ctg(\arcsin x) = x, \quad |x| \le 1 \ x \ne 0$.

184. $arc\cos x = \mathrm{arctg}\frac{x}{\sqrt{1-x^2}}, \quad |x| < 1$.

185. $\arcsin x = \begin{cases} \arccos\sqrt{1-x^2} & 0 < x < 1 \\ -\arccos\sqrt{1-x^2} & -1 < x < 0. \end{cases}$

186. $\mathrm{arc}cos\, x = \begin{cases} \arcsin\sqrt{1-x^2}, & 0 \le x \le 1 \\ \pi - \arcsin\sqrt{1-x^2}, & -1 \le x \le 0. \end{cases}$

187. $\mathrm{arc}tgx = \begin{cases} \arccos\frac{1}{\sqrt{1+x^2}}, & x \ge 0 \\ -\arccos\frac{1}{\sqrt{1+x^2}}, & x \le 0. \end{cases}$

188. $\arccos x = \begin{cases} \mathrm{arc}tg\frac{\sqrt{1-x^2}}{x}, & 0 \le x \le 1 \\ \pi + \mathrm{arc}tg\frac{\sqrt{1-x^2}}{x}, & -1 \le x \le 0. \end{cases}$

189. $\mathrm{arc}tgx = \begin{cases} \mathrm{arcc}tg\frac{1}{x}, & x > 0 \\ \mathrm{arcc}tg\frac{1}{x} - \pi, & x < 0. \end{cases}$

190. $\mathrm{arc}ctgx = \begin{cases} \mathrm{arc}tg\frac{1}{x}, & x > 0 \\ \pi + \mathrm{arc}tg\frac{1}{x}, & x < 0. \end{cases}$

V BOB. TRIGONOMETRIK TENGLAMALAR VA TENGSIZLIKLAR. TRIGONOMETRIK TENGLAMALAR VA TENGSIZLIKLAR SISTEMASI

1-§. Trigonometrik tenglamalar va ularni yechish usullari

Trigonometrik tenglamalar yechishda asosan, quyidagi eng sodda trigonometrik tenglamalarni yechishga keltiriladi:

a) $\sin x = a$
b) $\cos x = a$
c) $\mathrm{tg} x = a$
d) $\mathrm{ctg} x = a$

Bu tenglamalarni yechishni quyidagi jadvalda keltiramiz.

Tenglama	yechimi
$\sin x = a$ $\lvert a\rvert \le 1$ bo'lsa	$x = (-1)^n \arcsin a + \pi n,$ $n \in Z$
$\cos x = a$ $\lvert a\rvert \le 1$ bo'lsa	$x = \pm\arccos a + 2\pi n,$ $n \in Z$
$tgx = a$	$x = arctga + \pi n,$ $n \in Z$
$ctgx = a$	$x = arcctga + \pi n,$ $n \in Z$

Sodda trigonometrik tenglamalarni yechishda umumiy formulalardan foydalanilmagan holda ba'zi bir holatlardagi qiymatlarini eslatib o'tamiz.

1. $\sin x = 0.$ $\quad x = \pi n, \; n \in Z$
2. $\sin x = 1$ $\quad x = \frac{\pi}{2} + 2\pi n, \; n \in Z$
3. $\sin x = -1$ $\quad x = -\frac{\pi}{2} + 2\pi n, \; n \in Z$
4. $\cos x = 0$ $\quad x = \frac{\pi}{2} + \pi n, \; n \in Z$
5. $\cos x = 1$ $\quad x = 2\pi n, \; n \in Z$
6. $\cos x = -1$ $\quad x = \pi + 2\pi n, \; n \in Z$
7. $tgx = 0$ $\quad x = \pi n, \; n \in Z$

Umuman, trigonometrik tenglamalarni yechishning ko'pgina yo'llari va usullari mavjudki, ularni umumiy nazariya doirasida to'laligicha ko'zda tutilishining iloji yo'q.

O'zgaruvchilarni almashtirish, ko'paytuvchilarga ajratish, yordamchi argument kiritish ratsionallovchi o'rniga qo'yishlarni qo'llash, trigonometrik funksiya ko'paytmasini ularning yig'indisiga almashtirish va buning aksi, simmetrik ko'phadlar xossalaridan foydalanish, baholashdan va tengsizliklardan foydalanish, elementar funksiyalarning xossalaridan foydalanishga asoslangan usullar va hokazo. Trigonometrik tenglamalarni yechishda qo'llaniladigan ayrim usullarni misollarda ko'rsatamiz.

1-misol. Tenglamani yeching.

$\sin(\pi \cos 4x) = 1$

Yechilishi. (2) formulaga asosan,

$\pi \cos 4x = \frac{\pi}{2} + 2\pi n, \; n \in Z$ bundan

$\cos 4x = \frac{1}{2} + 2n, \; n \in Z$ lekin, $|\cos 4x| \le 1$. Shunga ko'ra $n = 0$.

Bundan $\cos 4x = 0{,}5$ va yechimi

$x=\pm\frac{\pi}{12}+\frac{2\pi k}{2},\ n\in Z$

2-misol. Tenglamani yeching.

$tgx\cdot\sin 2x=0$

Yechilishi. Bu tenglamani yechish uchun ulardan aqalli bittasi nolga teng bo'lishi kerak, shunga ko'ra,

$tgx=0$ yoki $\sin 2x=0$ bundan,

$tgx=0 \qquad\qquad \sin 2x=0$

$x=\pi n,\quad n\in Z \qquad x=\frac{\pi}{2}k,\quad k\in Z$

Bu yechimlarni umumlashtirib $x=\pi n,\quad n\in Z$ ga kelamiz.

3-misol. Tenglamani yeching.

$5\sin^2 x+3\sin x\cos x-3\cos^2 x=2$

Yechilishi. Bu bir jinsli tenglamaga keluvchi tenglama hisoblanadi. Tenglikning o'ng tomonini $2\cdot1=2\cdot(\sin^2 x+\cos^2 x)$ ko'rinishda yozib olamiz. U holda:

$5\sin^2 x+3\sin x\cos x-3\cos^2 x=2(\sin^2 x+\cos^2 x)\Rightarrow$

$\Rightarrow 3\sin^2 x+3\sin x\cos x-5\cos^2 x=0$

Bir jinsli tenglamaga kelamiz.

Tenglikni ikkala tarafini $\cos^2 x$ ga bo'lib, $3tg^2x+3tgx-5=0$ ga ega bo'lamiz. $tgx=t$ deb belgilash kiritib $3t^2+3t-5=0$, $t_{1,2}=\frac{-3\pm\sqrt{69}}{6}$ bundan

$tgx=\frac{-3\pm\sqrt{69}}{6}$

$x=arctg\frac{-3\pm\sqrt{69}}{6}+\pi n,\quad n\in Z$

4-misol. Ushbu $4\sin x+5\cos x=6$ tenglamani yeching.

Yechilishi. Bu ko'rinishdagi tenglamalarni yechish uchun, yordamchi argument kiritish uslidan foydalaniladi. Bunday tenglamalarga $\sin x$ va $\cos x$ ga nisbatan chiziqli tenglamalar kiradi, ya'ni:

$$a\sin x + b\cos x = c \qquad (a^2+b^2 \neq 0)$$

Bu tenglama uning barcha hadlari $\sqrt{a^2+b^2}$ ga bo'lingandan keyin

$\sin(x+\varphi)=\frac{c}{\sqrt{a^2+b^2}}$ ko'rinishga keladi, unda φ-yordamchi burchak,

uning uchun $\sin\varphi=\frac{b}{\sqrt{a^2+b^2}}$; $\cos\varphi=\frac{a}{\sqrt{a^2+b^2}}$. Tenglama $|c|\leq\sqrt{a^2+b^2}$ bo'lgan holdagina yechimga ega bo'ladi.

$\sqrt{4^2+5^2}=\sqrt{41}$ bo'lgani uchun berilgan tenglama.

$\frac{4}{\sqrt{41}}\sin x+\frac{5}{\sqrt{41}}\cos x=\frac{6}{\sqrt{41}}$ tenglamaga teng kuchli. $\left(\frac{4}{\sqrt{41}}\right)^2+\left(\frac{5}{\sqrt{41}}\right)^2=1$ bo'lganidan shunday φ burchak mavjudki, uning uchun

$$\cos\varphi=\frac{4}{\sqrt{41}};\quad \sin\varphi=\frac{5}{\sqrt{41}} \qquad \left(\mathrm{tg}\varphi=\frac{5}{4}\right)$$

Bundan,

$$\sin x\cos\varphi+\cos x\sin\varphi=\frac{6}{\sqrt{41}}$$

$$\sin(x+\varphi)=\frac{6}{\sqrt{41}} \qquad \left(\varphi=\arcsin\frac{5}{\sqrt{41}}\right)$$

$\left|\frac{6}{\sqrt{41}}\right|<1$ bo'lgani uchun, $x=-\arcsin\frac{6}{\sqrt{41}}+(-1)^k\arcsin\frac{6}{\sqrt{41}}+\pi k,\quad k\in Z$.

5-misol. $\sin x+7\cos x=5$ tenglamani yeching. Bu tenglamani yuqoridagi usuldan foydalanib ham yechish mumkin, hamda buni universal o'rniga qo'yish usuli bilan ham yechish mumkin.

Yechilishi. $\sin x$ va $\cos x$ larni $\mathrm{tg}\frac{x}{2}$ orqali ifodalaymiz, ya'ni $\sin x=\frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}}$, $\cos x=\frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}}$ formulalar yordami bilan $\mathrm{tg}\frac{x}{2}$ ga nisbatan algebraik tenglamaga keltirilishi mumkin. $\mathrm{tg}\frac{x}{2}=t$ belgilash kiritib quyidagi ratsional tenglamaga keltiramiz.

$\frac{2t}{1+t^2}+\frac{7(1-t^2)}{1+t^2}=5$ bu tenglamani yechib, $t_1=\frac{1}{2}$, $t_2=-\frac{1}{3}$ ga ega bo'lamiz.

$\mathrm{tg}\frac{x}{2}=\frac{1}{2}$, $\mathrm{tg}\frac{x}{2}=-\frac{1}{3}$ tenglamalardan $x=2arctg\frac{1}{2}+2\pi k$, $x=2arctg\left(-\frac{1}{3}\right)+2\pi n$

, bundan, $x=2arctg\frac{1}{2}+2\pi k$, $x=-2arctg\frac{1}{3}+2\pi n$, $n,k\in Z$.

6-misol. $8\cos^4 x+\cos 2x+4\sin^2 2x=3$ tenglamani yeching.

Yechilishi. Bu tenglamani yechish uchun quyidagi darajani pasaytirish formulalaridan foydalanamiz.

$\sin^2 x=\frac{1-\cos 2x}{2}$, $\cos^2 x=\frac{1+\cos 2x}{2}$

$\cos 2x=t$ deb belgilash kiritsak, $\sin^2 x=1-\cos^2 x$ ekanligidan foydalanib, $8\cdot\left(\frac{1+t}{2}\right)^2+t+4(1-t^2)=3$ tenglamani yechamiz. Bu tenglamada ba'zi bir ixchamlashtirishlar o'tkazgandan so'ng $2t^2-5t-3=0$ tenglama hosil bo'ladi. $|t|\le 1$ ekanini e'tiborga olib, $t=-\frac{1}{2}$ ni topamiz, ya'ni

$\cos 2x=-\frac{1}{2}$, $x=\pm\frac{\pi}{3}+\pi n$, $n\in z$.

Javob: $\pm\frac{\pi}{3}+\pi n$, $n\in Z$.

7-misol. Tenglamani yeching.

$$2\cos^2 6x+2\cos^2 8x+2\cos^2 10x=3$$

Yechilishi. $\cos^2 x=\frac{1+\cos 2x}{2}$ formuladan foydalanamiz.

$$1+\cos 12x++1+\cos 16x+1+\cos 20x=3$$
$$(\cos 12x+\cos 20x)+\cos 16x=0$$
$$2\cos 16x\cdot\cos 4x+\cos 16x=0$$
$$\cos 16x(2\cos 4x+1)=0$$

bundan, $\cos 16x=0$ yoki $2\cos 4x+1=0$.

$$\cos 16x=0 \quad x=\frac{\pi}{32}+\frac{\pi}{16}n, \quad n\in Z$$
$$\cos 4x=-\frac{1}{2} \quad x=\pm\frac{\pi}{6}+\frac{\pi}{2}k, \quad k\in Z$$

8-misol. Tenglamani yeching.

$$\mathrm{tg}x\cdot tg2x=tgx+tg2x$$

Yechilishi. Ta'rifdan va yig'indini ko'paytmaga almashtirish formulalaridan foydalanib, quyidagilarga ega bo'lamiz:

$$\frac{\sin x}{\cos x}\cdot\frac{\sin 2x}{\cos 2x}=\frac{\sin(x+2x)}{\cos x\cdot\cos 2x}$$
$$\sin x\cdot\sin 2x=\sin x\cdot\cos 2x+\sin 2x\cdot\cos x$$
$$\sin x(\sin 2x-\cos 2x-2\cos^2 x)=0$$

Ko'paytma nolga teng bo'lishi uchun ulardan aqalli bittasi nolga teng bo'lishi kerak. Shunga asosan,

$$\sin x=0; \quad \sin 2x-\cos 2x-2\cos^2 x=0$$

Birinchi tenglamadan $x=k\pi$, $k\in Z$. Ikkinchisi esa quyidagi ko'rinishga keladi.

$\sin^2 x-2\sin x\cos x-3\cos^2 x=0$ $/\cos^2 x$ bo'lib, $\mathrm{tg}^2 x+2tgx-3=0$ ko'rinishga keltiramiz. Bu tenglamani yechib, $\mathrm{tg}x=-3$. $\mathrm{tg}x=1$.

Bundan, $x=arctg(-3)+\pi n,\ x=\frac{\pi}{4}+\pi m,\ n,m\in Z$. Tenglamaning shartidagi $\cos x\neq 0,\ \cos 2x\neq 0$ edi. Shuning uchun $x=\frac{\pi}{4}+\pi m$, chet ildizlar sirasiga kiradi.

Javob: $x=\pi k,\ x=arctg(-3)+\pi n,\ k,n\in N$.

9-misol. Tenglamani yeching.

$$\sin^4 x+\cos^4 x=\frac{7}{8}$$

Yechilishi. 1-usul.

$$\left(\sin^2 x+\cos^2 x\right)^2-2\sin^2 x\cdot\cos^2 x=\frac{7}{8}$$

$$1\text{-}2\sin^2 x\cdot\cos^2 x=\frac{7}{8}$$

$$1\text{-}\frac{1}{2}\sin^2 2x=\frac{7}{8}$$

$$\sin^2 2x=\frac{1}{4}$$

$$\sin 2x=\mp\frac{1}{2}$$

$$2x=\pm\frac{\pi}{6}+\pi k,$$

$$x=\pm\frac{\pi}{12}+\frac{\pi k}{2},\quad k\in Z.$$

Javob: $\left\{\pm\frac{\pi}{12}+\frac{\pi k}{2},\quad k\in Z\right\}$

2-usul.

$$\left(\sin^2 x\right)^2+\left(\cos^2 x\right)^2=\frac{7}{8}$$

$$\left(\frac{1\text{-}\cos 2x}{2}\right)^2+\left(\frac{1+\cos 2x}{2}\right)^2=\frac{7}{8}$$

$$\cos^2 2x=\frac{3}{4}$$

$$\cos 2x=\pm\frac{\sqrt{3}}{2}$$

$$2k=\pm\frac{\pi}{6}+\pi k,$$

$$x=\pm\frac{\pi}{12}+\frac{\pi k}{2},\quad k\in Z$$

Mustaqil yechish uchun misollar

Trigonometrik tenglamalarni yeching

1. $\sin 2x=-\frac{\sqrt{3}}{2}$.

2. $ctg 2x=-\frac{\sqrt{3}}{3}$.

3. $\cos\frac{x}{2}=\frac{\sqrt{10}-1}{2}$.

4. $tg\left(x+15^0\right)+1=0$.

5. $\cos 2x+\cos x=0$.

6. $tg\left(x+\frac{\pi}{3}\right)+tg\left(\frac{\pi}{6}-x\right)=\frac{4\sqrt{3}}{3}$.

7. $4\sin^3 x+8\sin^2 x-\sin x+2=0$.

8. $2\sin^5 x+2\sin\frac{x}{2}\cos\frac{x}{2}=3\sin^3 x$.

9. $tg\left(x+45^0\right)=ctgx$.

10. $\sin^2\left(x+\frac{\pi}{4}\right)-\sin^2\left(\frac{\pi}{6}-x\right)-\sin\frac{\pi}{12}\cos\left(2x+\frac{\pi}{12}\right)=\sin x$.

11. $\sin^4 x+\cos^4 x=\frac{5}{8}$.

12. $19\sin^2 2x-30\sin 4x+25\cos^2 2x=25$.

13. $2\sin x-3\cos x=\frac{1}{2}$.

14. $4\sqrt{3}\cos(\pi+x)+12\sin x=\sqrt{3}\pi$.

15. $\sin x+ctg\frac{x}{2}=2$.

16. $4\sin\left(2x+20^0\right)-\cos\left(2x-20^0\right)=3$.

17. $\cos x-\cos 2x=\sin 3x$.

18. $1-\cos^2 2x=\sin 3x-\cos\left(\frac{\pi}{2}+x\right)$.

19. $\sin x+\cos x=1+\sin 2x$.

20. $\sin^2 x+\sin^2 2x=\sin^2 3x$.

21. $\left(\sin x-\frac{1}{2}\right)\left(\sin x+1\right)=0$.

22. $\left(\cos x - \frac{1}{\sqrt{2}}\right)\left(\sin x + \frac{1}{\sqrt{2}}\right) = 0$.

23. $\cos x \cdot tg3x = 0$.

24. $\frac{\sin x + \cos x}{\cos 2x} = 0$.

25. $\cos 5x = \sin 15x$.

26. $\sin^2 3x - 5\sin x + 4 = 0$.

27. $tg^3 x + tg^2 x - 3tgx - 3 = 0$.

28. $\cos^2 3x + 5\sin x - 4 = 0$.

29. $2\cos^2 x + \sin x - 2 = 0$.

30. $4\cos^2 2x + 8\cos^2 x - 7 = 0$.

31. $3(1 - \sin x) = 1 + \cos 2x$.

32. $\sin^2 x + 2\sin x \cos x = 2$.

33. $\sqrt{3}\sin 2x + \cos 2x = \sqrt{2}$.

34. $3\sin x - 5\sin\left(7x + \frac{\pi}{6}\right) = 4\cos 2x$.

35. $\sin 6x \cdot \cos 2x + \sin 2x = \sin 5x \cdot \cos 3x$.

36. $\sin x + \sin 2x + \sin 3x = 0$.

37. $\sin 5x + \sin x + 2\sin^2 x - 1 = 0$.

38. $3\sin 2x + \cos 2x = 2$.

39. $3tg\frac{x}{2} + ctgx = \frac{5}{\sin x}$.

40. $ctgx + \frac{\sin x}{1 + \cos x} = 2$.

41. $\left(2\sin^4\frac{x}{2} - 1\right)\frac{1}{\cos^4\frac{x}{2}} = 2$.

42. $2\sin^4 2x - \sin^2 2x \cdot \sin 4x = 2\sin^2 2x - \sin 4x$ bu yerda, $0 \le x \le \pi$.

43. $\dfrac{6\sin^2 x-6\sin x+\cos 2x+1}{12x^2-8\pi x+\pi^2}=0$.

44. $\sqrt{9-x^2}\left(\sin 2x-\cos 3x\right)=0$.

45. $4\cos^3\dfrac{x}{2}+3\sqrt{2}\sin x=8\cos\dfrac{x}{2}$.

46. $\dfrac{1-\sin x+...+(-1)^n\sin^n x+...}{1+\sin x+...+\sin^n x+...}=\dfrac{1-\cos 2x}{1+\cos 2x}$.

47. $\sin^5 x-\cos^5 x=\dfrac{1}{\cos x}-\dfrac{1}{\sin x}$.

48. $\sin^{10} x-\cos^{10} x=\dfrac{29}{10}\cos^4 2x$.

49. $\cos(3x-2)=-\dfrac{1}{2}$.

50. $\dfrac{\sin x}{1+\cos x}=-2-ctg(x+\pi)$.

51. $\dfrac{1+\cos(x-\pi)}{\sin x}=\sin\dfrac{x}{2}$.

52. $\cos 2x-tg^2(\pi-x)=\dfrac{1}{6\cos^2 x}$.

53. $\dfrac{7}{4}\cos\dfrac{x}{4}=\cos^3\dfrac{x}{4}+\sin\dfrac{x}{2}$.

54. $\sin^2 x+\cos 2x-2\sin^2\dfrac{\pi}{8}\cos x=\dfrac{1}{\sqrt{2}}$, bu yerda $-\pi<x<\dfrac{5\pi}{2}$.

55. $\dfrac{tg2x}{tgx}+\dfrac{tgx}{tg2x}=\dfrac{5}{2}$.

56. $\dfrac{\sin x\cdot tgx}{1-\cos x}=\dfrac{9}{4}$.

57. $\sqrt{1+\dfrac{1}{2}\sin x}=\cos x$.

58. $\sqrt{1+\cos x}-\sqrt{1-\cos x}=1+\sin x$.

59. $\sin x+\cos x=\sqrt{tgx+ctgx}$.

60. $4(\sin 3x \sin x)^2 - \sin 3x = 5$.

2-§. Teskari trigonometrik funksiyalar qatnashgan tenglamalar

Teskari trigonometrik tenglamalarni yechishda teng argumentlarda bir xil ismli trigonometrik funktsiyalarning qiymatlari ham teng bo'lishidan, ya'ni trigonometrik funksiyalarning bir qiymatlilik xossasidan foydalaniladi.

Albatta bunday tenglamalarni yechish jarayonida chet ildizlar paydo bo'lishi mumkin. Shuning uchun tenglama javoblarini albatta tekshirib ko'rish kerak.

1-misol. $\arccos\frac{x}{2} = 2arctg(x-1)$ tenglamani yeching.

Yechilishi. Tenglikni ikkala qismini kosinuslaymiz.

$$\cos\left(\arccos\frac{x}{2}\right) = \cos\left(2arctg(x-1)\right)$$

$$\cos 2\alpha = \frac{1-tg^2\alpha}{1+tg^2\alpha}$$

dan foydalanamiz.

$$\frac{x}{2} = \frac{1-tg^2\left(arctg(x-1)\right)}{1+tg^2\left(arctg(x-1)\right)}$$

$$\frac{x}{2} = \frac{1-(x-1)^2}{1+(x-1)^2}$$

bundan, $x = 0$, $\quad x = \pm\sqrt{2}$

Tekshirish:

1) $x=0$, $\arccos 0 \neq 2arctg(-1)$, shunga asosan , $\arccos 0=\frac{\pi}{2}$

$\text{arctg}(-1)=2\left(-\frac{\pi}{4}\right)=-\frac{\pi}{2}$

2) $x=-\sqrt{2}$, $\arccos\left(-\frac{\sqrt{2}}{2}\right)\neq 2arctg\left(-\sqrt{2}-1\right)$, bunda, $\arccos\left(-\frac{\sqrt{2}}{2}\right)>0$, lekin $2arctg\left(-\sqrt{2}-1\right)<0$

3) $x=\sqrt{2}$.

$\arccos\frac{\sqrt{2}}{2}=2arctg\left(\sqrt{2}-1\right)$

$\frac{\pi}{4}=2arctg\left(\sqrt{2}-1\right)$

Shunday qilib, $0<\sqrt{2}-1 \Rightarrow arctg\left(\sqrt{2}-1\right)\in\left(0;\frac{\pi}{4}\right)\Rightarrow 2arctg\left(\sqrt{2}-1\right)\in\left(0;\frac{\pi}{2}\right)$ va $x=\sqrt{2}$, tenglamani yechimi bo‘ladi.

Javob: $\{\sqrt{2}\}$.

2-misol. $\arcsin 2x+\arcsin x=\frac{\pi}{3}$ tenglamani yeching.

Yechilishi. Tenglikni ikkala qismini kosinuslaymiz.

$\cos(\arcsin 2x+\arcsin x)=\cos\frac{\pi}{3}$ u holda $\sqrt{1-4x^2}\cdot\sqrt{1-x^2}-2x\cdot x=\frac{1}{2}$ ni yechib $7x^2=\frac{3}{4}\Rightarrow x_1=\frac{1}{2}\sqrt{\frac{3}{7}},\ x_2=-\frac{1}{2}\sqrt{\frac{3}{7}}$ ga ega bo‘lamiz.

Tekshirish:

$\alpha=\arcsin 2x_1+\arcsin x_1$ ni o‘rniga qo‘yamiz.

$\cos\left(\arcsin\sqrt{\frac{3}{7}}+\arcsin\left(\frac{1}{2}\sqrt{\frac{3}{7}}\right)\right)=\cos\alpha$ bundan,

$$\cos\alpha=\sqrt{1-\frac{3}{7}}\cdot\sqrt{1-\frac{1}{4}\cdot\frac{3}{7}}-\sqrt{\frac{3}{7}}\cdot\frac{1}{2}\cdot\sqrt{\frac{3}{7}}$$

Demak, $\cos\alpha=\frac{1}{2}$.

Shunday qilib, $0<\sqrt{\frac{3}{7}}<\frac{\sqrt{2}}{2}$ va $0<\frac{1}{2}\sqrt{\frac{3}{7}}<\frac{\sqrt{2}}{2}$,

$0<\arcsin\sqrt{\frac{3}{7}}<\frac{\pi}{4}$ va $0<\arcsin\frac{1}{2}\sqrt{\frac{3}{7}}<\frac{\pi}{4}$,

U holda $0<\arcsin\sqrt{\frac{3}{7}}+\arcsin\left(\frac{1}{2}\sqrt{\frac{3}{7}}\right)<\frac{\pi}{2}$, demak, α birinchi chorakka tegishli.

$0<\alpha<\frac{\pi}{2}$ da $\cos\alpha=\frac{1}{2}$ bundan $\alpha=\frac{\pi}{3}$. Demak, $x_1=\frac{1}{2}\sqrt{\frac{3}{7}}$ tenglamaning yechimi bo'ladi.

Endi, $x_2=-\frac{1}{2}\sqrt{\frac{3}{7}}$ ni tekshirib ko'ramiz.

$\beta=\arcsin 2x_2+\arcsin x_2$ ni o'rniga qo'yamiz.

$$\arcsin\left(-\sqrt{\frac{3}{7}}\right)+\arcsin\left(-\frac{1}{2}\sqrt{\frac{3}{7}}\right)=\beta$$

$-1<-\sqrt{\frac{3}{7}}<0$ va $-1<-\frac{1}{2}\sqrt{\frac{3}{7}}<0$ bo'lganligi uchun $-\pi<\arcsin\left(-\sqrt{\frac{3}{7}}\right)+\arcsin\left(-\frac{1}{2}\sqrt{\frac{3}{7}}\right)<0$ yoki $-\pi<\beta<0$.

Demak, $\beta\neq\frac{\pi}{3}$ bundan esa $x_2=-\frac{1}{2}\sqrt{\frac{3}{7}}$ chet ildizligi kelib chiqadi.

Javob: $x=\frac{1}{2}\sqrt{\frac{3}{7}}$

Mustaqil yechish uchun misollar

61. $\mathrm{ar}ctgx=-\frac{3}{2}$.

62. $\mathrm{ar}ctg(x+2)-\mathrm{ar}ctg(x+1)=\frac{\pi}{4}$.

63. $\arccos x+\arcsin x=\frac{11\pi}{6}$.

64. $\arccos(1-x)+2\arcsin x=0$.

65. $\arccos^2 x+\arccos x=0$.

66. $\arcsin 6x+\arcsin 6\sqrt{3}x+\frac{\pi}{2}=0$.

67. $2\arccos\left(-\frac{x}{2}\right)=\arccos(x+3)$.

68. $\cos(4\arccos x)=-\frac{1}{2}$.

69. $\arcsin x+\arcsin\sqrt{3}x=\frac{\pi}{2}$.

70. $2\arccos x=\arcsin\left(2x\sqrt{1-x^2}\right)$.

71. $\arcsin\left(tg\frac{\pi}{4}\right)-\arcsin\sqrt{\frac{3}{x}}-\frac{\pi}{6}=0$.

72. $\arcsin^2 x+5\arcsin x+2=0$.

73. $4\mathrm{arc}tgx-6\mathrm{arcc}tgx-\pi=0$.

74. $2\arcsin x-2\arccos 2x=0$.

75. $\arcsin\frac{2}{3\sqrt{x}}-2\arcsin\sqrt{1-x}-\arcsin\frac{1}{3}=0$.

76. $\arcsin(1-x)-2\arcsin x=\frac{\pi}{2}$.

77. $\arcsin\left(\frac{1}{5}\arccos x\right)=1$.

78. $\arcsin(2x+1)=\arccos x$.

79. $\arcsin x+\arcsin(x\sqrt{3})=\frac{\pi}{2}$.

80. $\operatorname{arc}tgx+\operatorname{arc}tg3x=\frac{\pi}{2}$.

81. $\operatorname{arc}tg(x+1)-\operatorname{arc}tg(x-1)=\frac{\pi}{2}$.

82. $\arcsin x-\arccos x=\arccos\frac{\sqrt{3}}{2}$.

83. $\arcsin x+\arcsin 2x=\frac{\pi}{2}$.

84. $\operatorname{arc}ctgx+\operatorname{arc}ctg(x+1)=\frac{\pi}{4}$.

85. $\arcsin x+\arcsin\frac{x}{2}=\frac{\pi}{4}$.

86. $\arccos\frac{x}{2}=2\operatorname{arc}tg(x-1)$.

87. $\operatorname{arc}tg2x+\operatorname{arc}tg3x=\frac{3\pi}{4}$.

88. $\arccos|x|=\arcsin 2x$.

89. $2\operatorname{arc}tg(\cos x)=\operatorname{arc}tg\left(\frac{2}{\sin x}\right)$.

3-§. Trigonometrik tenglamalar sistemasi

Trigonometrik tenglamalar sistemalarini yechishda algebraik tenglamalar sistemasini yechishda qo'llaniladigan usullardan foydalanamiz.

Quyida tenglamalar sistemasini yechishni o'ziga xos xususiyatlari bilan tanishamiz.

1-misol. $\begin{cases} x+y=\dfrac{3\pi}{4} \\ \cos^2 x+\sin^2 y=1 \end{cases}$ tenglamalar sistemasini yeching.

Yechilishi.

$$\begin{cases} y=\dfrac{3\pi}{4}-x \\ \cos^2 x+\sin^2\left(\dfrac{3\pi}{4}-x\right)=1 \end{cases} \Rightarrow \begin{cases} y=\dfrac{3\pi}{4}-x \\ \dfrac{1+\cos 2x}{2}+\dfrac{1-\cos\left(\dfrac{3\pi}{2}-2x\right)}{2}=1 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} y=\dfrac{3\pi}{4}-x \\ \cos 2x+\sin 2x=0 \end{cases} \Rightarrow \begin{cases} y=\dfrac{3\pi}{4}-x \\ tg2x=1 \end{cases} \Rightarrow \begin{cases} y=\dfrac{3\pi}{4}-x \\ 2x=-\dfrac{\pi}{4}+\pi k \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} y=\dfrac{3\pi}{4}-x \\ x=-\dfrac{\pi}{8}+\dfrac{\pi k}{2} \end{cases} \Rightarrow \begin{cases} x=-\dfrac{\pi}{8}+\dfrac{\pi k}{2} \\ y=\dfrac{3\pi}{4}+\dfrac{\pi}{8}-\dfrac{\pi k}{2} \end{cases} \Rightarrow \begin{cases} x=-\dfrac{\pi}{8}+\dfrac{\pi k}{2} \\ y=\dfrac{7\pi}{8}-\dfrac{\pi k}{2} \end{cases}$$

Javob: $\left\{(x,y)/x=-\frac{\pi}{8}+\frac{\pi k}{2},\ y=\frac{7\pi}{8}-\frac{\pi k}{2},\ k\in z\right.$

2-misol. Tenglamalar sistemasinini yeching.

$$\begin{cases} \sin^3 x=\dfrac{1}{2}\sin y \\ \cos^3 x=\dfrac{1}{2}\cos y \end{cases} \qquad (1)$$

Yechilishi. Bu tenglamalar sistemasini har ikkala tomonini kvadratga oshiramiz va hadma-had qo'shamiz. Natijada quyidagi yangi sistemaga kelamiz.

$$\begin{cases} \sin^6 x + cos^6 x = \frac{1}{4} \\ \sin^3 x = \frac{1}{2}\sin y \end{cases} \qquad (2)$$

$\sin^6 x + cos^6 x = \frac{1}{4}$ tenglamani yechamiz.

Ba'zi bir almashtirishlarni bajarib,

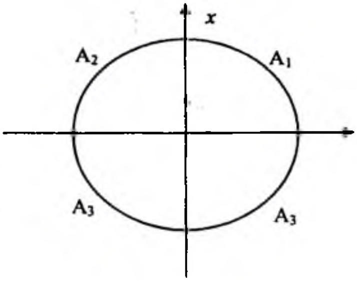
$$\left(\frac{1-\cos 2x}{2}\right)^3 + \left(\frac{1+\cos 2x}{2}\right)^3 = \frac{1}{2}, \ \cos 2x = 0, \ \ x = \frac{\pi}{4} + \frac{\pi}{2}k, \ \ k \in Z$$

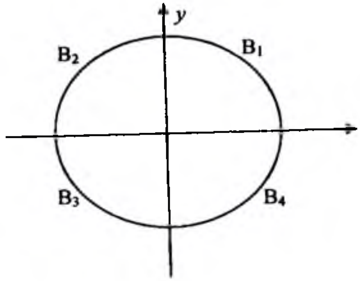
Shunday qilib, (2) tenglamalar sistemasini quyidagi tenglamalar sistemasiga keltiramiz.

$$\begin{cases} x = \frac{\pi}{4} + \frac{\pi}{2}k \\ \sin^3 x = \frac{1}{2}\sin y \end{cases} \ \text{Bundan,} \ \begin{cases} x = \frac{\pi}{4} + \frac{\pi}{2}k \\ \frac{1}{2}\sin y = \left(\pm\frac{\sqrt{2}}{2}\right)^3 \end{cases} \ \text{yoki} \ \begin{cases} x = \frac{\pi}{4} + \frac{\pi}{2}k \\ \sin y = \pm\frac{\sqrt{2}}{2} \end{cases}$$

$$\begin{cases} x = \frac{\pi}{4} + \frac{\pi}{2}k \\ y = \frac{\pi}{4} + \frac{\pi}{2}n \end{cases} \quad n, k \in Z \qquad (3)$$

(1) dan (2) ga o'tganda biz kvadratga oshirishni bajargandik, chet ildizlar paydo bo'lishi mumkin, albatta tekshirishlar o'tkazish kerak. (3) sistemadagi x, y ni qiymatlarini quyidagi ikkita aylanada tasvirlaymiz.





A_1 nuqtada $\sin x > 0, \cos x > 0$ u holda (3) sistemadan $\sin y > 0, \cos y > 0$ degan xulosaga kelamiz. B_1, B_2, B_3, B_4 nuqtalardan faqatgina B_1 nuqta musbat absissa va ordinataga ega. (A_1, B_1) (3) sistemaning geometrik yechimi ekan.

Demak, $\begin{cases} x_1 = \frac{\pi}{4} + 2\pi k \\ y_1 = \frac{\pi}{4} + 2\pi n \end{cases}$ (3) sistemani yechimi.

Sistemaning qolgan yechimlarini anologik tahlil qilib, mos ravishda (3) sistemaning $(A_2, B_2); (A_3, B_3); (A_4, B_4)$ geometrik yechimlarini topamiz.

$$\begin{cases} x_2 = \frac{3\pi}{4} + 2\pi k \\ y_2 = \frac{3\pi}{4} + 2\pi n \end{cases}; \quad \begin{cases} x_3 = \frac{5\pi}{4} + 2\pi k \\ y_3 = \frac{5\pi}{4} + 2\pi n \end{cases}; \quad \begin{cases} x_4 = \frac{7\pi}{4} + 2\pi k \\ y_4 = \frac{7\pi}{4} + 2\pi n, \end{cases} \qquad n, k \in Z$$

Shunday qilib, (3) tenglamalar sistemasini yechimi quyidagi birlashmalardan iborat.

$$\begin{cases} x_1 = \frac{\pi}{4} + 2\pi k \\ y_1 = \frac{\pi}{4} + 2\pi n \end{cases}; \quad \begin{cases} x_2 = \frac{3\pi}{4} + 2\pi k \\ y_2 = \frac{3\pi}{4} + 2\pi n \end{cases}; \quad \begin{cases} x_3 = \frac{5\pi}{4} + 2\pi k \\ y_3 = \frac{5\pi}{4} + 2\pi n \end{cases}; \quad \begin{cases} x_4 = \frac{7\pi}{4} + 2\pi k \\ y_4 = \frac{7\pi}{4} + 2\pi n \end{cases}$$

3-misol. Tenglamalar sistemasini yeching.

$$\begin{cases} \sin x+\sin y=a \\ \sin^2 x+\sin^2 y=b \end{cases}$$

Yechilishi. $\begin{cases} \sin x+\sin y=a \\ \sin^2 x+\sin^2 y=b \end{cases} \Rightarrow \begin{cases} \sin x=u \\ \sin y=v \\ u+v=a \\ u^2+v^2=b \end{cases} \Rightarrow \begin{cases} \sin x=u \\ \sin y=v \\ u+v=a \\ u^2+v^2=b \end{cases} \Rightarrow$

$$\begin{cases} \sin x=\dfrac{a+\sqrt{2b-a^2}}{2} \\ \sin y=\dfrac{a-\sqrt{2b-a^2}}{2} \\ b\ge\dfrac{a^2}{2} \\ \left|\dfrac{a\pm\sqrt{2b-a^2}}{2}\right|\le 1 \end{cases}$$

Agar $\left|\dfrac{a\pm\sqrt{2b-a^2}}{2}\right|\le 1$ shart bajarilsa,

$$\begin{cases} x=(-1)^n \arcsin\dfrac{a+\sqrt{2b-a^2}}{2}+\pi n \\ y=(-1)^k \arcsin\dfrac{a-\sqrt{2b-a^2}}{2}+\pi k. \end{cases} \vee \begin{cases} x=(-1)^n \arcsin\dfrac{a-\sqrt{2b-a^2}}{2}+\pi n \\ y=(-1)^k \arcsin\dfrac{a+\sqrt{2b-a^2}}{2}+\pi k. \end{cases} \quad k,n\in Z.$$

Mustaqil yechish uchun misollar

90. $\begin{cases} x+y=\dfrac{5\pi}{6} \\ \cos^2 x+\sin^2 y=\dfrac{1}{4} \end{cases}$

91. $\begin{cases} x-y=\dfrac{\pi}{3} \\ \cos x\sin x=\dfrac{1}{2} \end{cases}$

92. $\begin{cases} tgx+tgy=\dfrac{5\pi}{6} \\ ctgx+2ctgy=3 \end{cases}$

93. $\begin{cases} 2^{\sin x+\cos y}=1 \\ 16^{\cos^2 x+\sin^2 y}=4 \end{cases}$

94. $\begin{cases} tgx\cdot tgz=3 \\ tgy\cdot tgz=6 \\ x+y+y=\pi \end{cases}$

95. $\begin{cases} 2x-y=2\pi \\ 2\sin x+\sin y=0 \end{cases}$

96. $\begin{cases} x+y=-\dfrac{5\pi}{2} \\ tgx\cdot tgy=\dfrac{\sqrt{3}}{3} \end{cases}$

97. $\begin{cases} x-y=\dfrac{\pi}{4} \\ tgx+3tgy=0 \end{cases}$

98. $\begin{cases} \sqrt{2}\sin x=\sin y \\ \sqrt{2}\cos x=\sqrt{3}\cos y \end{cases}$

99. $\begin{cases} \cos x+\cos y=1 \\ \cos\dfrac{x}{2}+\cos\dfrac{y}{2}=\dfrac{\sqrt{2}}{2}+1 \end{cases}$

100. $\begin{cases} \cos(x-y)=2\cos(x+y) \\ \cos x\cos y=\dfrac{3}{2} \end{cases}$

101. $\begin{cases} x+y=\dfrac{\pi}{4} \\ \dfrac{\cos y\sin x}{\cos x\sin y}=\dfrac{1}{2} \end{cases}$

102. $\begin{cases} \sin x=\sin 2y \\ \cos x=\sin y \end{cases}$

103. $\begin{cases} \sin y=5\sin x \\ 3\cos x+\cos y=2 \end{cases}$

104. $\begin{cases} \cos^2 y+3\sin x\sin y=0 \\ 21\cos 2x-\cos 2y=10 \end{cases}$

105. $\begin{cases} x+y=135^0 \\ tgx-ctgx=2 \end{cases}$

106. $\begin{cases} y-x=30^0 \\ tgx+ctgx=\dfrac{2\sqrt{3}}{3} \end{cases}$

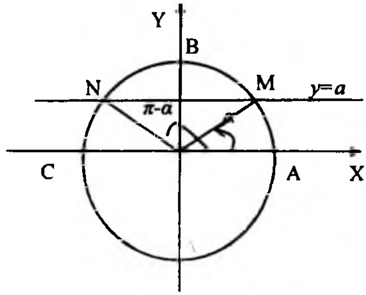
107. $\begin{cases} \cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2}=\dfrac{1}{2} \\ \cos x\cos y=\dfrac{1}{4} \end{cases}$

108. $\begin{cases} x+y=\dfrac{1}{3} \\ \sin\pi x+\sin\pi y=1 \end{cases}$

109. $\begin{cases} \sin\pi x\cdot\sin\pi y=\dfrac{3}{4} \\ tg\pi x\cdot tg\pi y=3 \end{cases}$

4-§. Trigonometrik tengsizliklar

Trigonometrik tengsizliklarni yechishda ham sodda trigonometrik tenglamalarni yechishga olib kelinadi va $\sin x > a$, $\sin x < a$, $\cos x > a$, $\cos x < a$, $\mathrm{tg} x > a$, $\mathrm{tg} x < a$, $\mathrm{ctg} x > a$, $\mathrm{ctg} x < a$ ko'rinishidagi tengsizliklarni yechishda koordinatali aylanadan yoki trigonometrik funksiyalarning grafiklaridan foydalanamiz.



1-misol. $\sin x > a$ tengsizlikni yeching.

Yechilishi. $\sin x > a$ ning yechimlar to'plami sinusoidaning $y = a$ dan yuqorida joylashgan bo'laklari bilan aniqlanadi. (1-chizma). Demak, $\arcsin a < x < \pi - \arcsin a$ qolgan yechimlari undan $2\pi k$, $n \in Z$ uzoqliklarda joylashgan oraliqlariga mos keladi, ya'ni $\arcsin a + 2\pi k < x < \pi - \arcsin a + n$, $n \in Z$

Shunga o'xshash, $\sin x < a$ ni yechadigan bo'lsak, $\pi - \arcsin a + 2\pi n < x < 2\pi + \arcsin a + 2\pi n$, $n \in Z$.

Xuddi shu kabi boshqa sodda trigonometrik tengsizliklarni yechimlarini topamiz.

2) $\cos x > a$

$\arccos a + 2\pi n < x < 2\pi - \arccos a + 2\pi n$, $n \in Z$

3) $\cos x < a$

$-\arccos a + 2\pi n < x < \arccos a + 2\pi n$, $n \in Z$

4) $\mathrm{tg} x > a$

$$\text{arc}tga+\pi n<x<\frac{\pi}{2}+\pi n,\quad n\in Z$$

5) $\text{tg}x<a$

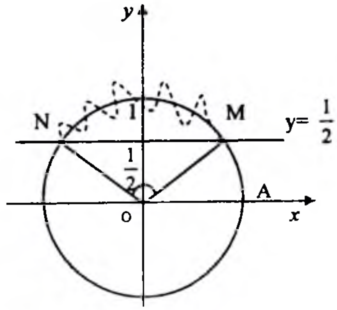
$$-\frac{\pi}{2}+\pi n<x<\text{arc}tga+\pi n,\quad n\in Z$$

6) $\text{ctg}x>a$

$$\pi k<x<arcctga+\pi k,\ k\in Z$$

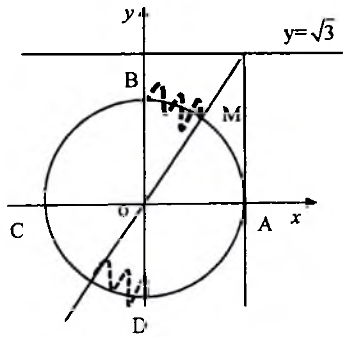
7) $\text{ctg}x<a$

$$arcctga+\pi k<x<\pi+\pi k,\quad k\in Z\ .$$



2-misol. $\sin x\geq\frac{1}{2}$ tengsizlikni yeching.

Yechilishi. Bu tengsizlikni yechishda birlik aylanadan foydalanamiz. Albatta bu yerda $y=\frac{1}{2}$ to'g'ri chiziqdan yuqori qismini olamiz.



Chunki $\frac{1}{2}$ dan katta qiymatlar $\smile MN$ yoyning yuqori qismida joylashadi, ya'ni

$$\frac{\pi}{6}+2\pi k\leq x\leq\frac{5\pi}{6}+2\pi k,\ k\in Z$$

3-misol. $tgx\geq\frac{1}{\sqrt{3}}$ tengsizlikni yeching.

Yechilishi. tgx $\quad x=\frac{\pi}{2}+\pi k$ qiymatda aniqlanmagan.

Bu chizmadagi B va D

nuqtalar. Bundan

ko'rinadiki,

$$arctg x \frac{1}{\sqrt{3}} + \pi k < x < \frac{\pi}{2} + \pi n, \quad n \in Z$$

4-misol. $tgx + ctgx > -3$ tengsizlikni yeching.

Yechilishi.

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} > -3 \Rightarrow \frac{\sin^2 x + \cos^2 x}{\sin x \cdot \cos x} > -3 \Rightarrow \frac{2}{\sin 2x} > -3 \Rightarrow \frac{\sin 2x}{2} = 3 > 0$$

$$\Rightarrow \frac{2 + 3\sin 2x}{\sin 2x} > 0 \Rightarrow (2 + 3\sin 2x)\sin 2x > 0$$

$$\sin 2x > 0 \vee \sin 2x < -\frac{2}{3}$$

$$\pi + \arcsin\frac{2}{3} + 2\pi k < 2x < 2\pi + \arcsin\frac{2}{3} + 2\pi k \vee 2\pi k < 2x < \pi + 2\pi k \Rightarrow$$

$$\Rightarrow \frac{\pi}{2} + \frac{1}{2}\arcsin\frac{2}{3} + \pi k < x < \pi - \frac{1}{2}\arcsin\frac{2}{3} + \pi k, \quad k \in Z$$

Trigonometrik tengsizliklarni yechishda intervallar usulini qo'llash yaxshi natija beradi. Albatta trigonometrik funksiyalarning davriylik xossasi bu intervallarni ularning umumiy davri qadar kamaytirib kelishga imkon beradi.

Mustaqil yechish uchun misollar

Trigonometrik tengsizliklar

110. $tgx \geq \sqrt{3}$.

111. $\cos(x+1) \leq -\frac{\sqrt{3}}{2}$.

112. $12\cos^2 x + 7\sin x - 13 < 0$.

113. $3\sin 2x - 1 > \sin x + \cos x$.

114. $tg^2x+ctg^2x<2$.

115. $\cos x\cos 3x<\cos 5x\cos 7x$.

116. $\cos^2 x+\sin x\cos x\geq 1$.

117. $\sqrt{\cos x-\sin x}\geq \sin x-\frac{1}{2}$, bu yerda $0\leq x\leq \pi$.

118. $\sin x>\frac{1}{3}$.

119. $\cos x>-\frac{1}{2}$.

120. $tgx\geq 2$.

121. $ctgx\geq -\sqrt{3}$.

122. $\sin(x-1)\leq -\frac{\sqrt{3}}{2}$.

123. $ctg(x-1)<-1$.

124. $\sin x+\cos x>-\sqrt{2}$.

125. $2tg2x\leq 3tgx$.

126. $1-\sin x<ctgx-\cos x$.

127. $\sin(\cos x)>0$.

128. $2\cos^2 x-\sin x+\sin 3x\leq 1$.

129. $\sin x\sin 2x-\cos x\cos 2x<\sin 6x$.

130. $\sqrt{3}\sin 2x+\cos 2x<1$.

131. $\sin 3x>\cos 3x$.

132. $\frac{\sin 3x-\cos 3x}{\sin 3x+\cos 3x}<0$.

133. $3\sin^2 x+\sin 2x-\cos^2 x\geq 2$.

134. $2\cos x\left(\cos x-\sqrt{8}tgx\right)<5$.

135. $ctgx+\frac{\sin x}{\cos x-2}\leq 2$.

136. $\sin x+\cos x>-\sqrt{2}\cos 2x$.

137. $\sin 2x\sin 3x-\cos 2x\cos 3x<\sin 10x$.

138. $4\sin x\sin 2x\sin 3x>\sin 4x$.

139. $\dfrac{\cos x+2\cos^2 x+\cos 3x}{\cos x+2\cos^2 x-1}>1$.

140. $\sqrt{3}\sec^2 x<4tgx$.

141. $\sin 4x+\cos 4xctg2x>1$.

142. $\dfrac{\cos^2 2x}{\cos^2 x}\geq 3tgx$.

143. $3\cos^2 x\sin x-\sin^2 x<\dfrac{1}{2}$.

144. $ctgx-tgx-2tg2x-4tg4x>8\sqrt{3}$.

TEST VARIANTLARIDAN NAMUNALAR

1-VARIANT

1. $\sqrt{x+2}-\sqrt{x-6}=2$ tenglamani yeching.

A) 7 B) -7 C) 1 D) 0

2. $P_{n+2}=30P_n$ tenglamani yeching.

A) 4 B) -7 C) -4; -7 D) 4; 7

3. $\frac{5!+6!}{4!}$ ni hisoblang.

A) 24 B) 38 C) 35 D) 30.

4. $\frac{\sqrt{x}+1}{x\sqrt{x}+x+\sqrt{x}}:\frac{1}{x^2-\sqrt{x}}$ ni soddalashtiring.

A) x+1 B) x-1 C) x+3 D) x-2

5. $x^3-3x-2=0$ ratsional ildizlarini toping.

A) -1; 2 B) 1; -2 C) 1; 2 D) -1; -2

6. $|-x+2|=2x+1$ tenglamani yeching.

A) $\frac{1}{3}$ B) 0 C) $\frac{2}{3}$ D) 3

7. p ning qanday butun qiymatlarida $2x^2+(2p-1)x-3=0$ va $6x^2-(2p-3)x-1=0$ tenglamalar umumiy ildizga ega?

A) 1 B) 2 C) 3 D) 1/2; 1/3

8. $\left(\frac{3}{7}\right)^{3x-7}=\left(\frac{7}{3}\right)^{7x-3}$ tenglamani yeching.

A) 1 B) 0 C) -1 D) $\frac{5}{2}$

9. $\log_3 x-2\log_{\frac{1}{3}} x=3$ tenglamani yeching.

A) -1 B) 3 C) 2 D) 4

10. $\log_3 x-2\log_{\frac{1}{3}} x=3$ tenglamani yeching.

A) -1 B) 3 C) 1 D) 2

11. $tg(\arcsin\frac{\sqrt{3}}{2}+arctg\sqrt{3})$ ni hisoblang.

A) $\sqrt{3}$ B) $-\frac{\sqrt{3}}{3}$ C) $-\sqrt{3}$ D) -1

12. $2\sin^2\frac{\pi}{4}+4\sin^2\frac{\pi}{3}+\cos^2\frac{\pi}{4}-\sin\frac{\pi}{6}$ ni qiymatini toping.

A) 4 B) 3 C) 2 D) 1

13. $\frac{\sin\alpha+\sin2\alpha-\sin(\pi+3\alpha)}{2\cos\alpha+1}$ soddalashtiring.

A) $\sin\alpha$ B) $\cos\alpha$ C) $\sin2\alpha$ D) $\cos2\alpha$

14. $y=3+\sin x$ qiymatlar sohasini toping.

A) [2; 4] B) (2; 4) C) [-1; 1] D) [0; 4]

15. $\sin3x+\cos3x=\sqrt{2}$ tenglamani yeching.

A) 15°+120°n B) 15°+360°n C) 45°+360°n D) 45°+120°n

2-VARIANT

1. $\sqrt{x+3}+\sqrt{3x-2}=7$ tenglamani yeching.

A) 6 B) 97 C) 7 D) Ø

2. $5C_n^3=C^4_{n+2}$ tenglamani yeching.

A) 3; 14 B) 3 C) 14 D) Ø

3. $\frac{3!-2!}{2!}$ ni hisoblang.

A) 9 B) 10 C) 2 D) 3

4. $\frac{(\sqrt[4]{m}+\sqrt[4]{n})^2+(\sqrt[4]{m}-\sqrt[4]{n})^2}{2(m-n)}:\frac{1}{\sqrt{m^3}-\sqrt{n^3}}-3\sqrt{mn}$ ni soddalashtiring.

A) $(\sqrt{m}-\sqrt{n})^2$ B) $\sqrt{n}-\sqrt{m}$ C) n^2-m^2 D) $\frac{\sqrt{n-m}}{2}$

5. $12x^4+1 = 9x^2+4x^3$ tenglamaning nechta ratsional ildizi bor?

A) 4 ta B) 3 ta C) 2 ta D) 1 ta

6. $|x-1|+|x-2|=1$ tenglamani yeching.

A) [1; 2] B) {1; 2} C) (1; 2) D) Ø

7. p ning qanday qiymatlarida $x^2+px-16=0$ tenglama ildizlarining nisbati -4 ga teng bo'ladi?

A) -4 va 4 B) -2 va 3 C) -6 ва 6 D) 1 va 7

8. $4^{x-2}-17\cdot 2^{x-4}+1=0$ tenglamani yeching.

A) 0; 4 B) 0 C) 4 D) 1

9. $\log_{16}x+\log_4 x\log_2 x=9$ tenglamani yeching.

A) 16 B) 4 C) 8 D) 1

10. $\log_8(5x-8)<\log_8(2x+7)$ tengsizlikni yeching.

A) (3,5; 5) B) (5; ∞) C) (1,6; 5) D) (-3,5; 1,6)

11. $2\arcsin(-\frac{1}{2})+\frac{1}{2}\arccos\frac{\sqrt{3}}{2}$ hisoblang.

A) $-\frac{\pi}{4}$ B) $\frac{5\pi}{4}$ C) $\frac{\pi}{4}$ D)

12. $arctg(tg(\frac{6\pi}{7}))$ ifodani qiymatini toping.

A) $\frac{\pi}{7}$ B) $-\frac{\pi}{7}$ C) $\frac{6\pi}{7}$ D) $-\frac{6\pi}{7}$

13. $\dfrac{4\cos^2 2\alpha-4\cos^2\alpha+3\sin^2\alpha}{4\cos^2(\frac{5\pi}{2}-\alpha)-\sin^2 2(\alpha-\pi)}$ ifodani soddalashtiring.

A) $\dfrac{3\cos\alpha}{4\sin^2\alpha}$ B) $\dfrac{8\cos 2\alpha+1}{2\cos 2\alpha-2}$ C) $4\cos 2\alpha-1$ D) $\dfrac{2\cos 2\alpha}{\sin^2\alpha}$

14. $y=tg\frac{2x}{3}-4ctg\frac{2x}{3}-2$ davrini ko'rsating.

A) 6π B) 2π C) 3π D) davriy emas

15. $\sin x\sin 7x=\sin 3x\sin 5x$ tenglamani yeching.

A) $\frac{n\pi}{4}$ B) $\frac{n\pi}{2}$ C) $n\pi$ D) $2n\pi$

3-VARIANT

1. $\sqrt{x}+\sqrt[4]{x}=12$ tenglamani yeching.

A) 81 B) 256 C) 3 D) - 4

2. $C_n^1+C^3_n=2C_n^2$ tenglamani yeching.

A) 2; 7 B) 2 C) 7 D) 0

3. $\mathbf{A_7^3 = 42x}$ tenglamani yeching

A) 4 B) 8 C) 4 D) 5

4. $\frac{x-y}{x^{\frac{3}{4}}+x^{\frac{1}{2}}y^{\frac{1}{4}}}\cdot\frac{x^{\frac{1}{2}}y^{\frac{1}{4}}+x^{\frac{1}{4}}y^{\frac{1}{2}}}{x^{\frac{1}{2}}+y^{\frac{1}{2}}}\cdot\frac{x^{\frac{1}{4}}y^{-\frac{1}{4}}}{x^{\frac{1}{2}}-2x^{\frac{1}{4}}y^{\frac{1}{4}}+y^{\frac{1}{2}}}$ ni soddalashtiring.

A) $\sqrt{x}$ B) $\sqrt[4]{x+y}$ C) $\frac{\sqrt[4]{x}+\sqrt[4]{y}}{\sqrt[4]{x}-\sqrt[4]{y}}$ D) 0

5. $x^5+x^3+x=0$ tenglamani nechta ratsional ildiz bor?

A) 1 ta B) 2 ta C) 3 ta D) 4 ta

6. $|-x^2+1|=-x^2+1$ tenglamani yeching.

A) [-1; 1] B) (-1; 1) C) {1; 2} D) Ø

7. p ning qanday qiymatlarida $(p-\frac{7}{4})x=3p$ tenglama yechimga ega emas?

A) 7/4 B) 3/4 C) 9/4 D) 3/2

8. $5^2\cdot5^4\cdot5^6\cdot...\cdot5^{2x}=0{,}04^{-28}$ tenglamani yeching.

A) 7 B) -8 C) -8; 7 D) 8

9. $\log_{16}x+\log_4 x\log_2 x=9$ tenglamani yeching.

A) 16 B) 4 C) 8 D) 1

10. $6^{x-1}<216$ tengsizlikni yeching.

A) (-4; 4) B) (4; ∞) C) (– 4;4] D) [– 4; 4)

11. $\lg tg22^0+\lg tg68^0$ hisoblang.

A) 0,5 B) 1 C) 0 D) 0,6

12. $\frac{\sin35^0+\cos65^0}{2\cos5^0}$ hisoblang.

A) 0,25 B) 0,3 C) 0,5 D) 0,6

13. $\frac{\sin2\alpha+\cos(\pi-\alpha)*\sin\alpha}{\sin(\frac{\pi}{2}-\alpha)}$ soddalashtiring.

A) $2\sin\alpha$ B) $\cos\alpha$ C) $\sin\alpha$ D) $\cos\alpha$

14. $y=\sin\frac{3x}{2}+\sin\frac{2x}{3}$ davrini ko'rsating.

A) 12π B) 2π C) 3π D) 6π

15. $\sin 3x+\cos 3x=\sqrt{2}$ tenglamani yeching.

A) 15°+120°n B) 15°+360°n C) 45°+360°n D) 45°+120°n

4-VARIANT

1. $\sqrt[3]{x}+2\sqrt[3]{x^2}=3$ tenglamani yeching.

A) 1; $-\frac{27}{8}$ B) 1; $-\frac{3}{2}$ C) 1 D) 1; $\frac{27}{8}$

2. $A_x^3:A_x^4=1:20$ tenglamani yeching.

A) 23 B) 21 C) 19 D) 25

3. $P_5=15x$ tenglamani yeching.

A) 8 B) 9 C) 3 D) 5

4. $\left(\frac{a+2}{\sqrt{2a}}-\frac{a}{\sqrt{2a}+2}+\frac{2}{a-\sqrt{2a}}\right)\cdot\left(\frac{\sqrt{a}-\sqrt{2}}{a+2}\right)$ ni soddalashtiring.

A) $\frac{1}{\sqrt{a}+\sqrt{2}}$ B) $\sqrt{a}+2$ C) $\frac{\sqrt{a}}{2}$ D) $\frac{\sqrt{a}+2}{3}$

5. $(x-2)^6-19(x-2)^3=216$ ratsional ildizlarini toping.

A) 0; 5 B) 0; -5 C) 0 D) 5

6. $|-x^2-1|=|x|+1$ tenglamani yeching.

A) -1; 0; 1 B) 0; 1 C) -1; 0 D) -1; 1

7. $x^2-2x+c=0$ tenglamada c ning shunday qiymatlarini topingki, bu qiymatlarda x_1 va x_2 ildizlar $7x_2-4x_1=47$ shartni qanoatlantirsin.

A) -15 B) –12 C) -14 D) -13

8. $0,5^{x^2}\cdot 2^{2x+2}=64^{-1}$ tenglamani yeching.

A) -2; 4 B) -2 C) 4 D) 2; 4

9. $\log_4(x+12)\cdot\log_x 2=1$ tenglamani yeching.

A) 4 B) -3 C) -3 D) 4

10. $\log_{0,3}(x^2+1)-\log_{0,3}2x<0$ tengsizlikni yeching.

A) $(1;\ \infty)$ B) $(0;\ 1)\cup(1;\infty)$ C) $(0;\ 1)$ D) $(0;\ 8)$

11. $128\cdot\sin^2 20^0\cdot\sin^2 40^0\cdot\sin^2 60^0\cdot\sin^2 80^0$ ko'paytmani hisoblang.

A) 4,5 B) 3,5 C) 5,5 D) 2,5

12. $tg(\arcsin\frac{\sqrt{3}}{2}+arctg\sqrt{3})$ hisoblang.

A) $-\sqrt{3}$ B) $\frac{-\sqrt{3}}{3}$ C) -1 D) $\frac{\sqrt{3}}{3}$

13. $\dfrac{\sin(2\alpha-\pi)}{1-\sin(\frac{3\pi}{2}+2\alpha)}$ soddalashtiring.

A) $\sin\alpha$ B) $\mathrm{ctg}\alpha$ C) $-\mathrm{tg}\alpha$ D) $-\mathrm{ctg}\alpha$

14. $y=\sin 2x+\cos 3x$ davrini ko'rsating.

A) 2π B) π C) 3π D) 5π

15. $\sin x+\sqrt{3}\cos x=1$ tenglamani yeching.

A) $90^\circ+360^\circ n;\ 360^\circ n-30^\circ$ B) $90^\circ+360^\circ n$ C) $90^\circ+180^\circ n; 180^\circ n-30^\circ$ D) $-30^\circ+360^\circ n$

5-VARIANT

1. $\sqrt{1+x\sqrt{x^2+24}}=x+1$ tenglamani yeching.

A) 0; 5 B) 5 C) 3 D) 3;5

2. $\left(\sqrt{a}-\sqrt{b}\right)^8$ yoyilmaning yettinchi hadini toping.

A) $28ab^3$ B) $28a^2b^2$ C) $56ab^3$ D) $56a^3b$

3. $\mathbf{B}_5^4=25x$ tenglamani yeching.

A) 25 B) 24 C) 5 D) 16

4. $z^{\frac{p-3}{p^2+3p}}:z^{\frac{12}{9-p^2}}\cdot z^{\frac{3}{3p-p^2}}$ ni soddalashtiring.

A) $z^{\frac{1}{p-3}}$ B) z^{p-3} C) $3p^2$ D) z^{3p-p^2}

5. $x^4+5x^3+2x^2+5x+1=0$ ratsional ildizlarini toping.

A) Ø B) -1 C) 1 D) ±1

6. $|2x+3|>|4x-3|$ tengsizlikni yeching.

A) (0;3) B) [0; 3] C) {0; 3} D) Ø

7. Agar $a+\frac{1}{a}=3$ bo'lsa, $\frac{a^3+1}{a^3}$ ning qiymatini toping.

A) 27 B) 24 C) 18 D) $21\frac{1}{3}$

8. $\left(\frac{2}{3}\right)^x\cdot\left(\frac{9}{8}\right)^x=\frac{27}{64}$ tenglamani yeching.

A) 3 B) 1 C) -1 D) -3

9. $\log_4\log_3\log_2 x=0$ tenglamani yeching.

A) 8 B) 1 C) 9 D) 64

10. $\log_2(x^2-3x)\le 2$ tengsizlikni yeching.

A) $[1;4]$ B) $(-1;4)$ C) $(0;1]\cup(3;4]$ D) $[-1;4)$

11. $tg9^0-tg63^0+tg81^0-tg27^0$ ni hisoblang.

A) 4 B) 5 C) 3 D) 6

12. Quyidagi sonlardan qaysi biri 12 ga qoldiqsiz bo'linmaydi?

A) 9216 B) 13626 C) 12024 D) 18312

13. $\sin 6x\cdot\cos^3 2x+\cos 6x\cdot\sin^3 2x$ ifodani soddalashtiring.

A) $0,25\sin 8x$ B) $0,75\sin 8x$ C) $0,75\cos 8x$ D) $0,125\cos 6x$

14. $y=\log_x(7-x)$ aniqlanish sohasini toping.

A) $(-\infty;0)$ B) $(0;\infty)$ C) $(0, 1)$ D) $(1;7)\cup(7;8)$

15. $\sin^4 x+\cos^4 x=\frac{7}{8}$ tenglamani yeching.

A) $\pm\frac{\pi}{18}+\frac{\pi m}{2}$ B) $\pm\frac{\pi}{3}+2\pi m$ C) $\pm\frac{\pi}{6}+\frac{\pi m}{2}$ D) $\pm\frac{\pi}{12}+\frac{\pi m}{2}$

6-VARIANT

1. $\frac{4}{x+\sqrt{x^2+x}}-\frac{1}{x-\sqrt{x^2+x}}=\frac{3}{x}$ tenglamani yeching.

A) -1; $\frac{9}{16}$ B) -1 C) $\frac{9}{16}$ D) 1; $\frac{9}{16}$

2. $\begin{cases} C_x^y = C_x^{y+2} \\ C_x^2 = 153 \end{cases}$ sistemani yeching.

A) (18; 8) B) (-17; 8) C) (8; 18) D) (8; -17)

3. $C_{15}^{14} = 3x$ tenglamani yeching.

A) 5 B) 6 C) 7 D) 25

4. $\frac{1}{2(1+\sqrt{a})}+\frac{1}{2(1-\sqrt{a})}-\frac{a^2+2}{1-a^3}$ ifodani sodalashtiring.

A) $\frac{-1}{a^2+a+1}$ B) $\frac{1}{a^2+a}$ C) $\frac{a}{1+a}$ D) 1

5. $x^4-13x^2+36 = 0$ ratsional ildizlarini toping.

A) ±2; ±3 B) 2; 3 C) -2; -3 D) Ø

6. $|2x-1|>|x-1|$. tengsizlikni yeching.

A) $(-\infty;0)\cup\left(\frac{2}{3};\infty\right)$ B) $(-\infty;0]\cup\left[\frac{2}{3};\infty\right)$ C) $(-\infty;0)$ D) $\left(\frac{2}{3};\infty\right)$

7. q ning qanday qiymatlarida $x^2-x-q=0$ tenglama ildizlari kublarining yig'indisi 19 ga teng bo'ladi?

A) 3 B) 4 C) 5 D) 6

8. $7^{2x}-8\cdot 7^x+7=0$ tenglamani yeching.

A) 1 B) -1,2 C) 1,0 D) 2

9. $\log_4\log_3\log_2 x=0$ tenglamani yeching.

A) 8 B) 1 C) 9 D) 64

10. $\left(\frac{3}{7}\right)^{3x-7} < \left(\frac{7}{3}\right)^{7x-3}$ tengsizlikni yeching.

A) $(-\infty;1)$ B) $[1;\infty)$ C) $(-\infty;\infty)$ D) $(1;\infty)$

11. $\sin\left(\arcsin\frac{3}{5}\right)$ ni hisoblang.

A) 3/5 B) 4/5 C) 2/5 D) -3/5

12. $1+\dfrac{tg^2(-\alpha)-1}{\sin(2\alpha+\frac{\pi}{2})}$ ni hisoblang.

A) $\text{ctg}^2\alpha$ B) $-tg^2\alpha$ C) $-ctg^2\alpha$ D) $\sin^2\alpha$

13. $\dfrac{\cos(2\alpha-\frac{\pi}{2})+\sin(3\pi-4\alpha)-\cos(\frac{5\pi}{2}+6\alpha)}{4\sin(5\pi-3\alpha)\cos(\alpha-2\pi)}$ ifodani soddalashtiring.

A) $\cos\alpha$ B) $\sin 2\alpha$ C) 0 D) $\cos 2\alpha$

14. $tg(2x+3)=\sqrt{3}$ tenglamani yeching.

A) $\dfrac{\pi(3n+1)-9}{6}$ B) $\dfrac{\pi(2n+1)-9}{3}$ C) $\pi n+1$ D) $\frac{\pi}{3}+\pi n$

15. $\cos x+\cos 2x=2$ tenglamani yeching.

A) $2n\pi$ *B)* $n\pi$ C) $\frac{\pi}{2}+2n\pi$ D) $\pm\frac{\pi}{2}+2n\pi$

7-VARIANT

1. $\sqrt{x^2-3x+5}+x^2=3x+7$ tenglamani yeching.

A) -1; 4 B) Ø C) -1 D) 4

2. $\begin{cases} A_x^2=20 \\ C_x^y=C_x^{y+1} \end{cases}$ sistemani yeching.

A) (5; 2) B) (2; 5) C) (6; 2) D) (2; 6)

3. $\mathbf{P_6=72x}$ tenglamani yeching.

A) 10 B) 2 C) 30 D) 40

4. $\sqrt{7+5\sqrt{2}}-\sqrt{5\sqrt{2}-7}$ hisoblang.

A) 4 B) 3 C) $10\sqrt{2}$-2 D) 1

5. $x^5+6x^4+9x^3-6x^2+8x=0$ ratsional ildizlarini toping.

A) 0; 2; 4 B) 0; 2 C) 0; 4 D) 2; 4

6. $|2x-3|-|3x+71|<0$ tengsizlikni yeching.

A) $\left(-10;-\frac{4}{5}\right)$ B) $\left[-10;-\frac{4}{5}\right]$ C) $\left\{-10;-\frac{4}{5}\right\}$ D) $(-10;\infty)$

7. $\begin{cases} x^2+y=5 \\ x+y^2=3 \end{cases}$ sistema nechta yechimga ega?

A) 1 B) 2 C) 3 D) 4

8. $3^x-2\cdot 3^{x-2}=63$ tenglamani yeching.

A) 6 B) 4 C) 5 D) 2

9. $\left(\frac{4}{9}\right)^x\cdot\left(\frac{27}{8}\right)^{x-1}=\frac{\lg 4}{\lg 8}$. tenglamani yeching.

A) 2 B) 0 C) 1 D) 1; 2

10. $\log_2(5-2x)>1$ tengsizlikni yeching.

A) (-8; 2,5) B) (2,5; ∞) C) (-∞; 1,5) D) (1,5; ∞)

11. $arctg1+\arccos\left(-\frac{1}{2}\right)+\arcsin\left(-\frac{1}{2}\right)$ hisoblang.

A) 2π/3 B) 3π C) π/4 D) 3π/4

12. $\arccos(-\frac{\sqrt{2}}{2})-arc\sin(-\frac{\sqrt{3}}{2})$ hisoblang.

A) $-\frac{5\pi}{12}$ B) $\frac{13\pi}{12}$ C) $\frac{8\pi}{12}$ D) $\frac{\pi}{12}$

13. $\sin^6\alpha+\cos^6\alpha+3\sin^2\alpha\cos^2\alpha$ ifodani soddalashtiring.

A) 1 B) $\cos\alpha$ C) $\sin\alpha$ D) 4

14. $\cos 2x-2\sin^2\left(\frac{x}{2}\right)=0$ tenglamani yeching.

A) $\pm\frac{\pi}{6}+2\pi m$ B) $\pm\frac{\pi}{3}+2\pi m$ C) $\frac{\pi}{2}+\pi m$ D) $(-1)^m\frac{\pi}{3}+2\pi m$

15. $\sin(x-60^\circ)=\cos(x+30^\circ)$ tenglamani yeching.

A) $60^\circ(3n+1)$ B) $60^\circ+360^\circ\cdot n$ C) $30^\circ+180^\circ\cdot n$ D) $30^\circ+360^\circ\cdot n$

8-VARIANT

1. $\sqrt{2x+1}+\sqrt{x-3}=2\sqrt{x}.$ tenglamani yeching.

A) 4 B) -4 C) 0 D) Ø

2. A_{15}^4 ni hisoblang.

A) 32760 B) 3394 C) 8830 D) 3400

3. $B_5^3=25x$ tenglamani yeching.

A) 5 B) 4 C) 3 D) 2

4. $\frac{x-1}{x+x^{\frac{1}{2}}+1}:\frac{x^{\frac{1}{2}}+1}{x^{\frac{1}{2}}-1}+\frac{2}{x^{-\frac{1}{2}}}$ ifodani sodalashtiring.

A) $x+1$ B) $x-1$ C) 0 D) x

5. $\frac{21}{x^2-4x+10}-x^2+4x=6$ tenglamaning ratsional ildizlarini toping.

A) 1;3 B) -1; -3 C) 2; 3 D) 1

6. $|2x+7|-|3x+5|>0$ tengsizlikni yeching.

A) (-2,4; 2) B) [-2,4; 2] C) {-2,4; 2} D) Ø

7. $\left(m^2-\frac{1+m^4}{m^2-1}\right):\frac{m^2+1}{m+1}$ ifodani soddalashtiring.

A) $\frac{1}{m-1}$ B) $\frac{1}{m+1}$ C) 1 D) $\frac{1}{1-m}$

8. $2^{2+x}-2^{2-x}=15$ tenglamani yeching.

A) $-\frac{1}{4}$ B) 2 C) $\frac{-1}{4}$; 2 D) $\frac{1}{4}$; 2

9. $\log_3^2 x-\log x=2$ tenglama ildizlari ko'paytmasini toping.

A) 1 B) 2 C) 3 D) 1

10. $2^x - 2^{x-2} \le 3$ tengsizlikni yeching.

A) (-∞; 2) B) $(-\infty;\ 2]$ C) (2; ∞) D) $[2;\infty)$

11. $\cos\frac{\pi}{7}\cdot\cos\frac{4\pi}{7}\cdot\cos\frac{5\pi}{7}$ koʻpaytmani hisoblang.

A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) 0,5 D) $\frac{1}{16}$

12. $\dfrac{\cos(\frac{\pi}{2}-2\alpha)}{\sin(\pi-\alpha)}$ hisoblang.

A) $2\sin\alpha$ B) $2\cos\alpha$ C) $\sin\alpha$ D) $tg\ \alpha$

13. $\dfrac{\sin 190^0+\cos(-320^0)-\sin(-170^0)-\cos(-140^0)}{ctg(-112^0)+ctg(-140^0)-tg(-338^0)+tg(230^0)}$ ifodani hisoblang.

A) 0 B) $\cos 10^0$ C) $\sin 40^0$ D) $tg 10^0$

14. $\sin 2x=\sqrt{2}\cos x$ tenglamani yeching.

A) $\pi m; (-1)^m\frac{\pi}{4}+\pi m$ B) $\frac{\pi}{2}+\pi m;\pm\frac{\pi}{3}+2\pi m$ C) $\pm\frac{\pi}{4}+2\pi m$

D) $\frac{\pi}{2}+\pi m;(-1)^m\frac{\pi}{4}+\pi m$

15. $\sin 3x=\cos 2x$ tenglamani yeching.

A) $\frac{\pi}{2}+2n\pi;\frac{\pi}{10}+\frac{2n\pi}{5}$ B) $\frac{\pi}{2}+2n\pi;\frac{\pi}{10}+\frac{2n\pi}{10}$ C) $\frac{\pi}{2}+n\pi;\frac{\pi}{5}+\frac{2n\pi}{5}$

D) $\frac{\pi}{2}+n\pi;\frac{\pi}{10}+\frac{4\pi}{5}$

9-VARIANT

1. $\sqrt{3x+1}-\sqrt{x-1}=2$ tenglamani yeching.

A) 1; 5 B) 2; 3 C) -1; -5 D) Ø

2. P_5 hisoblang.

A) 5 B) 120 C) 8 D) 25

3. $P_5 = 4x + 20$ tenglamani yeching.

A) 25 B) 20 C) 9 D) 10

4. $\dfrac{x-1}{x+x^{\frac{1}{2}}+1} : \dfrac{x^{\frac{1}{2}}+1}{x^{\frac{1}{2}}-1} + \dfrac{2}{x^{-\frac{1}{2}}}$ ifodani sodalashtiring.

A) $x+1$ B) $x-1$ C) 0 D) x

5. $(x^2-5x+7)^2 - (x-2)(x-3) = 1$ tenglamaning ratsional ildizlarini toping.

A) 2; 3 B) -2; -3 C) -2; 3 D) 2; -3

6. $|x-1|+|x-3|>2$ tengsizlikni yeching.

A) $(-\infty;+1)\cup(3;\infty)$ B) $(-\infty;1]\cup[3;\infty)$ C) $(-\infty;1)$ D) $[3;\infty)$

7. $\left(6\frac{1}{3}0,(5)+0,(4):\frac{3}{19}\right)\cdot 4\frac{5}{19}$ ifodani hisoblang.

A) 28 B) 27,5 C) 27 D) 2

8. $17^{x^2-5x+6}=1$ tenglamani yeching.

A) 2 B) 3 C) 2; 3 D) -3; 2

9. $\log_3 x - 2\log_{\frac{1}{3}} x = 3$ tenglamani yeching.

A) -1 B) 3 C) 2 D) 4

10. $\log_8(5x-8) < \log_8(2x+7)$ tengsizlikni yeching.

A) (-35,5) B) (1,6; 5) C) (3,5; 5) D) (5; ∞)

11. $\dfrac{\cos\frac{11\pi}{12} - \cos\frac{\pi}{12}}{\sin\frac{5\pi}{12}}$ hisoblang.

A) - 2 B) -3 C) -1 D) 2

12. $\dfrac{\cos 4\alpha}{\sin 5\alpha - \sin 3\alpha}$ ifodani soddalashtiring.

A) $\dfrac{1}{2\cos\alpha}$ B) $\dfrac{1}{\cos\alpha}$ C) $\dfrac{\cos 4\alpha}{\sin 2\alpha}$ D) $\dfrac{1}{2\sin\alpha}$

13. $4\cos^4 x-2\cos 2x-0,5\cos 4x$ ifodani soddalashtiring.

A) 1 B) 3 C) 1,5 D) 2

14. $\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x=1$ tenglamani yeching.

A) $\frac{\pi}{3}+2\pi m$ B) $\frac{-\pi}{3}+2\pi m$ C) $\frac{\pi}{6}+\pi m$ D) $\frac{\pi}{6}+2\pi m$

15. $\sin x>-\frac{1}{2}$ tengsizlikni yeching.

A) $(-\frac{\pi}{6}+\pi m;\frac{\pi}{6}+\pi m)$ B) $(-\frac{\pi}{6}+2\pi m;\frac{7\pi}{6}+2\pi m$ C) $x>-\frac{\pi}{6}$

D) $(-\frac{\pi}{6}+\pi n;\frac{7\pi}{6}+\pi m)$

10-VARIANT

1. $(16-x^2)\sqrt{x-3}=0$ tenglamani yeching.

A) 4 B) -4 C) -4; 4 D) Ø

2. B_5^4 ni hisoblang.

A) 1 B) 625 C) 20 D) 9

3. $P_6=84x-120$ tenglamani yeching.

A) 10 B) 100 C) 20 D) 30

4. $\frac{x-1}{x^{\frac{3}{4}}+x^{\frac{1}{2}}}\cdot\frac{x^{\frac{1}{2}}+x^{\frac{1}{4}}}{x^{\frac{1}{2}}+1}\cdot x^{\frac{1}{4}}+1$ ifodani soddalashtiring.

A) $\sqrt{x+1}$ B) $\sqrt{x}$ C) $\sqrt{\frac{x+1}{x}}$ D) $x+1$

5. $x^4+2x^3+5x^2+4x-12=0$ tenglamani ratsional ildizlarini toping.

A) -2; 1 B) 0; 1; 2 C) -1; 2 D) 0; -1; -2

6. $|x^2-3x+2|>3x-x^2-2$ tengsizlikni yeching.

A) $(-\infty;1)\cup(2;\infty)$ B) $(-\infty;1]\cup[2;\infty)$ C) $(-\infty;1)$ D) $(2;\infty)$

7. $\frac{\sqrt{32}+\sqrt{98}-\sqrt{50}}{\sqrt{72}}$ hisoblang.

A) 1 B) $\sqrt{2}$ C) $2\sqrt{2}$ D) 0,9988207

8. $2^{2+x}-2^{2-x}=15$ tenglamani yeching.

A) $-\frac{1}{2}$ B) $\frac{1}{4}$; 2 C) $-\frac{1}{4}$; 2 D) 2

9. $\lg^2 x=3-2\lg x$ tenglamani yeching:

A) 0,001;10 B) 0,001;100 C) 0,01;10 D) 0,1;10

10. $y=\log_x(10-x)$ aniqlanish sohasini toping.

A) (-∞; 0) B) (0; ∞) C) (0; 1) ∪(1;10) D) (1; 10)

11. $ctg\frac{\pi}{2}+tg\pi-\sin\frac{3\pi}{2}-\cos\left(-\frac{\pi}{2}\right)+\sin\pi$ ni qiymatini toping.

A) 1 B) 0 C) 2 D) -1

12. $\sin(\arcsin\frac{\sqrt{2}}{2}-\arccos\frac{\sqrt{2}}{2})$ ifodani hisoblang.

A) 1 B) 0 C) $\frac{\sqrt{2}}{2}$ D) $\frac{1}{2}$

13. $\dfrac{\sin\alpha\cdot\cos(\frac{\pi}{2}-\alpha)+\cos 2\alpha}{\sin(\frac{\pi}{2}+\alpha)}$ ifodani soddalashtiring.

A) $\sin\alpha$ B) $\cos\alpha$ C) $-\cos\alpha$ D) $\frac{\sqrt{2}}{2}$

14. $\cos^2 x+3\sin^2 x+2\sqrt{3}\sin x\cos x=1$ tenglamani yeching.

A) $n\pi; n\pi-\frac{\pi}{3}$ B) $+\frac{\pi}{3}+n\pi; n\pi$ C) $\pm\frac{\pi}{3}+n\pi$ D) $2n\pi-\frac{\pi}{3}+n\pi$

15. $tgx>-\sqrt{3}$ tengsizlikni yeching.

A) $(\frac{2\pi}{3}+\pi m;\frac{\pi}{3}+\pi m)$ B) $(-\pi+\pi m;\frac{\pi}{2}+\pi m)$ C) $(-\frac{\pi}{3}+\pi m;\frac{\pi}{2}+\pi m)$

D) $(-\frac{\pi}{3}+\pi m;\frac{\pi}{3}+\pi m)$

11-VARIANT

1. $\sqrt{\sqrt{5}-\sqrt{3-\sqrt{29-6\sqrt{20}}}}$ ifodaning qiymatini toping.

A) 1 B) $\sqrt[4]{5}$ C) $\sqrt{5}$ D) $-\sqrt{5}$

2. C_{15}^{14} hisoblang.

A) 1 B) 15 C) 9 D) $\frac{1}{15}$

3. $C_{20}^{19}=6x-340$ tenglamani yeching.

A) 60 B) 30 C) 40 D) 10

4. $\frac{1-x^{-2}}{x^{\frac{1}{2}}-x^{\frac{1}{2}}}-\frac{2}{x^{\frac{3}{2}}}+\frac{x^{-2}-x}{x^{\frac{1}{2}}-x^{-\frac{1}{2}}}$ ifodani soddalashtiring.

A) $-\sqrt{x}(1+\frac{2}{x^2})$ B) $\sqrt{x}(1+\frac{2}{x})$ C) $\sqrt{x+1}$ D) x

5. $\frac{10-x}{x+18}\geq 0$ tengsizlikni yeching.

A) $[-18:10]$ B) $(-18;10]$ C) $[-18;10)$ D) $(-18;10)$

6. $\frac{x^2+5x-6}{x+6}\geq 0$ tengsizlikni yeching.

A) $(-\infty;1]$ B) $[1;\infty)$ C) $(-\infty;-6)\cup[1;\infty)$ D) $(-\infty;-6]\cup[1;\infty)$

7. $y=3\sin^2 x+2\cos^2 x$ funksiyaning eng katta va eng kichik qiymatlarini toping.

A) 3 va 2 B) 2 va 1 C) 1 va 0 D) 5 va 1

8 . $2^{x+2}-2^x>96$ tengsizlikni yeching.

A) $(6;\infty)$ B) $(4;\infty)$ C) $(5;\infty)$ D) $(-\infty;\infty)$

9. $\log_2(x+13)=2\log_2(x+1)$ tenglamani yeching.

A) -2; 1 B) 3 C) -2 D) -1;2

10. $y=\log_x(5-x)$ funksiyaning aniqlanish sohasini toping.

A) (0;5) B) $(0;\infty)$ C) $(0;1)\cup(1;5)$ D) (1;5)

11. $tg^2\frac{\pi}{3}+ctg\frac{\pi}{6}-2\sin\frac{\pi}{3}+\sin\pi+4\cos\frac{3\pi}{2}-2\cos\frac{\pi}{3}$ ni qiymatini toping.

A) 2 B) 1 C) 0 D) -1

12. $2\arcsin(-\frac{1}{2})+\frac{1}{2}\arccos\frac{\sqrt{3}}{2}$ ifodaning quymatini toping.

A) $-\frac{\pi}{4}$ B) $\frac{5\pi}{4}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{6}$

13. $y=1-|\cos x|$ qiymatlar sohasini toping.

A) [0; 1] B) (0; 1) C) [-1; 1] D) (-1; 1).

14. $\sin^4\frac{x}{3}+\cos^4\frac{x}{3}=\frac{5}{8}$ tenglamani yeching.

A) $\pm\frac{\pi}{2}+\frac{3n\pi}{2}$ B) $\frac{\pi}{2}+\frac{3n\pi}{2}$ C) $\pm\frac{\pi}{2}+2n\pi$ D) $-\frac{\pi}{2}+\frac{3n\pi}{2}$

15. $2\sin^2 x-7\sin x+3>0$ tengsizlikni yeching.

A) $(\frac{7\pi}{6}+2\pi m;\pi+\pi n)$ B) $(\frac{5\pi}{6}+2\pi n;\frac{\pi}{6}+2\pi n)$ C) $(\frac{-7\pi}{6}+2\pi n;\frac{\pi}{6}+2\pi n)$

D) $\frac{\pi}{6}+2\pi n;\frac{5\pi}{6}+2\pi n$

12-VARIANT

1. p ning qanday qiymatlarida $(p-\frac{7}{4})x=3p$ tenglama yechimga ega emas?

A) 7/4 B) 3/4 C) 9/4 D) 3/2

2. $\frac{10!-8!}{89}$ hisoblang.

A) 402 B) 40620 C) 40320 D) 8080

3. $\left(1-\frac{1}{5^2}\right)\cdot\left(1-\frac{1}{6^2}\right)\cdot\ldots\cdot\left(1-\frac{1}{103^2}\right)$ hisoblang.

A) $\frac{67}{103}$ B) $\frac{69}{103}$ C) $\frac{415}{515}$ D) $\frac{416}{515}$

4. $\left(\frac{4}{6}\right)\cdot\left(\frac{8}{6}\right)^3\cdot\left(-\frac{3}{2}\right)^2\cdot(0,75)^3$ ni hisoblang.

A) 1,5 B) 2,75 C) 2 D) - 1,5

5. $3x^2-5x-2=0$ tenglamaning ildizlari kublari yig'indisini toping.

A) $\frac{144}{169}$ B) $\frac{215}{8}$ C) $\frac{169}{27}$ D) $\frac{215}{27}$

6. $\log_3|2x-7|<1$ tengsizlikni yeching.

A) (-∞; 5) B) (2; 5) C) (2; ∞) D) (5; ∞)

7. $x^3=\sin 3x$ tenglama nechta ildizgsa ega?

A) 1 B) 2 C) 3 D) 4

8. $5^{2x-1}+5^{x+1}>250$ tengsizlikni yeching.

A) (2; ∞) B) [2; 8) C) (-∞; 2) D) $(-\infty;\infty)$

9. $\log_2(x+12)=2\log_2 x$ tenglamani yeching.

A) -3; 4 B) -4 C) 4 D) 3;4

10. $y=\log_{x-1}(2x+3)$ aniqlanish sohasini toping.

A) $(1;2)\cup(2;\infty)$ B) (1; ∞) C) (1,5; 2)∪(2; ∞) D) (2; ∞)

11. $2\sin\frac{\pi}{3}+2\cos\frac{\pi}{4}-3tg\frac{\pi}{3}+ctg\frac{\pi}{2}$ ifodani qiymatini toping.

A) $\sqrt{2}-2\sqrt{3}$ B) $\sqrt{2}-\sqrt{3}$ C) $\sqrt{2}+\sqrt{3}$ D) $2\sqrt{2}-\sqrt{3}$

12. $\frac{\sin(2\alpha-\pi)}{1-\sin(\frac{3\pi}{2}+2\alpha)}$ ifodani soddalashtiring.

A) $\sin\alpha$ B) ctgα C) -tgα D) –ctgα

13. $y=2-\cos x$ qiymatlar sohasini toping.

A) [1; 3] B) (1; 3) C) (0; 2) D) [0; 2]

14. $\sin^4 x+\cos^4 x=\cos 4x$ tenglamani yeching.

A) $\frac{n\pi}{2}$ B) $\frac{n\pi}{4}$ C) $n\pi$ D) $2n\pi$

15. $2\sin^2 x-7\sin x+3>0$ tengsizlikni yeching.

A) $(\frac{7\pi}{6}+2\pi m;\pi+\pi n)$ B) $(\frac{5\pi}{6}+2\pi n;\frac{\pi}{6}+2\pi n)$ C) $(\frac{-7\pi}{6}+2\pi n;\frac{\pi}{6}+2\pi n)$

D) $\frac{\pi}{6}+2\pi n;\frac{5\pi}{6}+2\pi n$

JAVOBLAR VA KO'RSATMALAR

I BOB. BUTUN SONLAR VA KOMBINATORIKA

3-§. Sanoq sistemalari

87. 1100. **88**. 10011. **89**. 11010 **90**. 1000,101. **91**. 110,100. **92**. 1001,1001. **93**. 1110,10. **94**. 100. **95**. 1001. **96**. 101,01. **97**. 10110,10. **98**. 1100.01. **99**. 100,011 **100**. 1010,11. **101**. 10 1101,11. **102**. 10,1101. **103**. 1111. **104**. 11110. **105**. 10101. **106**. 10001111. **107**. 1001100,01. **108**. 1001,01011 **109**. 111100,0011. **110**. 11001,011. **111**. 11000110,111. **112**. 110. **113**. 110 **114**. 11. **115**. 101 **116**. 11,01. **117**. 1,0001. **118**. 1001000. **119**. 11111. **120**. 100100

131. $x=-1$. **132**. $x=1$. **133**. $x=1111(1010q)$. **134**. $x_1=110{,}1$ $x_2=1010$, $x_3=1101{,}1$ $x_4=10001$ $x_5=10100{,}1$. **135**. $x=110$.

136. $x=-10101$. **137**. $1(101q)$. **138**. $x=100$. **139**. $x=\frac{-101\pm\sqrt{10001}}{100}$.

141. $x=\pm\sqrt[10]{110001}$. **142**. $x=100$.

143. $S_1=101,\ S_2=111,\ S_3=1001,\ S_4=1011,\ S_5=1101$

144. $S_1=1011,\ S_2=10001,\ S_3=10111,\ S_4=11101,\ S_5=100011.$

147. $S_1=111,\ S_2=1001,\ S_3=1011,\ S_4=1101,\ S_5=1111.$

148. $S_1=110{,}1\ S_2=1010,\ S_3=1101{,}1,\ S_4=10001,\ S_5=10100{,}1.$

149. $S_1=111{,}1\ S_2=1100{,}1,\ S_3=101,\ S_4=10001{,}1,\ S_5=1101.$

150. $S_1=1000{,}01,\ S_2=101{,}10,\ S_3=110{,}11,\ S_4=1000,\ S_5=1001{,}01$

151. $\frac{1111}{10}$. **152**. 1111. **153**. 1111,1. **154**. 101. **155**. $\frac{110001}{10}$.

156. 100,1 **157**. 1100,01 **158**. 1000,01.

159. $S_{11}=110000$km, $S_{110}=1100000$km, $S_{1111}=11110000$km.

160. $S=100001\,km$. **161**. 1050207. **162**. $143205=2002203,\ 12103(1q)$

163. $16537=222034,\ 24(101q)$. **178**. $5\frac{33}{49}$. **179**. $74\frac{11}{64}$ **180**. $4\frac{337}{512}$.

181. $\frac{3383}{4096}$. **182.** $2\frac{11}{27}$. **183.** $7\frac{605}{729}$. **184.** $3\frac{51}{125}$. **185.** $4\frac{47}{108}$. **186.** $\left(2\frac{114}{1000}\right)$.

187. $\left(2\frac{114}{1000}\right)_8$. **188.** $\left(5\frac{442}{100}\right)_7$. **189.** $\left(74\frac{13}{100}\right)_8$. **190.** $\left(35\frac{13}{100}\right)_7$. **191.** $\left(4\frac{521}{1000}\right)_8$

192. $\left(\frac{6467}{10000}\right)_8$. **193.** $\left(2\frac{224}{1000}\right)_6$. **194.** $\left(7\frac{742}{1000}\right)_9$. **195.** $\left(3\frac{201}{1000}\right)_5$. **196.** $\left(4\frac{234}{1000}\right)_6$. **197.** $1\frac{1}{6}$. **198.** $\left(\frac{23}{30}\right)_4$. **199.** $\left(3\frac{141}{440}\right)_5$. **200.** $\left(4\frac{21}{40}\right)_5$. **201.** $\left(32\frac{123}{400}\right)_5$.

202. $\left(5\frac{2}{13}\right)_6$ **203.** $\left(2\frac{11}{60}\right)_7$. **204.** $\left(2\frac{53}{500}\right)_5$. **205.** $\left(1\frac{11}{20}\right)_3$. **206.** $\left(\frac{65}{70}\right)_8$. **207.** $1{,}12_3$. **208.** $0{,}311_5$. **209.** $0{,}5375$. **210.** $0{,}00(21312)_4$. **211.** $3{,}42_6$. **212.** $0{,}2213_6$.

213. $0{,}83_9$. **214.** $0{,}4_9$. **215.** $10{,}3_7$. **216.** $0{,}65$. **217.** $1{,}2_3$. **218.** $2{,}1(2)_7$. **219.** $4{,}2(3)_5$. **220.** $0{,}2_6$. **221.** $\left(0{,}(23)\right)_4$. **222.** $3{,}1_6$. **223.** $2{,}012_3$. **224.** $0{,}36_{14}$. **225.** $\left(16{,}1(25)\right)_8$. **226.** $17{,}78_8$.

4-§. Kombinatorika elementlari

227. a) 24; b) 28. **228.** $9\cdot10^6$. **229.** $10^3\cdot26^3=1757600$. **230.** O‘g‘il bolalar 27/25 marta ko‘p. **231**. a) $9\cdot10^4\cdot2$; b) $9\cdot10^5-5^6$; c) $9\cdot10^9-9\cdot9!$. **232**. a) $9\cdot10^2$; b) $9\cdot10^7\cdot2$; c) 5^4. **233.** 3^7. **234.** 38. **235.** 50 ta juftlikdan hammasida ayol kishi bo‘lishi mumkin emas. **236.** Hammasi bo‘lib n ta jamoa bo‘lsa, xar bir jamoa 0 ta dan n tagacha o‘yin o‘ynagan. Lekin bittasi n ta, ikkinchisi 0 ta o‘yin o‘ynagan jamoa mavjud emas. **237.** Induksiya metodi bilan isbotlaymiz. Agar 1 dan $2n-2$ ($n\geq3$) gacha $n+1$ ta turli son olingan bo‘lsa, u holda ulardan shunday uchta sonni ajratib olish mumkinki, ulardan ikkitasining yig‘indisi uchinchisiga teng bo‘ladi. $n=3$ bo‘lganda bu tasdiqning o‘rinli ekani ravshan. Bu tasdiqni $n=k$ uchun o‘rinli deb hisoblab, $n=k+1$ uchun isbotlaymiz. Agar tanlangan $k+1$ ta son 1 dan $2k-2$ gacha oraliqqa tegishli bo‘lsa, u holda induksiyaning farazi o‘rinli bo‘ladi. Agar aksincha bo‘lsa, u holda albatta $2k-1$ va $2k$ sonlar tanlanishi kerak. Boshqa k ta tanlangan sonlar 1 dan $2k-2$ gacha oraliqda yotadi. Bu oraliqni $(1, 2k-2)$, $(2, 2k-3)$,..., $(k-1,$

k) juftliklarga ajratib, ulardan biri tanlangan sonlardan iborat ekanligini hosil qilamiz. Ularning yig'indisi $2k-1$ ga teng. Agar 1002 sonini 1001 bilan almashtirilsa, yuqoridagi mulohaza o'rinli emas. Masalan, 1000, 1001, ..., 2000 sonlar tizimida ikkitasining yig'indisi uchinchisiga teng bo'ladigan uchta sonni ajratib olish mumkin emas.

238. 17!. **239.** $28\cdot 6!\cdot 1111111$. **240.** S_9^4 241. $2\cdot S_{10}^7+1\cdot S_{10}^6$. **242.** $S_7^5\cdot S_9^5$

243. a) 26; b) 51. **244.** $n=2a+1$ ni qarang. **245.** b) S_k^2-k . **246.** *Ko'rsatma.* Bunday sonlar 9876543210 dan 6 ta raqamni tanlab olish bilan hosil qilinadi. *Javob*:S_{10}^6. **247.** a) S_{999}^2; b) S_{1002}^2 . **249.** $n=2^k-1$. **250.** a) 3^5; b) 0; c) 2^n . **251.** a) tenglikning chap qismida: r ta element dan m ta elementni tanlash, keyin tanlangan m tadan yana k tasini tanlash usullari coni. Tenglikning o'ng qismida: r ta elementdan k ta elementni, qolgan elementlardan yana $m-k$ tasini tanlaymiz.

252. $x=2, y=3, n=5$. **253.** $m=3,\ n=2$. **254.** $S_{100}^6\cdot 4(\sqrt{3})^{64}$. **255.** $6^5\cdot 210+1$; **256.** a) 5; b) 8; c) 5; d) 3.

5-§. Birinchi darajali aniqmas tenglamalarni yechish

257. [1;9]. **258.** [0;2,15]. **259.** [-2;1,30,20]. **260.** [0;1,4,3,2]. **261.** [-3;1,1,2] **262.** [2;2,3,1]. **263.** [1;4,2,1,7]. **264.** [1;1,2,1,2,1,2]. **272.** Yechim yo'q. **273.** $x=13+44t,\ y=-70-237t$. **274.** $x=9+29t,\ y=-17-55t$. **275.** $x=7+8t,\ y=-2-3t$. **276.** $x=1+5t,\ y=1-2t$

II bob. Ayniy shakl almashtirishlar. Ayniyatlar va tengsizliklarni isbotlash

1-§. Ratsional ifodalar

1. 0,1. **2.** $\frac{b+1}{b-2a}$. **3.** 1. **4.** $\frac{1}{xy}$. **5.** $\frac{24}{5y-2x}$. **6.** 20. **7.** $2x-1$. **8.** $x>3$ da (x^2+x+1); $x<3,\ x\neq 1$ da $-(x^2+x+1)$. **9.** $x>-2$ da $\frac{x(x-1)}{2}$; $x<-2$ da $-\frac{x}{2}$ **10.** $a>3$ va $a<0$ da $\frac{1}{a+2}$; $0<a<3$ da $-\frac{1}{a+2}$. **11.** $\frac{a+2}{a+1}$ **12.** $a<0$ da $\frac{1}{1-3a}$; $0<a<1,\ a\neq\frac{1}{3}$ da $\frac{a+1}{(a-1)(3a-1)}$; $a>1$ da $\frac{1}{a-1}$ **13.** $\frac{x+2}{x^2(x-1)^2}$.

14. $x<-1$ da 2, $-1<x<1$ da $\frac{2x^2}{2x^2-1}$; $x>1$ da 0 **15.** $a<-1$ da -1; -$1<a<1$, $a\neq 0$ da. $\frac{2+a+a^3}{a^3+a}$; $a\geq 1$ da 1 **16.** $y<1$ da $-y-1$; $-1\leq y\leq 1$ da $y+1$; $y\geq 1$ da $2y^2+y-1$ **17.** $x<-1$ da $x-2$; $-1<x<1$ da. $\frac{x^2+4}{x-2}$; $1<x<2$ da $-(x+2)$; $x>2$ da $x+2$ **18.** $\frac{n+1}{n}$. **19.** $x<-1$ da $\frac{x}{x-1}$; $-1<x<0$ da $\frac{x}{1-x}$; $0\leq x<1$ da $-\frac{x}{x+1}$; $x>1$ da $\frac{x}{x+1}$. **20.** $\frac{x+y}{x-y}$ $\left\{(x;y)\in R^2 / y\neq x\wedge y\neq \frac{2x}{3}\right\}$ **21.** x^3+5 $R/\left\{0, -\sqrt[3]{3}, -\sqrt[3]{2}, \sqrt[3]{2}\right\}$. **22.** p.

23. $\frac{1}{2}(x+y+z)$ $\left\{(x,y,z)\in R^3 / x\neq y\vee y\neq z\vee z\neq x\right\}$.

24. $\left\{(x,y,z)\in R^3 / z\neq x-y\wedge z\neq -x-y\right\}$ **26.** $a+b+c$ **27.** 1 . **28.** x^3+y^3

29. $\frac{x^8-x^4+1}{x^8+x^4+1}$ **30.** 0. **31.** 0.

2-§. Irratsional ifodalarni ayniy shakl almashtirish

37. $\frac{(90-2\sqrt{30})(\sqrt{5}+\sqrt{6}-\sqrt{7})}{26}$ **38.** $\left\{\frac{1}{6}\sqrt[4]{27}(\sqrt[4]{3}-1)\right\}$. **39.** $\frac{\sqrt{5}(\sqrt{6}+1)}{5}$

40. $\frac{\sqrt[3]{225}+\sqrt[3]{105}+\sqrt[3]{49}}{8}$ **41.** $1+\sqrt{2}$. **42.** $\sqrt[3]{3}-\sqrt[3]{2}$. **49.** 2. **50.** -1 agar $0<a\leq 1$, $\frac{-(\sqrt{1-a^2}+1)}{a^2}$ agar $-1\leq a<0$. **51.** 1. **52.** 2a. **53.** 2.

54. x agar $\frac{a}{x^2+2a^2}>0$, $-x$ agar $\frac{a}{x^2-2a^2}<0$. **55.** $\frac{2\sqrt[6]{a^5}}{a}$. **56.** -1.

57. $4p-\sqrt{4p^2-1}$ **58.** $\frac{1}{xy}$. **59.** 1. **60.** $\frac{x+y}{2}$

61. $0<x<1$ da $\frac{1}{x-x^2}$; $x>1$ da $\frac{1}{x^2-x}$. **62.** $0\le x<9$ da $3-2\sqrt{x}$; $x>9$ da -3. **63.** x^2+1. **64.** $\frac{1}{x-\sqrt{2x}+1}$. **65.** $x<0$ da $-\frac{2x^2+2x-3}{x}$; $0<x<2$ *da* $\frac{3+2x}{x}$; $x\ge 2$ da $\frac{2x^2-2x+3}{x}$ **66.** $x<-1$ da $-\frac{x+1}{x-1}$; $-1<x<0$ da $\frac{x+1}{x-1}$; $x\ge 0$ *da* $\frac{x-1}{x+1}$

67. $0\le m\le\frac{1}{2}$ da $\sqrt{1-m}-\sqrt{m}$; $\frac{1}{2}<m\le 1$ da $\sqrt{m}-\sqrt{1-m}$

68. $x<-2$ da $\frac{2-x}{2}$; $-2\le x<0$ da $-\frac{x^2+2x+8}{2x}$; $x>0$ da $\frac{x^2+2x+8}{2x}$

3-§. Ko'rsatkichli va logarifmik ifodalarni ayniy shakl almashtirishlar

69. $\frac{1}{1-a}$. **70.** $\frac{2}{3}-\frac{a}{9}$. **71.** $\frac{a-2b+1}{2}$. **72.** $\frac{a+b}{2}$. **73.** $\frac{18}{3+2a}$. **74.** $\{0\}$

75. Agar $\begin{cases}0<a<1\\0<b<1\end{cases}\cup\begin{cases}a>1\\b>1\end{cases}$ bo'lganda $\{0\}$; agar $\begin{cases}a>1\\o<b<1\end{cases}\cup\begin{cases}0<a<1\\b>1\end{cases}$ bo'lganda $-2(\log_a b+\log_b a)$. **77.** $\log_a b$. **79.** 0. **80.** -1. **81.** $\frac{1}{\sqrt[3]{9}}$ **82.** $\frac{1}{\sqrt[4]{125}}$ **.83.** 24. **84.** $\frac{1}{3}$ **.85.** 0. **86.** 0. **87.** $\frac{a+3}{2(a+1)}$. **88.** $\frac{4(3-a)}{3+a}$. **89.** $\frac{3a-b+5}{a-b+1}$ **.90.** $\frac{a+1}{2a+b}$. **95.** $\log_a b$. **96.** $b^{\log_a b}$ **.97.** Agar $0<a<1$ bo'lsa $-a$, agar $a>1$ bo'lsa $a-2$ **98.** Agar 1<a<b bo'lsa 2, agar 1<b<a bo'lsa 2 $\log_a b$ **99.** 0. **100.** $\alpha=10^{\frac{1}{1-\lg\lambda}}$.

III-BOB. ALGEBRAIK TENGLAMA VA TENGSIZLIKLAR

1-§. Ratsional tenglamalar. Teng kuchli tenglamalar

1. Ha. **2.** Yo'q. **3.** Ha **4.** Ha. **5.** Yo'q. **6.** Ha. **7.** Ha. **8.** Yo'q. **9.** Ha. **10.** Yo'q. **11.** Yo'q **12.** Ha. **13.** {1;-1}. **14.** {2;-2}. 15. ∅. **16.** $\{\sqrt{10}-4;-\sqrt{10}-4;-4\}$ **17.** (-2;1). **18.** $\left\{-1;\frac{1}{3};3\right\}$ **19.** {0;1}. **20.** {-1;2}. **21.** {-5;3}. **22.** {4.5;5.5}. **23.** $-\frac{1}{2}$. **24.** $\{1;-\frac{1}{2}\}$. **25.** {1;2;2.5;5} **26.** {-4; -3; 3}. **27.** $\{-1-\sqrt{6};\ -1+\sqrt{6};\ -i\sqrt{3};\ i\sqrt{3}\}$. **28.**

$\left\{-\frac{1}{2};\ 1+i\sqrt{3};\ 1-i\sqrt{3}\right\}$. **30.** $\left\{-3;\ 1;\ -1-i\sqrt{3};\ -1+i\sqrt{3}\right\}$. **31.** $\{1;\frac{-3\pm\sqrt{5}}{2}\}$. **32.** $\{-1;$ $-\frac{1}{4}\}$. **33.** $\{-1;9;\frac{5\pm\sqrt{61}}{2}\}$. **34.** $\{-2;-1;0;1\}$ **35.** $\{1;2\}$. **36.** $\left\{0;-\frac{5}{2}\right\}$. **37.** $\left\{-\frac{2}{5}\right\}$ **38.** $x=-5$. **39.** $\{-3;3\}$. **40.** $\left\{1\frac{6}{13}\right\}$. **41.** $\{5\}$ **42.** $\left\{\frac{12}{13}\right\}$. **44.** $\left\{\frac{2}{3}\right\}$. **50.** $\left\{6;-2;3;\pm\sqrt{21}\right\}$.

2-§. Qaytma tenglamalar

53. $\{-1;1;\ i;-i\}$. **55.** $i;\ -i;\ \frac{\sqrt{3}+i}{2};\ \frac{\sqrt{3}-i}{2};\ \frac{-\sqrt{3}+i}{2};\frac{-\sqrt{3}-i}{2}$. **56.** $\{-2;\ 3;\ -1\}$.

57. $\left\{-\frac{1}{2};\ 1+i\sqrt{3};1-i\sqrt{3}\right\}$. **61.** $\left(-2;\ \frac{1}{2};\ \frac{2}{3};3\right)$. **62.** $(\pm i;0;2;4)$. **63.** $-1;\ 2;\ -3+i\sqrt{3};\ -3-i\sqrt{3}$

64. $2;\ -2;\ \frac{3\sqrt{21}}{7};-\frac{3\sqrt{21}}{7}$. **65.** $2;3;\frac{5+i\sqrt{3}}{2};\frac{5-i\sqrt{3}}{2}$. **66.** $-1;1+\sqrt{10};\ 1-\sqrt{10}$.

67. $2;\frac{1}{2};\ \frac{1+2i\sqrt{6}}{5};\frac{1-2i\sqrt{6}}{5}$. **68.** $\left\{-6;1;\frac{-5\pm\sqrt{39}i}{2}\right\}$. **69.** $\frac{11}{2};\frac{9}{2};\frac{10\pm i\sqrt{7}}{2}$

70. $-3;-5;-4\pm i\sqrt{7}$. **71.** $1;\frac{1}{2};\frac{2}{3}$. **72.** $-3;\frac{3\pm i\sqrt{83}}{4}$. **73.** $2;\frac{1}{2};\frac{1\pm i\sqrt{3}}{2}$. **74.** $2;\frac{1}{2};\frac{-11\pm\sqrt{105}}{4}$. **75.** $2;6;\frac{-3\pm i\sqrt{39}}{2}$ **76.** $\frac{3\pm\sqrt{5}}{2};\frac{-1\pm i\sqrt{3}}{2}$. **77.** $2;-1;\frac{-3\pm\sqrt{17}}{2}$.

78. $-1;-\frac{1}{4};\frac{3\pm i\sqrt{17}}{8}$. **79.** $\left(3;\frac{2}{3};-\frac{5}{2}\right)$. **80.** $-1;\frac{-1\pm i\sqrt{15}}{4}$. **81.** $-1;3;\frac{1}{3}$. **82.** $\{0;\ 1;\ -1;\ -20\}$. **83.** $1;2;\frac{9\pm i\sqrt{51}}{6}$. **84.** $\left\{\frac{1}{2}\right\}$. **85.** $\left\{\frac{1}{2}\right\}$ **86.** $\{-1;-1.5\}$. **87.** $\varnothing$. **88.** $\{-13\}$.

3-§. Modul qatnashgan tenglamalar

89. $\{-1.5\}$. **90.** $\{0.1;-1\}$. **91.** $\{1\}$. **92.** $\left\{1\frac{3}{4};2\frac{1}{2};3\frac{1}{4};2\frac{1}{2}\right\}$. **93.** $\{5\}$. **94.** $\left\{\frac{1-\sqrt{3}}{2}\right\}$.

96. $[-3;-2]\cup[2;3]$. **100.** $\left\{\frac{1}{3}\right\}$. **101.** $\left\{\frac{17}{19};3\right\}$. **102.** $\{-8;2\}$. **103.** $\left\{-4;\ 0;\ 2;\ 2\frac{2}{3}\right\}$

106. $\{x\le -2 \vee x\ge 2\}$. **107.** $\{1\}$. **108.** $\left\{1\frac{3}{4};2\frac{1}{2};3\frac{1}{4};2\frac{1}{2}\right\}$. **109.** $\left(-\infty;\frac{7}{4}\right]$. **110.** $\{-1\}$

.**111.** $-5;\frac{1}{5}$. **112.** $1\pm\sqrt{2}$. **113.** $-3;3$. **114.** $\{-3;-2;0;1\}$. **115.** $[2;\infty)$. **116.** $-\frac{14}{3};0$

.**117.** $\frac{2}{3};\frac{4}{3}$. **118.** -1. **119.** $[-3;-2]\cup[2;3]$. **120.** $\varnothing$. **121.** $\frac{11}{5};-2$. **122.** $[0;1]$.**123.**

$\frac{1}{2};1;\frac{3}{2}$. **126.** $\left\{-\frac{1}{2}\right\}$. **127.** $\left\{\frac{1}{2};1-\sqrt{3}\right\}$. **128.** $\{5\}$.

4-§. Modul qatnashgan tengsizliklar

129. $\left[-2;3\frac{2}{3}\right]$. **130.** $(-\infty;1)\cup(7;\infty)$. **131.** $(-\infty;1]\cup[3;\infty)$. **132.** $(2;3)\cup(3;\infty)$.**133.**

$\left[3;3\frac{1}{3}\right]$. **134.** $(-\infty;-2)\cup(-2;-1)\cup(-1;0)$. **135.** $\left[-2+\sqrt{6};1\right)\cup(1;4]$.**136.**

$\left(-\infty;\frac{1+\sqrt{17}}{4}\right]$. **137.** $[-1;0)\cup(0;1]$. **138.** $\left(2;\frac{10}{3}\right)$. **139.** $\left(-\infty;-\frac{8}{3}\right)\cup(2;\infty)$ **140.**

$(-\infty;-2.5)\cup(-1.5;0.5)\cup(0.5;1.5)\cup(2.5;\infty)$. **141.** $\left(\frac{11-\sqrt{57}}{4};\frac{11+\sqrt{57}}{4}\right)$.**142.**

$(-\infty;1)\cup(2;\infty)$. **143.** $\left[0.1;\frac{3}{5}\right]\cup\left[2\frac{1}{2};\infty\right)$. **144.** $(-\infty;2]$. **145.** $(-\infty;-16)\cup(6;\infty)$.**146.**

$[1.5;2.5]$. **147.** $(-\infty;-4)\cup(-4;2)\cup(6;\infty)$. **148.** $(4.5;\infty)$. **149.**$(-\infty;1]\cup[1.5;\infty)$

.**150.** $(0;0.4)$. **151.** $\left(-\infty;\frac{3-\sqrt{65}}{4}\right)\cup\left(\frac{3-\sqrt{33}}{4};\frac{3+\sqrt{33}}{4}\right)\cup\left(\frac{3+\sqrt{65}}{4};\infty\right)$.

152. $\left[\frac{-1-\sqrt{11}}{2};-1\right)\cup(-1;1)\cup\left(1;\frac{-1+\sqrt{11}}{2}\right]$. **153.** $(-\infty;-2]\cup[-1;\infty)$.

154. $(-\infty;-4]\cup[1;\infty)$. **155.** $(-0,5;\ 2,75)$. **156.** $(1;3)$. **157.** $(-\infty;-0.5)\cup(0;4)$.

158. $\left[\frac{1}{3};3\right]$. **159.** $(-\infty;2]\cup[4;\infty)$. **160.** $(-\infty;1.75)\cup(2.5;\infty)$.

5-§. Tenglamalar sistemasi

161. $(6;6),\left(\frac{-3+3\sqrt{5}}{2};\frac{-3-3\sqrt{5}}{2}\right),\left(\frac{-3-3\sqrt{5}}{2};\frac{-3+3\sqrt{5}}{2}\right)$. **162.** $(3;1),(-1;-3),(\sqrt[3]{13};-\sqrt[3]{13})$.**163.**$(4;2),(-4;-2)$.**165.** $\{(2;3),(3;2)\};\left(-\frac{3}{4}-\sqrt{\frac{103}{48}};-\frac{3}{4}+\sqrt{\frac{103}{48}}\right),\left(-\frac{3}{4}+\sqrt{\frac{103}{48}};-\frac{3}{4}-\sqrt{\frac{103}{48}}\right)$ **166.** $\{(3+2\sqrt{2};3-2\sqrt{2}),(3-2\sqrt{2};3+2\sqrt{2})\};\left\{\left(\frac{\sqrt{129}+1}{2};\frac{\sqrt{129}-1}{2}\right),\left(\frac{\sqrt{129}-1}{2};\frac{\sqrt{129}+1}{2}\right)\right\}$

170. $(25;9);\left(\frac{49}{4};\frac{81}{4}\right)$. **171.** $(1;3),(-1;-3),(3;1),(-3;-1)$. **172.** $(2;1),(-2;-1)$. **173.** $\left(\frac{1}{2};4\right)$. **174.** $(1;1;1),(-2;-2;-2)$. **175.** $(2;2)$. **176.** $(3;5),(5;3)$. **178.** $(4;2),(-4;-2)$. **178.** $(1;-1;2),(1;2;-1),(-1;1;2),(-1;1;2),(2;1;-1),(2;-1;1)$. **179.** $(2;1),(-1;2),(-2;1),(1;-2)$. **180.** $(1;1)$. **181.** $(3;-2),(-3;2)$. **182.** $(9;12)$, $(-12;-9)$ **183.** $(2;1)$, $(1;2)$, $(0;-3)$, $(-3;0)$, $(-2;1)$, $(1;-2)$. **184.** $(-1;-3)$, $(3;1)$. **185.** $(3;4)$, $(4;3)$, $(6+\sqrt{29};\ 6-\sqrt{29});(6-\sqrt{29};\ 6+\sqrt{29})$. **186.** $(1;4)$, $(4;1)$; $\left(\frac{-5+\sqrt{41}}{2};\frac{-5-\sqrt{41}}{2}\right),\left(\frac{-5-\sqrt{41}}{2};\frac{-5+\sqrt{41}}{2}\right)$. **187** $(0;0;0)$, $(1;2;1)$, $(2;1;1)$, $\left(1+\sqrt{\frac{2}{3}};1-\sqrt{\frac{2}{3}};\frac{2}{3}\right);\left(1-\sqrt{\frac{2}{3}};1+\sqrt{\frac{2}{3}};\frac{2}{3}\right)$ **188.** $(1;0;0)$, $(0;1;0)$, $(0;0;12)$. **189.** $\left(\frac{2}{3}\sqrt{6}-1;\ \frac{1}{2}\sqrt{6}-1;\ \sqrt{6}-1\right);\ \left(-\frac{2}{3}\sqrt{6}-1;\ -\frac{1}{2}\sqrt{6}-1;\ -\sqrt{6}-1\right)$.

7-§. Kasr ratsional va yuqori darajali tengsizliklar

190. $(-\infty;1)\cup\left(\frac{3}{2};5\right)\cup\left(\frac{7}{2};4\right)$. **193.** $\left(-\frac{4}{5};-\frac{3}{4}\right)\cup\left(-\frac{2}{3};-\frac{1}{2}\right)\cup(0;\infty)$. **194.** $(-\infty;-1)\cup(-1;2]$. **195.** $(-\infty;7]\cup(-1;0)\cup(0;1]\cup(3;\infty)$. **196.**

$(-\infty;2)\cup[3,5;4)\cup[7;\infty)$. **197.** $\left(-\infty;-\frac{4}{3}\right)\cup\left(-\frac{79}{75};\frac{3}{2}\right)\cup(2;\infty)$. **200.** $\left(-2;-\frac{5}{4}\right)\cup(-1;1)\cup(5;\infty)$. **201.** $(-\infty;-4)\cup\left(-3;-\frac{5}{2}\right)\cup(-2;-1)\cup(0;\infty)$ **202.** $\left(\frac{1}{4};8\right)$.

203. $(-2;0)\cup(6;\infty)$. **204.** $(-1;2)$ **205.** $(-\infty;-6]\cup[-2;\infty)$. **206.** $(-\infty;1)$. **207.** $[-1;1]$. **208.** $(-\infty;-4]\cup\left[\frac{1-\sqrt{13}}{2};\frac{1+\sqrt{13}}{2}\right]\cup[4;\infty)$. **209.** $\left[-3;\frac{1-\sqrt{41}}{4}\right]\cup\left[0;\frac{1+\sqrt{41}}{4}\right]\cup\{3\}$.

210. $(-\infty;-3)\cup(2-\sqrt{6};3)\cup(2+\sqrt{6};\infty)$. **211.** $\left(-2;\frac{-1-\sqrt{5}}{2}\right)\cup\left(\frac{-1+\sqrt{5}}{2};2\right)$. **212.** $\left(-\infty;\frac{3}{2}\right)\cup\left(\frac{7}{3};\infty\right)$. **213.** $(-3;-2)\cup(-1;1)$. **214.** $\left(-1\frac{1}{2};4\right)$. **215.** $\varnothing$. **216.** $(-\infty;-2)\cup\left(\frac{1}{2};3\right)\cup(3;4)$. **217.** $(-\infty;-1]\cup[3;4]$. **218.** $(-\infty;-3)\cup(-2;-1)\cup(0;\infty)$.

219. $\left[-\frac{1}{2};-\frac{1}{4}\right]\cup\left[1;1\frac{1}{4}\right]$. **220.** $\{-3\}\cup\left[-\frac{2}{5};\frac{1}{3}\right)\cup(2;\infty)$. **221.** $\left(-1;-\frac{1}{2}\right)\cup\left(0;\frac{1}{2}\right)\cup(2;\infty)$. **222.** $(-\infty;-3)\cup\left[-2\frac{3}{5};2\right)\cup(2;\infty)$. **223.** $\left(-2;\frac{1}{3}\right)\cup\left(2\frac{1}{5};\infty\right)$. **224.** $(-\infty;-2)\cup(-1;1)\cup(3;\infty)$. **225.** $\left[3\frac{1}{7};4\right)\cup(5;\infty)$. **226.** $\left(1-\sqrt{3};-\frac{1}{2}\right)\cup(1;1+\sqrt{3})$.

7-§. Irratsional tenglama va tenglamalar sistemasi

227. $x=3$ **228.** $x=7$. **229.** $x=9$. **230.** $\{1\}$. **231.** $x_1=6,\ x_2=-2$. **232.** $x=12$.**233.** $x=2$ **234.** $x_1=0,\ x_2=-1$. **235.** $x=1$. **236.** $x=3$. **237.** $x_1=-6,\ x_2=-5,5$ **238.** $x=64$. **239.** $x=5$. **240.** $x_1=1,\ x_{1,2}=-1\pm 2i$. **241.** $-2;\frac{14}{9}$. **242.** 1;2. **243.** -4;4. **244.** 1024. **245.** $\frac{9+\sqrt{73}}{4}$. **246.** 1. **247.** 1. **248.** 0. **249.** 1. **250.** 5. **251.** -4;4. **252.** $\varnothing$. **254.** $\pm1;\pm\sqrt{6}$. **255.** 0. **256.** $\varnothing$. **257.** -1;4. **258.** $[2;+\infty)$. **259.** 2. **260.** 4.

261. -3. **262.** $\frac{\sqrt{5}}{2}$. **263.** 0;–2. **264.** $\left\{(4;4),\left(\frac{9}{2};\frac{7}{2}\right)\right\}$. **265.** $\left\{(-5;-3),(3;1),\left(\sqrt{10}-1;\frac{4}{5}\sqrt{10}-1\right),\left(-\sqrt{10}-1;-\frac{4}{5}\sqrt{10}-1\right)\right\}$ **266.** $\left\{(0;0),\left(\sqrt{2};\frac{\sqrt{2}}{2}\right),\left(\frac{3\sqrt{3}}{4};\frac{\sqrt{3}}{4}\right)\right\}$. **267.** $\{(25;36),(36;25)\}$. **268.** (3;1). **269.** $\{(1;4),(4;1)\}$. **270.** $\{(-8;-1),(-8;1),(8;-1),(8;1)\}$. **271.** $\left(\frac{12}{7};\frac{5}{3}\right)$. **272.** $(3;4,5)$. **273.** $\{(1;27),(27;1)\}$. **274.** $(16;1)$. **275.** $(5;4)$. **276.** $(-4;5;3)$.

8-§. Irratsional tengsizliklar

277. $\left(\frac{1}{6}\left(7+\sqrt{13}\right);2\right)\cup[3;+\infty)$. **278.** $(-\infty;-1]\cup\left(8\frac{4}{5};+\infty\right)$. **279.** $[5;6)$.

280. $\left(1;1\frac{9}{16}\right)$. **281.** $(-1;0)\cup[1;\infty)$. **282.** $[2,6;4)$. **283.** (-1;3).

284. $(-\infty;-2)\cup[20,5;\infty)$ **285.** $(-9;4)$. **286.** $(-2;-1]\cup\left[-\frac{2}{3};-\frac{1}{3}\right)$.

287. $[-2;0)\cup(0;2]$. **288.** $(5;\infty)$. **289.** $\left(\frac{1}{2};+\infty\right)$. **290.** $(5;\infty)$

291. $\left(\frac{1}{2};+\infty\right)$. **292.** $x<\frac{3}{4};\ 4<x<7$. **293.** $x>4$. **294.** $-3<x<1$.

295. $0\le x\le 3$. **296.** $x<2\sqrt{2}-4$. **297.** $x\le 0;\ x>4,5$. **298.** $1\le x\le\frac{2}{\sqrt{3}}$.

299. $2<x\le 8$. **300.** $\frac{4}{5}$. **301.** $x<-2,\ 0<x<1,\ x>1$. **303.** $\left(-\infty;-\frac{1}{3}\right)\cup(3;\infty)$. **304.** $\left(1;\frac{1+\sqrt{5}}{2}\right)\cup\left(\frac{1+\sqrt{5}}{2};\infty\right)$. **305.** $\left[-2;\frac{2}{3}\right)\cup\left(\frac{4\sqrt{2}}{3};2\right]$. **306.** $\left(1;\frac{5}{4}\right)\cup\left(\frac{5}{3};\infty\right)$. **307.** $\left[-\frac{1}{2};\frac{11-\sqrt{153}}{4}\right)\cup(0;2)$.

9-§. Ko'rsatkichli va logarifmik tenglamalar

308. $\left\{0, \frac{\lg 1,5}{\lg 2,5}\right\}$. **309.** $\left\{2; \frac{1}{2}\right\}$. **310.** 25.

311. $\left\{-1-\sqrt{\frac{1}{2}\lg\left(1+\sqrt{11}\right)};\ -1+\sqrt{\frac{1}{2}\lg\left(1+\sqrt{11}\right)}\right\}$ **312.** $\{10\}$. **313.** $\{0;5\}$.

314. $\{1\}$. **315.** $\{2\lg 3+0,5\lg 7\}$. **316.** $x_1=3;\ x_2=-\frac{1}{5}$. **317.** $x=\pm\frac{1}{2}$.

318. $x_1=1000;\ x_2=0,1$. **319.** $x_1=\sqrt{2};\ x_2=\sqrt[4]{2}$. **320.** $x=-2$.

321. $x_{1,2}=1\pm\sqrt{1+\log_{2+\sqrt{3}}10}$. **322.** $\{-3;1\}$. **323.** $x=-\frac{5}{2}$. **324.** $x=\log_{\frac{3}{4}}\frac{21}{13}$. **325.** $x=3$. **326.** $\{-1;1\}$. **327.** $x=0$. **328.** $\{0;\log_{1,5}3\}$. **329.** $\left\{\frac{4}{3};\frac{5}{3}\right\}$.

330. $x=\varnothing$. **331.** $\{9;2\}$. **332.** $\{7;-1\}$. **333.** $\frac{1}{2}\left(\log_3 2\pm\sqrt{\log_3 2-4\log_3\frac{56}{153}}\right)$.

334. $\{2;-1-\log_5 2\}$. **335.** $x=0$. **336.** $\left\{\frac{lg19-lg3}{lg19+lg3-lg37};\frac{lg3-lg13}{lg19+lg3-lg37}\right\}$.

337. $\{3\}$. **338.** $x=2+\sqrt{3}$. **339.** $\{8\}$. **340.** $\{2\}$. **341.** $\{16\}$. **343.** $\sqrt{3}$. **344.** $\left\{\frac{1}{4\sqrt[5]{8}}\right\}$.

345. $x=1$. **346.** $x=29$. **347.** $x=9$. **348.** $x=-2$.

349. $x_1=9\ x_2=\frac{1}{9}$. **350.** $x=\sqrt[10]{10}$. **351.** $x_1=0,1;\ x_2=100;\ x_3=\sqrt{10}$. **352.** $x=1$ **353.** $x=4$. **354.** $x=4$. **355.** $x=-100$. **356.** $x=41$. **357.** $x=\frac{1}{9}$.

358. $\left\{\frac{3-2\sqrt{2}}{8};\frac{3+2\sqrt{2}}{8}\right\}$. **359.** $\{10;10^5\}$. **360.** $\sqrt{626}$. **361.** $x=\frac{1}{9}$. **362.** $\{1;2\}$.

10-§. Ko'rsatkichli va logarifmik tenglamalar sistemasi

363. $(1;1)$. **364.** $(3;2)$. **365.** $\left(\frac{1}{2};\frac{\sqrt{2}}{5}\right)$. **366.** $(4;1)$. **367.** $(3;2)$.

369. $\{(1;1),(4;2)\}$. 370. $\{(1;1),(2;4),(-2;4)\}$. 371. $\{(16;-28),(1;2)\}$. 372. $\left(\frac{\sqrt{5}-1}{2};\frac{3-\sqrt{5}}{2}\right)$. 373. $\{(1;2),(2;1)\}$. 374. $\left(\frac{1}{4};64\right),(8;2)$. 375. $(9;7)$. 376. $\{(2;32),(32;2)\}$. 377. $\left\{\left(\frac{1}{3};\frac{7}{3}\right),(3;1)\right\}$. 378. $(7;3)$. 379. $(17;9)$. 380. $(2;6)$.

381. $\{(125;4),(625;3)\}$. 382. $\{(3;27),(27;3)\}$. 383. $(5,5;2,5)$. 384. $\left(\frac{11}{5};3\right)$. 385. $\left(\sqrt{\frac{\sqrt{5}+1}{2}};\sqrt{\frac{\sqrt{5}-1}{2}}\right)$. 388. $(1;1),\left(\frac{\lg b}{\lg a};\frac{\lg a}{\lg b}\right)$. 389. $(2;6)$.

390. $(12;10),(-10;-12)$. 391. $(10;4),(4;10)$. 392. $\left(\frac{1}{1000};\frac{1}{2}\right),\left(1000;-\frac{1}{2}\right)$. 393. $(1;1),\left(\frac{27}{8};\frac{9}{4}\right)$.

11-§. Ko'rsatkichli va logarifmik tengsizlik

394. $(-\infty;-2)\cup\left(-\frac{3}{2};-1\right)$. 395. $(0;\infty)$. 396. $(-1;1)\cup(2;+\infty)$. 397. $(-\infty;2)$. 398. $(-2;2)$. 399. $(-1;1)$. 400. $(-\infty;1)$. 401. $\left(-\infty;\frac{1}{2}\right)\cup(1;+\infty)$. 402. $[0;4)$. 403. $\left(-\frac{1}{2};0\right)$.

404. $\left(-\infty;-2\frac{1}{2}\right)$. 405. $(-\infty;-\log_3 5)\cup(-1;-\log_3 2)$.

406. $(0;\log_3 2)\cup\left(1\frac{1}{2};\infty\right)$. 407. $\left(\frac{1}{2}\log_5 6;\log_6 5\right)$. 408. $\left(0;\frac{1}{2}\right)\cup(2;+\infty)$.

409. $(-\infty;0)$. 410. $(0;1)$. 411. $0<x\leq 2$. 412. $-\sqrt{3}\leq x\leq\sqrt{3}$. 413. $0<x<4$ 414. $0\leq x\leq 0,5$. 415. $x>2$. 416. $-3\leq x\leq -\sqrt{6}$. 417. $0<x<2$.

418. $-1<x<-\frac{1}{\sqrt{2}}$. 419. $-2<x\leq -1;-\frac{1}{2}\leq x\leq 0$. 420. $(-\infty;14)$. 421. $(3;+\infty)$. 422. $\left(-2;\frac{5}{2}\right)$. 423. $(2;\infty)$. 424. $(2;\infty)$.

425. $(-\infty;0)\cup(2;3)\cup(3;3,5)\cup(4;\infty)$. 426. $\frac{23}{32}$. 427. $[\log_5 7;2]$.

428. $\left(-1;0\right)\cup\left(1;2\right)$. **429.** $\left(-1;\frac{91}{9}\right)$. **430.** $\left(1;1,04\right)\cup\left(26;\infty\right)$.

431. $\left(-\infty;-2\right)\cup\left(6;\infty\right)$. **432.** $\left(1;3\right)$. **433.** $\left(2;\infty\right)$. **434.** $\left(\log_5\left(\sqrt{2}+1\right);\log_5 3\right)$ **435.** $\left(\log_5\left(\sqrt{2}+1\right);\log_5 3\right)$. **436.** $\left(-\sqrt{12};-2\right),\left(2;\sqrt{12}\right)$. **437.** $\left(-\infty;2\frac{1}{2}\right)\cup\left(3;\infty\right)$ **438.** $\left(4;6\right),\sqrt{22}$. **439.** $\left(2;\infty\right)$. **440.** $\left(0;0,75\right),\left(1,25;2\right)$. **441.** $(-\infty;0)\cup(1;2)\cup(2;3)\cup(4;\infty)$. **442.** $\left(\frac{2}{3};\infty\right)$. **443.** $\left(4^{\log_{0,5}0,2};\infty\right)$.

444. $\left(0;\frac{1}{2}\right),(1;2),(3;6)$. **445.** $\left(\sqrt{2};-1\right),\left(1;\sqrt{2}\right)$. **446.** $\left(-1\frac{1}{2};-1\right)\cup(4;5)$.

447. $(2;3)$. **448.** $\left(0;\frac{1}{8}\right)\cup(1;2)$. 449. $\left(\frac{1}{2};1\right)\cup(2;\infty)$. **450.** $\left(\sqrt{3};9\right)\cup\left(81;\infty\right)$. **451.** $\left(-\infty;-0,5\right]\cup\left[0,5;\infty\right)$. **452.** $(-2;-1)\cup(-1;0)\cup(2;\infty)$. **453.** $(1;4)$. **454.** $(20;\infty)$.

455. $\left[0;\frac{1}{3}\right)\cup\left(3;\frac{10}{3}\right]$.

12-§. Parametr qatnashgan tenglama va tengsizliklar

456. $a=1$. **457.** $\left(0;\frac{1}{2}\right)\cup\left(\frac{9}{2};5\right)$. **458.** $a\in(4,5;5)$. **459.** $a\in\left(\frac{2}{3};1\right)$ **460.** $b=-15,5$.

461. $p=1$. **462.** $a=\pm3$. **463.** $p=-6,\ p=3$. **464.** $c=\frac{1}{3}$. **465.** Agar $a\in\left(-\infty;0\right)$ bo'lsa, ildizlarga ega emas; $a=0$ bo'lsa, 2 ta diz; $a\in\left(0;8\right)$ bo'lsa, 4 ta ildiz; $a=8$ bo'lsa, 3 ta ildiz; $a\in\left(8;+\infty\right)$ bo'lsa, 2 ta ildiz. **466.** $a=-6$. **467.** $a=0$. **468.** $a\in\left[-8;\frac{4}{3}\right]; a=\frac{8}{13}, a=-4;\ -3,-1,1,3$ yoki $3,1,-1,-3$. **469.** $a\in\left[-10;\frac{2}{3}\right]; a=-\frac{23}{3}, a=-1;\ -2,-1,0,1,2$ yoki $2,1,0,-1,-2$. **470.** $a=-8;\ x_1=-4;\ x_2=2;\ x_3=-1$. **471.** $a=-2;\ x=1$. **472.** $a=0;\ x=\frac{2}{9}$. **473.** $a=\frac{3}{4}$

bo'lganda, $x=-\frac{1}{2}$ $a>1$ bo'lganda, $x=-\frac{1}{2}+\sqrt{a-\frac{3}{4}}$. **474.** $a<-3$ bo'lganda, $\emptyset$, $a=-3$ bo'lganda, $x=1$. $-3<a\le-2$ bo'lganda, $x_{1,2}=\frac{2\pm\sqrt{a+3}}{2}$ $a>-2$ bo'lganda, $x_{1,2}=\frac{2+\sqrt{a+3}}{2}$.

475. $a\in(-\infty;0)\cup(0;1)$ bo'lganda, $\emptyset$, $a=0$ bo'lganda, $x=0$ $a\in[1;+\infty)$ bo'lganda, $x=\frac{2a-1-\sqrt{4a-1}}{2}$. **476.** $a\in(-\infty;0)$ $a\in(-\infty;0)$ bo'lganda, $\emptyset$, $a=0$ bo'lganda, $x=0$, $a\in(0;1)$ $a\in(0;1)$ $a\in(0;1)$ bo'lganda, $\emptyset$, $a\in[1;+\infty)$ bo'lganda, $x=\frac{1}{2}(a-1)^2$. **477.** $a<2$ bo'lganda, $\emptyset$, $a\ge2$ bo'lganda, $x=\frac{a}{2}+\sqrt{a-1}$. **478.** $a<0$ bo'lganda, $\emptyset$, $a\ge0$ bo'lganda, $x=\frac{a-1+\sqrt{2a+1}}{2}$. **479.** Ixtiyoriy a uchun $x_1=-63-a, x_2=1-a$. **480.** $a<\frac{4}{3}$ bo'lganda, $x=12-2a+4\sqrt{8-3a}$. $\frac{4}{3}\le a\le\frac{8}{3}$. bo'lganda, $x_{1,2}=12-2a\pm4\sqrt{8-3a}$. $a=\frac{8}{3}$ bo'lganda, $x=\frac{20}{3}$. $a>\frac{8}{3}$. bo'lganda, $\varnothing$ **481.** $a\le-5$ bo'lganda, $\varnothing$, $a>-5$ bo'lganda, $x=(a+2)(a+8)$, **482.** $a<-1$ bo'lganda, , $-1\le a<0$ bo'lganda, $x=\frac{a^2+1}{2a}$, $0\le a<1$ bo'lganda, $\varnothing$, $a\ge1$ bo'lganda, $x=\frac{a^2+1}{2a}$. **483.** $a<0$ bo'lganda, $\varnothing$, $a>0$ bo'lganda, $x=\frac{3a}{4}$. **484.** $a+b=0$ bo'lganda, $\varnothing$, $a+b\ne0$ bo'lganda, $x=\frac{a-b}{2}$. **485.** Agar $a\le-1$ bo'lsa, yechim yo'q; agar $-1<a\le0$ bo'lsa, $x=\log_2(a+1)$ agar $a>0$ bo'lsa, $x_1=\log_2(a+1)$; $x_2=\log_2 a$. **486.** Agar $a<0$ bo'lsa, yechim yo'q; agar $0\le a<1$ bo'lsa, $x_{1,2}=a+2\pm2\sqrt{a}$; agar $a\ge0$ bo'lsa, $x=a+2+2\sqrt{a}$. **487.** Agar $a\le0$ bo'lsa, yechim yo'q; agar $0<a<1$ bo'lsa, $x_{1,2}=-1\pm\sqrt{1-\sqrt{a}}$; $x_3=-1+\sqrt{1+\sqrt{a}}$ agar $a=1$ bo'lsa, yechim yo'q; agar $a>1$

bo‘lsa, $x=-1+\sqrt{1+\sqrt{a}}$ **488.** Agar $a<0;\ a=1$ bo‘lsa, yechim yo‘q; agar $0<a<1; a>1$ bo‘lsa, $x=\frac{1}{a^2}$. **489.** $a\in[0;2]$.

490. $a\in\left[\frac{7}{5};3\right]\cup\{5\}$ **491.** $a\in\left[-\frac{4}{3};\frac{14}{9}\right)\cup\left(\frac{14}{9};3\right)$. **492.** $a\neq 0$: $x\in(-\infty;-4)\cup(0;+\infty)$. $a=0$: $x\in(-\infty;-4)\cup(0;1)\cup(1;+\infty)$. **493.** Agar $a=10$ bo‘lsa, $x<8$; $a<10$; bo‘lsa, $x<\frac{4a}{5}$; $x>\frac{16a+3}{20a-2a}$; $a>10$; bo‘lsa, $\frac{16a+3}{20a-2a}<x<\frac{4a}{5}$. **494.** Agar $a=1,\ a=-3,\ a=3$ bo‘lsa, yechim yo‘q; agar $-3<x<1$ bo‘lsa, $x>\frac{a+1}{a-3}$ agar $a<-3;\ 1<a<3;\ a>3$ bo‘lsa, $x<\frac{a+1}{a-3}$ **495.** $|x|\geq -\frac{|a|}{2}$ **496.** Agar $a=0$,bo‘lsa, yechim mavjud emas; agar $a\neq 0$ bo‘lsa, $-\frac{|a|}{\sqrt{5}}<x\leq|a|$.**497.** Agar $a<0$ bo‘lsa, yechim mavjud emas; agar $0<a<1$ bo‘lsa, $1-2\sqrt{a}<x<1+2\sqrt{a}$ agar $a>1$ bo‘lsa, $-a<x<1+2\sqrt{a}$ **498.** Agar $a<-2$ bo‘lsa, $|x|\leq 1$; agar $|a|\leq 2$ bo‘lsa, $-1\leq x\leq -\frac{-2a+\sqrt{5-a^2}}{5}$ agar $2<a\leq\sqrt{5}$ bo‘lsa, $\frac{-2a-\sqrt{5-a^2}}{5a}\leq x\leq\frac{-2a+\sqrt{5-a^2}}{5a}$ agar $a>\sqrt{5}$ bo‘lsa, yechim mavjud emas. **499.** Agar $a<4$ bo‘lsa, $x\geq\frac{4-a}{9}$; agar $a=4$ bo‘lsa, $x\geq 0$ agar $a>4$ bo‘lsa, $x\geq a-4$ **500.** Agar $a<5$ bo‘lsa, $x\geq 5-a$; agar $a=5$ bo‘lsa, $x\geq 0$ agar $a>5$ bo‘lsa, $x\geq\frac{a-5}{7}$ **501.** Agar $0<a<1$ bo‘lsa, $x<-\log_a(a+2)$; agar $a=1$ bo‘lsa, $x\in R$ agar $a>1$ bo‘lsa, $x>-\log_a(a+2)$ **502.** Agar $a\leq 0$ bo‘lsa, $x\in R$ agar $a>0$ bo‘lsa, $x\in(-\infty;-\log_4(a-1))\cup(-\log_4 a;+\infty)$ **503.** Agar $a<0$ bo‘lsa, $x\in(-\infty;-\log_2(-a-1))$; agar $a=0$ bo‘lsa, yechim yo‘q; agar $a>0$ bo‘lsa, **504.**

$x\in\left(-\infty;-\log_2(a-2)\right)$ Agar $a\le 4$ bo'lsa, $x\in\left(-\infty;\log_2\left(-\frac{a}{5}\right)\right)$; agar $-4<a<-2-3<-2-4<a<-2$ bo'lsa, $x\in\left(\log_5\frac{a+4}{5};\log_5\left(-\frac{a}{5}\right)\right)$; agar $a=-2$ bo'lsa, yechim yo'q; agar $-2<a<0$ bo'lsa, $x\in\left(\log_5\left(-\frac{a}{5}\right);\log_5\frac{a+4}{5}\right)$; agar $a\ge 0$ bo'lsa, $x\in\left(-\infty;\log_5\frac{a+4}{5}\right)$ **505.** Agar $a\le -1$ bo'lsa, $x\in\left[\log_3\left(-\frac{a}{3}\right);+\infty\right)$; agar $x\in\left[\log_3\left(-\frac{a}{3}\right);+\infty\right)$ bo'lsa, $x\in\left[-\infty;\log_3\frac{a+1}{3}\right)\cup\left[\log_2\left(-\frac{a}{3}\right);+\infty\right)$; agar $a=-\frac{1}{2}$ bo'lsa, $x\in R$; Agar $-\frac{1}{2}<a<0$ bo'lsa, $x\in\left(-\infty;\log_3\left(-\frac{a}{3}\right)\right]\cup\left[\log_3\frac{a+1}{3};+\infty\right)$; agar $a\ge 0$ bo'lsa, $\left[\log_3\frac{a+1}{3};+\infty\right)$; **506.** Agar $0<a<1$ bo'lsa, $x\in\left(a;\frac{1}{a}\right)$; agar $a>1$ bo'lsa, $x\in\left(0;\frac{1}{a}\right)\cup(a;+\infty)$; **507.** Agar $0<a<1$ bo'lsa, $x\in\left(1+\sqrt{1+a};+\infty\right)$; agar $a>1$ bo'lsa, $x\in\left(2;1+\sqrt{1+a}\right)$; **508.** Agar $a\le -8$ bo'lsa, yechim yo'q; agar $-8<a\le -3$ bo'lsa, $x\in\left[\frac{a-4}{6};1-\sqrt{1-a}\right)$; agar $-3<a<-2$ bo'lsa, $x\in\left[\frac{a-4}{6};-1\right]$; agar $a=-2$ bo'lsa, yechim yo'q; agar $a>-2$ bo'lsa, $x\in\left(-1;\frac{a-4}{6}\right]$

13-§. Parametr qatnashgan tenglama va tengsizliklar sistemasi

509. $a\ne -2$. **510.** $a<-5-4\sqrt{2};a>0$ **511.** $a\ge\frac{3}{4}$. **512.** $a\le -3;a\ge\frac{3}{4}$

513. $a<1;a=2$ **514.** $a=\frac{11}{12};a=1;a=3$. **515.** $a=\frac{1}{4}$. **516.** $a=-1;a=0$.

517. $(0;a),(a;0), a\in R$. 518. $a<0$ da $\varnothing$, $a\geq 0$ da $(9a^2;a^2)$. 519. $a<1$ da $a>1$ da $\left(\frac{(a+1)^2}{4};\frac{(a+1)^2}{4}\right)$. 520. $a\leq -2, a\geq 0$. 521. $-6\leq a\leq -5; a=-2; a=-1$. 522. $a=\frac{\sqrt{3}}{\sqrt{2}}$ bo'lganda, $x=3-\sqrt{6}$, $y=-3-\sqrt{6}$, $a=-\frac{\sqrt{3}}{\sqrt{2}}$ bo'lganda, $x=3+\sqrt{6}; y=-3+\sqrt{6}$, $-4<a<-2-3<a<-2$ $a=-1$, bo'lganda, $x=3; y=0$, $a=1$ bo'lganda, $x=0; y=-3$. 523. $a\leq -1$ bo'lganda, $\varnothing$; $-1<a<0$ bo'lganda, $-a<x<-\frac{1}{3}$ $a=0$; bo'lganda, $x>0; 0<a\leq 1$ bo'lganda, $x>-a$; $a>1$ bo'lganda, $a>1$. 524. $a<-3$ bo'lganda, $a+3<x<0; a=-3$ bo'lganda, $\varnothing$; $-3<a<-2$ bo'lganda, $0<x<a+3; -2\leq a\leq 0$ bo'lganda, $0<x<1; 0<a<1$; bo'lganda, $a<x<1$; $a=1$ bo'lganda, $\varnothing$; $a>1$ bo'lganda, $1<x<a$ 525. Agar $a-b<1$ bo'lsa, yechim mavjud emas; agar $a-b\geq 1$ bo'lsa, $x=y=\frac{(a-b)^2-2(a+b)+1}{4}$

526. $a=-6$ 527. $a>-1$ 528. $a\in(-\infty;-4]\cup[4;+\infty)$.. 529. $a\in\left[-\frac{7}{2};1\right]$.

IV bob. Trigonometrik funksiyalar va ular orasidagi munosabatlar

1-§. Haqiqiy argumentli tigonometrik funksiyalar

1. $1-p^2$. 2. 0.96. 3. $\frac{23}{32}$. 4. $\frac{3}{4}$ 5. $\frac{4\sqrt{6}}{23}$ 6. $-1\frac{41}{44}$. 7. $-\frac{50}{7}$ 8. $\frac{3-n^2}{2}.n$ 9. $-\frac{9}{4}$

11. $\frac{1}{\cos\alpha}$ 12. $tg\left(\frac{\alpha}{2}-\frac{\pi}{8}\right)$ 13. $\frac{\sqrt{2}}{2}\sin\alpha$ 14. $\sin^2 35$ 15. $ctg6\alpha$ 16. 7.

17. 2 agar $2\pi k<x<\pi+2\pi k(k\in Z)$, -2 agar $2\pi k<x<2\pi+2\pi k(k\in Z)$

18. $2\sin\alpha$. 19. $-\cos 2\alpha\cos\frac{\alpha}{3}$. 20. $\frac{tg2\alpha}{2}$. 21. $tg^2\alpha$ 22. $\frac{\sin 2\alpha}{\sqrt{2}}$. 23. $\cos 4\alpha$ 24. $ctg4\alpha$. 25. $tg4\alpha$. 26. 1. 27. $\cos(40+2\alpha)$. 28. $\sin 4\alpha$. 29. $8\sqrt{3}$. 30. $2|\mathrm{cosec}2\alpha|$. 31. 1.

Yig'indini ko'paytmaga keltirishga oid misollar

78. $a)2\sin55^0\cos25^0$ $b)2\sin25^0\cos55^0$ **79.**

$a)2\cos61{,}5^0\cos13{,}5^0$ $b)-2\sin52{,}5^0\sin22{,}5^0$ **80.** $a)2\sin1\cos4,\ b)-2\sin x\sin4x$

81. $a)\sqrt{2}\cos\left(\frac{\pi}{4}-2x\right), b)-2\sin3\sin x$.

82. $a)2-\sqrt{3},\ b)tg10^0$ **83.** $a)\frac{2}{\cos130^0},\ b)2\cos50^0$ **84.** $a)\frac{\sqrt{2}}{\sin2x},\ b)\frac{\sqrt{2}}{\cos2x}$

85. $a)1-2\cos100^0,\ b)\frac{1}{2}\sin70^0$ **86.** $a)\frac{1}{2}(\cos\beta-\cos\alpha)$,

$b)1+tg\alpha=\frac{\sqrt{2}\cos\left(\frac{\pi}{4}-\alpha\right)}{\cos\alpha}, 1-tg\alpha=\frac{\sqrt{2}\sin\left(\frac{\pi}{4}-\alpha\right)}{\cos\alpha}$ **88.** $a)0$, $b)0$

89. $a)4\cos7^0\sin18^0\cos36^0,\ b)4\sin7^0\sin18^0\cos18^0$ **90.** $\sin1^0$ **91.** $\frac{4}{\sqrt{3}}+\frac{2}{\cos10^0}$

92. $2\cos\left(\frac{\pi}{6}+\frac{\alpha}{2}\right)\cos\left(\frac{\pi}{6}-\frac{\alpha}{2}\right)$ $b)\frac{2\sin\left(\frac{\pi}{3}+\alpha\right)}{\cos\alpha}$ **93.** $a)4\sin\left(\frac{\pi}{6}-\frac{\alpha}{2}\right)\cos\left(\frac{\pi}{6}+\frac{\alpha}{2}\right)$

$b)\frac{2\sqrt{3}\sin\left(\frac{\pi}{3}+\alpha\right)}{\cos\alpha}$ **94.** $a)16\sin\left(\frac{\pi}{6}-\frac{\alpha}{2}\right)\cos\left(\frac{\pi}{6}+\frac{\alpha}{2}\right)\sin\left(\frac{\pi}{6}+\frac{\alpha}{2}\right)\cos\left(\frac{\pi}{6}-\frac{\alpha}{2}\right)$

$b)\frac{4\sin\left(\frac{\pi}{3}+\alpha\right)\sin\left(\frac{\pi}{3}-\alpha\right)}{\cos^2\alpha}$ **95.** $2\cos^2\frac{\alpha}{2}tg\alpha$, $b)2\sin^2\frac{\alpha}{2}tg\alpha$ **96**

$a)-2\sqrt{2}\cos\frac{\alpha}{2}\sin(\frac{\alpha}{2}-\frac{\pi}{4})$, $b)2\sqrt{2}\sin\frac{\alpha}{2}\sin(\frac{\alpha}{2}-\frac{\pi}{4})$ **97.** $\sin\left(\frac{\pi}{4}+\alpha\right)\cos\left(\frac{\pi}{4}-\alpha\right)$

Ba'zi trigonometrik yig'indi va ko'paytmalarni hisoblash

118. $a)\frac{5-12\sqrt{3}}{26}$. **119.** $\frac{\sqrt{2}}{2}$. **121.** $\frac{1}{\sqrt{10}};\frac{3}{\sqrt{5}};\frac{1}{3}$. **122.** $-\frac{4}{5};-\frac{3}{5};\frac{4}{3}$. **124.**

$a)\frac{t^2-1}{2}$ $b)\sqrt{2-t^2}$ $v)\frac{2-(t^2-1)}{2}$. **125.** $a)t^2-2$ $b)t^3-3t$. **126.** 2. **127.**

$\frac{4}{2-\sqrt{3}}$. **128.** $\frac{7\sqrt{2}}{26}$. **129.** $\frac{26}{87}$. **130.** $\frac{3}{2}$. **131.** $-\frac{1}{\sqrt{2-\sqrt{2}}}$. **134.** $-\frac{9}{4}$.

11-§. Teskari trigonometrik funksiyalar va ularning asosiy xossalari, grafigi

157. $\frac{\sqrt{3}}{6}-\frac{\sqrt{2}}{2}$. 158. $-\frac{33}{56}$. 159. $\frac{a}{\sqrt{2}}$. 160. $-\frac{\sqrt{3}}{3}$. 161. $a)\frac{\pi}{4}$ $b)\frac{\pi}{4}$. 162. $\sqrt{\frac{3}{11}}$.

163. $\frac{8\sqrt{5}}{81}$. 164. $\frac{56}{65}$. 165 0. 166. $\frac{4}{5}$.

V.BOB. Trigonometrik tenglamalar va tengsizliklar. Trigonometrik tenglamalar va tengsizliklar sistemasi

1-§. Trigonometrik tenglamalar va ularni yechish usullari

1. $(-1)^{n+1}\frac{\pi}{6}+\frac{\pi n}{2},\ n\in\mathbb{Z}$. 2. $\frac{\pi}{3}+\frac{\pi n}{2},\ n\in\mathbb{Z}$. 3. $\varnothing$. 4. $-\frac{\pi}{3}+\pi n,\ n\in\mathbb{Z}$. 5. $\pm\frac{\pi}{3}+2\pi n,\ n\in\mathbb{Z}$. 6. $\pi n,\ n\in\mathbb{Z}$. 7. $\pm\frac{\pi}{6}+2\pi n,\ n\in\mathbb{Z}$. 8. $\pi n;\ \frac{\pi}{2}\pm\pi n,\ n\in\mathbb{Z}$. 9. $\frac{\pi}{8}+\frac{\pi n}{2},\ n\in\mathbb{Z}$. 10. $2\pi n;\ \frac{\pi}{3}+2\pi n,\ n\in\mathbb{Z}$. 11. $\pm\frac{\pi}{6}+\frac{\pi n}{2},\ n\in\mathbb{Z}$. 12. $\pm\frac{\pi}{6}+\pi n,\ n\in\mathbb{Z}$.

13. $arctg\frac{3}{2}+(-1)^n\arcsin\frac{\sqrt{13}}{16}+\pi n,\ n\in\mathbb{Z}$ 14. $\frac{\pi}{6}+(-1)^n\arcsin\frac{\pi}{8}+\pi n,\ n\in\mathbb{Z}$ 15. $\frac{\pi}{2}+2\pi n,\ n\in\mathbb{Z}$. 16. $-\frac{\pi}{18}+arctg\frac{-2\pm\sqrt{2}}{2}+\pi n,\ n\in\mathbb{Z}$. 17. $\frac{2\pi n}{3};\ \frac{\pi}{2}+2\pi n;\ \frac{\pi}{4}+\pi n,\ n\in\mathbb{Z}$.

18. $-\frac{\pi n}{2},\ n\in\mathbb{Z}$ 19. $2\pi n,\ \frac{\pi}{4}+\pi n,\ \frac{\pi}{2\pi}+2\pi n,\ n\in\mathbb{Z}$. 20. $\frac{\pi n}{2};\ \pm\frac{\pi}{6}+\pi n,\ n\in\mathbb{Z}$.

21. $\frac{\pi}{6}+\frac{\pi n}{3},\ n\in\mathbb{Z}$. 22. $\frac{\pi}{4}+\frac{\pi k}{2},\ k\in 4n+1,\ n\in\mathbb{Z}$. 23. $\frac{\pi n}{3},\ n\in\mathbb{Z}$. 24. $\varnothing$. 25. $\frac{\pi}{40}+\frac{\pi n}{10},\ \frac{\pi}{20}+\frac{\pi n}{5},\ n\in\mathbb{Z}$. 26. $\frac{\pi}{6}+\frac{2\pi n}{3},\ n\in\mathbb{Z}$. 27. $\pm\frac{\pi}{3}+\pi n,\ n\in\mathbb{Z}$. 28. $(-1)^n\frac{\pi}{6}+\pi n,\ n\in\mathbb{Z}$. 29. $\pi n,\ n\in\mathbb{Z}$. 30. $\pm\frac{\pi}{6}+\pi n,\ n\in\mathbb{Z}$. 31. $\frac{\pi}{2}+2\pi n,\ (-1)^n\frac{\pi}{6}+2\pi n,\ n\in\mathbb{Z}$. 32. $arctg(-1\pm\sqrt{3})+\pi n,\ n\in\mathbb{Z}$. 33. $\frac{\pi}{24}+\pi n,\ \frac{7\pi}{24}+\pi n,\ n\in\mathbb{Z}$. 34. $\frac{2}{3}-\frac{\pi}{36}+\frac{\pi n}{3},\ \frac{1}{2}+\frac{5\pi}{40}+\frac{\pi n}{4},\ n\in\mathbb{Z}$.

35. $\frac{\pi n}{3}, \frac{\pi}{2}+\pi n, n\in \mathbb{Z}$. 36. $\frac{\pi n}{2}, \pm\frac{\pi n}{3}+2\pi n, n\in \mathbb{Z}$. 37. $\frac{\pi}{4}+\pi n, (-1)^n\frac{\pi}{18}+\frac{\pi n}{3}, n\in \mathbb{Z}$. 38. $arctg\frac{3\pm\sqrt{6}}{3}+\pi n, n\in \mathbb{Z}$. 39. $\varnothing$.

40. $(-1)^n\frac{\pi}{6}+\pi n, n\in \mathbb{Z}$. 41. $\pm\frac{2\pi}{3}+2\pi n, n\in \mathbb{Z}$. 42. $0; \frac{\pi}{8}; \frac{\pi}{4}; \frac{\pi}{2}; \frac{5\pi}{8}; \frac{3\pi}{4}; \pi$.

43. $\frac{\pi}{2}+2\pi n, n=\pm1,\pm2$ $(-1)^n\frac{\pi}{6}+\pi n, n=\pm1,\pm2,\pm3$. 44. $\pm\frac{\pi}{2}+2\pi n, n\in \mathbb{Z}$

45. $\pi+2\pi n, (-1)^n\frac{\pi}{2}+2\pi n, n\in \mathbb{Z}$. 46. $(-1)^n\frac{\pi}{6}+\pi n, n\in \mathbb{Z}$. 47. $\frac{\pi}{4}+\pi n, n\in \mathbb{Z}$.

48. $\frac{\pi}{8}+\frac{\pi n}{4}, n\in \mathbb{Z}$ 49. $\frac{2}{3}\pm\frac{2\pi}{3}+\frac{2\pi n}{3}, n\in \mathbb{Z}$

50. $\left\{-\frac{\pi}{6}+2\pi n, n\in \mathbb{Z}\right\}\cup\left\{-\frac{\pi}{6}+2\pi n, n\in \mathbb{Z}\right\}=(-1)^n\frac{\pi}{6}+\pi n, n\in \mathbb{Z}$

51. $\{2\pi n, n\in \mathbb{Z}\}\cup\left\{-\frac{\pi}{4}+\pi n, n\in \mathbb{Z}\right\}$. 52. $\pm\frac{1}{2}\arccos\frac{\sqrt{21}-3}{3}+\pi n, n\in Z$.

53. $\{2\pi+4\pi n, n\in \mathbb{Z}\}\cup\{(-1)^n 2\pi+4\pi n, n\in \mathbb{Z}\}$. 54. $\left\{-\frac{3\pi}{4}, 0, \frac{3\pi}{4}, \frac{5\pi}{4}, 2\pi\right\}$.

55. $\varnothing$. 57. $\{2\pi n, n\in \mathbb{Z}\}\cup\left\{-\frac{\pi}{6}+2\pi n, n\in \mathbb{Z}\right\}$. 58. $\left\{-\frac{\pi}{2}+2\pi n, n\in \mathbb{Z}\right\}\cup\{\arcsin(\sqrt{5}-2)+2\pi n\}\cup\left\{-\frac{\pi}{6}+2\pi n, n\in \mathbb{Z}\right\}$.

59. $\left\{\frac{\pi}{4}+2\pi n, n\in \mathbb{Z}\right\}$. 60. $\left\{-\frac{\pi}{2}+2\pi n, n\in \mathbb{Z}\right\}$.

2-§. Teskari trigonometrik funksiyalar qatnashgan tenglamalar

61. $-tg\frac{3}{2}$. 62. $\{-2;-1\}$. 63. $\varnothing$. 64. $\{0\}$. 65. $\{1\}$. 66. $\left\{-\frac{1}{12}\right\}$. 76. $\{2\}$. 68. $\left\{-\frac{1}{2}; \frac{1}{2}; -\frac{\sqrt{3}}{2}; \frac{\sqrt{3}}{2}\right\}$. 69. $\left\{\frac{1}{2}\right\}$. 70. $\left[\frac{\sqrt{2}}{2}; 1\right]$. 71. 4. 72. $\sin\frac{1}{2}$ 73. $tg\frac{2\pi}{3}$ 74. $\frac{-1\pm\sqrt{3}}{2}$

75. $\frac{2}{3}$ 76. 0 77. $\varnothing$. 78. 0 79. $\frac{1}{2}$ 80. $\frac{1}{\sqrt{3}}$ 81. 0 82. $\frac{\sqrt{3}}{2}$ 83. $\frac{1}{\sqrt{5}}$ 84. 2 85. $\sqrt{\frac{2}{17}(5-2\sqrt{2})}$. 86. $\sqrt{2}$ 87. 1 va $-\frac{1}{6}$ 88. $\frac{1}{\sqrt{5}}$

89. $\frac{\pi}{4}+\pi k, k\in\mathbb{Z}.$

3-§. Trigonometrik tenglamalar sistemasi

90. $x=\pm\frac{\pi}{2}+\frac{5\pi}{2}+\pi n;\quad y=\pm\frac{\pi}{2}+\frac{5\pi}{12}+\pi n,\quad n\in\mathbb{Z}$

91. $x=\pm\frac{\pi}{6}+\frac{\pi}{6}+\pi n;\quad y=\pm\frac{\pi}{6}-\frac{\pi}{6}+\pi n,\quad n\in\mathbb{Z}$ **92.**

$\left(\frac{\pi}{4}+\pi n,\ -\frac{\pi}{4}+\pi n\right)\cup\left(arctg\frac{2}{3}+\pi n,\ arctg\frac{4}{3}+\pi n\right),\quad n\in\mathbb{Z}.$

93. $x_1=(-1)^n\frac{\pi}{6}+\pi n;\quad x_2=(-1)^{n+1}\frac{\pi}{6}+\pi n,$

$y_1=\pm\frac{2\pi}{3}+2\pi n,\quad y_2=\pm\frac{\pi}{3}+2\pi n,\quad n\in\mathbb{Z}$

94 $x_1=\frac{\pi}{4}+\pi n,\quad x_2=-\frac{\pi}{4}+\pi n,\quad y_1=arctg2+2\pi n,\quad y_2=-arctg2+2\pi n,$

$z_1=\frac{3\pi}{4}-arctg2-\pi(k+n),\quad z_2=\frac{5\pi}{4}+arctg2-\pi(k+n),\quad n\in\mathbb{Z}.$

95. $\{\pi;-2\pi+2\pi n\},\quad n\in\mathbb{Z}.$ **96.**

$\left\{-\frac{\pi}{4}+\pi n,-\frac{\pi}{6}-\pi n\right\}\cup\left\{-\frac{\pi}{6}+\pi n,-\frac{\pi}{6}-\pi n\right\}\cup$

$\left\{-\frac{\pi}{6}+\pi n,-\frac{\pi}{4}-\pi n\right\}, n\in\mathbb{Z}.\ \left(\frac{\pi}{6}+2\pi n;\ \frac{\pi}{4}+2\pi n\right),\ \left(\frac{5\pi}{6}+2\pi n;\ \frac{3\pi}{4}+2\pi n\right);$

97. $\left(\frac{\pi}{6}+2\pi n;\ \frac{\pi}{4}+2\pi n\right),\ \left(\frac{5\pi}{6}+2\pi n;\ \frac{3\pi}{4}+2\pi n\right);$

98. $\left(\frac{\pi}{6}+2\pi n;\ \frac{\pi}{4}+2\pi n\right),\ \left(\frac{5\pi}{6}+2\pi n;\ \frac{3\pi}{4}+2\pi n\right);$

$\left(-\frac{\pi}{6}+2\pi n;\ -\frac{\pi}{4}+2\pi n\right),\ \left(-\frac{5\pi}{6}+2\pi n;\ -\frac{3\pi}{4}+2\pi n\right),\quad n\in\mathbb{Z}.$

99. $\left(\frac{\pi}{2}+4\pi n;\ 4\pi n\right),\ \left(4\pi n;\ \pm\frac{\pi}{2}+4\pi n\right),\quad n\in\mathbb{Z}.$

100. $\left(\frac{\pi}{6}+\pi(n+k);\ \frac{\pi}{6}+\pi(n-k)\right),\ \left(-\frac{\pi}{6}+\pi(n+k);\ -\frac{\pi}{6}+\pi(n-k)\right),\quad k,n\in\mathbb{Z}$

101. $\left(\alpha+\pi k;\ \frac{\pi}{4}-\alpha-\pi k\right),\ \left(\beta+\pi k;\ \frac{\pi}{4}-\beta-\pi k\right)$ bunda

$\alpha=\mathrm{arctg}\frac{-7+\sqrt{97}}{8},\quad \beta=arctg\frac{-7-\sqrt{97}}{8}$

102. $(2\pi n;\pi+2\pi n)$, $\left(\pi+2\pi n;-\frac{\pi}{2}+2\pi n\right)$, $\left(\frac{\pi}{3}+2\pi n;\frac{\pi}{6}+2\pi n\right)$, $\left(\frac{2\pi}{3}+2\pi n;\frac{7\pi}{6}+2\pi n\right)$, $\left(-\frac{\pi}{3}+2\pi n;\frac{5\pi}{6}+2\pi n\right)$, $\left(\frac{4\pi}{3}+2\pi n;-\frac{\pi}{6}+2\pi n\right)$, $n\in\mathbb{Z}$.

103. $(2\pi n;\ \pi+2\pi n)$, $n\in\mathbb{Z}$. **104.**

$\left(\left(\frac{\pi}{6}+\pi n, -\frac{\pi}{6}+\pi n\right), \left(-\frac{\pi}{6}+\pi n, \frac{\pi}{6}+\pi n\right)\right)$, $n\in\mathbb{Z}$.

105. $x=75^\circ\pm180^\circ k$, $y=60^\circ\pm180^\circ k$, $k\in\mathbb{Z}$.

106. $x=30^\circ\pm180^\circ k$, $y=60^\circ\pm180^\circ k$, $k\in\mathbb{Z}$. **107.**

$x=\pm\frac{\pi}{3}+\pi k$, $y=\pm\frac{\pi}{3}+\pi k$, $k\in\mathbb{Z}$. **108.** $x=2k+\frac{1}{2}$, $y=\frac{1}{6}-2k$, $k\in\mathbb{Z}$. **109.**

$x_1=\frac{1}{3}+k$, $y_1=\frac{1}{3}+k$, $x_2=-\frac{1}{3}+k$, $y_2=-\frac{1}{3}+k$, $k\in\mathbb{Z}$.

4-§. Trigonometrik tengsizliklar

110. $\mathrm{arctg}\,3+\pi n\le x<\frac{\pi}{2}+2\pi n$, $n\in\mathbb{Z}$. **111**

$\frac{5\pi}{6}-1+2\pi n\le x\le\frac{7\pi}{6}-1+2\pi n$, $n\in\mathbb{Z}$. **112.**

$-\pi-\arcsin\frac{1}{4}+2\pi n<x<2\pi n+\arcsin\frac{1}{4}$, $\arcsin\frac{1}{3}+2\pi n<x<\pi-\arcsin\frac{1}{3}+2\pi n$, $n\in\mathbb{Z}$

113.

$\pi+2\pi n<x<\frac{3\pi}{2}+2\pi n$, $\arcsin\frac{2\sqrt{2}}{3}-\frac{\pi}{4}+2\pi n<x<\frac{3\pi}{4}-\arcsin\frac{2\sqrt{2}}{3}+2\pi n$, $n\in\mathbb{Z}$.

114. $\varnothing$. **115.** $\frac{\pi}{8}+\frac{\pi n}{2}<x<\frac{3\pi}{8}+\frac{\pi n}{2}$, $x\ne\frac{\pi}{4}+\frac{\pi n}{3}$, $n\in\mathbb{Z}$.

116. $\pi n\le x<\frac{\pi}{4}+\pi n$, $n\in\mathbb{Z}$ **117.** $\left[0;\arccos\frac{\sqrt{6}-1}{2}\right]$. **118.**

$\left\{x\,/\,\mathrm{argsin}\frac{1}{3}+2\pi n<x<\pi-\mathrm{arg\,sin}\frac{1}{3}+2\pi n,\ n\in\mathbb{Z}.\right\}$

119. $\left\{-\frac{2\pi}{3}+2\pi n<x<\frac{2\pi}{3}+2\pi n,\quad n\in\mathbb{Z}.\right\}$

120. $\left\{x\,/\,\mathrm{argtg}\,2+\pi n\le x<\frac{\pi}{2}+\pi n,\quad n\in\mathbb{Z}.\right\}$

121. $\left\{x/\ \pi n<x<\frac{5\pi}{6}+\pi n,\ \ n\in\mathbb{Z}.\right\}$

122. $\left\{x/1-\frac{2\pi}{3}+2\pi n\le x\le 1-\frac{\pi}{3}+2\pi n,\quad n\in\mathbb{Z}.\right\}$

123. $\left\{x/1-\frac{3\pi}{4}+\pi n<x<1+\pi+\pi n,\ \ n\in\mathbb{Z}.\right\}$ 124. $R/\left\{\frac{5\pi}{4}+2\pi n/\ \ n\in\mathbb{Z}\right\}.$

125. $\left\{x/-\frac{\pi}{4}+\pi n<x\le\pi n\right\}\cup\left\{x/\frac{\pi}{4}+\pi n<x<\frac{\pi}{2}+\pi n,\quad n\in\mathbb{Z}\right\}$

126. $\left\{x/\ \pi n<x<\frac{\pi}{4}+\pi n,\quad n\in\mathbb{Z}.\right\}$ 127. $\left\{x/-\frac{\pi}{2}+2\pi n<x<\frac{\pi}{2}+2\pi n,\quad n\in\mathbb{Z}.\right\}$

128. $\left\{x/-\frac{5\pi}{6}+2\pi n<x<-\frac{3\pi}{4}+2\pi n\right\}\cup\left\{x/-\frac{\pi}{4}+2\pi n<x<-\frac{\pi}{6}+2\pi n\right\};\ n\in\mathbb{Z}$

129. $\left\{x/\frac{\pi}{6}+\frac{2\pi n}{3}<x<\frac{7\pi}{18}+\frac{2\pi n}{3}\right\}\cup\left\{x/\frac{\pi}{2}+\frac{2\pi n}{3}<x<\frac{11\pi}{8}+\frac{2\pi n}{3}\right\};\quad n\in\mathbb{Z}$ 130.

$\frac{\pi}{3}+\pi n<x<\pi+2\pi n;\ n\in\mathbb{Z}$ 131. $\frac{\pi}{12}+\frac{2\pi n}{3}<x<\frac{5\pi}{12}+\frac{2\pi n}{3};\quad n\in\mathbb{Z}$

132. $-\frac{\pi}{12}+\frac{\pi n}{3}<x<\frac{\pi}{12}+\frac{\pi n}{3};\ \ n\in\mathbb{Z}$ 133. $\frac{\pi}{4}+\pi n<x<\operatorname{arctg}(\frac{1}{3})+\pi n;\ \ n\in\mathbb{Z}$

134. $-\frac{\pi}{4}+2\pi n<x<\frac{\pi}{2}+2\pi n;\ \ \frac{\pi}{2}+2\pi n<x<\frac{5\pi}{4}+2\pi n,\quad n\in\mathbb{Z}$

135. $-\frac{\pi}{6}+2\pi n\le x<2\pi n;\ \frac{\pi}{3}+2\pi n\le x<\pi+2\pi n,\ \ n\in\mathbb{Z}$

136. $\frac{\pi}{18}+2\pi n<x<\frac{3\pi}{4}+2\pi n;\ \ \frac{7\pi}{12}+2\pi n<x<\frac{7\pi}{4}+2\pi n,\quad n\in\mathbb{Z}$

137. $-\frac{\pi}{10}+\frac{2\pi n}{5}<x<-\frac{\pi}{30}+\frac{2\pi n}{5};\quad \frac{\pi}{10}+\frac{2\pi n}{5}<x<\frac{7\pi}{10}+\frac{2\pi n}{5},\quad n\in\mathbb{Z}.$

138. $-\frac{\pi}{8}+\pi n<x<\pi n;\ \ \frac{\pi}{2}+\pi n<x<\frac{5\pi}{8}+\pi n;\ \ \frac{\pi}{8}+\pi n<x<\frac{3\pi}{8}+\pi n,\quad n\in\mathbb{Z}.$

139. $\frac{\pi}{3}+2\pi n<x<\frac{\pi}{3}+2\pi n,\quad n\in\mathbb{Z}.$

Test variantlari javoblari

1-variant

1.A 2.B 3.C 4.B 5.A 6.A 7.B 8.A 9.B 10.B 11.C 12.A 13.C 14.A 15.A

2-variant

1.A 2.A 3.C 4.A 5.A 6.A 7.C 8.A 9.A 10.C 11.A 12.B 13.B 14.A 15.A

3-variant

1.A 2.A 3.D 4.C 5.A 6.A 7.A 8.A 9.A 10.A 11.C 12.C 13.C 14.A 15.A

4-variant

1.A 2.A 3.A 4.A 5.A 6.A 7.A 8.A 9.A 10.B 11.A 12.A 13.C 14.A 15.A

5-variant

1.A 2.A 3.A 4.A 5.A 6.A 7.C 8.A 9.A 10.A 11.A 12.B 13.B 14.D 15.D

6-variant

1.A 2.A 3.A 4.A 5.A 6.A 7.D 8.C 9.A 10.D 11.B 12.B 13.D 14.A 15.A

7-variant

1.A 2.A 3.A 4.C 5.A 6.A 7.D 8.B 9.A 10.C 11.D 12.D 13.A 14.B 15.A

8-variant

1.A 2.A 3.A 4.A 5.A 6.A 7.D 8.B 9.C 10.B 11.A 12.B 13.C 14.D 15.A

9-variant

1.A 2.B 3.A 4.A 5.A 6.A 7.C 8.C 9.B 10.B 11.A 12.D 13.C 14.D 15.B

10-variant

1.A 2.B 3.A 4.B 5.A 6.A 7.A 8.D 9.A 10.C 11.A 12.B 13.B 14.A 15.C

11-variant

1.A 2.B 3.A 4.A 5.B 6.B 7.A 8.C 9.B 10.C 11.A 12.A 13.A 14.A 15.C

12-variant

1.A 2.C 3.D 4.A 5.D 6.B 7.C 8.A 9.C 10.A 11.A 12.C 13.A 14.A 15.C

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MUNDARIJA

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ALGEBRA. TRIGONOMETRIYA

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