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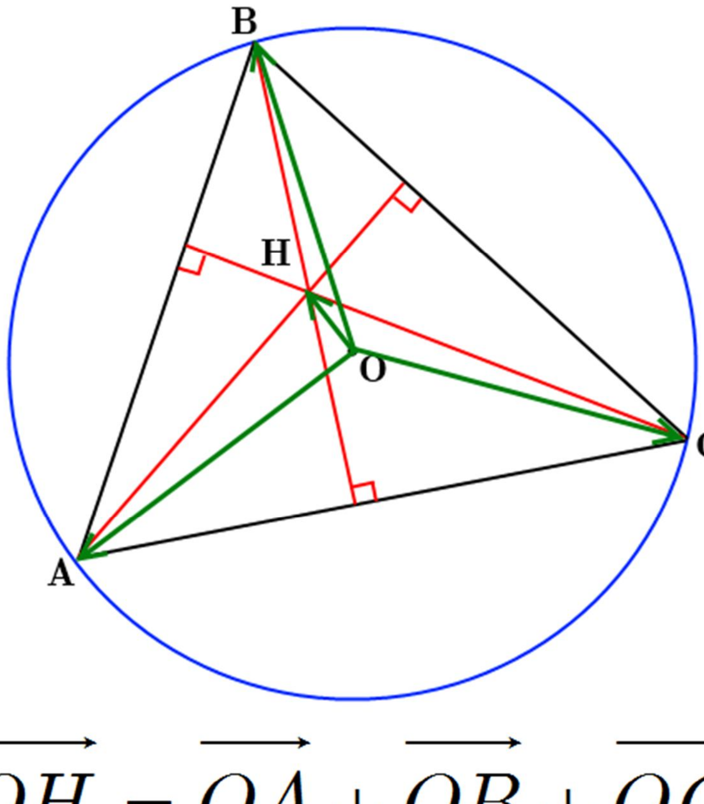
MATEMATIKADAN SIRTQI OLIMPIADA MASALALARI



$$\overrightarrow{OH} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$$

O'ZBEKISTON RESPUBLIKASI OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

URGANCH DAVLAT UNIVERSITETI

XORAZM VILOYATI XALQ TA'LIMI BOSHQARMASI

Kamalov Ne'matjon Bahodirovich
Olimbayev To'lqin G'ayrat o'g'li

MATEMATIKADAN SIRTQI OLIMPIADA MASALALARI

(Uslubiy qo'llanma)

Uslubiy qo'llanma Xorazm viloyati Xalq ta'limi xodimlarini qayta tayyorlash va ularning malakasini oshirish hududiy markazining 2020-yil 30-iyundagi 4-sonli ilmiy-metodik kengashi yig'ilishi bayonnomasiga asosan nashrga tavsiya etilgan.

Urganch–2020

Ne'matjon Kamalov, To'lqin Olimbayev
Matematikadan sirtqi olimpiada masalalari

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Mazkur qo'llanmada Xalq ta'limi vazirligining eduportal.uz va Urganch davlat universitetining olimp.urdu.uz saytlarida o'tkazilgan sirtqi Matematika olimpiadalarida taklif qilingan masalalar, tuman, viloyat va respublika fan olimpiadalarida taklif qilingan masalalar joy olgan.

Qo'llanma 4 bobdan iborat bo'lib, 1-bobda berilgan 300 ta masalaning to'liq yechimlari 4-bobda berilgan. 2-bobda mustaqil yechishga berilgan 150 ta masalada javoblar va ko'rsatmalar keltirilgan. 3-bob 200 ta testdan iborat bo'lib, testlarning kalitlari berilgan.

Qo'llanma umumta'lim va davlat ixtisoslashtirilgan maktablarning 9-11-sinf o'quvchilari va akademik litseylarning 1-2-kurs talabalari uchun mo'ljallangan.

Mas'ul muharrir **Quvondiq Kamolov**, oliy toifali matematika fani o'qituvchisi.

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M U N D A R I J A

SO‘ZBOSHI

Mamlakatimizda Matematika sohasini rivojlantirishga juda katta e’tibor qaratilmoqda. Muhtaram Prezidentimiz tomonidan 2020-yil 7-may kuni qabul qilingan ”Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida” gi qarorini alohida ta’kidlash mumkin. Mazkur qaror O‘zbekiston matematiklari uchun yangi davrni boshlab beradi. O‘quvchilar xalqaro matematika olimpiadalarida sovrinli o‘rinlarni qo‘lga kiritsa, o‘quvchilar va ularning ustozlariga katta miqdordagi pul mukofotlari berish yo‘lga qo‘yildi. Bu o‘z navbatida olimpiadalarga bo‘lgan qiziqishni yanada orttiradi.

Mazkur qo‘llanma olimpidada qatnashib, g‘olib bo‘lish istagida bo‘lgan iqtidorli o‘quvchilar uchun yaratildi. Qo‘llanma mustaqil o‘rganuvchilar uchun qulay bo‘lib, undagi masalalar yechimlari bilan berilgan. Mustaqil yechishga berilgan masalalarni yechishda kitobxonga qulaylik yaratish maqsadida javoblar va ko‘rsatmalar berilgan.

Qo‘llanma orqali o‘z bilimingizni boyitib, olimpiadalarda g‘olib bo‘lsangiz biz bundan xursandmiz.

Mualliflar

1-BOB. MATEMATIKADAN SIRTQI OLIMPIADA MASALALARI

1. Yig'indini toping: $\sin x + 2\sin 2x + 3\sin 3x + \ldots + 2018\sin 2018x$

2. Agar $a, b \in \mathbb{Q}$ va $n \in \mathbb{N}$ sonlar uchun $a^{2n+1} + b^{2n+1} = 2a^n b^n$ tenglik o'rinli bo'lsa, $1 - ab$ ifoda biror ratsional sonning kvadrati ekanligini isbotlang.

3. Agar x_1, x_2, x_3 sonlari $x^3 - 3x + 1 = 0$ tenglamaning yechimlari bo'lsa, $x_1^5 + x_2^5 + x_3^5$ ifodaning qiymatini toping.

4. Ushbu $\left[\frac{x}{2017}\right] = \left[\frac{x}{2018}\right] + 1$ tenglamaning natural yechimlari sonini aniqlang.

5. ΔABC da $AB = 7$, $BC = 6$ va $CA = 5$ bo'lsin. ABC uchburchakka ichki chizilgan aylana AB, BC, CA tomonlarga mos ravishda C_1, A_1, B_1 nuqtalarda urinsa $A_1B_1C_1$ uchburchak yuzini toping.

6. $ABCD$ qavariq to'rtburchakning BC va DA qarama-qarshi tomonlarida M va N nuqtalar shunday olinganki, bunda ushbu $\frac{|BM|}{|MC|} = \frac{|AN|}{|ND|} = \frac{|AB|}{|CD|}$ tenglik o'rinli. MN to'g'ri chiziq AB va CD tomonlar yordamida hosil qilingan burchak bissektrisasiga parallel bo'lishini isbotlang.

7. Agar $z^2 + y^2 = a^2$ va $u^2 + v^2 = b^2$ bo'lsa, $zu + yv$ ifodaning eng kichik qiymatini toping. Bu yerda $ab \ge 0$.

8. Agar $(x; y)$ juftliklar $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ tenglikni qanoatlantirsa, $\sqrt{x^2 + \sqrt[3]{x^4y^2}} + \sqrt{y^2 + \sqrt[3]{x^2y^4}} = a$ ekanini isbotlang.

9. Tenglamalar sistemasini yeching:

$$\begin{cases} \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \\ 4xy(2x^2 - a^2) = a^3b \end{cases}$$

10. 101 ta tanga bor. Ularning 50 tasi qalbaki. Qalbaki tangalar haqiqiysidan 1 grammga farq qiladi. Sitora 1 ta tangani oldi va uning haqiqiy ekanligini pallalaridagi yuklarning ayirmasini ko'rsatuvchi tarozida bir marta o'lchash yordamida aniqlamoqchi bo'ldi. U buni uddalay oladimi?

11. Ushbu $24x - 17y = 2$ tenglamaning barcha butun yechimlarini toping.

12. Agar p va q sonlari 2 dan katta natural sonlar bo‘lsa, quyidagi tengsizlikni isbotlang:

$$\left(\left[\frac{p}{2}\right]+1\right)\left(\left[\frac{q}{2}\right]+1\right) \le \left[\frac{pq}{2}\right]+1$$

Bu yerda $[\,]$-sonning butun qismi.

13. Qavariq $ABCDEF$ oltiburchakda ichki burchaklar o‘zaro teng. Agar $AB = 3$, $BC = 4$, $CD = 5$, $EF = 3$ bo‘lsa, AF tomon uzunligini toping.

14. O nuqta ABC uchburchakning medianalar kesishgan nuqtasi. BC tomonda D nuqta shunday olinganki, $OD \parallel AC$ shart o‘rinli. $AODB$ to‘rtburchak yuzining $AODC$ to‘rtburchak yuziga nisbatini toping.

15. Agar $a, b, c > 0$ sonlari uchun $ab + bc + ac = abc$ tenglik o‘rinli bo‘lsa, quyidagi tengsizlikni isbotlang:

$$\frac{a^4 + b^4}{ab(a^3 + b^3)} + \frac{b^4 + c^4}{bc(b^3 + c^3)} + \frac{a^4 + c^4}{ac(a^3 + c^3)} \ge 1$$

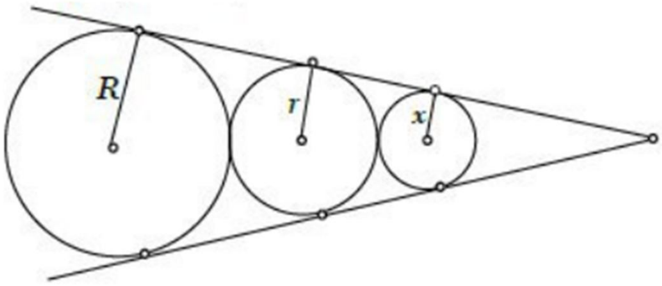
16. Tenglamalar sistemasini yeching: $\begin{cases} x + \dfrac{3x - y}{x^2 + y^2} = 3 \\ y - \dfrac{x + 3y}{x^2 + y^2} = 0 \end{cases}$

17. Ushbu $\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{3}} + \dfrac{1}{\sqrt{4}} + \ldots + \dfrac{1}{\sqrt{10000}}$ yig‘indining butun qismini toping.

18. Yuk tashuvchi tashkilotdan 67 tonna yukni bir qatnovda tashib berish iltimos qilindi. Bu tashkilot yukni tashish uchun yuk ko‘tarish quvvati ikki xil 2,5 va 6,5 tonnali avtomashinalardan ajratdi. Bu tashkilot har bir tur mashinadan nechtadan ajratgan?

19. Kvadratga ikkita aylana ichki chizilgan. Radiusi 1 ga teng aylana birinchi aylanaga va kvadratning ikki qo‘shni tomonlariga urinadi, radiusi 3 ga teng bo‘lgan aylana ikkinchi aylanaga va kvadratning qolgan ikki tomoniga va birinchi aylanaga urinadi. Kavdratning diagonalini toping.

20. Quyidagi chizmada $R = 5$ va $r = 3$ bo‘lsa, u holda eng kichik aylana uzunligini toping:



21. Agar $0 < x < \frac{\pi}{2}, m > 0, n > 0$ sonlar uchun $\frac{\sin(x-\alpha)}{\sin(x-\beta)} = m$ va $\frac{\cos(x-\alpha)}{\cos(x-\beta)} = n$ ekanligi ma'lum bo'lsa, $\cos(\alpha-\beta)$ ni toping.

22. $tg20^\circ + tg40^\circ + \sqrt{3}tg20^\circ \cdot tg40^\circ$ ni hisoblang.

23. $\frac{1}{\cos\alpha\cos2\alpha} + \frac{1}{\cos2\alpha\cos3\alpha} + \ldots + \frac{1}{\cos2020\alpha\cos2021\alpha}$ ni soddalashtiring.

24. Yig'indini hisoblang. $\sin\alpha + \sin2\alpha + \sin3\alpha + \ldots + \sin2021\alpha$

25. Yig'indini hisoblang. $\cos\alpha + \cos2\alpha + \cos3\alpha + \ldots + \cos2021\alpha$

26. Tenglamani yeching: $\frac{\cos x}{|\cos x|} + \frac{|\sin x|}{\sin x} = -2$

27. Agar $x = \sin18^\circ$ bo'lsa, $4x^2 + 2x = 1$ tenglikni isbotlang.

28. $\forall n > 1, (n \in \mathbb{N})$ da quyidagi tenglikni isbotlang:

$$\sin\frac{\pi}{n} \cdot \sin\frac{2\pi}{n} \cdot \sin\frac{3\pi}{n} \cdot \ldots \cdot \sin\frac{(n-1)\pi}{n} = \frac{n}{2^{n-1}}$$

29. Har bir a haqiqiy son uchun quyidagi tenglamani yeching:

$$(a-1)\left(\frac{1}{\sin x} + \frac{1}{\cos x} + \frac{1}{\sin x\cos x}\right) = 2$$

30. Agar $\forall n > 1, (n \in \mathbb{N})$ bo'lsa, $\cos\frac{2\pi}{n} + \cos\frac{4\pi}{n} + \cos\frac{6\pi}{n} + \ldots + \cos\frac{2n\pi}{n}$ yig'indi nimaga teng?

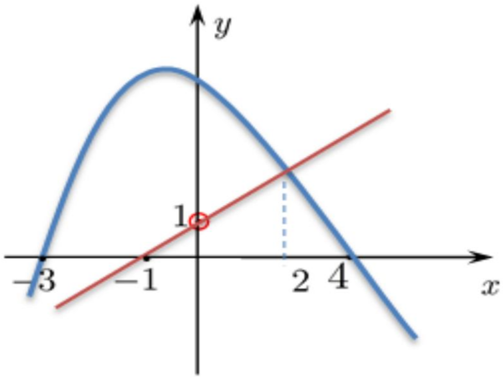
31. Ifodaning eng katta qiymatini toping. $\sin^2 x \cdot \cos^4 x \cdot (2 - \sin^2 x)$

32. Agar α, β, γ lar o'tmas burchakli bo'lmagan uchburchakning burchaklari bo'lsa, u holda quyidagi tengsizlikni isbotlang:

$$\sin\alpha + \sin\beta + \sin\gamma > \cos\alpha + \cos\beta + \cos\gamma$$

33. $x, y \geq 0$ sonlari uchun $(x+y)\left(\sqrt{xy}+1\right) \geq 2\sqrt{xy(1+x)(1+y)}$ tengsizlikni isbotlang.

34. Quyidagi chizmada to‘g‘ri chiziq va parabola kesishmasi grafigi berilgan. Parabola tenglamasini toping.



35. Tengsizlikni isbotlang: $\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \ldots \cdot \frac{2017}{2018} < \frac{1}{44}$

36. Agar a, b, c, d sonlar $a^2+b^2+c^2+d^2=4$ bo‘lsa, u holda $(2+a)(2+b) \geq cd$ o‘rinli bo‘lishini isbotlang.

37. Agar uchburchakning ichki burchaklari α, β, γ bo‘lsa, $\sin^2\alpha + \sin^2\beta + \sin^2\gamma \leq \frac{9}{4}$ tengsizlikni isbotlang.

38. α, β, γ lar uchburchakning ichki burchaklari bo‘lsa, ushbu $\cos^2\alpha + \cos^2\beta + \cos^2\gamma \geq \frac{3}{4}$ tengsizlikni isbotlang.

39. Quyidagi tenglamalar sistemasini yeching: $\begin{cases} x^2 - xy + y^2 = 21 \\ y^2 - 2xy + 15 = 0 \end{cases}$

40. Tenglamani yeching: $(6x+7)^2(3x+4)(x+1)=1$

41. Tenglamani yeching: $(x-1)^4 + (x+3)^4 = 82$

42. Ixtiyoriy nomanfiy a, b, c sonlar uchun quyidagi tengsizlikni isbotlang:

$$(2\sqrt[4]{a} + 2\sqrt[4]{b} + 2\sqrt[4]{c}) - (\sqrt{a} + \sqrt{b} + \sqrt{c}) \leq 3$$

43. Tayinlangan $a, b, c \in \mathbb{Z}$ uchun x^2+ax+b, x^2+bx+c ko‘phadlar $x+1$ ga, x^3-4x^2+x+6 ko‘phad x^2+ax+b, x^2+bx+c ko‘phadlarga bo‘linadi. $a+b+c$ ni toping.

44. $x^2 \leq [2x]\cdot\{2x\}$ tengsizlikni yeching. (Bu yerda$[\,]$-sonning butun, $\{\,\}$-sonning kasr qismi.)

45. $P(x)$ ko‘phad uchun $(x^2+2)P(x)+ax+b=x^7+2x^5+3x^4+3x^3-2x+5$ munosabat o‘rinli bo‘lsa, $a+b=?$

46. Ushbu $\sqrt{2017+\sqrt{2017+\sqrt{2017+\ldots+\sqrt{2017}}}}$ ifodaning butun qismini toping. Bu yerda ildizlar soni cheksiz ko‘p

47. $9\cdot 99\cdot 999\cdot\ldots\cdot\underbrace{99\ldots9}_{2021\,ta}\equiv x(\mathrm{mod}\,1000)$ bo‘lsa, x ni toping.

48. Yig‘indini hisoblang: $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ldots+\frac{2016}{2017!}$

49. Agar $k=2019^2+2^{2019}$ bo‘lsa, $(k^2+2^k)^{2019}$ ning oxirgi raqamini toping.

50. $N=100^2+99^2-98^2-97^2+\ldots+4^2+3^2-2^2-1^2$ va $N\equiv x\,\mathrm{mod}(1000)$ bo‘lsa, x ni toping.

51. ABC uchburchakning AB tomoniga teng tomonli ABC_1 uchburchak shunday yasalganki, uning C va C_1 uchlari AB to‘g‘ri chiziqning bir tomonida joylashgan. Ushbu $|CC_1|=\frac{1}{2}(a^2+b^2+c^2)-2\sqrt{3}S$ tenglikni isbotlang. Bu yerda a,b,c-uchburchak tomonlari va S-uchburchak yuzasi.

52. ABC uchburchakning BD balandligi($BD=24$) AC tomonni A uchidan boshlab hisoblaganda 3:8 nisbatda bo‘ladi. Shu balandlikka parallel va uchburchak yuzini teng ikkiga bo‘luvchi kesma uzunligini toping.

53. Ko‘paytuvchilarga ajrating: $5x^4+9x^3-2x^2-4x-8$

54. $7777^{2222}+2222^{7777}$ sonining 9 ga bo‘linishini isbotlang.

55. Idishda 64 kg un bor. Ikki pallali tarozida toshlardan foydalanmasdan 23 kg unni qanday o‘lchash mumkin?

56. $A=99!$ va $B=50^{99}$ sonlaridan qaysi biri katta?

57. Tenglamani yeching: $\sqrt{3(x+y+z)}+\sqrt{y}+\sqrt{z}=\sqrt{x}$

58. Tengsizlikni isbotlang: $\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{(m-1)^2}+\frac{1}{m^2}<\frac{m-1}{m}$, bu yerda $m\geq 2,\ m\in\mathbb{N}$.

59. Kasrni qisqartiring: $\frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ac}$

60. Agar $EKUK(a;b;c)=2^{2011}$ bo‘lsa, $EKUB(ab;bc;ac)$ ifoda nechta butun qiymat qabul qila oladi?

61. Tenglamani yeching: $\left[\sqrt[3]{1}\right]+\left[\sqrt[3]{2}\right]+\ldots\left[\sqrt[3]{x^3-1}\right]=400$

62. Ixtiyoriy ABC uchburchak uchun $h_a\leq\sqrt{p\left(p-a\right)}$ tengsizlikni isbotlang. Bu yerda h_a-uchburchakning $BC=a$ tomoniga tushirilgan balandligi, p-yarim perimetri.

63. To‘g‘ri to‘rtburchakning ikki uchidan diagonalga tushirilgan perpendikulyar diagonalni teng uch bo‘lakka bo‘ladi. Agar to‘g‘ri to‘rtburchakning bir tomoni 2 ga teng bo‘lsa, uning ikkinchi tomoni va yuzini toping.

64. a, b, c lar arifmetik progressiyani hosil qiladi. Tomonlari a, b, c bo‘lgan uchburchakka ichki chizilgan aylana radiusini toping.

65. a,b,c sonlari ushbu $x^3 - x+1=0$ tenglamaning ildizlari bo‘lsa, $\frac{1}{a+1}+\frac{1}{b+1}+\frac{1}{c+1}$ ning qiymatini toping.

66. $a+b+c\neq 0$ shartni qanoatlantiruvchi a,b,c sonlari uchun $a(b^3-c^3)+b(c^3-a^3)+c(a^3-b^3)$ ifodaning $a+b+c$ ga bo‘linishini isbotlang.

67. Tenglamalar sistemasini yeching:

$$\begin{cases}(x+y)^3=z\\(y+z)^3=x\\(x+z)^3=y\end{cases}$$

68. Tengsizlikni yeching: $|x-3|^{2x^2-7x}>1$

69. $a,b,c>0$ sonlari uchun $\frac{a^8+b^8+c^8}{a^3b^3c^3}\geq\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ tengsizlikni isbotlang.

70. Tenglamani yeching: $\frac{1}{\sqrt{3}}\left(\frac{xy+yz+zx}{\sqrt{xyz}}\right)=\sqrt{\frac{xy}{z}+\frac{yz}{x}+\frac{zx}{y}}$

71. Uchburchakning perimetri 2013 ga teng va $\frac{\sqrt{p-a}+\sqrt{p-b}+\sqrt{p-c}}{\sqrt{p}}=\sqrt{3}$ tenglik o‘rinli. Bu yerda a,b,c-uchburchakning tomonlari, p-yarim perimetr. Uchburchakning yuzini toping.

72. ABC uchburchakning ichidagi M nuqta qanday joylashganda $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=\frac{a+b+c}{r}$ tenglik o‘rinli bo‘ladi? Bu yerda a,b,c-uchburchakning tomonlari, x,y,z-mos ravishda M nuqtadan BC, AC, AB tomonlargacha bo‘lgan masofalar.

73. Agar uchburchakda $36S^2=(a^2+b^2+c^2)(h_a^2+h_b^2+h_c^2)$ tenglik o‘rinli bo‘lsa, uning burchaklarini toping. Bu yerda a,b,c-uchburchakning tomonlari, h_a,h_b,h_c-mos ravishda shu tomonlarga tushirilgan balandliklar, S-uchburchakning yuzi.

74. Teng yonli uchburchakning yon tomoniga o‘tkazilgan bissektrisa yon tomonni uchidan boshlab AK va KC kesmalarga ajratadi. Agar $AK+BK=BC$ tenglik o‘rinli bo‘lsa, uchburchakning burchaklarini toping.

75. O‘tkir burchakli ABC uchburchakka ichki va tashqi chizilgan aylana radiuslari mos r va R ga teng bo‘lib, $\angle BAC=\alpha$ bo‘lsa, uchburchakning yuzini toping.

76. $p^3-q^7=p-q$ tenglamani qanoatlantiruvchi barcha (p,q) tub sonlar juftligini toping.

77. Aylanaga ichki chizilgan $ABCD$ to‘rtburchakda $AB\perp AD$. BC va CD tomonlarda mos ravishda M va N nuqtalar olingan bo‘lib, $MN=BM+DN$ tenglikni qanoatlantiradi. Ma’lumki, AB va AD to‘g‘ri chiziqlar aylanani ikkinchi marta P va Q nuqtalarda kesib o‘tadi. APQ uchburchak balandliklarining kesishish nuqtasi MN kesmada yotishini isbotlang.

78. Doskada $1,2,3,\ldots,n$ sonlari yozilgan($n>2$). Har minutda doskadan ikkita son o‘chirilib, ularning o‘rniga yig‘indining eng kichik tub bo‘luvchisi yoziladi. Ma’lumki, oxirida faqat 97 soni qoldi. n eng kami bilan nechaga teng bo‘la oladi.

79. Bir davrali futbol musobaqasida ,,Barcelona" jamoasi boshqa hamma jamoalardan ko'p gol o'tkazib yubordi va ko'p gol urdi. U oxirgi o'rinni egallashi mumkinmi?

80. Agar ABC uchburchakka tashqi chizilgan aylana markazi O va bu uchburchak medianalari kesishgan nuqtasi M bo'lsa, ushbu $\overrightarrow{OM}=\frac{1}{3}(\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC})$ tenglikni isbotlang.

81. Quyida berilganlarga ko'ra $f(x)$ va $g(x)$ funksiyalarni toping:

$$\begin{cases} f(2x+2)+2g(4x+7)=x-1 \\ f(x-1)+g(2x+1)=2x \end{cases}$$

82. Quyida berilganlarga ko'ra $f(x)$ va $g(x)$ funksiyalarni toping:

$$\begin{cases} f(4x+3)+xg(6x+4)=2 \\ f(2x+1)+g(3x+1)=x+1 \end{cases}$$

83. Ushbu $f(x)+xf(\frac{x}{2x-1})=2$ tenglamadan $f(x)$ ni toping.

84. Bir idishda (probilkada) oq , ikkinchisida qizil ichimlik bor . Qizil ichimlikdan oqiga bir tomchi tomizamiz . So'ngra hosil bo'lgan aralashmadan qizil ichimlikka bir tomchi tomizamiz. Qizil ichimlikdagi oq ichimlik ko'pmi yoki oqidagi qizilmi?

85. Tenglamani yeching: $x^3-(\sqrt{3}+1)x^2+3=0$

86. Tenglamani yeching: $4x^2+4x+17=\frac{12}{x^2-x+1}$

87. Agar r,s,t lar ushbu $8x^3+1001x+2008=0$ tenglamaning ildizlari bo'lsa, $(r+s)^3+(s+t)^3+(t+r)^3$ ifodaning qiymatini toping.

88. Agar a,b,c sonlar $P(x)=x^3-x-1$ ko'phadning ildizlari bo'lsa, $\frac{1-a}{1+a}+\frac{1-b}{1+b}+\frac{1-c}{1+c}$ ning qiymatini toping.

89. ABC uchburchakning ichida ixtiyoriy M nuqta olingan va bu nuqtadan AM, BM, CM to'g'ri chiziqlar o'tkazilgan. Bu to'g'ri chiziqlar uchburchak tomonlarini, mos ravishda A_1, B_1, C_1 nuqtalarda kesib o'tadi. Quyidagi tengsizlikni isbotlang:

$$\frac{AM}{A_1M}+\frac{BM}{B_1M}+\frac{CM}{C_1M}\geq 6$$

90. ABC uchburchak ichida ixtiyoriy O nuqta olingan va bu nuqtadan uchburchak tomonlariga parallel to‘g‘ri chiziqlar o‘tkazilgan. $AB \parallel DE$, $BC \parallel MN$, $AC \parallel FK$. Bu yerda $F,M\in AB$, $E,K\in BC$, $D,N\in AC$. U holda quyidagi tenglikni isbotlang:

$$\frac{AF}{AB}+\frac{BE}{BC}+\frac{CN}{AC}=1$$

91. ABC uchburchakda $AB=c, BC=a, AC=b$ va $\angle A=30^0, \angle B=50^0$ bo‘lsa, a ni b va c orqali ifodalang.

92. $x^n+y^n=z^{n+1}, n\in\mathbb{N}$ tenglamani natural sonlarda yeching.

93. *a)* k^2+k sonlari orasida nechta to‘la kvadrat bo‘ladigan $k\in\mathbb{N}$ soni bor?
b) Agar $k\in\mathbb{Z}$ bo‘lsa-chi?

94. Agar $[x]\cdot\{x\}=100$ bo‘lsa, u holda $[x^2]-[x]^2$ ifodaning qiymatini toping. Bunda $[x]$ belgi x ning butun, $\{x\}$ belgi x ning kasr qisimini bildiradi.

95. Yig‘indini hisoblang: $\left[\frac{1}{3}\right]+\left[\frac{2}{3}\right]+\left[\frac{2^2}{3}\right]+\ldots+\left[\frac{2^{1000}}{3}\right]$ (bunda $[a]$-a ning butun qisimi).

96. Tenglamani yeching: $\sqrt[3]{a+x}-\sqrt[3]{a-x}=\sqrt[6]{a^2-x^2}$

97. $\forall n\geq 2$, $a_1,a_2,\ldots,a_n\geq 0$ sonlari uchun quyidagi tengsizlikni isbotlang:

$$a_1+a_2+\ldots+a_n-n\sqrt[n]{a_1a_2\ldots a_n}\geq(\sqrt{a_1}-\sqrt{a_2})^2$$

98. Agar $|x|<1$ bo‘lsa ushbu $1+2x+3x^2+4x^3+\ldots$ yig‘indini toping.

99. a,b,c lar to‘g‘ri burchakli uchburchakning tomonlari bo‘lsa, (c-gipotenuza) ushbu $ab(a+b+c)<\frac{5}{4}c^3$ tengsizlikni isbotlang.

100. $[x]+[2x]+[3x]=6$ tenglamani yeching, bu yerda $[x]$-x sonning butun qismi.

101. Agar $a,b>0$ va $x_i\in[a,b], i=1,2,\ldots,n$ bo‘lsa, quyidagi tengsizlikni isbotlang:

$$(x_1 + x_2 + \ldots + x_n)\left(\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}\right) \le \frac{(a+b)^2}{4ab} \cdot n^2$$

102. Musbat $a, b, c \neq 1$ sonlari uchun $\log_a b + \log_b c + \log_c a = 0$ bo'lsa, $(\log_a b)^3 + (\log_b c)^3 + (\log_c a)^3$ ning qiymatini toping.

103. $a, b, c > 0$ va $a^2 + b^2 + c^2 = \frac{5}{3}$ bo'lsa, $\frac{1}{a} + \frac{1}{b} - \frac{1}{c} < \frac{1}{abc}$ tengsizlikni isbotlang.

104. Hisoblang: $\sqrt{1 + \frac{1}{2^2} + \frac{1}{3^2}} + \sqrt{1 + \frac{1}{3^2} + \frac{1}{4^2}} + \ldots + \sqrt{1 + \frac{1}{2019^2} + \frac{1}{2020^2}}$

105. Taqqoslang. $\sqrt{1 \cdot 2} + \sqrt{3 \cdot 4} + \sqrt{5 \cdot 6} + \ldots + \sqrt{2019 \cdot 2020}$ va $2021 \cdot 505$

106. $\frac{1 \cdot 3!}{3} + \frac{2 \cdot 4!}{3^2} + \ldots + \frac{n \cdot (n+2)!}{3^n}$ yig'indini toping.

107. $A = 1! + 2! + 3! + 4! + 5! + \ldots + 2021!$ yig'indini 12 ga bo'lgandagi qoldiqni toping.

108. Yig'indini hisoblang: $\frac{3}{1! + 2! + 3!} + \frac{4}{2! + 3! + 4!} + \ldots + \frac{2020}{2018! + 2019! + 2020!}$

109. Yig'indini hisoblang: $2 \cdot 2020 + 3 \cdot 2020^2 + 4 \cdot 2020^3 + \ldots + 2020^{2020}$

110. Ifodaning qiymatini toping:

$$1! \cdot 3 - 2! \cdot 4 + 3! \cdot 5 - 4! \cdot 6 + \ldots - 2018! \cdot 2020 + 2019!$$

111. $(x_1; y_1), (x_2; y_2), (x_3; y_3)$ sonlari $x^3 + 3xy^2 = 2021$ va $y^3 + 3x^2y = 2020$ tenglamaning ildizlari bo'lsa, $\left(1 - \frac{x_1}{y_1}\right)\left(1 - \frac{x_2}{y_2}\right)\left(1 - \frac{x_3}{y_3}\right)$ ni toping.

112. p, q, r sonlar $x^3 + ax^2 + bx + c = 0$ tenglamaning ildizlari bo'lsa, $(pq)^2 + (qr)^2 + (pr)^2$ ifodani a, b, c lar orqali ifodalang.

113. $f(x) = x^2 + 12x + 30$ kvadrat funksiya berilgan. $\underbrace{f(f(\ldots(f(x))\ldots))}_{n\,ta} = 0$ tenglamani yeching.

114. Agar uchburchak tomonlarining uzunliklari a, b, c va yarim perimetri p ga teng bo‘lsa $\sqrt{(p-a)(p-b)} + \sqrt{(p-a)(p-c)} + \sqrt{(p-b)(p-c)} \le p$ ekanligini isbotlang

115. Agar $m * n = \dfrac{m+n}{mn+4}$ bo‘lsa, $\left(\ldots\left((2020 * 2019) * 2018\right) * \ldots * 1\right) * 0$ ni toping.

116. Ixtiyoriy butun musbat a va b sonlari uchun $*$ algebraik amali quyidagi shartlarni qanoatlantiradi:

i) $a * a = a^2 + 2019$

ii) $a * b = b * a$

iii) $\dfrac{a * (a+b)}{a * b} = \dfrac{a^2 + b^2}{ab}$

Yuqoridagi berilganlarga asosan $3 * 5$ ni toping.

117. $\int\limits_{-\pi}^{\pi} \sin^7 x \cos^7 x dx$ ni hisoblang.

118. Aniq integralni hisoblang: $\int\limits_{-\pi/4}^{\pi/4} \dfrac{x^{2n+1} + x^{2n-1} + \ldots + x^5 + 3x^3 - x + 1}{\cos^2 x} dx = ?$

119. Aniqmas integralni hisoblang: $\int \dfrac{dx}{\sin(x+a)\sin(x+b)}$

120. Aniqmas integralni hisoblang: $\int \dfrac{dx}{\sin(x+a)\cos(x+b)}$

121. Aniqmas integralni hisoblang: $\int \dfrac{dx}{\cos(x+a)\cos(x+b)}$

122. Aniqmas integralni hisoblang: $\int \dfrac{dx}{\sin^4 x}$

123. Agar $\int\limits_0^1 f(x)dx = a$ bo‘lsa, $\int\limits_0^1 xf(x^2)dx$ ni hisoblang.

124. Hisoblang: $\int\limits_0^{\pi} x \operatorname{sgn}(\cos x) dx$

125. $\int_0^3 \text{sgn}(x-x^3)dx$ integralni hisoblang.

126. Aniq integralni hisoblang: $\int_0^6 [x]\sin\frac{\pi x}{6}\,dx$.

127. Hisoblang: $\int_1^{2021} \ln[x]dx$. Bu yerda [ ]-sonning butun qismi.

128. Tenglamalar sistemasini yeching: $\begin{cases} 1-5y=\frac{x}{y}-6\sqrt{x-y} \\ \sqrt{x-\sqrt{x-y}}=x-5y-6 \end{cases}$

129. Agar $0<x<\frac{\pi}{2}$ bo‘lsa, $(tgx)^{\sin x}+(ctgx)^{\cos x}$ ifodaning eng kichik qiymatini toping.

130. $a,b,c>0$ bo‘lgan hol uchun ushbu $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\geq\frac{1}{\sqrt{ab}}+\frac{1}{\sqrt{ac}}+\frac{1}{\sqrt{bc}}$ tengsizlikni isbotlang.

131. Trapetsiyaning katta asosiga yopishgan burchaklarining yig‘indisi 90^0 ga teng bo‘lsa, asoslarining o‘rtalarini tutashtiruvchi kesma katta va kichik asoslar ayirmasining yarmiga teng ekanligini isbotlang.

132. Teng yonli trapetsiyaning asoslari a va b, yon tomoni c va diagonali d ga teng bo‘lsa, $d=\sqrt{ab+c^2}$ ekanini isbotlang.

133. a,b,c-natural sonlar ko‘rsatilgan tartibda geometrik progressiyani tashkil qilsa, $\left(EKUB(a;b)\right)^2=a\cdot EKUB(a;c)$ ekanligini isbotlang.

134. Aytaylik a va b to‘g‘ri burchakli uchburchakning katetlari, c gipotenuzasi, h gipotenuzaga tushirilgan balandligi bo‘lsin. U holda $c+h>a+b$ tengsizlikni isbotlang.

135. Aniq integralni hisoblang: $\int_0^{2016} x(x-4)(x-8)\cdot\ldots\cdot(x-2016)dx$

136. $x^2+px-\frac{1}{2p^2}$ ko‘phadning ildizlari x_1 va x_2 bo‘lsa, $x_1^4+x_2^4$ ifodaning eng kichik qiymatini toping. Bu yerda $p\in R, p\neq 0$.

137. Biror uchburchakning ichki burchaklari A, B, C ekanligini bilgan holda quyidagi ayniyatni isbotlang:

$$\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

138. Ushbu $f(x) = |x-1| + |2x-1| + |3x-1| + \ldots + |119x-1|$ funksiyaning eng kichik qiymatini toping.

139. Biror uchburchakning ichki burchaklari A, B, C ekanligini bilgan holda quyidagi ayniyatni isbotlang:

$$tg\frac{A}{2}tg\frac{B}{2} + tg\frac{B}{2}tg\frac{C}{2} + tg\frac{C}{2}tg\frac{A}{2} = 1$$

140. 2^{2020} sonini 127 ga bo‘lgandagi qoldiqni toping.

141. Diagonallari a va b ga teng bo‘lgan qavariq to‘rtburchakning eng katta tomoni uzunligi $\sqrt{\frac{a^2+b^2}{8}}$ dan katta ekanligini isbotlang.

142. 201920202021 sonining raqamlari o‘rinlarini almashtirib, ko‘pi bilan nechta o‘nikki xonali son tuzish mumkin?

143. Mohinur noldan farqli uchta turli raqam o‘yladi. Akmaljon bu raqamlardan tuzish mumkin bo‘lgan barcha ikki xonali sonlarni qo‘shib chiqdi. Agar yig‘indi 231 ga teng bo‘lsa, Mohinur o‘ylagan raqamlarni toping.

144. Berilgan oltita 1,2,3,4,5, va 6 raqamlaridan bitta bir xonali, bitta ikki xonali va bitta uch xonali son tuzilmoqda. Bunda har bir raqam bir marta ishlatiladi. Bir xonali va ikki xonali sonlarning yig‘indisi 47 ga, ikki xonali va uch xonali sonlarning yig‘indisi 358 ga teng. Shu uchta son yig‘indisini toping.

145. Natural son o‘zi tashkil topgan raqamlariga ketma-ket ko‘paytirildi. Ko‘paytmada 1995 hosil bo‘ldi. Shu sonning raqamlari yig‘indisini toping.

146. Burchaklari tub sonlar bilan ifodalanuvchi nechta uchburchak mavjud?

147. Agar $\alpha + \beta + \gamma = 2\pi$ bo‘lsa, $tg\frac{\alpha}{2} + tg\frac{\beta}{2} + tg\frac{\gamma}{2} = tg\frac{\alpha}{2} \cdot tg\frac{\beta}{2} \cdot tg\frac{\gamma}{2}$ ni isbotlang.

148. Agar $x, y > 0$ va $xy \ge 1$ bo‘lsa, ushbu $\frac{1}{x^2+1} + \frac{1}{y^2+1} \ge \frac{2}{xy+1}$ tengsizlikni isbotlang.

149. Barcha haqiqiy $a, b, c, d \ge 1$ sonlar uchun quyidagi tengsizlikni isbotlang:

$$\frac{1}{a^4+1}+\frac{1}{b^4+1}+\frac{1}{c^4+1}+\frac{1}{d^4+1}\geq\frac{4}{abcd+1}$$

150. Bittadan muzqaymoq olish uchun Ra'noga 7 so'm, Guliga 1 so'm yetmadi. Ular pullarini qo'shganda ham pullari bitta muzqaymoqqa yetmadi. Agar muzqaymoqning bahosi natural sonda ifodalansa, u qancha turadi?

151. Og'irliklari $1^2g,\ 2^2g,\ 3^2g,\dots,\ 81^2g$ bo'lgan 81 ta tosh berilgan. Shu toshlarni 3 ta guruhga shunday ajratingki, ularning massalari teng bo'lsin

152. ABC uchburchakda $\angle A=20^0$, $\angle C=45^0$ ekanligi ma'lum. BM mediana davom ettirilib, unda K nuqta qo'yilgan, bunda $BM=MK$, BH balandlik davom ettirilib, unda N nuqta qo'yilgan, bunda $BH=HN$ tengliklar o'rinli. KAN burchakni toping.

153. Muntazam ABC uchburchakning ichida shunday O nuqta olinganki, $\angle AOB=110^0$, $\angle BOC=117^0$. Tomonlari OA, OB, OC kesmalarga teng bo'lgan uchburchakning burchaklarini toping.

154. Quyidagi tenglamalar sistemasini yeching:

$$\begin{cases} x^3+y^3+z^3=8 \\ x^2+y^2+z^2=22 \\ \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=-\frac{z}{xy} \end{cases}$$

155. $2011^{2011^{2011^{2011}}}$ ni 19 ga bo'lgandagi qoldiqni toping.

156. O'tkir burchakli ABC uchburchak berilgan. AB tomonning o'rtasidan A va B burchaklarning bissektrissalariga perpendikulyar to'g'ri chiziqlar o'tkazilgan. Bu to'g'ri chiziqlar AC va BC tomonlarni mos ravishda K va M nuqtalarda kesib o'tadi. $AK=BM$ ekanligini isbotlang.

157. Hovuz turli mehnat unumdorligiga ega bo'lgan uchta quvur orqali suv bilan to'ldiriladi. Har kuni ertalab soat 8^{00} da bo'sh hovuzga suv quyish boshlanadi. Birinchi kuni faqat birinchi quvur ishga tushirildi, u ikkinchi va uchinchi quvurlar birgalikda ishlab, hovuzning $\frac{2}{3}$ qismini to'ldirishga ketadigan vaqtga teng vaqt ishlagandan so'ng o'chirilib ikkinchi quvur ishga tushirildi, u birinchi va uchinchi quvurlar birgalikda ishlab hovuzning $\frac{1}{5}$ qismini to'ldirishga ketadigan vaqtga teng vaqt ishlagandan so'ng o'chirildi va uchinchi quvur ishga tushirildi, u birinchi va

ikkinchi quvurlar birgalikda ishlab, hovuzning $\frac{1}{3}$ qismini to‘ldirishga ketadigan vaqtga teng vaqt ishlagandan so‘ng o‘chirildi. Shunda hovuz to‘lgan va soat 19^{00} edi. Ikkinchi kuni uchta quvur birgalikda yoqildi va hovuz to‘lgach o‘chirildi. Ikkinchi kuni hovuz to‘lib, quvurlar o‘chirilganda soat necha edi?

158. $ABCD$ parallelogrammda E nuqta BC tomonning o‘rtasi, F nuqta AD tomonning o‘rtasi. AC diagonal BF va ED kesmalarni mos ravishda G va H nuqtalarda kesib o‘tadi. $AG = GH = HC$ ekanligini isbotlang.

159. $ABCD$ trapetsiyada $BC \parallel AD$ bo‘lib, diagonallari O nuqtada kesishadi. U holda $S_{\Delta AOB} = S_{\Delta COD}$ tenglikni isbotlang.

160. $ABCD$ trapetsiyaning AB va CD asoslarida K va L nuqtalar olingan, bunda $E-AL$ va DK, $F-BL$ va CK kesmalarning kesishish nuqtalari. AED va BFC uchburchaklar yuzlarining yig‘indisi $EKFL$ to‘rtburchakning yuziga teng bo‘lishini isbotlang.

161. x, y, z musbat sonlari $\begin{cases} x^2 + xy + y^2 = a^2 \\ y^2 + yz + z^2 = b^2 \\ x^2 + xz + z^2 = c^2 \end{cases}$ tenglamalar sistemasini qanoatlantirsa, ushbu $xy + yz + xz$ ifodaning qiymatini toping. Bunda $a > 0$, $b > 0$, $c > 0$.

162. Musbat x, y, z sonlari quyidagi sistemani qanoatlantiradi:

$$\begin{cases} x^2 + xy + \dfrac{y^2}{3} = 25 \\ \dfrac{y^2}{3} + z^2 = 9 \\ z^2 + zx + x^2 = 16 \end{cases}$$

U holda $xy + 2yz + 3xz$ ifodaning qiymatini toping.

163. Agar x va y haqiqiy sonlar bo‘lsa, u holda quyidagi tengsizlikni isbotlang:

$$|\cos x| + |\cos y| + |\cos(x + y)| \geq 1$$

164. Agar $x_1, x_2, x_3, x_4, x_5 \in \mathbb{R}$ sonlari uchun, $x_1 + x_2 + x_3 + x_4 + x_5 = 0$ bo‘lsa, u holda quyidagi tengsizlikni isbotlang:

$$|\cos x_1| + |\cos x_2| + |\cos x_3| + |\cos x_4| + |\cos x_5| \geq 1$$

165. Ikki xonali $\overline{bc}$ son qandaydir sonning kvadrati bo‘lsin. $a = b + c$ tenglikni qanoatlantiradigan barcha 11 ga karrali to‘rt xonali $\overline{abcd}$ sonlarni toping.
166. Madina uchta kartochkaning har biriga bittadan raqam yozib, so‘ng shu kartochkalar yordamida barcha uch xonali sonlarni tuzdi (bunda kartochkalar uchchalasi ham ishlatildi) va ularning yig‘indisini hisobladi. Natijada 3159 soni hosil bo‘ldi. Qo‘shish jarayonida Madina bitta sonni hisobga olmagani aniqlandi. Shu sonni toping.
167. Bir qatorga dastlabki 2018 ta son yozilgan. Dastlab barcha toq sonlar o‘chirildi. Qolgan juft sonlar yana bir qatorga yozilib, toq o‘rinlarda turgan barcha sonlar o‘chirildi. So‘ng bu ish yana takrorlandi. Eng oxirida qanday son qoladi?
168. $\{a_n\}$ ketma-ketlik rekurrent usulda quyidagicha aniqlangan:

$$a_1 = 1,\ a_n = \frac{n+1}{n-1} \cdot (a_1 + a_2 + \ldots + a_{n-1})$$

U holda a_{2020} ni toping.
169. ABC uchburchak yuzi 12 ga teng. B uchdan C burchakning bissektrisasiga BM perpendikulyar o‘tkazilgan. AMC uchburchak yuzini toping.
170. Raqamlari yig‘indisidan 20 marta katta bo‘lgan barcha uch xonali sonlarni toping.
171. Do‘konda uchta turli rangdagi sharchalar sotilmoqda: qizil, ko‘k va yashil. Siz 10 ta sharcha sotib olishingiz kerak. Shu 10 ta sharchani sotib olish uchun nechta variant bor?
172. d_1 va d_2 sonlari n natural sonning bo‘luvchilari bo‘lib, $d_1 > d_2$ shart bajarilsa, u holda $d_1 > d_2 + \frac{d_2^{\,2}}{n}$ ekanligini isbotlang.
173. $a,b,c \ge 0$ sonlari uchun $abc \ge (a+b-c)(a-b+c)(b+c-a)$ tengsizlik o‘rinli ekanini isbotlang.
174. $a,b,c \ge 0$ sonlari uchun $a^3 + b^3 + c^3 + 5abc \ge (a+b)(b+c)(a+c)$ tengsizlikni isbotlang.
175. Uchburchakda $abc(a+b+c) \ge 16S^2$ tengsizlikni isbotlang. Bu yerda a,b,c lar uchburchakning tomonlari, S-uchburchak yuzi.
176. Uchburchakda $a^4 + b^4 + c^4 \ge 16S^2$ tengsizlikni isbotlang. Bu yerda a,b,c lar uchburchakning tomonlari, S-uchburchak yuzi.

177. Agar uchburchakning burchaklari α, β, γ, yuzi S va tashqi chizilgan aylana radiusi R bo‘lsa, ushbu $\sin^4 \alpha + \sin^4 \beta + \sin^4 \gamma \geq \frac{S^2}{R^4}$ tengsizlikni isbotlang.

178. Agar uchburchakka ichki va tashqi chizilgan aylana radiuslari mos ravishda r va R ga teng bo‘lsa, $R \geq 2r$ ekanligini isbotlang.

179. To‘g‘ri burchakli uchburchakda $1{,}5c > a + b$ ekanligini isbotlang. Bunda a va b lar katetlar, c -gipotenuza.

180. Uchburchakning yuzi S, yarim perimetri p va ichki chizilgan aylana radiusi r bo‘lsa, quyidagilarni isbotlang:

a) $S \leq \frac{p^2}{3\sqrt{3}}$ *b)* $S < \frac{p^2}{4}$ *c)* $p > 4r$

d) $S > 4r^2$ *e)* $a^2 + b^2 + c^2 \geq 4\sqrt{3}S$

181. Agar $a, b, c > 0$ va $a + b + c = 1$ bo‘lsa, $5(a^2 + b^2 + c^2) \leq 6(a^3 + b^3 + c^3) + 1$ tengsizlikni isbotlang.

182. Agar $a, b, c > 0$ sonlari uchun $a^3 + b^3 + c^3 + 3abc = 8$ bo‘lsa, quyidagi tengsizlikni isbotlang:

$$\sqrt{(ab)^3} + \sqrt{(bc)^3} + \sqrt{(ac)^3} \leq 4$$

183. a, b, c lar uchburchakning tomonlari bo‘lsa, quyidagi tengsizlikni isbotlang:

$$(a + b - c)^a \cdot (b + c - a)^b \cdot (a + c - b)^c \leq a^a \cdot b^b \cdot c^c$$

184. Barcha nomanfiy a, b, c sonlar uchun quyidagi tengsizlikni isbotlang:

$$(a^2 + 2)(b^2 + 2)(c^2 + 2) \geq 9(ab + bc + ac)$$

185. Tomoni a ga teng bo‘lgan $ABCD$ kvadrat berilgan. BC va CD tomonlarda M va N nuqtalar mos ravishda shunday olinganki, bunda $BM = 3MC$ va $2CN = ND$ tengliklar o‘rinli. AMN uchburchakka ichki chizilgan aylana radiusini toping.

186. $\sqrt{a^2 + b^2} + \sqrt{(\sqrt{2020} - a)^2 + (\sqrt{5} - b)^2}$ ifodaning eng kichik qiymatini toping.

187. Ifodaning eng kichik qiymatini toping: $2^x + 2^{1-x-y} + 2^y$

188. Nechta $(a;b)$ natural sonlar uchun $EKUB(a;b) = 1$ va $\frac{a}{b} + \frac{14b}{9a} \in \mathbb{N}$ bo‘ladi?

189. λ ning qanday qiymatida $x^3 - \lambda x + 2 = 0$ va $x^2 + \lambda x + 2 = 0$ tenglamalar umumiy ildizga ega bo‘ladi?

190. Nechta nomanfiy butun (a, b, c) sonlar uchligi $2^a + 2^b = c!$ tenglamani qanoatlantiradi?

191. K nuqta $ABCD$ kvadrat AB tomonining o‘rtasi, L nuqta AC diagonalni $AL : LC = 3 : 1$ kabi nisbatda bo‘ladi. KLD burchakni toping.

192. To‘g‘ri burchakli uchburchakning yuzi $\frac{\sqrt{3}}{12}(a^2 + 3b^2)$ kvadrat birlikka teng. Uning o‘tkir burchaklarini toping. Bunda a va b uchburchakning katetlari.

193. Istalgan natural $n \geq 3$ soni uchun $n^{n+1} > (n+1)^n$ tengsizlikni isbotlang

194. $\forall n \geq 2$ da $\log_n(n+1) > \log_{n+1}(n+2)$ tengsizlik o‘rinli ekanini isbotlang

195. Qaysi biri katta? π^e yoki e^π

196. ABC uchburchakda BH balandlik va BM medianalar. Agar $AB = 1$, $BC = 2$ va $AM = BM$ bo‘lsa, $\angle MBH$ burchakni hisoblang.

197. Ushbu $(x-y)^3 + (y-z)^3 + (z-x)^3 = 30$ tenglamani qanoatlantiruvchi (x, y, z) butun sonlar uchliklari nechta?

198. Uchburchakka tashqi chizilgan aylana radiusi 2 ga teng bo‘lsa, shu uchburchak medianalari kvadratlari yig‘indisining eng katta qiymatini toping.

199. Burchaklaridan hech biri o‘tmas bo‘lmagan uchburchakka tashqi chizilgan aylana radiusi 5 ga teng bo‘lsa, u holda shu uchburchak medianalari yig‘indisining eng katta qiymatini toping.

200. $\varphi(n)$ orqali n dan kichik va n bilan o‘zaro tub sonlarning sonini belgilasak, $\varphi(2020)$ ni toping. $\varphi(n)$-Eyler funksiyasi deyiladi.

201. ABC uchburchakda $AC = 3$, $BC = 4$ va $AB = 5$. Uchburchakning CD bissektrissasi o‘tkazilgan. ACD va BCD uchburchaklarga ichki chizilgan aylana radiuslari r_a va r_b bo‘lsa, u holda $\frac{r_a}{r_b}$ ni hisoblang.

202. $7^{x+7} = 8^x$ tenglama ildizi $x = \log_b 7^7$ bo‘lsa, b ni toping.

203. ABC uchburchakda $\cos(2A - B) + \sin(A + B) = 2$ va $AB = 4$ tengliklar o‘rinli bo‘lsa, uchburchakning yuzini toping.

204. $\{a_n\}$-ketma-ketlik ixtiyoriy natural $n \geq 1$ uchun $a_1 = 1$ va $a_{n+1} = \dfrac{a_n}{1 + na_n}$ tenglikni qanoatlantiradi. U holda a_{2021} ni hisoblang.

205. $\{a_n\}$ ketma-ketlikda($n \in \mathbb{N}$) $a_1 = 0$ va $a_{n+1} = \dfrac{n}{n+1}(a_n + 1)$ bo‘lsa a_{2021} ni toping.

206. $\{a_n\}$ ketma ketlikda($n \in \mathbb{N}$) $a_1 = 2$, $a_2 = 3$ va $a_{n+2} = \dfrac{a_{n+1}}{a_n}$ shartlar o‘rinli bo‘lsa, a_{2021} ni toping.

207. $\{a_n\}$ ketma ketlik $a_1 = 1$ va $a_{n+1} = a_n + \dfrac{1}{a_n^2}$ shartlar bilan berilgan. $a_{9000} > 30$ tengsizlikni isbotlang.

208. $a_0, a_1, a_2, ..., a_{100}$-natural sonlar va $a_1 > a_0$, $a_2 = 3a_1 - 2a_0$, $a_3 = 3a_2 - 2a_1$, ..., $a_{100} = 3a_{99} - 2a_{98}$ ekanligi malum bo‘lsa, $a_{100} > 2^{99}$ ekanini isbotlang.

209. $a_1, a_2, ...$ sonlar ketma-ketligi uchun $n \in \mathbb{N}$ da $a_{n+1} - 2a_n + a_{n-1} = 1$ tenglik bajariladi. $\{a_n\}$ ni a_1, a_2 va n lar orqali ifodalang.

210. Hisoblang: $\sum_{n=1}^{2019} i^n$ (i-mavhum birlik)

211. Tenglamani yeching: $(x^2 - 2x)^3 + x\sqrt{x(x-2)^3} = 2$

212. Agar $f(x) = \dfrac{1}{1-x}$, $f^{k+1}(x) = f\left(f^k(x)\right)$, $f^1(x) = f(x)$ bo‘lsa $f^{2020}(2020)$ ni toping.

213. ABC uchburchakda $A(3;0)$ va $B(0;3)$ bo‘lib, C uchi $x + y = 7$ to‘g‘ri chiziqda yotsa, $S_{\Delta ABC}$ ni toping.

214. Ushbu $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{2020}$ tenglamaning natural yechimlari sonini toping.

215. Muntazam ABC uchburchakning BC tomonini kesib o‘tuvchi AP nurda P nuqta shunday olinganki, $\angle APB = 20^0$ va $\angle APC = 30^0$. $\angle BAP$ ni toping.

216. Tengsizlikni yeching: $\left|\sin x - \sin y\right| + \sin x \cdot \sin y \geq 1$

217. AC va BD diognallari o‘zaro perpendikular bo‘lgan $ABCD$ to‘rtburchakka radiusi 2 ga teng bo‘lgan aylana tashqi chizilgan. Agar $AB = 3$ bo‘lsa, CD ni toping

218. Tenglamani yeching: $\left(x - 2\sqrt{2}\right)\left(x + 2\sqrt{2}\right) = \dfrac{x^2}{1 - x}$

219. Agar $f(x) = \dfrac{x}{\sqrt{1 + x^2}}$ bo‘lsa $\underbrace{f(f(f(...f(2020)...)))}_{2021\,ta}$ ni hisoblang.

220. Agar $a, b \in \mathbb{N}$ va $a^2 - b^4 = 2009$ bo‘lsa, $a + b$ ning qiymatini toping.

221. $P(x) = (1 + ix)^{2020}$ ko‘phadning barcha haqiqiy koeffitsientlari yig‘indisini toping. Bu yerda $i^2 = -1$

222. Agar $a, b, x, y \in \mathbb{R}$ sonlari uchun quyidagi tenglik o‘rinli bo‘lsa, $ax^5 + by^5$ ning qiymatini toping:

$$\begin{cases} ax + by = 3 \\ ax^2 + by^2 = 7 \\ ax^3 + by^3 = 16 \\ ax^4 + by^4 = 42 \end{cases}$$

223. $a = 5^{56}$, $b = 10^{51}$, $c = 17^{35}$, $d = 31^{28}$ sonlarni o‘sib borish tartibida joylashtiring.

224. Hisoblang: $n + \dfrac{1}{2}\left((n-1) + \dfrac{1}{2}\left((n-2) + ... + \dfrac{1}{2}\left(3 + \dfrac{1}{2} \cdot 2\right)...\right)\right)$

225. p, q-natural sonlar uchun $\dfrac{2018}{2019} < \dfrac{p}{q} < \dfrac{2019}{2020}$ munosabat o‘rinli bo‘lsa, $p_{\min}$ ni toping.

226. 17^{2021} ning oxirgi ikkita raqamini toping

227. $f(x) = \sin^6\left(\dfrac{x}{4}\right) + \cos^6\left(\dfrac{x}{4}\right)$ funksiyaning 2020–tartibli hosilasining $x = 0$ nuqtadagi qiymatini toping

228. $a,b,c,d \in \mathbb{R}$ sonlar uchun $a^2+d^2=1$, $b^2+c^2=1$, $ac+bd=\dfrac{1}{3}$ tengliklar o‘rinli bo‘lsa, $ab-cd$ ni toping

229. Nechta $(a;b)$ butun sonlar juftligi $\begin{cases} a^2+b^2<16 \\ a^2+b^2<8a \\ a^2+b^2<8b \end{cases}$ sistemani qanoatlantiradi?

230. Hisoblang: $\left[\dfrac{2020^3}{2018\cdot 2019}-\dfrac{2018^3}{2019\cdot 2020}\right]$ (bunda $[\,]$ belgi sonning butun qismi)

231. $x,y,k \in \mathbb{R}^+$ sonlari uchun $k^2\left(\dfrac{x^2}{y^2}+\dfrac{y^2}{x^2}\right)+k\left(\dfrac{x}{y}+\dfrac{y}{x}\right)=3$ bo‘lsa, $k_{\max}$ ni toping.

232. Hisoblang: $\dfrac{\sin 10^0+\sin 20^0+\ldots+\sin 70^0+\sin 80^0}{\cos 5^0 \cos 10^0 \cos 20^0}$

233. Bizga har birida 1000 tadan oltin tangasi bo‘lgan 10 ta qop berilgan. Berilgan qoplarning faqat bittasida qalbaki tanglalar mavjud. Agar haqiqiy tangalarning har biri 10gramm va qalbaki tangalarning harbiri 9 grammdan bo‘lsa, faqat bir marta elektron tarozida o‘lchash yordamida qalbaki tangalar qaysi qopda ekanligini aniqlash mumkinmi? Javobingizni asoslang.

234. ACE uchburchakda B nuqta AC kesmada, D nuqta CE kesmada shunday olinganki, bunda $AE \parallel BD$. AE kesmada Y nuqta olingan bo‘lib, CY va BD kesmalar X nuqtada kesishadi. $CX=5$ va $CY=8$ bo‘lsa, $\dfrac{S_{ABDE}}{S_{BCD}}$ nisbatni toping.

235. Aylanada ketma-ket olingan A, B, C va D nuqtalar uchun $AB=11$, $CD=19$. AB kesmadagi P nuqta uchun $AP=6$ va CD kesmadagi Q nuqta uchun $CQ=7$. P va Q nuqtalardan o‘tuvchi to‘g‘ri chiziq aylanani X va Y nuqtalarda kesadi. Agar $PQ=27$ bo‘lsa, XY ni toping.

236. ABC uchburchakning B burchagi to‘g‘ri burchak va BC tomondagi D nuqta uchun $3\cdot\angle BAD=\angle BAC$, $AC=2$, $CD=1$ tengliklar o‘rinli bo‘lsa, BD ni toping.

237. Tenglamani yeching: $x=\sqrt{x-\frac{1}{x}}+\sqrt{1-\frac{1}{x}}$

238. $231m^2=130n^2$ tenglama butun sonlar to‘plamida nechta yechimga ega?

239. $\frac{2020!}{2020^n}$ ifoda butun son bo‘ladigan n ning eng katta qiymatini toping.

240. $7^{2048}-1\vdots 2^n$ bo‘ladigan $n_{\max}$ ni toping.

241. $ABCD$ qavariq to‘rtburchakda $AB=BC=7$, $CD=5$, $AD=3$ va $\angle ABC=60^0$ bo‘lsa, BD ni toping

242. $ABCD$ to‘rtburchakda E,F,G,H nuqtalar mos ravishda, AB, BC, CD, DA tomonlarining o‘rtalari. $EG=12$, $FH=15$ bo‘lsa, $\left(S_{ABCD}\right)_{\max}$ ni toping.

243. $a,b,c\in\mathbb{N}$, $a>b>c$ va $\frac{1}{a-1}+\frac{1}{b-1}+\frac{1}{c-1}=1$ bo‘lsa, $a+2b+3c$ ning qiymatini toping.

244. Tenglamani yeching: $a\cdot 2^x+(x+1)\cdot a+2^x\cdot(x+1)=2^{2x}+(x+1)^2+a^2$

245. Tenglamani yeching: $\sqrt{x^2-\frac{7}{x^2}}+\sqrt{x-\frac{7}{x^2}}=x$

246. Taqqoslang:

$$\frac{1}{\sqrt{1\cdot 2012}}+\frac{1}{\sqrt{2\cdot 2011}}+\ldots+\frac{1}{\sqrt{k\cdot(2012-k+1)}}+\ldots+\frac{1}{\sqrt{2012\cdot 1}} \text{ va } 2\cdot\frac{2012}{2013}$$

247. Quyidagi yig‘indilarni hisoblang, bunda $C_n^m=\frac{n!}{m!\cdot(n-m)!}$

a) $C_n^0+C_n^2+C_n^4+C_n^6+\ldots$ *b)* $C_n^1+C_n^3+C_n^5+C_n^7+\ldots$

248. Tenglamani yeching:

$$\frac{1}{x^2+2x+2}+\frac{3}{x^2+2x+4}+\frac{5}{x^2+2x+6}+\ldots+\frac{2019}{x^2+2x+2020}=1010$$

249. Ushbu $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{a+b+c}$ tenglikdan $\frac{1}{a^{2021}}+\frac{1}{b^{2021}}+\frac{1}{c^{2021}}=\frac{1}{a^{2021}+b^{2021}+c^{2021}}$ tenglik kelib chiqishini isbotlang.

250. 3×3 kvadratda a,b,c,d,e,f,g,h,i sonlar joylashtirilgan(rasmga qarang)

a	b	c
d	e	f
g	h	i

Bu sehrli kvadrat bo‘lib, har bir qator, ustun va diagonaldagi sonlar yig‘indisi o‘zaro teng. $2(a+c+g+i)=b+d+f+h+4e$ tenglik o‘rinli bo‘lishini isbotlang.

251. Tenglamani yeching:

$$(x^2+x+1)(x^{10}+x^9+...+x+1)=(x^6+x^5+...+x+1)^2$$

252. Tenglamani yeching: $x^4+2y^4+4z^4=8xyz-2$

253. Ixtiyoriy a soni uchun $3(1+a^2+a^4)\geq(1+a+a^2)^2$ tengsizlikni isbotlang.

254. $ABCD$ qavariq to‘rtburchakda $\angle A=60^0$, $\angle B=40^0$, $\angle C=120^0$ va $CD=AD$ bo‘lsa, $BC+CD=AB$ tenglikni isbotlang.

255. Har bir x,y sonlar juftligi uchun $x*y$ son aniqlangan va ixtiyoriy x,y,z sonlari uchun quyidagi xossalar o‘rinli:

i) $x*x=0$

ii) $x*(y*z)=(x*y)+z$

U holda $2021*2020$ nimaga teng?

256. $\int_0^1 xf(x)dx=\int_0^1 x^3f(x)dx=\int_0^1 x^5f(x)dx=0$ tenglik o‘rinli bo‘ladigan uchinchi darajali $f(x)$ ko‘phadni toping.

257. $ABCD$ parallelogrammda AD va DC tomonlarining o‘rtalari mos ravishda M va N nuqtalar bo‘lsin. CM va BN kesmalar O nuqtada kesishadi. U holda $\frac{BO}{ON}\cdot\frac{CO}{OM}$ ni toping

258. Tomoni 4 ga teng $ABCD$ kvadratga tashqi chizilgan aylanadagi AB va BC yoylarning o'rtalari mos ravishda P va Q bo'lsin. Agar DP va DQ kesmalar AB va BC ni mos ravishda M va N nuqtada kessa, MN ni toping.

259. Aylananing AB vatari o'rtasi bo'lgan C nuqtadan ikkita KL va MN vatarlar o'tkazilgan (K va M nuqtalar AB vatardan bir tomonda yotadi). Agar Q nuqta AB va KN hamda P nuqta AB va ML vatarlarning kesishish nuqtalari bo'lsa, $QC = CP$ tenglikni isbotlang.

260. Agar $a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n$ ko'phadning barcha ildizlari haqiqiy bo'lsa, u holda $a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$ ko'phadning ham barcha ildizlari haqiqiy bo'lishini isbotlang. Bunda $a_0 \cdot a_n \neq 0$.

261. $\{a_n\}$ va $\{b_n\}$ ketma-ketliklar hadlari natural sonlardan iborat arifmetik progressiya tashkil qiladi. $a_1 = b_1 = 1 < a_2 \le b_2$ va $a_nb_n = 2020$ bo'lsa, n ni toping.

262. a,b,c,d nomanfiy haqiqiy sonlar bo'lib, $a^2 + b^2 + c^2 + d^2 = 1$ tenglikni qanoatlantirsa, quyidagi tengsizlikni isbotlang:

$$(1-a)(1-b)(1-c)(1-d) \ge abcd$$

263. $x, y \in \mathbb{R}$ uchun ushbu $\sqrt{(x-1)^2 + y^2} + \sqrt{x^2 + (y-1)^2}$ ifodaning eng kichik qiymatini toping.

264. $\dfrac{k^3-1}{k^3+1}$ ko'rinishidagi 2020 ta kasrning ko'paytmasi $\dfrac{2}{3}$ dan katta ekanini isbotlang. Bunda $k = 2, 3, 4, \ldots, 2021$

265. ABC uchburchak ichidan ixtiyoriy M nuqta olingan. Agar uchburchakning yuzi S ga teng bo'lsa, $4S \le AM \cdot BC + BM \cdot AC + CM \cdot AB$ ni isbotlang

266. Agar $(x+5)^2 + (y-12)^2 = 196$ bo'lsa, $x^2 + y^2$ ifodaning eng kichik qiymatini toping.

267. $0 < d \le c \le b \le a$ va $a + b + c + d \le 1$ bo'lsa, $a^2 + 3b^2 + 5c^2 + 7d^2 \le 1$ ni isbotlang.

268. a,b,c,x,y,z sonlar uchun $a = \dfrac{b+c}{x-2}$, $b = \dfrac{c+a}{y-2}$, $c = \dfrac{a+b}{z-2}$, $xy + yz + zx = 67$ va $x + y + z = 2020$ tengliklar o'rinli bo'lsa xyz ni toping.

269. 2^{2019} sonining oxirgi ikkita raqamini toping.

270. ABC uchburchakning AC va AB tomonlarida mos ravishda E va F nuqtalar olingan. BE va CF kesmalar X nuqtada kesishadi. $\frac{AF}{FB} = \left(\frac{AE}{EC}\right)^2$ va X nuqta BE kesmaning o‘rtasi bo‘lsa, $\frac{CX}{XF}$ ni toping.

271. Shunday p va q tub sonlarini topingki $(5^p - 2^p)(5^q - 2^q)$ ifoda pq ga bo‘linsin.

272. Agar $a^2 + a + 1 = 0$ bo‘lsa, $a^{2020} + \frac{1}{a^{2020}}$ ning qiymatini toping.

273. Tenglamani yeching:

$$\left(\sqrt{\sqrt{x^2 - 8x + 9} + \sqrt{x^2 - 8x + 7}}\right)^x + \left(\sqrt{\sqrt{x^2 - 8x + 9} - \sqrt{x^2 - 8x + 7}}\right)^x = 2^{1+\frac{x}{4}}$$

274. N sonining raqamlari yig‘indisi 2021 ga teng. N soni biror natural sonning kvadrati bo‘la oladimi?

275. * amali ushbu $a * b = a + b - \frac{2019}{2}$ xossaga ega. $1 * 2 * 3 * \ldots * 2019$ ni toping.

276. ABC uchburchakda $AB = 1$, $BC = \sqrt{7}$ va $CA = \sqrt{3}$. ℓ_1 to‘g‘ri chiziq A nuqtadan o‘tib, AB ga perpendikulyar, ℓ_2 to‘g‘ri chiziq esa B nuqtadan o‘tib, AC ga perpendikular. ℓ_1 va ℓ_2 to‘g‘ri chiziqlar P nuqtada kesishadi, u holda PC ni toping.

277. $a, b, c, d \in \mathbb{N}$, $a > b > c > d$, $a + b + c + d = 2020$ va $a^2 - b^2 + c^2 - d^2 = 2020$ bo‘lsa, a ning mumkin bo‘lgan qiymatlarini toping.

278. ABC uchburchak ichida O nuqta shunday tanlanganki, $\angle ABO = \angle BCO = \angle CAO = \alpha$ tenglik o‘rinli. Agar S uchburchakning yuzi va $AB^2 + BC^2 + AC^2 = m$ bo‘lsa, $ctg\alpha$ ni toping.

279. Markazi O bo‘lgan aylanada AB va CD o‘zaro perpendikular vatarlar E nuqtada kesishadi. N va T nuqtalar, mos ravishda AC va BD kesmalarning o‘rtalari bo‘lsa, $ENOT$ to‘rtburchak parallelogramm ekanligini isbotlang.

280. Agar $0 < a, b, c < 1$ bo‘lsa, u holda $\sqrt{abc} + \sqrt{(1-a)(1-b)(1-c)} < 1$ tengsizlikni isbotlang.

281. Tomonlarining uzunliklari a,b,c bo‘lgan ABC uchburchak berilgan. $x=\sqrt{a(-a+b+c)}$, $y=\sqrt{b(a-b+c)}$ va $z=\sqrt{c(a+b-c)}$ deb belgilash kiritilgan bo‘lsin. U holda:
a) Tomonlarining uzunliklari x,y,z ga teng bo‘lgan XYZ uchburchak mavjudligini isbotlang
b) Ushbu XYZ uchburchakning perimetri ABC uchburchakning perimetridan katta bo‘la olmasligini isbotlang
c) Agar $S_{\Delta ABC}=2021$ bo‘lsa, u holda $S_{\Delta XYZ}$ ni toping, bu yerda S-uchburchakning yuzi.
282. Tenglamani tub sonlarda yeching: $41x-yz=2009$
283. Darajasi 2021 dan oshmaydigan $P(x)$ ko‘phadning 2021-darajasi oldidagi koeffitsienti 1 ga teng. $P(0)=2020$, $P(1)=2019$,…, $P(2020)=0$ bo‘lsa, $P(2021)$ ni toping.
284. Aylanaga ichki chizilgan $ABCDEF$ oltiburchakda $AB=BC=2$, $CD=DE=9$, $EF=FA=12$ bo‘lsa, aylana radiusini toping.
285. $x_1,x_2,\ldots,x_n$ nomanfiy sonlari uchun $\frac{x_1}{\sqrt{1}}+\frac{x_2}{\sqrt{2}}+\ldots+\frac{x_n}{\sqrt{n}}=1$ bo‘lsa, $x_1^2+x_2^2+\ldots+x_n^2$ ning eng kichik qiymatini toping.
286. Ushbu $(1+x+x^2+x^3+\ldots+x^{100})^3$ ko‘phadning yoyilmasidagi x^{100} oldidagi koeffitsientni toping.
287. Ixtiyoriy natural $n>1$ lar uchun n^n-n sonining $(n-1)^2$ ga bo‘linishini isbotlang.
288. Ushbu $\sqrt{n+2020^k}+\sqrt{n}=(\sqrt{2021}+1)^k$ tenglamaning barcha natural yechimlarni toping.
289. Ushbu $P(x)=x^{2019}-ax^{2018}+bx-1$ ko‘phad a va b ning qanday qiymatlarida $(x-1)^2$ ga bo‘linadi?
290. Ushbu $x^2+y^3+z^6=w^7$ tenglama natural sonlarda cheksiz ko‘p yechimga ega ekanligini isbotlang
291. a va b irratsional sonlar bo‘lib, a^b ifoda ratsional son bo‘lishi mumkinmi?

292. $x, y \in \mathbb{R}$ bo‘lsa, $|x-y|+\sqrt{(x-3)^3+(y+1)^2}$ ifodaning eng kichik qiymatini toping

293. $\dfrac{x^2+y}{y^2-x}$ va $\dfrac{y^2+x}{x^2-y}$ ifodalarning har biri butun son bo‘ladigan barcha x va y natural sonlarni toping.

294. $4k+3$ ko‘rinishidagi tub sonlar cheksiz ko‘pligini isbotlang, bunda $k \in \mathbb{N}$

295. p_1 va p_2 lar ketma-ket kelgan toq tub sonlar bo‘lsin. Ushbu $q=\dfrac{p_1+p_2}{2}$ tenglikni qanoatlantiruvchi q soni murakkab son ekanini isbotlang

296. $f(x)=\dfrac{4^x}{4^x+2}$ funksiya uchun quyidagini hisoblang:

$$f(0)+f\left(\frac{1}{2020}\right)+f\left(\frac{2}{2020}\right)+\ldots+f\left(\frac{2019}{2020}\right)+f(1)$$

297. $\forall x, y \in \mathbb{Q}$ uchun $f(1)=2$ va $f(xy)=f(x)f(y)-f(x+y)+1$ shartlarni qanoatlantiruvchi barcha $f:\mathbb{Q}\rightarrow\mathbb{Q}$ funksiyalarni toping.

298. ABC uchburchakda $\angle A=60^0$. BK va CL bissektrisalar O nuqtada kesishadi. $OK=OL$ tenglikni isbotlang.

299. ABC uchburchakda O nuqta tashqi chizilgan aylana markazi va H nuqta ortomarkaz bo‘lsa, $\overrightarrow{OH}=\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}$ tenglikni isbotlang

300. Ushbu $\overline{abc}\cdot\overline{bca}\cdot\overline{cab}\geq\overline{aaa}\cdot\overline{bbb}\cdot\overline{ccc}$ tengsizlikni isbotlang, bu yerda $\overline{xyz}$ ifoda uch xonali son.

2-BOB. MATEMATIKADAN OLIMPIADA TESTLARI

1. $\sin 1^0, \sin 10^0, \sin 100^0, \sin 1000^0, \ldots$ ketma-ketlikning nechta hadi musbat?

A) 3 ta B) 4 ta C) barchasi D) cheksiz ko‘p

2. $f(x) = \sum_{n=1}^{5} \frac{|x-n|}{x-n}$ funksiyaning qiymatlar sohasi nechta butun sondan iborat?

A) 11 B) 10 C) 8 D) 6

3. $\int_{-4}^{4} x^3 |x| dx$ integralni hisoblang

A) 0 B) $\frac{1}{4}$ C) $\frac{1}{16}$ D) $-\frac{1}{4}$

4. 3^{101} ni 101 ga bo‘lgandagi qoldiqni toping(*Ko‘rsatma: Ferma teoremasidan foydalaning*)

A) 1 B) 3 C) 9 D) 100

5. $(x-1)(x-2)+(x-2)(x-3)-(x-3)(x-1)=2$ tenglamani yeching.

A) 1;2 B) 3 C) 2,3 D) 1;3

6. $x^3(x^3+1)(x^3+2)(x^3+3)$ ifodaning eng kichik qiymatini toping.

A) -1 B) 2 C) -2 D) 1

7. $x^{100}-2x^{51}+1$ ko‘phadni x^2-1 ga bo‘lgandagi qoldiqni toping.

A) $-2x$ B) 0 C) $2-2x$ D) $2+2x$

8. $P^2(x+1)=P(x^2)+2x+1$ ayniyatni qanoatlantiradigan $P(x)$ ko‘phadni toping.

A) $P(x)=x$ B) $P(x)=1$ C) $P(x)=-x$ D) $P(x)=x^2+1$

9. $\begin{cases} x+y=2 \\ xy-z^2=1 \end{cases}$ sistema nechta haqiqiy ildizga ega?

A) 2 ta B) 1 ta C) $\varnothing$ D) 3 ta

10. $x^2+y^2+ay=0$ $(a>0)$ aylana markazidan $y=2(a-x)$ to‘g‘ri chiziqqacha bo‘lgan masofani toping.

A) $\frac{a\sqrt{5}}{4}$ B) $\frac{a\sqrt{3}}{4}$ C) $\frac{a\sqrt{5}}{2}$ D) $\frac{\sqrt{5}}{2a}$

11. $\int \frac{dx}{\sin x}$ integralni hisoblang.

A) $\ln\left|\frac{1+\cos x}{1-\cos x}\right|+C$ B) $\frac{1}{2}\ln\left|\frac{\cos x-1}{\cos x+1}\right|+C$

C) $\ln\left|\frac{1+\cos x}{\cos x}\right|+C$ D) $\ln\left|\sin x\right|+C$

12. $\int \frac{dx}{e^x-1}$ ni hisoblang.

A) $\ln\left|\frac{e^x-1}{e^x}\right|+C$ B) $\ln\left|\frac{e^x}{e^x-1}\right|+C$ C) $\ln\left|\frac{e^x+1}{e^x-1}\right|+C$ D) $\ln\left|\frac{e^x-1}{e^x+1}\right|+C$

13. Agar $\vec{a}, \vec{b}, \vec{c}$ lar birlik vektorlar bo‘lib, $\vec{a}+\vec{b}+\vec{c}=0$ bo‘lsa, $\vec{a}\vec{b}+\vec{b}\vec{c}+\vec{a}\vec{c}$ ning qiymatini toping.

A) 0 B) 1,5 C) -1,5 D) -1

14. $\vec{a}$ vektor $\vec{b}=(1;2;3)$ va $\vec{c}=(-2;4;1)$ vektorlarga perpendikulyar bo‘lib, ushbu $\vec{a}\cdot(\vec{i}-2\vec{j}+\vec{k})=6$ shartni qanoatlantirsa, $\vec{a}$ ni toping.

A) $\vec{a}(1;2;-1)$ B) $\vec{a}(-5;3,5;4)$ C) $\vec{a}(5;1;-4)$ D) $\vec{a}(5;3,5;-4)$

15. $(x^2-x-3)^4$ ifoda yoyilmasida x ning juft darajalari oldidagi koeffitsiyentlar yig‘indisini toping.

A)40 B) 41 C)42 D) 43

16. $f(x)=x^3-3x+\lambda$ ko‘phad λ ning qanday qiymatlarida karrali ildizga ega?

A) ±3 B) ±1 C) ±2 D) ±4

17. $z=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ bo‘lsa, z^4 ni toping. Bunda $i^2=-1$

A)-1 B)4 C)-2 D)-4

18. (a,b,c) nuqtadan koordinata o‘qlaridan a,b va c uzunlikdagi kesmalar ajratuvchi tekislikkacha bo‘lgan masofa topilsin.

A) $\frac{abc}{\sqrt{a^2b^2+a^2b^2+b^2c^2}}$ B) $\frac{abc}{2\sqrt{a^2b^2+a^2b^2+b^2c^2}}$

C) $\dfrac{4abc}{\sqrt{a^2b^2+a^2b^2+b^2c^2}}$ D) $\dfrac{2abc}{\sqrt{a^2b^2+a^2b^2+b^2c^2}}$

19. $\arccos x$ funksiyani juft va toq funksiyalar yig'indisi ko'rinishida yozib, juft qismini ko'rsating.

A) $\dfrac{\pi}{2}$ B) $\dfrac{\arccos x+\arcsin x}{2}$ C) $\arcsin x$ D) $\dfrac{\arccos x-\arcsin x}{2}$

20. ABC uchburchakda AC tomonga tushirilgan balandligi 2 ga AB tomoni 5 ga, ABC uchburchakka tashqi chizilgan aylana radiusi 5 ga teng bo'lsa, BC tomonining uzunligini toping.

A) 2 B) 5 C) 4 D) $\sqrt{21}$

21. Uchlari $A(0;0)$, $B(2;0)$, $C(0;-4)$ nuqtalarda bo'lgan uchburchakka tashqi chizilgan aylana markazining koordinatalarini toping.

A) (1;-3) B) (1;-2) C) (-2;1) D) (3;4)

22. ABC uchburchakning B va C burchaklari ayirmasi $\dfrac{\pi}{2}$ ga teng. Agar AC va AB tomonlarining uzunliklari yig'indisi k ga, A uchidan tushirilgan balandlik h ga teng bo'lsa, uchburchakning C burchagini toping.

A) $2\arcsin 2hk$ B) $\arccos\dfrac{k}{h}$ C) $\dfrac{1}{2}\arcsin\dfrac{2h}{k}$ D) $\dfrac{1}{2}\arcsin\dfrac{2h(h+\sqrt{h^2+k^2})}{k^2}$

23. ABC uchburchakning balandliklari, $AA_1=h_a$, $BB_1=h_b$, va C burchagining bissektrisasi $CC_1=l$ ga teng bo'lsa, uchburchakning C burchagini toping.

A) $\arccos\dfrac{lh_b}{h_a^2+h_b^2}$ B) $arctg\dfrac{2lh_b}{h_a^2+h_b^2}$

C) $2\arcsin\dfrac{h_ah_b}{l(h_a+h_b)}$ D) $\dfrac{1}{2}\arcsin\dfrac{h_a}{l(h_a+h_b)}$

24. ABC uchburchakning B va C burchaklari nisbati 3:1, A burchakdagi bissektrisasi uchburchakning yuzini 2:1 nisbatda bo'lsa, uchburchak burchaklarini toping.

A) $\dfrac{\pi}{3},\dfrac{\pi}{3},\dfrac{\pi}{3}$ B) $\dfrac{\pi}{2},\dfrac{\pi}{2},0$ C) $\dfrac{\pi}{4},\dfrac{\pi}{2},\dfrac{\pi}{4}$ D) $\dfrac{\pi}{3},\dfrac{\pi}{6},\dfrac{\pi}{2}$

25. ABC uchburchakda AM va BN bissektrisalari O nuqtada kesishadi. Agar $AO:OM=\sqrt{3}:1$ va $BO:ON=1:(\sqrt{3}-1)$ bo'lsa, ABC uchburchakning burchaklarini toping.

A) $\frac{\pi}{4},\frac{\pi}{3},\frac{7\pi}{2}$ B) $\frac{\pi}{4},\frac{\pi}{2},\frac{3\pi}{4}$ C) $\frac{\pi}{3},\frac{\pi}{2},\frac{\pi}{6}$ D) $\frac{\pi}{6},\frac{\pi}{6},\frac{2\pi}{3}$

26. Limitni hisoblang. $\lim\limits_{x\to 1}\frac{x^4-4x+3}{x^2-3x+2}$

A) 1 B) 0 C) 0,5 D) aniqlab bo'lmaydi

27. Hosilani hisoblang: $y(x)=x^x+arctg(x^2+1)$

A) $x^x(1+2\ln x)+\frac{2x}{1+(x^2+1)^2}$ B) $x^x(1+\ln x)+\frac{x}{1+(x^2+1)^2}$

C) $x^x(1+\ln x)+\frac{2x}{1+(x^2+1)^2}$ D) $x^x(1+\ln x)+\frac{x}{2(1+(x^2+1))^2}$

28. $\int(2^x+3^x)^2dx$ ni hisoblang.

A) $\frac{6^x}{\ln 6}+2+C$ B) $\frac{4^x}{\ln 4}+2\frac{6^x}{\ln 6}+\frac{9^x}{\ln 9}+C$

C) $\frac{4^x}{\ln 5}+2\frac{6^x}{\ln 6}+\frac{9^x}{\ln 9}+C$ D) $\frac{4^x}{\ln 4}+\frac{6^x}{\ln 6}+\frac{9^x}{\ln 9}+C$

29. $\int tgxdx$ ni hisoblang

A) $-\ln|\cos x|+C$ B) $\ln|\cos x|+C$ C) $\ln|\sin x|+C$ D) $-\ln|\sin x|+C$

30. Agar $a_i>0, b_i>0$ $(i=1,2,...,n)$ sonlari uchun $\sum\limits_{i=1}^{n}a_i=\sum\limits_{i=1}^{n}b_i=1$ bo'lsa, $\sum\limits_{i=1}^{n}\frac{a_i^2}{a_i+b_i}$ ning eng katta qiymatini toping.

A) 0 B) aniqlab bo'lmaydi C) 0,5 D) 1

31. Aniq integralni hisoblang: $\int\limits_{-1}^{1}\frac{x}{\cos^3 x}dx$

A) 1 B) $\frac{2}{\cos 1}$ C) $\frac{1}{\cos 2}$ D) 0

32. $\varphi(n)$ orqali n dan kichik va n bilan o'zaro tub sonlar sonini belgilasak, $\varphi(1996)$ ni toping.

A) 1995 B) 1996 C) 996 D) 1001

33. $f_1(x) = x^2$ va $f_2(x) = x - 1$ funksiyalar grafiklari orasidagi eng qisqa masofani toping.

A) 1 B) $\frac{\sqrt{3}}{4}$ C) $\frac{3\sqrt{2}}{3}$ D) $\frac{3\sqrt{2}}{8}$

34. Muntazam tetraedrning qirrasi 1 ga teng. Tetraedr ichidagi ixtiyoriy nuqtadan uning yoqlarigacha bo'lgan masofalar yig'indisini toping.

A) 1 B) $\frac{\sqrt{6}}{3}$ C) $\frac{\sqrt{6}}{2}$ D) $\sqrt{12}$

35. Hisoblang: $\left(\frac{1+i\sqrt{3}}{1-i}\right)^{20}$

A) $2^9(1-i\sqrt{3})$ B) $2^{10}(1-i\sqrt{3})$ C) $-2^9(1-i\sqrt{3})$ D) $2^9(1+i\sqrt{3})$

36. Tengsizlikni yeching: $\frac{4x^2}{(1-\sqrt{1+2x})^2} < 2x+9$

A) $-\frac{1}{2} < x < 5\frac{5}{8}$ B) $-\frac{1}{2} < x \le 5\frac{5}{8}$ C) $0 \le x \le 5$ D) $-\frac{1}{2} \le x < 5\frac{5}{8}$

37. $\sqrt{\log_m a + 1}$, $\sqrt{\log_n a + 1}$, $\sqrt{\log_l a + 1}$ larning nisbati mos ravishda 5:6:7 kabi va yig'indisi 36 ga teng bo'lsa, $\log_{mnl} a$ ni toping.

A) 45 B) 42 C) 22,5 D) 21

38. i^i ni hsoblang.

A) $e^{-\frac{\pi}{2}}$ B) $e^{\frac{\pi}{2}}$ C) $\frac{\pi}{2}$ D) 1

39. To'g'ri burchakli ABC uchburchakning($AC \perp CB$) A uchidan AK mediana, B uchidan BD bissektrisa tushirilgan va ular O nuqtada kesishadi, $BO : OD = 8 : 5$ bo'lsa, B burchagining kosinusini toping.

A) 0,5 B) 0,6 C) 0,8 D) 0,28

40. Agar $f(x) = x \cdot (x-1) \cdot (x-2) \cdot \ldots \cdot (x-100)$ bo'lsa, $f'(0)$ ni toping.

A) 0 B) 100 C) 100! D) 1000!

41. Integralni hisoblang. $\int \frac{dx}{\cos(x+9)\cdot\cos(x+7)}$

A) $\frac{2}{\sin 2}\ln\left|\frac{\cos(x+9)}{\cos(x+7)}\right|$ B) $\frac{2}{\sin 16}\ln\left|\frac{\cos(x+7)}{\cos(x+9)}\right|$

C) $\frac{2}{\sin 2}\ln\left|\frac{\cos(x+7)}{\cos(x+9)}\right|$ D) $\frac{1}{\sin 2}\ln\left|\frac{\cos(x+7)}{\cos(x+9)}\right|$

42. $0,5[x]+10\{x\}=10$ tenglamaning barcha ildizlari yig'indisini toping.

A) 219,5 B) 220 C) 210 D) 199,5

43. Agar $f(x)=x^2+14x+42$, bo'lsa $f(f(f(f(x))))=0$ tenglamani yeching.

A) ildizi yo'q B) $\pm\sqrt[16]{7}-7$ C) $\pm\sqrt[32]{7}+7$ D) $\pm\sqrt[16]{7}+7$

44. $\sin x=\frac{x}{100}$ tenglama nechta yechimga ega?

A) 31 B) 32 C) 61 D) 63

45. ABC uchburchakning AB va BC tomonidan $AC_1:C_1B=1:4$ va $BA_1:A_1C=1:3$ shartni qanoatlantiruvchi C_1 va A_1 nuqtalar olingan. Agar AA_1 va CC_1 kesmalar P nuqtada kesishsa, $AP_1:PA$ va $CP:PC_1$ nisbatlarni toping.

A) 12:1 va 7:1 B) 15:1 va 4:1 C) 7:1 va 4:1 D) 16:1 va 9:1

46. $|x|+\left|\frac{x+1}{3x-1}\right|=a$ tenglama a ning qanday qiymatlarida 3 ta ildizga ega bo'ladi?

A) 2; $\frac{16}{9}$ B) 5; $\frac{2}{11}$ C) 9 D) 2

47. $x+y+\frac{2}{x+y}+\frac{1}{2xy}$ $(x>0, y>0)$ ifodaning eng katta qiymatini toping.

A) $\frac{47}{12}$ B) $\frac{7}{2}$ C) $\frac{15}{2}$ D) $\frac{\sqrt{845}}{3}$

48. Tenglamaning haqiqiy sonlarda nechta yechimi bor?

$$(2x-1)(3x+1)(5x+1)(30x+1)=10$$

A) 1 B) 2 C) 4 D) 3

49. $x,y,z\in\mathbb{R}^+$ sonlar uchun $xy+z=(x+z)(y+z)$ bo'lsa, $xyz_{\max}$ ni toping.

A) $\frac{1}{27}$ B) $\frac{3}{2}$ C) $\frac{1}{3}$ D) $\frac{9}{4}$

50. Aziz yonidagi pulning $\frac{1}{3}$ qismiga kitob, qolgan pulning $\frac{3}{4}$ qismiga daftar sotib oldi. Shundan so‘ng hisoblab qarasa, yonida qolgan pul kitob sotib olish uchun ishlatilgan pulning 40 foizidan 120 so‘m ko‘p ekan. Aziz daftar sotib olish uchun qancha pul ishlatgan?

A) 1800 so‘m B) 1200 so‘m C) 2400 so‘m D) 600 so‘m

51. $\cos\frac{\pi x}{9}\cos\frac{2\pi x}{9}\cos\frac{4\pi x}{9}=\frac{1}{8}$ tenglamani yeching.

A) $x=1$ B) $x=\frac{18n}{7}, n\in\mathbb{Z}$ C) $x=2n+1, n\in\mathbb{Z}$ D) B va C

52. Agar $\cos\alpha+\cos\beta=a$ va $\sin\alpha+\sin\beta=b$ va $a^2+b^2\neq 0$ bo‘lsa, $\cos(\alpha+\beta)$ ni toping.

A) $\frac{a^2-2b^2}{a^2+2b^2}$ B) $\frac{a^2-b^2}{a^2+b^2}$ C) $\frac{a^2-b^2}{2a^2+2b^2}$ D) $\frac{a^2+b^2}{a^2-b^2}$

53. $x^{99}+x^3+10x+5$ ko‘phadni x^2+1 ga bo‘lgandagi qoldiqni toping.

A) $8x-5$ B) $-8x+5$ C) $-8x-5$ D) $8x+5$

54. $(x^5-2x^4-3x^3+4x^2+5x-6)^{2021}$ ko‘phad standart ko‘rinishda yozilgandagi koeffitsientlar yig‘indisini toping.

A) 1 B) 2^{2021} C) -3^{2021} D) -1

55. Diagonali d ga teng kvadrat uchidan a masofada($a<\frac{d}{2}$) diagonaliga parallel to‘g‘ri chiziq bilan kesilgan. Hosil bo‘lgan uchburchak yuzini toping.

A) $0,5ad$ B) $0,25ad$ C) $0,5a^2$ D) a^2

56. $\frac{1}{1,\underbrace{000...00}_{100\,ta}1}$ sonni o‘nli kasr shaklida ifodalansa, verguldan keyingi 200-raqamni toping.

A) 0 B) 1 C) 2 D) 9

57. S uchburchak yuzi, P uning yarim perimetri bo‘lsa, to‘g‘ri tengsizlikni aniqlang.

A) $S \le \frac{P^2}{6}$ B) $S \le \frac{P^2}{3\sqrt{3}}$ C) $S \le \frac{P^2}{9}$ D) $S \le \frac{P^2}{9\sqrt{3}}$

58. O‘suvchi geometrik progressiyaning dastlabki uchta hadi yig‘indisi 35 ga, shu uchta hadning kvadratlari yig‘indisi 525 ga teng bo‘lsa, progressiyaning uchinchi hadini toping.

A) 22 B) 26 C) 20 D) 24

59. $(x+1)(x+2)(x+3)(x+4)$ ifodaning eng kichik qiymatini toping.

A) -2 B) 1 C) -1 D) 3

60. Geometrik progressiyada $S_n = b_1 + b_2 + \ldots + b_n$ va $S_n' = \frac{1}{b_1} + \frac{1}{b_2} + \ldots + \frac{1}{b_n}$ bo‘lsa, to‘g‘ri tasdiqni toping.

A) $S_n' \cdot S_n = 1$ B) $S_n = b_1 b_n S_n'$ C) $b_1 S_n' = b_n S_n$ D) $b_n S_n' = b_1 S_n$

61. Ikkita firma bitta binoni ijaraga olishgan va har oyda 6000 so‘m to‘lashadi. Agar birinchi firma soat 8^{00} dan 14^{30} gacha, ikkinchisi esa, soat 16^{30} dan 22^{00} gacha binoni band qilishsa, ikkinchi firma qancha ijara puli to‘laydi?

A) 2750 so‘m B) 3000 so‘m C) 2850 so‘m D) 3250 so‘m

62. To‘g‘ri tengliklarni aniqlang: 1) $\frac{171717}{252525} = \frac{1717}{2525}$ 2) $\frac{313131}{757575} = \frac{3131}{7575}$ 3) $\frac{1771}{2552} = \frac{17}{25}$

A) 1, 2, 3 B) 1, 2 C) 1, 3 D) 2, 3

63. $x^4 + x^3 + 2x^2 + ax + b$ to‘la kvadrat bo‘lsa, $\frac{a}{b}$ ning qiymatini toping.

A) $\frac{8}{7}$ B) $\frac{7}{8}$ C) $\frac{5}{8}$ D) 1

64. $x^2 - 4003x + 4006002 = 0$ tenglama ildizlari ayirmasining modulini toping.

A) 1 B) 2 C) 3 D) 4

65. $4arctg\frac{1}{5} - arctg\frac{1}{239}$ ning qiymatini toping

A) $arctg\frac{2}{3}$ B) $arctg\frac{7}{17}$ C) $arctg\frac{23}{23}$ D) $arctg\frac{9}{46}$

66. Uchburchak burchaklarining kosinuslari kvadratlari yig‘indisi 1 ga teng. Uchburchakka ichki va tashqi chizilgan aylanalar radiuslari mos ravishda $\sqrt{3}$ va $3\sqrt{2}$ ga teng bo‘lsa, uchburchakning yuzini toping.

A) $6\sqrt{6}$ B) $3\sqrt{6}$ C) $3+6\sqrt{6}$ D) $4\sqrt{6}$

67. Yig‘indini hisoblang: $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ldots+\frac{999}{1000!}$

A) $\frac{1}{1000!}$ B) $1-\frac{1}{1000!}$ C) $1+\frac{1}{1000!}$ D) 1

68. $\left(\overline{ab}\right)^c=\overline{cde}$ tenglik o‘rinli bo‘lsa, $c+d+e$ ni toping. Bunda turli harflar turli raqamlarni, bir xil harflar bir xil raqamlarni bildiradi.

A) 3 B) 9 C) 13 D) 19

69. Radiuslari r va $3r$ ga teng bo‘lgan aylanalar o‘zaro tashqi urinadi. Aylanalar va ularga o‘tkaziladigan umumiy urinma orasidagi soha yuzini toping.

A) $\frac{r^2(24\sqrt{3}-11\pi)}{6}$ B) $\frac{r^2(12\sqrt{3}-8\pi)}{3}$ C) $\frac{r^2(24\sqrt{3}-11\pi)}{3}$ B) $\frac{r^2(24\sqrt{3}-9\pi)}{6}$

70. $2x-17y=9$ tenglamani butun sonlar to‘plamida yeching.

A) $x=2a-1, y=17a+4, a\in\mathbb{Z}$ B) $x=17a+4, y=2a-1, a\in\mathbb{Z}$

C) $x=17a-4, y=2a-1, a\in\mathbb{Z}$ D) $(x;y)\in\left\{(-4;-1),(13;1)\right\}$

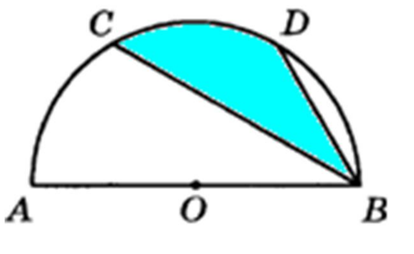
71. $2\ln x=ax$ tenglama a ning qanday qiymatlarida ikkita ildizga ega?

A) $a>0$ B) $0<a<1$ C) $0<a<\frac{1}{e}$ D) $0<a<\frac{2}{e}$

72. $EKUB(2n-3;n+2)=7$ bo‘lsa, n natural sonining $2000<n<2016$ oraliqdagi barcha qiymatlari yig‘indisini toping.

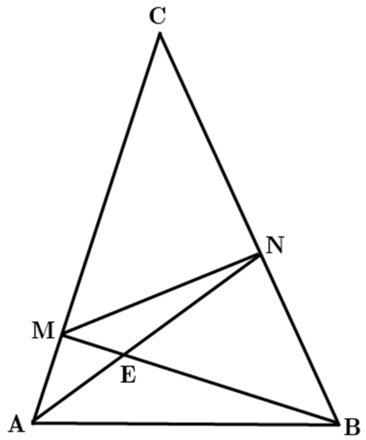
A) 4019 B) 2014 C) 2007 D) 4021

73. C va D nuqtalar AB diametrli yarim aylanani uchta teng qismga ajratadi. Aylananing radiusi 6 sm bo‘lsa, bo‘yalgan soha yuzini toping.



A) 8π B) 4π C) 5π D) 6π

74. Quyidagi chizmada $\angle NAB = 50^0$, $\angle \mathrm{NAM} = 30^0$, $\angle \mathrm{ABM} = 20^0$, $\angle \mathrm{CBM} = 60^0$, ekanligi ma'lum. $\angle \mathrm{CMN}$ ni toping.



A) 30^0 B) 45^0 C) 60^0 D) 40^0

75. $P(x)$ ko'phad uchun $(x^2+2)P(x)+ax+b = x^7+2x^5+3x^4+3x^3-2x+5$ munosabat o'rinli bo'lsa, $a+b$ ni toping

A) -23 B) 9 C) 12 D) 23

76. $f(4)=3$ va $f^{-1}(2)=1$ bo'lsa, $\frac{f^{-1}(3)}{f(1)}$ ni toping. Bunda $f^{-1}(x)$ funksiya $f(x)$ funksiyaga teskari funksiya

A) 1 B) 2 C) 3 D) 4

77. $2x^2+4y^2+z^2+4xy+2x-4$ ifodaning eng kichik qiymatini toping.

A) -5 B) 2 C) -3 D) -4

78. O'tmas burchakli uchburchakning eng katta tomoni 16 ga, qolgan ikki tomoni 8 va $3x+1$ ga teng. Noma'lum x ning qabul qilishi mumkin bo'lgan eng katta butun qiymatini toping.

A) 3 B) 4 C) 5 D) 6

79. $12 \cdot 125^{13} \cdot 16^{10} + 27^{20}$ soni necha xonali?

A) 39 B) 43 C) 40 D) 41

80. $3+7+11+\ldots+4x-3=820$ bo'lsa, x ni toping.

A) 79 B) 20 C) 20,5 D) 41

81. $\sqrt{3}+\sqrt{5}+\sqrt{7}+\ldots+\sqrt{79}=a-2$ bo'lsa, $\sqrt{12}+3+\sqrt{15}+\sqrt{21}+\ldots+\sqrt{237}$ ni toping.

A) $a+\sqrt{3}$ B) $\sqrt{3}a-3$ C) $3a$ D) $\sqrt{3}a$

82. Uchburchak ikki medianasining uzunligi 18 va 13,5 ga teng bo'lib ular o'zaro perpendikulyar. Uchinchi mediana uzunligini toping.
A) 18 B) 15 C) 17,5 D) 22,5

83. Agar $a+4b-c=0$ bo'lsa, $\dfrac{a^2-b^2-c^2-2bc}{a^2+b^2-c^2+2ab}$ ifodaning qiymatini toping.
A) 0 B) -4 C) 1,(6) D) 2

84. Yig'indini hisoblang: $17+20+23+...+(9n+8)$
A) $\dfrac{(3n-2)(9n+25)}{2}$ B) $(2n-6)(9n+25)$
C) $\dfrac{3(n-3)(5n+23)}{2}$ D) $3(n-1)(9n+25)$

85. $P(x)=(x^2-5x-3)Q(x-1)+3x-4$ ko'phad berilgan. $P(x)$-ko'phadning koeffitsiyentlari yig'indisi 13 ga teng bo'lsa, $Q(x)$ ko'phadning ozod hadini toping.
A) 2 B) -1 C) -2 D) 1

86. $P(x)$ ko'phad berilgan. $P(1)=5$, $P(-2)=2$ bo'lsa, $P(x)$ ko'phadni x^2+x-2 ga bo'lgandagi qoldiqni toping.
A) $x+2$ B) $2x+3$ C) $6x-1$ D) $x+4$

87. $\dfrac{\sqrt{x^2+x+\sqrt{x^2+x+\sqrt{x^2+x+...}}}}{\sqrt[3]{x^2\cdot\sqrt{x\cdot\sqrt[3]{x^2\cdot\sqrt{x\cdot...}}}}}=5$ bo'lsa, x ning musbat qiymatini toping.
A) 1 B) 0,5 C) 0,25 D) $\dfrac{1}{3}$

88. $x(y+z)^2+y(z+x)^2+z(x+y)^2-4xyz$ ifodani ko'paytuvchilarga ajrating.
A) $(x+y)(y+z)(x-2z)$ B) $(x+y)(x+2z)(y-z)$
C) $(x+2y)(y+2z)(x-2z)$ D) $(x+y)(y+z)(x+z)$

89. Qaysi nuqta $y=x^3+5x-2$ funksiyaga teskari funksiya grafigiga tegishli?
A) (4;1) B) (0;-2) C) (-2;1) D) (2;1)

90. Hisoblang: $\sqrt{2020^2+2020^2\cdot2021^2+2021^2}$
A) 4082421 B) 4072429 C) 4082429 D) 4072421

91. Toq sonlardan (1), (3,5), (7,9,11), … kabi guruhlar tuzilgan. 100-guruhdagi sonlar yig‘indisini toping.

A) 9702998 B) 922368 C) 1000000 D) 10000000

92. Agar $x \neq y$ va $x^4 + y^4 + 2x^2y + 2xy^2 + 2 = x^2 + y^2 + 2x + 2y$ bo‘lsa, $x + y$ ni toping.

A) 1 B) 2 C) 3 D) 4

93. $\frac{1}{x^2} + \frac{1}{xy} + \frac{1}{y^2} = 1$ tenglama nechta musbat butun yechimga ega?

A) 4 B) cheksiz ko‘p C) $\varnothing$ D) 1

94. $|x| + |y| < 100$ tengsizlik butun sonlarda nechta yechimga ega?

A) 19601 B) 19701 C) 19801 D) 10

95. Tenglamaning ildizlari yig‘indisini toping: $\frac{x}{y+z} = \frac{y}{x+z+1} = \frac{z}{x+y-1} = x + y + z$

A) 1,5 B) 2,5 C) 0,5 D) 0,8

96. Agar $x = \sqrt[3]{7+5\sqrt{2}} - \frac{1}{\sqrt[3]{7+5\sqrt{2}}}$ bo‘lsa, $x^3 + 3^x - 14$ ifodaning qiymatini toping.

A) 0 B) 1 C) 2 D) 3

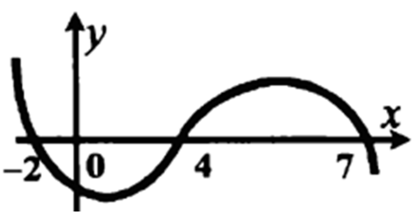
97. Agar $(x^2 + y^2)(y^2 + 1) + 9 = 6(x + y)$ bo‘lsa, $x^2 + y^2$ ni toping.

A) 4 B) 5 C) 7 D) 8

98. ABC to‘g‘ri burchakli uchburchakning AB gipotenuzasida M va N nuqtalar olingan. Bu nuqtalar uchun $AC = AM$ va $BC = BN$ shartlar bajarilsa, $\angle MCN$ ni toping.

A) 30^0 B) 40^0 C) 45^0 D) 60^0

99. Quyidagi chizmada $f(x)$ funksiyaning grafigi keltirilgan bo‘lsa, $f(x^2 + x + 1) = 0$ tenglamaning barcha haqiqiy yechimlari yig‘indisini toping.



A) -1 B) 2 C) -2 D) -3

100. $xy-2y-7x+19=0$ tenglamani qanoatlantiruvchi barcha butun x va y sonlar yig‘indisini toping.

A) 18 B) 14 C) 32 D) 36

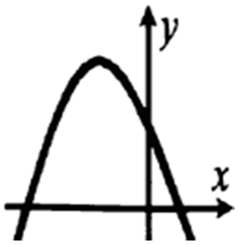
101. Agar $x^2+4xy+5y^2-2y+1=0$ bo‘lsa, xy ko‘paytmaning qiymatini toping

A) 1 B) 2 C) -2 D) 4

102. $x-2=\sqrt{2(b-1)x+1}$ tenglama yagona yechimga ega bo‘ladigan b ning barcha qiymatlarini toping.

A) $b\in(0;\infty)$ B) $b\in[1;\infty)$ C) $b\in\left[\frac{3}{4};\infty\right)$ D) $b\in\left[-\frac{2}{3};\infty\right)$

103. Quyidagi $y=ax^2+bx+c$ funksiya grafigidan foydalanib a,b,c larning ishoralarini aniqlang.



A) $a<0,b>0,c<0$ B) $a>0,b<0,c>0$

C) $a<0,b<0,c>0$ D) $a>0,b>0,c<0$

104. $y=|x|+\sqrt{4x^2-16x+16}$ funksiyaning eng kichik qiymatini toping.

A) 1 B) 2 C) 3 D) 4

105. Hisoblang: $\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}$

A) 3 B) 2 C) 1 D) 4

106. Hisoblang: $\sqrt[3]{1-27\sqrt[3]{26}+9\sqrt[3]{26^2}}+\sqrt[3]{26}$

A) 3 B) 2 C) 1 D) 4

107. Tenglama nechta ildizga ega?

$$\sqrt{3x^2+6x+7}+\sqrt{5x^2+10x+14}=4-2x-x^2$$

A) -1 B) 4 C) 2 D) 1

108. Teng yonli trapetsiyaning diagonali uni ikkita teng yonli uchburchakka ajratadi. Trapetsiya burchaklarini toping.

A) 72^0, 108^0 B) 45^0, 135^0 C) 80^0, 100^0 D) 82^0, 98^0

109. $a+b+c<0$ va $ax^2+bx+c=0$ tenglama haqiqiy yechimga ega emasligi ma'lum. c ning ishorasini aniqlang.

A) $c>0$ B) $c<0$ C) $c=0$ D) aniqlab bo'lmaydi

110. Agar A, B, C lar uchburchakning burchaklari bo'lib $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=a$ shartni qanoatlantisa, a ning eng kichik qiymatini toping.

A) $\frac{1}{8}$ B) $\frac{3}{2}$ C) $\frac{3}{4}$ D) $\frac{1}{2}$

111. Teng yonli ABC uchburchakda B burchak 110^0 ga teng. Uchburchakning ichida shunday M nuqta olinganki, bunda $\angle MAC=30^0$ va $\angle MCA=25^0$. $\angle BMC$ ni toping.

A) 75^0 B) 80^0 C) 85^0 D) 90^0

112. Hisoblang: $\frac{1}{\pi}\left(\arccos\left(\cos\frac{\pi}{3}\right)+\arccos\left(\cos\frac{2\pi}{3}\right)+\ldots+\arccos\left(\cos\frac{300\pi}{3}\right)\right)$

A) 150 B) 300 C) 15050 D) 30100

113. Tengsizlikni yeching: $x^2-7x+12<|x-4|$

A) (2;4) B) $\varnothing$ C) (3;4) D) (2;3)

114. Agar teng yonli trapetsiyaning balandligi h, yon tomoni esa unga tashqi chizilgan aylana markazidan α burchak ostida ko'rinsa, trapetsiyaning yuzini toping.

A) $S=h^2\cos\frac{\alpha}{2}$ B) $S=h^2ctg\frac{\alpha}{2}$ C) $S=h^2\sin\frac{\alpha}{2}$ D) $S=\frac{1}{2}h^2ctg\alpha$

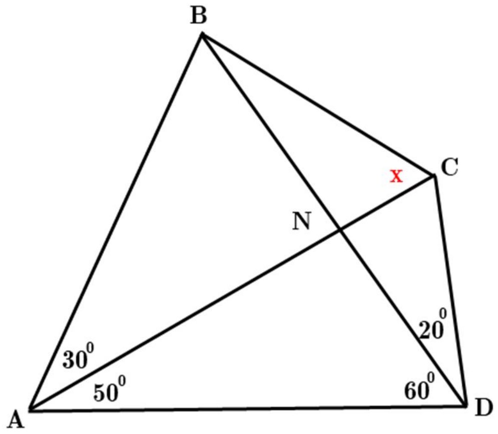
115. Agar $a=\sqrt[3]{20}+\sqrt[3]{50}$ bo'lsa, u holda a^3-30a ni hisoblang.

A) 35 B) 70 C) 85 D) 75

116. Agar $f(x)$ ko'phadni $x-1$ ga bo'lganda 3 qoldiq, $x-2$ ga bo'lganda 5 qoldiq qolsa, u holda $(x-1)(x-2)$ ga bo'lganda qanday qoldiq qoladi?

A) $3x+1$ B) $3x-2$ C) $2x-1$ D) $2x+1$

117. Quyidagi chizmaga ko'ra noma'lum burchak x ni toping.



A) 40^0 B) 60^0 C) 70^0 D) 80^0

118. $x^2+1=\log_3(x+2)+3x$ tenglamaning nechta ildizi bor?

A) 2 B) 1 C) 3 D) $\varnothing$

119. Hisoblang: $\sin 47^0+\sin 61^0-\sin 11^0-\sin 25^0$

A) 1 B) 0,5 C) $\cos 7^0$ D) $\dfrac{1}{2}\cos 7^0$

120. Teng yonli uchburchakda $\dfrac{r}{R}$ nisbat eng katta qiymatga ega bo‘lsa, burchaklar qanday qiymatga ega bo‘ladi(r, R –ichki va tashqi chizilgan aylana radiuslari)?

A) teng tomonli B) to‘g‘ri burchakli C) aniqlab bo‘lmaydi D) uchidagi burchak 120^0

121. Uchburchakning balandliklari 12, 15 va 20 ga teng. Bu qanday uchburchak?

A) to‘g‘ri burchakli B) o‘tmas burchakli C) o‘tkir burchakli D) teng yonli

122. Agar $a+b=1$ bo‘lsa, a^4+b^4 ifodaning eng kichik qiymatini toping.

A) 1 B) 0,5 C) 0,25 D) 0,125

123. a ning qanday qiymatlarida $-3<\dfrac{x^2+ax-2}{x^2-x+1}<2$ tengsizlik x ning barcha qiymatlarida o‘rinli bo‘ladi?

A) $-1<a<2$ B) $-3<a<2$ C) $-2<a<1$ D) $a>0$

124. $\begin{cases} x^2+y^2+2x\le 1 \\ x-y+a=0 \end{cases}$ sistema yagona yechimga ega bo‘ladigan a ning barcha qiymatlarini toping.

A) $a=3, a=-1$ B) $a=3, a=1$ C) $a=-1$ D) $a=1$

125. $|x|\cdot(x^2-4)=-1$ tenglama nechta ildizga ega?

A) 1   B) 2   C) 3   D) 4

126. $x^2 = y^2 + 2y + 13$ tenglama nechta butun yechimga ega?

A) 1   B) 2   C) 4   D) 0

127. Ifodaning eng katta qiymatini toping: $\sqrt{x-y} + \sqrt{x+2y-18} + \sqrt{36-2x-y}$

A) $3\sqrt{6}$ B) $3\sqrt{2}$ C) $2\sqrt{3}$ D) 6

128. $\sqrt[3]{x+1} - \sqrt{x-3} = 0! - \left(\frac{1}{5}\right)^{-1} + 4$ tenglama nechta haqiqiy ildizga ega?

A) 7   B) 1   C) 2   D) 0

129. ABC teng yonli uchburchakda $AB = BC = 5$, $AC = 6$ bo'lsa, uchburchakning ortomarkazi bilan og'irlik markazi orasidagi masofani toping.

A) $\frac{11}{24}$ B) $\frac{5}{6}$ C) $\frac{7}{12}$ D) $\frac{11}{12}$

130. Tenglamalar sistemasini yeching: $\begin{cases} 3-(y+1)^2 = \sqrt{x-y} \\ x+8y = \sqrt{x-y-9} \end{cases}$

A) (64;0)   B) (62;-2)   C) ∅   D) (8;-1)

131. $(x^2+30x+30)(x^2+x+30) = 30x^2$ tenglama ildizlari yig'indisini toping.

A) -31   B) -30   C) 31   D) 30

132. Yig'indini hisoblang: $1\cdot 1! + 2\cdot 2! + 3\cdot 3! + \ldots + n\cdot n!$

A) $(n+1)!$ B) $(n+1)!-1$ C) $n!+1$ D) $(n+1)! + \frac{n(n+1)}{2}$

133. Agar $a_n = \frac{1}{(n+1)\cdot\sqrt{n} + n\cdot\sqrt{n+1}}$ bo'lsa, ushbu $a_1 + a_2 + a_3 + \ldots + a_{99}$ yig'indini hisoblang.

A) 0,81   B) 0,1   C) 0,9   D) 0,01

134. ABC to'g'ri burchakli uchburchakda ($\angle C = 90^0$) CD balandlik o'tkazilgan. Agar ACD va BCD uchburchaklarga ichki chizilgan aylanalarning radiuslari mos ravishda r_1 va r_2 bo'lsa, ABC uchburchakka ichki chizilgan aylana radiusini toping.

A) $\sqrt{r_1^2 + r_2^2}$ B) $r_1 + r_2$ C) $r_1 + r_2 + \sqrt{r_1^2 + r_2^2}$ D) $2r_1 + 2r_2$

135. Ko'paytuvchilarga ajrating: $x^3 + y^3 + z^3 - 3xyz$

A) $(x+y+z)(x^2+y^2+z^2+xy+yz+zx)$

B) $(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

C) $(x+y+z)(x^2+y^2+z^2)$

D) $(x+y+z)^2(xy+yz+zx)$

136. Yig‘indini hisoblang: $1^2-2^2+3^2-4^2+\ldots+99^2-100^2+101^2$

A) 5050 B) 5151 C) 10201 D) 5051

137. Tenglama nechta yechimga ega? $\sin x = x^2+x+1$

A) 1 ta B) 2 ta C) 3 ta D) $\varnothing$

138. p ning qanday musbat qiymatida $3x^2-4px+9=0$ va $x^2-2px+5=0$ tenglamalar umumiy ildizga ega?

A) 3 B) 2 C) 1 D) -2

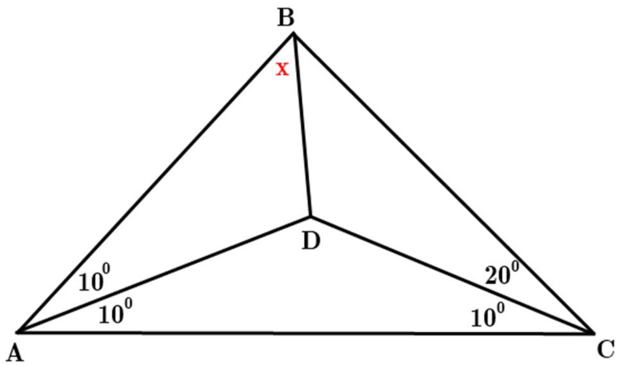
139. $n^2-7n+10$ kvadrat uchhadning absolyut qiymati tub son bo‘ladigan barcha n butun sonlar yig‘indisini toping.

A) 9 B) 10 C) 7 D) 12

140. $4038-\cfrac{2019^2}{4038-\cfrac{2019^2}{4038-\cfrac{2019^2}{\ddots}}}$ ni hisoblang

A) 2020 B) 2019 C) 2018 D) 2021

141. Quyidagi chizmaga ko‘ra noma’lum x ni toping.



A) 30^0 B) 45^0 C) 40^0 D) 20^0

142. $f(x) = \ln(x-4)$ va $g(x) = 2x+1$ bo‘lsa, $f^{-1}(g(2))$ ni toping. Bunda $f^{-1}(x)$ funksiya $f(x)$ funksiyaga teskari funksiya

A) $4+e^5$ B) 0 C) $4-e^5$ D) 1

143. Agar $arctga + arctgb + arctgc = \pi$ bo‘lsa, $a+b+c$ ni toping.

A) 0 B) $3\sqrt{abc}$ C) abc D) $\dfrac{ab}{c}$

144. $y = arctg\left(\arcsin\dfrac{\sin x - \cos x}{\sin x + \cos x}\right)$ funksiyaning aniqlanish sohasini toping.

A) $\left(\pi k; \dfrac{\pi}{2} + \pi k\right], k \in \mathbb{Z}$ B) $\left[\pi k; \dfrac{\pi}{2} + \pi k\right], k \in \mathbb{Z}$

C) $\left(\pi k; \dfrac{\pi}{2} + \pi k\right), k \in \mathbb{Z}$ D) $\left[\pi k; \dfrac{\pi}{2} + \pi k\right), k \in \mathbb{Z}$

145. 1 dan 300 gacha bo‘lgan natural sonlar ko‘paytmasi 6^n ga qoldiqsiz bo‘linsa, n ning qabul qilishi mumkin bo‘lgan eng katta qiymatini toping.

A) 59 B) 148 C) 256 D) 196

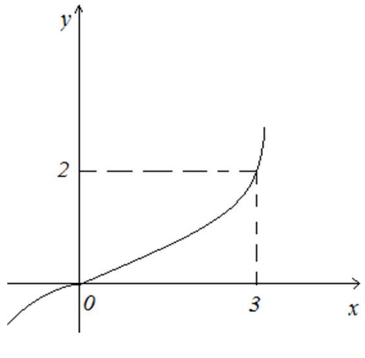
146. Agar $2x + 4y = 1$ bo‘lsa, $x^2 + y^2$ ning eng kichik qiymatini toping.

A) 1 B) $\dfrac{1}{10}$ C) $\dfrac{1}{20}$ D) $\dfrac{1}{5}$

147. Hisoblang. $\displaystyle\int_1^3 \frac{6x-2}{3x^2-2x+1}dx$

A) ln12,5 B) 21 C) ln11 D) ln21

148. Quyida chizmada $f(x)$ funksiyaning grafigi keltirilgan. $\displaystyle\int_0^3 f^2(x)\cdot f'(x)dx$ ni toping.



A) $\frac{8}{3}$ B) $\frac{4}{3}$ C) 9 D) $\frac{2}{3}$

149. $7+9+11+...+(2n+1)=an^2+bn+c$ bo‘lsa, $a+b+c$ ning qiymatini toping.

A) 0 B) 4 C) -5 D) -6

150. $f(x)=x^{1+x^2}$ bo‘lsa, $f'(1)$ ni toping.

A) 2 B) 1,5 C) 1 D) 0,5

151. ABC $(AB=BC)$ teng yonli uchburchakning BC tomonida N va M (N nuqta M ga qaraganda B ga yaqinroq) nuqtalar shunday olinganki, $NM=AM$ va $\angle MAC=\angle BAN$ tengliklar o‘rinli. $\angle BAN$ ni toping.

A) 45^0 B) 60^0 C) 40^0 D) 30^0

152. Tenglamani yeching: $\frac{\lg x^2}{(\lg x)^2}+\frac{\lg x^3}{(\lg x)^3}+\frac{\lg x^4}{(\lg x)^4}+...=8$

A) $10\sqrt{10}$ B) $2\sqrt{10}$ C) $10\sqrt{2}$ D) 3

153. Hisoblang: $\arcsin\frac{1}{3}+\arcsin\frac{1}{3\sqrt{11}}+\arcsin\frac{3}{\sqrt{11}}$

A) $\frac{\pi}{3}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{2}$ D) $\frac{\pi}{8}$

154. Hisoblang: $\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}+\sin^4\frac{7\pi}{8}$

A) 1,5 B) 0,5 C) 0,75 D) 6,5

155. a ning qanday qiymatida $\frac{x^2+2}{\sqrt{x^2+1}}\geq a$ tengsizlik x ning istalgan qiymatida o‘rinli bo‘ladi?

A) 3 B) 2 C) 9 D) 7

156. a ning qanday qiymatlarida $x^2-4x-\log_2 a=0$ tenglamaning ildizlari haqiqiy sonlar bo‘ladi?

A) $[2^4;\infty)$ B) $[2^{-4};\infty)$ C) $(0;\infty)$ D) $\varnothing$

157. $y=tgx\cdot ctgx+2\cos^2\frac{x}{2}$ funksiyaning qiymatlar sohasini toping.

A) $(1;2)\cup(2;3)$ B) $(2;3)$ C) $(1;2)\cup(2;4]$ D) $(1;3)$

158. Agar $x^2 + mx + m^2 + a = 0$ tenglamaning ildizlari a va b bo‘lsa, $a^2 + ab + b^2 + a$ ning qiymatini toping.

A) $m^2 - a$ B) 0 C) $2m^2$ D) $-m^2 - a$

159. Agar ikkita sonning ayirmasi, kvadratlarining ayirmasi, kublarining ayirmasi 1:3:7 kabi nisbatda ekanligi ma’lum bo‘lsa, shu sonlarning o‘rta geometrik qiymatini toping.

A) 1,5 B) $\sqrt{5}$ C) $\sqrt{2}$ D) $\sqrt{3}$

160. Uch xonali sonni xonalari teskari tartibda yozilsa, 99 ga kamayadi. Raqamlari yig‘indisi 14 ga teng va o‘rtada turgan raqam qolgan raqamlari yig‘indisiga teng. Shu sonning raqamlari ko‘paytmasini toping.

A) 48 B) 84 C) 120 D) 68

161. $y = 8 - x^2$ va $y = x^2$ parabolalar qanday burchak ostida kesishadi?

A) $arctg\dfrac{5}{7}$ B) $arctg\dfrac{15}{11}$ C) $arctg\dfrac{8}{15}$ D) aniqlab bo‘lmaydi

162. $\cos\left(\lg\left(2 - 3^{x^2}\right)\right) = 3^{x^2}$ tenglama nechta ildizga ega?

A) 0 B) cheksiz ko‘p C) 1 D) 2

163. $arctg\dfrac{x}{2} + arctg\dfrac{x}{3} = arctgx$ tenglama nechta yechimga ega?

A) 2 B) 3 C) 4 D) 5

164. Yig‘indini hisoblang: $lg\left(2tg1^0\right) + lg\left(2^3tg3^0\right) + \ldots + lg\left(2^{89}tg89^0\right)$

A) $\lg 2^{3025}$ B) $\lg 2^{1025}$ C) $\lg 2^{4025}$ D) $\lg 2^{2025}$

165. $x^2 + xy - 2y^2 - 7 = 0$ tenglamani qanoatlantiruvchi barcha x va y natural yechimlar yig‘indisini toping.

A) 4 B) 7 C) 3 D) 5

166. $x^2 - y^2 = 12$ va $2x^2 - 3xy + y^2 = 12$ egri chiziqlar kesishadigan barcha nuqtalar koordinatalarining ko‘paytmasini toping.

A) 8 B) -8 C) -64 D) 64

167. $xy - 3y - 5x + 8 = 0$ tenglamani qanoatlantiruvchi barcha butun x va y sonlar yig‘indisini toping.

A) 14 B) 34 C) 32 D) 36

168. To‘g‘ri burchakli uchburchakning katetlari uzunliklari $5x^2-9x+1=0$ tenglamaning ildizlariga teng. Shu uchburchakka tashqi chizilgan doiraning yuzini toping.
A) $0,71\pi$ B) $0,7\pi$ C) $0,76\pi$ D) $0,79\pi$

169. Agar $x^2-3xy+2y^2=0$ bo‘lsa, $\frac{2x-3y}{5x+3y}$ kasrning eng katta qiymatini toping.

A) -0,125 B) $\frac{1}{13}$ C) $\frac{8}{13}$ D) 4

170. $x^2+y^2\le 4$ to‘plamda $x+y$ ifodaning eng kichik qiymatini toping.

A) -2 B) $2-\sqrt{2}$ C) $-\sqrt{2}$ D) $-2\sqrt{2}$

171. $a^2+b^2+2ab-2a-2b+7$ ifodaning eng kichik qiymatini toping.
A) 6 B) 7 C) -7 D) 4

172. $\begin{cases} x^2+y^2=16 \\ |x|+|y|=5 \end{cases}$ tenglamalar sistemasi nechta yechimga ega?

A) 4 B) 2 C) 8 D) 1

173. Ifodaning eng kichik qiymatini toping: $2x^2-2xy+2y^2+2x+2y$
A) 0 B) 1 C) 2 D) -2

174. Trapetsiyaning diagonallari uni to‘rtta uchburchakka ajratadi. Agar trapetsiya asosiga yopishgan uchburchaklarning yuzlari S_1 va S_2 ga teng bo‘lsa, trapetsiyaning yuzini toping.

A) $\left(\sqrt{S_1}+\sqrt{S_2}\right)^2$ B) $\sqrt{S_1S_2}$ C) $S_1+S_2+\sqrt{S_1S_2}$ D) S_1+S_2

175. $3\cdot 2^x+1=y^2$ tenglamani qanoatlantiruvchi $(x;y)$ butun sonlar juftligini toping.
A) (0;2), (3;5) B) (0;2), (4;7) C) (0;2), (3;5), (4;7) D) cheksiz ko‘p yechimga ega

176. Ko‘paytuvchilarga ajrating: x^5+x+1

A) $(x^2-x+1)(x^3+x^2+1)$ B) $(x^2+x+1)(x^3-x+1)$

C) $(x^2+x-1)(x^3-x^2-1)$ D) $(x^2+x+1)(x^3-x^2+1)$

177. Tenglama nechta butun yechimga ega? $2ab+3a+b=0$
A) 1 B) 2 C) 4 D) cheksiz ko‘p

178. Nomanfiy a,b,c sonlari uchun $a+b+c=12$ tenglik o‘rinli bo‘lsa, $ab+bc+ac+abc$ ifodaning eng katta qiymatini toping.

A) 112 B) 108 C) 64 D) 128

179. Yig‘indini hisoblang: $1\cdot 2+2\cdot 3+3\cdot 4+...+(n-1)\cdot n$

A) $\dfrac{n(n+1)(n+2)}{3}$ B) $\dfrac{n(n-1)(n+1)}{3}$ C) $\dfrac{n(n+1)(2n+1)}{6}$ D) $\dfrac{n(n-1)(n+1)}{6}$

180. $x^2+px+q=0$ kvadrat tenglamaning ildizlari butun sonlar va $p+q=198$ ekani ma’lum. Berilgan tenglama ildizlari yig‘indisini toping.

A) 198 yoki 202 B) -198 C) 202 D) -198 yoki 202

181. Agar $a+b+c=36$ bo‘lsa, $\sqrt{2a+1}+\sqrt{2b+3}+\sqrt{2c+5}$ ifodaning eng katta qiymatini toping.

A) $3\sqrt{3}$ B) 36 C) 42 D) 45

182. Aniqmas integralni hisoblang: $\int e^x\cdot\cos x dx$

A) $\dfrac{e^x(\sin x+\cos x)}{2}+C$ B) $\dfrac{e^x(\sin x-\cos x)}{2}+C$

C) $\dfrac{e^x(\cos x-\sin x)}{2}+C$ D) $e^x(\sin x+\cos x)+C$

183. $f(x)+2f(\dfrac{1}{x})=x$ tenglikdan $f(x)$ ni toping.

A) $f(x)=\dfrac{2+3x}{x}$ B) $f(x)=\dfrac{3-2x^2}{2x}$ C) $f(x)=\dfrac{2-x^2}{3x}$ D) $f(x)=\dfrac{3+2x^2}{3x}$

184. $(2020+2017^{2017})^{2019}$ ni 8 ga bo‘lgandagi qoldiqni toping.

A) 1 B) 7 C) 5 D) 3

185. Chelakda bir oz suv bor. Agar chelakka 3 litr suv quyilsa, chelakning yarmi to‘ladi. Aksincha, 3 litr suv to‘kilsa, undagi suv chelakning $\dfrac{1}{8}$ qismini egallaydi. Dastlab chelakda necha litr suv bo‘lgan?

A) 3 B) 9 C) 7 D) 5

186. Birlar xonasidagi raqamidan 57 marta katta bo‘lgan eng kichik natural sonning raqamlari yigindisini toping.

A) 16 B) 12 C) 14 D) 15

187. $a = \lg 11$ va $b = \dfrac{1}{\lg 9}$ sonlarni taqqoslang

A) $a > b$ B) $a < b$ C) $a = b$ D) aniqlab bo‘lmaydi

188. ABC to‘g‘ri burchakli uchburchakda $\angle A = 90^0$, AH-balandlik, AL-bissektirisa va $\dfrac{2}{AL^2} = \dfrac{1}{5} + \dfrac{1}{AH^2}$ tenglik o‘rinli bo‘lsa, ABC uchburchak yuzini toping.

A) 10 B) 5 C) 25 D) 20

189. Tomonlari 2016 va 2017 bo‘lgan to‘g‘ri to‘rtburchaklardan kamida nechtasi birlashtirib kvadrat hosil qilish mumkin.

A) 4070306 B) 4066172 C) 4066272 D) 4046102

190. α, β, γ lar uchburchakning burchaklari bo‘lib, bu burchaklar sinuslarining nisbati mos ravishda 5:12:13 kabi bo‘lsa, $\cos\beta$ ni hisoblang.

A) $\dfrac{5}{13}$ B) $\dfrac{12}{13}$ C) $\dfrac{5}{12}$ D) aniqlab bo‘lmaydi

191. Agar $\dfrac{22}{x} = \dfrac{m^2}{n^2 - m^2}$ va $\dfrac{n^2}{n^2 - m^2} = 12$ bo‘lsa, x ni toping

A) 1 B) 2 C) 3 D) 12

192. $y = \dfrac{2x}{x-1}$ funksiyaning grafigi qaysi choraklardan o‘tadi?

A) I, II B) I, II, IV C) I, III, IV D)I,II,III, IV

193. Kema 8 soatdan ko‘p bo‘lmagan vaqt davomida daryo oqimi bo‘yicha 45 km yurishi va orqaga qaytishi kerak. Agar daryo oqimining tezligi 3 km/soat bo‘lsa, kemaning turgun suvdagi tezligi kamida qanday bo‘lishi kerak?

A) 9 km/soat B) 15 km/soat C) 12 km/soat D) 7,5 km/soat

194. $A(-1;-1)$ nuqtadan $y = 4x^2 - 4x + 3{,}25$ funksiyagacha bo‘lgan eng qisqa masofani toping.

A) $\dfrac{\sqrt{205}}{4}$ B) $\sqrt{17}$ C) $\dfrac{\sqrt{195}}{4}$ D) $\dfrac{\sqrt{203}}{4}$

195. Hisoblang: $1 + 4 \cdot 2 + 7 \cdot 2^2 + \ldots + 67 \cdot 2^{22}$

A) $2^{28} + 1$ B) $2^{29} - 1$ C) $2^{29} - 5$ D) $2^{29} + 5$

196. $a=-\frac{1}{3}$ bo‘lsa, $\left(a+\left(1+\left(\frac{3-a}{a+1}\right)^{-1}\right)^{-1}\right)^{-1}$ ni hisoblang.

A) -2 B) 0 C) 1 D) 2

197. Agar $(3-a)(3-b)(3-c)=(3+a)(3+b)(3+c)$ bo‘lsa, $(ab)^{-1}+(bc)^{-1}+(ac)^{-1}$ ni hisoblang. Bunda $abc\neq 0$

A) $\frac{1}{27}$ B) $\frac{1}{9}$ C) $-\frac{1}{9}$ D) $-\frac{1}{3}$

198. Tenglama nechta yechimga ega? $2^x+3^x+4^x=9^x$

A) 1 B) 2 C) $\varnothing$ D) cheksiz ko‘p

199. $ABCD$ trapetsiyada $\angle A=90^0$, $\angle D=30^0$. Markazi AD asosda bo‘lgan aylana AB,BC,CD tomonlarga urinadi. Agar trapetsiyaning o‘rta chizig‘i $6-\sqrt{3}$ ga teng bo‘lsa, aylana radiusini toping.

A) 1 B) 2 C) 3 D) 4

200. $ABCD$ kvadrat ichida M nuqta olingan. Agar $\angle MCD=15^0$ va $\angle MAB=60^0$ bo‘lsa, $\angle MBC$ ni toping.

A) 30^0 B) 45^0 C) 60^0 D) 75^0

3-BOB. MUSTAQIL YECHISH UCHUN MASALALAR

Yangi o'zgaruvchi kiritish orqali quyidagi tenglamalarni yeching(1-10)

1. $\sqrt{x+1}-\sqrt{a-x}=1$

2. $\sqrt[3]{1-x}+\sqrt[3]{1+x}=p$

3. $2\sqrt[3]{x-1}+\sqrt[3]{27-14x}=1$

4. $x=\sqrt{a-x}\cdot\sqrt{b-x}+\sqrt{b-x}\cdot\sqrt{c-x}+\sqrt{c-x}\cdot\sqrt{a-x}$

5. $7\sqrt{4x^2+5x-1}-14\sqrt{x^2-3x+3}=17x-13$

6. $(x^2+3x-4)^3+(2x^2-5x+3)^3=(3x^2-2x-1)^3$

7. $\frac{x^2}{x-1}+\sqrt{x-1}+\frac{\sqrt{x-1}}{x^2}=\frac{x-1}{x^2}+\frac{1}{\sqrt{x-1}}+\frac{x^2}{\sqrt{x-1}}$

8. $\sqrt{3-x}-\sqrt{\frac{1-x}{2-x}}=1$

9. $\sqrt{7x^2+8x+10}-\sqrt{7x^2-8x+10}=2x$

10. $(x^2+2x-5)^2+2(x^2+2x-5)=x+5$

11. $P(x)$ haqiqiy koeffitsientli kvadrat uchhad uchun $x^2-2x+2\le P(x)\le 2x^2-4x+3$ va $P(11)=181$ bo'lsa, $P(16)$ ni toping.

Ko'rsatma: $P(x)=a(x-1)^2+1$ *deb oling*

12. Tenglamalar sistemasini natural sonlarda yeching: $\begin{cases} a^3-b^3-c^3=3abc \\ a^2=2(b+c) \end{cases}$

Javob: $a=2, b=1, c=1$

13. $ABCD$ kvadrat ichida M nuqta olingan. $\angle MAB+\angle MBC+\angle MCD+\angle MDA>135^0$ tengsizlikni isbotlang.

14. $(a;b;c)$ sonlar uchligi ushbu $\begin{cases} x^3-xyz=2 \\ y^3-xyz=6 \\ z^3-xyz=20 \end{cases}$ sistemaning yechimlari bo'lib, $a^3+b^3+c^3=\frac{m}{n}$ tenglikni qanoatlantiradi($m,n\in\mathbb{N}$ va $EKUB(m;n)=1$). U holda $m+n$ ni toping.

Ko'rsatma: Sistemadagi xyz larni o'ng tomonga o'tkazib, tenglamalarni bir-biriga ko'paytiring va $xyz = t$ deb belgilash kiriting

15. l to'g'ri chiziqning AB kesmasida yotmagan 2021 ta nuqta olingan. Bu nuqtalardan A nuqtagacha bo'lgan masofalar yig'indisi B nuqtagacha bo'lgan masofalar yig'indisiga teng bo'lmasligini isbotlang.

16. ABC uchburchakning tomonlari $AB = 6\sqrt{2}$, $BC = 8$, $AC = 14$ ga teng. Uchburchakning B uchidan mediana va balandlik o'tkazilgan. C uchidan o'tkazilgan p to'g'ri chiziq mediananing davomini K nuqtada, balandlikning davomini M nuqtada va BA ning davomini N nuqtada kesadi. $\frac{KM}{CN}$ nisbat eng katta bo'lishi uchun p va AC to'g'ri chiziqlar orasidagi burchak nimaga teng bo'lishi kerak?

17. ABC uchburchak ichida olingan P nuqta uchun $\angle PBC = \angle PCA < \angle PAB$ bo'lsin. PB to'g'ri chiziq ΔABC ga tashqi chizilgan aylanani B va E nuqtalarda, CE tog'ri chiziq esa ΔAPE ga tashqi chizilgan aylanani E va F nuqtalarda kessin. U holda quyidagi tengliklarni isbotlang:

a) $S_{APE} = S_{AEC}$

b) $\frac{S_{APEF}}{S_{ABP}} = \left(\frac{AC}{AB}\right)^2$

18. $ax^2 + bx + c = 0$ va $(a+1)x^2 + (b+1)x + c + 1 = 0$ kvadrat tenglamalarning har biri ikkitadan butun ildizga ega bo'ladigan a, b, c butun sonlar mavjudmi?

19. $f(x)$ kvadrat funksiya ikkita turli haqiqiy nolga ega, hamda ixtiyoriy haqiqiy a, b sonlar uchun $f(a^2 + b^2) \geq f(2ab)$ tengsizlik o'rinli. $f(x)$ funksiya nollaridan kamida bittasi manfiy ekanligini isbotlang.

20. Tenglamaning barcha butun yechimlari sonini toping:

$$\cos\left(\frac{\pi}{8}\left(3x - \sqrt{9x^2 + 160x + 800}\right)\right) = 1$$

21. Ushbu $(1 + x + x^2 + \ldots + x^{27})(1 + x + x^2 + \ldots + x^{14})^2$ ko'phadning yoyilmasidagi x^{28} oldidagi koeffitsientini toping.

Javob: 434

22. Nechta $(a, b, c, d, e) \in \mathbb{N}$ sonlari uchun $abcde \leq a + b + c + d + e \leq 10$ tengsizlik bajariladi?

Javob: 116

23. Tekislikda n ta nuqta olinib, har ikkitasidan to‘g‘ri chiziq o‘tkazildi. Natijada 11 ta turli to‘g‘ri chiziq hosil bo‘ldi. n ning eng kichik qiymati nechaga teng?

24. Agar $a,b > 0$ va m natural son bo‘lsa, $\left(1+\frac{a}{b}\right)^m + \left(1+\frac{b}{a}\right)^m \geq 2^{m+1}$ tengsizlikni isbotlang.

Ko‘rsatma: Koshi tengsizligini qo‘llang

25. Taqqoslang: $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{2021}}$ va $\sqrt{2021}$

26. Hisoblang: $\frac{1}{2[\sqrt{1}]+1}+\frac{1}{2[\sqrt{2}]+1}+...+\frac{1}{2[\sqrt{100}]+1}$ (bunda $[\,]$ belgi sonning butun qismi). *Javob:* $\frac{190}{21}$

27. ABC uchburchakda $BC=a$ va $AC=b$. Agar bu tomonlarga tushirilgan medianalar o‘zaro perpendikular bo‘lsa, uchburchakning uchinchi tomonini toping.

Javob: $\sqrt{\frac{a^2+b^2}{5}}$

28. x haqiqiy son uchun $x^3+4x=8$ bo‘lsa, u holda x^7+64x^2 nimaga teng bo‘ladi?

Javob: 128

29. Qanday natural n larda n^3+2n^2+9n+8 ifoda biror natural sonning kubi bo‘ladi?

Javob: 7

30. Olim tirga bordi. Olimga 10 ta o‘q berilib, nishonga tekkan har bir o‘q uchun yana 3 ta o‘q beriladi. Olim 14 marta o‘q uzib, teng yarmini nishonga tekkizdi. Unda qancha o‘q qoldi?

31. Agar $0 \leq a \leq b \leq c \leq d \leq e \leq 100$ bo‘lsa, $\frac{a+b+c+d+e}{5}-c$ ifodaning eng katta qiymatini toping.

Javob: 40

32. Agar $1 \leq a,b,c \leq 2$ va $a+b+c=5$ bo‘lsa, $\frac{1}{a+b}+\frac{1}{b+c}$ ifodaning eng kichik qiymatini toping.

Javob: $\frac{4}{7}$

33. Taqqoslang: $\sin 1$ va $\log_3 \sqrt{7}$

34. Futbol to‘pi 32 ta charm bo‘lakdan tikilgan: oq oltiburchaklar va qora beshbo‘rchaklar. Har qanday qora bo‘lak faqat oq bo‘laklar bilan, har qanday oq bo‘lak esa uchta qora va uchta oq bo‘lak bilan chegaradosh. Oq rangdagi bo‘laklar nechta?

35. ABC uchburchakning AA_1 va BB_1 balandliklari uchun A_1 nuqtadan AC va AB tomonga tushirilgan perpendikulyar va B_1 nuqtadan BC va BA tomonlarga tushirilgan perpendikulyarlar asoslari teng yonli trapetsiya hosil qilishini isbotlang.

36. ABC uchburchakning tomonlari a, b, c bo‘lib, uning A uchidan AM mediana va AL bissektrisa tushirilgan. BC kesmada S nuqta shunday tanlanganki, AL nur MAS burchakning bissektrisasi bo‘ladi. $\frac{BS}{SC}$ nisbatni toping.

37. Ushbu $x_1 + x_2 + x_3 + x_4 = 11$ tenglama nechta nomanfiy butun yechimga ega?

38. Ixtiyoriy $x \in \mathbb{R}$ uchun $f(f(x)) + xf(x) = 1$ shartni qanoatlantiruvchi barcha $f(x)$ funksiyalarni toping

39. $ABCD$ to‘rtburchakka ichki aylana chizish mumkin. A uchidan chiquvchi a to‘g‘ri chiziq BC tomonni M nuqtada va DC nurni esa N nuqtada kesadi. I_1, I_2, I_3 nuqtalar mos ravishda ABM, MNC va NDA uchburchaklarga ichki chizilgan aylana markazlari bo‘lsin. a to‘g‘ri chiziq $I_1I_2I_3$ uchburchakning ortomarkazidan o‘tishini isbotlang.

40. A to‘plam elementlari quyidagicha tuzilgan. $x \in A$ haqiqiy son $x \neq 0$ va $x \neq 1$ uchun $\frac{x+1}{x} \in A$ va $\frac{2x-1}{x-1} \in A$ shart bajariladi. Agar $2 \in A$ ekanligi ma’lum bo‘lsa A to‘plamning barcha ratsional elementlari birdan katta ekanligini isbotlang.

41. $f(x+y)=f(x)+f(y)-2xy$ tenglikni qanoatlantiruvchi barcha $f(x)$ funksiyalarni toping.

42. $f(x)$ funksiya $[0;\infty)$ da aniqlangan va $f\left(f\left(f(x)\right)\right)=x^3$ tenglik o‘rinli bo‘lsa, $f(x)$ funksiyani toping.

43. Ushbu $\sqrt{x^2-6x+13}+\sqrt{x^2-14x+58}$ ifodaning eng kichik qiymatini toping.

Javob: $\sqrt{41}$

44. ABC uchburchakning BC va AC tomonlarida mos ravishda D va E nuqtalar shunday olinganki, bunda $\angle BAD=50^0$, $\angle ABE=30^0$. Agar $\angle ABC=\angle ACB=50^0$ bo‘lsa, $\angle BED$ ni toping.

45. a,b,c,d,e lar bir-biridan farqli butun sonlar bo‘lib, ushbu $(6-a)(6-b)(6-c)(6-d)(6-e)=45$ tenglik o‘rinli bo‘lsa, $a+b+c+d+e$ ni toping

Javob: 25

46. To‘g‘ri burchakli ABC uchburchakning C burchagi to‘g‘ri burchak bo‘lib, BC kateti D va E nuqtalar orqali(D nuqta B ga yaqin) teng uch bo‘lakka bo‘lingan. Agar $BC=3AC$ bo‘lsa, $\angle ADC$ va $\angle ABC$ burchaklarning yig‘indisini toping.

47. Tekislikning biror nuqtasidan to‘g‘ri to‘rtburchakning uchlarigacha bo‘lgan masofalar 5, 12, 13 ga teng. To‘g‘ri to‘rtburchakning yuzini toping.

Javob: 60

48. $2^{99}+2^9$ sonini 49 ga bo‘lgandagi qoldiqni toping.

Javob: 9

49. 2^{p^2} sonini 18 ga bo‘lgandagi qoldiqni toping ($p>3$-tub son)

50. $\left[\frac{1^2}{2010}\right];\left[\frac{2^2}{2010}\right];...;\left[\frac{2009^2}{2010}\right]$ ketma-ketlikda nechta turli sonlar uchraydi? Bu yerda $[\ ]$ belgi soning butun qismini bildiradi.

51. $a=\log_{2019}2020$ va $b=\log_{2020}2021$ sonlarni taqqoslang.

Ko‘rsatma: 194-masalaning yechimiga qarang

52. a,b,c lar uchburchak tomonlarining uzunliklari. Uchburchakning ichidagi biror nuqtadan o‘tuvchi 3 ta kesishuvchi to‘g‘ri chiziq uchburchakning tomonlariga

parallel. Kesishuvchi to‘g‘ri chiziqlarning uchburchak tomonlari bilan kesilishdan hosil bo‘lgan kesmalarning har biri x ga teng bo‘lsa, x ni toping.

53. $ABCD$ qavariq to‘rtburchakda ABC, BCD, CDA, DAB uchburchaklarning og‘irlik markazlari ketma-ket tutashtirilgan. Agar $ABCD$ to‘rtburchakning yuzi S bo‘lsa, u holda hosil bo‘lgan to‘rtburchakning yuzini toping.

54. $n \in \mathbb{N}$ uchun $i + 2i^2 + 3i^3 + \ldots + ni^n = 48 + 49i$ tenglik o‘rinli bo‘lsa, n ni toping. Bu yerda $i^2 = -1$

Javob: $n = 97$

55. Tomoni n ga teng bo‘lgan kvadrat vertikal va gorizontal bo‘lgan n^2 ta har xil kvadratlarga bo‘lingan. Hosil bo‘lgan chizmada nechta kvadrat sanash mumkin?

Javob: $\dfrac{n(n+1)(2n+1)}{6}$

56. p-fazodagi kesmaning uzunligi, a, b, c lar esa uning koordinata tekisliklardagi proeksiyalari bo‘lsin. U holda $\dfrac{a+b+c}{p}$ nisbatning mumkin bo‘lgan eng katta qiymatini toping.

Javob: $\sqrt{3}$

57. $\log_{2020}\dfrac{1}{2}\log_{2019}\dfrac{1}{3}\ldots\log_2\dfrac{1}{2020}$ ni hisoblang.

58. $f(x) = 2^x$ bo‘lsa $f^{(n)}(x)$ ni toping. Bu yerda $f^{(n)}(x)$ ifoda $f(x)$ funksiyaning n-tartibli hosilasi.

59. $x^4 + (x-2)^4 = 34$ tenglama ildizlari yig‘indisini toping.

Ko‘rsatma: $x - 1 = a$ *deb belgilash kiriting*

60. a, b, c sonlari $x^3 - 9x^2 + 11x - 1 = 0$ tenglamaning ildizlari bo‘lib, $n = \sqrt{a} + \sqrt{b} + \sqrt{c}$ bo‘lsa, $n^4 - 18n^2 - 8n$ ning qiymatini toping

Javob: -37

61. x_1, x_2, x_3 sonlari $x^3 - ax^2 + ax - a = 0$ tenglamaning ildizlari bo‘lsa, $x_1^3 + x_2^3 + x_3^3 - 3x_1x_2x_3$ ni toping.

62. Hisoblang: $\dfrac{\sqrt{2021 + \sqrt{2021 + \sqrt{2021 + \ldots}}}}{\sqrt{2020 + \sqrt{2020 + \sqrt{2020 + \ldots.}}}}$

63. $y=|x-1|+|x-2|+...+|x-2021|$ ifodaning eng kichik qiymatini toping.

64. $a,b,c\in\left(0;\frac{\pi}{2}\right)$ va $\cos a=a$, $\sin(\cos b)=b$, $\cos(\sin c)=c$ tengliklar o‘rinli bo‘lsa, a,b,c sonlarni o‘sish tartibida joylashtiring.

65. Aniqmas integralni hisoblang: $\int \operatorname{sgn} x dx$

66. Hisoblang: $\left[\frac{2020!+2017!}{2019!+2018!}\right]$. Bunda [ ] belgi sonning butun qismi

67. ABC uchburchakning AB va BC tomonlari olingan D va E nuqtalar uchun $AC=25$, $DB=6$, $BE=20$ va $AD=EC=x$ bo‘lib, $S_{DBE}=S_{ADEC}$ shart bajarilsa x ni toping.

68. Uchlari $A(-2;-4)$, $B(2;8)$ va $C(10;2)$ nuqtalarda bo‘lgan uchburchak yuzini toping.

69. Uchlari $A(2;2;2)$, $B(4;3;3)$, $C(4;5;4)$ va $D(5;5;6)$ nuqtalarda bo‘lgan uchburchakli piramida hajmini hisoblang.

70. BD-ABC uchburchak B burchagining bissektrisasi. E nuqta shunday tanlanganki, bunda $\angle EAB=\angle ACB$, $AE=DC$ va ED kesma AB kesma bilan K nuqtada kesishadi. $KE=KD$ ekanligini isbotlang.

71. $x^2+y^2+z^2=2xyz$ tenglama nechta butun yechimga ega?

72. Tenglamani yeching: $x^3-[x]=3$. Bunda [ ] belgi sonning butun qismi

Javob: $\sqrt[3]{4}$

73. $ABCD$ trapetsiyada BC va AD asoslar. E-diagonallar kesishgan nuqta. O-AED uchburchakka tashqi chizilgan aylana markazi. B va C uchlaridan AC va BD diagonallarga tushirilgan perpendikulyarlarning asoslari mos ravishda K va L bo‘lsa, u holda $KL\perp OE$ ekanligini isbotlang.

74. B burchagi to‘g‘ri bo‘lgan $ABCD$ qavariq to‘rtburchakda $AB=2$, $BC=3$, $AD=6$, $CD=7$ va bu to‘rtburchakka aylana ichki chizish mumkin bo‘lsa, uning radiusini aniqlang.

75. $x\geq 0$, $y\geq 0$, $z\geq 0$ sonlari uchun quyidagi tengsizlikni isbotlang:

$$xy+yz+zx\geq\sqrt{3xyz(x+y+z)}$$

Ko'rsatma: Ikkala tomonini xyz *ga bo'ling va* $\frac{1}{x}=a$, $\frac{1}{y}=b$, $\frac{1}{z}=c$ *deb belgilang*

76. ABC uchburchakning ichida O nuqta shunday olinganki, bunda $\angle ABO=\angle CAO$, $\angle BAO=\angle BCO$, $\angle BOC=90^0$. U holda $AC:OC$ nisbatni toping.

77. ABC o'tkir burchakli uchburchakning AA_1 va BB_1 balandliklari o'tkazilgan. Ma'lumki, ABC uchburchakka tashqi chizilgan aylana markazi A_1B_1 kesmada yotadi. U holda $\sin\angle A\cdot\sin\angle B\cdot\cos\angle C$ ning qiymatini toping.

78. -1 dan katta a,b,c sonlar uchun $ab+a+b=11$, $bc+b+c=5$ va $ac+a+c=1$ tengliklar o'rinli bo'lsa, $ab+bc+ac+a+b+c$ ning qiymatini toping.

79. $f(x)$ funksiya uchun $f(x+1)=f(x)+2x+1$ va $f(0)=-5$ bo'lsa, $f(18)$ ni toping.

80. Tenglamani butun sonlarda yeching: $2^x+1=y^2$

81. *a)* 1,2,2,3,3,3,4,4,4,4,5,... ketma-ketlikning dastlabki 100 ta hadi yig'indisini toping.

b) 1,2,2,3,3,3,4,4,4,4,5,... ketma-ketlikning 2020-hadini toping.

Ko'rsatma: Umumiy hadi formulasi $a_n=\left[\frac{1+\sqrt{8n-7}}{2}\right]$ *ekanini keltirib chiqaring.*

82. $9997\cdot n$ ko'paytmaning o'nli yozuvida faqat toq raqamlar bo'ladigan eng kichik natural $n(n>1)$ sonini toping.

83. ABC uchburchakda $AB=65$, $BC=33$, $AC=56$ bo'lib, w aylana AC, BC tomonlarga va ABC uchburchakka tashqi chizilgan aylanaga urinadi. w aylananing radiusini toping.

84. $a>0$ va butun koeffitsientli $P(x)$ ko'phad uchun $P(1)=P(3)=P(5)=P(7)=a$ va $P(2)=P(4)=P(6)=P(8)=-a$ tengliklar o'rinli bo'lsa, a ni toping.

85. 1 bilan boshlanadigan va 1 raqami oxiriga o'tkazilsa, 3 marta kattalashadigan sonni toping.

86. $ABCD$ to'rtburchak aylanaga ichki chizilgan. $AB=AD$, $AC=1$ va $\angle ACD=60^0$ bo'lsa, $ABCD$ to'rtburchak yuzini toping.

87. $f(x)=x^4+ax^3+bx^2+cx+d$ ko‘phadning barcha ildizlari nomusbat butun sonlar bo‘lib, $a+b+c+d=2020$ bo‘lsa, d ni toping.
Javob: $d=0$
88. $ACDE$ to‘rtburchakda D va E nuqtalar AC to‘g‘ri chiziqqa nisbatan bir tomonda yotadi. AC tomonda shunday B nuqta olinganki, asosi BC bo‘lgan teng yonli uchburchak hosil bo‘lgan. BDC, DBE, ADE burchaklar 80^0 dan bo‘lsa, AED burchakni toping.
89. O‘tma sburchakli uchburchakning barcha burchaklari tub sonlardan iborat. O‘zaro o‘xshash bo‘lmagan bunday uchburchaklar nechta?
90. a va b natural sonlar uchun $104<a+b<108$ va $0,91<\frac{a}{b}<0,92$ bo‘lsa, $2a+b$ ning qiymatini toping.
91. $|x|+|x+1|+|x+2|+\ldots+|x+2018|=x^2+2018x-2019$ tenglama nechta yechimga ega?
92. $x^3-y^3=xy+61$ tenglamani natural sonlarda yeching.
93. $x+2y=20$ va $y+2x=16$ to‘g‘ri chiziqlarning $y=\frac{1}{x}$ giperbola bilan kesishish nuqtalari bitta aylanada yotishini isbotlang.
94. Ixtiyoriy $0<x,y,z<1$ sonlar uchun $x(1-y)+y(1-z)+z(1-x)<1$ tengsizlikni isbotlang.
95. Qavariq $ABCDE$ beshburchakda ABC, BCD, CDE, DEA uchburchaklarning har birining yuzi S ga, EAB uchburchakning yuzi esa $1,5S$ ga teng. Beshburchakning yuzini toping.
96. Butun x,y,z,t sonlar yig‘indisi nolga teng bo‘lsa, $\frac{x^4+y^4+z^4+t^4}{2}+2xyzt$ ifoda biror butun sonning kvadrati ekanligini isbotlang.
97. Musbat a,b,c sonlar uchun $abc=1$ munosabat o‘rinli bo‘lsa, quyidagi tengsizlikni isbotlang:

$$\frac{1}{a^3(b+c)}+\frac{1}{b^3(a+c)}+\frac{1}{c^3(a+b)}\geq\frac{3}{2}$$

98. ABC muntazam uchburchak ichidan D nuqta shunday tanlanganki, bu nuqtadan uchburchak uchlarigacha bo‘lgan masofalar 3, 4 va 5 ga teng. ABC uchburchak yuzini toping.

99. $\begin{cases} x^2+y^2=4 \\ z^2+t^2=9 \\ xt+yz=6 \end{cases}$ sistemaning barcha yechimlari orasidan $x+z$ yig‘indi qabul qilishi mumkin bo‘lgan eng katta qiymatini toping.

100. a,b,c -uchburchak tomonlari bo‘lsin. Uchburchakka tashqi chizilgan aylana markazi O bilan uchburchakning og‘irlik markazi G orasidagi masofani toping.

101. Yig‘indini hisoblang:

$$arctg\frac{x}{x^2+2}+arctg\frac{x}{x^2+2+4}+\ldots+arctg\frac{x}{x^2+2+4+\ldots+2020}$$

102. Biror uchburchak burchaklarining sinuslari ratsional. Bu burchaklarning kosinuslari ham ratsional bo‘lishini isbotlang.

103. Uzunliklari a,b,c,d larga teng bo‘lgan to‘rtta kesma berilgan. Uzunligi $\sqrt{a^2+b^2-c^2+d^2}$ ga teng bo‘lgan kesma yasang.

104. Sistema yechimlari ko‘paytmasini toping: $\begin{cases} x^2y^2-2x+y^2=0 \\ 2x^2-4x+3+y^3=0 \end{cases}$

105. $[x^2-4x]=x-6$ tenglamaning eng katta va eng kichik ildizlari nisbatiga ildizlarini qo‘shing.

106. $a^2+b^2-8c=6$ tenglamani butun sonlarda yeching

107. $1^1+2^2+3^3+\ldots+999^{999}+1000^{1000}$ yig‘indining birinchi 3 ta raqamini toping.

108. Barcha natural sonlar bir qator qilib, ketma-ket yozilganda 123456789101112... dastlabki ketma-ket kelgan 3 ta 7 ajratib olindi. Bu 7 lar nechanchi o‘rinda turishini aniqlang.

109. 1111112111111 tub sonmi yoki murakkab?

110. a) Barcha shunday p,q,r tub sonlarni topingki, $p^4+q^4+r^4-3$ ham tub son bo‘lsin.

b) Barcha shunday p,q,r,s tub sonlarni topingki, $p^4+q^4+r^4+14=s^4$ tenglik o‘rinli bo‘lsin.

111. O‘tkir burchakli uchburchak tomonlarining uzunliklari ketma-ket kelgan butun sonlar bilan ifodalanadi. O‘rtacha uzunlikdagi tomonga tushirilgan balandlik bu tomonni uzunliklarining farqi 4 ga teng bo‘lgan kesmalarga ajratishini isbotlang.

112. $a > b > 1$ sonlari uchun $a^{b^{a}} > b^{a^{b}}$ tengsizlik o‘rinli ekanini isbotlang

113. Istalgan natural n uchun $\left(1+\frac{1}{n}\right)^{n+1} > \left(1+\frac{1}{n+1}\right)^{n+2}$ tengsizlikni isbotlang.

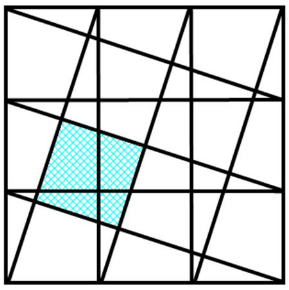
(*Ko‘rsatma*: $x > 0$ oraliqda $f(x) = (1+x)\ln\left(1+\frac{1}{x}\right)$ funksiyadan foydalaning)

114. Ixtiyoriy $n > 1$ uchun $\sqrt[n]{n} \le \sqrt[3]{3}$ tengsizlik o‘rinli ekanini isbotlang.

(*Ko‘rsatma*: $x \ge 3$ oraliqda $f(x) = \frac{\ln x}{x}$ funksiyadan foydalaning)

115. 123456789 sonining bitta yoki bir nechta raqamini o‘chirib, natijada 11 ga bo‘linadigan son hosil qilish mumkinmi?

116. Tomoni 1 ga teng bo‘lgan kvadrat berilgan bo‘lib, uning har bir tomoni uchta teng bo‘lakka bo‘lingan. Kvadratning tomonlari bo‘lingan nuqtalardan kesmalar o‘tkazilgan (rasmga qarang). Bo‘yalgan kvadratchaning yuzini toping.



117. $\{a_n\}$ ketma-ketlikda $a_1 = 1$, $a_2 = \frac{1}{\sqrt{3}}$ va $a_{n+2} = \frac{a_n + a_{n+1}}{1 - a_n a_{n+1}}$ bo‘lsa, $|a_{2021}|$ ni toping.

118. $(a_1;b_1)$, $(a_2;b_2)$, $(a_3;b_3)$,... nuqtalar ketma-ketligi uchun $\left(a_{n+1};b_{n+1}\right) = \left(\sqrt{3}a_n - b_n;\sqrt{3}b_n + a_n\right)$ tenglik o‘rinli bo‘lib, $(a_{100};b_{100}) = (2;4)$ bo‘lsa $(a_1;b_2)$ ni toping

119. $\{a_n\}$-geometrik progressiyada $a_1 = \sin x$, $a_2 = \cos x$, $a_3 = tgx$ va $a_n = 1 + \cos x$ bo‘lsa, n ni toping. Bunda $x \in \mathbb{R}$.

120. Agar $a_1 > 0$ va $n \ge 1$ uchun $a_{n+1} = a_n + \frac{n}{a_n}$ bo‘lsa, $n \ge 2$ bo‘lganda $a_n \ge n$ ekanini isbotlang.

121. Agar a_1, a_2, a_3, ..., a_{2021} butun sonlar bo‘lsa, quyidagi ko‘paytmaning juft son ekanligini isbotlang:

$$\left|(a_1-a_2)(a_2-a_3)(a_3-a_4)\dots(a_{2020}-a_{2021})(a_{2021}-a_1)\right|$$

122. $\{a_n\}$ ketma-ketlikda $a_0=\frac{6}{7}$ va $a_{n+1}=\begin{cases} a_n<\frac{1}{2} \Rightarrow 2a_n \\ a_n\geq\frac{1}{2} \Rightarrow 2a_n-1 \end{cases}$ u holda a_{2020} ni toping.

123. Tenglamaning natural yechimlari sonini toping: $pq+qr+pr-pqr=2$

Javob: 7 ta

124. x va y sonlari uchun $x^2+y^2-3y-1=0$ bo‘lsa $(x+y)_{\max}$ ni toping.

125. $|x|+|y|+|x+y|\leq 1$ sohaning yuzini toping.

Javob: $\frac{3}{4}$

126. ABC uchburchakda $AB=13$, $BC=15$ va $CA=14$ bo‘lsin.Agar D nuqta BC ning, E nuqta AD ning, F nuqta BE ning va G nuqta DF ning o‘rtalari bo‘lsa, EFG uchburchakning yuzini toping.

127. Nechta $(p;q)$ tub sonlar juftligi uchun p^2+pq+q^2 ifoda aniq kvadrat bo‘ladi?

Javob: 2 ta

128. $(2010!)!\,\vdots\,((n!)!)!$ shartni qanoatlantiruvchi n-natural sonni toping.

Javob: 3

129. $ABCD$ to‘rtburchak va uning AD, DC, CB tomonlariga mos ravishda K, L, M nuqtalarda urinuvchi aylana berilgan. L nuqtadan o‘tib AD tomonga parallel bo‘lgan to‘g‘ri chiziq KM kesmani N nuqtada kesadi. LN va KC to‘g‘ri chiziqlar P nuqtada kesishadi. $|PL|=|PN|$ tenglikni isbotlang.

130. n ning ixtiyoriy natural qiymatida $19\cdot 8^n+17$ sonining murakkab son ekanligini isbotlang.

131. Manfiy bo‘lmagan x,y,z sonlar $x+y+z=1$ tenglikni qanoatlantiradi. Quyidagi tengsizlikni isbotlang:

$$0\leq xy+xz+yz-2xyz\leq\frac{7}{27}$$

Ko'rsatma: Ushbu $(x+y)(y+z)(x+z)=(x+y+z)(xy+yz+xz)-xyz$ *ayniyatdan foydalaning.*

132. $ABCD$ parallelogrammning AB va AD tomonlari o'rtasidan mos ravishda E va F nuqtalar olingan. CE va BF kesmalar K nuqtada kesishadi. M nuqta CE kesmada yotadi va $BM \parallel KD$. KFD uchburchak va $KMBD$ trapetsiyalarning yuzlari teng bo'lishini isbotlang.

133. $100!$ ni 101 ga bo'lgandagi qoldiqni toping.

Javob: 100

134. $a,b,c>0$ sonlar bo'lsa, $\left[\frac{a+b}{c}\right]+\left[\frac{b+c}{a}\right]+\left[\frac{c+a}{b}\right]$ ifodaning eng kichik qiymatini toping(bunda $[\,]$ belgi sonning butun qismi)

Javob: 4

135. Agar $3a+2b+4d=10$, $6a+5b+4c+3d+2e=8$, $a+b+2c+5e=3$, $2c+3d+3e=4$ va $a+2b+3c+d=7$ bo'lsa, $a+b+c+d+e$ ifodaning son qiymatini toping.

Javob: 4

136. ABC uchburchakda $\angle A=90^0$. AB katetdagi D nuqta uchun $CD=1$. AE-gipotenuzaga tushirilgan balandlik. Agar $BD=BE=1$ bo'lsa, AD ni toping.

Javob: $\sqrt[3]{2}-1$

137. Agar a va b ratsional sonlar bo'lib, $\sqrt{a}+\sqrt{b}=c$ tenglik o'rinli bo'lsa(c - ratsional), u holda $\sqrt{a}$ va $\sqrt{b}$ lar ham ratsional bo'lishini isbotlang.

138. Agar d soni istalgan natural n larda $(n+19)(n+98)(n+1998)$ ning bo'luvchisi bo'lsa, shunday d larning eng katta natural qiymati topilsin.

139. ABC uchburchak tomonlariga tashqaridan ABB_1A_2, BCC_1B_2 va CAA_1C_2 kvadratlar qo'yilgan. A_1A_2, B_1B_2 va C_1C_2 kesmalar yordamida uchburchak qurish mumkinligini isbotlang.

140. Xorazm viloyatidagi shaxsiy transport vositalarining taniqlik raqamlari 90 kodi bilan boshlanishini (masalan: *90 P 212 JA*) bilgan holda, nechta shaxsiy transport vositasiga taniqlik raqami berish mumkinligini hisoblab toping.

Ko'rsatma: Ingliz alifbosida A dan Z gacha 26 ta harf borligidan foydalaning.

Eslatma! 000 raqami mavjud emas deb hisoblang.

Javob: 17558424

Koshi-Bunyakovskiy tengsizligidan foydalanib, quyidagi tenglamalar sistemasini yeching(141-145)

141. Tenglamalar sistemasini yeching: $\begin{cases} m^2 + n^2 = 1 \\ a^2 + b^2 + c^2 = 1 \\ |ma + nb + c| = \sqrt{2} \end{cases}$

Ko'rsatma: $a_1 = m,\ a_2 = n,\ a_3 = 1,\ b_1 = a,\ b_2 = b,\ b_3 = c$ *deb oling*

142. Tenglamalar sistemasini yechin:. $\begin{cases} x_1 + x_2 + \ldots + x_n = 1 \\ x_1^2 + x_2^2 + \ldots + x_n^2 = \dfrac{1}{n} \end{cases}$

Ko'rsatma: $a_1 = x_1,\ a_2 = x_2, \ldots, a_n = x_n,\ b_1 = b_2 = \ldots = b_n = 1$ *deb oling*

143. Tenglamalar sistemasini yechin:. $\begin{cases} \sqrt{x^2 + 4y^2 + z^2} = 3 \\ \left|\sqrt{3}\left(x + 2y + z\right)\right| = 9 \end{cases}$

Ko'rsatma: $a_1 = x,\ a_2 = 2y,\ a_3 = z,\ b_1 = 1,\ b_2 = 1,\ b_3 = 1$ *deb oling*

144. Ushbu $\begin{cases} \sin^2 x + \sin^2 y + \sin^2 z = 2 \\ \left|\cos x \sin z + \cos y \sin x + \cos z \sin y\right| = \sqrt{2} \end{cases}$ tenglamalar sistemasini qanoatlantiruvchi barcha (x, y, z) juftliklarni toping.

Ko'rsatma: $a_1 = \cos x,\ a_2 = \cos y,\ a_3 = \cos z,\ b_1 = \sin z,\ b_2 = \sin x,\ b_3 = \sin y$ *deb oling*

145. Tenglamani yechin: $(x + y + 1)^2 = (xy)^2 + 2x^2 + 2y^2 + 4$

Ko'rsatma: $a_1 = x,\ a_2 = y,\ a_3 = 2,\ b_1 = 1,\ b_2 = 2,\ b_3 = 2$ *deb oling*

146. Yig'indini hisoblang:

$$\frac{2^4 + 2^2 + 1}{2^7 - 2} + \frac{3^4 + 3^2 + 1}{3^7 - 3} + \frac{4^4 + 4^2 + 1}{4^7 - 4} + \ldots + \frac{2021^4 + 2021^2 + 1}{2021^7 - 2021} + \frac{1}{2 \cdot 2021 \cdot 2022}$$

Ko'rsatma: $\dfrac{k^4 + k^2 + 1}{k^7 - k} = \dfrac{1}{2}\left(\dfrac{1}{(k-1)k} - \dfrac{1}{k(k+1)}\right)$ *dan foydalaning*

147. L, M, N sonlari mos ravishda geometrik progressiyaning l-, m-, n-nomerli hadlari bo'lsa, $L^{m-n} \cdot M^{n-l} \cdot N^{l-m} = 1$ tenglikni isbotlang.

148. Tenglamani yeching:

$$(x+2)^{2017}+(x+2)^{2016}(x-3)+(x+2)^{2015}(x-3)^2+...+(x+2)(x-3)^{2016}+(x-3)^{2017}=0$$

Ko‘rsatma: Tenglamaning har ikkala tomonini $(x+2)-(x-3)=5$ *ga ko‘paytiring*

149. ABC uchburchakning medianalari O nuqtada kesishsa, quyidagi tenglikni isbotlang:

$$AB^2+BC^2+CA^2=3(OA^2+OB^2+OC^2)$$

Ko‘rsatma: Medianalar kesishish nuqtasida 2:1 nisbatda bo‘linishidan va mediana formulasidan foydalaning

150. a,b,c–natural sonlar bo‘lib, $a-b$ ayirma tub son va $3c^2=c(a+b)+ab$ bo‘lsa, $8c+1$ aniq kvadrat bo‘lishini isbotlang.

4-BOB. JAVOBLAR, YECHIMLAR VA KO‘RSATMALAR

1. Berilgan yig‘indini S deb belgilab olamiz va tenglikning har ikkala tomonini $2\sin\frac{x}{2}$ ga ko‘paytiramiz:

$$2\sin\frac{x}{2}\sin x+2\cdot 2\sin\frac{x}{2}\sin 2x+3\cdot 2\sin\frac{x}{2}\sin 3x+\ldots+$$

$$+2018\cdot 2\sin\frac{x}{2}\sin 2018x=2S\sin\frac{x}{2}$$

$$\cos\frac{x}{2}-\cos\frac{3x}{2}+2\cos\frac{3x}{2}-2\cos\frac{5x}{2}+3\cos\frac{5x}{2}-3\cos\frac{7x}{2}+\ldots+2017\cos\frac{4033x}{2}-$$

$$-2017\cos\frac{4035x}{2}+2018\cos\frac{4035x}{2}-2018\cos\frac{4037x}{2}=2S\sin\frac{x}{2}$$

$$\cos\frac{x}{2}+\cos\frac{3x}{2}+\cos\frac{5x}{2}+\ldots+\cos\frac{4035x}{2}-2018\cos\frac{4037x}{2}=2S\sin\frac{x}{2}$$

Oxirgi tenglikning ikkala tomonini yana bir marta $2\sin\frac{x}{2}$ ga ko‘paytiramiz va quyidagilarga ega bo‘lamiz:

$$2\sin\frac{x}{2}\cos\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{3x}{2}+\ldots+2\sin\frac{x}{2}\cos\frac{4035x}{2}-$$

$$-2018\cdot 2\sin\frac{x}{2}\cos\frac{4037x}{2}=4S\sin^2\frac{x}{2}$$

$$\sin x+\sin 2x-\sin x+\sin 3x-\sin 2x+\ldots+\sin 2018x-\sin 2017x-$$

$$-2018\sin 2019x+2018\sin 2018x=4S\sin^2\frac{x}{2}$$

$$2019\sin 2018x-2018\sin 2019x=4S\sin^2\frac{x}{2}\ \Rightarrow$$

$$\Rightarrow\ S=\frac{2019\sin 2018x-2018\sin 2019x}{4\sin^2\frac{x}{2}}$$

Javob: $\dfrac{2019\sin 2018x-2018\sin 2019x}{4\sin^2\frac{x}{2}}$

2. Berilgan tenglikning ikkala tomonini $a^n b^n$ ga bo‘lamiz va quyidagicha shakl almashtirishlar bajaramiz:

$$a^{2n+1}+b^{2n+1}=2a^nb^n$$

$$\frac{a^{2n+1}+b^{2n+1}}{a^nb^n}=2$$

$$\frac{a\cdot a^{2n}}{a^nb^n}+\frac{b\cdot b^{2n}}{a^nb^n}=2$$

$$a\cdot\left(\frac{a}{b}\right)^n+b\cdot\left(\frac{b}{a}\right)^n=2$$

Oxirgi tenglikning ikkala tomonini kvadratga ko‘taramiz va ikkala tomonidan $4ab$ ni ayiramiz:

$$a^2\cdot\left(\frac{a}{b}\right)^{2n}+2ab+b^2\cdot\left(\frac{b}{a}\right)^{2n}=4$$

$$a^2\cdot\left(\frac{a}{b}\right)^{2n}-2ab+b^2\cdot\left(\frac{b}{a}\right)^{2n}=4-4ab$$

$$\left(a\cdot\left(\frac{a}{b}\right)^n-b\cdot\left(\frac{b}{a}\right)^n\right)^2=4(1-ab)$$

$$1-ab=\left(\frac{a\cdot\left(\frac{a}{b}\right)^n-b\cdot\left(\frac{b}{a}\right)^n}{2}\right)^2=\left(\frac{a^{2n+1}-b^{2n+1}}{2a^nb^n}\right)^2$$

Bundan $1-ab$ soni biror ratsional sonning kvadrati ekanligi kelib chiqadi. Shuni isbotlash talab qilingan edi.

3. $x^3-3x+1=0$ tenglamada Viet teoremasiga ko‘ra quyidagilarni yoza olamiz:

$$\begin{cases}x_1+x_2+x_3=0\\ x_1x_2+x_2x_3+x_1x_3=-3\\ x_1x_2x_3=-1\end{cases}$$

Ushbu $x_1+x_2=-x_3$ tenglikning ikkala tomonini 5-darajaga ko‘taramiz:

$$(x_1+x_2)^5=(-x_3)^5$$

$$x_1^5+5x_1^4x_2+10x_1^3x_2^2+10x_1^2x_2^3+5x_1x_2^4+x_2^5=-x_3^5$$

$$x_1^5 + x_2^5 + x_3^5 = -5(x_1^4 x_2 + 2x_1^3 x_2^2 + 2x_1^2 x_2^3 + x_1 x_2^4)$$

Oxirgi tenglikning o‘ng tomonini x_3 ga ko‘paytiramiz va bo‘lamiz:

$$x_1^5 + x_2^5 + x_3^5 = \frac{-5(x_1^3 \cdot x_1x_2x_3 + 2x_1^2x_2 \cdot x_1x_2x_3 + 2x_1x_2^2 \cdot x_1x_2x_3 + x_2^3 \cdot x_1x_2x_3)}{x_3}$$

$$x_1^5 + x_2^5 + x_3^5 = \frac{5(x_1^3 + 2x_1^2x_2 + 2x_1x_2^2 + x_2^3)}{x_3}$$

$$x_1^5 + x_2^5 + x_3^5 = \frac{5\Big((x_1 + x_2)(x_1^2 - x_1x_2 + x_2^2) + 2x_1x_2(x_1 + x_2)\Big)}{x_3}$$

$$x_1^5 + x_2^5 + x_3^5 = \frac{5(x_1 + x_2)(x_1^2 + x_1x_2 + x_2^2)}{x_3} = \frac{5 \cdot (-x_3)(x_1^2 + x_1x_2 + x_2^2)}{x_3} =$$

$$= -5(x_1^2 + x_1x_2 + x_2^2) = -5\Big((x_1 + x_2)^2 - x_1x_2\Big) = -5\Big((x_1 + x_2) \cdot (-x_3) - x_1x_2\Big) =$$

$$= -5(-x_1x_2 - x_2x_3 - x_1x_3) = 5(x_1x_2 + x_2x_3 + x_1x_3) = 5 \cdot (-3) = -15$$

Javob: -15

4. x ni bir vaqtda $x = 2017a + b$ va $x = 2018c + d$ ko‘rinishda izlaymiz, bu yerda $a \geq 0$, $c \geq 0$, $0 \leq b < 2017$ va $0 \leq d < 2018$. Masala shartidan $a = c + 1$ ekanligi kelib chiqadi. U holda quyidagilarga ega bo‘lamiz:

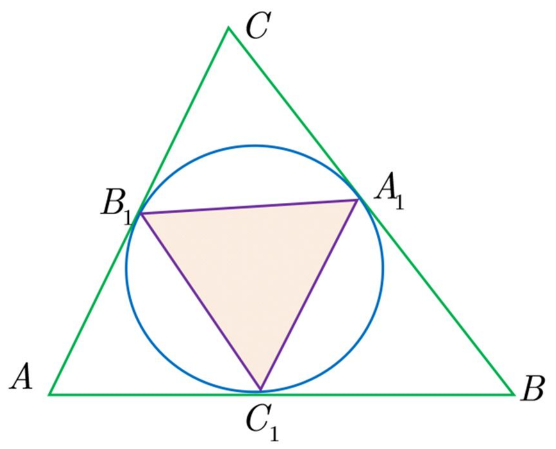
$$2017(c + 1) + b = 2018c + d$$

$$c = 2017 + b - d$$

Agar $c \geq 0$, $0 \leq b < 2017$ va $0 \leq d < 2018$ ekaniligini hisobga olsak, oxirgi tenglikdagi b ning 2017 ta har bir qiymatiga d ning 2018 ta qiymati mos keladi. Bundan tenglamaning natural sonlarda $2017 \cdot 2018 = 4070306$ ta yechimi borligini topamiz.

Javob: 4070306 ta

5. Aylanaga bir nuqtadan o‘tkazilgan urinmalar teng ekanligidan $AB_1 = AC_1$, $CA_1 = CB_1 = 5 - AB_1$, $BA_1 = BC_1 = 7 - AC_1 = 7 - AB_1$ ekanligi kelib chiqadi.



Agar $CA_1 + A_1B = CB$ ekanini hisobga olsak, $5 - AB_1 + 7 - AB_1 = 6$, bundan $AB_1 = AC_1 = 3$, $CB_1 = CA_1 = 2$, $BA_1 = BC_1 = 4$ ekanini topish mumkin. Geron formulasiga ko‘ra $S_{ABC} = 6\sqrt{6}$. Boshqa tomondan:

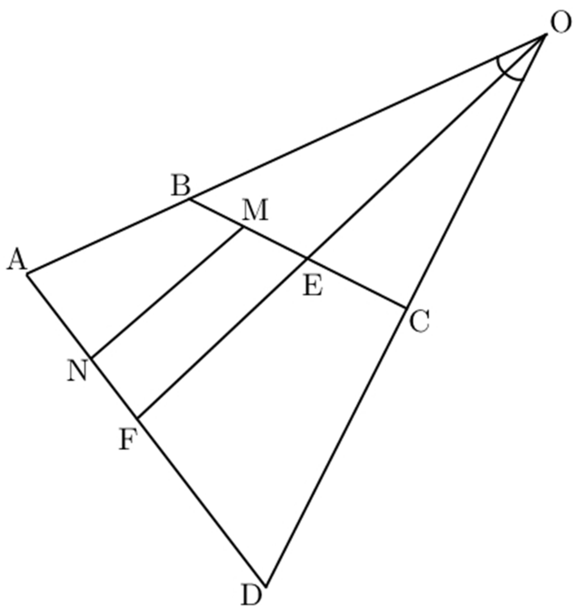
$$S_{ABC} = \frac{1}{2} \cdot 7 \cdot 5 \cdot \sin A = \frac{1}{2} \cdot 7 \cdot 6 \cdot \sin B = \frac{1}{2} \cdot 6 \cdot 5 \cdot \sin C = 6\sqrt{6}$$

Bundan $\sin A = \frac{12\sqrt{6}}{35}$, $\sin B = \frac{2\sqrt{6}}{7}$, $\sin C = \frac{2\sqrt{6}}{5}$ tengliklarga egamiz. U holda ushbu $S_{AB_1C_1} = \frac{1}{2} \cdot 3 \cdot 3 \cdot \sin A = \frac{54\sqrt{6}}{35}$, $S_{BA_1C_1} = \frac{1}{2} \cdot 4 \cdot 4 \cdot \sin B = \frac{16\sqrt{6}}{7}$ va $S_{CA_1B_1} = \frac{1}{2} \cdot 2 \cdot 2 \cdot \sin C = \frac{4\sqrt{6}}{5}$ tengliklar o‘rinli.

Bundan $S_{A_1B_1C_1} = 6\sqrt{6} - \left(\frac{54\sqrt{6}}{35} + \frac{16\sqrt{6}}{7} + \frac{4\sqrt{6}}{5} \right) = \frac{48\sqrt{6}}{35}$ ekani kelib chiqadi.

Javob: $S_{A_1B_1C_1} = \frac{48\sqrt{6}}{35}$

6. $\frac{|BM|}{|MC|} = \frac{|AN|}{|ND|} = \frac{|AB|}{|CD|} = \lambda$ deb olamiz(rasmga qarang).



U holda $\overrightarrow{BM} = \lambda\overrightarrow{MC}$, $\overrightarrow{AN} = \lambda\overrightarrow{ND}$ lardan $\overrightarrow{BM} = \frac{\lambda}{\lambda+1}\overrightarrow{BC}$ va $\overrightarrow{AN} = \frac{\lambda}{\lambda+1}\overrightarrow{AD}$ ekanligi kelib chiqadi. Bularga ko‘ra quyidagini yoza olamiz:

$$\overrightarrow{MN} = \overrightarrow{MB} + \overrightarrow{BA} + \overrightarrow{AN} = -\frac{\lambda}{\lambda+1}\overrightarrow{BC} + \overrightarrow{BA} + \frac{\lambda}{\lambda+1}\overrightarrow{AD} =$$

$$= \frac{\lambda}{\lambda+1}(\overrightarrow{AD} - \overrightarrow{BC}) + \overrightarrow{BA}$$

Ushbu $|BA|\cdot\overrightarrow{CD}$ va $|CD|\cdot\overrightarrow{BA}$ vektorlarning uzunliklari o‘zaro teng bo‘lgani uchun ularning yig‘indisi, ya’ni $\vec{p} = |BA|\cdot\overrightarrow{CD} + |CD|\cdot\overrightarrow{BA} = |CD|\cdot(\lambda\overrightarrow{CD} + \overrightarrow{BA})$ vektor BA va CD tomonlar yordamida hosil qilingan burchak bissektrisasi bo‘yicha yo‘naladi.

$\overrightarrow{CD} = -\overrightarrow{BC} + \overrightarrow{BA} + \overrightarrow{AD}$ bo‘lgani uchun quyidagi tenglik o‘rinli:

$$\overline{p} = |CD|\cdot\left(\lambda(\overrightarrow{AD} - \overrightarrow{BC}) + (\lambda+1)\overrightarrow{BA}\right) =$$

$$= |CD|\cdot(\lambda+1)\left(\frac{\lambda}{\lambda+1}(\overrightarrow{AD} - \overrightarrow{BC}) + \overrightarrow{BA}\right) = |CD|\cdot(\lambda+1)\cdot\overrightarrow{MN}$$

Demak, $\vec{p}$ va $\overrightarrow{MN}$ o‘zaro parallel ekan. Da’vo isbotlandi.

7. Qulaylik uchun quyidagi belgilashni kiritamiz:

$$\begin{cases} z = a\cos\alpha \\ y = a\sin\alpha \end{cases} \text{va} \begin{cases} u = b\cos\beta \\ v = b\sin\beta \end{cases}$$

bunda $\alpha, \beta \in \mathbb{R}$. Ma'lumki, yuqoridagi belgilashlar masala shartidagi (1) va (2) tengliklarni bajaradi. U holda $zu + yv$ ifodani qaraymiz:

$$zu + yv = ab\cos\alpha\cos\beta + ab\sin\alpha\sin\beta = ab\cos(\alpha - \beta)$$

Ushbu $-1 \le \cos(\alpha - \beta) \le 1$ qo'sh tengsizlikdan $-ab \le zu + yv \le ab$ ekanligi kelib chiqadi. Bundan $zu + yv$ ifodaning eng katta qiymati ab va eng kichik qiymati $-ab$ ekanini topamiz.

Javob: $(zu + yv)_{\max} = ab$ va $(zu + yv)_{\min} = -ab$

8. Masala shartidagi $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ tenglikdan foydalangan holda quyidagicha shakl almashtirishlar bajaramiz:

$$\sqrt{x^2 + \sqrt[3]{x^4y^2}} + \sqrt{y^2 + \sqrt[3]{x^2y^4}} = \sqrt{x^2 + x^{\frac{4}{3}}y^{\frac{2}{3}}} + \sqrt{y^2 + x^{\frac{2}{3}}y^{\frac{4}{3}}} =$$

$$= \sqrt{x^{\frac{4}{3}}\left(x^{\frac{2}{3}} + y^{\frac{2}{3}}\right)} + \sqrt{y^{\frac{4}{3}}\left(x^{\frac{2}{3}} + y^{\frac{2}{3}}\right)} = \sqrt{x^{\frac{4}{3}} \cdot a^{\frac{2}{3}}} + \sqrt{y^{\frac{4}{3}} \cdot a^{\frac{2}{3}}} =$$

$$= x^{\frac{2}{3}} \cdot a^{\frac{1}{3}} + y^{\frac{2}{3}} \cdot a^{\frac{1}{3}} = a^{\frac{1}{3}}(x^{\frac{2}{3}} + y^{\frac{2}{3}}) = a^{\frac{1}{3}} \cdot a^{\frac{2}{3}} = a$$

Shuni isbotlash talab qilingan edi.

9. Qulaylik uchun $x = a\cos\alpha$ va $y = b\sin\alpha$ belgilashni kiritamiz, bunda $\alpha \in \mathbb{R}$. Bu belgilash berilgan sistemaning birinchi tenglamasini qanoatlantirishini ko'rish qiyin emas. Belgilashni sistemaning ikkinchi tenglamasiga qo'yamiz:

$$4ab(2a^2\cos^2\alpha - a^2)\sin\alpha\cos\alpha = a^3b$$

$$4a^3b(2\cos^2\alpha - 1)\sin\alpha\cos\alpha = a^3b$$

$$4(2\cos^2\alpha - 1)\sin\alpha\cos\alpha = 1$$

$$2\sin 2\alpha\cos 2\alpha = 1$$

$$\sin 4\alpha = 1$$

$$4\alpha = \frac{\pi}{2} + 2\pi n, n \in \mathbb{Z}$$

$$\alpha = \frac{\pi}{8} + \frac{\pi n}{2}, n \in \mathbb{Z}$$

$$\sin\frac{\pi}{8} = \sqrt{\frac{1 - \cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$$

$$\cos\frac{\pi}{8}=\sqrt{\frac{1+\cos\frac{\pi}{4}}{2}}=\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}=\frac{\sqrt{2+\sqrt{2}}}{2}$$

ekanidan va sinus, kosinuslarning choraklardagi ishoralaridan quyidagi yechimlarga ega bo‘lamiz:

$$\left(\frac{\sqrt{2-\sqrt{2}}}{2}a;\frac{\sqrt{2+\sqrt{2}}}{2}b\right),\left(\frac{\sqrt{2-\sqrt{2}}}{2}a;-\frac{\sqrt{2+\sqrt{2}}}{2}b\right),\left(-\frac{\sqrt{2-\sqrt{2}}}{2}a;-\frac{\sqrt{2+\sqrt{2}}}{2}b\right)$$

va $\left(-\frac{\sqrt{2-\sqrt{2}}}{2}a;\frac{\sqrt{2+\sqrt{2}}}{2}b\right)$

10. Qolgan tangalarni tarozi pallalariga 50 tadan joylaydi. Agar tarozi juft sonni ko‘rsatsa, Sitoradagi tanga haqiqiy, aks holda qalbaki bo‘ladi.

Javob: Ha, uddalay oladi.

11. $24x-17y=2 \Rightarrow y=\frac{24x-2}{17}=x+\frac{7x-2}{17}$. x va y larning butun son ekanidan $\frac{7x-2}{17}$ ning ham butun son bo‘lishi kelib chiqadi. Agar $\frac{7x-2}{17}=a,(a\in\mathbb{Z})$ desak, bundan $x=\frac{17a+2}{7}=2a+\frac{3a+2}{7}$ ekanligi kelib chiqadi. Xuddi yuqorigidek fikr yuritib va $\frac{3a+2}{7}=b,(b\in\mathbb{Z})$ deb belgilab olib, $a=\frac{7b-2}{3}=2b+\frac{b-2}{3}$ va nihoyat $\frac{b-2}{3}=c,(c\in\mathbb{Z})$ deb belgilab olib, $b=3c+2$ ekanini topib olamiz. Belgilashlarga qaytib, $a=7c+4 \Rightarrow y=24c+14 \Rightarrow x=17c+10$ yechimlarga ega bo‘lamiz.

Javob: $x=17c+10;y=24c+14$ *bunda* $c\in\mathbb{Z}$.

12. Shartga ko‘ra $p>2$ va $q>2$ bo‘lgan natural sonlar. Quyidagi hollarni qaraymiz.

1-hol: $p=2n$ va $q=2m$ bo‘lsin. Bunda $n>1,m>1$ shartni qanoatlantiruvchi natural sonlar. U holda berilgan tengsizlik quyidagi ko‘rinishga keladi:

$$(n+1)(m+1)\le 2nm+1$$

$$nm+n+m+1\le 2nm+1$$

$$n+m\le nm$$

$$n \geq \frac{m}{m-1} = 1 + \frac{1}{m-1} \text{ to‘g‘ri.}$$

2-hol: $p = 2n$ va $q = 2m + 1$ bo‘lsin. Bunda $n > 1, m \geq 1$ shartni qanoatlantiruvchi natural sonlar. U holda berilgan tengsizlik quyidagi ko‘rinishga keladi:

$$(n+1)(m+1) \leq 2nm + n + 1$$

$$nm + n + m + 1 \leq 2nm + n + 1$$

$$m \leq nm$$

$$1 \leq n \text{ to‘g‘ri.}$$

3-hol: $p = 2n + 1$ va $q = 2m$ bo‘lsin. Bunda $n \geq 1, m > 1$ shartni qanoatlantiruvchi natural sonlar. U holda berilgan tengsizlik quyidagi ko‘rinishga keladi:

$$(n+1)(m+1) \leq 2nm + m + 1$$

$$nm + n + m + 1 \leq 2nm + m + 1$$

$$n \leq nm$$

$$1 \leq m \text{ to‘g‘ri.}$$

4-hol: $p = 2n + 1$ va $q = 2m + 1$ bo‘lsin. Bunda $n \geq 1, m \geq 1$ shartni qanoatlantiruvchi natural sonlar. U holda berilgan tengsizlik quyidagi ko‘rinishga keladi:

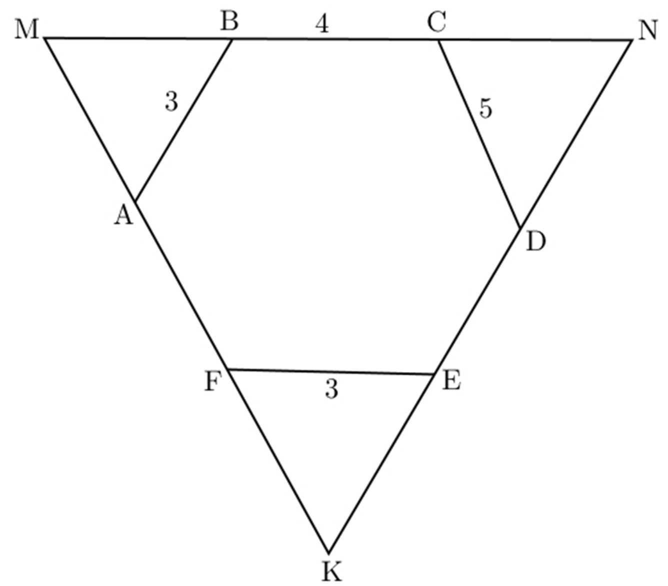
$$(n+1)(m+1) \leq 2nm + n + m + 1$$

$$nm + n + m + 1 \leq 2nm + n + m + 1$$

$$0 \leq nm \text{ to‘g‘ri.}$$

Demak, ixtiyoriy $p > 2$ va $q > 2$ natural sonlari uchun $\left(\left[\frac{p}{2}\right]+1\right)\left(\left[\frac{q}{2}\right]+1\right) \leq \left[\frac{pq}{2}\right]+1$ tengsizlik o‘rinli. Tenglik sharti $p = q = 4$ bo‘lganda bajariladi. Isbot tugadi.

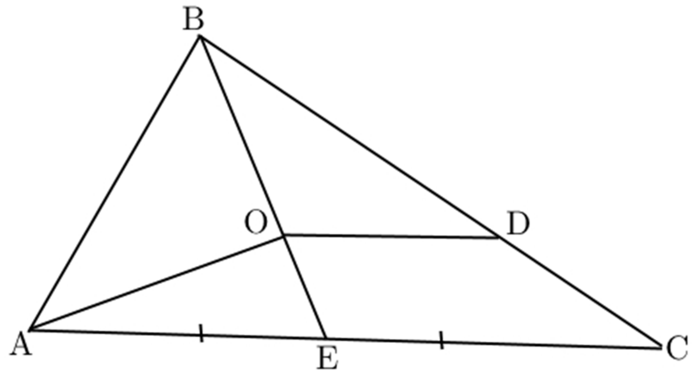
13. Oltiburchakning ichki burchaklari tengligidan ularning har biri 120^0 ga tengligini topish qiyin emas. BC, AF, ED kesmalarni ikkala tomonga davom ettiraylik va ular M, N, K nuqtalarda kesishsin(chizmaga qarang).



Natijada muntazam MNK uchburchak hosil bo‘lishini ko‘rish mumkin. Bundan $MB = MA = 3$, $CN = 5$ va $FK = 3$ ekanligi kelib chiqadi. U holda $MB + BC + CN = MA + AF + FK$ tenglikdan $AF = 6$ ekanligi kelib chiqadi.

Javob: $AF = 6$

14. Qulaylik uchun AOE uchburchakning yuzini S va BOD uchburchakning yuzini Q orqali belgilaylik(chizmaga qarang).



Ma’lumki, uchburchakning medianalari kesishish nuqtada 2:1 nisbatda bo‘linadi ya’ni, $BO : OE = 2 : 1$. Bundan AOB uchburchakning yuzi $2S$ ga tengligi kelib chiqadi. BE mediana uchburchak yuzini teng ikkiga bo‘lishini hisobga olsak, BEC uchburchakning yuzi $3S$ ekanligi kelib chiqadi. $OD / /AC$ ekanidan BOD va BEC uchburchaklarning o‘xshash ekanligini aniqlab olamiz. Shunga ko‘ra $\frac{S_{BEC}}{S_{BOD}} = \left(\frac{BE}{BO}\right)^2 \Rightarrow \frac{3S}{Q} = \frac{9}{4} \Rightarrow Q = \frac{4}{3}S$ ekani kelib chiqadi. U holda

$S_{AODB} = 2S + Q = 2S + \frac{4}{3}S = \frac{10}{3}S$ va $S_{AODC} = S + 3S - Q = \frac{8}{3}S$ ekanini bilgan holda, $\frac{S_{AODB}}{S_{AODC}} = \frac{\frac{10}{3}S}{\frac{8}{3}S} = \frac{5}{4}$ tenglikka ega bo‘lamiz.

Javob: 5:4

15. Musbat x, y sonlar uchun $\frac{x^4 + y^4}{x^3 + y^3} \geq \frac{x + y}{2}$ (*) tengsizlik o‘rinli. Chunki,

$$2(x^4 + y^4) \geq (x + y)(x^3 + y^3) \Rightarrow x^4 + y^4 \geq x^3y + y^3x \Rightarrow$$

$$\Rightarrow (x - y)^2(x^2 + xy + y^2) \geq 0$$

Endi (*) dan foydalansak va $ab + bc + ac = abc \Leftrightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$ ekanini hisobga olsak,

$$\frac{a^4 + b^4}{ab(a^3 + b^3)} + \frac{b^4 + c^4}{bc(b^3 + c^3)} + \frac{c^4 + a^4}{ac(a^3 + c^3)} \geq \frac{a + b}{2ab} + \frac{b + c}{2bc} + \frac{a + c}{2ac} =$$

$$= \frac{1}{2a} + \frac{1}{2b} + \frac{1}{2c} + \frac{1}{2b} + \frac{1}{2c} + \frac{1}{2a} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$$

munosabat hosil bo‘ladi. Tengsizlikda tenglik sharti $a = b = c = 3$ bo‘lganda bajariladi. Shuni isbotlash talab qilingan edi.

16. Sistemadagi birinchi tenglamani y ga, ikkinchi tenglamani x ga ko‘paytirib, hosil bo‘lgan tenglamalarni qo‘shib, quyidagiga ega bo‘lamiz:

$$2xy + \frac{(3x - y)y - (x + 3y)x}{x^2 + y^2} = 3y \Rightarrow$$

$$\Rightarrow 2xy + \frac{3xy - y^2 - x^2 - 3xy}{x^2 + y^2} = 3y \Rightarrow 2xy - 1 = 3y$$

bu yerda $y \neq 0$ bo‘lgani uchun $x = \frac{3}{2} + \frac{1}{2y}$. Bu munosabatni berilgan sistemadagi ikkinchi tenglamaga qo‘yib:

$y\left(\left(\frac{3}{2} + \frac{1}{2y}\right)^2 + y^2\right) - \left(\frac{3}{2} + \frac{1}{2y}\right) - 3y = 0$ tenglamani hosil qilamiz. Bundan

$$y\left(\frac{9}{4}+\frac{3}{2y}+\frac{1}{4y^2}+y^2\right)-\frac{3}{2}-\frac{1}{2y}-3y=0 \Rightarrow$$

$$\Rightarrow \frac{9}{4}y+\frac{3}{2}+\frac{1}{4y}+y^3-\frac{3}{2}-\frac{1}{2y}-3y=0 \Rightarrow$$

$$\Rightarrow y^3-\frac{3}{4}y-\frac{1}{4y}=0 \Rightarrow 4y^4-3y^2-1=0$$

tenglama kelib chiqadi. Bu tenglamani yechib, $y_1=-1$ va $y_2=1$ yechimlarni topib olamiz. U holda $x_1=1$ va $x_2=2$ ekanligi kelib chiqadi.

Javob: $(1;-1),(2;1)$

17. Dastlab ixtiyoriy n natural soni uchun quyidagi tengsizlikning o‘rinli ekanini isbot qilamiz.

$$2(\sqrt{n+1}-1)<1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{n}}\le 2\sqrt{n}-1 \qquad (1)$$

Buning uchun matematik induksiya metodidan foydalanamiz. Dastlab tengsizlikning o‘ng qismini isbot qilaylik.

$n=1$ da $1\le 2\sqrt{1}-1=1$ to‘g‘ri;

$n=k$ da $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{k}}\le 2\sqrt{k}-1$ munosabatni to‘g‘ri deb faraz qilib, bu tasdiqning to‘g‘riligini $n=k+1$ bo‘lganda isbotlaymiz.

$n=k+1$ da $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}\le 2\sqrt{k}-1+\frac{1}{\sqrt{k+1}}<2\sqrt{k+1}-1$

biz oxirgi tengsizlikni isbotlashimiz kifoya. Bu esa quyidagi almashtirishdan oson kelib chiqadi:

$$2\sqrt{k}-1+\frac{1}{\sqrt{k+1}}<2\sqrt{k+1}-1 \Rightarrow 2\sqrt{k^2+k}+1<2(k+1) \Rightarrow$$

$$\Rightarrow 2\sqrt{k^2+k}<2k+1 \Rightarrow 4k^2+4k<4k^2+4k+1 \Rightarrow 0<1$$

Demak, ixtiyoriy n natural soni uchun $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{n}}\le 2\sqrt{n}-1$ tengsizlik o‘rinli. (1) tengsizlikning chap qismi ham xuddi yuqoridagidek isbot etiladi. U holda (1) munosabatdan $2\sqrt{n+1}-3<\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{n}}\le 2\sqrt{n}-2$

ekanligi kelib chiqadi. Agar $n = 10000$ ekanini hisobga olsak, quyidagiga ega bo‘lamiz:

$$197 = 2\sqrt{10000} - 3 < 2\sqrt{10000+1} - 3 <$$
$$< \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{10000}} < 2\sqrt{10000} - 2 = 198$$

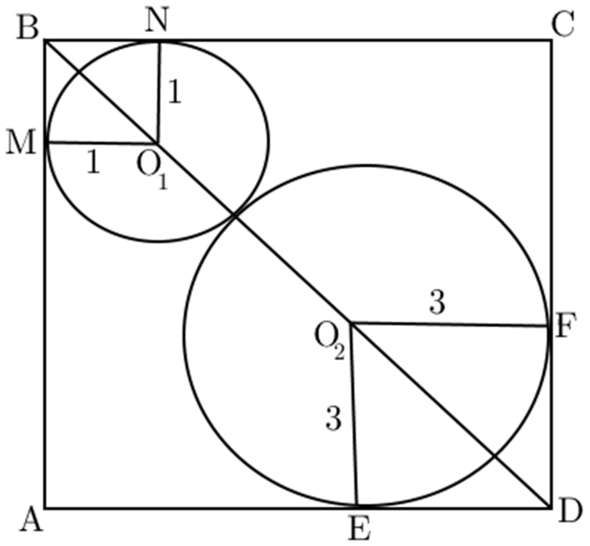
Shunga ko‘ra $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{10000}}$ ifodaning butun qismi 197 ekanligini topishimiz mumkin.

Javob: 197

18. Yukni tashish uchun 2,5 tonnali avtomashinalardan x tasi va 6,5 tonnali avtomashinalardan y tasi kerak bo‘lsin. Natijada masala $2,5x + 6,5y = 67 \Rightarrow 25x + 65y = 670 \Rightarrow 5x + 13y = 134$ tenglamani natural sonlarda yechishga keladi. Oxirgi tenglamadan $x = \frac{134 - 13y}{5} = 26 - 3y + \frac{2y+4}{5}$ ekanini topib olamiz. x va y larning natural sonlar ekanligidan $\frac{2y+4}{5}$ ning ham natural son bo‘lishi kelib chiqadi. Agar $\frac{2y+4}{5} = n, (n \in \mathbb{N})$ deb belgilasak, $y = \frac{5n-4}{2} = 2n - 2 + \frac{n}{2}$ ekanligi va xuddi yuqorigidek fikr yuritib $n = 2m, (m \in \mathbb{N})$ ko‘rinishida bo‘lishi kelib chiqadi. Belgilashlarga qaytib, $y = 5m - 2$ va $x = 32 - 13m$ ekanligini topib olamiz. x va y larning natural sonlar ekanligidan $m = 1$ va $m = 2$ qiymatlarni qabul qilishi kelib chiqadi. Bundan $x = 19$, $y = 3$ yoki $x = 6$, $y = 8$ ekanini topish mumkin.

Javob: 2,5 tonnalidan 19 ta, 6,5 tonnalidan 3 ta yoki 2,5 tonnalidan 6 ta, 6,5 tonnalidan 8 ta avtomashina kerak

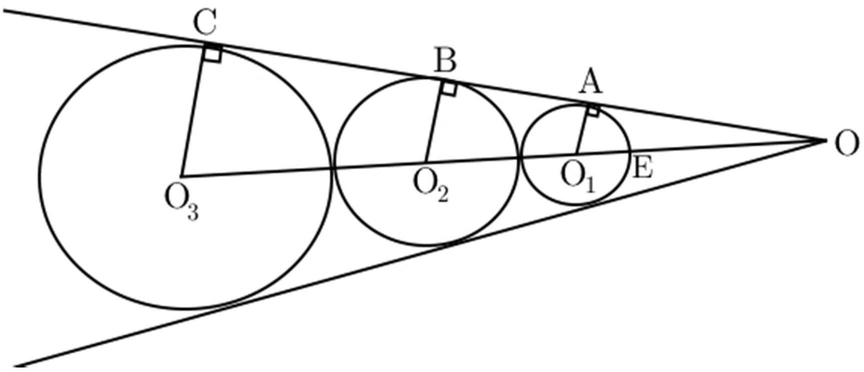
19. Masala shartidan kelib chiqib, quyidagi chizmani chizib olamiz:



Chizmadan $BO_1 = \sqrt{1+1} = \sqrt{2}$, $O_2D = \sqrt{9+9} = 3\sqrt{2}$ va $O_1O_2 = 1+3 = 4$ ekanini topib olamiz. Bundan kvadratning diagonali $BD = BO_1 + O_1O_2 + O_2D = \sqrt{2} + 4 + 3\sqrt{2} = 4\sqrt{2} + 4$ ga tengligi kelib chiqadi.

Javob: $4\sqrt{2} + 4$

20. Quyidagi chizmadan AOO_1, BOO_2 va COO_3 to‘g‘ri burchakli uchburchaklar bitta o‘tkir burchagi umumiy bo‘lgani uchun ($\angle AOO_1 = \angle BOO_2 = \angle COO_3$) ular o‘xshash.



Qulaylik uchun $OE = a$ va kichik aylananing radiusini r orqali belgilaylik. U holda AOO_1 va BOO_2 uchburchaklarning o‘xshashligidan

$$\frac{AO_1}{BO_2} = \frac{OO_1}{OO_2} \Rightarrow \frac{r}{3} = \frac{a+r}{a+2r+3} \qquad (1)$$

AOO_1 va COO_3 uchburchaklarning o‘xshashligidan

$$\frac{AO_1}{CO_3} = \frac{OO_1}{OO_3} \Rightarrow \frac{r}{5} = \frac{a+r}{a+2r+11} \qquad (2)$$

tengliklar kelib chiqadi. (1) tenglikni (2) tenglikka bo‘lsak, $\frac{5}{3}=\frac{a+2r+11}{a+2r+3} \Rightarrow a+2r=9$ ekani kelib chiqadi. Oxirgi tenglikni (1) ga qo‘ysak, $\frac{r}{3}=\frac{a+r}{12} \Rightarrow a=3r$ ekani kelib chiqadi. U holda $a+2r=9$ dan $r=1,8$ va kichik aylananing uzunligi $2\pi r=2\pi \cdot 1,8=3,6\pi$ ekanini topish mumkin.

Javob: $3,6\pi$

21. Birinchi va ikkinchi tengliklardan foydalanib tgx ni topib olamiz.

$$\frac{\sin(x-\alpha)}{\sin(x-\beta)}=\frac{\sin x\cos\alpha-\cos x\sin\alpha}{\sin x\cos\beta-\cos x\sin\beta}=m$$

$$\sin x\cos\alpha-\cos x\sin\alpha=m\sin x\cos\beta-m\cos x\sin\beta$$

$$\sin x\cos\alpha-m\sin x\cos\beta=\cos x\sin\alpha-m\cos x\sin\beta$$

$$(\cos\alpha-m\cos\beta)\sin x=(\sin\alpha-m\sin\beta)\cos x$$

$$tgx=\frac{\sin\alpha-m\sin\beta}{\cos\alpha-m\cos\beta} \qquad (1)$$

Xuddi shunga o‘xshash

$$\frac{\cos(x-\alpha)}{\cos(x-\beta)}=\frac{\cos x\cos\alpha+\sin x\sin\alpha}{\cos x\cos\beta+\sin x\sin\beta}=n$$

$$\cos x\cos\alpha+\sin x\sin\alpha=n\cos x\cos\beta+n\sin x\sin\beta$$

$$\sin x\sin\alpha-n\sin x\sin\beta=n\cos x\cos\beta-\cos x\cos\alpha$$

$$(\sin\alpha-n\sin\beta)\sin x=(n\cos\beta-\cos\alpha)\cos x$$

$$tgx=\frac{n\cos\beta-\cos\alpha}{\sin\alpha-n\sin\beta} \qquad (2)$$

(1) va (2) ifodalarning o‘ng tomonini tenglashtirsak quyidagilarga ega bo‘lamiz:

$$\frac{\sin\alpha-m\sin\beta}{\cos\alpha-m\cos\beta}=\frac{n\cos\beta-\cos\alpha}{\sin\alpha-n\sin\beta}$$

$$(\sin\alpha-m\sin\beta)(\sin\alpha-n\sin\beta)=(n\cos\beta-\cos\alpha)(\cos\alpha-m\cos\beta)$$

$$\sin^2\alpha-n\sin\alpha\sin\beta-m\sin\alpha\sin\beta+mn\sin^2\beta=$$

$$=n\cos\alpha\cos\beta-mn\cos^2\beta-\cos^2\alpha+m\cos\alpha\cos\beta$$

bundan esa $\sin^2\alpha+\cos^2\alpha+mn(\sin^2\beta+\cos^2\beta)=(m+n)(\cos\alpha\cos\beta+\sin\alpha\sin\beta)$ ya'ni $1+mn=(m+n)\cos(\alpha-\beta)\ \Rightarrow\ \cos(\alpha-\beta)=\dfrac{1+mn}{m+n}$ ekanini topish mumkin.

Javob: $\cos(\alpha-\beta)=\dfrac{1+mn}{m+n}$

22. $tg60^0=tg(20^0+40^0)=\dfrac{tg20^0+tg40^0}{1-tg20^0tg40^0}=\sqrt{3}$ ekani ma'lum. Bundan $tg20^0+tg40^0=\sqrt{3}-\sqrt{3}tg20^0tg40^0\ \Rightarrow\ tg20^0+tg40^0+\sqrt{3}tg20^0tg40^0=\sqrt{3}$ ekani kelib chiqadi.

Javob: $\sqrt{3}$

23. Quyidagi tenglikdan foydalanamiz:

$$\frac{1}{\cos k\alpha\cos(k+1)\alpha}=\frac{1}{\sin\alpha}\left(\frac{\sin\big((k+1)-k\big)\alpha}{\cos k\alpha\cos\big(k+1\big)\alpha}\right)=$$

$$=\frac{1}{\sin\alpha}\left(\frac{\sin(k+1)\alpha\cos k\alpha-\cos(k+1)\alpha\sin k\alpha}{\cos k\alpha\cos(k+1)\alpha}\right)=\frac{1}{\sin\alpha}(tg(k+1)\alpha-tgk\alpha)$$

U holda quyidagi tenglik kelib chiqadi:

$$\frac{1}{\cos\alpha\cos2\alpha}+\frac{1}{\cos2\alpha\cos3\alpha}+\ldots+\frac{1}{\cos2020\alpha\cos2021\alpha}=$$

$$=\frac{1}{\sin\alpha}\Big(tg2\alpha-tg\alpha+tg3\alpha-tg2\alpha+\ldots+tg2021\alpha-tg2020\alpha\Big)=$$

$$=\frac{1}{\sin\alpha}\Big(tg2021\alpha-tg\alpha\Big)=\frac{\sin2020\alpha}{\sin\alpha\cos\alpha\cos2021\alpha}=\frac{2\sin2020\alpha}{\sin2\alpha\cos2021\alpha}$$

Javob: $\dfrac{2\sin2020\alpha}{\sin2\alpha\cos2021\alpha}$

24. Ushbu $2\sin\dfrac{\alpha}{2}\sin k\alpha=\cos\left(k-\dfrac{1}{2}\right)\alpha-\cos\left(k+\dfrac{1}{2}\right)\alpha$ tenglikdan foydalanamiz.

Shunga ko'ra:

$$\sin\alpha+\sin2\alpha+\sin3\alpha+\ldots+\sin2021\alpha=$$

$$=\frac{2(\sin\alpha+\sin 2\alpha+\sin 3\alpha+\ldots+\sin 2021\alpha)\sin\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}}=$$

$$=\frac{2\sin\frac{\alpha}{2}\sin\alpha+2\sin\frac{\alpha}{2}\sin 2\alpha+2\sin\frac{\alpha}{2}\sin 3\alpha+\ldots+2\sin\frac{\alpha}{2}\sin 2021\alpha}{2\sin\frac{\alpha}{2}}=$$

$$=\frac{\cos\frac{\alpha}{2}-\cos\frac{3\alpha}{2}+\cos\frac{3\alpha}{2}-\cos\frac{5\alpha}{2}+\ldots+\cos\frac{4041\alpha}{2}-\cos\frac{4043\alpha}{2}}{2\sin\frac{\alpha}{2}}=$$

$$=\frac{\cos\frac{\alpha}{2}-\cos\frac{4043\alpha}{2}}{2\sin\frac{\alpha}{2}}=\frac{2\sin 1010,5\alpha\sin 1011\alpha}{2\sin\frac{\alpha}{2}}=$$

$$=\frac{\sin 1010,5\alpha\sin 1011\alpha}{\sin\frac{\alpha}{2}},\ \alpha\neq 2\pi n, n\in\mathbb{Z}.$$

Javob: $\dfrac{\sin 1010,5\alpha\sin 1011\alpha}{\sin\frac{\alpha}{2}},\ \alpha\neq 2\pi n, n\in\mathbb{Z}$

25. Ushbu $2\sin\frac{\alpha}{2}\cos k\alpha=\sin\left(k+\frac{1}{2}\right)\alpha-\sin\left(k-\frac{1}{2}\right)\alpha$ tenglikdan foydalanamiz.

Shunga ko‘ra:

$$\cos\alpha+\cos 2\alpha+\cos 3\alpha+\ldots+\cos 2021\alpha=$$

$$=\frac{2(\cos\alpha+\cos 2\alpha+\cos 3\alpha+\ldots+\cos 2021\alpha)\sin\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}}=$$

$$=\frac{2\sin\frac{\alpha}{2}\cos\alpha+2\sin\frac{\alpha}{2}\cos 2\alpha+2\sin\frac{\alpha}{2}\cos 3\alpha+\ldots+2\sin\frac{\alpha}{2}\cos 2021\alpha}{2\sin\frac{\alpha}{2}}=$$

$$=\frac{\sin\frac{3\alpha}{2}-\sin\frac{\alpha}{2}+\sin\frac{5\alpha}{2}-\sin\frac{3\alpha}{2}+\ldots+\sin\frac{4043\alpha}{2}-\sin\frac{4041\alpha}{2}}{2\sin\frac{\alpha}{2}}=$$

$$=\frac{\sin\frac{4043\alpha}{2}-\sin\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}}=\frac{2\sin 1010,5\alpha\cos 1011\alpha}{2\sin\frac{\alpha}{2}}=$$

$$=\frac{\sin 1010,5\alpha\cos 1011\alpha}{\sin\frac{\alpha}{2}},\quad \alpha\neq 2\pi n, n\in\mathbb{Z}.$$

Javob: $\dfrac{\sin 1010,5\alpha\cos 1011\alpha}{\sin\frac{\alpha}{2}},\quad \alpha\neq 2\pi n, n\in\mathbb{Z}$

26. $\dfrac{\cos x}{|\cos x|}+\dfrac{|\sin x|}{\sin x}=-2$ tenglama faqat $\begin{cases}\sin x<0\\ \cos x<0\end{cases}$ bo‘lganda yechimga ekanligini ko‘rish qiyin emas. Bu esa x burchakning faqat III chorakda ekanligini bildiradi. Demak, $x\in\left(\pi+2\pi n;\dfrac{3\pi}{2}+2\pi n\right)$, $n\in\mathbb{Z}$.

Javob: $x\in\left(\pi+2\pi n;\dfrac{3\pi}{2}+2\pi n\right)$, $n\in\mathbb{Z}$

27. Oldin $\cos 36^0-\cos 72^0$ ni hisoblaymiz:

$$\cos 36^0-\cos 72^0=\frac{2(\cos 36^0-\cos 72^0)\sin 36^0}{2\sin 36^0}=$$

$$=\frac{2\sin 36^0\cos 36^0-2\sin 36^0\cos 72^0}{2\sin 36^0}=$$

$$=\frac{\sin 72^0-(\sin 108^0-\sin 36^0)}{2\sin 36^0}=\frac{\sin 72^0-\sin 108^0+\sin 36^0}{2\sin 36^0}=$$

$$=\frac{\sin 72^0-\sin(180^0-72^0)+\sin 36^0}{2\sin 36^0}=\frac{\sin 72^0-\sin 72^0+\sin 36^0}{2\sin 36^0}=\frac{1}{2}$$

Demak, $\cos 36^0-\cos 72^0=\frac{1}{2}$ bundan $1-2\sin^2 18^0-\cos(90^0-18^0)=\frac{1}{2}$ yoki $4\sin^2 18^0+2\sin 18^0-1=0$ ekanligi kelib chiqadi. Agar $\sin 18^0=x$ deb belgilasak, u holda $4x^2+2x=1$ ekanligini ko‘rish mumkin. Shuni isbotlash talab qilingan edi.

28. 1 soning n-darajali ildizi quyidagi ko‘rinishda:

$$z^{n-1}+z^{n-2}+...+z+1=(z-w)\cdot(z-w^2)\cdot...\cdot(z-w^{n-1})$$

bu yerda, $w=e^{\frac{2\pi i}{n}}$ ekani bizga ma’lum. U holda $z=1$ bo‘lsa, $n=(1-w)(1-w^2)...(1-w^{n-1})$ tenglik o‘rinli bo‘ladi.

$1-w^k=1-e^{(2\pi ik/n)}=-e^{(\pi ik/n)}$ va $e^{(\pi ik/n)}-e^{(-\pi ik/n)}=-2ie^{(\pi ik/n)}\sin\frac{\pi k}{n}$ ga ko‘ra(bunda $k=1,2,...,n-1$):

$$\prod_{k=1}^{n-1}(1-w^k)=2^{n-1}(-1)^{n-1}i^{n-1}e^{\left(\frac{\pi i}{n}(1+2+...+n-1)\right)}\prod_{k=1}^{n-1}\sin\frac{\pi k}{n}=$$

$$=2^{n-1}(-1)^{n-1}i^{n-1}e^{\left(\frac{\pi i}{2}(n-1)\right)}\prod_{k=1}^{n-1}\sin\frac{\pi k}{n}=2^{n-1}(-1)^{n-1}i^{n-1}i^{n-1}\prod_{k=1}^{n-1}\sin\frac{\pi k}{n}=$$

$$=2^{n-1}(-1)^{n-1}(i^2)^{n-1}\prod_{k=1}^{n-1}\sin\frac{\pi k}{n}=2^{n-1}\prod_{k=1}^{n-1}\sin\frac{\pi k}{n}$$

u holda $\prod_{k=1}^{n-1}\sin\frac{\pi k}{n}=\frac{n}{2^{n-1}}$ tenglik o‘rinli. Isbotlandi.

29. Tenglamaning o‘ng qismini soddalashtirsak:

$$(a-1)\left(\frac{1}{\sin x}+\frac{1}{\cos x}+\frac{1}{\sin x\cos x}\right)=(a-1)\left(\frac{\sin x+\cos x+1}{\sin x\cos x}\right)=$$

$$=(a-1)\left(\frac{\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)+1}{\sin^2\left(x+\frac{\pi}{4}\right)-\frac{1}{2}}\right)$$

kelib chiqadi, bu yerda $\sin^2\left(x+\frac{\pi}{4}\right)\neq\frac{1}{2}$.

U holda $\frac{2(a-1)}{\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)-1}=2 \Rightarrow \sin\left(x+\frac{\pi}{4}\right)=\frac{a}{\sqrt{2}}\neq\pm\frac{1}{\sqrt{2}}$ ekani ma'lum.

Oxirgi munosabatdan ko‘rinadiki, tenglama $|a|=1$ va $|a|>\sqrt{2}$ da yechimga ega emas. Tenglama faqat $|a|\in[0;1)\cup(1;\sqrt{2}]$ bo‘lgandagina yechimga ega bo‘ladi va bu yechim $x=(-1)^n\arcsin\left(\frac{a}{\sqrt{2}}\right)-\frac{\pi}{4}+\pi n$ bunda $n\in\mathbb{Z}$ da $|a|\in[0;1)\cup(1;\sqrt{2})$ va $\frac{n}{2}\in\mathbb{Z}$ da $|a|=\sqrt{2}$

30. Qulaylik uchun $\frac{\pi}{n}=\alpha$ deb belgilab olamiz. U holda masala $\cos 2\alpha+\cos 4\alpha+\cos 6\alpha+\ldots+\cos 2n\alpha$ ni hisoblashga keladi. Xuddi 5-masaladagi usuldan foydalansak:

$$\cos 2\alpha+\cos 4\alpha+\cos 6\alpha+\ldots+\cos 2n\alpha=$$

$$=\frac{2(\cos 2\alpha+\cos 4\alpha+\cos 6\alpha+\ldots+\cos 2n\alpha)\sin\alpha}{2\sin\alpha}=$$

$$=\frac{2\sin\alpha\cos 2\alpha+2\sin\alpha\cos 4\alpha+2\sin\alpha\cos 6\alpha+\ldots+2\sin\alpha\cos 2n\alpha}{2\sin\alpha}=$$

$$=\frac{\sin 3\alpha-\sin\alpha+\sin 5\alpha-\sin 3\alpha+\ldots+\sin(2n+1)\alpha-\sin(2n-1)\alpha}{2\sin\alpha}=$$

$$=\frac{\sin(2n+1)\alpha-\sin\alpha}{2\sin\alpha}=\frac{2\sin n\alpha\cos(n+1)\alpha}{2\sin\alpha}$$

ekanligi kelib chiqadi. Agar α ning o‘rniga $\frac{\pi}{n}$ ni qo‘ysak va $\sin\pi=0$ ekanini hisobga olsak:

$\cos\frac{2\pi}{n}+\cos\frac{4\pi}{n}+\cos\frac{6\pi}{n}+\ldots+\cos\frac{2n\pi}{n}\alpha=\frac{2\sin\pi\cos\frac{(n+1)\pi}{n}}{2\sin\frac{\pi}{n}}=0$ ekanini ko‘rishimiz mumkin.

Javob: 0

31. Quyidagicha shakl almashtirishlar bajaramiz:

$$(2-\sin^2 x)\sin^2 x\cos^4 x = (2-\sin^2 x)(1-\sin^2 x)^2\sin^2 x =$$

$$= (2\sin^2 x - \sin^4 x)(1-2\sin^2 x + \sin^4 x) = \left|\sin^2 x = t\right| =$$

$$= (2t-t^2)(1-(2t-t^2)) = \left|2t-t^2 = z\right| = z(1-z) = z - z^2 = -\left(z-\frac{1}{2}\right)^2 + \frac{1}{4} \le \frac{1}{4}$$

Bundan ko'rinadiki, $(2-\sin^2 x)\sin^2 x\cos^4 x$ ifodaning eng katta qiymati $\frac{1}{4}$ ga teng. Bu qiymatga $z=\frac{1}{2} \Rightarrow t=\sqrt{2}-1 \Rightarrow x = \pm\arcsin\sqrt{\sqrt{2}-1}+\pi n, n\in\mathbb{Z}$ da erishadi.

Javob: $\frac{1}{4}$

32. Biz oldin burchaklari $\alpha,\beta,\gamma \le 90^0$ shartni qanoatlantiruvchi uchburchak uchun $\begin{cases}\cos\frac{\alpha-\beta}{2} < 2\cos\frac{\gamma}{2}\\ \cos\frac{\alpha-\gamma}{2} < 2\cos\frac{\beta}{2}\\ \cos\frac{\beta-\gamma}{2} < 2\cos\frac{\alpha}{2}\end{cases}$ tengsizliklarni isbotlaymiz. Buning uchun $\frac{\gamma}{2} < 60^0$ bo'lgan holda $\cos\frac{\alpha-\beta}{2} < 2\cos\frac{\gamma}{2}$ tengsizlikni isbotlash yetarli. Bu isbot esa quyidagicha:

$\cos\frac{\alpha-\beta}{2} \le 1 = 2\cos 60^0 < 2\cos\frac{\gamma}{2}$.

Yuqoridagilarga asosan:

$$\cos\alpha+\cos\beta+\cos\gamma =$$

$$= \cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} + \cos\frac{\alpha+\gamma}{2}\cos\frac{\alpha-\gamma}{2} + \cos\frac{\beta+\gamma}{2}\cos\frac{\beta-\gamma}{2} <$$

$$< 2\sin\frac{\gamma}{2}\cos\frac{\gamma}{2} + 2\sin\frac{\beta}{2}\cos\frac{\beta}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} = \sin\alpha+\sin\beta+\sin\gamma$$

tengsizlik kelib chiqadi. Shuni isbotlash talab qilingan edi.

33. Koshi tengsizligi va shakl almashtirishlardan foydalanamiz:

$$(x+y)(1+\sqrt{xy}) = (x+y)(1+\sqrt{xy}) + x + y - x - y \ge$$

$$\geq (x+y)(1+\sqrt{xy})+2\sqrt{xy}-x-y=$$
$$=x+y+(x+y)\sqrt{xy}+2\sqrt{xy}-x-y=$$
$$=\sqrt{xy}(1+x+1+y)\geq \sqrt{xy}\cdot 2\sqrt{(1+x)(1+y)}$$

Bundan $(x+y)(1+\sqrt{xy})\geq 2\sqrt{xy(1+x)(1+y)}$ ekanligi kelib chiqadi. Tenglik sharti $x=y\geq 0$ bo‘lganda bajariladi.

34. Berilgan chizmadan ma’lumki, $f(x)$ kvadrat funksiyaning nollari -3 va 4 ga teng. U holda biz kvadrat funksiyani $f(x)=a(x+3)(x-4)$ ko‘rinishida izlashimiz mumkin. Bundan tashqari $y=kx+l$ to‘g‘ri chiziq (0;1) va (-1;0) nuqtalardan o‘tganligini hisobga olsak, k=1 va l=1 ekanligini topish qiyin emas. Demak, $y=x+1$ va $f(x)=a(x+3)(x-4)$ funksiyalar absissasi 2 ga teng bo‘lgan nuqtada kesishayotgan ekan. U holda $x=2$ da funksiyalarning har birining qiymati 3 ga teng ekanligidan $3=a(2+3)(2-4) \;\Rightarrow\; a=-0,3$ ekanini topish mumkin. Bundan izlanayotgan kvadrat funksiyaning $f(x)=-0,3(x+3)(x-4)$ ya’ni, $f(x)=-0,3x^2+0,3x-0,36$ ekanligi kelib chiqadi.

Javob: $f(x)=-0,3x^2+0,3x-0,36$

35. Quyidagi baholashlardan foydalanamiz:

$$\begin{cases} \frac{1}{2}<\frac{2}{3} \\ \frac{3}{4}<\frac{4}{5} \\ \cdots \\ \frac{2017}{2018}<\frac{2018}{2019} \end{cases}$$

bularni hadma-had ko‘paytirsak, $\frac{1}{2}\cdot\frac{3}{4}\cdot\ldots\cdot\frac{2017}{2018}<\frac{2}{3}\cdot\frac{4}{5}\cdot\ldots\cdot\frac{2018}{2019}$ ekani kelib chiqadi. Oxirgi tengsizlikning har ikkala tomoniga $\frac{1}{2}\cdot\frac{3}{4}\cdot\ldots\cdot\frac{2017}{2018}$ ni ko‘paytirsak:

$$\left(\frac{1}{2}\cdot\frac{3}{4}\cdot\ldots\cdot\frac{2017}{2018}\right)^2<\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}\cdot\ldots\cdot\frac{2017}{2018}\cdot\frac{2018}{2019}=\frac{1}{2019}<\frac{1}{1936}=\frac{1}{44^2}$$

bundan esa, $\left(\frac{1}{2}\cdot\frac{3}{4}\cdot\ldots\cdot\frac{2017}{2018}\right)^2 < \frac{1}{44^2}$ ya'ni $\frac{1}{2}\cdot\frac{3}{4}\cdot\ldots\cdot\frac{2017}{2018} < \frac{1}{44}$ ekani kelib chiqadi.

36. Har qanday a va b haqiqiy sonlari uchun $(a+b+2)^2 \geq 0$ tengsizlikning doimo o'rinli ekanligi bizga ma'lum. Shunga asosan quyidagilarga ega bo'lamiz:

$$(a+b+2)^2 \geq 0 \ \Rightarrow\ a^2+b^2+4+2ab+4a+4b \geq 0 \ \Rightarrow$$

$$\Rightarrow\ 2ab+4a+4b+4 \geq -a^2-b^2$$

Oxirgi tengsizlikning har ikkala tomoniga 4 ni qo'shamiz:

$$2ab+4a+4b+8 \geq 4-a^2-b^2 \ \Rightarrow\ 2a(b+2)+4(b+2) \geq c^2+d^2 \ \Rightarrow$$

$$\Rightarrow\ 2(a+2)(b+2) \geq c^2+d^2 \ \Rightarrow\ (a+2)(b+2) \geq \frac{c^2+d^2}{2}$$

Oxirgi tengsizlikda $\frac{c^2+d^2}{2} \geq cd$ ekanligini hisobga olsak, $(a+2)(b+2) \geq cd$ ekanligi kelib chiqadi. Tenglik sharti $a=b=-1$ va $c=d=1$ da yoki $a=b=-1$ va $c=d=-1$ bo'lganda bajariladi. Shuni isbotlash talab qilingan edi.

37. Tekislikda bitta nuqtadan chiquvchi $\vec{r_1}$, $\vec{r_2}$, $\vec{r_3}$ birlik vektorlarni olamiz. Ular orasidagi burchaklar 2α, 2β va 2γ bo'lsin. U holda ushbu $(\vec{r_1}+\vec{r_2}+\vec{r_3})^2 \geq 0$ tengsizlikka ko'ra:

$$3+2\cos 2\alpha+2\cos 2\beta+2\cos 2\gamma \geq 0 \ \Rightarrow$$

$$\Rightarrow\ 3+2(3-2\sin^2\alpha-2\sin^2\beta-2\sin^2\gamma) \geq 0$$

ekanligini ko'rish qiyin emas. Oxirgi tengsizlikdan $\sin^2\alpha+\sin^2\beta+\sin^2\gamma \leq \frac{9}{4}$ ekanligi kelib chiqadi. Tenglik sharti $\alpha=\beta=\gamma=\frac{\pi}{3}$ da bajariladi. Isbot tugadi.

38. 37-masalaga asosiy trigonometrik ayniyatni qo'llaymiz:

$$\sin^2\alpha+\sin^2\beta+\sin^2\gamma \leq \frac{9}{4} \ \Rightarrow\ 1-\cos^2\alpha+1-\cos^2\beta+1-\cos^2\gamma \leq \frac{9}{4} \ \Rightarrow$$

$$\Rightarrow\ \cos^2\alpha+\cos^2\beta+\cos^2\gamma \geq \frac{3}{4}$$

Tenglik sharti $\alpha=\beta=\gamma=\frac{\pi}{3}$ da bajariladi.

39. Sistemaning birinchi tenglamasidan ikkinchisini ayiramiz va y ni topib olamiz:

$$-\begin{cases} x^2 - xy + y^2 = 21 \\ y^2 - 2xy + 15 = 0 \end{cases} \Rightarrow x^2 + xy = 36 \Rightarrow y = \frac{36 - x^2}{x}$$

Topilgan y ni sistemaning ikkinchi tenglamasiga qo‘ysak:

$$y^2 - 2xy + 15 = 0$$

$$\left(\frac{36 - x^2}{x}\right)^2 - 2x \cdot \frac{36 - x^2}{x} + 15 = 0$$

$$1296 - 72x^2 + x^4 - 72x^2 + 2x^4 + 15x^2 = 0$$

$$3x^4 - 129x^2 + 1296 = 0$$

$$x^4 - 43x^2 + 432 = 0 \Rightarrow (x^2 - 16)(x^2 - 27) = 0 \Rightarrow x_{1,2} = \pm 4; \quad x_{3,4} = \pm 3\sqrt{3}$$

$$\frac{1296 - 72x^2 + x^4}{x^2} - 72 + 2x^2 + 15 = 0$$

yechimlarni topamiz. Bulardan esa, $y_{1,2} = \pm 5$ va $y_{3,4} = \pm\sqrt{3}$ ekanligi kelib chiqadi.

Javob: $(x;y) \in \left\{(4;5),(-4;-5),(3\sqrt{3};\sqrt{3}),(-3\sqrt{3};-\sqrt{3})\right\}$

40. Qavslarni quyidagicha ochamiz:

$$(6x+7)^2(3x+4)(x+1) = 1$$

$$(36x^2 + 84x + 48 + 1)(3x^2 + 7x + 4) = 1$$

$$(12(3x^2 + 7x + 4) + 1)(3x^2 + 7x + 4) = 1$$

Endi $3x^2 + 7x + 4 = a$ deb belgilab olamiz, natijada quyidagi tenglamaga kelamiz:

$$(12a+1)a = 1 \Rightarrow 12a^2 + a - 1 = 0 \Rightarrow a_1 = \frac{1}{4}, a_2 = -\frac{1}{3}$$

Belgilashga qaytsak, ushbu $3x^2 + 7x + 4 = \frac{1}{4}$ va $3x^2 + 7x + 4 = -\frac{1}{3}$ kvadrat tenglamalar hosil bo‘ladi. Bularning birinchisidan $x_1 = -\frac{5}{6}$ va $x_2 = -1,5$ ikkinchisidan $x \in \varnothing$ ekanligini topishimiz mumkin.

Javob: $x_1 = -\frac{5}{6}$ *va* $x_2 = -1,5$

41. $x+1=a$ deb belgilash kiritamiz. U holda tenglama quyidagi ko‘rinishga keladi:

$$(x+1-2)^4+(x+1+2)^4=82$$

$$(a-2)^4+(a+2)^4=82$$

$$a^4-8a^3+24a^2-32a+16+a^4+8a^3+24a^2+32a+16=82$$

$$2a^4+48a^2-50=0 \Rightarrow a^4+24a^2-25=0$$

Oxirgi bikvadrat tenglamadan $a^2=1$ va $a^2=-25$ ekanligi kelib chiqadi. Belgilashga qaytib, $x_1=0$ va $x_2=-2$ yechimlarga ega bo‘lamiz.

Javob: $x_1=0$ *va* $x_2=-2$

42. Nomanfiy a,b,c sonlari uchun $(\sqrt[4]{a}-1)^2\geq 0$, $(\sqrt[4]{b}-1)^2\geq 0$, $(\sqrt[4]{c}-1)^2\geq 0$ tengsizliklarning bajarilishi ma’lum. Bu uchta tengsizlikda qavslarni ochib, bir-biriga qo‘shsak:

$$\begin{cases}(\sqrt[4]{a}-1)^2\geq 0\\(\sqrt[4]{b}-1)^2\geq 0\\(\sqrt[4]{c}-1)^2\geq 0\end{cases}\Rightarrow\begin{cases}\sqrt{a}-2\sqrt[4]{a}+1\geq 0\\\sqrt{b}-2\sqrt[4]{b}+1\geq 0\\\sqrt{c}-2\sqrt[4]{c}+1\geq 0\end{cases}\Rightarrow\begin{cases}2\sqrt[4]{a}-\sqrt{a}\leq 1\\2\sqrt[4]{b}-\sqrt{b}\leq 1\\2\sqrt[4]{c}-\sqrt{c}\leq 1\end{cases}\Rightarrow$$

$$\Rightarrow (2\sqrt[4]{a}+2\sqrt[4]{b}+2\sqrt[4]{c})-(\sqrt{a}+\sqrt{b}+\sqrt{c})\leq 3$$

ekanligi kelib chiqadi. Tenglik sharti faqat $a=b=c=1$ bo‘lganda bajariladi. Isbot tugadi.

43. Masala shartiga ko‘ra x^2+ax+b va x^2+bx+c ko‘phadlar $x+1$ ga bo‘lingani uchun ushbu $x^2+ax+b=(x+1)(x+b) \Rightarrow a=b+1$ va $x^2+bx+c=(x+1)(x+c) \Rightarrow b=c+1$ tengliklarni yozishimiz mumkin. Bundan tashqari

$$x^3-4x^2+x+6=x^3+x^2-5x^2+5+x+1=$$

$$=x^2(x+1)-5(x+1)(x-1)+(x+1)=$$

$$=(x+1)(x^2-5x+6)=(x+1)(x-2)(x-3)$$

ko‘phadning $x^2+ax+b=(x+1)(x+b)$ va $x^2+bx+c=(x+1)(x+c)$ ko‘phadlarga bo‘linishidan

$$\begin{cases} x+b=x-2 \\ x+c=x-3 \end{cases} \Rightarrow \begin{cases} b=-2 \\ c=-3 \end{cases} \Rightarrow a=-1 \qquad (1)$$

yoki

$$\begin{cases} x+b=x-3 \\ x+c=x-2 \end{cases} \Rightarrow \begin{cases} b=-3 \\ c=-2 \end{cases} \Rightarrow a=-2 \qquad (2)$$

ekanligili kelib chiqadi. (2) da $b=c+1$ ning bajarilmasligini hisobga olsak, $a+b+c=-1-2-3=-6$ ekani kelib chiqadi.

Javob: -6

44. Tengsizlikning har ikkala tomonini 4 ga ko'paytiramiz:

$$4x^2 \le 4\cdot[2x]\cdot\{2x\} \;\Rightarrow\; (2x)^2 \le 4\cdot[2x]\cdot\{2x\}$$

Agar $2x=[2x]+\{2x\}$ ekanini hisobga olsak, tengsizlik quyidagi ko'rinishga keladi:

$$([2x]+\{2x\})^2 \le 4\cdot[2x]\cdot\{2x\}$$

$$[2x]^2+2\cdot[2x]\cdot\{2x\}+\{2x\}^2 \le 4\cdot[2x]\cdot\{2x\}$$

$$[2x]^2-2\cdot[2x]\cdot\{2x\}+\{2x\}^2 \le 0$$

$$([2x]-\{2x\})^2 \le 0$$

Oxirgi tengsizlikda faqat tenglik sharti bajarilishi ma'lum. U holda ushbu $([2x]-\{2x\})^2=0 \;\Rightarrow\; [2x]-\{2x\}=0 \;\Rightarrow\; [2x]=\{2x\}$ tenglamaga kelamiz. Bunda $0\le\{2x\}<1$ va $[2x]\in\mathbb{Z}$ ekanidan $[2x]=\{2x\}=0 \;\Rightarrow\; x=0$ ekanligi kelib chiqadi.

Javob: $x=0$

45. Berilgan masala $x^7+2x^5+3x^4+3x^3-2x+5$ ko'phadni x^2+2 ko'phadga bo'lgandagi qoldiqni topish masalasiga keladi. U holda Bezu teoremasiga ko'ra qoldiqni topamiz:

$$x^2+2=0 \;\Rightarrow\; x^2=-2 \text{ (kompleks yechim)}$$

$$x^7+2x^5+3x^4+3x^3-2x+5=(x^2)^3\cdot x+2(x^2)^2\cdot x+3(x^2)^2+3\cdot x^2\cdot x-2x+5=$$
$$=-8x+8x+12-6x-2x+5=-8x+17$$

Bundan $a=-8$ va $b=17$ ekanini topish qiyin emas. U holda $a+b=9$.

Javob: 9

46. $\sqrt{2017+\sqrt{2017+\sqrt{2017+\ldots+\sqrt{2017+\sqrt{2017}}}}}=x$ deb belgilaymiz ($x>0$) va ikkala tomonini kvadratga oshiramiz. Natijada quyidagi kvadrat tenglamaga ega bo‘lamiz:

$$2017+x=x^2 \Rightarrow x_1=\frac{1+\sqrt{1+4\cdot 2017}}{2};\ x_2=\frac{1-\sqrt{1+4\cdot 2017}}{2}$$

Agar $x>0$ ekanini hisobga olsak, $[x]=\left[\frac{1+\sqrt{1+4\cdot 2017}}{2}\right]=45$ yechimga ega bo‘lamiz. Demak, $\left[\sqrt{2017+\sqrt{2017+\sqrt{2017+\ldots+\sqrt{2017+\sqrt{2017}}}}}\right]=45$ ekan.

Javob: 45

47. Quyidagicha shakl almashtirishlarni bajaramiz:

$$9\cdot 99\cdot 999\cdot\ldots\cdot\underbrace{999\ldots9}_{2021ta}=(10-1)(100-1)(1000-1)\ldots(1\underbrace{00\ldots0}_{2021ta}-1)=$$

$$=(10-1)(100-1)(1000A-1)=(1000-109)(1000A-1)=1000B+109$$

Yuqoridagilarga ko‘ra $x=109$.

Javob: $x=109$

48. Ixtiyoriy natural n soni uchun ushbu $\frac{n}{(n+1)!}=\frac{n+1-1}{(n+1)!}=\frac{n+1}{(n+1)!}-\frac{1}{(n+1)!}=\frac{1}{n!}-\frac{1}{(n+1)!}$ tenglikning o‘rinli ekanidan:

$$\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ldots+\frac{2016}{2017!}=\frac{1}{1!}-\frac{1}{2!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{3!}-\frac{1}{4!}+\ldots+$$

$$+\frac{1}{2016!}-\frac{1}{2017!}=1-\frac{1}{2017!}$$

ekanligi kelib chiqadi.

Javob: $1-\frac{1}{2017!}$

49. Bizga ma’lumki, 2 ning darajalaridagi oxirgi raqamlar har 4 sikldan takrorlanadi. Agar biz $2^{2019}=2^{4\cdot 504+3}=\ldots 8$ va $2019^2+2^{2019}=(2020-1)^2+2^{2019}=4A+1$ ekanligini hisobga olsak, $k^2+2^k=(2019^2+2^{2019})^2+2^{2019^2+2^{2019}}=(\ldots1+\ldots8)^2+2^{4A+1}=\ldots1+\ldots2=\ldots3$

ekanligi kelib chiqadi. Endi biz 3^{2019} ning oxirgi raqamini topishimiz qoldi. 3 ning darajalaridagi oxirgi raqamlar ham har 4 sikldan takrorlanishini bilgan holda $3^{2019} = 3^{4\cdot 504+3} = \ldots 7$ ekanligini oson topib olamiz.

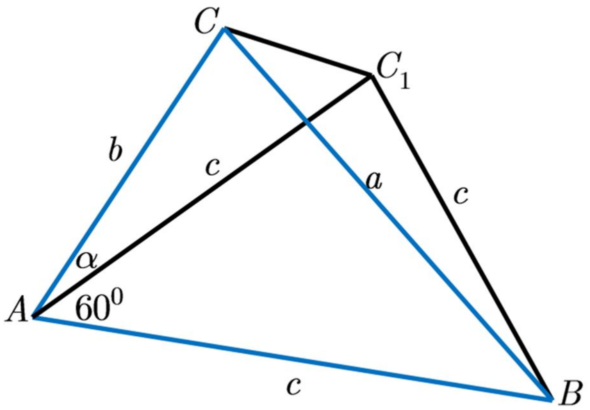
Javob: 7

50. Oddiygina shakl almashtirish bajaramiz:

$$N = 100^2 + 99^2 - 98^2 - 97^2 + \ldots + 4^2 + 3^2 - 2^2 - 1^2 = (100-98)(100+98) +$$
$$+(99-97)(99+97) + \ldots + (4-2)(4+2) + (3-1)(3+1) =$$
$$= 2(198 + 196 + 190 + 188 + \ldots + 6 + 4) =$$
$$= 2\big((196 + 188 + 180 + \ldots + 4) + (198 + 190 + 182 + \ldots + 6)\big) =$$
$$= 2\left(\frac{196+4}{2}\cdot 25 + \frac{198+6}{2}\cdot 25\right) = 50 \cdot 202 = 10100 = 10000 + 100$$
$$N \equiv x(\mathrm{mod}\,1000) \quad \Rightarrow \quad x = 100$$

Javob: $x = 100$

51. α orqali $\angle CAC_1$ ni belgilaylik. U holda $S_{ABC} = \dfrac{bc\sin(60^0 + \alpha)}{2}$ ekanidan va ΔACC_1 da kosinuslar teoremasiga ko‘ra:



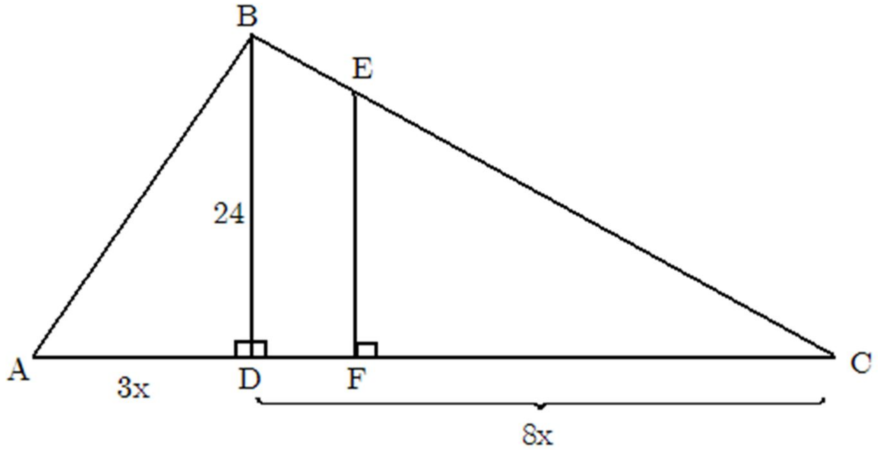
$$\left|CC_1\right|^2 = b^2 + c^2 - 2bc\cos\alpha = b^2 + c^2 - 2bc\cos(60^0 + \alpha - 60^0) =$$
$$= b^2 + c^2 - 2bc\Big(\cos 60^0 \cdot \cos(60^0 + \alpha) + \sin 60^0 \cdot \sin(60^0 + \alpha)\Big) =$$
$$= b^2 + c^2 - 2bc\left(\frac{1}{2}\cos(60^0 + \alpha) + \frac{\sqrt{3}}{2}\sin(60^0 + \alpha)\right) =$$
$$= b^2 + c^2 - \frac{2bc\cos(60^0 + \alpha)}{2} - \frac{2\sqrt{3}bc\sin(60^0 + \alpha)}{2} =$$

$$=\frac{2b^2+2c^2-2bc\cos(60^0+\alpha)}{2}-2\sqrt{3}S=$$

$$=\frac{b^2+c^2+b^2+c^2-2bc\cos(60^0+\alpha)}{2}-2\sqrt{3}S=\frac{a^2+b^2+c^2}{2}-2\sqrt{3}S$$

Demak, $\left|CC_1\right|^2=\dfrac{a^2+b^2+c^2}{2}-2\sqrt{3}S$. Isbot tugadi.

52. Qulaylik uchun $AD=3x$ va $DC=8x$ deb belgilab olamiz.



U holda $S_{ABC}=\dfrac{1}{2}\cdot 11x\cdot 24=132x$, $S_{ABD}=\dfrac{1}{2}\cdot 3x\cdot 24=36x$ ekanligini topish mumkin. Shartga ko‘ra $S_{EFC}=\dfrac{S_{ABC}}{2}=\dfrac{132x}{2}=66x$, bundan $S_{BEFD}=66x-36x=30x$ va $S_{BDC}=66x+30x=96x$ ekanligi kelib chiqadi. Bundan tashqari ΔBDC va ΔEFC lar o‘xshash ekanligidan: $\dfrac{S_{BDC}}{S_{EFC}}=\left(\dfrac{BD}{EF}\right)^2\Rightarrow\dfrac{96x}{66x}=\left(\dfrac{24}{EF}\right)^2\Rightarrow EF=24\cdot\sqrt{\dfrac{66}{96}}=6\sqrt{11}$ ekani kelib chiqadi.

Javob: $6\sqrt{11}$

53. Quyidagicha shakl almashtirishlar bajaramiz:

$$P_4(x)=5x^4+9x^3-2x^2-4x-8=5x^4+10x^3-x^3-2x^2-4x-8=$$

$$=5x^3(x+2)-x^2(x+2)-4(x+2)=(x+2)(5x^3-x^2-4)=$$

$$=5x^3(x+2)-x^2(x+2)-4(x+2)=(x+2)(5x^3-x^2-4)=$$

$$=(x+2)(x^3-x^2+4x^3-4)=(x+2)\left(x^2(x-1)+4(x^3-1)\right)=$$

$$= (x+2)\left(x^2(x-1)+4(x-1)(x^2+x+1)\right) = (x+2)(x-1)(x^2+4x^2+4x+4) =$$

$$= (x+2)(x-1)(5x^2+4x+4)$$

Javob: $P_4(x) = (x+2)(x-1)(5x^2+4x+4)$

54. Berilgan ifodani quyidagicha yozib olamiz:

$$7777^{2222} + 2222^{7777} = (7777^{2222} - 1^{2222}) + (2222^{7777} + 1^{7777})$$

Bezu teoremasiga ko‘ra birinchi qavsdagi son $7777 - 1 = 7776$ ga, ikkinchi qavsdagi son $2222 + 1 = 2223$ ga bo‘linadi. 7776 va 2223 lar 9 ga karrali ekanidan berilgan ifoda ham 9 ga bo‘linadi.

55. 64 kg unni dastlab 2 ta teng bo‘lakka bo‘lamiz.

1-qadam: 32=32

Chapdagi 32 kg unni qoldirib, o‘ngdagi 32 kg unni yana 2 ta teng bo‘lakka bo‘lamiz.

2-qadam: 16=16

Bu jarayonni davom ettiramiz.

3-qadam: 8=8
4-qadam: 4=4
5-qadam: 2=2
6-qadam: 1=1

O‘ng tomonda turgan unlardan olamiz. $16\,kg + 4\,kg + 2\,kg + 1\,kg = 23\,kg$.

56. B sonining shaklini o‘zgartirib yozamiz.

$$B = 50^{99} = \left(\frac{99+1}{2}\right)^{99}$$

Ikki sonning o‘rta arifmetigi va o‘rta geometrigi haqidagi teoremadan foydalanamiz.

$$A = 99! = 1\cdot 2\cdot 3\cdot \ldots \cdot 98\cdot 99$$

$$\begin{cases} \sqrt{1\cdot 99} < \dfrac{1+99}{2} \\ \sqrt{2\cdot 98} < \dfrac{2+98}{2} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \sqrt{98\cdot 2} < \dfrac{98+2}{2} \\ \sqrt{99\cdot 1} < \dfrac{99+1}{2} \end{cases}$$

Tengsizliklarni hadma-had ko‘paytiramiz:

$$\sqrt{(99!)^2} < 50^{99}$$

$$99! < 50^{99}$$

$$A < B$$

Javob: $A < B$

57. ***Koshi-Bunyakovskiy tengsizligi:*** *Faraz qilaylik* $(a_1, a_2, ..., a_n)$ *va* $(b_1, b_2, ..., b_n)$– *haqiqiy sonlarning istalgan ketma-ketliklari bo‘lsin. U holda quyidagi tengsizlik o‘rinli:*

$$(a_1b_1 + a_2b_2 + ... + a_nb_n)^2 \le (a_1^2 + a_2^2 + ... + a_n^2)(b_1^2 + b_2^2 + ... + b_n^2)$$

tengsizlikda tenglik sharti $\frac{a_1}{b_1} = \frac{a_2}{b_2} = ... = \frac{a_n}{b_n}$ *bo‘lganda bajariladi.*

Berilgan tenglamani yechish uchun $a_1 = 1$, $a_2 = -1$, $a_3 = -1$, $b_1 = \sqrt{x}$, $b_2 = \sqrt{y}$ va $b_3 = \sqrt{z}$ deb belgilab olamiz. U holda Koshi-Bunyakovskiy tengsizligiga ko‘ra quyidagilarga ega bo‘lamiz:

$$(\sqrt{x} - \sqrt{y} - \sqrt{z})^2 \le (x + y + z)(1 + 1 + 1)$$

$$\sqrt{x} - \sqrt{y} - \sqrt{z} \le \sqrt{3(x + y + z)}$$

$$\sqrt{x} \le \sqrt{3(x + y + z)} + \sqrt{y} + \sqrt{z}$$

Oxirgi tengsizlikda tenglik sharti $\frac{\sqrt{x}}{1} = \frac{\sqrt{y}}{-1} = \frac{\sqrt{z}}{-1} \Rightarrow \sqrt{x} = -\sqrt{y} = -\sqrt{z}$ bo‘lganda bajariladi. Bundan $x = y = z = 0$ ekanligi kelib chiqadi.

Javob: $x = y = z = 0$

58. Ushbu $\frac{1}{m^2} < \frac{1}{m^2 - m} = \frac{1}{m(m-1)} = \frac{1}{m-1} - \frac{1}{m}$ tengsizlikdan foydalanamiz.

$$\begin{cases} \dfrac{1}{2^2} < \dfrac{1}{1} - \dfrac{1}{2} \\ \dfrac{1}{3^2} < \dfrac{1}{2} - \dfrac{1}{3} \\ \dots\dots\dots\dots\dots \\ \dfrac{1}{(m-1)^2} < \dfrac{1}{m-2} - \dfrac{1}{m-1} \\ \dfrac{1}{m^2} < \dfrac{1}{m-1} - \dfrac{1}{m} \end{cases}$$

Tengsizliklarni hadma-had qo‘shamiz:

$$\frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{(m-1)^2} + \frac{1}{m^2} < \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \dots + \frac{1}{m-2} - \frac{1}{m-1} + \frac{1}{m-1} - \frac{1}{m}$$

$$\frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{(m-1)^2} + \frac{1}{m^2} < \frac{m-1}{m}$$

59. Quyidagicha shakl almashtirishlar bajaramiz:

$$\frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ac} =$$

$$= \frac{a^3 + b^3 + c^3 - 3abc + a^2b + ab^2 + a^2c + ac^2 + b^2c + bc^2}{a^2 + b^2 + c^2 - ab - bc - ac} +$$

$$+ \frac{-a^2b - ab^2 - a^2c - ac^2 - b^2c - bc^2}{a^2 + b^2 + c^2 - ab - bc - ac} =$$

$$= \frac{(a^3 + a^2b + a^2c) + (ab^2 + b^3 + b^2c) + (ac^2 + bc^2 + c^3)}{a^2 + b^2 + c^2 - ab - bc - ac} -$$

$$- \frac{(a^2b + ab^2 + abc) + (abc + b^2c + bc^2) + (a^2c + abc + ac^2)}{a^2 + b^2 + c^2 - ab - bc - ac} =$$

$$= \frac{a^2(a + b + c) + b^2(a + b + c) + c^2(a + b + c)}{a^2 + b^2 + c^2 - ab - bc - ac} +$$

$$+ \frac{-ab(a + b + c) - bc(a + b + c) - ac(a + b + c)}{a^2 + b^2 + c^2 - ab - bc - ac} =$$

$$= \frac{(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)}{a^2 + b^2 + c^2 - ab - bc - ac} = a + b + c$$

Javob: $a + b + c$

60. $EKUK(a;b;c)=2^{2011}$ ekanidan $a=2^{2011}$, $b=2^n$ va $c=2^m$ deb olishimiz mumkin, bunda $0\le n\le 2011$ va $0\le m\le 2011$. U holda $ab=2^{2011+n}$, $bc=2^{n+m}$ va $ac=2^{2011+m}$ tengliklar o‘rinli bo‘ladi. Agar $2011\le 2011+n\le 4022$, $0\le n+m\le 4022$ va $2011\le 2011+m\le 4022$ ekanligini hisobga olsak, $EKUB(ab;bc;ac)$ ifoda $2^0,2^1,\ldots,2^{4022}$ qiymatlarni qabul qilishi mumkinligi kelib chiqadi. Bundan $EKUB\left(ab;bc;ac\right)$ ifodaning jami 4023 ta qiymat qabul qilishini topishimiz mumkin.

Javob: 4023

61. 1 dan 7 gacha bo‘lgan sonlar kub ildizining butun qismi 1 ga, 8 dan 26 gacha bo‘lgan sonlar kub ildizining butun qismi 2 ga, 27 dan 63 gacha bo‘lgan sonlar kub ildizining butun qismi 3 ga, 64 dan 124 gacha bo‘lgan sonlar kub ildizining butun qismi 4 ga teng chiqishi ma’lum. U holda quyidagi tenglikdan:

$$\underbrace{1+1+\ldots+1}_{7\,ta}+\underbrace{2+2+\ldots+2}_{19\,ta}+\underbrace{3+3+\ldots+3}_{37\,ta}+\underbrace{4+4+\ldots+4}_{61\,ta}=400$$

$x^3-1=124 \Rightarrow x=5$ ekanligi kelib chiqadi.

Javob: $x=5$

62. Nomanfiy bo‘lgan x va y sonlari uchun ushbu $(\sqrt{x}-\sqrt{y})^2\ge 0 \Rightarrow \sqrt{xy}\le \dfrac{x+y}{2}$ tengsizlikning o‘rinli ekanidan va uchburchakning balandligini topish formulasidan foydalanamiz:

$$h_a=\frac{2}{a}\sqrt{p(p-a)(p-b)(p-c)}=\frac{2}{a}\sqrt{p(p-a)}\cdot\sqrt{(p-b)(p-c)}\le$$

$$\le\frac{2}{a}\sqrt{p(p-a)}\cdot\frac{p-b+p-c}{2}=\frac{2}{a}\sqrt{p(p-a)}\cdot\frac{2p-b-c}{2}=$$

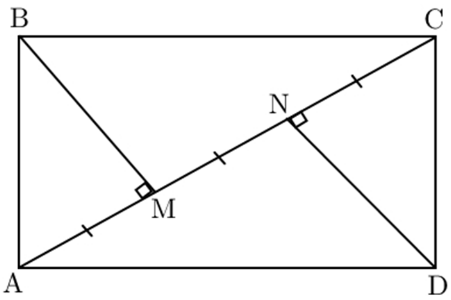
$$=\frac{2}{a}\sqrt{p(p-a)}\cdot\frac{a+b+c-b-c}{2}=\frac{2}{a}\sqrt{p(p-a)}\cdot\frac{a}{2}=\sqrt{p(p-a)}.$$

Bundan $h_a\le\sqrt{p(p-a)}$ tengsizlikka ega bo‘lamiz. Tengsizlikda tenglik sharti $a=b=c$ bo‘lganda ya’ni, muntazam uchburchakda bajariladi. Shuni isbotlash talab qilingan edi.

63. Masalalada quyidagi ikki hol bo‘lishi mumkin:

1-hol: To‘g‘ri to‘rtburchakning kichik tomoni 2 ga teng bo‘lsin.

$ABCD$ to‘g‘ri to‘rtburchakda $AB=CD=2$ va $AM=MN=CN$ tengliklar o‘rinli bo‘lsin.

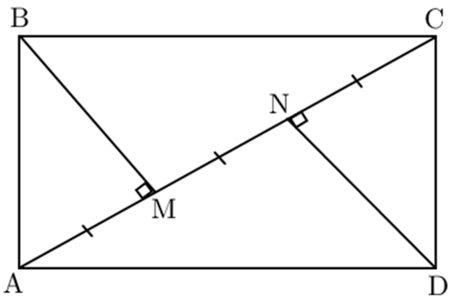


U holda ABM va BMC to‘g‘ri burchakli uchburchaklarda Pifagor teoremasini qo‘llab, quyidagilarga ega bo‘lamiz:

$BM^2 = 4 - AM^2$ va $BM^2 = BC^2 - CM^2 = BC^2 - 4AM^2$. Bundan tashqari ABC uchburchak uchun $BC^2 = AC^2 - 4 = 9AM^2 - 4$ tenglik o‘rinli. Yuqoridagilardan $4 - AM^2 = 9AM^2 - 4 - 4AM^2$ tenglik, bundan $AM^2 = \dfrac{4}{3}$ ekanligi kelib chiqadi. U holda $BC = 2\sqrt{2}$ ga teng bo‘lib, to‘g‘ri to‘rtburchakning yuzi $S = 2 \cdot 2\sqrt{2} = 4\sqrt{2}$ ga teng bo‘ladi.

2-hol: To‘g‘ri to‘rtburchakning katta tomoni 2 ga teng bo‘lsin.

$ABCD$ to‘g‘ri to‘rtburchakda $BC = AD = 2$ va $AM = MN = CN$ tengliklar o‘rinli bo‘lsin.



U holda ABM va BMC to‘g‘ri burchakli uchburchaklarda Pifagor teoremasini qo‘llab, quyidagilarga ega bo‘lamiz:

$BM^2 = AB^2 - AM^2$ va $BM^2 = 4 - CM^2 = 4 - 4AM^2$. Bundan tashqari ABC uchburchak uchun $AB^2 = AC^2 - 4 = 9AM^2 - 4$ tenglik o‘rinli. Yuqoridagilardan $9AM^2 - 4 - AM^2 = 4 - 4AM^2$ tenglik, bundan $AM^2 = \dfrac{2}{3}$ ekanligi kelib chiqadi. U holda $AB = \sqrt{2}$ ga teng bo‘lib, to‘g‘ri to‘rtburchakning yuzi $S = 2 \cdot \sqrt{2} = 2\sqrt{2}$ ga teng bo‘ladi.

Javob: $2\sqrt{2}$ *va* $4\sqrt{2}$ *yoki* $\sqrt{2}$ *va* $2\sqrt{2}$

64. a,b,c lar arifmetik progressiyaning ketma-ket hadlari ekanidan $a+c=2b$ tenglik o‘rinli. U holda uchburchakning yarim perimetri $p=\frac{a+b+c}{2}=\frac{3b}{2}$ ga teng bo‘ladi. Geron formulasidan foydalanib, uchburchakning yuzini topib olamiz:

$$S=\sqrt{p(p-a)(p-b)(p-c)}=\sqrt{\frac{3b}{2}\cdot\left(\frac{3b}{2}-a\right)\left(\frac{3b}{2}-b\right)\left(\frac{3b}{2}-c\right)}=$$

$$=\sqrt{\frac{3b}{2}\cdot\frac{3b-2a}{2}\cdot\frac{b}{2}\cdot\frac{3b-2c}{2}}=\sqrt{\frac{3b}{2}\cdot\frac{b+2b-2a}{2}\cdot\frac{b}{2}\cdot\frac{b+2b-2c}{2}}=$$

$$=\sqrt{\frac{3b}{2}\cdot\frac{b+a+c-2a}{2}\cdot\frac{b}{2}\cdot\frac{b+a+c-2c}{2}}=\frac{b}{4}\sqrt{3(b+c-a)(b+a-c)}.$$

Endi uchburchakka ichki chizilgan aylana radiusini topamiz:

$$r=\frac{2S}{a+b+c}=\frac{2\cdot\frac{b}{4}\sqrt{3(b+c-a)(b+a-c)}}{3b}=\frac{1}{6}\sqrt{3(b+c-a)(b+a-c)}$$

Javob: $r=\frac{1}{6}\sqrt{3(b+c-a)(b+a-c)}$

65. Ushbu $x^3+ax^2+bx+c=0$ kubik tenglamaning x_1,x_2,x_3 ildizlari uchun quyidagi Viet teoremasi o‘rinli:

$$\begin{cases}x_1+x_2+x_3=-a\\ x_1x_2+x_1x_3+x_2x_3=b\\ x_1x_2x_3=-c\end{cases}$$

Shunga ko‘ra $x^3-x+1=0$ tenglamaning a,b,c ildizlari uchun quyidagini yoza olamiz:

$$\begin{cases}a+b+c=0\\ ab+bc+ac=-1\\ abc=-1\end{cases}$$

Shunga asoslanib, quyidagiga ega bo‘lamiz:

$$\frac{1}{a+1}+\frac{1}{b+1}+\frac{1}{c+1}=\frac{(b+1)(c+1)+(a+1)(c+1)+(a+1)(b+1)}{(a+1)(b+1)(c+1)}=$$

$$=\frac{(ab+bc+ac)+2(a+b+c)+3}{abc+(ab+bc+ac)+(a+b+c)+1}=\frac{-1+2\cdot0+3}{-1-1+0+1}=-2$$

Javob: -2

66. $a^3-b^3=-(b^3-c^3)-(c^3-a^3)$ ekanidan foydalanib, quyidagilarga ega bo‘lamiz:

$$a(b^3-c^3)+b(c^3-a^3)+c(a^3-b^3)=$$
$$=a(b^3-c^3)+b(c^3-a^3)-c(b^3-c^3)-c(c^3-a^3)=$$
$$=(b^3-c^3)(a-c)+(c^3-a^3)(b-c)=$$
$$=(b-c)(a-c)(b^2+bc+c^2)-(b-c)(a-c)(c^2+ac+a^2)=$$
$$=(b-c)(a-c)(b^2+bc+c^2-c^2-ac-a^2)=$$
$$=(b-c)(a-c)((b-a)(b+a)+c(b-a))=(b-c)(a-c)(b-a)(a+b+c)$$

Bundan ko‘rinadiki, berilgan ifoda istalgan natural a,b,c sonlar uchun $a+b+c$ ga karrali.

67. Umumiylikka zarar yetkazmagan holda $x>y$ deb faraz qilamiz. U holda sistemasining ikkinchi tenglamasidan $(y+z)^3=x>y$ va uchinchi tenglamasidan $y=(x+z)^3>(y+z)^3$ ekanligi kelib chiqadi. Bu esa ziddiyat. Agar $x<y$ deb faraz qilsak ham ziddiyatga kelamiz. Bundan $x=y$ ekanligi kelib chiqadi. Xuddi shunga o‘xshash $y=z$ va $x=z$ ekanini topish mumkin. U holda ushbu $(x+x)^3=x \Rightarrow 8x^3-x=0$ tenglama hosil bo‘ladi. Uni yechib, $x=0$ va $x=\pm\dfrac{1}{2\sqrt{2}}$ yechimlarga ega bo‘lamiz.

Javob: $(x;y;z)\in\left\{(0;0;0),\left(\dfrac{1}{2\sqrt{2}};\dfrac{1}{2\sqrt{2}};\dfrac{1}{2\sqrt{2}}\right),\left(-\dfrac{1}{2\sqrt{2}};-\dfrac{1}{2\sqrt{2}};-\dfrac{1}{2\sqrt{2}}\right)\right\}$

68. Berilgan tengsizlikni ushbu $|x-3|^{2x^2-7x}>1=|x-3|^0$ ko‘rinishda yozib olib, quyidagi ikki holni qaraymiz:

1-hol: $|x-3|>1$ bo‘lsin.

$$\begin{cases}|x-3|>1\\2x^2-7x>0\end{cases}\Rightarrow\begin{cases}\left[\begin{array}{l}x-3>1\\x-3<-1\end{array}\right.\\2x(x-3,5)>0\end{cases}\Rightarrow\begin{cases}\left[\begin{array}{l}x>4\\x<2\end{array}\right.\\x<0;x>3,5\end{cases}\Rightarrow x\in(-\infty;0)\cup(4;\infty)$$

2-hol: $0<|x-3|<1$ bo‘lsin.

$$\begin{cases}0<|x-3|<1\\2x^2-7x<0\end{cases}\Rightarrow\begin{cases}x\neq 3\\x-3<1\\x-3>-1\\2x(x-3,5)<0\end{cases}\Rightarrow\begin{cases}x\neq 3\\x<4\\x>2\\0<x<3,5\end{cases}\Rightarrow x\in(2;3)\cup(3;3,5)$$

Javob: $x\in(-\infty;0)\cup(2;3)\cup(3;3,5)\cup(4;\infty)$

69. Ma'lumki, $x>0, y>0, z>0$ sonlari uchun quyidagi tengsizliklar o'rinli:

$$\begin{cases}(x-y)^2\geq 0\\(y-z)^2\geq 0\\(x-z)^2\geq 0\end{cases}\Rightarrow\begin{cases}x^2+y^2\geq 2xy\\y^2+z^2\geq 2yz\\x^2+z^2\geq 2xz\end{cases}\Rightarrow x^2+y^2+z^2\geq xy+yz+xz$$

Oxirgi tengsizlikni 3 marta qo'llasak, quyidagilarga ega bo'lamiz:

$$\frac{a^8+b^8+c^8}{a^3b^3c^3}=\frac{a^5}{b^3c^3}+\frac{b^5}{a^3c^3}+\frac{c^5}{a^3b^3}\geq\sqrt{\frac{a^5b^5}{a^3b^3c^6}}+\sqrt{\frac{a^5c^5}{a^3b^6c^3}}+\sqrt{\frac{b^5c^5}{a^6b^3c^3}}=$$

$$=\frac{ab}{c^3}+\frac{ac}{b^3}+\frac{bc}{a^3}\geq\sqrt{\frac{a^2bc}{b^3c^3}}+\sqrt{\frac{ab^2c}{a^3c^3}}+\sqrt{\frac{abc^2}{a^3b^3}}=$$

$$=\frac{a}{bc}+\frac{b}{ac}+\frac{c}{ab}\geq\sqrt{\frac{ab}{abc^2}}+\sqrt{\frac{ac}{ab^2c}}+\sqrt{\frac{bc}{a^2bc}}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$$

Tengsizlikda tenglik sharti $a=b=c$ bo'lganda bajariladi. Da'vo isbotlandi.

70. 57-masaladagiga o'xshash $a_1=xy$, $a_2=yz$, $a_3=zx$, $b_1=1$, $b_2=1$, $b_3=1$ almashtirishdan foydalanamiz. U holda Koshi-Bunyakovskiy tengsizligiga ko'ra quyidagilarga ega bo'lamiz:

$$(xy+yz+xz)^2\leq(x^2y^2+y^2z^2+x^2z^2)(1+1+1)$$

$$xy+yz+xz\leq\sqrt{3(x^2y^2+y^2z^2+x^2z^2)}$$

$$\frac{1}{\sqrt{3}}\left(\frac{xy+yz+xz}{\sqrt{xyz}}\right)\leq\sqrt{\frac{x^2y^2+y^2z^2+x^2z^2}{xyz}}$$

$$\frac{1}{\sqrt{3}}\left(\frac{xy+yz+zx}{\sqrt{xyz}}\right)\leq\sqrt{\frac{xy}{z}+\frac{yz}{x}+\frac{zx}{y}}$$

Oxirgi tengsizlikda tenglik sharti $\frac{xy}{1}=\frac{yz}{1}=\frac{xz}{1}$ bo‘lganda bajariladi. Bundan $x=y=z>0$ ekanligi kelib chiqadi.

Javob: $x=y=z>0$

71. 57-masaladagiga o‘xshash quyidagicha almashtirish bajaramiz:

$$a_1=\sqrt{p-a}, a_2=\sqrt{p-b}, a_3=\sqrt{p-c}, b_1=1, b_2=1, b_3=1$$

Koshi-Bunyakovskiy tengsizligini qo‘llab, quyidagiga ega bo‘lamiz:

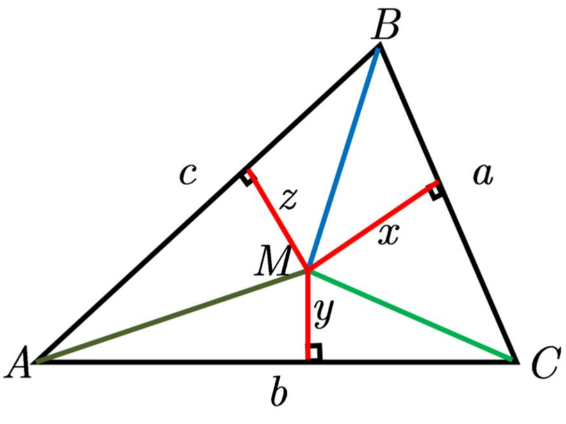
$$(\sqrt{p-a}\cdot 1+\sqrt{p-b}\cdot 1+\sqrt{p-c}\cdot 1)^2\le(p-a+p-b+p-c)(1+1+1)=3p,$$

$$\frac{\sqrt{p-a}+\sqrt{p-b}+\sqrt{p-c}}{\sqrt{p}}=\sqrt{3}$$

Tenglik sharti $\frac{\sqrt{p-a}}{1}=\frac{\sqrt{p-b}}{1}=\frac{\sqrt{p-c}}{1}$ da bajariladi. Bundan uchburchakning teng tomonli ekanligi kelib chiqadi U holda $a=b=c=\frac{2013}{3}=671$ tenglikka ega bo‘lamiz. Bundan muntazam uchburchkning yuzi $S=\frac{a^2\sqrt{3}}{4}=\frac{671^2\cdot\sqrt{3}}{4}=\frac{450241\sqrt{3}}{4}$ ga tengligini topamiz.

Javob: $\frac{450241\sqrt{3}}{4}$

72. Quyidagi chizmadan foydalanamiz:



$S_{AMB}=\frac{1}{2}cz$, $S_{AMC}=\frac{1}{2}by$, $S_{CMB}=\frac{1}{2}ax$ va $S_{ABC}=\frac{1}{2}ah_a=\frac{1}{2}bh_b=\frac{1}{2}ch_c$ ekani ma’lum. Bundan $\frac{x}{h_a}+\frac{y}{h_b}+\frac{z}{h_c}=\frac{S_{AMB}+S_{AMC}+S_{CMB}}{S_{ABC}}=1$ ekanligi kelib chiqadi.

U holda $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right)\left(\frac{x}{h_a}+\frac{y}{h_b}+\frac{z}{h_c}\right)$ ko‘paytmaga Koshi-Bunyakovskiy tengsizligini qo‘llaymiz:

$$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right)\left(\frac{x}{h_a}+\frac{y}{h_b}+\frac{z}{h_c}\right)\geq\left(\sqrt{\frac{a}{h_a}}+\sqrt{\frac{b}{h_b}}+\sqrt{\frac{c}{h_c}}\right)^2=$$

$$=\left(\frac{a}{\sqrt{2S}}+\frac{b}{\sqrt{2S}}+\frac{c}{\sqrt{2S}}\right)^2=\frac{(a+b+c)^2}{2S}=\frac{(a+b+c)^2}{2\cdot\frac{1}{2}(a+b+c)r}=\frac{a+b+c}{r}$$

Koshi-Bunyakovskiy tengsizligida tenglik sharti $\frac{\frac{a}{x}}{\frac{x}{h_a}}=\frac{\frac{b}{y}}{\frac{y}{h_b}}=\frac{\frac{c}{z}}{\frac{z}{h_c}}\Rightarrow\frac{ah_a}{x^2}=\frac{bh_b}{y^2}=\frac{ch_c}{z^2}\Rightarrow x=y=z=r$ bo‘lganda bajariladi. U holda M nuqta uchburchakning bissektrisalari kesishgan nuqtasi ekanini topamiz.

Javob: Uchburchakning bissektrisalari kesishgan nuqtada joylashganda

73. 57-masaladagiga o‘xshash ushbu $a_1=a$, $a_2=b$, $a_3=c$, $b_1=h_a$, $b_2=h_b$, $b_3=h_c$ almashtirishdan foydalanamiz. Uchburchakning yuzi uchun $S=\frac{1}{2}ah_a=\frac{1}{2}bh_b=\frac{1}{2}ch_c$ ekanidan va Koshi-Bunyakovskiy tengsizligiga ko‘ra quyidagilarga ega bo‘lamiz:

$$(ah_a+bh_b+ch_c)^2\leq(a^2+b^2+c^2)(h_a^2+h_b^2+h_c^2)$$

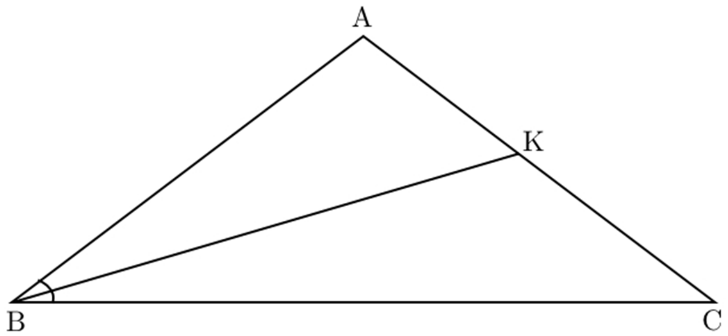
$$(2S+2S+2S)^2\leq(a^2+b^2+c^2)(h_a^2+h_b^2+h_c^2)$$

$$36S^2\leq(a^2+b^2+c^2)(h_a^2+h_b^2+h_c^2)$$

Oxirgi tengsizlikda tenglik sharti $\frac{a}{h_a}=\frac{b}{h_b}=\frac{c}{h_c}$ bo‘lganda bajariladi. Bundan uchburchakning teng tomonli ekanligi kelib chiqadi. U holda uning har bir ichki burchagi 60^0 ga teng bo‘ladi.

Javob: 60^0, 60^0, 60^0

74. Shartga ko‘ra $AB=AC$. Qulaylik uchun $\angle ABK=\angle KBC=\alpha$ deb olaylik.



U holda $\angle ACB = 2\alpha$, $\angle BKC = 180^0 - 3\alpha$ va $\angle BAC = 180^0 - 4\alpha$ tengliklar o‘rinli bo‘ladi. ABK va BKC uchburchaklarda sinuslar teoremasiga ko‘ra quyidagi tengliklar o‘rinli:

$$\begin{cases} \dfrac{AK}{\sin\alpha} = \dfrac{BK}{\sin(180^0 - 4\alpha)} \\ \dfrac{BC}{\sin(180^0 - 3\alpha)} = \dfrac{BK}{\sin 2\alpha} \end{cases} \Rightarrow \begin{cases} \dfrac{AK}{BK} = \dfrac{\sin\alpha}{\sin 4\alpha} \\ \dfrac{BC}{BK} = \dfrac{\sin 3\alpha}{\sin 2\alpha} \end{cases} \Rightarrow \begin{cases} \dfrac{AK}{BK} = \dfrac{\sin\alpha}{\sin 4\alpha} \\ \dfrac{AK + BK}{BK} = \dfrac{\sin 3\alpha}{\sin 2\alpha} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \dfrac{AK}{BK} = \dfrac{\sin\alpha}{\sin 4\alpha} \\ \dfrac{AK}{BK} + 1 = \dfrac{\sin 3\alpha}{\sin 2\alpha} \end{cases} \Rightarrow \begin{cases} \dfrac{AK}{BK} = \dfrac{\sin\alpha}{\sin 4\alpha} \\ \dfrac{AK}{BK} = \dfrac{\sin 3\alpha}{\sin 2\alpha} - 1 \end{cases} \Rightarrow \frac{\sin\alpha}{\sin 4\alpha} = \frac{\sin 3\alpha}{\sin 2\alpha} - 1 \Rightarrow$$

$$\Rightarrow \frac{\sin\alpha}{2\sin 2\alpha \cos 2\alpha} = \frac{\sin 3\alpha - \sin 2\alpha}{\sin 2\alpha} \Rightarrow \sin\alpha = 2(\sin 3\alpha - \sin 2\alpha)\cos 2\alpha \Rightarrow$$

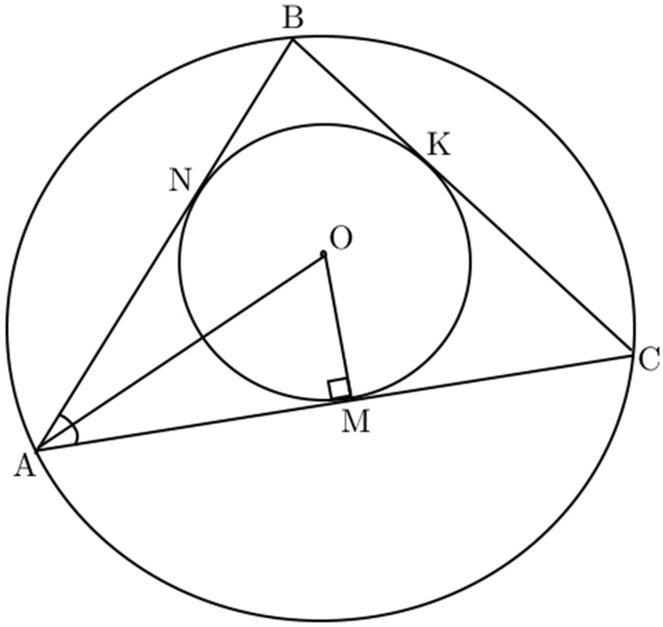
$$\Rightarrow 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} = 2\cos 2\alpha \cdot 2\sin\frac{\alpha}{2}\cos\frac{5\alpha}{2} \Rightarrow \cos\frac{\alpha}{2} = 2\cos 2\alpha \cos\frac{5\alpha}{2} \Rightarrow$$

$$\Rightarrow \cos\frac{\alpha}{2} = \cos\frac{9\alpha}{2} + \cos\frac{\alpha}{2} \Rightarrow \cos\frac{9\alpha}{2} = 0 \Rightarrow \frac{9\alpha}{2} = 90^0 \Rightarrow \alpha = 20^0$$

Bundan uchburchakning burchaklari 40^0, 40^0 va 100^0 ekanligi kelib chiqadi.

Javob: 40^0, 40^0, 100^0

75. Masala shartiga mos chizmani chizib olamiz.



Shartga ko‘ra $\angle BAC = \alpha$. Sinuslar teoremasining natijasidan $BC = 2R\sin\alpha$ ekani ma’lum. AOM uchburchakdan $tg\frac{\alpha}{2} = \frac{OM}{AM} \Rightarrow AM = \frac{r}{tg\frac{\alpha}{2}}$ tenglikka ega bo‘lamiz. Aylanaga o‘tkazilgan urinmalar tengligidan $AN = AM$, $BN = BK$ va $CK = CM$ tengliklar o‘rinli. U holda ABC uchburchakning perimetri quyidagiga teng bo‘ladi:

$$p = AB + BC + AC = AN + NB + BC + AM + MC =$$
$$= 2AM + BC + (NB + CM) =$$
$$= 2AM + BC + BC = 2(AM + BC) =$$
$$= 2\left(\frac{r}{tg\frac{\alpha}{2}} + 2R\sin\alpha\right) = 2(rctg\frac{\alpha}{2} + 2R\sin\alpha)$$

Bundan ABC uchburchakning yuzini topib olamiz:

$$S = \frac{1}{2}pr = \frac{1}{2}\cdot 2\left(rctg\frac{\alpha}{2} + 2R\sin\alpha\right)\cdot r = r^2ctg\frac{\alpha}{2} + 2Rr\sin\alpha$$

Javob: $r^2ctg\frac{\alpha}{2} + 2Rr\sin\alpha$

76. *1-usul:* Ravshanki . $p^3-1>p^3-p>q(q^6-1)>q^6-1>q^3-1$, demak, $p>q$. Shundan $p^3-q^7=p-q>0$ bo‘lgani bois $p>q^{\frac{7}{3}}$ tengsizlikka ega bo‘lamiz. Tenglamani quyidagicha yozamiz:

$$q(q^2-1)(q^2-q+1)(q^2+q+1)=p(p^2-1)$$

p sonining tubligidan hamda $p>q$ tengsizligidan quyidagi uchta hol vujudga kelishi mumkin:

$q^2-1\,\vdots\,p$ yoki $q^2-q+1\,\vdots\,p$ yoki $q^2+q+1\,\vdots\,p$

Barcha hollarda $q^2+q+1\geq p>q^{\frac{7}{3}}$.

Agar $q\geq 5$ bo‘lsa, quyidagilarni hosil qilamiz:

$$q>1,5^3,\ q^{\frac{7}{3}}>1,5q^2,\ q^2+q+1>1,5q^2,\ 0,5q^2-q-1<0$$

Oxirgi tengsizlikdan $q<1+\sqrt{3}$. Ziddiyat.

Demak, $q\leq 3$

1-hol: $q=2$ bo‘lsa $q^3-p=126$ tenglama hosil bo‘ladi.

Ravshanki, $p=2,3,5$ qiymatlar uni qanoatlantirmaydi.

x^3-x funksiya $x\geq\frac{1}{\sqrt{3}}$ uchun o‘suvchi, demak, $p\geq 7$ larda

$p^3-p\geq 7^3-7>126$

Demak, $q=2$ hol o‘rinli emas.

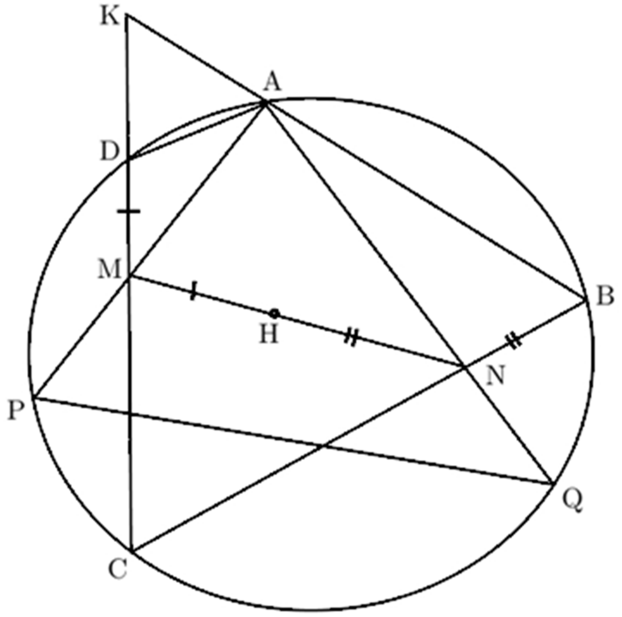
2-hol: $q=3$ bo‘lsa $p^3-p=2184$ tenglama hosil bo‘ladi va bu tenglamani $p=13$ qanoatlantiradi. Chunki $p>13$ uchun $p^3-p>2184$ bo‘ladi.

2-usul: Xuddi birinchi yechimdagidek $p>q^{\frac{7}{3}}>q^2$, bundan $p>q$, $p>q^2-q+1$. Demak, $q^2+q+1\,\vdots\,p$. Bundan $2p>2q^2>q^2+q+1$ bo‘lgani uchun $q^2+q+1=p$ tenglamani hosil qilamiz. Birinchi tenglamada $q(q-1)(q^2-q+1)=(q^2+q)(q^2+q+2)$ yoki $q^3-2q^2+2q-1=q^2+q+2$

Demak, $q^3-3q^2+q-3=(q^2+1)(q-3)=0$ va $q=3$. $q^2+q+1=p$ tenglikdan $p=13$ qiymatini hosil qilamiz.

Javob: $p=13,\ q=3$

77. MD kesmaning D nuqtadan davomida $DK = NB$ tenglikni qanoatlantiruvchi K nuqtani olaylik.



Demak $\angle KDA = 180^\circ - \angle ADC = \angle ABN$, $DA = AB$ va $DK = NB$, bundan $\Delta KDA = \Delta MBA$. Demak $KA = AN$ va $MK = MD + DK = MD + NB = MN$. Bundan $\Delta KDA = \Delta MBA$ va $\angle DMA = \angle NMA$ munsobatlarini hosil qilamiz. Xuddi shunday, $\angle MNA = \angle BNA$ tenglikni hosil qilish mumkin. MN kesmada $MH = MD$ tenglikni qanoatlantiradigan H nuqtani olamiz. Ravshanki, $NH = BN$. $\angle DMA = \angle HMA$ va $MD = MH$ bo‘lgani uchun D va H nuqtalar AP ga nisbatan simmetrik. Xuddi shunday B va H nuqtalar AN ga nisbatan simmetrik. Demak, $\angle DAB = 2\angle MAN$ va $\angle HBA = \angle DPA = \angle ABD = 90^0 - \frac{1}{2}\angle DAB = 90^0 - \angle MAN$. Bundan $PH \perp AQ$. Xuddi shunday $PH \perp AP$ isbotlanadi. Bundan APQ uchburchak balandliklari H nuqtada yotishi kelib chiqadi. Isbot tugadi.

78. 1-qadam: Doskada 97 ni hosil qilishimiz uchun, eng avvalo, doskadagi sonlar yig‘indisi 97 dan kichik bo‘lmasligi kerak. Demak, $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2} \geq 97$, ya’ni $n \geq 14$.

2-qadam: Endi doskadagi sonlar yig‘indisi toq ekanligini isbotlaymiz. Agar biz tanlagan 2 ta sonning yig‘indisi juft bo‘lsa, ular o‘rniga 2 ni yozamiz. Agar biz tanlagan 2 ta sonning yig‘indisi toq bo‘lsa, biz ular o‘rniga toq sonni yozamiz. Ya’ni biz juft son o‘rniga juft sonni toq son o‘rniga toq sonni yozyapmiz. Demak, biz doskada 97 hosil qilishimiz uchun doskadagi sonlar yig‘indisi toq bo‘lishi kerak. Aks holda oxirgi raqamda doskada 2 hosil bo‘ladi.

3-qadam: $n=14$ bo‘lmasligini isbotlaymiz. Faraz qilaylik $n=14$ bo‘lsin. Biz 97 hosil qilishimiz uchun qandaydir bitta toq son va juft sonlardan foydalanishimiz kerak. Ya’ni har bir qadamdan keyin tub son hosil qilish orqali maqsadimizga tez erishamiz. Lekin biz doskada eng ko‘pi bilan $13+2+4+6+8+10+12+14+2+2+2=75$ hosil qilishimiz mumkin xolos. Oxirgi ikkilarni 1, 3, 5, 7, 9, 11 lar hosil qilgan. Demak, $n=14$ bo‘lganida masala yechimga ega emas.

4-qadam: $n=15$ va $n=16$ da doskadagi sonlar yig‘indisi juft sondir. Demak bu holatlar ham 2-qadamga ko‘ra ziddiyatdir. Demak, $n \geq 17$.

5-qadam: $n=17$ da bo‘lmasligini isbotlaymiz. $n=17$ da 17 dan kichik bo‘lgan barcha juft natural sonlar yig‘indisi 72 ga teng. 17 dan kichik bo‘lgan toq sonlar jufti bilan 4 ta 2 ni hosil qiladi. $(1;3)\to 2$, $(5;7)\to 2$, $(9;11)\to 2$, $(13,15)\to 2$. Demak biz 17, 4, 6, 8, 10, 12, 14, 16, 2, 2, 2, 2, 2 sonlarini ketma-ket qo‘shish orqali 97 sonini hosil qilib bo‘lmasligini isbotlaymiz. Jarayon 17 da boshlanadi va har bir qadamda tub son hosil bo‘lishi kerak. 17 dan 97 gacha bo‘lgan tub sonlar orasida ayirmasi 2 ga teng tub sonlar 5 ta va quyidagilar:

$$(17;19),\ (29;31),\ (41;143),\ (59;61),\ (71;73)$$

Demak, biz yig‘indimizda shu sonlarni hosil qilamiz va 2 larni shu sonlar orasiga qo‘yamiz. Endi quyidagi sonlarni qaraymiz:

$$(19;29),\ (31;41),\ (61;71)$$

Bu sonlar orasidagi ayirma 10 ga teng. Demak, bizga yig‘indisi 10 ga teng 3 ta son kerak. Lekin bizda atigi 2 ta 10 bor. Demak $n=17$ da ham hosil qilib bo‘lmaydi.

6-qadam: $n=18$ bo‘lsin.

Dastlab $(3;5)$, $(7;9)$, $(11;13)$, $(15;17)$ juftliklardan 4 ta 2 ni hosil qilamiz. Bundan keyin quyidagicha ish qilamiz:

$$(1,2)\to 3;(3,2)\to 5;(5,2)\to 7;(7,4)\to 11;(11,2)\to 13;(13,6)\to 19;(19,10)\to$$
$$\to 29;(29,8)\to 37;(37,16)\to 53;(53,14)\to 67;(67,12)\to 79;(79,18)\to 97$$

Javob: $n=18$

79. *Javob: Mumkin*

Faraz qilaylik 4 ta jamoa qatnashsin(Sevilya, Barcelona, Real-Madrid, Atletiko). 4 ta jamoa qolgan jamoalar bilan bittadan o‘yin o‘tkazsin.

Sevilya:Barcelona-4:3

Real-Madrid:Barcelona-3:3

Atletiko:Barcelona-3:3

Sevilya:Real-Madrid-1:3

Real-Madrid:Atletiko-1:0

Atletiko:Sevilya-1:0

Natijada musobaqa jadvali quyidagicha bo‘ladi:

Jamoalar	Kiritilgan to‘plar soni	O‘tkazib yuborilgan to‘plar soni	Ochko
Real Madrid	7	4	7
Atletiko	4	4	4
Sevilya	5	7	3
Barcelona	9	10	2

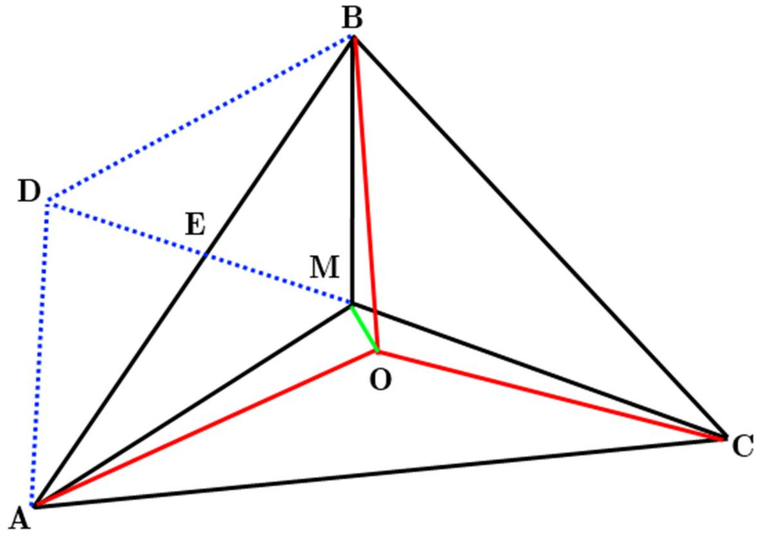
80. ***1-usul:*** Ushbu $\overrightarrow{OM}=\overrightarrow{OA}+\overrightarrow{AM}$, $\overrightarrow{OM}=\overrightarrow{OB}+\overrightarrow{BM}$, $\overrightarrow{OM}=\overrightarrow{OC}+\overrightarrow{CM}$ tengliklarni qo‘shsak, $3\cdot\overrightarrow{OM}=(\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC})+(\overrightarrow{AM}+\overrightarrow{BM}+\overrightarrow{CM})$ tenglik kelib chiqadi. Endi, uchburchak medianalari kesishish nuqtasida 2:1 nisbatda bo‘linishini inobatga olib, $\overrightarrow{AM}+\overrightarrow{BM}+\overrightarrow{CM}=0$ ekanligini ko‘rsatamiz:

$$\overrightarrow{AM}+\overrightarrow{BM}+\overrightarrow{CM}=\frac{2}{3}\overrightarrow{AA_1}+\frac{2}{3}\overrightarrow{BB_1}+\frac{2}{3}\overrightarrow{CC_1}=$$

$$=\frac{1}{3}(\overrightarrow{AB}+\overrightarrow{AC})+\frac{1}{3}(\overrightarrow{BA}+\overrightarrow{BC})+\frac{1}{3}(\overrightarrow{CA}+\overrightarrow{CB})=0$$

Shunga asosan $\overrightarrow{OM}=\frac{1}{3}(\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC})$ ekanligi kelib chiqadi.

2-usul: Masala shartiga mos chizmani chizamiz va $AMBD$ parallelogrammni yasaymiz(rasmga qarang).



Uning diagonallari E nuqtada kesishsin. $|CM|=2\cdot|ME|$, $|ME|=|ED|$ va $\overrightarrow{CM}\uparrow\uparrow\overrightarrow{MD}$ ekanidan $\overrightarrow{CM}=\overrightarrow{MD}$. Bundan tashqari $\overrightarrow{BM}=\overrightarrow{DA}$. U holda AMD uchburchakda $\overrightarrow{DA}+\overrightarrow{AM}=\overrightarrow{DM}=-\overrightarrow{MD}$ bundan $\overrightarrow{DA}+\overrightarrow{AM}+\overrightarrow{MD}=0$. Agar

$\overrightarrow{CM} = \overrightarrow{MD}$ va $\overrightarrow{BM} = \overrightarrow{DA}$ ekanini hisobga olsak, $\overrightarrow{BM} + \overrightarrow{AM} + \overrightarrow{CM} = 0$ tenglik o‘rinli. Bundan quyidagilarga ega bo‘lamiz:

$$\overrightarrow{BM} + \overrightarrow{AM} + \overrightarrow{CM} = 0$$

$$(\overrightarrow{BO} + \overrightarrow{OM}) + (\overrightarrow{AO} + \overrightarrow{OM}) + (\overrightarrow{CO} + \overrightarrow{OM}) = 0$$

$$3 \cdot \overrightarrow{OM} = -\overrightarrow{AO} - \overrightarrow{BO} - \overrightarrow{CO}$$

$$\overrightarrow{OM} = \frac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$$

Isbot tugadi.

81. Ushbu $2x + 2 = t - 1$ almashtirishdan $x = \dfrac{t-3}{2}$ ekanligi kelib chiqadi va berilgan ifodalar quyidagi ko‘rinishga keladi:

$$\begin{cases} f(t-1) + 2g(2t+1) = \dfrac{t-5}{2} \\ f(t-1) + g(2t+1) = 2t \end{cases}$$

Bu sistemani yechib va t ni x ga almashtirib, $f(x) = 3,5x + 6$ va $g(x) = -1,5x - 3,5$ ekanligini topish mumkin.

82. Sistemaning birinchi tenglamasida $4x + 3 = 2t + 1$ almashtirishdan $x = \dfrac{t-1}{2}$ ekanligi kelib chiqadi. Ikkinchi tenglamasida x ni t ga almashtirish natijasida quyidagi sistemaga ega bo‘lamiz:

$$\begin{cases} f(2t+1) + \left(\dfrac{t-1}{2}\right) g(3t+1) = 2 \\ f(2t+1) + g(3t+1) = t + 1 \end{cases}$$

Bu sistemani yechib va t ni x ga almashtirib, $f(x) = \dfrac{x^2 - 2x - 19}{2(x-7)}$ va $g(x) = \dfrac{6(4-x)}{x-10}$ ekanligini topish mumkin.

83. $f(x) + xf(\dfrac{x}{2x-1}) = 2$ tenglamada $\dfrac{x}{2x-1} = t$ deb almashtirish olsak, $x = \dfrac{t}{2t-1}$ ekanligi kelib chiqadi va x o‘rniga $\dfrac{t}{2t-1}$ ni qo‘ysak

$f(t)+\frac{t}{2t-1}f(\frac{t}{2t-1})=2$ ni hosil qilamiz. Agar t va x larni almashtirsak, quyidagi tenglamalar sistemasi kelib chiqadi:

$$\begin{cases} f(x)+\frac{x}{2x-1}f(\frac{x}{2x-1})=2 \\ f(x)+xf(\frac{x}{2x-1})=2 \end{cases}$$

bu tenglamalar sistemasidan $f(x)=\frac{4x-2}{x-1}$ ekanini topish mumkin.

Javob: $f(x)=\frac{4x-2}{x-1}$

84. Qizil ichimlikda p ta tomchi, oq ichimlikda q ta tomchi bo‘lsin deylik. Birinchi quyishdan so‘ng qizil ichimlikli idishda $p-1$ ta tomchi, ikkinchi idishda $q+1$ ta tomchi bo‘ladi. Keyin ikkinchi idishdan birinchisiga bir tomchi qaytarganimizdan keyin bir tomchining $\frac{1}{p+1}$ qismi qizil va $\frac{p}{p+1}$ qismi oq bo‘ladi.

Ikkinchi idishda esa, $1-\frac{1}{p+1}=\frac{p}{p+1}$ qizil tomchi qoladi. Demak, qaralayotgan qismlar teng ekan.

85. $\sqrt{3}=a$ deb belgilash kiritamiz:

$$x^3-(a+1)x^2+a^2=0$$

$$a^2-ax^2+x^3-x^2=0$$

a ga nisbatan kvadrat tenglamani yechamiz:

$$a^2-ax^2+x^3-x^2=(a-x)(a-x^2+x)=0$$

$$x^3-(\sqrt{3}+1)x^2+3=(x-\sqrt{3})(x^2-x-\sqrt{3})=0$$

$x_1=\sqrt{3}$, $x_2=\frac{1+\sqrt{1+4\sqrt{3}}}{2}$, $x_3=\frac{1-\sqrt{1+4\sqrt{3}}}{2}$

86. Tenglamani quyidagicha yozib olamiz:

$$\left(x+\frac{1}{2}\right)^2+4=\frac{3}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}$$

Bundan ko‘rinadiki, $\left(x+\frac{1}{2}\right)^2+4\geq 0+4=4$ va $\frac{3}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}\leq\frac{3}{0+\frac{3}{4}}=4$

ekanidan masala quyidagi sistemani yechishga keladi:

$$\begin{cases}\left(x+\frac{1}{2}\right)^2+4=4\\ \left(x-\frac{1}{2}\right)^2+\frac{3}{4}=\frac{3}{4}\end{cases}\Rightarrow\begin{cases}x=-\frac{1}{2}\\ x=\frac{1}{2}\end{cases}\Rightarrow x\in\varnothing$$

Javob: $\varnothing$.

87. Qavslarni ochamiz va Viet teoremasidan foydalanamiz:

$$(r+s)^3+(s+t)^3+(t+r)^3=2(r^3+s^3+t^3)+$$

$$+3(r^2s+r^2t+s^2r+s^2t+t^2r+t^2r)=$$

$$=(r+s+t)^3+r^3+s^3+t^3-6rst=\left|\begin{matrix}r+s+t=0, r+s=-t\\ rst=-\frac{2008}{8}=-251\\ rs+st+rt=\frac{1001}{8}\end{matrix}\right|=$$

$$=(r+s)(r^2-rs+s^2)+t^3-6rst=-t((r+s)^2-3rs)+t^3-6rst=$$

$$=-t^3+3rst+t^3-6rst=-3rst=753$$

Javob: 753

88. Agar $P(x)=x^3-x-1$ ko‘phadning ildizlari a,b,c bo‘lsa, u holda $P(x-1)=(x-1)^3-(x-1)-1=x^3-3x^2+2x-1$ ko‘phadning ildizlari $a+1, b+1, c+1$ ga teng bo‘ladi. Viet teoremasiga ko‘ra $(1+a)(1+b)+(1+a)(1+c)+(1+b)(1+c)=2$ va $(1+a)(1+b)(1+c)=1$ tengliklar o‘rinli. Quyidagicha shakl almashtiramiz:

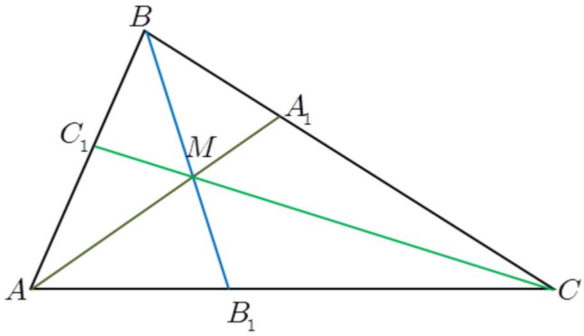
$$\frac{1-a}{1+a}+\frac{1-b}{1+b}+\frac{1-c}{1+c}=\frac{2-(1+a)}{1+a}+\frac{2-(1+b)}{1+b}+\frac{2-(1+c)}{1+c}=$$

$$=2\left(\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}\right)-3=$$

$$=2\left(\frac{(1+a)(1+b)+(1+a)(1+c)+(1+b)(1+c)}{(1+a)(1+b)(1+c)}\right)-3=2\cdot\frac{2}{1}-3=1$$

Javob: 1

89. Umumiylikka zarar yetkazmasdan tengsizlikning har ikkala tomoniga 3 ni qo‘shamiz:



$\frac{AM}{A_1M}+1+\frac{BM}{B_1M}+1+\frac{CM}{C_1M}+1\ge 6+3 \Rightarrow \frac{AM}{A_1M}+\frac{BM}{B_1M}+\frac{CM}{C_1M}\ge 9$ ni isbotlash kifoya.

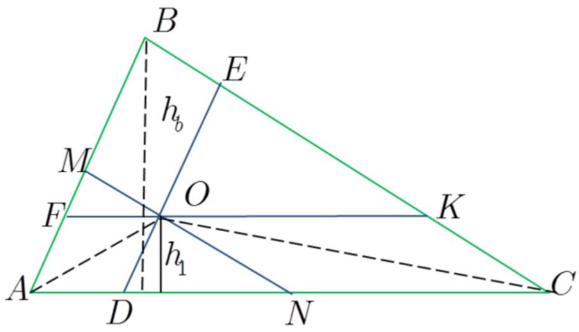
$S_{BMC}=\frac{1}{2}\cdot A_1M\cdot BC\cdot\sin\angle MA_1B,\quad S_{ABC}=\frac{1}{2}\cdot AA_1\cdot BC\cdot\sin\angle MA_1B$ bundan $\frac{AA_1}{A_1M}=\frac{S_{ABC}}{S_{AMC}}$ tenglikka ega bo‘lamiz. Xuddi shunga o‘xshash $\frac{BB_1}{B_1M}=\frac{S_{ABC}}{S_{BMC}}$, $\frac{CC_1}{C_1M}=\frac{S_{ABC}}{S_{AMB}}$ tengliklarni hosil qilishimiz mumkin.

$$\frac{AM}{A_1M}+\frac{BM}{B_1M}+\frac{CM}{C_1M}=S_{ABC}\cdot\left(\frac{1}{S_{AMC}}+\frac{1}{S_{BMC}}+\frac{1}{S_{AMB}}\right)=$$

$$=\left(S_{AMC}+S_{BMC}+S_{AMB}\right)\left(\frac{1}{S_{AMC}}+\frac{1}{S_{BMC}}+\frac{1}{S_{AMB}}\right)\ge 9$$

oxirgi tengsizlik har bir qavs ichiga Koshi tengsizligining $n=3$ holini qo‘llash orqali hosil qilinadi. Tenglik sharti M nuqta medianalar kesishgan nuqtada bo‘lganda bajariladi. Isbot tugadi.

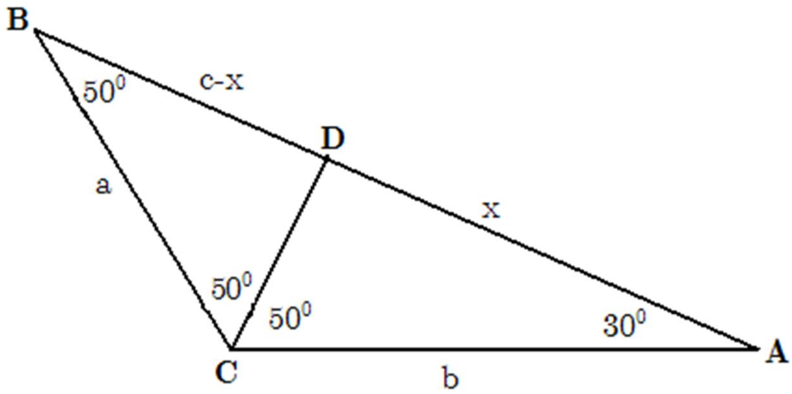
90. Masala shartiga mos chizma chizib olamiz:



$$\frac{S_{AOC}}{S_{ABC}}=\frac{\frac{AC}{2}\cdot h_1}{\frac{AC}{2}\cdot h_b}=\frac{h_1}{h_b}=\frac{AF}{AB}$$ chunki, $\sin A=\frac{h_b}{AB}=\frac{h_1}{OD}=\frac{h_1}{AF}$. Xuddi shunga o‘xshash $\frac{S_{BOC}}{S_{ABC}}=\frac{CN}{AC}$ va $\frac{S_{AOB}}{S_{ABC}}=\frac{BE}{BC}$ tengliklarni hosil qilamiz. Bundan isbotlanishi kerak bo‘lgan tenglik kelib chiqadi:

$$\frac{AF}{AB}+\frac{BE}{BC}+\frac{CN}{AC}=\frac{S_{AOC}+S_{AOB}+S_{BOC}}{S_{ABC}}=\frac{S_{ABC}}{S_{ABC}}=1$$

91. ABC uchburchakning CD bissektrisasini o‘tkazamiz, natijada ΔABC ga o‘xshash bo‘lgan ΔACD hosil bo‘ladi.



Birinchidan, bissektrisa xossasiga asosan:

$$\frac{a}{c-x}=\frac{b}{x}\Rightarrow x=\frac{bc}{a+b}$$

Ikkinchidan, ΔABC va ΔACD uchburchaklarning o‘xshashligidan:

$$\frac{AC}{AD}=\frac{AB}{AC}\quad\Rightarrow\quad\frac{b}{x}=\frac{c}{b}\quad\Rightarrow\quad b^2=cx\quad\Rightarrow\quad b^2=\frac{bc^2}{a+b}\quad\Rightarrow$$

$$\Rightarrow\ ab+b^2=c^2\quad\Rightarrow\quad a=\frac{c^2-b^2}{b}$$

Javob: $a = \dfrac{c^2 - b^2}{b}$

92. Berilgan tenglamaning ikkala tomonini z^n ga bo‘lib yuboramiz:

$$\left(\frac{x}{z}\right)^n + \left(\frac{y}{z}\right)^n = z$$

Agar $x = a \cdot z$ va $y = b \cdot z$ desak($a, b \in \mathbb{N}$), $z = a^n + b^n$ ekanini topamiz. Bundan $x = a(a^n + b^n)$ va $y = b(a^n + b^n)$ yechimlarga bo‘lamiz. Berilgan tenglama natural sonlarda cheksiz ko‘p yechimga ega ekanligi kelib chiqadi:

Javob: $x = a(a^n + b^n)$, $y = b(a^n + b^n)$, $z = a^n + b^n$, bu yerda $a, b, n \in \mathbb{N}$

93. *a)* $k^2 < k^2 + k < k^2 + 2k + 1 = (k+1)^2$ bundan ko‘rinadiki, $k^2 + k$ ifoda ikkita ketma-ket kelgan natural sonning kvadratlari orasida joylashgan. Ma’lumki, bu oraliqda biror natural sonning kvadrati bo‘la oladigan natural son yo‘q.

b) $k^2 + k = k(k+1)$ ekanini hisobga olsak, ikkita ketma-ket kelgan butun sonlar ko‘paytmasi faqat $k = -1$ va $k = 0$ bo‘lgandagina biror sonning kvadrati bo‘la olishini topamiz.

94. $x = [x] + \{x\}$ va $\left[\{x\}^2\right] = 0$ ekanligidan foydalanamiz:

$$[x^2] - [x]^2 = \left[\left([x] + \{x\}\right)^2\right] - [x]^2 = \left[[x]^2 + 2 \cdot \underbrace{[x] \cdot \{x\}}_{100} + \{x\}^2\right] - [x]^2 =$$

$$= \left[[x]^2 + 200 + \{x\}^2\right] - [x]^2 = [x]^2 + 200 - [x]^2 = 200$$

Javob: 200

95. $[a] = a - \{a\}$ va 2 ning toq darajalarini 3 ga bo‘lganda 2 qoldiq, 2 ning juft darajalarini 3 ga bo‘lganda 1 qoldiq qolishidan foydalanib, quyidagilarni yoza olamiz:

$$\left[\frac{1}{3}\right] + \left[\frac{2}{3}\right] + \left[\frac{2^2}{3}\right] + \ldots + \left[\frac{2^{1000}}{3}\right] =$$

$$= \frac{1}{3} - \left\{\frac{1}{3}\right\} + \frac{2}{3} - \left\{\frac{2}{3}\right\} + \frac{2^2}{3} - \left\{\frac{2^2}{3}\right\} + \ldots + \frac{2^{1000}}{3} - \left\{\frac{2^{1000}}{3}\right\} =$$

$$= \frac{1+2+2^2+\ldots+2^{1000}}{3} - \left(\left\{\frac{1}{3}\right\} + \left\{\frac{2}{3}\right\} + \left\{\frac{2^2}{3}\right\} + \ldots + \left\{\frac{2^{1000}}{3}\right\} \right) =$$

$$= \frac{\frac{1\cdot(2^{1001}-1)}{2-1}}{3} - - \left(\underbrace{\left(\frac{1}{3}+\frac{2}{3}\right) + \left(\frac{1}{3}+\frac{2}{3}\right) + \ldots + \left(\frac{1}{3}+\frac{2}{3}\right)}_{500\,ta} + \frac{1}{3} \right) =$$

$$= \frac{2^{1000}-1}{3} - 500 - \frac{1}{3} = \frac{2^{1000}-2}{3} - 500$$

Javob: $\frac{2^{1000}-2}{3} - 500$

96. $a + x = u$ va $a - x = v$ deb belgilab olamiz. U holda $\sqrt[3]{u} - \sqrt[3]{v} = \sqrt[6]{u^2v^2}$ tenglik o‘rinli. Oxirgi tenglikning ikkala tomonini kubga oshiramiz:

$$u - v - 3\sqrt[3]{uv}(\sqrt[3]{u} - \sqrt[3]{v}) = \sqrt{uv} \;\Rightarrow\; u - v - 3\sqrt[3]{uv}\cdot\sqrt[6]{uv} = \sqrt{uv}$$

bundan $u - v = 4\sqrt{uv}$ yoki $\begin{cases} u - v \geq 0 \\ (u-v)^2 = 16uv \end{cases}$ ekani kelib chiqadi. Agar belgilashlarga qaytsak:

$$\begin{cases} 2x \geq 0 \\ 4x^2 = 16(a^2 - x^2) \end{cases} \Rightarrow \begin{cases} x \geq 0 \\ x = \pm\frac{2a}{\sqrt{5}} \end{cases} \Rightarrow \begin{cases} x = \frac{2a}{\sqrt{5}}, a \geq 0 \\ x = -\frac{2a}{\sqrt{5}}, a \leq 0 \end{cases}$$

ekanligini topishimiz mumkin.

Javob: $a \geq 0$ da $x = \frac{2a}{\sqrt{5}}$, $a \leq 0$ da $x = -\frac{2a}{\sqrt{5}}$

97. Quyidagicha belgilash kiritamiz:

$$A_n = a_1 + a_2 + \ldots + a_n - n\sqrt[n]{a_1a_2\ldots a_n}$$

Bundan quyidagini topamiz:

$$A_{n+1} - A_n = a_{n+1} + n\sqrt[n]{a_1a_2\ldots a_n} - (n+1)\sqrt[n+1]{a_1a_2\ldots a_na_{n+1}} =$$

$$= a_{n+1} + n\sqrt[n]{a_1a_2\ldots a_n} - (n+1)(\sqrt[n]{a_1a_2\ldots a_na_n})^{\frac{n}{n+1}} \cdot (a_{n+1})^{\frac{1}{n+1}}$$

Agar $x = (a_{n+1})^{\frac{1}{n+1}}$ va $y = (\sqrt[n]{a_1a_2\ldots a_na_n})^{\frac{1}{n+1}}$ desak:

$$A_{n+1} - A_n = x^{n+1} + ny^{n+1} - (n+1)xy^n = y^{n+1}\left(\left(\frac{x}{y}\right)^{n+1} - (n+1)\frac{x}{y} + n\right)$$

Ushbu $\frac{x}{y} = 1 + z$ belgilashdan esa:

$$A_{n+1} - A_n = y^{n+1}((1+z)^{n+1} - 1 - (n+1)z)$$

tenglikka ega bo‘lamiz.

Bernulli tengsizligiga ko‘ra, $(1+z)^{\alpha} > 1 + \alpha z$ ekanidan:

$$A_{n+1} - A_n = y^{n+1}((1+z)^{n+1} - 1 - (n+1)z) > y^{n+1}(1 + (n+1)z - 1 - (n+1)z) = 0$$

$$A_{n+1} > A_n$$

Xuddi shunga o‘xshash, $A_{n+1} > A_n > A_{n-1} > \ldots > A_2 > A_1$ va $A_2 = (\sqrt{a_1} - \sqrt{a_2})^2$ ekanini e’tiborga olsak, $A_n \geq (\sqrt{a_1} - \sqrt{a_2})^2$, ya’ni berilgan tengsizlikning isboti kelib chiqadi. Tenglik ishorasi faqat $a_1 = a_2 = \ldots = a_n \geq 0$ da bajariladi.

98. Bu yig‘indini S deb belgilab olamiz va tenglikning har ikkala tomonini x ga ko‘paytirib, quyidagilarga ega bo‘lamiz:

$$\begin{cases} S = 1 + 2x + 3x^2 + 4x^3 + \ldots \\ Sx = x + 2x^2 + 3x^3 + 4x^4 + \ldots \end{cases}$$

1-ifodadan 2-ifodani ayiramiz:

$$S(1-x) = 1 + x + x^2 + x^3 + x^4 + \ldots \quad (|x| < 1)$$

$$S(1-x) = \frac{1}{1-x}$$

$$S = \frac{1}{(1-x)^2}$$

ekani kelib chiqadi.

Javob: $\frac{1}{(1-x)^2}$

99. O‘rta qiymatlar haqidagi teorema va Pifagor teoremasiga ko‘ra $ab \leq \frac{a^2+b^2}{2} = \frac{c^2}{2}$ va $a + b = \sqrt{a^2 + b^2 + 2ab} \leq \sqrt{c^2 + 2\cdot\frac{c^2}{2}} = c\sqrt{2}$ tengsizliklarni yoza olamiz. Shularga asosan quyidagiga ega bo‘lamiz:

$$ab(a+b+c) \le \frac{c^2}{2}\cdot(c\sqrt{2}+c) = \frac{\sqrt{2}+1}{2}\cdot c^3 = \frac{2\sqrt{2}+2}{4}\cdot c^3 < \frac{3+2}{4}\cdot c^3 = \frac{5}{4}\cdot c^3$$

Bundan $ab(a+b+c) < \frac{5}{4}c^3$ tengsizlikning o‘rinli ekanligi kelib chiqadi. Isbot tugadi.

100. $x = k + \alpha$ bo‘lsin. Bunda $k = [x]$ va $\alpha = \{x\}$. U holda tenglama quyidagi ko‘rinishga keladi:

$$[k+\alpha] + [2k+2\alpha] + [3k+3\alpha] = 6$$
$$k + 2k + [2\alpha] + 3k + [3\alpha] = 6$$
$$6k + [2\alpha] + [3\alpha] = 6$$

Quyidagi hollarni qaraymiz:

1-hol: $0 \le \alpha < \frac{1}{3} \Rightarrow 0 \le 3\alpha < 1$ va $0 \le 2\alpha < \frac{2}{3}$

Demak, $[2\alpha] = [3\alpha] = 0$, $6k = 6 \Rightarrow k = 1$, $x = k + \alpha \Rightarrow x = 1 + \alpha \Rightarrow x \in [1; \frac{4}{3})$

2-hol: $\frac{1}{3} \le \alpha < \frac{1}{2} \Rightarrow 1 \le 3\alpha < \frac{3}{2}$ va $\frac{2}{3} < 2\alpha < 1$

Demak, $[3\alpha] = 1$, $[2\alpha] = 0$, $6k + 1 = 6 \Rightarrow k = \frac{5}{6} \notin \mathbb{Z}$, Bu holda yechim yo‘q.

3-hol: $\frac{1}{2} \le \alpha < \frac{2}{3} \Rightarrow \frac{3}{2} \le 3\alpha < 2$ va $1 \le 2\alpha < \frac{4}{3}$

Demak, $[3\alpha] = 1$, $[2\alpha] = 1$, $6k + 1 + 1 = 6 \Rightarrow k = \frac{4}{6} \notin \mathbb{Z}$. Bu holda ham yechim yo‘q.

4-hol: $\frac{2}{3} \le \alpha < 1 \Rightarrow 2 \le 3\alpha < 3$ va $\frac{4}{3} \le 2\alpha < 2$

Demak, $[3\alpha] = 2$, $[2\alpha] = 1$, $6k + 2 + 1 = 6 \Rightarrow k = \frac{1}{2} \notin \mathbb{Z}$. Bu holda ham yechim yo‘q.

Javob: $x \in [1; \frac{4}{3})$

101. Masala shartiga ko‘ra $a \le x_i \le b$, $(i = 1, 2, \ldots, n)$. U holda $0 \ge (x_i - a)(x_i - b)$ yoki $x_i(a+b) \ge x_i^2 + ab$ tengsizlik o‘rinli. Bu tengsizlikni har bir x_i lar uchun yozib chiqamiz:

$$+\begin{cases} x_1(a+b) \ge x_1^2 + ab \\ x_2(a+b) \ge x_2^2 + ab \\ \dots \\ x_n(a+b) \ge x_n^2 + ab \end{cases} \Rightarrow \begin{cases} a+b \ge x_1 + \dfrac{ab}{x_1} \\ a+b \ge x_2 + \dfrac{ab}{x_2} \\ \dots \\ a+b \ge x_n + \dfrac{ab}{x_n} \end{cases}$$

Oxirgi tengsizliklarni hadma-had qo‘shib, o‘rta qiymatlar haqidagi teoremani qo‘llasak, quyidagi ifodaga ega bo‘lamiz:

$$n(a+b) \ge (x_1 + x_2 + \dots + x_n) + ab\left(\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}\right) \ge$$

$$\ge 2\sqrt{ab(x_1 + x_2 + \dots + x_n)\left(\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}\right)}$$

Ikkala tomonini kvadratga oshirib, isbotlanishi kerak bo‘lgan tengsizlikni hosil qilamiz:

$$(x_1 + x_2 + \dots + x_n)\left(\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}\right) \le \frac{n^2(a+b)^2}{4ab}$$

Tenglik sharti $x_1 = x_2 = \dots = x_n = a = b$ bo‘lganda bajariladi.

102. Agar $x + y + z = 0$ bo‘lsa, $x^3 + y^3 + z^3 = 3xyz$ bo‘lishini isbotlash qiyin emas. Shunga asosan $(\log_a b)^3 + (\log_b c)^3 + (\log_c a)^3 = 3 \cdot \log_a b \cdot \log_b c \cdot \log_c a = 3$ ekanini oson topish mumkin.

Javob: 3

103. Ma’lumki, $(a + b - c)^2 \ge 0$ tengsizlik o‘rinli. Shunga asosan quyidagilarga ega bo‘lamiz:

$$(a+b-c)^2 \ge 0$$

$$a^2 + b^2 + c^2 + 2(ab - bc - ac) \ge 0$$

$$\frac{5}{3} + 2(ab - bc - ac) \ge 0 \Rightarrow bc + ac - ab \le \frac{5}{6} < 1$$

Agar $abc > 0$ ekanini hisobga olsak, isbotlanishi kerak bo‘lgan tengsizlik kelib chiqadi:

$$bc + ac - ab < 1 \Rightarrow \frac{bc + ac - ab}{abc} < \frac{1}{abc} \Rightarrow \frac{1}{a} + \frac{1}{b} - \frac{1}{c} < \frac{1}{abc}$$

104. $a^2+b^2=(a-b)^2+2ab$ tenglikni quyidagi ifodaga qo‘llaymiz:

$$\sqrt{1+\frac{1}{n^2}+\frac{1}{(n+1)^2}}=\sqrt{1+\left(\frac{1}{n}-\frac{1}{n+1}\right)^2+2\cdot\frac{1}{n}\cdot\frac{1}{n+1}}=$$

$$=\sqrt{1+\frac{2}{n(n+1)}+\left(\frac{1}{n(n+1)}\right)^2}=\sqrt{\left(1+\frac{1}{n(n+1)}\right)^2}=1+\frac{1}{n}-\frac{1}{n+1}$$

Shunga asosan va ildizlar soni 2018 ta ekanidan quyidagi natijaga ega bo‘lamiz:

$$\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\sqrt{1+\frac{1}{3^2}+\frac{1}{4^2}}+\ldots+\sqrt{1+\frac{1}{2019^2}+\frac{1}{2020^2}}=$$

$$=1+\frac{1}{2}-\frac{1}{3}+1+\frac{1}{3}-\frac{1}{4}+\ldots+1+\frac{1}{2019}-\frac{1}{2020}=$$

$$=1\cdot 2018+\frac{1}{2}-\frac{1}{2020}=2018\frac{2019}{2020}$$

Javob: $2018\frac{2019}{2020}$

105. O‘rta qiymatlar haqidagi teoremaga ko‘ra $\sqrt{ab}\le\frac{a+b}{2}$ ekanini bilamiz(bu yerda $a,b\ge 0$). Shunga asosan quyidagilarni yoza olamiz:

$$\begin{cases}\sqrt{1\cdot 2}<\frac{1+2}{2}\\ \sqrt{3\cdot 4}<\frac{3+4}{2}\\ \ldots\\ \sqrt{2019\cdot 2020}<\frac{2019+2020}{2}\end{cases}$$

Yuqoridagi tengsizliklarni hadma-had qo‘shamiz:

$$\sqrt{1\cdot 2}+\sqrt{3\cdot 4}+\ldots+\sqrt{2019\cdot 2020}<\frac{1+2+3+\ldots+2019+2020}{2}$$

$$\sqrt{1\cdot 2}+\sqrt{3\cdot 4}+\ldots+\sqrt{2019\cdot 2020}<2021\cdot 505$$

106. Quyidagi tenglikni yig‘indining har bir hadi uchun qo‘llaymiz:

$$\frac{k\cdot(k+2)!}{3^k}=\frac{(k+3-3)\cdot(k+2)!}{3^k}=\frac{(k+3)!-3(k+2)!}{3^k}=\frac{(k+3)!}{3^k}-\frac{(k+2)!}{3^{k-1}}$$

$$\frac{1\cdot 3!}{3}+\frac{2\cdot 4!}{3^2}+\ldots+\frac{n\cdot(n+2)!}{3^n}=\frac{4!}{3^1}-\frac{3!}{3^0}+\frac{5!}{3^2}-\frac{4!}{3^1}+\ldots+$$

$$+\frac{(n+2)!}{3^{n-1}}-\frac{(n+1)!}{3^{n-2}}+\frac{(n+3)!}{3^n}-\frac{(n+2)!}{3^{n-1}}=\frac{(n+3)!}{3^n}-\frac{3!}{3^0}=\frac{(n+3)!}{3^n}-6$$

Javob: $\frac{(n+3)!}{3^n}-6$

107. $4!=1\cdot 2\cdot 3\cdot 4$ ekanidan $n\geq 4$ shartni qanoatlantiruvchi har qanday natural n soni uchun $n!\vdots 12$ ekanligini topish qiyin emas. U holda $1!+2!+3!=1+2+6=9$ ekanidan $A=12M+9$ tenglikni yoza olamiz. Bundan yig'indini 12 ga bo'lgandagi qoldiq 9 ekanligi kelib chiqadi.

Javob: 9

108. Istalgan $k\in\mathbb{N}$ soni uchun quyidagi ifodani qaraymiz:

$$\frac{k}{(k-2)!+(k-1)!+k!}=\frac{k}{(k-2)!\cdot(1+k-1+(k-1)k)}=\frac{k}{(k-2)!\cdot k^2}=$$

$$=\frac{1}{(k-2)!\cdot k}=\frac{k-1}{(k-2)!\cdot(k-1)\cdot k}=\frac{k-1}{k!}=\frac{k}{k!}-\frac{1}{k!}=\frac{1}{(k-1)!}-\frac{1}{k!}$$

Yuqoridagi tenglikni yig'indining har bir hadi uchun yozib chiqamiz:

$$\frac{3}{1!+2!+3!}+\frac{4}{2!+3!+4!}+\ldots+\frac{2020}{2018!+2019!+2020!}=$$

$$=\frac{1}{2!}-\frac{1}{3!}+\frac{1}{3!}-\frac{1}{4!}+\ldots+\frac{1}{2019!}-\frac{1}{2020!}=\frac{1}{2}-\frac{1}{2020!}$$

Javob: $\frac{1}{2}-\frac{1}{2020!}$

109. Berilgan yig'indini S orqali belgilab, tenglikning ikkala tomonini 2020 ga ko'paytiramiz va hosil bo'lgan tengliklarni hadma-had ayiramiz:

$$-\begin{cases} S=2\cdot 2020+3\cdot 2020^2+4\cdot 2020^3+\ldots+2019\cdot 2020^{2018}+2020^{2020} \\ 2020S=2\cdot 2020^2+3\cdot 2020^3+4\cdot 2020^4+\ldots+2019\cdot 2020^{2019}+2020^{2021} \end{cases}$$

Natijada quyidagi ifodadan S ni oson topib olamiz:

$$-2019S=2\cdot 2020+2020^2+2020^3+\ldots+2020^{2019}-2020^{2021}$$

$$-2019S=2020+(2020+2020^2+2020^3+\ldots+2020^{2019})-2020^{2021}$$

$$-2019S=2020+\frac{2020\cdot(2020^{2019}-1)}{2020-1}-2020^{2021}$$

$$-2019S = 2020 + \frac{2020^{2020} - 2020}{2019} - 2020^{2021}$$

$$S = \frac{2020^{2021} - 2020}{2019} - \frac{2020^{2020} - 2020}{2019^2}$$

$$S = \frac{2020^{2020}(2019 \cdot 2020 - 1) - 2020 \cdot 2018}{2019^2}$$

Javob: $\frac{2020^{2020}(2019 \cdot 2020 - 1) - 2020 \cdot 2018}{2019^2}$

110. Ushbu $n! \cdot (n+2) = n! \cdot (n+1+1) = (n+1)! + n!$ tenglikdan foydalanamiz.

$$1! \cdot 3 - 2! \cdot 4 + 3! \cdot 5 - 4! \cdot 6 + \ldots + 2017! \cdot 2019 - 2018! \cdot 2020 + 2019! =$$
$$= 2! + 1! - 3! - 2! + 4! + 3! - 5! - 4! + \ldots + 2018! + 2017! - 2019! - 2018! + 2019! = 1! = 1$$

Javob: 1

111. Birinchidan, tenglamalarni bir-biridan ayirib, $(x-y)^3 = 1 \Rightarrow y - x = -1$ ekanini topamiz. Bu o‘z navbatida $y_1 - x_1 = -1$, $y_2 - x_2 = -1$, $y_3 - x_3 = -1$ ekanligini bildiradi.

Ikkinchidan, $y^3 + 3x^2y = 2020$ kubik tenglamada Viet teoremasiga ko‘ra $y_1y_2y_3 = 2020$ ekanini topishmiz mumkin. U holda quyidagiga ega bo‘lamiz:

$$\left(1 - \frac{x_1}{y_1}\right)\left(1 - \frac{x_2}{y_2}\right)\left(1 - \frac{x_3}{y_3}\right) = \frac{(y_1 - x_1)(y_2 - x_2)(y_3 - x_3)}{y_1y_2y_3} = \frac{-1}{2020}$$

Javob: $-\frac{1}{2020}$

112. Viet teoremasiga ko‘ra $p + q + r = -a$, $pq + qr + pr = b$ va $pqr = -c$ munosabatlar o‘rinli.

$$(pq)^2 + (qr)^2 + (pr)^2 = (pq + qr + pr)^2 - 2(p^2qr + pq^2r + pqr^2) =$$
$$= (pq + qr + pr)^2 - 2pqr(p + q + r) = b^2 - 2ac$$

Javob: $b^2 - 2ac$

113. Oldin $n = 1$ bo‘lgandagi $f(x) = 0$ tenglamani yechib olamiz.

$$f(x) = 0$$

$$x^2 + 12x + 30 = 0$$

$$(x+6)^2 - 6 = 0$$

$$x = \pm\sqrt{6} - 6$$

Endi $n = 2$ holdagi $f(f(x)) = 0$ tenglamani yechaylik.

$$f(f(x)) = 0$$

$$f^2(x) + 12f(x) + 30 = 0$$

$$f(x) = \pm\sqrt{6} - 6$$

$$(x+6)^2 - 6 = \pm\sqrt{6} - 6$$

$$x = \pm\sqrt[4]{6} - 6$$

Matematik induksiya metodi orqali ushbu $\underbrace{f(f(...f(x)...))}_{n\,ta} = 0$ tenglamaning

yechimi $x = \pm\sqrt[2^n]{6} - 6$ ekanini oson isbotlash mumkin.

Javob: $x = \pm\sqrt[2^n]{6} - 6$

114. O‘rta arifmetik va o‘rta geometrik qiymatlar haqidagi Koshi tengsizligiga ko‘ra $\sqrt{ab} \le \frac{a+b}{2}$ ekanligi ma’lum($a, b \ge 0$). Shunga asosan quyidagini yoza olamiz:

$$\begin{cases} \sqrt{(p-a)(p-b)} \le \dfrac{p-a+p-b}{2} \\ \sqrt{(p-a)(p-c)} \le \dfrac{p-a+p-c}{2} \\ \sqrt{(p-b)(p-c)} \le \dfrac{p-b+p-c}{2} \end{cases}$$

Hosil bo‘lgan tengsizliklarni qo‘shsak va $a + b + c = 2p$ ekanini hisobga olsak, isbotlanishi kerak bo‘lgan tengsizlikka ega bo‘lamiz:

$$\sqrt{(p-a)(p-b)} + \sqrt{(p-a)(p-c)} + \sqrt{(p-b)(p-c)} \le$$

$$\le \frac{6p - 2(a+b+c)}{2} = \frac{6p - 4p}{2} = p$$

Tenglik sharti muntazam uchburchakda bajariladi.

115. $m * 2 = \frac{m+2}{2m+4} = \frac{1}{2}$ ekanidan $\left(...\left(2020 * 2019\right) * 2018 * ...\right) * 2 = \frac{1}{2}$

ekanligini topib olamiz. U holda biz $\left(\frac{1}{2} * 1\right) * 0$ ni hisoblashimiz kifoya.

$$\left(\frac{1}{2}*1\right)*0=\left(\frac{\frac{1}{2}+1}{\frac{1}{2}\cdot 1+4}\right)*0=\frac{1}{3}*0=\frac{\frac{1}{3}+0}{\frac{1}{3}\cdot 0+4}=\frac{1}{12}$$

Javob: $\frac{1}{12}$

116. Ushbu $\frac{a*(a+b)}{a*b}=\frac{a^2+b^2}{ab}$ shartdan $a*(a+b)=\frac{a^2+b^2}{ab}\cdot(a*b)$ ekanini topib olamiz. Shuni va berilgan shartlarni qo‘llab, quyidagilarga ega bo‘lamiz:

$$3*5=3*(3+2)=\frac{3^2+2^2}{3\cdot 2}\cdot(3*2)=\frac{13}{6}\cdot(2*3)=\frac{13}{6}\cdot(2*(2+1))=$$

$$=\frac{13}{6}\cdot\frac{2^2+1^2}{2\cdot 1}\cdot(2*1)=\frac{13}{6}\cdot\frac{5}{2}\cdot(1*2)=\frac{13}{6}\cdot\frac{5}{2}\cdot(1*(1+1))=$$

$$=\frac{13}{6}\cdot\frac{5}{2}\cdot\frac{1^2+1^2}{1\cdot 1}\cdot(1*1)=\frac{13}{6}\cdot\frac{5}{2}\cdot\frac{2}{1}\cdot(1^2+2019)=\frac{65650}{3}$$

Javob: $\frac{65650}{3}$

117. ***Lemma:*** *Toq funksiyaning simmetrik oraliqda olingan aniq integrali nolga teng. Ya'ni, agar* $f(x)$*-toq funksiya uchun* $\int_{-a}^{a} f(x)dx=0$ *tenglik o‘rinli. Bu yerda* $a\in\mathbb{R}$.

Isbot: $F(x)$ funksiya $f(x)$ funksiyaning boshlang‘ich funksiyasi bo‘lsin. $f(x)$ toq funksiya ekanidan $F(x)$-juft funksiya bo‘lishi kelib chiqadi. Ya’ni, $F(-a)=F(a)$ tenglik o‘rinli. U holda $\int_{-a}^{a} f(x)dx=F(x)\Big|_{-a}^{a}=F(a)-F(-a)=0$ tenglikka ega bo‘lamiz.

Bizning masalada $\sin^7 x\cos^7 x$ funksiyaning toq funksiya ekanini ko‘rish qiyin emas. Chunki, $\sin^7(-x)\cos^7(-x)=-\sin^7 x\cos^7 x$ tenglik o‘rinli. U holda yuqorida keltirilgan lemmaga ko‘ra bu funksiyaning $[-\pi;\pi]$ simmetrik oraliqda olingan aniq integrali nolga teng. Ya’ni,

$$\int_{-\pi}^{\pi}\sin^7 x\cos^7 x dx=0$$

Javob: 0

118. Berilgan aniq integralni quyidagicha yozib olamiz:

$$\int_{-\pi/4}^{\pi/4} \frac{x^{2n+1} + x^{2n-1} + \ldots + x^5 + 3x^3 - x + 1}{\cos^2 x} dx =$$

$$= \int_{-\pi/4}^{\pi/4} \left(\frac{x^{2n+1} + x^{2n-1} + \ldots + x^5 + 3x^3 - x}{\cos^2 x} + \frac{1}{\cos^2 x} \right) dx =$$

$$= \int_{-\pi/4}^{\pi/4} \frac{x^{2n+1} + x^{2n-1} + \ldots + x^5 + 3x^3 - x}{\cos^2 x} dx + \int_{-\pi/4}^{\pi/4} \frac{1}{\cos^2 x} dx$$

117-masalada keltirilgan lemmaga ko‘ra $\int_{-\pi/4}^{\pi/4} \frac{x^{2n+1} + x^{2n-1} + \ldots + x^5 + 3x^3 - x}{\cos^2 x} dx = 0$ ekanligi ma’lum. U holda

$\int_{-\pi/4}^{\pi/4} \frac{1}{\cos^2 x} dx = tgx\Big|_{-\pi/4}^{\pi/4} = 1 - (-1) = 2$ ekanligi kelib chiqadi.

Javob: 2

119. Quyidagi tengliklarga egamiz:

$$ctg(x+a) - ctg(x+b) = \frac{\cos(x+a)}{\sin(x+a)} - \frac{\cos(x+b)}{\sin(x+b)} = \frac{\sin(b-a)}{\sin(x+a)\sin(x+b)}$$

$$\int ctgxdx = \int \frac{\cos x}{\sin x} dx = \int \frac{1}{\sin x} d(\sin x) = \left|\sin x = t\right| =$$

$$= \int \frac{1}{t} dt = \ln\left|t\right| + C = \ln\left|\sin x\right| + C$$

Yuqoridagilarga asosan berilgan aniqmas integralni hisoblaymiz:

$$\int \frac{dx}{\sin(x+a)\sin(x+b)} = \int \frac{ctg(x+a) - ctg(x+b)}{\sin(b-a)} dx =$$

$$= \frac{1}{\sin(b-a)} \int (ctg(x+a) - ctg(x-b))dx =$$

$$= \frac{1}{\sin(b-a)} \cdot \left(\ln\left|\sin(x+a)\right| - \ln\left|\sin(x+b)\right| \right) + C$$

$$= \frac{1}{\sin(b-a)} \cdot \ln\left|\frac{\sin(x+a)}{\sin(x+b)}\right| + C, \;\; C = const$$

Javob: $\frac{1}{\sin(b-a)} \cdot \ln\left|\frac{\sin(x+a)}{\sin(x+b)}\right| + C, \;\; C = const$

120. 119-masalaga o‘xshash quyidagicha hisoblaymiz:

$$\int \frac{dx}{\sin(x+a)\cos(x+b)} = \int \frac{ctg(x+a) + tg(x+b)}{\cos(a-b)} dx =$$

$$= \frac{1}{\cos(a-b)} \int (ctg(x+a) + tg(x+b)) dx =$$

$$= \frac{1}{\cos(a-b)} \cdot \left(\ln|\sin(x+a)| - \ln|\cos(x+b)|\right) + C$$

$$= \frac{1}{\cos(a-b)} \cdot \ln\left|\frac{\sin(x+a)}{\cos(x+b)}\right| + C, \;\; C = const$$

Javob: $\frac{1}{\cos(a-b)} \cdot \ln\left|\frac{\sin(x+a)}{\cos(x+b)}\right| + C, \;\; C = const$

121. 119-masalaga o‘xshash quyidagicha hisoblaymiz:

$$\int \frac{dx}{\cos(x+a)\cos(x+b)} = \int \frac{tg(x+a) - tg(x+b)}{\sin(a-b)} dx =$$

$$= \frac{1}{\sin(a-b)} \int (tg(x+a) - tg(x+b)) dx =$$

$$= \frac{1}{\sin(a-b)} \cdot \left(-\ln|\cos(x+a)| + \ln|\cos(x+b)|\right) + C$$

$$= \frac{1}{\sin(a-b)} \cdot \ln\left|\frac{\cos(x+b)}{\cos(x+a)}\right| + C, \;\; C = const$$

Javob: $\frac{1}{\sin(a-b)} \cdot \ln\left|\frac{\cos(x+a)}{\cos(x+b)}\right| + C, \;\; C = const$

122. Ushbu $\sin^2 x + \cos^2 x = 1$ ayniyatdan foydalanib, aniqmas integralni quyidagicha hisoblaymiz:

$$\int \frac{dx}{\sin^4 x} = \int \frac{\sin^2 x + \cos^2 x}{\sin^4 x} dx = \int \left(\frac{1}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} \cdot \frac{1}{\sin^2 x} \right) dx =$$

$$= \int \frac{1}{\sin^2 x} dx + \int ctg^2 x \cdot \frac{1}{\sin^2 x} dx = -ctgx + C_1 + \int ctg^2 x d(-ctgx) =$$

$$= -ctgx + C_1 - \frac{ctg^3 x}{3} + C_2 = -ctgx - \frac{ctg^3 x}{3} + C, \;\; C = C_1 + C_2 = const$$

Javob: $-ctgx - \dfrac{ctg^3 x}{3} + C, \;\; C = const$

123. x ni differensial ostiga kiritish orqali masalani oson yechamiz:

$$\int_0^1 xf(x^2)dx = \frac{1}{2} \cdot \int_0^1 f(x^2)d(x^2) = \left| \begin{matrix} x^2 = t \\ 0 \le x \le 1 \\ 0 \le t \le 1 \end{matrix} \right| = \frac{1}{2} \cdot \int_0^1 f(t)dt = \frac{a}{2}$$

Javob: $\dfrac{a}{2}$

124. Oldin $\operatorname{sgn}(x) = \begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases}$ ekanligini yodga solaylik. Shunga asosan masalani quyidagicha yechamiz:

$$\int_0^\pi x \operatorname{sgn}(\cos x) dx = \int_0^{\pi/2} x \operatorname{sgn}(\cos x) dx + \int_{\pi/2}^{\pi} x \operatorname{sgn}(\cos x) dx =$$

$$= \int_0^{\pi/2} x \cdot 1 dx + \int_{\pi/2}^{\pi} x \cdot (-1) dx = \left. \frac{x^2}{2} \right|_0^{\pi/2} - \left. \frac{x^2}{2} \right|_{\pi/2}^{\pi} = \left(\frac{\pi^2}{8} - 0 \right) - \left(\frac{\pi^2}{2} - \frac{\pi^2}{8} \right) = -\frac{\pi^2}{4}$$

Javob: $-\dfrac{\pi^2}{4}$

125. Ushbu $x - x^3$ ifodaning $[0;1]$ oraliqda musbat, $[1;3]$ oraliqda manfiy qiymatlar qabul qilishini hisobga olgan holda, aniq integralni quyidagicha hisoblaymiz:

$$\int_0^3 \operatorname{sgn}(x - x^3) dx = \int_0^1 \operatorname{sgn}(x - x^3) dx + \int_1^3 \operatorname{sgn}(x - x^3) dx =$$

$$= \int_0^1 1 dx + \int_1^3 (-1) dx = x \Big|_0^1 - x \Big|_1^3 = (1 - 0) - (3 - 1) = -1$$

Javob: -1

126. *Javob:* $\frac{30}{\pi}$

$$\int_0^6[x]\sin\frac{\pi x}{6}dx=\int_0^1[x]\sin\frac{\pi x}{6}dx+\int_1^2[x]\sin\frac{\pi x}{6}dx+\int_2^3[x]\sin\frac{\pi x}{6}dx+$$

$$+\int_3^4[x]\sin\frac{\pi x}{6}dx+\int_4^5[x]\sin\frac{\pi x}{6}dx+\int_5^6[x]\sin\frac{\pi x}{6}dx=\int_0^1 0\cdot\sin\frac{\pi x}{6}dx+$$

$$+\int_1^2 1\cdot\sin\frac{\pi x}{6}dx+\int_2^3 2\cdot\sin\frac{\pi x}{6}dx+\int_3^4 3\cdot\sin\frac{\pi x}{6}dx+\int_4^5 4\cdot\sin\frac{\pi x}{6}dx+\int_5^6 5\cdot\sin\frac{\pi x}{6}dx=$$

$$=0-\frac{6}{\pi}\cos\frac{\pi x}{6}\Big|_1^2-\frac{12}{\pi}\cos\frac{\pi x}{6}\Big|_2^3-\frac{18}{\pi}\cos\frac{\pi x}{6}\Big|_3^4-\frac{24}{\pi}\cos\frac{\pi x}{6}\Big|_4^5-\frac{30}{\pi}\cos\frac{\pi x}{6}\Big|_5^6=$$

$$=-\frac{6}{\pi}\cdot\left(\cos\frac{2\pi}{6}-\cos\frac{\pi}{6}\right)-\frac{12}{\pi}\cdot\left(\cos\frac{3\pi}{6}-\cos\frac{2\pi}{6}\right)-\frac{18}{\pi}\cdot\left(\cos\frac{4\pi}{6}-\cos\frac{3\pi}{6}\right)-$$

$$-\frac{24}{\pi}\cdot\left(\cos\frac{5\pi}{6}-\cos\frac{4\pi}{6}\right)-\frac{30}{\pi}\cdot\left(\cos\frac{6\pi}{6}-\cos\frac{5\pi}{6}\right)=\frac{30}{\pi}$$

127. *Javob:* $\ln 2020!$

$$\int_1^{2021}\ln[x]dx=\int_1^2\ln[x]dx+\int_2^3\ln[x]dx+\int_3^4\ln[x]dx+\ldots+\int_{2020}^{2021}\ln[x]dx=$$

$$=\int_1^2\ln 1dx+\int_2^3\ln 2dx+\int_3^4\ln 3dx+\ldots+\int_{2020}^{2021}\ln 2020dx=0+x\ln 2\Big|_2^3+x\ln 3\Big|_3^4+$$

$$+\ldots+x\ln 2020\Big|_{2020}^{2021}=\ln 2+\ln 3+\ldots+\ln 2020=\ln(2\cdot 3\cdot\ldots\cdot 2020)=\ln 2020!$$

128. Ma'lumki, $x\ge y$ va $y\ne 0$. Qulaylik uchun $\sqrt{x-y}=a$ deb belgilash kiritamiz($a\ge 0$). U holda sistemaning birinchi tenglamasidan quyidagiga ega bo‘lamiz:

$$1-5y=\frac{x}{y}-6\sqrt{x-y}\ \Rightarrow\ x-y-6y\sqrt{x-y}+5y^2=0\ \Rightarrow\ a^2-6ay+5y^2=0$$

Oxirgi tenglamani a ga nisbatan yechib, $a=y$ va $a=5y$ ekanini topamiz.

1-hol: $a=y\ \Rightarrow\ \sqrt{x-y}=y\ \Rightarrow\ x=y^2+y$ bo‘lsin. x ni ikkinchi tenglamaga qo‘yamiz:

$$\sqrt{y^2+y-y}=y^2+y-5y-6$$

$$|y|=y^2-4y-6 \Rightarrow y_1=6,\ y_2=\frac{3-3\sqrt{5}}{2} \Rightarrow x_1=42,\ x_2=10-6\sqrt{5}$$

$x \ge y$ ekanidan $x_1=42$ va $y_1=6$ yechimga ega bo‘lamiz.

2-hol: $a=5y \Rightarrow \sqrt{x-y}=5y \Rightarrow x=25y^2+y$ bo‘lsin. x ni ikkinchi tenglamaga qo‘yamiz:

$$\sqrt{25y^2+y-5y}=25y^2+y-5y-6$$

$$\sqrt{25y^2-4y}=25y^2-4y-6 \Rightarrow \sqrt{25y^2-4y}=t,(t\ge 0) \Rightarrow t^2-t-6=0$$

$$t_1=3, t_2=-2 \Rightarrow \sqrt{25y^2-4y}=3 \Rightarrow y_1=\frac{2+\sqrt{29}}{25}, y_2=\frac{2-\sqrt{29}}{25} \Rightarrow$$

$$\Rightarrow x_1=\frac{47+\sqrt{229}}{5}, x_2=\frac{47-\sqrt{229}}{5}$$

Javob: $(x;y)\in\left\{(42;6),\left(\frac{47+\sqrt{229}}{5};\frac{2+\sqrt{229}}{25}\right),\left(\frac{47-\sqrt{229}}{5};\frac{2-\sqrt{229}}{25}\right)\right\}$

129. O‘rta arifmetik va o‘rta geometrik haqidagi Koshi tengsizligini qo‘llaymiz:

$$(tgx)^{\sin x}+(ctgx)^{\cos x}=(tgx)^{\sin x}+\frac{1}{(tgx)^{\cos x}}\ge 2\sqrt{(tgx)^{\sin x-\cos x}}$$

Quyidagi 3 ta holni qaraymiz:

1-hol: $0<x<\frac{\pi}{4}$

$$\begin{cases}0<tgx<1\\ \sin x<\cos x\end{cases} \Rightarrow \begin{cases}0<tgx<1\\ \sin x-\cos x<0\end{cases} \Rightarrow (tgx)^{\sin x-\cos x}>1 \Rightarrow$$

$$\Rightarrow (tgx)^{\sin x}+(ctgx)^{\cos x}\ge 2\sqrt{(tgx)^{\sin x-\cos x}}>2$$

2-hol: $\frac{\pi}{4}<x<\frac{\pi}{2}$

$$\begin{cases}0<tgx<+\infty\\ \sin x>\cos x\end{cases} \Rightarrow \begin{cases}0<tgx<+\infty\\ \sin x-\cos x>0\end{cases} \Rightarrow (tgx)^{\sin x-\cos x}>1 \Rightarrow$$

$$\Rightarrow (tgx)^{\sin x}+(ctgx)^{\cos x}\ge 2\sqrt{(tgx)^{\sin x-\cos x}}>2$$

3-hol: $x = \frac{\pi}{4}$

$$(tgx)^{\sin x} + (ctgx)^{\cos x} \geq 2\sqrt{(tgx)^{\sin x - \cos x}} = 2$$

Yuqoridagilarga asosan berilgan ifodaning $\left(0; \frac{\pi}{2}\right)$ oraliqdagi eng kichik qiymati 2 ga tengligi kelib chiqadi.

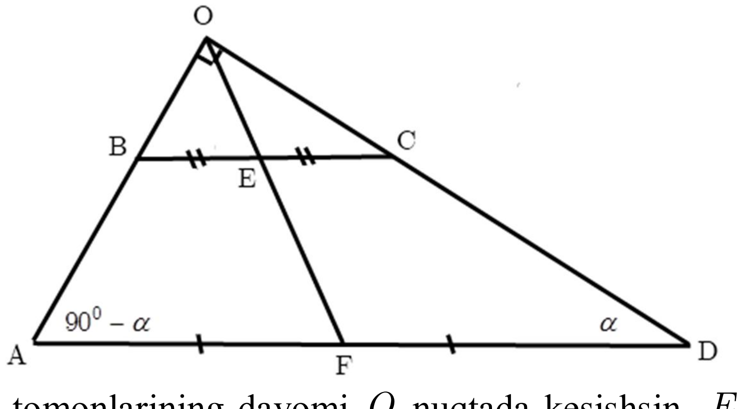
Javob: 2

130. Ma'lumki, har qanday musbat a va b sonlari uchun ushbu $\left(\frac{1}{\sqrt{a}} - \frac{1}{\sqrt{b}}\right)^2 \geq 0$ tengsizlik o'rinli. Bundan $\frac{1}{a} + \frac{1}{b} \geq \frac{2}{\sqrt{ab}}$ ekanligi kelib chiqadi. Xuddi shunga o'xshash $\frac{1}{b} + \frac{1}{c} \geq \frac{2}{\sqrt{bc}}$ va $\frac{1}{a} + \frac{1}{c} \geq \frac{2}{\sqrt{ac}}$ tengsizliklarni yozishimiz mumkin. Oxirgi uchta tengsizlikni hadma-had qo'shsak, isbotlanishi kerak bo'lgan tengsizlik kelib chiqadi:

$$\frac{2}{a} + \frac{2}{b} + \frac{2}{c} \geq \frac{2}{\sqrt{ab}} + \frac{2}{\sqrt{bc}} + \frac{2}{\sqrt{ac}} \Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq \frac{1}{\sqrt{ab}} + \frac{1}{\sqrt{bc}} + \frac{1}{\sqrt{ac}}$$

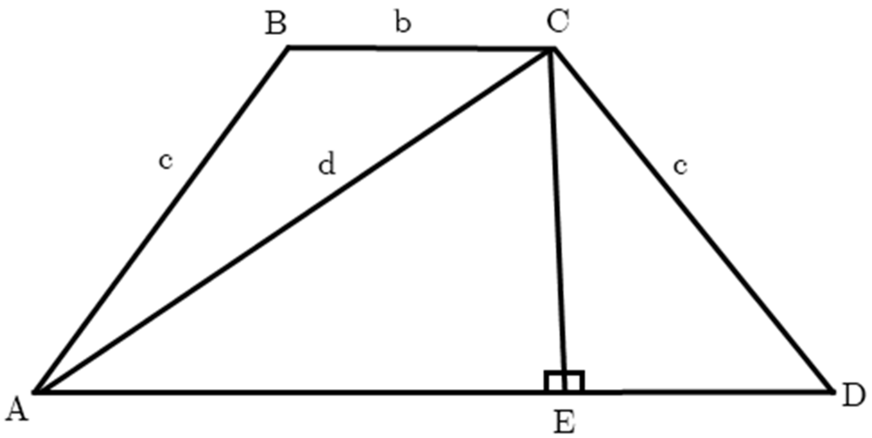
Tenglik sharti $a = b = c > 0$ da bajariladi.

131. $ABCD$ trapetsiyada $\angle D = \alpha$ deb belgilab olaylik. U holda $\angle A = 90^0 - \alpha$ bo'ladi.



AB va DC yon tomonlarining davomi O nuqtada kesishsin. E va F nuqtalar mos ravishda BC va AD asoslarning o'rtalari bo'lsin. Ma'lumki, ΔAOD-to'g'ri burchakli. Gipotenuzaga tushirilgan mediana gipotenuzaning yarmiga teng ekanligidan $OF = \frac{AD}{2}$ va $OE = \frac{BC}{2}$ tengliklar o‘rinli. U holda $EF = OF - OE = \frac{AD - BC}{2}$ tenglik o‘rinli. Isbot tugadi.

132. $ABCD$ trapetsiyaning CE balandligini va AC diagonalini o‘tkazamiz.



Ma’lumki, $AE = \frac{a+b}{2}$ va $ED = \frac{a-b}{2}$ tengliklar o‘rinli. ΔACE va ΔCED larga Pifagor teoremasini qo‘llab, so‘ralgan tenglikni hosil qilamiz:

$$\begin{cases} CE^2 = d^2 - \left(\frac{a+b}{2}\right)^2 \\ CE^2 = c^2 - \left(\frac{a-b}{2}\right)^2 \end{cases} \Rightarrow d^2 - \left(\frac{a+b}{2}\right) = c^2 - \left(\frac{a-b}{2}\right)^2 \Rightarrow d = \sqrt{c^2 + ab}$$

133. a, b, c lar geometrik progressiyaning ketma-ket hadlari bo‘lgani uchun $b = aq$ va $c = aq^2$ deb olaylik($q \in \mathbb{N}$). Quyidagilarni topib olamiz:

$EKUB(a;b) = EKUB(a;aq) = a$ va $EKUB(a;c) = EKUB(a;aq^2) = a$

U holda ushbu $\left(EKUB(a;b)\right)^2 = a \cdot EKUB(a;c)$ tenglikning o‘rinli ekanini ko‘rish qiyin emas.

134. To‘g‘ri burchakli uchburchakda $h = \frac{ab}{c}$ va $c^2 = a^2 + b^2$ tengliklar o‘rinli ekanligi ma’lum. Shularga asosan quyidagilarni yoza olamiz:

$$\left(\frac{ab}{c}\right)^2 > 0$$

$$c^2+2ab+\left(\frac{ab}{c}\right)^2>c^2+2ab$$

$$c^2+2\cdot c\cdot\frac{ab}{c}+\left(\frac{ab}{c}\right)^2>a^2+b^2+2ab$$

$$\left(c+\frac{ab}{c}\right)^2>(a+b)^2\Rightarrow c+\frac{ab}{c}>a+b\Rightarrow c+h>a+b$$

Da’vo isbotlandi.

135. Quyidagicha almashtirish bajaramiz:

$$\begin{cases}0\le x\le 2016\\ x-1008=t\\ dx=dt\\ -1008\le t\le 1008\end{cases}$$

U holda berilgan integral quyidagi ko‘rinishga keladi:

$$\int_{-1008}^{1008}(t+1008)\cdot(t+1004)\cdot\ldots\cdot(t-4)\cdot t\cdot(t+4)\cdot\ldots\cdot(t-1004)\cdot(t-1008)dt=$$

$$\int_{-1008}^{1008}t\cdot(t^2 1008^2)\cdot(t^2-1004^2)\cdot\ldots\cdot(t^2-4^2)dt=$$

Ushbu $t\cdot(t^2 1008^2)\cdot(t^2-1004^2)\cdot\ldots\cdot(t^2-4^2)$ funksiya t ga nisbatan toq funksiya ekani ma’lum. 117-masaladagi lemmaga ko‘ra oxirgi aniq integralning qiymati nolga teng.

Javob: 0

136. Viet teoremasiga ko‘ra $x_1+x_2=-p$ va $x_1x_2=-\frac{1}{2p^2}$ tengliklarga egamiz.

$$x_1^4+x_2^4=(x_1^2+x_2^2)^2-2x_1^2x_2^2=\left((x_1+x_2)^2-2x_1x_2\right)^2-2x_1^2x_2^2=$$

$$\left(p^2+\frac{1}{p^2}\right)^2-\frac{1}{2p^4}=p^4+2+\frac{1}{p^4}-\frac{1}{2p^4}=\left(p^4+\frac{1}{2p^4}\right)+2\ge 2\sqrt{p^4\cdot\frac{1}{2p^4}}+2=2+\sqrt{2}$$

Bu qiymatga $p^4=\frac{1}{2p^4}\Rightarrow p=\pm\sqrt[8]{\frac{1}{2}}$ da bajariladi.

Javob: $2+\sqrt{2}$

137. $A+B+C=\pi$ va trigonometriyaning ba'zi formulalaridan foydalanamiz:

$$\sin A+\sin B+\sin C=2\sin\frac{A+B}{2}\cos\frac{A-B}{2}+\sin(\pi-(A+B))=$$

$$=2\sin\frac{A+B}{2}\cos\frac{A-B}{2}+2\sin\frac{A+B}{2}\cos\frac{A+B}{2}=$$

$$=2\sin\frac{A+B}{2}\left(\cos\frac{A-B}{2}+\cos\frac{A+B}{2}\right)=2\sin\frac{\pi-C}{2}\cdot 2\cos\frac{A}{2}\cos\frac{B}{2}=$$

$$=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

138. Quyidagilardan foydalanib berilgan funksiya x qanday bo'lganda eng kichik qiymat qabul qilishini topib olamiz:

$$1+2+...+119=\frac{1+119}{2}\cdot 119=119\cdot 60$$

$$1+2+...+n=\frac{119\cdot 60}{2}$$

$$\frac{1+n}{2}\cdot n=\frac{119\cdot 60}{2}$$

$$(n+1)\cdot n=119\cdot 60\ \Rightarrow\ n=84\ \Rightarrow\ 84x-1=0\ \Rightarrow\ x=\frac{1}{84}$$

Endi funksiyaning eng kichik qiymatini topamiz:

$$f\left(\frac{1}{84}\right)=\left|\frac{1}{84}-1\right|+\left|\frac{2}{84}-1\right|+...+\left|\frac{83}{84}-1\right|+\left|\frac{84}{84}-1\right|+\left|\frac{85}{84}-1\right|+...+\left|\frac{119}{84}-1\right|=$$

$$=\left(1-\frac{1}{84}+1-\frac{2}{84}+...+1-\frac{83}{84}+1-\frac{84}{84}\right)+\left(\frac{85}{84}-1+...+\frac{119}{84}-1\right)=$$

$$=84\cdot 1-35\cdot 1=49$$

Javob: 49

139. $A+B+C=\pi$ ekanidan va ba'zi trigonometrik formulalardan foydalanamiz:

$$tg\frac{C}{2}\cdot ctg\frac{C}{2}=1\ \Rightarrow\ tg\frac{C}{2}\cdot tg\left(\frac{\pi}{2}-\frac{C}{2}\right)=1\ \Rightarrow\ tg\frac{C}{2}\cdot tg\frac{\pi-C}{2}=1\ \Rightarrow$$

$$\Rightarrow\ tg\frac{C}{2}\cdot tg\frac{A+B}{2}=1\ \Rightarrow\ tg\frac{C}{2}\cdot tg\left(\frac{A}{2}+\frac{B}{2}\right)=1\ \Rightarrow\ tg\frac{C}{2}\cdot\frac{tg\frac{A}{2}+tg\frac{B}{2}}{1-tg\frac{A}{2}tg\frac{B}{2}}=1\ \Rightarrow$$

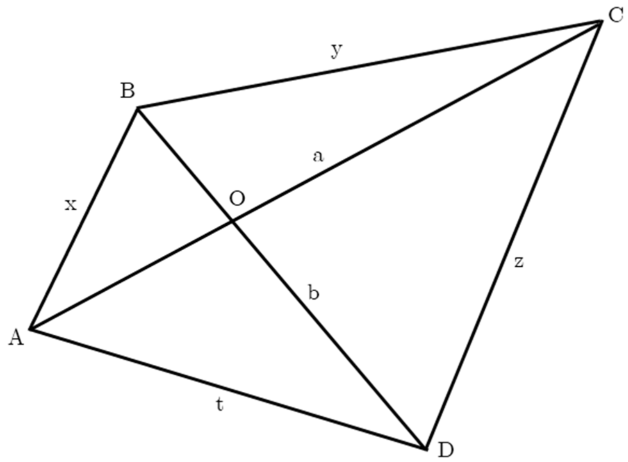
$$\Rightarrow\ tg\frac{A}{2}tg\frac{C}{2}+tg\frac{B}{2}tg\frac{C}{2}=1-tg\frac{A}{2}tg\frac{B}{2}\ \Rightarrow\ tg\frac{A}{2}tg\frac{B}{2}+tg\frac{B}{2}tg\frac{C}{2}+tg\frac{A}{2}tg\frac{C}{2}=1$$

140. Bizga ma'lumki, $2^7\equiv 1(\bmod 127)$ taqqoslama o'rinli. Agar $2020=288\cdot 7+4$ ekanini hisobga olsak, quyidagilarga ega bo'lamiz:

$$\left(2^7\right)^{288}=1^{288}(\bmod 127)\ \Rightarrow\ 2^{2016}\equiv 1(\bmod 127)\ \Rightarrow\ 2^{2020}\equiv 2^4(\bmod 127)$$

Bundan 2^{2020} ni 127 ga bo'lgandagi qoldiq 16 ekanligi kelib chiqadi.
Javob: 16

141. Qavariq to'rtburchakning tomonlari uzunliklarini x,y,z,t orqali belgilaylik va eng uzun tomoni z bo'lsin.



$\Delta ABC,\Delta BCD,\Delta CDA,\Delta DAB,$ larda uchburchak tengsizligini va $z>x,y,t$ ekanini hisobga olsak,

$$\begin{cases} a<x+y\le z+z=2z\\ b<y+z\le z+z=2z\\ a<z+t\le z+z=2z\\ b<x+t\le z+z=2z\end{cases}\ \Rightarrow\ +\begin{cases} a^2<4z^2\\ b^2<4z^2\\ a^2<4z^2\\ b^2<4z^2\end{cases}\ \Rightarrow\ 2(a^2+b^2)<16z^2\ \Rightarrow\ z>\sqrt{\frac{a^2+b^2}{8}}$$

ekanligi kelib chiqadi. Shuni isbotlash talab qilingan edi.

142. Masala takroriy o'rin almashtirish qoidasi orqali oson yechiladi. Berilgan 201920202021 son 12 xonali($n=12$) bo'lib, sonda 0 raqami 4 marta($m=4$), 1 raqami 2 marta($p=2$), 2 raqami 5 marta($l=5$) va 9 raqami 1 marta($q=1$) takrorlanadi. Demak, jami takroriy o'rin almashtirishlar soni $P_n(T)=\dfrac{n!}{m!\cdot p!\cdot l!\cdot q!}=\dfrac{12!}{4!\cdot 2!\cdot 5!\cdot 1!}=83160$ ga teng(bunda (T) belgi takroriy ekanini bildiradi). Bu sondan birinchi raqami 0 bo'lgan takroriy o'rin almashtirishlar soni $\frac{11!}{3!\cdot 2!\cdot 5!\cdot 1!} = 27720$ ni ayiramiz va so‘ralgan natijaga ega bo‘lamiz:

83160-27720=55440

Javob: 55440 ta

143. Mohinur o‘ylagan noldan farqli raqamlarni a, b, c orqali belgilaylik. U holda Akmaljon tuzgan ikki xonali sonlar yig‘indisini yozamiz:

$$\overline{ab} + \overline{ba} + \overline{bc} + \overline{cb} + \overline{ac} + \overline{ca} + \overline{aa} + \overline{bb} + \overline{cc} = 231$$
$$33(a + b + c) = 231$$
$$a + b + c = 7$$

a, b, c lar turli raqamlar ekanidan Mohinur o‘ylagan raqamlar 1, 2, 4 ekanligini oson topish mumkin.

Javob: 1, 2, 4

144. Izlanayotgan sonlarni a, $\overline{bc}$, $\overline{def}$ deb olaylik. Masala shartidan $a + \overline{bc} = 47$, va $\overline{bc} + \overline{def} = 358$ tengliklarni yozishimiz mumkin. Bizda faqat 1,2,3,4,5,6 raqamlari borligini hisobga olsak, $b = 4$, $a + c = 7$, $d = 3$, $e = 1$ va $c + f = 8$ ekanligini topishimiz mumkin. Ishlatilmagan 2,5,8 raqamlari qoldi. $a + c = 7$ va $c + f = 8$ ifodalardan $c = 2$, $a = 5$, va $f = 6$ ekanligi kelib chiqadi. U holda biz izlayotgan sonlar 5, 42, 316 va ularning yig‘indisi 363 ekanini topamiz.

Javob: 363

145. 1995 ni $1995 = 3 \cdot 5 \cdot 7 \cdot 19$ ko‘rinishda yozish mumkin. Biz izlayotgan ko‘paytma $\overline{abc} \cdot a \cdot b \cdot c = 1995$ ko‘rinishida emas chunki, 1995 soni ushbu 665, 399, 285, 105 uch xonali sonlarga bo‘linadi va bu sonlar masala shartini qanoatlantirmaydi. Demak biz ko‘paytmani $\overline{ab} \cdot a \cdot b = 1995$ ko‘rinishida izlaymiz. 3, 5, 7, 19 sonlaridan foydalanib, $57 \cdot 5 \cdot 7 = 1995$ ekanini topish mumkin. U holda so‘ralgan yig‘indi *57+5+7=69* ga teng.

Javob: 69

146. Masala ushbu $\alpha + \beta + \gamma = 180$ tenglamani tub sonlarda yechishga keladi. Ma’lumki, 2 dan boshqa tub sonlar toq hamdir. Uchta toq sonning yig‘indisi juft son bo‘la olmagani uchun noma’lumlardan biri 2 ga tengligi kelib chiqadi. $\gamma = 2$ bo‘lsin. U holda ushbu $\alpha + \beta = 178$ tenglamani tub sonlarda yechishimiz kerak. Ma’lumki, 3 dan katta har qanday tub sonni 6 ga bo‘lganda 1 yoki 5 qoldiq qoladi. 178 ni 6 ga bo‘lganda 4 qoldiq qoladi. Bu esa α va β larni 6 ga bo‘lganda bir vaqtda 5 qoldiq qolishi kerakligini bildiradi. Biz *178:2=89* dan katta bo‘lmagan va

6 ga bo‘lganda 5 qoldiq qoladigan tub sonlarni qarashimiz kifoya. Shu yo‘l orqali (5;173), (11;167), (29;149), (41;137), (47;131), (71;107) va (89;89) yechimlarni topamiz.

Demak, burchaklari tub sonlar bilan ifodalanadigan uchburchaklar quyidagilar:

$(2^0;5^0;173^0)$, $(2^0;11^0;167^0)$, $(2^0;29^0;149^0)$, $(2^0;41^0;137^0)$, $(2^0;47^0;131^0)$, $(2^0;71^0;107^0)$ va $(2^0;89^0;89^0)$

Javob: 7 ta

147. $tg\pi = 0$ va qo‘shish formulasidan foydalanamiz:

$$tg\frac{\alpha+\beta+\gamma}{2} = 0 \Rightarrow tg\left(\frac{\alpha}{2}+\frac{\beta}{2}+\frac{\gamma}{2}\right) = 0 \Rightarrow \frac{tg\left(\frac{\alpha}{2}+\frac{\beta}{2}\right)+tg\frac{\gamma}{2}}{1-tg\left(\frac{\alpha}{2}+\frac{\beta}{2}\right)tg\frac{\gamma}{2}} = 0 \Rightarrow$$

$$\Rightarrow tg\left(\frac{\alpha}{2}+\frac{\beta}{2}\right)+tg\frac{\gamma}{2} = 0 \Rightarrow \frac{tg\frac{\alpha}{2}+tg\frac{\beta}{2}}{1-tg\frac{\alpha}{2}tg\frac{\beta}{2}}+tg\frac{\gamma}{2} = 0 \Rightarrow$$

$$tg\frac{\alpha}{2}+tg\frac{\beta}{2}+tg\frac{\gamma}{2}-tg\frac{\alpha}{2}tg\frac{\beta}{2}tg\frac{\gamma}{2} = 0 \Rightarrow tg\frac{\alpha}{2}+tg\frac{\beta}{2}+tg\frac{\gamma}{2} = tg\frac{\alpha}{2}tg\frac{\beta}{2}tg\frac{\gamma}{2}$$

148. $xy \geq 1$ ning ikkala tomonini $(x-y)^2$ ga ko‘paytiramiz va qavslarni ochamiz:

$$xy(x-y)^2 \geq (x-y)^2$$

$$xy(x^2+y^2)-2x^2y^2 \geq x^2+y^2-2xy$$

$$xy(x^2+y^2)+2xy \geq 2x^2y^2+x^2+y^2$$

$$xy(x^2+y^2+2) \geq 2x^2y^2+x^2+y^2$$

Oxirgi tengsizlikning ikkala tomoniga x^2+y^2+2 ni qo‘shamiz va shakl almashtirishlar bajaramiz:

$$xy(x^2+y^2+2)+(x^2+y^2+2) \geq 2x^2y^2+2x^2+2y^2+2$$

$$(x^2+y^2+2)(xy+1) \geq 2(x^2+1)(y^2+1)$$

$$\frac{x^2+y^2+2}{(x^2+1)(y^2+1)} \geq \frac{2}{xy+1}$$

$$\frac{1}{x^2+1}+\frac{1}{y^2+1} \geq \frac{2}{xy+1}$$

Tenglik sharti $x = y$ bo‘lganda bajariladi.

149. 148-masalada keltirilgan tengsizlikni 2 marta qo‘llash orqali berilgan tengsizlikni isbot qilamiz:

$$\frac{1}{a^4+1}+\frac{1}{b^4+1}+\frac{1}{c^4+1}+\frac{1}{d^4+1}\geq\frac{2}{a^2b^2+1}+\frac{2}{c^2d^2+1}\geq\frac{4}{abcd+1}$$

Tenglik sharti $a=b=c=d$ da bajariladi.

150. Muzqaymoqning bahosini x so‘m desak(bunda $x\geq 7$), masala shartidan Ra’noda $x-7$ so‘m va Gulida $x-1$ so‘m pul borligi kelib chiqadi. U holda quyidagi tengsizlik o‘rinli:

$$x-7+x-1<x \Rightarrow x<8$$

Bundan $x=7$ ekanini topish mumkin(Ra’noning puli yo‘q, Gulida 6 so‘m bo‘lgan).

Javob: 7 so‘m

151. Dastlab massalari n^2, $(n+1)^2$, ..., $(n+8)^2$ grammdan bo‘lgan 9 ta toshni quyidagicha 3 ta guruhga ajratamiz:

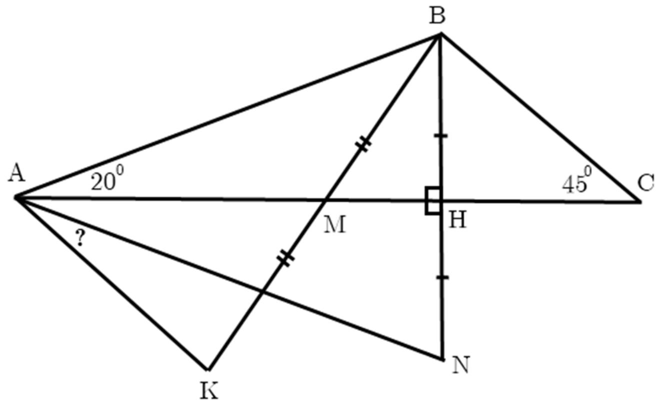
1-guruh: $\left\{n^2,(n+5)^2,(n+7)^2\right\}$

2-guruh: $\left\{(n+1)^2,(n+3)^2,(n+8)^2\right\}$

3-guruh: $\left\{(n+2)^2,(n+4)^2,(n+6)^2\right\}$

Bunda 1- va 2-guruhdagi toshlarning massalari yig‘indisi teng, 3-guruhniki 18 grammga yengil. Keyingi 9 ta toshni shunday 3 ta guruhga ajratamizki, bunda 1- va 3-guruhlar bir xil vaznga ega bo‘lib, 2-guruhniki 18 grammga yengil. Nihoyat keyingi 9 ta toshni shunday 3 ta guruhga ajratamizki, bunda 2- va 3-guruhlar bir xil vaznga ega bo‘lib, 1-guruhniki 18 grammga yengil bo‘lsin. U holda uchala holdagi barcha 1-guruhlarni alohida, barcha 2-guruhlarni alohida va barcha 3-guruhlarni alohida olganimizda bu guruhlardagi toshlarning massalari yig‘indisi teng bo‘ladi. Bu bilan biz dastlabki 27 ta toshni massalari teng bo‘lgan 3 ta guruhga ajratdik. Qolgan 54 ta toshni ham xuddi shu usul bilan massalari teng bo‘lgan guruhlarga ajratamiz. Bu bilan biz berilgan 81 ta toshni massalari teng bo‘lgan 3 ta guruhga ajratgan bo‘lamiz.

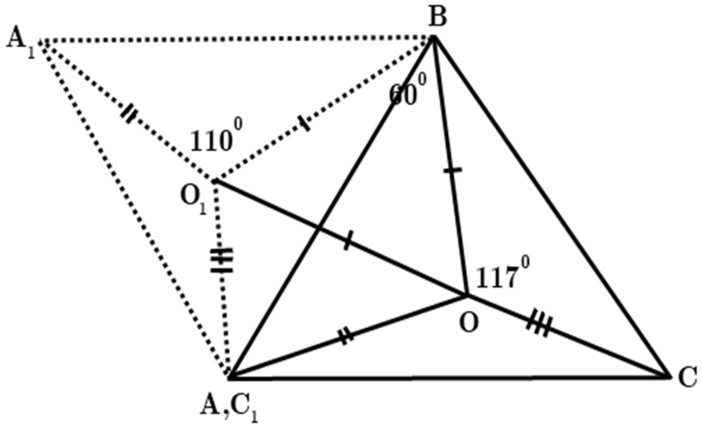
152. Birinchidan, $BH=HN$, AH-umumiy bo‘lgani uchun to‘g‘ri burchakli uchburchaklar tengligining KK(katet-katet) alomatiga ko‘ra $\Delta ABH=\Delta ANH$ tenglik o‘rinli. Bundan $\angle NAH=20^0$ ekanligi kelib chiqadi.



Ikkinchidan, $AM = MC$, $\angle AMK = \angle BMC$ va $BM = MK$ bo‘lgani uchun uchburchaklar tengligining TBT(tomon-burchak-tomon) alomatiga ko‘ra $\Delta AMK = \Delta BMC$ tenglik o‘rinli. Bundan $\angle MAK = 45^0$ ekanini topamiz. U holda $\angle KAN = 45^0 - 20^0 = 25^0$ ekanligi kelib chiqadi.

Javob: 25^0

153. Berilgan ABC muntazam uchburchakni B uchi atrofida soat strelkasi yo‘nalishida 60^0 ga buramiz. Natijada A_1BC_1 muntazam uchburchak hosil bo‘ladi(chizmaga qarang).



Ma’lumki, bunday burishda $\angle OBO_1 = 60^0$ bo‘lib, $OB = OB_1$ ekanidan ΔOBO_1 ning muntazam ekanligi kelib chiqadi. U holda biz izlayotgan uchburchak OC_1O_1 bo‘lib qoladi chunki, $O_1C_1 = OC$, va $OO_1 = OB$ tengliklar o‘rinli. ΔOC_1O_1 ning ichki burchaklari $117^0 - 60^0 = 57^0$, $110^0 - 60^0 = 50^0$ va $180^0 - (57^0 + 50^0) = 73^0$ ekanligini topib olamiz. Bular o‘z navbatida tomonlari OA, OB, OC bo‘lgan uchburchakning ham burchaklaridir.

Javob: 57^0, 50^0, 73^0

154. Sistemaning oxirgi tenglamasini soddalashtirsak, quyidagi ko‘rinishga keladi:

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=-\frac{z}{xy} \Rightarrow xy+yz+xz+z^2=0 \Rightarrow (x+z)(y+z)=0$$

1-hol: $x=-z$. Buni sistemaning birinchi va ikkinchi tenglamasiga qo‘yib, $y=2$, $x=\pm3$, $z=\pm3$ yechimlarni topamiz.

2-hol: $y=-z$. Bu holatda ham sistemaning birinchi va ikkinchi tenglamasidan $x=2$, $y=\pm3$, $z=\pm3$ yechimlarga ega bo‘lamiz.

Javob: $(3;2;-3),(-3;2;3),(2;3;-3),(2;-3;3)$

155. Masalani taqqoslamalardan foydalanib yechamiz.

$$2011^1\equiv16(\mathrm{mod}\,19),\ 2011^2\equiv9(\mathrm{mod}\,19),\ 2011^3\equiv11(\mathrm{mod}\,19),$$

$$2011^4\equiv5(\mathrm{mod}\,19),\ 2011^5\equiv4(\mathrm{mod}\,19),\ 2011^6\equiv7(\mathrm{mod}\,19),\ 2011^7\equiv17(\mathrm{mod}\,19)$$

$$,\ 2011^8\equiv6(\mathrm{mod}\,19),\ 2011^9\equiv1(\mathrm{mod}\,19)$$

Biz $2011^{2011^{2011}}$ ni 9 ga bo‘lgandagi qoldiqni topishimiz kerak.

$$2011^1\equiv4(\mathrm{mod}\,9),\ 2011^2\equiv7(\mathrm{mod}\,9),\ 2011^3\equiv1(\mathrm{mod}\,9)$$

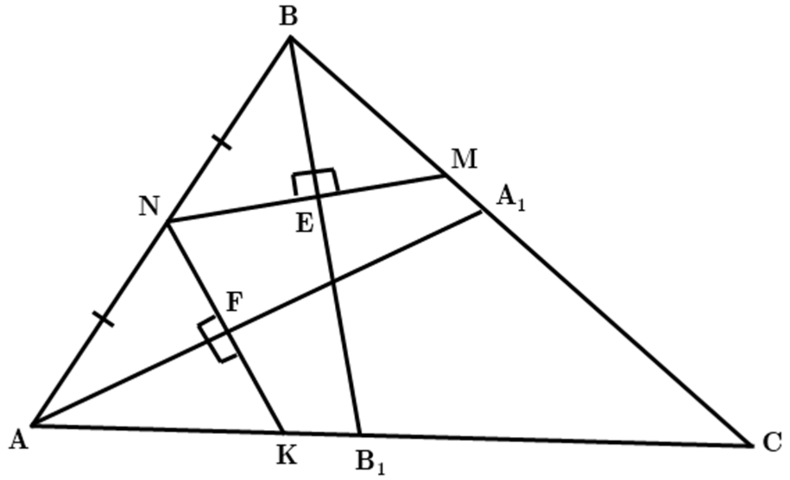
Endi 2011^{2011} ni 3 ga bo‘lgandagi qoldiqni topamiz.

$$2011^1\equiv1(\mathrm{mod}\,3) \Rightarrow 2011^{2011}\equiv1(\mathrm{mod}\,3) \Rightarrow 2011^{2011^{2011}}\equiv4(\mathrm{mod}\,9) \Rightarrow$$

$$\Rightarrow 2011^{2011^{2011^{2011}}}\equiv5(\mathrm{mod}\,19)$$

Javob: 5

156. Uchburchakning AA_1 va BB_1 bissektrisalarini o‘tkazamiz.



N nuqta AB tomonning o‘rtasi bo‘lsin. Shu nuqtadan AA_1 va BB_1 bissektrisalarga NK va NM perpendikulyarlar o‘tkazamiz. Bu perpendikulyarlar bissektrisalarni mos ravishda F va E nuqtalarda kesib o‘tsin.

Birinchidan, AF-umumiy va $\angle NAF = \angle KAF$ ekanidan to'g'ri burchakli uchburchaklar tengligining KB(katet-burchak) alomatiga ko'ra $\Delta ANF = \Delta AKF$ tenglik o'rinli. Bundan $AK = AN$ ekanligi kelib chiqadi.
Ikkinchidan, BE-umumiy va $\angle NBE = \angle MBE$ ekanidan to'g'ri burchakli uchburchaklar tengligining KB(katet-burchak) alomatiga ko'ra $\Delta BNE = \Delta BME$ tenglik o'rinli. Bundan $BM = BN$ ekanligi kelib chiqadi. Shuni isbotlash talab qilingan edi.
Agar $AN = BN$ ekanini hisobga olsak, $AK = BM$ tenglikka ega bo'ladi. Isbot tugadi.
157. Birinchi quvur yolg'iz o'zi bo'sh hovuzni x soatda, ikinchisi y soatda, uchinchisi z soatda va uchala quvur birgalikda bo'sh hovuzni t soatda to'ldirsin deylik. Birinchi kuni hovuz 11 soatda to'lganini hisobga olsak, masala shartidan quyidagi tenglamalar sistemasiga ega bo'lamiz:

$$\begin{cases} \dfrac{1}{x}\cdot\dfrac{\frac{2}{3}}{\frac{1}{y}+\frac{1}{z}}+\dfrac{1}{y}\cdot\dfrac{\frac{1}{5}}{\frac{1}{x}+\frac{1}{z}}+\dfrac{1}{z}\cdot\dfrac{\frac{1}{3}}{\frac{1}{x}+\frac{1}{y}}=1 \\ \dfrac{\frac{2}{3}}{\frac{1}{y}+\frac{1}{z}}+\dfrac{\frac{1}{5}}{\frac{1}{x}+\frac{1}{z}}+\dfrac{\frac{1}{3}}{\frac{1}{x}+\frac{1}{y}}=11 \\ \dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}=\dfrac{1}{t} \end{cases}$$

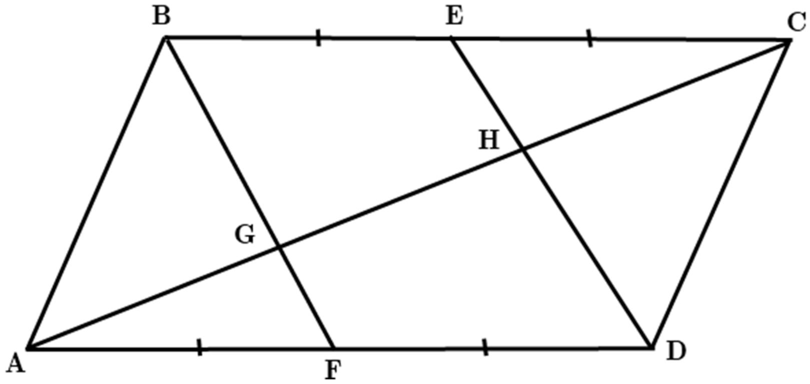
$$\begin{cases} \dfrac{1}{x}\cdot\dfrac{2}{3}\cdot\dfrac{1}{\frac{1}{t}-\frac{1}{x}}+\dfrac{1}{y}\cdot\dfrac{1}{5}\cdot\dfrac{1}{\frac{1}{t}-\frac{1}{y}}+\dfrac{1}{z}\cdot\dfrac{1}{3}\cdot\dfrac{1}{\frac{1}{t}-\frac{1}{z}}=1 \\ \dfrac{2}{3}\cdot\dfrac{1}{\frac{1}{t}-\frac{1}{x}}+\dfrac{1}{5}\cdot\dfrac{1}{\frac{1}{t}-\frac{1}{y}}+\dfrac{1}{3}\cdot\dfrac{1}{\frac{1}{t}-\frac{1}{z}}=11 \end{cases}$$

$$+\begin{cases} \dfrac{2t}{3(x-t)}+\dfrac{t}{5(y-t)}+\dfrac{t}{3(z-t)}=1\,/\cdot(-t) \\ \dfrac{2tx}{3(x-t)}+\dfrac{ty}{5(y-t)}+\dfrac{tz}{3(z-t)}=11 \end{cases} \Rightarrow \dfrac{2t}{3}+\dfrac{t}{5}+\dfrac{t}{3}=11-t \Rightarrow t=5$$

Demak uchala quvur birgalikda bo‘sh hovuzni 5 soatda to‘ldirar ekan. Hovuz to‘lganda soat millari 13^{00} (8+5=13) ni ko‘rsatadi.

Javob: Hovuz soat 13^{00} da to‘lgan

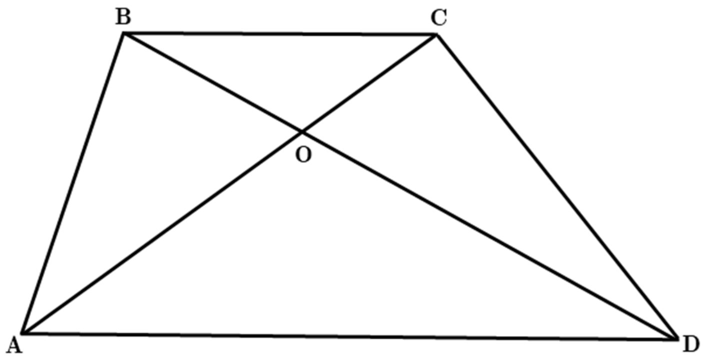
158. Masala shartiga mos chizmani chizib olamiz.



Uchburchaklar tengligining TBT(tomon-burchak-tomon) alomatiga ko‘ra $\Delta ABF = \Delta CED$ ekanligi ma’lum. Bundan $BF = ED$ ekanligi kelib chiqadi. U holda qarama-qarshi tomonlari jufti-jufti bilan teng bo‘lgan to‘rtburchak parallelogramm bo‘lishligi alomatidan $BEFD$ to‘rtburchak parallelogramm bo‘ladi. Bu esa $BF \parallel ED$ ekanini bildiradi.

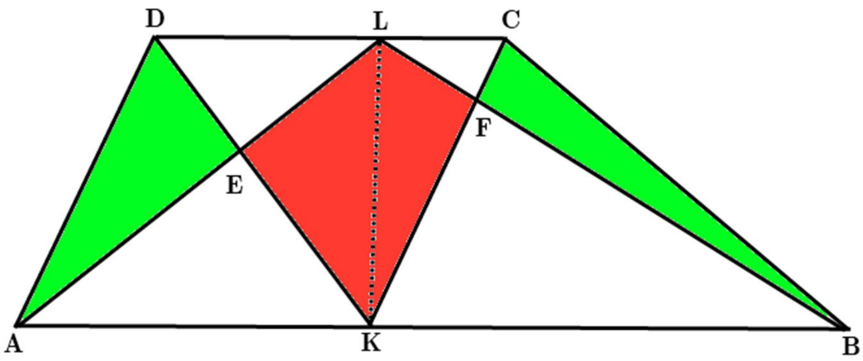
CAD va BCA burchaklarda Fales teoremasiga ko‘ra $AG = GH$ va $GH = HC$ tengliklar o‘rinli. Bu esa $AG = GH = HC$ ekanligini bildiradi. Isbot tugadi.

159. AOD va BOC uchburchaklarning o‘xshash ekanligidan $\frac{OA}{OC} = \frac{OD}{OB}$ yoki $OA \cdot OB = OD \cdot OC$ tenglik o‘rinli.



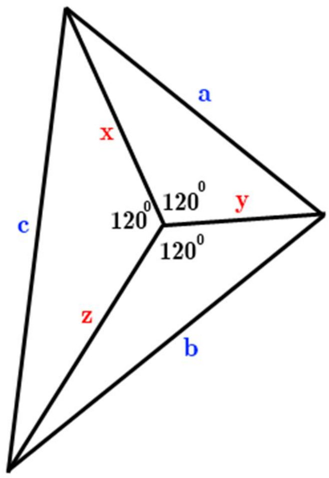
$\angle AOB$ va $\angle COD$ burchaklar vertikal bo‘lgani uchun teng. $S_{AOB} = \frac{1}{2} \cdot OA \cdot OB \cdot \sin \angle AOB$ va $S_{COD} = \frac{1}{2} \cdot OC \cdot OD \cdot \sin \angle COD$ ekanligidan $S_{AOB} = S_{COD}$ tenglikning o‘rinli ekani kelib chiqadi.

160. L va K nuqtalarni tutashtiramiz. Natijada $ADLK$ va $LCBK$ trapetsiyalar hosil bo‘ladi.



159-masalada isbotlanganiga ko‘ra $S_{AED}=S_{KEL}$ va $S_{KFL}=S_{BFC}$ tengliklar o‘rinli. Bundan $S_{ELFK}=S_{KEL}+S_{KFL}=S_{AED}+S_{BFC}$ ekanligi kelib chiqadi.

161. Bir nuqtadan chiquvchi va oralaridagi burchak 120^0 dan bo‘lgan x,y,z kesmalarni olamiz. Kesmalarning ikkinchi uchlarini ketma-ket tutashtirib tomonlari a,b,c bo‘lgan uchburchakni hosil qilamiz(chizmaga qarang).



Hosil bo‘lgan kichik uchburchaklarga kosinuslar teoremasini qo‘llasak $\begin{cases} x^2+xy+y^2=a^2 \\ y^2+yz+z^2=b^2 \\ x^2+xz+z^2=c^2 \end{cases}$ tenglamalar sistemasi hosil bo‘ladi. Endi kichik uchburchaklar yuzlarining yig‘indisi katta uchburchak yuziga teng bo‘lishidan foydalanib, quyidagilarga ega bo‘lamiz:

$$\frac{1}{2}xy\sin 120^0+\frac{1}{2}yz\sin 120^0+\frac{1}{2}xz\sin 120^0=$$

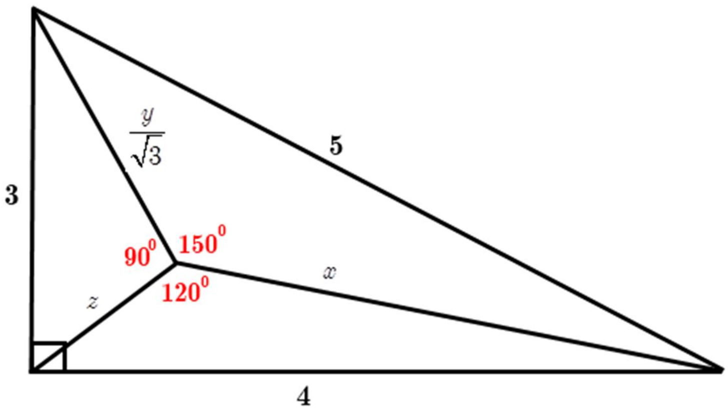
$$=\sqrt{\frac{a+b+c}{2}\cdot\frac{-a+b+c}{2}\cdot\frac{a-b+c}{2}\cdot\frac{a+b-c}{2}}$$

$$\frac{\sqrt{3}}{4}(xy+yz+xz)=\frac{\sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)}}{4}$$

$$xy + yz + xz = \frac{\sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)}}{3}$$

Javob: $\frac{\sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)}}{3}$

162. Katetlari 3 va 4 ga teng bo‘lgan to‘g‘ri burchakli uchburchakning ichidan biror nuqta olib, undan orasidagi burchaklari 150^0, 90^0 va 120^0 bo‘lgan kesmalarni x, $\frac{y}{\sqrt{3}}$ va z kesmalarni uchburchakning uchlari bilan tutashtiramiz(chizmaga qarang).



Xuddi 161-masaladagi usul bilan $xy + 2yz + 3xz = 24\sqrt{3}$ ekanini topish mumkin.

Javob: $24\sqrt{3}$

163. Ixtiyoriy $\alpha \in \mathbb{R}$ uchun $|\cos\alpha| \geq \cos^2\alpha$, $|\cos\alpha| \geq \cos\alpha$ va $\cos\alpha \geq -1$ tengsizliklar o‘rinli. Shular va ba’zi trigonometrik formulalarga asosan tengsizlikni isbotlaymiz:

$$|\cos x| + |\cos y| + |\cos(x+y)| \geq \cos^2 x + \cos^2 y + \cos(x+y) =$$

$$= \frac{1+\cos 2x}{2} + \frac{1+\cos 2y}{2} + \cos(x+y) = 1 + \cos(x-y)\cos(x+y) + \cos(x+y) \geq$$

$$\geq 1 + (-1)\cdot\cos(x+y) + \cos(x+y) = 1$$

164. $a, b, x \in \mathbb{R}$ uchun $|a| + |b| \geq |a+b|$ va $1 \geq |\sin x|$ tengsizliklarga ko‘ra quyidagi yordamchi tengsizlikka ega bo‘lamiz:

$$|\cos\alpha| + |\cos\beta| = |\cos\alpha|\cdot 1 + |\cos\beta|\cdot 1 \geq |\cos\alpha|\cdot|\sin\beta| + |\cos\beta|\cdot|\sin\alpha| =$$

$$= |\cos\alpha\sin\beta| + |\cos\beta\sin\alpha| \geq |\cos\alpha\sin\beta + \cos\beta\sin\alpha| = |\sin(\alpha+\beta)|$$

Bundan $\alpha, \beta \in \mathbb{R}$ uchun $\left|\cos\alpha\right| + \left|\cos\beta\right| \geq \left|\sin(\alpha+\beta)\right|$ tengsizlikning o‘rinli ekani kelib chiqadi. Shunga asosan:

$$\left|\cos x_1\right| + \left|\cos x_2\right| + \left|\cos x_3\right| + \left|\cos x_4\right| + \left|\cos x_5\right| \geq$$

$$\geq \left|\sin(x_1 + x_2)\right| + \left|\sin(x_3 + x_4)\right| + \left|\cos x_5\right| =$$

$$= \left|\cos\left(\frac{\pi}{2} - (x_1 + x_2)\right)\right| + \left|\cos\left(\frac{\pi}{2} - (x_3 + x_4)\right)\right| + \left|\cos x_5\right| \geq$$

$$\geq \left|\sin(\pi - (x_1 + x_2 + x_3 + x_4))\right| + \left|\cos x_5\right| =$$

$$= \left|\cos\left(\frac{\pi}{2} - (x_1 + x_2 + x_3 + x_4)\right)\right| + \left|\cos(-x_5)\right| \geq$$

$$\geq \left|\sin\left(\frac{\pi}{2} - (x_1 + x_2 + x_3 + x_4 + x_5)\right)\right| = \left|\sin\frac{\pi}{2}\right| = 1$$

165. Biror natural sonning kvadrati bo‘lgan ikki xonali sonlar 16, 25, 36, 49, 64, 81 ekanligidan va 11 ga bo‘linish qoidasidan foydalanib, quyidagilarni topamiz:

1) $b = 1, c = 6 \;\Rightarrow\; a = 7 \;\Rightarrow\; \overline{716d} \;\Rightarrow\; d = 1$

2) $b = 2, c = 5 \;\Rightarrow\; a = 7 \;\Rightarrow\; \overline{725d} \;\Rightarrow\; d \in \varnothing$

3) $b = 3, c = 6 \;\Rightarrow\; a = 9 \;\Rightarrow\; \overline{936d} \;\Rightarrow\; d = 1$

4) $b = 4, c = 9 \;\Rightarrow\; a = 13$, a raqam emas

5) $b = 6, c = 4 \;\Rightarrow\; a = 10$, a raqam emas

6) $b = 8, c = 1 \;\Rightarrow\; a = 9 \;\Rightarrow\; \overline{981d} \;\Rightarrow\; d = 2$

Javob: 7161, 9361, 9812

166. Madina yozgan raqamlarni a, b, c deylik. U holda Madina tuzishi mumkin bo‘lgan uch xonali sonlar $\overline{abc}$, $\overline{acb}$, $\overline{bac}$, $\overline{bca}$, $\overline{cab}$, $\overline{cba}$ ko‘rinishida bo‘ladi. Agar hamma sonni qo‘shganda edi, yig‘indi $222(a+b+c)$ ko‘rinishida bo‘lib, 222 ga karrali bo‘lardi. Biz 3159 dan katta va 3159 ni ayirganda ayirma uch xonali son bo‘ladigan 222 ga karrali sonlarni izlaymiz.

1) *3330-3159=171*, bunda raqamlar takrorlanadi, masala shartiga zid
2) *3552-3159=393*, bunda raqamlar takrorlanadi, masala shartiga zid
3) *3774-3159=615*, bunda $222 \cdot (6+1+5) - 615 = 2049 \neq 3159$ ziddiyat
4) *3996-3159=837*, bunda $222 \cdot (8+3+7) - 837 = 3159$

Javob: 837

167. Bunday o‘chirishda 1-qadamda 1, 3, 5, ..., 2-qadamda $2\cdot1$, $2\cdot3$, $2\cdot5$, ..., 3-qadamda $2^2\cdot1$, $2^2\cdot3$, $2^2\cdot5$, ... va hokazo(2 ning darajalari bilan toq sonlar ko‘paytmasi) ko‘rinishidagi sonlar o‘chib boradi. Oxirgi qadamda $2^n\cdot1$ (bunda $2^n\cdot1\le2018$) ko‘rinishidagi bitta son qoladi. Bundan $n=10$ yoki $2^{10}=1024$ ekanini topish mumkin.

Javob: 1024

168. Oldin ketma-ketlikning $(n+1)$-hadini topib olamiz:

$$a_{n+1}=\frac{n+2}{n}\cdot(a_1+a_2+\ldots+a_{n-1}+a_n)=\frac{n+2}{n}\cdot\left(\frac{n-1}{n+1}\cdot a_n+a_n\right)=$$

$$=\frac{n+2}{n}\cdot\frac{2n}{n+1}\cdot a_n=\frac{2(n+2)}{n+1}\cdot a_n$$

Bundan quyidagi natijaga ega bo‘lamiz:

$$a_{n+1}=\frac{2(n+2)}{n+1}\cdot a_n=\frac{2(n+2)}{n+1}\cdot\frac{2(n+1)}{n}\cdot a_{n-1}=$$

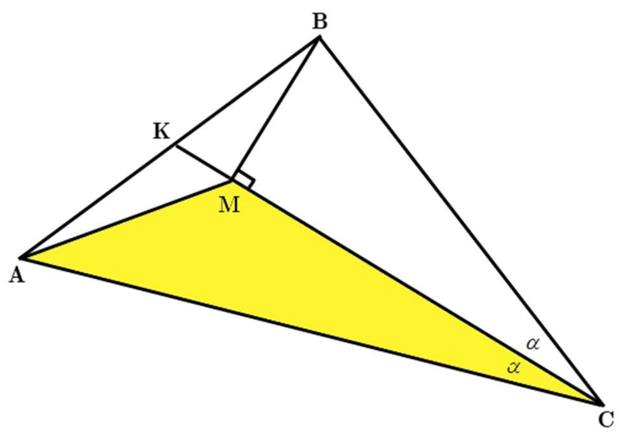
$$=\frac{2(n+2)}{n+1}\cdot\frac{2(n+1)}{n}\cdot\frac{2n}{n-1}\cdot a_{n-2}=\ldots=$$

$$=\frac{2(n+2)}{n+1}\cdot\frac{2(n+1)}{n}\cdot\frac{2n}{n-1}\cdot\ldots\cdot\frac{2\cdot3}{2}\cdot a_1=\frac{2^n(n+2)}{2}\cdot a_1=2^{n-1}(n+2)\cdot a_1$$

Agar $n=2019$ desak, $a_{2020}=2^{2019-1}(2019+2)\cdot1=2021\cdot2^{2018}$ ekanligi kelib chiqadi.

Javob: $a_{2020}=2021\cdot2^{2018}$

169. Uchburchakning CK bissektrisasini o‘tkazamiz va unga BM perpendikulyar tushirazmiz.



$\angle ACM = \angle BCM = \alpha$ deb belgilab olaylik. U holda ABC uchburchakning yuzi 12 ga tengligidan $S_{ABC} = \frac{1}{2} \cdot AC \cdot BC \cdot \sin 2\alpha = 12$ tenglikka ega bo‘lamiz. To‘g‘ri burchakli BMC uchburchakda $\cos\alpha = \frac{CM}{BC} \Rightarrow CM = BC\cos\alpha$ ekanligi ma’lum. Shularga asosan AMC uchburchakning yuzini quyidagicha topamiz:

$$S_{AMC} = \frac{1}{2} \cdot AC \cdot CM \cdot \sin\alpha = \frac{1}{2} \cdot AC \cdot BC \cdot \cos\alpha \cdot \sin\alpha =$$

$$= \frac{1}{2} \cdot \frac{1}{2} \cdot AC \cdot BC \cdot \sin 2\alpha = \frac{1}{2} \cdot S_{ABC} = \frac{1}{2} \cdot 12 = 6$$

Javob: 6

170. Masala shartini qanoatlantiruvchi uch xonali sonlarni $\overline{abc}$ ko‘rinishda izlaymiz. U holda quyidagi tengliklar o‘rinli:

$$\overline{abc} = 20(a + b + c)$$

$$100a + 10b + c = 20a + 20b + 20c$$

$$10(8a - b) = 19c$$

Oxirgi tenglikning chap tomoni 10 ga bo‘lingani uchun uning o‘ng tomoni ham 10 ga bo‘linishi lozimligi kelib chiqadi. Bu esa faqat $c = 0$ bo‘lganda bajariladi. Bundan $8a = b$ tenglik hosil bo‘ladi va $a = 1,\ b = 8$ ekanini topishimiz mumkin. Demak, biz izlayotgan uch xonali son 180 ga teng.

Javob: 180

171. 10 ta sharchaning har birini tanlashda 3 xil variant bor. U holda kombinatorikaning asosiy qoidasi(ko‘paytirish qoidasi)ga ko‘ra jami $\underbrace{3 \cdot 3 \cdot \ldots \cdot 3}_{10\,ta} = 3^{10}$ ta variant bor.

Javob: 3^{10}

172. $n = d_1 x$ va $n = d_2 y$ deb olaylik, bunda $x, y \in \mathbb{N}$. Ushbu $d_1 > d_2$ munosabatdan $\frac{n}{x} > \frac{n}{y}$ yoki $y > x$ ga ega bo‘lamiz. Agar $x, y \in \mathbb{N}$ ekanini hisobga olsak, $y - x \geq 1$ tengsizlik o‘rinli. U holda $y > x$ va $y - x \geq 1$ tengsizliklarni hadma-had ko‘paytirib, $y(y-x)>x$ ni hosil qilamiz. Oxirgi tengsizlikka $x=\frac{n}{d_1}$ va $y=\frac{n}{d_2}$ larni qo‘yamiz:

$$y(y-x)>x \Rightarrow \frac{n}{d_2}\left(\frac{n}{d_2}-\frac{n}{d_1}\right)>\frac{n}{d_1} \Rightarrow \frac{n^2(d_1-d_2)}{d_1d_2^2}>\frac{n}{d_1} \Rightarrow d_1>d_2+\frac{d_2^2}{n}$$

Shuni isbotlash talab qilingan edi.

173. $(a-b)^2\geq 0$ tengsizlikning o‘rinli ekanini yaxshi bilamiz. Ikkala tomoniga c^2 ni qo‘shamiz va shakl almashtirishlar natijasida quyidagilarga ega bo‘lamiz:

$$c^2+(a-b)^2\geq c^2 \Rightarrow c^2\geq c^2-(a-b)^2 \Rightarrow c^2\geq(c-a+b)(c+a-b)$$

Xuddi shunga o‘xshash $a^2\geq(a-c+b)(a+c-b)$ va $b^2\geq(b-a+c)(b+a-c)$ tengsizliklar o‘rinli ekanini topish mumkin. Hosil bo‘lgan tengsizliklarni hadma-had ko‘paytirsak, quyidagi muhim tengsizlikka ega bo‘lamiz:

$$a^2b^2c^2\geq(a+b-c)^2(a-b+c)^2(b+c-a)^2$$

$$abc\geq(a+b-c)(a-b+c)(b+c-a)$$

Tengsizlikda tenglik sharti $a=b=c$ bo‘lganda bajariladi.

174. 173-masaladagi tengsizlikning o‘ng tomonidagi qavslarni ochib, soddalashtiramiz:

$$abc\geq(ab+ac-a^2+b^2+bc-ab-bc-c^2+ac)(b+c-a)$$

$$abc\geq(2ac-a^2+b^2-c^2)(b+c-a)$$

$$abc\geq 2a^2c+2ac^2-2abc-a^3-a^2c+a^2b+ab^2+b^2c-b^3-ac^2-c^3+bc^2$$

$$a^3+b^3+c^3+3abc\geq a^2c+ac^2+b^2c+bc^2+a^2b+ab^2$$

$$a^3+b^3+c^3+5abc\geq a^2c+ac^2+b^2c+bc^2+a^2b+ab^2+2abc$$

$$a^3+b^3+c^3+5abc\geq(a+b)(b+c)(a+c)$$

Tengsizlikda tenglik sharti $a=b=c$ bo‘lganda bajariladi.

175. 173-masalada a,b,c larni uchburchak tomonlari deb olsak ham natija o‘zgarmaydi. U holda 173-masaladagi tengsizlikning ikkala tomonini $\frac{a+b+c}{16}$ ga ko‘paytirib, shakl almashtirishlar orqali quyidagilarga ega bo‘lamiz:

$$\frac{abc(a+b+c)}{16}\geq\frac{(a+b+c)}{2}\frac{(a+b-c)}{2}\frac{(a-b+c)}{2}\frac{(b+c-a)}{2}$$

Tengizlikning o‘ng tomoniga uchburchak yuzi uchun Geron formulasini qo‘llasak, isbotlanishi kerak bo‘lgan $abc(a+b+c) \geq 16S^2$ tengsizlik hosil bo‘ladi. Tengsizlikda tenglik sharti uchburchak muntazam bo‘lganda bajariladi.

176. $x, y, z \geq 0$ sonlari uchun o‘rinli bo‘lgan ushbu $(x-y)^2 + (y-z)^2 + (x-z)^2 \geq 0$ tengsizlikda qavslarni ochib soddalashtirsak, quyidagi tengsizlikka ega bo‘lamiz:

$$x^2 + y^2 + z^2 \geq xy + yz + xz$$

Shunga asosan

$$a^4 + b^4 + c^4 \geq a^2b^2 + b^2c^2 + a^2c^2 \geq a^2bc + ab^2c + abc^2 = abc(a+b+c)$$

ekanligini topamiz. Agar 175-masaladagi munosabatni hisobga olsak, $a^4 + b^4 + c^4 \geq 16S^2$ tengsizlik kelib chiqadi. Tenglik sharti muntazam uchburchakda bajariladi.

177. Uchburchakda $a = 2R\sin\alpha$, $b = 2R\sin\beta$ va $c = 2R\sin\gamma$ formulalarni 176-masaladagi munosabatga qo‘yib, shakl almashtirishlar bajarsak, quyidagi tengsizlikka ega bo‘lamiz:

$$(2R\sin\alpha)^4 + (2R\sin\beta)^4 + (2R\sin\gamma)^4 \geq 16S^2$$

$$16R^4(\sin^4\alpha + \sin^4\beta + \sin^4\gamma) \geq 16S^2 \;\Rightarrow\; \sin^4\alpha + \sin^4\beta + \sin^4\gamma \geq \frac{S^2}{R^4}$$

178. Uchburchak uchun $R = \dfrac{abc}{4S}$ va $r = \dfrac{2S}{a+b+c}$ formulalarni 175-masaladagi ifodaga qo‘llaymiz:

$$abc(a+b+c) \geq 16S^2 \;\Rightarrow\; \frac{abc}{4S} \geq 2\cdot\frac{2S}{a+b+c} \;\Rightarrow\; R \geq 2r$$

Tengsizlikda tenglik sharti muntazam uchburchakda bajariladi.

179. To‘g‘ri burchakli uchburchakda $R = \dfrac{c}{2}$ va $r = \dfrac{a+b-c}{2}$ formulalarni 178-masaladagi ifodaga qo‘llaymiz:

$$\frac{c}{2} > 2\cdot\frac{a+b-c}{2} \;\Rightarrow\; c > 2a + 2b - 2c \;\Rightarrow\; 3c > 2a + 2b \;\Rightarrow\; 1{,}5c > a + b$$

Isbot tugadi.

180. *a)* O'rta arifmetik va o'rta geometrik haqidagi Koshi tengsizligiga ko'ra $a+b+c \ge 3\sqrt[3]{abc}$ ekanligini ma'lum. Buni $abc \le \frac{(a+b+c)^3}{27}$ ko'rinishda ham yozish mumkin. Oxirgi ifodani 175-masaladagi tengsizlikka qo'yamiz:

$$16S^2 \le abc(a+b+c) \le \frac{(a+b+c)^3}{27}\cdot(a+b+c) \Rightarrow 16S^2 \le \frac{(a+b+c)^4}{27}$$

Agar $a+b+c=2p$ ekanini hisobga olsak,

$$16S^2 \le \frac{(2p)^4}{27} \Rightarrow 16S^2 \le \frac{16p^4}{27} \Rightarrow S^2 \le \frac{p^4}{27} \Rightarrow S \le \frac{p^2}{3\sqrt{3}}$$

ekanligi kelib chiqadi. Tenglik sharti muntazam uchburchakda bajariladi.

b) Oddiy baholashdan foydalanamiz:

$$S \le \frac{p^2}{3\sqrt{3}} = \frac{p^2}{\sqrt{27}} < \frac{p^2}{\sqrt{16}} = \frac{p^2}{4} \Rightarrow S < \frac{p^2}{4}$$

c) Agar $S = pr$ (p-yarim perimetr) formulani $S \le \frac{p^2}{3\sqrt{3}}$ ifodaga qo'ysak, quyidagi tengsizlikka ega bo'lamiz:

$$S \le \frac{p^2}{3\sqrt{3}} \Rightarrow pr \le \frac{p^2}{3\sqrt{3}} \Rightarrow p \ge 3\sqrt{3}r$$

d) Agar $S = pr$ (p-yarim perimetr) formulani $S < \frac{p^2}{4}$ ifodaga qo'ysak, quyidagi tengsizlik hosil bo'ladi:

$$S < \frac{p^2}{4} \Rightarrow pr < \frac{p^2}{4} \Rightarrow p > 4r$$

e) 176-masalada foydalanilgan yordamchi tengsizlikka asosan $a^2+b^2+c^2 \ge ab+bc+ac$ munosabat o'rinli. Bundan $(a+b+c)^2 \le 3(a^2+b^2+c^2)$ ekanligi kelib chiqadi. $a+b+c=2p$ va *a)* ifodaga ko'ra:

$$4p^2 \le 3(a^2+b^2+c^2) \Rightarrow \frac{a^2+b^2+c^2}{\sqrt{3}} \ge \frac{4p^2}{3\sqrt{3}} \ge 4S \Rightarrow a^2+b^2+c^2 \ge 4\sqrt{3}S$$

181. $a+b+c=1$ ni 173-masaladagi tengsizlikka qo'llaymiz:

$$(a+b-c)(a-b+c)(b+c-a) \le abc \Rightarrow (1-2a)(1-2b)(1-2c) \le abc$$

$$1-2(a+b+c)+4(ab+bc+ac)-8abc \le abc$$

$$4(ab+bc+ac)\le 9abc+1 \;\Rightarrow\; 8(ab+bc+ac)\le 18abc+2$$

$a+b+c=1$ ni kvadratga oshirib $a^2+b^2+c^2=1-2(ab+bc+ac)$ ni hosil qilamiz.

$$8(ab+bc+ac)\le 18abc+2 \;\Rightarrow\; 6(ab+bc+ac)\le 18abc+1+1-2(ab+bc+ac)$$

$$6(ab+bc+ac)\le 18abc+1+a^2+b^2+c^2$$

$$6(ab+bc+ac)\le 18abc+1+6(a^2+b^2+c^2)-5(a^2+b^2+c^2)$$

$$5(a^2+b^2+c^2)-1\le 6(a^2+b^2+c^2)-6(ab+bc+ac)+18abc$$

Endi 59-masaladagi $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ac)$ ayniyatdan foydalanamiz. $a+b+c=1$ ekanidan $a^3+b^3+c^3=a^2+b^2+c^2-(ab+bc+ac)+3abc$ ekanligi kelib chiqadi. Uning ikkala tomonini 6 ga ko‘paytirib, yuqoridagilardan foydalansak quyidagi tengsizlikni hosil qilamiz:

$$6(a^3+b^3+c^3)=6(a^2+b^2+c^2)-6(ab+bc+ac)+18abc\ge 5(a^2+b^2+c^2)-1$$

$$6(a^3+b^3+c^3)\ge 5(a^2+b^2+c^2)-1 \;\Rightarrow\; 5(a^2+b^2+c^2)\le 6(a^3+b^3+c^3)+1$$

182. 174-masaladagi $a^3+b^3+c^3+3abc\ge a^2c+ac^2+b^2c+bc^2+a^2b+ab^2$ munosabatdan va Koshi tengsizligidan foydalanamiz:

$$a^3+b^3+c^3+3abc\ge (a+b)ab+(b+c)bc+(a+c)ac\ge 2(ab\sqrt{ab}+bc\sqrt{bc}+ac\sqrt{ac})$$

$a^3+b^3+c^3+3abc=8$ ekanini hisobga olib, quyidagi tengsizlikni hosil qilamiz:

$$\sqrt{(ab)^3}+\sqrt{(bc)^3}+\sqrt{(ac)^3}\le 4$$

183. Koshi tengsizligini qo‘llaymiz:

$$\sqrt[a+b+c]{\left(\frac{a+b-c}{a}\right)^a\cdot\left(\frac{b+c-a}{b}\right)^b\cdot\left(\frac{c+a-b}{c}\right)^c}\le$$

$$\le\frac{1}{a+b+c}\cdot\left(a\cdot\frac{a+b-c}{a}+b\cdot\frac{b+c-a}{b}+c\cdot\frac{c+a-b}{c}\right)=\frac{1}{a+b+c}\cdot(a+b+c)=1$$

$$\left(\frac{a+b-c}{a}\right)^a\cdot\left(\frac{b+c-a}{b}\right)^b\cdot\left(\frac{c+a-b}{c}\right)^c\le 1$$

$$(a+b-c)^a\cdot(b+c-a)^b\cdot(a+c-b)^c\le a^a\cdot b^b\cdot c^c$$

Tenglik sharti muntazam uchburchakda bajariladi.

184. Berilgan tengsizlikning chap qismidagi qavslarni ochib Koshi tengsizligini qo‘llaymiz. U holda quyidagilarni topamiz:

$$(a^2+2)(b^2+2)(c^2+2)=a^2b^2c^2+2(a^2b^2+b^2c^2+a^2c^2)+4(a^2+b^2+c^2)+8=$$

$$=2(a^2b^2+1)+2(b^2c^2+1)+2(a^2c^2+1)+3(a^2+b^2+c^2)+a^2b^2c^2+$$

$$+2+a^2+b^2+c^2\geq 4(ab+bc+ca)+3(ab+bc+ca)+(a^2+b^2+c^2)+$$

$$+2+a^2b^2c^2=a^2b^2c^2+a^2+b^2+c^2+2+7(ab+bc+ac)$$

Biz quyidagi tengsizlikni isbotlasak, masala yechiladi:

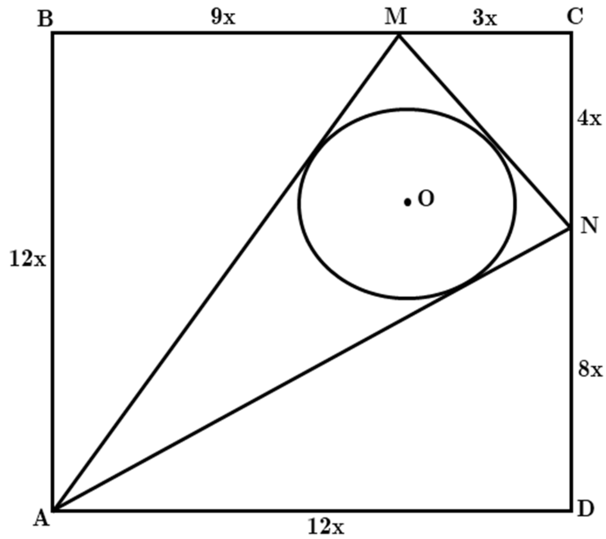
$$a^2b^2c^2+a^2+b^2+c^2+2\geq 2ab+2bc+2ac$$

182-masaladagi ushbu $a^3+b^3+c^3+3abc\geq 2(ab\sqrt{ab}+bc\sqrt{bc}+ac\sqrt{ac})$ munosabatga ko‘ra $a^2+b^2+c^2+3(abc)^{\frac{2}{3}}\geq 2ab+2bc+2ac$ tengsizlikni hosil qilamiz. Bu yerdan quyidagi

$$2ab+2bc+2ac\leq a^2+b^2+c^2+3(abc)^{\frac{2}{3}}\leq a^2+b^2+c^2+(abc)^2+2$$

munosabatni, ya’ni $a^2b^2c^2+a^2+b^2+c^2+2\geq 2ab+2bc+2ac$ ning to‘g‘ri ekanligini topamiz. Isbot tugadi.

185. Qulaylik uchun $a=12x$ deb belgilash kiritamiz. Berilganlarga ko‘ra, $BM=9x$, $MC=3x$, $CN=4x$ va $ND=8x$ ekanini topamiz.



Pifagor teoremasini qo‘llab, $AM=15x$, $MN=5x$ va $AN=4\sqrt{13}x$ larga ega bo‘lamiz. Endi AMN uchburchakning yuzini va perimetrini topamiz:

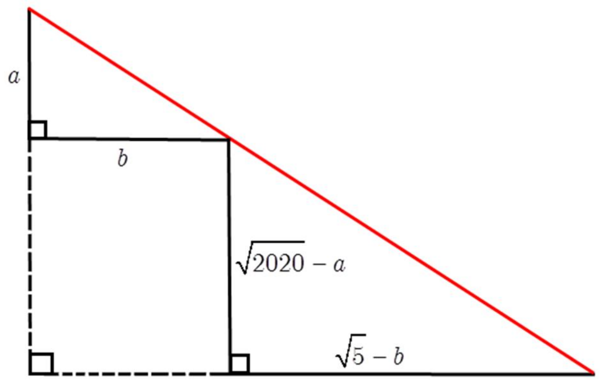
$$S_{AMN}=144x^2-\left(\frac{9x\cdot 12x}{2}+\frac{3x\cdot 4x}{2}+\frac{8x\cdot 12x}{2}\right)=36x^2 \text{ va } P_{AMN}=4(5+\sqrt{13})x$$

U holda $r = \dfrac{2S}{P}$ formulaga ko‘ra:

$$r = \frac{2 \cdot 36x^2}{4(5+\sqrt{13})} = \frac{18x}{5+\sqrt{13}} = \frac{18a}{12(5+\sqrt{13})} = \frac{3a}{2(5+\sqrt{13})}$$ ekanligi kelib chiqadi.

Javob: $\dfrac{3a}{2(5+\sqrt{13})}$

186. Katetlari a va b hamda $\sqrt{2020}-a$ va $\sqrt{5}-b$ bo‘lgan ikkita to‘g‘ri burchakli uchburchaklarni rasmdagidek joylashtiramiz.



Natijada katetlari $a+\sqrt{2020}-a=\sqrt{2020}$ va $b+\sqrt{5}-b=\sqrt{5}$ ga teng bo‘lgan katta to‘g‘ri burchakli uchburchak hosil bo‘ladi. Kichik uchburchaklar gipotenuzalarining yig‘indisi katta uchburchak gipotenuzasiga tengligidan foydalanib, quyidagiga ega bo‘lamiz:

$$\sqrt{a^2+b^2} + \sqrt{(\sqrt{2020}-a)^2 + (\sqrt{5}-b)^2} = \sqrt{(\sqrt{2020}^{2}) + (\sqrt{5})^2} = \sqrt{2025} = 45$$

Javob: 45

187. Koshi tengsizligidan foydalanamiz:

$$2^x + 2^{1-x-y} + 2^y \geq 3 \cdot \sqrt[3]{2^x \cdot 2^{1-x-y} \cdot 2^y} = 3\sqrt[3]{2}$$

Berilgan ifoda bu qiymatga $x = 1-x-y = y \;\Rightarrow\; x = y = \dfrac{1}{3}$ bo‘lganda erishadi.

Javob: $3\sqrt[3]{2}$

188. Masala ushbu $\dfrac{a}{b} + \dfrac{14b}{9a} = n$ tenglamani natural sonlarda yechishga keladi($n \in \mathbb{N}$).

$$\frac{a}{b} + \frac{14b}{9a} = n \;\Rightarrow\; 9a^2 - 9bna + 14b^2 = 0$$

Oxirgi tenglamani a ga nisbatan kvadrat tenglama sifatida yechamiz:

$$D = 81b^2n^2 - 4\cdot 9\cdot 14b^2 = 9b^2(9n^2 - 56)$$

Diskriminant to‘la kvadrat bo‘lishi uchun $9n^2 - 56 = m^2$ deb olamiz($m \in \mathbb{Z}$) va bu tenglamani natural sonlarda yechamiz. Quyidagi hollar bo‘lishi mumkin:

1) $\begin{cases} 3n - m = 1 \\ 3n + m = 56 \end{cases} \Rightarrow n, m \notin \mathbb{N}$

2) $\begin{cases} 3n - m = 2 \\ 3n + m = 28 \end{cases} \Rightarrow n = 5, m = 13$

3) $\begin{cases} 3n - m = 4 \\ 3n + m = 14 \end{cases} \Rightarrow n = 3, m = 9$

4) $\begin{cases} 3n - m = 7 \\ 3n + m = 8 \end{cases} \Rightarrow n, m \notin \mathbb{N}$

Kvadrat tenglamaga qaytib, quyidagi yechimlarga ega bo‘lamiz:

$$n = 5 \Rightarrow \begin{cases} a = \dfrac{14b}{3} \Rightarrow a = 14, b = 3 \\ a = \dfrac{b}{3} \Rightarrow a = 1, b = 3 \end{cases} \text{ va } n = 3 \Rightarrow \begin{cases} a = \dfrac{7b}{3} \Rightarrow a = 7, b = 3 \\ a = \dfrac{2b}{3} \Rightarrow a = 2, b = 3 \end{cases}$$

Javob: 4 ta

189. Tenglamalarning chap qismlarini bir-biriga tenglab x ni topib olamiz va tenglamalardan istalgan biriga qo‘yib, λ ni topamiz:

$$x^3 - \lambda x + 2 = x^2 + \lambda x + 2 \Rightarrow x(x^2 - x - 2\lambda) = 0$$

1) $x = 0 \Rightarrow \lambda \in \varnothing$

2) $x^2 - x - 2\lambda = 0$

$$x^2 = x + 2\lambda \Rightarrow x + 2\lambda + \lambda x + 2 = 0 \Rightarrow (\lambda + 1)(x + 2) = 0$$

$$\begin{cases} \lambda_1 = -1 \\ x = -2 \Rightarrow (-2)^2 - 2\lambda + 2 = 0 \Rightarrow \lambda_2 = 3 \end{cases}$$

Javob: $\lambda_1 = -1,\ \lambda_2 = 3$

190. Umumiylikka zarar yetkazmagan holda $a \geq b \geq 0$ deb olaylik($b \geq a \geq 0$ bo‘lganda ham xuddi shunday ko‘rsatiladi). U holda tenglamani quyidagicha yozamiz:

$$2^a + 2^b = c! \Rightarrow 2^b(2^{a-b} + 1) = c!$$

$c > 4$ bo‘lganda $c!\vdots 15$ ekanligi ma’lum. Ammo 2 ning darajalarini 15 ga bo‘lganda 1, 2, 4 yoki 8 qoldiqlar qolib, $2^b(2^{a-b} + 1)$ ifoda a va b larning hech bir qiymatida 15 ga bo‘linmaydi. Bundan $0 \le c \le 4$ ekanligi kelib chiqadi.

1) $c = 0 \Rightarrow 2^b(2^{a-b} + 1) = 0! = 1 \Rightarrow 2^{a-b} + 1 \ge 2 \Rightarrow a, b \in \varnothing$

2) $c = 1 \Rightarrow 2^b(2^{a-b} + 1) = 1! = 1 \Rightarrow 2^{a-b} + 1 \ge 2 \Rightarrow a, b \in \varnothing$

3) $c = 2 \Rightarrow 2^b(2^{a-b} + 1) = 2! = 2 \Rightarrow \begin{cases} 2^b = 1 \\ 2^{a-b} + 1 = 2 \end{cases} \Rightarrow a = b = 0$

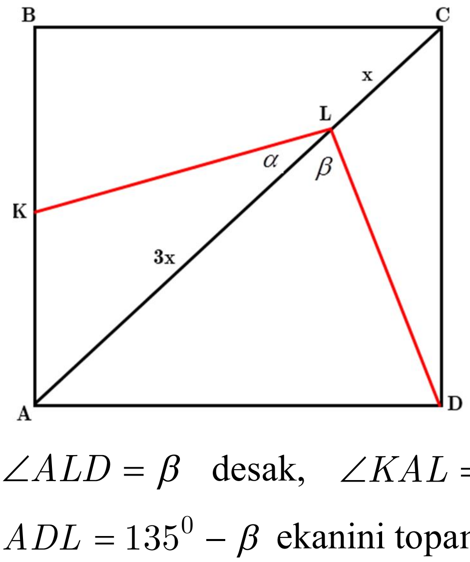
4) $c = 3 \Rightarrow 2^b(2^{a-b} + 1) = 3! = 6 \Rightarrow \begin{cases} 2^b = 2 \\ 2^{a-b} + 1 = 3 \end{cases} \Rightarrow a = 2; b = 1$

5) $c = 4 \Rightarrow 2^b(2^{a-b} + 1) = 4! = 24 \Rightarrow \begin{cases} 2^b = 8 \\ 2^{a-b} + 1 = 3 \end{cases} \Rightarrow a = 4; b = 3$

Demak, masala shartini $(0;0;2),(2;1;3),(1;2;3),(4;3;4),(3;4;4)$ sonlar uchligi qanoatlantiradi.

Javob: 5 ta

191. $LC = x$ deb belgilab olaylik. U holda $AL = 3x$ va kvadratning tomoni $2\sqrt{2}x$ ekanligi kelib chiqadi.



Agar $\angle KLA = \alpha$ va $\angle ALD = \beta$ desak, $\angle KAL = \angle DAL = 45^0$ ekanidan $\angle AKL = 135^0 - \alpha$ va $\angle ADL = 135^0 - \beta$ ekanini topamiz.

ΔAKL da sinuslar teoremasiga ko‘ra:

$$\frac{3x}{\sin(135^0-\alpha)}=\frac{\sqrt{2}x}{\sin\alpha}\Rightarrow 3\sin\alpha=\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos\alpha+\frac{1}{\sqrt{2}}\sin\alpha\right)\Rightarrow ctg\alpha=2$$

ΔADL da sinuslar teoremasiga ko‘ra:

$$\frac{3x}{\sin(135^0-\beta)}=\frac{2\sqrt{2}x}{\sin\beta}\Rightarrow 3\sin\beta=2\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos\beta+\frac{1}{\sqrt{2}}\sin\beta\right)\Rightarrow ctg\beta=\frac{1}{2}$$

Shularga asosan

$$\angle KLD=\alpha+\beta=arcctg2+arcctg\frac{1}{2}=arcctg\frac{2\cdot\frac{1}{2}-1}{2+\frac{1}{2}}=arcctg0=90^0$$

Javob: 90⁰

192. To‘g‘ri burchakli uchburchakning yuzi $\frac{ab}{2}$ formula yordamida topilishidan foydalanamiz:

$$\frac{\sqrt{3}}{12}(a^2+3b^2)=\frac{ab}{2}\Rightarrow a^2-2\sqrt{3}ab+3b^2=0\Rightarrow(a-\sqrt{3}b)^2=0\Rightarrow a=\sqrt{3}b$$

U holda burchak tangensi ta’rifidan uchburchakning o‘tkir burchaklari 30^0 va 60^0 ekanligi kelib chiqadi.

Javob: 30^0, 60^0

193. Buning uchun $f(x)=\frac{\ln x}{x}$ funksiyani $x\geq 3$ oraliqda qaraymiz. Uning hosilasini tekshiramiz:

$f'(x)=\frac{\frac{1}{x}\cdot x-\ln x}{x^2}=\frac{1-\ln x}{x^2}<0$ chunki, $x\geq 3$ da $\ln x\geq\ln e=1$ yoki $1-\ln x<0$. Demak, $f(x)=\frac{\ln x}{x}$ funksiya $x\geq 3$ da kamayuvchi funksiya ekan. Ya’ni, $x\geq 3$ oraliqdagi $x_1\geq x_2$ nuqtalar uchun $f(x_1)\leq f(x_2)$ tengsizlik o‘rinli. U holda $n+1>n$ ekanidan quyidagi tengsizlik o‘rinli:

$$f(n+1)<f(n)\Rightarrow\frac{\ln(n+1)}{n+1}<\frac{\ln n}{n}\Rightarrow n\ln(n+1)<(n+1)\ln n\Rightarrow$$

$$\Rightarrow (n+1)^n<n^{n+1}$$

Isbot tugadi.

194. $x \geq 2$ oraliqda ushbu $f(x) = \log_x(x+1)$ funksiyani qaraymiz. Uning hosilasini tekshiramiz:

$$f'(x) = \left(\frac{\ln(x+1)}{\ln x}\right)' = \frac{\frac{1}{x+1}\cdot\ln x - \frac{1}{x}\cdot\ln(x+1)}{\ln^2 x} = \frac{\ln x^{\frac{1}{x+1}} - \ln(x+1)^{\frac{1}{x}}}{\ln^2 x}$$

$x \geq 2$ ekanidan $(x+1)^{\frac{1}{x}} > x^{\frac{1}{x}} > x^{\frac{1}{x+1}}$ tengsizlik o‘rinli. Demak, $f'(x) < 0$. Bu esa $f(x)$ funksiyaning $x \geq 2$ oraliqda kamayuvchi ekanini bildiradi. Funksiyaning berilgan oraliqda monoton kamayuvchi ekanligidan quyidagilarga ega bo‘lamiz:

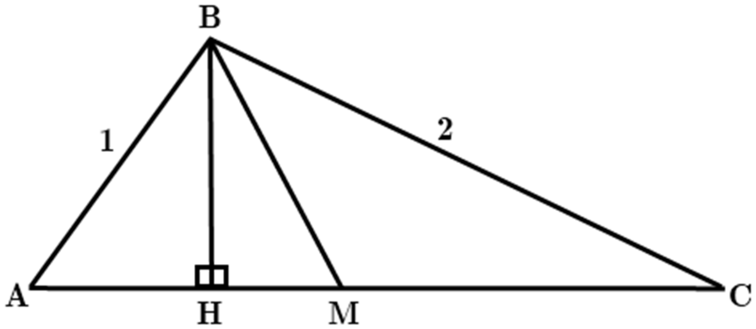
$$n+1 > n \;\Rightarrow\; f(n+1) < f(n) \;\Rightarrow\; \log_{n+1}(n+2) < \log_n(n+1)$$

Shuni isbotlash talab qilingan edi.

195. Xuddi 193-masaladagi kabi $x \geq e$ oraliqda $f(x) = \dfrac{\ln x}{x}$ funksiya kamayuvchi va $\pi > e$ ekanidan $f(\pi) < f(e) \;\Rightarrow\; \dfrac{\ln \pi}{\pi} < \dfrac{\ln e}{e} \;\Rightarrow\; \pi^e < e^\pi$ ekani kelib chiqadi.

Javob: $\pi^e < e^\pi$

196. $BM = AM = MC$ ekanidan $\angle B = 90^0$ ekanligi kelib chiqadi(to‘g‘ri burchakli uchburchakda gipotenuzaga tushirilgan mediana gipotenuzaning yarmiga teng).



Pifagor teoremasiga ko‘ra $AC = \sqrt{5}$ bundan $BM = \dfrac{\sqrt{5}}{2}$ ni topamiz.

$BH = \dfrac{2\cdot 1}{\sqrt{5}} = \dfrac{2}{\sqrt{5}}$ ekanidan

$$\cos\angle MBH = \frac{BH}{BM} = \frac{\frac{\sqrt{5}}{2}}{\frac{2}{\sqrt{5}}} = \frac{4}{5} = 0,8 \;\Rightarrow\; \angle MBH = \arccos 0,8$$

ekanligi kelib chiqadi.
Javob: $\arccos 0,8$

197. Agar $a+b+c=0$ bo‘lsa, u holda $a^3+b^3+c^3=3abc$ bo‘lishini isbotlash oson. Qulaylik uchun $x-y=a$, $y-z=b$, $z-x=c$ deb belgilab olamiz. Bundan quyidagi sistema kelib chiqadi:

$$\begin{cases} a+b+c=0 \\ 3abc=30 \end{cases} \Rightarrow \begin{cases} a+b+c=0 \\ abc=10 \end{cases}$$

10 ning butun bo‘luvchilari bo‘lgan -10, -5, -2, -1, 1, 2, 5, 10 sonlaridan yuqoridagi sistemani qanoatlantiruvchi sonlar uchligini tuzib bo‘lmasligidan, berilgan tenglama butun sonlarda yechimga ega emasligi kelib chiqadi.
Javob: $\varnothing$

198. Uchburchakdagi $\sin\alpha=\dfrac{a}{2R}$, $\sin\beta=\dfrac{b}{2R}$, $\sin\gamma=\dfrac{c}{2R}$ tengliklarni 37-masalada isbotlangan ushbu $\sin^2\alpha+\sin^2\beta+\sin^2\gamma\le\dfrac{9}{4}$ tengsizlikka qo‘ysak, $a^2+b^2+c^2\le 9R^2$ yoki $R=2$ ekanligidan $a^2+b^2+c^2\le 36$ tengsizlikka ega bo‘lamiz. Uchburchak medianasi formulasidan medianalar kvadratlari yig‘indisini hisoblaymiz:

$$m_a^2+m_b^2+m_c^2=\frac{3}{4}(a^2+b^2+c^2)\le\frac{3}{4}\cdot 36=27$$

Bundan medianalar kvadratlari yig‘indisining eng katta qiymati 27 ga tengligi kelib chiqadi. Bu qiymatga uchburchak muntazam bo‘lganda erishadi.
Javob: 27

199. Istalgan $x,y,z\ge 0$ sonlari uchun doim o‘rinli bo‘lgan ushbu $x^2+y^2+z^2\ge xy+yz+xz$ tengsizlikdan va 198-masaladagi munosabatdan foydalanamiz:

$$(m_a+m_b+m_c)^2=m_a^2+m_b^2+m_c^2+2(m_am_b+m_bm_c+m_am_c)\le$$

$$\le m_a^2+m_b^2+m_c^2+2(m_a^2+m_b^2+m_c^2)=3(m_a^2+m_b^2+m_c^2)=$$

$$=3\cdot\frac{3}{4}(a^2+b^2+c^2)\le 3\cdot\frac{3}{4}\cdot 9R^2=\frac{81R^2}{4}$$

Bundan $m_a+m_b+m_c\le\dfrac{9R}{2}$ yoki $R=5$ ekanligidan $m_a+m_b+m_c\le 22,5$ ekanini topamiz. Shunga ko‘ra medianalar yig‘indisining eng katta qiymati 22,5 ga teng ekanligi kelib chiqadi. Bu qiymatga uchburchak muntazam bo‘lganda erishadi.

Javob: 22,5

200. Agar n natural sonining kanonik yoyilmasi $n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot \ldots \cdot p_n^{\alpha_n}$ ko‘rinishida bo‘lsa(bu yerda $p_1, p_2, \ldots, p_n$ -tub sonlar va $\alpha_1, \alpha_2, \ldots, \alpha_n \in \mathbb{N}$) u holda Eyler funksiyasi quyidagicha aniqlanadi:

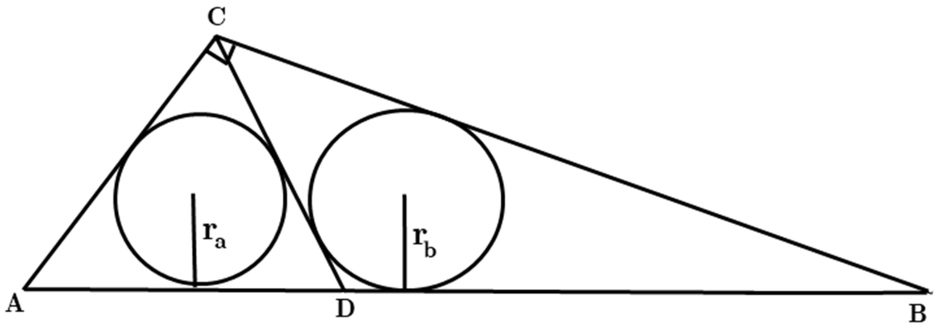
$$\varphi(n) = n \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdot \ldots \cdot \left(1 - \frac{1}{p_n}\right)$$

Endi $2020 = 2^2 \cdot 5 \cdot 101$ ekanini hisobga olib, $\varphi(2020)$ ni hisoblaymiz:

$$\varphi(2020) = 2020 \cdot \left(1 - \frac{1}{2}\right) \cdot \left(1 - \frac{1}{5}\right) \cdot \left(1 - \frac{1}{101}\right) = 800$$

Javob: 800

201. Uchburchakning to‘g‘ri burchakli ekanligi ma’lum.



Bissektrisa xossasiga ko‘ra:

$$\frac{3}{AD} = \frac{4}{5 - AD} \Rightarrow AD = \frac{15}{7} \Rightarrow DB = \frac{20}{7}$$

Bundan tashqari $CD = \dfrac{2 \cdot 3 \cdot 4 \cdot \cos\dfrac{90^0}{2}}{3 + 4} = \dfrac{12\sqrt{2}}{7}$ va

$\dfrac{S_{ACD}}{S_{BCD}} = \dfrac{\dfrac{1}{2} \cdot AC \cdot CD \cdot \sin 45^0}{\dfrac{1}{2} \cdot BC \cdot CD \cdot \sin 45^0} = \dfrac{AC}{BC}$ tengliklar o‘rinli. U holda $r = \dfrac{2S}{P}$

formulaga ko‘ra quyidagini topamiz:

$$\frac{r_a}{r_b} = \frac{2S_{ACD}}{P_{ACD}} : \frac{2S_{BCD}}{P_{BCD}} = \frac{S_{ACD}}{S_{BCD}} \cdot \frac{P_{BCD}}{P_{ACD}} =$$

$$=\frac{3}{4}\cdot\frac{\frac{48+12\sqrt{2}}{7}}{\frac{36+12\sqrt{2}}{7}}=\frac{3}{4}\cdot\frac{4+\sqrt{2}}{3+\sqrt{2}}=\frac{3}{28}(10-\sqrt{2})$$

Javob: $\frac{3}{28}(10-\sqrt{2})$

202. Quyidagicha shakl almashtirishlar bajaramiz:

$$7^{x+7}=8^x \Rightarrow 7^x\cdot 7=8^x \Rightarrow 7^7=\left(\frac{8}{7}\right)^x$$

$$x=\log_b 7^7=\log_b\left(\frac{8}{7}\right)^x=x\cdot\log_b\frac{8}{7} \Rightarrow 1=\log_b\frac{8}{7} \Rightarrow b=\frac{8}{7}$$

Javob: $\frac{8}{7}$

203. Ushbu $\cos(2A-B)+\sin(A+B)=2$ tenglik faqat va faqat $\cos(2A-B)=\sin(A+B)=1$ bo‘lganda bajarilishini hisobga olib, $\begin{cases}2A-B=0^0\\A+B=90^0\end{cases} \Rightarrow A=30^0, B=60^0$ ekanini topib olamiz. Bundan uchburchakning to‘g‘ri burchakli ekanligi kelib chiqadi. Gipotenuza $AB=4$ ekanidan katetlar 2 va $2\sqrt{3}$ ga tengligi, bundan uchburchakning yuzi $S_{ABC}=\frac{2\cdot 2\sqrt{3}}{2}=2\sqrt{3}$ ekanligini oson topish mumkin.

Javob: $2\sqrt{3}$

204. a_{n+1} ni almashtirirish bajarib quyidagicha yozib olamiz.

$$\frac{1}{a_{n+1}}=\frac{1}{a_n}+n=\frac{1}{a_{n-1}}+(n-1)+n=\frac{1}{a_{n-2}}+(n-2)+(n-1)+n=$$

$$=\ldots=\frac{1}{a_1}+1+2+\ldots+(n-1)+n=\frac{1}{a_1}+\frac{n(n+1)}{2}$$

Bundan $a_{n+1}=\frac{2a_1}{2+a_1n(n+1)}$ bo‘ladi. Endi a_{2021} ni hisoblaymiz:

$$a_{2021}=\frac{2}{2+2020(2020+1)}=\frac{1}{1010\cdot 2021+1}=\frac{1}{2041211}$$

Javob: $\dfrac{1}{2041211}$

205. Matematik induksiya metodidan foydalanamiz:

$n=1$ da $a_2=\dfrac{1}{1+1}\cdot(a_1+1)=\dfrac{1}{2}$

$n=2$ da $a_3=\dfrac{2}{1+2}\cdot(a_2+1)=\dfrac{2}{2}$

$n=3$ da $a_4=\dfrac{3}{1+3}\cdot(a_3+1)=\dfrac{3}{2}$

$n=k-1$ da $a_k=\dfrac{k-1}{2}$ deb faraz qilamiz.

$n=k$ uchun isbotlaymiz.

$$a_{k+1}=\frac{k}{k+1}\cdot(a_k+1)=\frac{k}{k+1}\cdot(\frac{k-1}{2}+1)=\frac{k}{k+1}\cdot\frac{k+1}{2}=\frac{k}{2}$$

Demak, $\forall n\in\mathbb{N}$ lar uchun $a_n=\dfrac{n-1}{2}$ tenglik o‘rinli ekan, bundan $a_{2021}=1010$ kelib chiqadi.

Javob: 1010

206. Ketma-ketlikning bir nechta hadlarini yozib olaylik.

$$a_1=2, a_2=3, a_3=\frac{3}{2}, a_4=\frac{1}{2}, a_5=\frac{1}{3}, a_6=\frac{2}{3}, a_7=2, a_8=3, a_9=\frac{3}{2},\ldots$$

Bundan ko‘rinib turibdiki, ketma-ketlikning hadlari har 6 sikldan takrorlanadi (matematik induksiya metodi orqali isbotlash oson). $2021=6\cdot 336+5$ tenglikdan $a_{2021}=\dfrac{1}{3}$ ekanini topamiz.

Javob: $\dfrac{1}{3}$

207. Matematik induksiya metodidan foydalanamiz.

$n=1$ da $a_2=a_1+\dfrac{1}{a_1^2}=1+\dfrac{1}{1^2}=2\ \Rightarrow\ a_2^3=2^3=8>3\cdot 2$

$n=2$ da $a_3=a_2+\dfrac{1}{a_2^2}=2+\dfrac{1}{2^2}=\dfrac{9}{4}\ \Rightarrow\ a_3^3=\dfrac{729}{64}>3\cdot 3$

$n=k$ da $a_k^3>3k$ tasdiqni to‘g‘ri deb faraz qilamiz.

$n=k+1$ uchun isbotlaymiz.

$$a_{k+1}^3 = (a_k + \frac{1}{a_k^2})^3 = a_k^3 + \frac{3}{a_k^3} + 3 + \frac{1}{a_k^6} > a_k^3 + 3 > 3k + 3 = 3(k+1)$$

Demak, $\forall n \in \mathbb{N}$ uchun $a_n^3 > 3n \Rightarrow a_{9000}^3 > 3 \cdot 9000 \Rightarrow a_{9000} > 30$. Isbot tugadi.

208. Shartga ko‘ra quyidagi tengliklar o‘rinli:

$$\begin{cases} a_1 - a_0 > 0 \\ a_2 - a_1 = 2(a_1 - a_0) \\ \dots \\ a_{100} - a_{99} = 2(a_{99} - a_{98}) \end{cases}$$

Shularga asosan $a_2 - a_1$, $a_3 - a_2$, ... , $a_{100} - a_{99}$ ayirmalarning musbat ekanligi kelib chiqadi. Shu bilan birga quyidagi nisbatlarni yoza olamiz:

$$\begin{cases} \dfrac{a_2 - a_1}{a_1 - a_0} = 2 \\ \dfrac{a_3 - a_2}{a_2 - a_1} = 2 \\ \dots \\ \dfrac{a_{100} - a_{99}}{a_{99} - a_{98}} = 2 \end{cases}$$

Bu 99 ta tenglikni bir-biriga ko‘paytirsak, $\frac{a_{100} - a_{99}}{a_1 - a_0} = 2^{99} \Rightarrow a_{100} = a_{99} + 2^{99}(a_1 - a_0)$ ni topamiz. Agar $a_{99} > 0$ va $a_1 - a_0 \geq 1$ ekanligini hisobga olsak, $a_{100} > 2^{99}$ tengsizlikning o‘rinli bo‘lishi kelib chiqadi.

209. $P_n = a_n - a_{n-1}$ deb belgilash kiritamiz. U holda masala shartiga ko‘ra $P_n = P_{n-1} + 1$ bo‘lib, bundan P_n ketma-ketlik ayirmasi 1 ga teng bo‘lgan arifmetik progressiya tashkil etishi kelib chiqadi. Shuning uchun $P_n = P_{n-1} + 1 = P_{n-2} + 1 + 1 = \dots = P_2 + n - 2$ tenglik o‘rinli. U holda quyidagiga ega bo‘lamiz:

$$a_n = (a_n - a_{n-1}) + (a_{n-1} - a_{n-2}) + \dots + (a_2 - a_1) + a_1 =$$
$$= P_n + P_{n-1} + \dots + P_2 + a_1 =$$
$$= (n-1)P_2 + (n-2) + (n-3) + \dots + 1 + a_1 =$$

$$= (n-1)(a_2 - a_1) + \frac{(n-2)(n-1)}{2} + a_1$$

Demak, $a_n = (n-1)a_2 - (n-2)a_1 + \dfrac{(n-2)(n-1)}{2}$

210. $i^2 = -1$ va geometrik progressiyaning dastlabki n ta hadi yig‘indisi formulasidan foydalanamiz:

$$\sum_{n=1}^{2019} i^n = i + i^2 + i^3 + \ldots + i^{2019} = \frac{i(i^{2019}-1)}{i-1} = \frac{i(i \cdot (i^2)^{1009} - 1)}{i-1} =$$

$$= \frac{i(-i-1)}{i-1} = \frac{-i^2 - i}{i-1} = \frac{1-i}{i-1} = -1$$

Javob: -1

211. Tenglamani quyidagicha yozib olamiz:

$$(x^2-2x)^3 + x\sqrt{x(x-2)^3} = 2 \Rightarrow (x^2-2x)^3 + \sqrt{x^3(x-2)^3} = 2$$

Bunda $x \in (-\infty; 0] \cup [2; \infty)$. Agar $\sqrt{(x^2-2x)^3} = a$ deb belgilash kiritsak($a \geq 0$) tenglama $a^2 + a - 2 = 0$ ko‘rinishga keladi. Bundan $a_1 = 1$ va $a_2 = -2$ ekanini topamiz. Belgilashga qaytib, $\sqrt{(x^2-2x)^3} = 1 \Rightarrow x^2 - 2x - 1 = 0 \Rightarrow x_{1,2} = 1 \pm \sqrt{2}$ yechimga ega bo‘lamiz.

Javob: $1 \pm \sqrt{2}$

212. Berilganlardan foydalanib, quyidagilarni topib olamiz:

$$f^1(x) = f(x) = \frac{1}{1-x}$$

$$f^2(x) = f(f(x)) = \frac{1}{1-f^1(x)} = \frac{1}{1-\dfrac{1}{1-x}} = \frac{1-x}{-x}$$

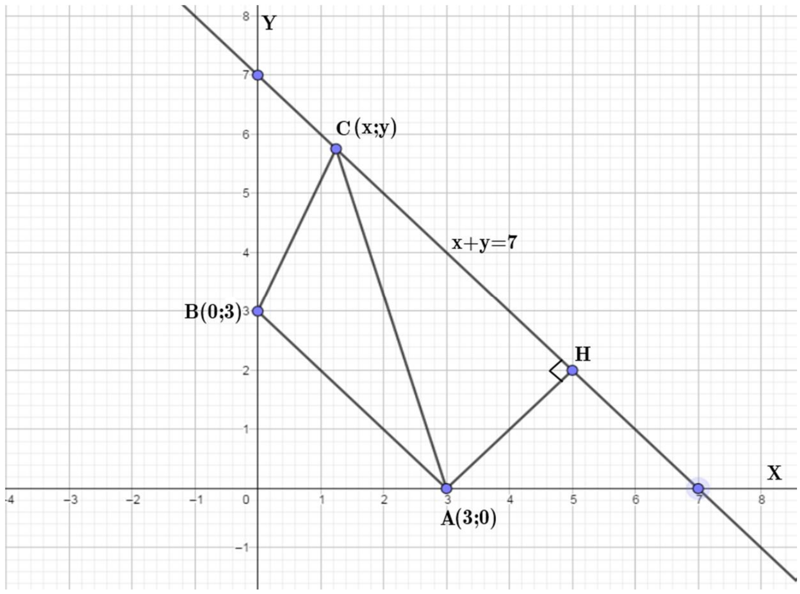
$$f^3(x) = f(f^2(x)) = \frac{1}{1-f^2(x)} = \frac{1}{1-\dfrac{1-x}{-x}} = x$$

$$f^4(x) = f(f^3(x)) = \frac{1}{1-f^3(x)} = \frac{1}{1-x} \text{ va hokazo.}$$

Bundan ko‘rinadiki, $f^k(x)$ funksiyalar 3 xil ko‘rinishda tasvirlanadi. Agar $2020 = 3 \cdot 673 + 1$ ekanini hisobga olsak, $f^{2020}(x) = \dfrac{1}{1-x}$ bo‘lib, bundan $f^{2020}(2020) = \dfrac{1}{1-2020} = -\dfrac{1}{2019}$ ekanligi kelib chiqadi.

Javob: $-\dfrac{1}{2019}$

213. Dekart koordinatalar sistemasida berilgan nuqtalarni joylashtirib, funksiya grafigini chizamiz.



C nuqtani $x + y = 7$ funksiya grafigining istalgan nuqtasiga qo‘yamiz. Pifagor teoremasiga ko‘ra $AB = 3\sqrt{2}$ ekani ma’lum. A va B nuqtalardan o‘tuvchi to‘g‘ri chiziqdan $x + y = 7$ to‘g‘ri chiziqqacha bo‘lgan masofa ABC uchburchakning balandligi bo‘la olishidan $A(3;0)$ nuqtadan $x + y = 7$ to‘g‘ri chiziqqacha bo‘lgan AH masofani topamiz:

$$AH = \frac{|3+0-7|}{\sqrt{1^2+1^2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

Bundan ABC uchburchakning yuzi $S_{ABC} = \dfrac{3\sqrt{2} \cdot 2\sqrt{2}}{2} = 6$ ekanligi kelib chiqadi.

Javob: 6

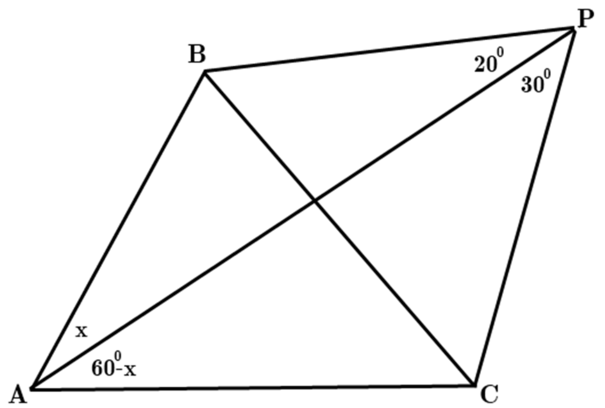
214. Tenglamani quyidagi ko‘rinishda yozib olamiz:

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{2020} \Rightarrow 2020x+2020y=xy \Rightarrow (x-2020)(y-2020)=2020^2$$

Agar $2020^2=2^4\cdot 5^2\cdot 101^2$ ekanini hisobga olsak, oxirgi tenglama $NBS(2020^2)=(4+1)(2+1)(2+1)=45$ ta tenglamalar sistemasini yechishga keladi. Bundan berilgan tenglamaning natural sonlarda 45 ta yechimga ega ekanligi kelib chiqadi.

Javob: 45 ta

215. $\angle BAP=x$ deb olib, ABP va ACP uchburchaklarga sinuslar teoremasini qo'llaymiz.



$$\frac{AB}{\sin 20^0}=\frac{AP}{\sin(180^0-(20^0+x))} \text{ va } \frac{AC}{\sin 30^0}=\frac{AP}{\sin(180^0-(30^0+60^0-x))}$$

Har ikkala ifodadagi AP larni tenglashtirib, $\frac{\sin(20^0+x)}{\sin 20^0}=\frac{\cos x}{\sin 30^0}$ yoki $\sin(20^0+x)=2\sin 20^0\cos x$ tenglamaga ega bo'lamiz. Tenglamani yechib, noma'lum x ning qiymatini topamiz:

$$\sin 20^0\cos x+\cos 20^0\sin x=2\sin 20^0\cos x \Rightarrow \sin(20^0-x)=0 \Rightarrow x=20^0$$

Javob: 20^0

216. $\sin x\le 1$ va $\sin y\le 1$ ekanidan $\sin x\sin y\le 1$ yoki $1-\sin x\sin y\ge 0$ ekanligi bevosita kelib chiqadi. $\sin x\sin y$ ifodani tengsizlikning o'ng tomoniga o'tkazib, ikkala tomonini kvadratga oshiramiz:

$$\left|\sin x-\sin y\right|^2\ge(1-\sin x\sin y)^2$$

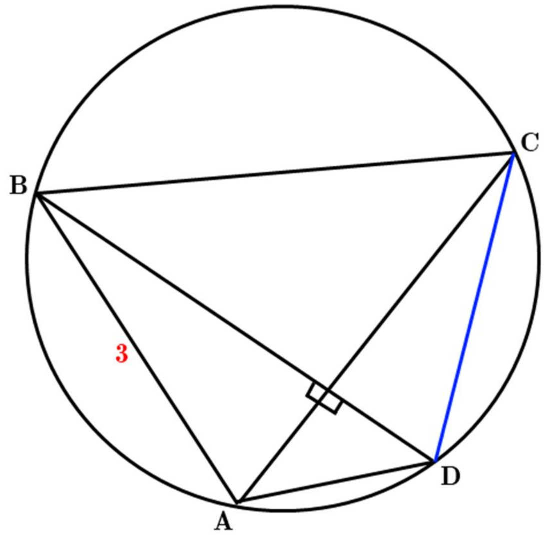
$$\sin^2 x-2\sin x\sin y+\sin^2 y\ge 1-2\sin x\sin y+\sin^2\sin^2 y$$

$$1-\sin^2 x-\sin^2 y+\sin^2 x\sin^2 y\le 0$$

$$(1-\sin^2 x)(1-\sin^2 y)\le 0$$

$\sin^2 x \leq 1$ va $\sin^2 y \leq 1$ ekanini hisobga olsak, oxirgi tengsizlik faqat $\sin^2 x = 1$ va $\sin^2 y = 1$ bo‘lganda bajarilishi kelib chiqadi. Bundan $x = \frac{\pi}{2} + \pi n, n \in \mathbb{Z}$ va $y = \frac{\pi}{2} + \pi m, m \in \mathbb{Z}$ yechimlarni topishimiz mumkin.

217. Agar $\angle ADB = \beta$ desak, $\angle DAC = 90^0 - \beta$ bo‘ladi.



Uchburchak uchun o‘rinli bo‘lgan $\frac{a}{\sin\alpha} = 2R$ formuladan foydalanamiz:

ABD uchburchakda $\frac{3}{\sin\beta} = 2 \cdot 2 \;\Rightarrow\; \sin\beta = \frac{3}{4} \;\Rightarrow\; \cos\beta = \frac{\sqrt{7}}{4}$

ACD uchburchakda $\frac{CD}{\sin(90^0 - \beta)} = 2 \cdot 2 \;\Rightarrow\; CD = 4\cos\beta = 4 \cdot \frac{\sqrt{7}}{4} = \sqrt{7}$

Javob: $\sqrt{7}$

218. Berilgan tenglamaning ikkala tomoniga 4 ni qo‘shamiz:

$$(x - 2\sqrt{2})(x + 2\sqrt{2}) + 4 = \frac{x^2}{1 - x} + 4$$

$$x^2 - 4 = \frac{(x-2)^2}{1-x} \;\Rightarrow\; (x-2)(x+2)(1-x) - (x-2)^2 = 0 \;\Rightarrow$$

$$\Rightarrow\; (x-2)(4 - 2x - x^2) = 0$$

Oxirgi tenglamadan $x_1 = 2$ va $x_{2,3} = -1 \pm \sqrt{5}$ yechimlarni topamiz.

Javo: $x_1 = 2$, $x_{2,3} = -1 \pm \sqrt{5}$

219. Berilganlarga ko‘ra quyidagilarni topamiz:

$$f(x)=\frac{x}{\sqrt{1+x^2}}$$

$$f(f(x))=\frac{f(x)}{\sqrt{1+f^2(x)}}=\frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+\frac{x}{1+x^2}}}=\frac{x}{\sqrt{1+2x^2}}$$

$$f(f(f(x)))=\frac{f(f(x))}{\sqrt{1+f^2(f(x))}}=\frac{\frac{x}{\sqrt{1+2x^2}}}{\sqrt{1+\frac{x}{1+2x^2}}}=\frac{x}{\sqrt{1+3x^2}}$$

Matematik induksiya metodi orqali $\underbrace{f(f(f(...f(x)...)))}_{n\,ta}=\frac{x}{\sqrt{1+nx^2}}$ ekanini isbotlash mumkin. U holda $\underbrace{f(f(f(...f(2020)...)))}_{2021\,ta}=\frac{2020}{\sqrt{1+2021\cdot 2020^2}}$ tenglik o‘rinli bo‘ladi.

220. $2009=7^2\cdot 41$ ekani ma’lum. Quyidagi hollar bo‘lishi mumkin:

1) $\begin{cases} a-b^2=1 \\ a+b^2=2009 \end{cases} \Rightarrow a=1005; b^2=1004 \Rightarrow b\notin\mathbb{N}$

2) $\begin{cases} a-b^2=7 \\ a+b^2=287 \end{cases} \Rightarrow a=147; b^2=140 \Rightarrow b\notin\mathbb{N}$

3) $\begin{cases} a-b^2=41 \\ a+b^2=49 \end{cases} \Rightarrow a=45; b^2=4 \Rightarrow b=2 \Rightarrow a+b=47$

Javob: 47

221. $i^2=-1$ bo‘lgani uchun berilgan ko‘phadni Nyuton binomi formulasi bo‘yicha ochib chiqqanimizda x ning juft darajalari oldidagi koeffitsientlar haqiqiy, toq darajalari oldidagi koeffitsientlar esa mavhum ekanini topamiz. U holda so‘ralgan yig‘indi quyidagicha topiladi:

$$\frac{P(1)+P(-1)}{2}=\frac{(1+i)^{2020}+(1-i)^{2020}}{2}=\frac{((1+i)^2)^{1010}+((1-i)^2)^{1010}}{2}=$$

$$=\frac{(2i)^{1010}+(-2i)^{1010}}{2}=\frac{2\cdot 2^{1010}\cdot(i^2)^{505}}{2}=-2^{1010}$$

Javob: -2^{1010}

222. Berilgan ifodalarni ikkinchisidan boshlab $x+y$ ga ko‘paytirish orqali quyidagilarga ega bo‘lamiz:

$$(ax^2+by^2)(x+y)=7(x+y)\ \Rightarrow\ 16+(ax+by)xy=7(x+y)\ \Rightarrow$$

$$\Rightarrow\ 16+3xy=7(x+y)$$

$$(ax^3+by^3)(x+y)=16(x+y)\ \Rightarrow\ 42+(ax^2+by^2)xy=16(x+y)\ \Rightarrow$$

$$\Rightarrow\ 42+7xy=16(x+y)$$

Hosil bo‘lgan tengliklardan $x+y=-14$ va $xy=-38$ ekanini topib olamiz. Xuddi yuqoridagidek amallarni bajarib, so‘ralgan qiymatni topamiz:

$$(ax^4+by^4)(x+y)=42(x+y)\ \Rightarrow\ ax^5+by^5+(ax^3+by^3)xy=42(x+y)\ \Rightarrow$$

$$\Rightarrow\ ax^5+by^5+16xy=42(x+y)\ \Rightarrow\ ax^5+by^5=42\cdot(-14)-16\cdot(-38)=20$$

Javob: 20

223. Quyidagi baholashlardan foydalanamiz:

$$a=5^{56}=25^{28}<31^{28}=d$$

$$d=31^{28}<32^{28}=2^{140}=(2^4)^{35}=16^{35}<17^{35}=c$$

$$c=17^{35}<20^{35}=2^{19}\cdot 2^{16}\cdot 10^{35}<2^{32}\cdot 2^{16}\cdot 10^{35}=$$

$$=4^{16}\cdot 2^{16}\cdot 10^{35}<5^{16}\cdot 2^{16}\cdot 10^{35}=10^{51}=b$$

Shularga ko‘ra, $a<d<c<b$

224. Matematik induksiya metodidan foydalanamiz.

$n=3$ da $3+\frac{1}{2}\cdot 2=4=2\cdot(3-1)$

$n=4$ da $4+\frac{1}{2}\left(3+\frac{1}{2}\cdot 2\right)=4+2=6=2\cdot(4-1)$

$n=k$ da $k+\frac{1}{2}\left((k-1)+\frac{1}{2}\left((k-2)+\ldots+\frac{1}{2}\left(3+\frac{1}{2}\cdot 2\right)\ldots\right)\right)=2(k-1)$ deb faraz qilamiz.

$n=k+1$ da

$$(k+1)+\frac{1}{2}\left(k+\frac{1}{2}\left((k-1)+\ldots+\frac{1}{2}\left(3+\frac{1}{2}\cdot 2\right)\ldots\right)\right)=k+1+\frac{1}{2}\cdot 2(k-1)=2k.$$

Demak, istalgan natural $n \geq 3$ uchun quyidagi tenglik o‘rinli:

$$n+\frac{1}{2}\left((n-1)+\frac{1}{2}\left((n-2)+\ldots+\frac{1}{2}\left(3+\frac{1}{2}\cdot 2\right)\ldots\right)\right)=2(n-1)$$

Javob: $2(n-1)$

225. $\frac{q}{p}$ ifodani qaraymiz:

$$\frac{2018}{2019}<\frac{p}{q}<\frac{2019}{2020} \Rightarrow \frac{2020}{2019}<\frac{q}{p}<\frac{2019}{2018} \Rightarrow 1\frac{1}{2019}<\frac{q}{p}<1\frac{1}{2018} \Rightarrow$$

$$\Rightarrow 1\frac{2}{4038}<\frac{q}{p}<1\frac{2}{4036}$$

Oxirgi munosabatni kamida $\frac{q}{p}=1\frac{2}{4037}=\frac{4039}{4037}$ tenglik qanoatlantirishidan $p_{\min}=4037$ ekanligini topamiz.

Javob: 4037

226. Oldin oxirgi raqami 1 bilan tugaydigan sonlar darajasining oxirgi ikki raqamini topish qoidasini keltiramiz:

$a_1, a_2, \ldots, a_n$***-raqamlar va*** $m \in \mathbb{N}$ ***soni uchun ushbu*** $\left(\overline{a_1 a_2 \ldots a_n 1}\right)^m = \overline{\ldots(a_n \cdot m)1}$ ***tenglik o‘rinli(bunda*** $a_n \cdot m$ ***ko‘paytmaning oxirgi raqami olinadi)***

$$17^{2021}=(17^4)^{505}\cdot 17=83521^{505}\cdot 17=\overline{\ldots(2\cdot 505)1}\cdot 17=\ldots 01\cdot 17=\ldots 17$$

Javob: 17

227. Oldin berilgan funksiyani soddaroq ko‘rinishda yozib olaylik:

$$f(x)=\sin^6\frac{x}{4}+\cos^6\frac{x}{4}=1-\frac{3}{4}\sin^2\frac{x}{2}=1-\frac{3}{4}\cdot\frac{1-\cos x}{2}=\frac{5}{8}+\frac{3}{8}\cos x$$

Endi hosilalarini qaraymiz:

$f'(x)=-\frac{3}{8}\sin x,\qquad f''(x)=-\frac{3}{8}\cos x,\qquad f'''(x)=\frac{3}{8}\sin x,\qquad f^{(4)}(x)=\frac{3}{8}\cos x,$

$f^{(5)}(x)=-\frac{3}{8}\sin x$ va hokazo. Bundan ko‘rinadiki, hosilalar har 4 sikldan takrorlanadi. U holda $2020 = 4 \cdot 505$ ekanligidan $f^{(2020)}(x) = \frac{3}{8}\cos x$ bo‘lib, bundan $f^{(2020)}(0) = \frac{3}{8}\cos 0 = \frac{3}{8}$ ekanini topish mumkin.

Javob: $\frac{3}{8}$

228. $a^2 + d^2 = 1$ va $b^2 + c^2 = 1$ tengliklarga asosan $a = \sin\alpha$, $d = \cos\alpha$, $b = \sin\beta$ va $c = \sin\beta$ deb belgilash kiritamiz. U holda $ac + bd = \frac{1}{3}$ tenglikdan $\sin\alpha\cos\beta + \cos\alpha\sin\beta = \frac{1}{3}$ yoki $\sin(\alpha+\beta) = \frac{1}{3}$ ekanligi kelib chiqadi. Shularga asosan quyidagini topamiz:

$$ab - cd = \sin\alpha\sin\beta - \cos\alpha\cos\beta = -\cos(\alpha+\beta) = -\sqrt{1-\sin^2(\alpha+\beta)} = \pm\frac{2\sqrt{2}}{3}$$

Javob: $\pm\frac{2\sqrt{2}}{3}$

229. Umumiylikka zarar yetkazmagan holda $a \ge b$ deb olamiz($b \ge a$ bo‘lgan holda ham xuddi shunga o‘xshash ko‘rsatiladi). $a^2 + b^2 < 16$ ga asosan $|a| < 4$ va $|b| < 4$ ekanligi ma’lum.

$$\begin{cases} 8b > a^2 + b^2 \ge b^2 + b^2 = 2b^2 \;\Rightarrow\; 8b > 2b^2 \;\Rightarrow\; b \in (0;4) \\ 16 > a^2 + b^2 \ge b^2 + b^2 = 2b^2 \;\Rightarrow\; b^2 < 8 \end{cases} \Rightarrow\; b = 1; b = 2$$

$a \ge b$ va $|a| < 4$ munosabatlardan $a = 1$, $a = 2$ yoki $a = 3$ ekanligini topamiz. Bundan masala shartini qanoatlantiruvchi ushbu $(1;1),(1;2),(2;1),(2;2),(3;1),(1;3)$ 6 ta juftlikni hosil qilamiz.

Javob: 6 ta

230. Quyidagicha shakl almashtirishlar bajarib, ushbu $2 < \frac{1010}{1009} + \frac{1009}{1010} < 2,25$ munosabatdan foydalanamiz:

$$\left[\frac{2020^3}{2018\cdot 2019} - \frac{2018^3}{2019\cdot 2020}\right] = \left[\frac{2020^4 - 2018^4}{2018\cdot 2019\cdot 2020}\right] =$$

$$=\left[\frac{2\cdot 4038\cdot(2020^2+2018^2)}{2018\cdot 2019\cdot 2020}\right]=\left[\frac{4\cdot(2020^2+2018^2)}{2018\cdot 2020}\right]=\left[4\cdot\left(\frac{2020}{2018}+\frac{2018}{2020}\right)\right]=$$

$$=\left[4\cdot\left(\frac{1010}{1009}+\frac{1009}{1010}\right)\right]=8$$

Javob: 8

231. O'rta qiymatlar haqidagi teoremadan foydalanib, berilgan ifodani quyidagicha tasvirlaymiz:

$$3=k^2\left(\frac{x^2}{y^2}+\frac{y^2}{x^2}\right)+k\left(\frac{x}{y}+\frac{y}{x}\right)\geq 2k^2+2k \;\Rightarrow\; 2k^2+2k-3\leq 0$$

Oxirgi tengsizlikni yechib, $k\in\left[\frac{-1-\sqrt{7}}{2};\frac{\sqrt{7}-1}{2}\right]$ ekanini topamiz. Bundan

$k_{\max}=\frac{\sqrt{7}-1}{2}$ tenglikka ega bo'lamiz.

Javob: $\frac{\sqrt{7}-1}{2}$

232. Trigonometrik formulalardan foydalanamiz:

$$\frac{\sin 10^0+\sin 20^0+\ldots+\sin 70^0+\sin 80^0}{\cos 5^0\cos 10^0\cos 20^0}=$$

$$=\frac{(\sin 10^0+\sin 80^0)+\ldots+(\sin 40^0+\sin 50^0)}{\cos 5^0\cos 10^0\cos 20^0}=$$

$$=\frac{2\sin 45^0(\cos 35^0+\cos 25^0+\cos 15^0+\cos 5^0)}{\cos 5^0\cos 10^0\cos 20^0}=$$

$$=\frac{\sqrt{2}((\cos 35^0+\cos 5^0)+(\cos 25^0+\cos 15^0))}{\cos 5^0\cos 10^0\cos 20^0}=$$

$$=\frac{\sqrt{2}(2\cos 20^0\cos 15^0+2\cos 20^0\cos 5^0)}{\cos 5^0\cos 10^0\cos 20^0}=\frac{2\sqrt{2}\cos 20^0(\cos 15^0+\cos 5^0)}{\cos 5^0\cos 10^0\cos 20^0}=$$

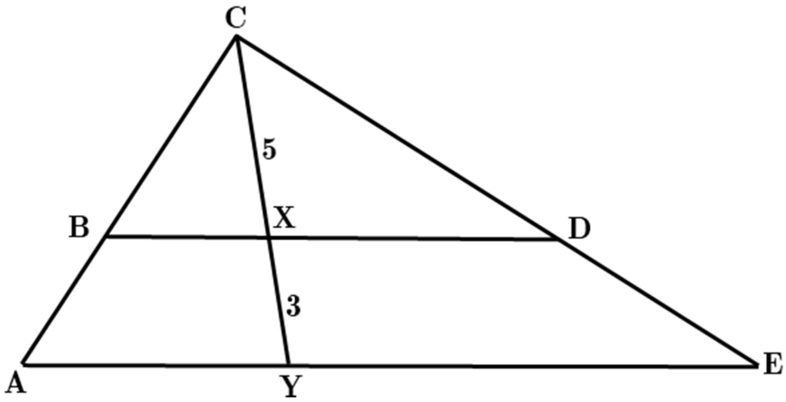
$$=\frac{4\sqrt{2}\cos 10^0\cos 5^0}{\cos 5^0\cos 10^0}=4\sqrt{2}$$

Javob: $4\sqrt{2}$

233. *Javob: Mumkin*

Qoplarni almashtirib yubormaslik maqsadida ularni 1 dan 10 gacha nomerlab chiqamiz. Shundan keyin qo‘limizga birinchi qopdan 1 ta tanga, ikkinchi qopdan 2 ta tanga va hokazo o‘ninchi qopdan 10 ta tanga olamiz. Qo‘limizda jami 55 ta tanga bo‘ldi. Agar hammasi har bir tanga 10 grammdan bo‘lganda edi, qo‘limizdagi 55 ta tanganing og‘irligi 550 gramm bo‘lar edi. Endi shu 55 ta tangani elektron taroziga birdaniga qo‘yamiz. Agar tarozi 549 grammni ko‘rsatsa, qalbaki tangalar 1-qopda, 548 grammni ko‘rsatsa qalbaki tangalar 2-qopda va hokazo 540 grammni ko‘rsatsa qalbaki tangalar 10-qopda ekanini aniqlash mumkin.

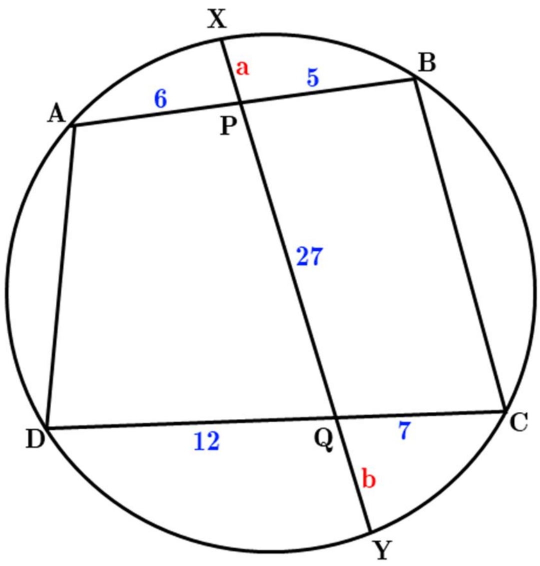
234. Masala shartiga mos chizma chizib olamiz.



ACY va BCX uchburchaklarning o‘xshashligidan $\frac{AC}{BC}=\frac{CY}{CX}=\frac{8}{5}$ tenglikni topib olamiz. ACE va BCD uchburchaklarning o‘xshashligidan esa, $\frac{S_{BCD}}{S_{ACE}}=\left(\frac{BC}{AC}\right)^2=\frac{25}{64}\Rightarrow S_{BCD}=\frac{25}{64}S_{ACE}$ ni topish mumkin. U holda $S_{ABDE}=S_{ACE}-S_{BCD}=\frac{39}{64}S_{ACE}$ tenglikka ko‘ra $\frac{S_{ABDE}}{S_{BCD}}=\frac{39}{25}$ ekanligi kelib chiqadi.

Javob: $\frac{39}{25}$

235. Masala shartiga mos chizma chizamiz. $PX=a$ va $QY=b$ deb olaylik.



Aylanada kesishuvchi vatarlar xossasini qo‘llaymiz:

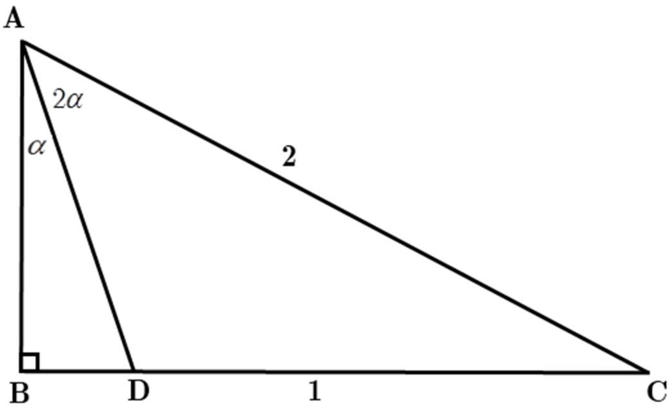
$$AP \cdot PB = XP \cdot PY \Rightarrow 6 \cdot 5 = a \cdot (27 + b)$$

$$DQ \cdot QC = YQ \cdot QX \Rightarrow 12 \cdot 7 = b \cdot (27 + a)$$

Bundan $a = 1$ va $b = 3$ ekanini topamiz. U holda $XY = 1 + 27 + 3 = 31$.

Javob: 31

236. $\angle BAD = \alpha \Rightarrow \angle CAD = 2\alpha$ va $BD = x$ deb olaylik.



$\angle ADC = 90^0 + \alpha$ ekanidan ΔADC da sinuslar teoremasini qo‘llaymiz:

$$\frac{2}{\sin(90^0 + \alpha)} = \frac{1}{\sin 2\alpha} \Rightarrow \sin\alpha = \frac{1}{4}$$

ΔABD da $\sin\alpha = \dfrac{x}{AD} = \dfrac{1}{4} \Rightarrow AD = 4x \Rightarrow AB = \sqrt{15}x$ ekanini topib olamiz.

Endi ΔABC da Pifagor teoremasiga ko‘ra $15x^2 + (x+1)^2 = 2^2 \Rightarrow x = \dfrac{3}{8}$ ekanligi kelib chiqadi.

Javob: $\frac{3}{8}$

237. Tenglamadan $x \geq 1$ ekanini topish qiyin emas.

$$x = \sqrt{x - \frac{1}{x}} + \sqrt{1 - \frac{1}{x}} \Rightarrow x - \sqrt{1 - \frac{1}{x}} = \sqrt{x - \frac{1}{x}} \Rightarrow$$

$$\Rightarrow x^2 - 2x\sqrt{1 - \frac{1}{x}} + 1 - \frac{1}{x} = x - \frac{1}{x} \Rightarrow x^2 - x - 2\sqrt{x^2 - x} + 1 = 0 \Rightarrow$$

$$\left(\sqrt{x^2 - x} - 1\right)^2 = 0 \Rightarrow x^2 - x = 1 \Rightarrow x = \frac{\sqrt{5} + 1}{2}$$

Javob: $x = \frac{\sqrt{5} + 1}{2}$

238. Tenglama butun sonlarda yechimga ega bo‘lishi uchun m soni juft ya’ni, $m = 2m_1$ ko‘rinishida bo‘lishi kerak($m_1 \in \mathbb{Z}$). U holda $231 \cdot 4m_1^2 = 130n^2 \Rightarrow 231 \cdot 2m_1^2 = 65n^2$ tenglama hosil bo‘ladi. Oxirgi tenglamada $n = 2n_1$ ko‘rinishida bo‘lishi kelib chiqadi($n_1 \in \mathbb{Z}$). Bundan $231m_1^2 = 130n_1^2$ tenglamaga ega bo‘lamiz. Bu jarayon cheksiz davom qiladi. 2 ning istalgan darajasiga bo‘linadigan son esa faqat 0 ga teng ekanligidan $m = n = 0$ yagona yechimga ega bo‘lamiz.

Javob: 1 ta

239. $n!$ soniga p tub soni $\left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \ldots$ daraja bilan kirishidan foydalanamiz.

$2020 = 2^2 \cdot 5 \cdot 101$ ekani ma’lum. Demak, $2020!$ soniga 2, 5 va 101 sonlari qanday daraja bilan kirishini topamiz.

$$\left[\frac{2020}{2}\right] + \left[\frac{2020}{2^2}\right] + \left[\frac{2020}{2^3}\right] + \left[\frac{2020}{2^4}\right] + \left[\frac{2020}{2^5}\right] + \left[\frac{2020}{2^6}\right] =$$

$$= \left[\frac{2020}{2^7}\right] + \left[\frac{2020}{2^8}\right] + \left[\frac{2020}{2^9}\right] + \left[\frac{2020}{2^{10}}\right] =$$

$$= 1010 + 505 + 252 + 126 + 63 + 31 + 15 + 7 + 3 + 1 = 2013$$

bundan $2^{2013} = (2^2)^{1006} \cdot 2$ (2^2 dan 1006 ta)

$$=\left[\frac{2020}{5}\right]+\left[\frac{2020}{5^2}\right]+\left[\frac{2020}{5^3}\right]+\left[\frac{2020}{5^4}\right]=404+80+16+3=503$$

bundan 5^{503} (5 dan 503 ta)

$\left[\frac{2020}{101}\right]=20$, bundan 101^{20} (101 dan 20 ta)

Biz yuqoridagilarga asosan ko'pi bilan 2020^{20} ni hosil qila olamiz.

Javob: $n_{\max}=20$

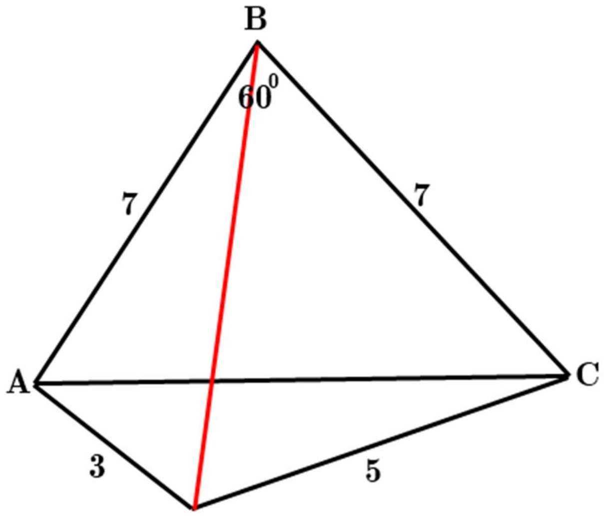
240. Berilgan ifodani quyidagicha ko'paytuvchilarga ajratamiz:

$$7^{2048}-1=(7-1)(7+1)(7^2+1)(7^4+1)(7^8+1)(7^{16}+1)(7^{32}+1)\times$$
$$\times(7^{64}+1)(7^{128}+1)(7^{256}+1)(7^{512}+1)(7^{1024}+1)$$

$7^{2n}=(8-1)^{2n}=8A+1$ ekanidan 7 ning juft darajalarini 4 ga va 8 ga bo'lganda 1 qoldiq qolishi kelib chiqadi. Shunga asosan ko'paytmadagi oxirgi 10 ta qavs ichidagi ifodaning har biri 2 ga bo'linadi(4 yoki 8 ga bo'linmaydi). Bundan tashqari $7-1=6=2\cdot 3$ va $7+1=8=2^3$ ekanini hisobga olsak, $n_{\max}=14$ natijaga ega bo'lamiz.

Javob: 14

241. $ABCD$ to'rtburchakning diagonallarini o'tkazamiz.



ABC uchburchakning muntazam ekani ma'lum. Bundan $AC=7$ ekanini topamiz. ADC uchburchakda kosinuslar teoremasini qo'llaymiz:

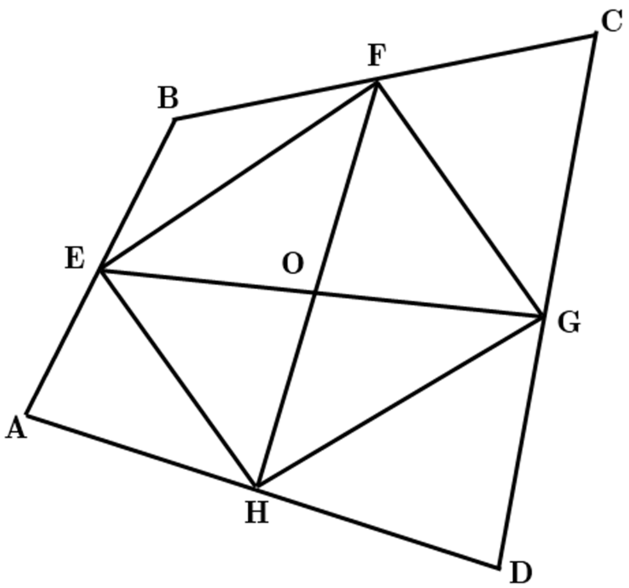
$$7^2=3^2+5^2-2\cdot 3\cdot 5\cdot\cos\angle D\ \Rightarrow\ \cos\angle D=-\frac{1}{2}\ \Rightarrow\ \angle D=120^0$$

Qavariq to‘rtburchakda $\angle B + \angle D = 180^0$ ekanligidan unga tashqi aylana chizish mumkinligi kelib chiqadi. U holda bu to‘rtburchak uchun Ptolomey teoremasi o‘rinli. Shunga ko‘ra:

$$AC \cdot BD = AB \cdot CD + AD \cdot BC \Rightarrow 7 \cdot BD = 7 \cdot 5 + 3 \cdot 7 \Rightarrow BD = 8$$

Javob: 8

242. Varinyon teoremasiga ko‘ra $EFGH$-parallelogramm va $S_{EFGH} = \frac{S_{ABCD}}{2}$ ekanligidan foydalanamiz.



Ushbu $S_{EFGH} = \frac{1}{2} \cdot EG \cdot FH \cdot \sin \angle EOH \leq \frac{1}{2} \cdot 12 \cdot 15 \cdot 1 = 90$ munosabatdan $\left(S_{ABCD}\right)_{\max} = 2 \cdot 90 = 180$ ekanligi kelib chiqadi. Bu qiymatga $EG \perp FH$ bo‘lganda erishadi.

Javob: 180

243. $a > b > c$ ekanidan $\frac{1}{a-1} < \frac{1}{c-1}$ va $\frac{1}{b-1} < \frac{1}{c-1}$ tengsizliklar o‘rinli. Shunga ko‘ra:

$$1 = \frac{1}{a-1} + \frac{1}{b-1} + \frac{1}{c-1} < \frac{1}{c-1} + \frac{1}{c-1} + \frac{1}{c-1}$$

$$1 < \frac{3}{c-1}$$

$$c < 4$$

munosabatga ega bo‘lamiz. $c \in \mathbb{N}$ ekanini hisobga olsak, quyidagi ikki hol bo‘lishi mumkin:

1-hol: $c = 2$.

$$\frac{1}{a-1} + \frac{1}{b-1} = 0 \Rightarrow a + b = 2 \Rightarrow a, b \in \varnothing$$

2-hol: $c = 3$.

$$\frac{1}{a-1} + \frac{1}{b-1} = \frac{1}{2} \Rightarrow 2a + 2b - 4 = ab - a - b + 1 \Rightarrow 3a + 3b - ab - 5 = 0 \Rightarrow$$

$$\Rightarrow (a-3)(b-3) = 4 \Rightarrow \begin{cases} a - 3 = 4 \\ b - 3 = 1 \end{cases} \Rightarrow a = 7, b = 4$$

U holda $a + 2b + 3c = 7 + 2 \cdot 4 + 3 \cdot 3 = 24$ ekanini topish mumkin.

Javob: 24

244. Ushbu $u = 2^x$, $v = x + 1$ va $w = a$ belgilashlarni kiritib olamiz. U holda berilgan tenglama $u^2 + v^2 + w^2 = uv + uw + vw$ ko‘rinishga keladi. Bundan $2u^2 + 2v^2 + 2w^2 = 2uv + 2uw + 2vw$ yoki $(u-v)^2 + (u-w)^2 + (v-w)^2 = 0$ ekanini topish mumkin. Oxirgi tenglik faqat $u = v = w$ da bajarilishi ma’lum. Belgilashlarga qaytsak, $\begin{cases} 2^x = x + 1 \\ x + 1 = a \end{cases}$ bundan $\begin{cases} x = 0 \\ a = x + 1 \end{cases}$ va $\begin{cases} x = 1 \\ a = x + 1 \end{cases}$ yoki $\begin{cases} a = 1 \\ x = 0 \end{cases}$ va $\begin{cases} a = 2 \\ x = 1 \end{cases}$ yechimlar kelib chiqadi.

Javob: $a = 1$ da $x = 0$, $a = 2$ da $x = 1$, $a \in (-\infty; 1) \cup (1; 2) \cup (2; \infty)$ da $x \in \varnothing$

245. $\sqrt{x^2 - \frac{7}{x^2}} = u \geq 0$, $\sqrt{x - \frac{7}{x^2}} = v \geq 0$ deylik. U holda ushbu $\begin{cases} u^2 - v^2 = x^2 - x \\ u + v = x \end{cases}$ sistema hosil bo‘ladi. Bunda $u^2 - v^2 = x^2 - x$ tenglama $x(u - v) = x(x - 1)$ ko‘rinishga keladi. $x \neq 0$ ekanidan u va v lar uchun quyidagi munosabatlar o‘rinli:

$$\begin{cases} u - v = x - 1 \\ u + v = x \end{cases} \Rightarrow \begin{cases} u = x - \frac{1}{2} \\ v = \frac{1}{2} \end{cases}$$

Belgilashlarga qaytib, $4x^3 - x^2 - 28 = 0$ tenglamani hosil qilamiz. Oxirgi tenglamaning chap qismini quyidagicha ko‘paytuvchilarga ajratamiz:

$$4x^3 - x^2 - 28 = (4x^3 - 8x^2) + (7x^2 - 14x) + (14x - 28) =$$

$$= (x-2)(x^2 + 7x + 14) = 0$$

Bundan $x = 2$ ekanligini topishimiz mumkin.

Javob: $x = 2$

246. $a, b \geq 0$ sonlari uchun doimo o‘rinli bo‘lgan $\frac{a+b}{2} \geq \sqrt{ab}$ tengsizlikdan foydalanamiz.

$$\begin{cases} \frac{1}{\sqrt{1 \cdot 2012}} > \frac{1}{\frac{1+2012}{2}} = \frac{2}{2013} \\ \frac{1}{\sqrt{2 \cdot 2011}} > \frac{1}{\frac{2+2011}{2}} = \frac{2}{2013} \\ \ldots \\ \frac{1}{\sqrt{k \cdot (2012-k+1)}} > \frac{1}{\frac{k+2012-k+1}{2}} = \frac{2}{2013} \\ \ldots \\ \frac{1}{\sqrt{2012 \cdot 1}} > \frac{1}{\frac{2012+1}{2}} = \frac{2}{2013} \end{cases}$$

Agar ularni hadma-had qo‘shsak, quyidagi tengsizlik kelib chiqadi:

$$\frac{1}{\sqrt{1 \cdot 2012}} + \frac{1}{\sqrt{2 \cdot 2011}} + \ldots + \frac{1}{\sqrt{k \cdot (2012-k+1)}} + \ldots + \frac{1}{\sqrt{2012 \cdot 1}} > 2 \cdot \frac{2012}{2013}$$

247. Nyuton binomi formulasidan foydalanamiz:

$$2^n = (1+1)^n = C_n^0 + C_n^1 + C_n^2 + C_n^3 + \ldots$$

$$0^n = (1-1)^n = C_n^0 - C_n^1 + C_n^2 - C_n^3 + \ldots$$

bu tengliklarni qo‘shsak, $2(C_n^0 + C_n^2 + C_n^4 + \ldots) = 2^n$ yoki $C_n^0 + C_n^2 + C_n^4 + \ldots = 2^{n-1}$ ga, birinchi tenglikdan ikkinchisini ayirsak, $2(C_n^1 + C_n^3 + C_n^5 + \ldots) = 2^n$ yoki $C_n^1 + C_n^3 + C_n^5 + \ldots = 2^{n-1}$ ga ega bo‘lamiz.

Javob: a) 2^{n-1} *b)* 2^{n-1}

248. Tenglamadagi har bir kasr uchun quyidagi munosabatlar o‘rinli:

$$+\begin{cases} \dfrac{1}{x^2+2x+2} = \dfrac{1}{(x+1)^2+1} \le 1 \\ \dfrac{3}{x^2+2x+4} = \dfrac{3}{(x+1)^2+3} \le 1 \\ \dots \\ \dfrac{2019}{x^2+2x+2020} = \dfrac{2019}{(x+1)^2+2019} \le 1 \end{cases} \Rightarrow$$

$$\Rightarrow \frac{1}{x^2+2x+2} + \frac{3}{x^2+2x+4} + \dots + \frac{2019}{x^2+2x+2020} \le 1010$$

Tenglik sharti faqat $(x+1)^2 = 0 \Rightarrow x = -1$ da bajariladi.

Javob: $x = -1$

249. Oldin $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{a+b+c}$ ifodadani quyidagicha yozib olamiz:

$$(ab+bc+ac)(a+b+c) = abc \Rightarrow (a+b)(b+c)(a+c) = 0$$

Oxirgi tenglikdan $a = -b$ yoki $b = -c$ yoki $a = -c$ ekanligi kelib chiqadi. Bu uch holatning istalgan birida $\frac{1}{a^{2021}} + \frac{1}{b^{2021}} + \frac{1}{c^{2021}} = \frac{1}{a^{2021} + b^{2021} + c^{2021}}$ tenglik bajariladi. Isbot tugadi.

250. Har bir qator, ustun va diagonaldagi sonlar yig‘indisi o‘zaro teng ekanligidan unumli foydalanamiz.

$$a+b+c = b+e+h \Rightarrow a+c = e+h \Rightarrow +\begin{cases} a+c = e+h \\ g+h+i = b+e+h \end{cases} \Rightarrow$$

$$\Rightarrow a+c+g+h+i = e+h+b+e+h \Rightarrow$$

$$+\begin{cases} a+c+g+h+i = e+h+b+e+h \\ a+b+c = d+e+f \\ g+h+i = b+e+h \end{cases} \Rightarrow$$

$$\Rightarrow 2(a+c+g+i) + b + 2h = 2b + d + f + 3h + 4e \Rightarrow$$

$$\Rightarrow 2(a+c+g+i) = b + d + f + h + 4e$$

251. Tenglamaning ikkala qismini ham $(x-1)^2$ ga ko‘paytiramiz:

$$(x-1)(x^2+x+1)(x-1)(x^{10}+x^9+\ldots+x+1)=\left((x-1)(x^6+x^5+\ldots+x+1)\right)^2$$

$$(x^3-1)(x^{11}-1)=(x^7-1)^2$$

$$x^{14}-x^{11}-x^3+1=x^{14}-2x^7+1$$

$$x^{11}-2x^7+x^3=0$$

$$x^3(x^8-2x^4+1)=0$$

$$x^3(x^4-1)^2=0 \Rightarrow x_1=0, x_2=-1, x_3=1$$

$x_3=1$ berilgan tenglamaning yechimi emasligidan $x_1=0$ va $x_2=-1$ yechimlarga ega bo‘lamiz.

Javob: $x_1=0$ *va* $x_2=-1$

252. Koshi tengsizligiga ko‘ra $x^4+2y^4+4z^4+2\geq 4\cdot\sqrt[4]{x^4\cdot 2y^4\cdot 4z^4\cdot 2}$ munosabat o‘rinli bo‘lib, bundan $x^4+2y^4+4z^4+2\geq 8xyz$ tengsizlik kelib chiqadi. Tenglik sharti faqat $x^4=2y^4=4z^4=2$ bo‘lganda bajarilishidan $x=\pm\sqrt[4]{2}$, $y=\pm 1$, $z=\pm\sqrt[4]{\frac{1}{2}}$ yechimlarga ega bo‘lamiz.

Javob: $x=\pm\sqrt[4]{2}$, $y=\pm 1$, $z=\pm\sqrt[4]{\frac{1}{2}}$

253. Ma’lumki, istalgan $a\in\mathbb{R}$ soni uchun $(a-1)^2\geq 0$ munosabat o‘rinli. Bunda qavslarni ochib, ikkala tomonini 2 ga ko‘paytirib, ikkala tomoniga a^2+a+1 ni qo‘shamiz:

$$(a-1)^2\geq 0$$

$$a^2-2a+1\geq 0$$

$$2a^2-4a+2\geq 0$$

$$2a^2-4a+2+a^2+a+1\geq a^2+a+1$$

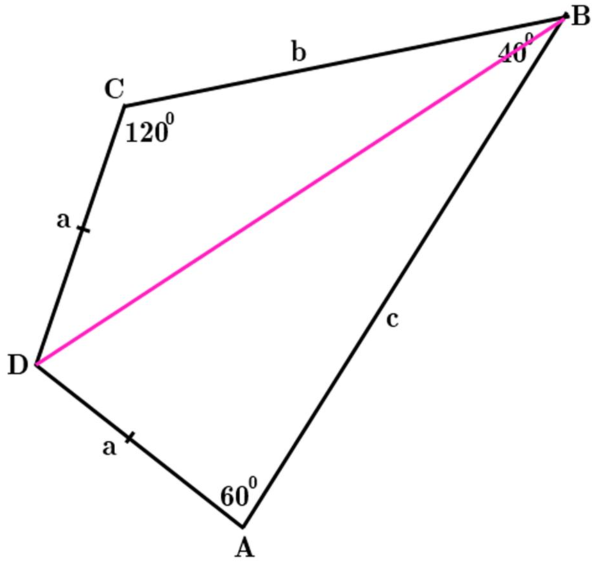
$$3(a^2-a+1)\geq a^2+a+1$$

Oxirgi tengsizlikning ikkala tomonini $a^2+a+1>0$ ifodaga ko‘paytirsak, isbotlanishi kerak bo‘lgan tengsizlik hosil bo‘ladi:

$$3(a^2-a+1)(a^2+a+1)\geq(a^2+a+1)^2 \Rightarrow 3(1+a^2+a^4)\geq(1+a+a^2)^2$$

Tenglik sharti $a=1$ bo‘lganda bajariladi.

254. Qulaylik uchun $AD = CD = a$, $BC = b$, $AB = c$ deb olaylik. To‘rtburchakning BD diagonalini o‘tkazamiz.



ABD va BCD uchburchaklarga kosinuslar teoremasini qo‘llaymiz:

$$BD^2 = a^2 + b^2 - 2ab\cos 120^0 \text{ va } BD^2 = a^2 + c^2 - 2ac\cos 60^0$$

Bu ifodalarni tenglashtirib, so‘ralgan tenglikni hosil qilamiz:

$$a^2 + b^2 + ab = a^2 + c^2 - ac \Rightarrow a(b+c) = (c-b)(c+b) \Rightarrow a + b = c \Rightarrow$$

$$\Rightarrow BC + CD = AB$$

255. Ushbu $x*(y*z) = (x*y) + z$ xossadan $x*y = x*(y*z) - z$ ekani ma’lum. Agar $x = 2021$, $y = 2020$ va $z = 2020$ desak, quyidagiga ega bo‘lamiz:

$$2021*2020 = 2021*(2020*2020) - 2020 =$$

$$= 2021*0 - 2020 = 2021*(2021*2021) - 2020 =$$

$$= (2021*2021) + 2021 - 2020 = 0 + 2021 - 2020 = 1$$

Javob: 1

256. Uchinchi darajali ko‘phadni $f(x) = nx^3 + mx^2 + kx + l$ ko‘rinishda izlaymiz($n \neq 0$).

$$\int_0^1 xf(x)dx = \int_0^1 (nx^4 + mx^3 + kx^2 + lx)dx = 0 \Rightarrow \left.\frac{nx^5}{5} + \frac{mx^4}{4} + \frac{kx^3}{3} + \frac{lx^2}{2}\right|_0^1 = 0$$

$$\int_0^1 x^3 f(x)dx = \int_0^1 (nx^6 + mx^5 + kx^4 + lx^3)dx = 0 \Rightarrow \left.\frac{nx^7}{7} + \frac{mx^6}{6} + \frac{kx^5}{5} + \frac{lx^4}{4}\right|_0^1 = 0$$

$$\int_0^1 x^5 f(x)dx = \int_0^1 (nx^8 + mx^7 + kx^6 + lx^5)dx = 0 \Rightarrow \left.\frac{nx^9}{9} + \frac{mx^8}{8} + \frac{kx^7}{7} + \frac{lx^6}{6}\right|_0^1 = 0$$

Bulardan quyidagi tenglamalar sistemasi kelib chiqadi:

$$\begin{cases} \frac{n}{5} + \frac{m}{4} + \frac{k}{3} + \frac{l}{2} = 0 \\ \frac{n}{7} + \frac{m}{6} + \frac{k}{5} + \frac{l}{4} = 0 \\ \frac{n}{9} + \frac{m}{8} + \frac{k}{7} + \frac{l}{6} = 0 \end{cases}$$

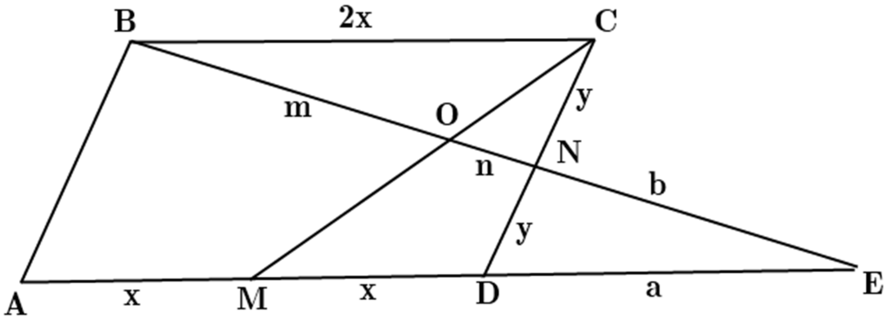
Agar $\frac{m}{n} = a$, $\frac{k}{n} = b$ va $\frac{l}{n} = c$ desak, sistema quyidagi ko‘rinishga keladi:

$$\begin{cases} \frac{1}{5} + \frac{a}{4} + \frac{b}{3} + \frac{c}{2} = 0 \\ \frac{1}{7} + \frac{a}{6} + \frac{b}{5} + \frac{c}{4} = 0 \\ \frac{1}{9} + \frac{a}{8} + \frac{b}{7} + \frac{c}{6} = 0 \end{cases}$$

Bu sistemani yechib, $a = -\frac{64}{35}$, $b = 1$ va $c = -\frac{16}{105}$ ekanini topamiz. Bundan $m = -\frac{64}{35}n$, $k = n$ va $l = -\frac{16}{105}n$ ekanligi kelib chiqadi. U holda biz izlayotgan uchinchi darajali ko‘phad $f(x) = nx^3 - \frac{64}{35}nx^2 + nx - \frac{16}{105}n$ ko‘rinishda bo‘ladi. Bunda $n \neq 0$.

Javob: $f(x) = nx^3 - \frac{64}{35}nx^2 + nx - \frac{16}{105}n$, *bunda* $n \neq 0$

257. BN va AD chiziqlar E nuqtada kesishsin. Qulaylik uchun $AM = MD = x$, $CN = ND = y$, $DE = a$, $EN = b$, $BO = m$ va $ON = n$ deb belgilab olaylik.



Birinchidan, BNC va DNE uchburchaklarning o‘xshashligidan:

$$\frac{m+n}{b}=\frac{y}{y}=\frac{2x}{a} \Rightarrow b=m+n,\ a=2x$$

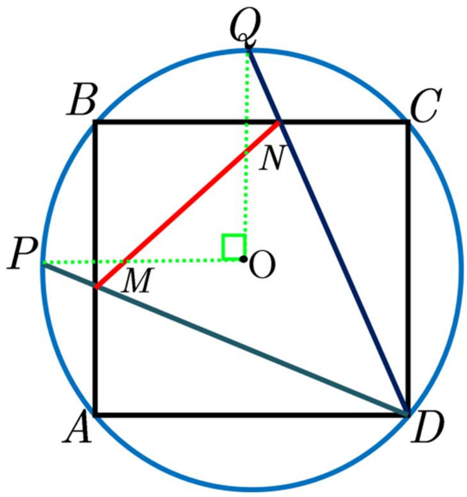
Ikkinchidan, BOC va MOE uchburchaklarning o‘xshashligidan:

$$\frac{m}{n+b}=\frac{2x}{3x}=\frac{CO}{OM} \Rightarrow \frac{CO}{OM}=\frac{2}{3},\ m=4n$$

Shularga ko‘ra $\frac{BO}{ON}\cdot\frac{CO}{OM}=\frac{m}{n}\cdot\frac{2}{3}=4\cdot\frac{2}{3}=\frac{8}{3}$ tenglik o‘rinli.

Javob: $\frac{8}{3}$

258. Aylana markazini O nuqta bilan belgilab, PO va QO kesmalarni o‘tkazamiz(rasmga qarang).



P va Q nuqtalar AB va BC yoylarning o‘rtalari ekanidan $\angle POQ=90^0$ ekani kelib chiqadi. U holda $\angle PDQ=\frac{90^0}{2}=45^0$ va $\angle CDN=\angle ADM=22,5^0$ ekanini topamiz. ΔCDN da $tg22,5^0=\frac{CN}{4} \Rightarrow CN=4(\sqrt{2}-1)$ ga ega bo‘lamiz.

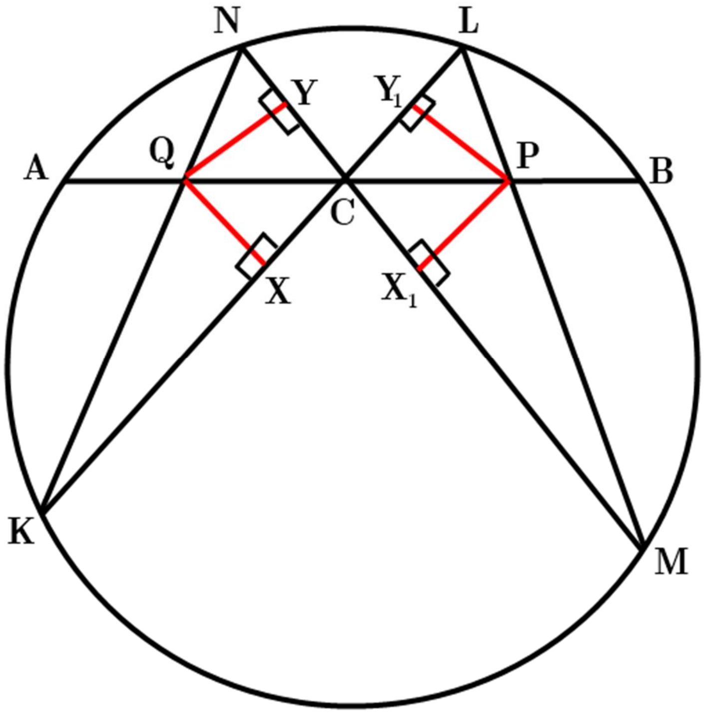
Bundan quyidagini topamiz:

$$BM=BN=4-4(\sqrt{2}-1)=8-4\sqrt{2}$$

ΔBMN ga Pifagor teoremasini qo‘llasak, $MN=8(\sqrt{2}-1)$ ekanligi kelib chiqadi.

Javob: $8(\sqrt{2}-1)$

259. Q nuqtadan LK va NM vatarlarga mos ravishda QX va QY perpendikulyarlar, P nuqtadan NM va LK vatarlarga mos ravishda PX_1 va PY_1 perpendikulyarlar tushiramiz.



QXC va CY_1P uchburchaklarning o‘xshashligidan $\frac{QC}{CP}=\frac{QY}{PX_1}$, QYC va CPX_1 uchburchaklarning o‘xshashligidan $\frac{QC}{CP}=\frac{QX}{PY_1}$ ekanligi, bulardan $\frac{QC^2}{CP^2}=\frac{QY\cdot QX}{PX_1\cdot PY_1}$ ekanligi kelib chiqadi. $\angle N=\angle L$ va $\angle K=\angle M$ ekanligi ma’lum(bir vatarga tiralgan burchaklar). U holda QNY va Y_1LP uchburchaklarning o‘xshashligidan $\frac{NQ}{LP}=\frac{QY}{PY_1}$, QXK va PMX_1 uchburchaklarning o‘xshashligidan $\frac{QK}{PM}=\frac{QX}{PX_1}$ ekanligi, bulardan $\frac{NQ\cdot QK}{LP\cdot PM}=\frac{QY\cdot QX}{PY_1\cdot PX_1}=\frac{QC^2}{CP^2}$ ekanligi kelib chiqadi. Kesishuvchi vatarlar xossasiga ko‘ra $NQ\cdot QK=AQ\cdot QB$ va $LP\cdot PM=BP\cdot PA$ tengliklar o‘rinli. Ushbu $AQ=AC-CQ$, $QB=BC+QC=AC+QC$,

$BP = BC - CP = AC - CP$ va $PA = AC + CP$ tengliklarni hisobga olib, quyidagilarni yoza olamiz:

$$\frac{QC^2}{CP^2} = \frac{AQ \cdot QB}{BP \cdot PA} = \frac{AC^2 - CQ^2}{AC^2 - CP^2}$$

$$AC^2 \cdot QC^2 - CP^2 \cdot QC^2 = AC^2 \cdot CP^2 - CQ^2 \cdot CP^2$$

$$AC^2 \cdot QC^2 = AC^2 \cdot CP^2$$

$$QC^2 = CP^2$$

$$QC = CP$$

Shuni isbotlash talab qilingan edi.

260. Ushbu $a_0 \cdot a_n \neq 0$ shartdan $a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n$ ko‘phadning ildizlaridan hech biri 0 ga teng emasligi kelib chiqadi. Bu ildizlarni karralisi bilan hisoblaganda x_i deb belgilaylik, bunda $i = 1, 2, \ldots, n$ va masala shartiga ko‘ra $x_i \in \mathbb{R}$. Endi $a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$ ko‘phadning ildizlari $\frac{1}{x_i}$ ko‘rinishida ekanligini ko‘rsatamiz. Buning uchun x ning o‘rniga $\frac{1}{x_i}$ ni qo‘yamiz:

$$a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0 = 0$$

$$\frac{a_n}{x_i^n} + \frac{a_{n-1}}{x_i^{n-1}} + \ldots + \frac{a_1}{x_i} + a_0 = 0$$

$$a_n + a_{n-1}x_i + \ldots + a_1x_i^{n-1} + a_0x_i^n = 0$$

Oxirgi tenglikning o‘rinli ekanligi $a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n$ ko‘phaning ildizlari x_i ko‘rinishida ekanligidan kelib chiqadi. Demak, $a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$ ko‘phadning ildizlari bo‘lgan $\frac{1}{x_i}$ sonlar $x_i \in \mathbb{R}$ bo‘lgani uchun haqiqiydir.

261. $\{a_n\}$ progressiyaning ayirmasini d_1 va $\{b_n\}$ progressiyaning ayirmasini d_2 deb olaylik. Ushbu $a_2 \leq b_2$ shartdan $1 \leq d_1 \leq d_2$ ekanligi kelib chiqadi. Berilganlarga ko‘ra quyidagilarni yozamiz:

$$a_nb_n = 2020$$

$$(1 + d_1(n-1))(1 + d_2(n-1)) = 2020$$

$$1+d_2(n-1)+d_1(n-1)+d_1d_2(n-1)^2=2020$$

$$(n-1)(d_1+d_2+d_1d_2(n-1))=2019$$

Oxirgi tenglikda quyidagi 4 ta hol bo‘lishi mumkin:

1-hol:

$$\begin{cases} n-1=1 \\ d_1+d_2+d_1d_2=2019 \end{cases} \Rightarrow \begin{cases} n=2 \\ (d_1+1)(d_2+1)=2020 \end{cases}$$

Bu holatda masala shartini qanoatlantiruvchi d_1 va d_2 lar mavjud. Masalan, $d_1=3$ va $d_2=504$.

2-hol:

$$\begin{cases} n-1=2019 \\ d_1+d_2+2019d_1d_2=1 \end{cases} \Rightarrow \begin{cases} n=2020 \\ d_1,d_2\in\varnothing \end{cases}$$

3-hol:

$$\begin{cases} n-1=3 \\ 3d_1+3d_2+9d_1d_2=2019 \end{cases} \Rightarrow \begin{cases} n=4 \\ (3d_1+1)(3d_2+1)=2020 \end{cases}$$

Bu holatda ham masala shartini qanoatlantiruvchi d_1 va d_2 larni topish mumkin. Masalan, $d_1=1$ va $d_2=168$.

4-hol:

$$\begin{cases} n-1=673 \\ d_1+d_2+673d_1d_2=3 \end{cases} \Rightarrow \begin{cases} n=674 \\ d_1,d_2\in\varnothing \end{cases}$$

Javob: $n=2$ *yoki* $n=4$

262. Berilganlar va Koshi tengsizligiga ko‘ra $1-a^2-b^2=c^2+d^2\geq 2cd$ yoki $1-a^2-b^2\geq 2cd$ munosabat o‘rinli. Shunga ko‘ra ushbu $(1-a)(1-b)\geq cd$ tengsizlikni isbotlaymiz:

$$2(1-a)(1-b)-2cd\geq 2(1-a)(1-b)-1+a^2+b^2=$$

$$=2-2a-2b+2ab-1+a^2+b^2=1+a^2+b^2-2a-2b+2ab=$$

$$=(1-a-b)^2\geq 0$$

Bundan $2(1-a)(1-b)-2cd\geq 0$ yoki $(1-a)(1-b)\geq cd$ ekanligi kelib chiqadi. Xuddi shunga o‘xshash $(1-c)(1-d)\geq ab$ ekanligini ham isbotlash mumkin. U holda $(1-a)(1-b)\geq cd$ va $(1-c)(1-d)\geq ab$ tengsizliklarni ko‘paytirib, isbotlanishi kerak bo‘lgan tengsizlikni hosil qilamiz. Tenglik sharti a, b, c, d lardan istalgan bittasi 1, qolganlari 0 bo‘lganda yoki $a = b = c = d = \frac{1}{2}$ bo‘lganda bajariladi.

263. ***Minkovskiy tengsizligi:*** *Ixtiyoriy musbat a_i, b_i $(i = 1, 2, \ldots, n)$ sonlari va p natural soni uchun quyidagi tengsizlik o‘rinli:*

$$(a_1^p + a_2^p + \ldots + a_n^p)^{\frac{1}{p}} + (b_1^p + b_2^p + \ldots + b_n^p)^{\frac{1}{p}} \geq$$

$$\geq \left((a_1 + b_1)^p + (a_2 + b_2)^p + \ldots + (a_n + b_n)^p\right)^{\frac{1}{p}}$$

Minkovskiy tengsizligida $p = 2$ desak, quyidagi tengsizlik o‘rinli:

$$\sqrt{(x-1)^2 + y^2} + \sqrt{x^2 + (y-1)^2} = \left((x-1)^2 + y^2\right)^{\frac{1}{2}} + \left(x^2 + (y-1)^2\right)^{\frac{1}{2}} \geq$$

$$\geq \left((x - 1 - x)^2 + (-y + y - 1)^2\right)^{\frac{1}{2}} = \sqrt{2}$$

Tenglik sharti $x = y = \frac{1}{2}$ da bajariladi.

Javob: $\sqrt{2}$

264. Ushbu $(k+1)^2 - (k+1) + 1 = k^2 + k + 1$ tenglikdan foydalanamiz:

$$\frac{2^3 - 1}{2^3 + 1} \cdot \frac{3^3 - 1}{3^3 + 1} \cdot \ldots \cdot \frac{2020^3 - 1}{2020^3 + 1} \cdot \frac{2021^3 - 1}{2021^3 + 1} = \frac{(2-1)(2^2 + 2 + 1)}{(2+1)(2^2 - 2 + 1)} \times$$

$$\times \frac{(3-1)(3^2 + 3 + 1)}{(3+1)(3^2 - 3 + 1)} \cdot \ldots \cdot \frac{(2020-1)(2020^2 + 2020 + 1)}{(2020+1)(2020^2 - 2020 + 1)} \times$$

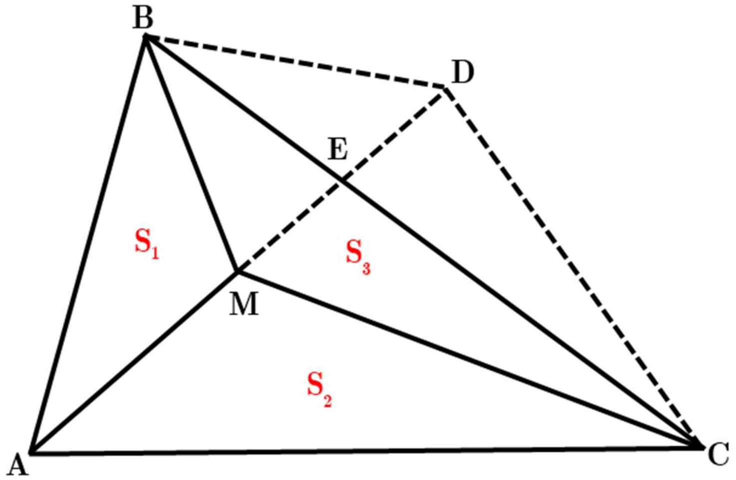
$$\times \frac{(2021-1)(2021^2 + 2021 + 1)}{(2021+1)(2021^2 - 2021 + 1)} = \frac{1 \cdot 2 \cdot \ldots \cdot 2020 \cdot (2021^2 + 2021 + 1)}{3 \cdot 4 \cdot \ldots \cdot 2022 \cdot (2^2 - 2 + 1)} =$$

$$= \frac{1 \cdot 2 \cdot (2021^2 + 2021 + 1)}{2021 \cdot 2022 \cdot 3} = \frac{2}{3} \cdot \frac{2021 \cdot 2022 + 1}{2021 \cdot 2022} =$$

$$= \frac{2}{3} \cdot \left(1 + \frac{1}{2021 \cdot 2022}\right) > \frac{2}{3} \cdot 1 = \frac{2}{3}$$

Da’vo isbotlandi.

265. AM to‘g‘ri chiziqda $AM = MD$ bo‘ladigan qilib D nuqtani olamiz. BC va MD kesmalar E nuqtada kesishsin. $S_{ABM} = S_1$, $S_{AMC} = S_2$ va $S_{BMC} = S_3$ deb olaylik($S = S_1 + S_2 + S_3$).



U holda $S_{BMD} = S_1$ va $S_{CMD} = S_2$ tengliklar o‘rinli bo‘ladi(ΔABD da BM va ΔACD da CM mediana). $S_{BMCD} = S_1 + S_2 = S_{BEM} + S_{BED} + S_{CED} + S_{CEM}$ ni topamiz:

$$S_1 + S_2 = \frac{1}{2} \cdot BE \cdot EM \cdot \sin\angle BEM + \frac{1}{2} \cdot BE \cdot ED \cdot \sin\angle BED +$$

$$+\frac{1}{2} \cdot CE \cdot ED \cdot \sin\angle CED + \frac{1}{2} \cdot CE \cdot EM \cdot \sin\angle CEM \leq$$

$$\leq \frac{1}{2} \cdot BE \cdot EM + \frac{1}{2} \cdot BE \cdot ED + \frac{1}{2} \cdot CE \cdot ED + \frac{1}{2} \cdot CE \cdot EM =$$

$$= \frac{1}{2} \cdot BE \cdot (EM + ED) + \frac{1}{2} \cdot CE \cdot (EM + ED) =$$

$$= \frac{1}{2} \cdot (EM + ED) \cdot (BE + CE) = \frac{1}{2} \cdot MD \cdot BC = \frac{1}{2} \cdot AM \cdot BC$$

Bundan $S_1 + S_2 \leq \frac{1}{2} \cdot AM \cdot BC$ tengsizlik kelib chiqadi. Xuddi shunga o‘xshash $S_1 + S_3 \leq \frac{1}{2} \cdot BM \cdot AC$ va $S_2 + S_3 \leq \frac{1}{2} \cdot CM \cdot AB$ tengsizliklarni hosil qilamiz. Oxirgi uchta tengsizliklarni hadma-had qo‘shamiz:

$$2(S_1 + S_2 + S_3) \leq \frac{1}{2} \cdot (AM \cdot BC + BM \cdot AC + CM \cdot AB)$$

$$4S \leq AM \cdot BC + BM \cdot AC + CM \cdot AB$$

Tenglik sharti M nuqta uchburchakning ortomarkazi ya'ni, balandliklari kesishish nuqtasi bo'laganda bajariladi.

266. Berilgan ifodaning ikkala tomonini 196 ga bo'lib, $\left(\frac{x+5}{14}\right)^2+\left(\frac{y-12}{14}\right)^2=1$ ni hosil qilamiz. $\frac{x+5}{14}=\sin\alpha$ va $\frac{y-12}{14}=\cos\alpha$ deb belgilash kiritamiz($\alpha\in\mathbb{R}$). Bundan $x=14\sin\alpha-5$ va $y=14\cos\alpha+12$ larni topib olamiz.

$$x^2+y^2=(14\sin\alpha-5)^2+(14\cos\alpha+12)^2=196(\sin^2\alpha+\cos^2\alpha)+$$
$$+28(12\cos\alpha-5\sin\alpha)+25+144=365+28(12\cos\alpha-5\sin\alpha)$$

Agar $-\sqrt{12^2+5^2}\le 12\cos\alpha-5\sin\alpha\le\sqrt{12^2+5^2}$ munosabatni hisobga olsak, quyidagiga ega bo'lamiz:

$$365-28\cdot 13\le x^2+y^2\le 365+28\cdot 13\ \Rightarrow\ 1\le x^2+y^2\le 729$$

Bundan x^2+y^2 ifodaning eng kichik qiymati 1 ga teng ekanligi kelib chiqadi.

Javob: 1

267. Ushbu $0<d\le c\le b\le a$ shartdan $2b^2\le 2ab$, $2c^2\le 2ac$, $2c^2\le 2bc$, $2d^2\le 2ad$, $2d^2\le 2bd$, $2d^2\le 2cd$ munosabatlarga ega bo'lamiz. Shularga ko'ra quyidagini yozamiz:

$$a^2+3b^2+5c^2+7d^2=a^2+b^2+c^2+d^2+2b^2+2c^2+2c^2+2d^2+2d^2+2d^2\le$$
$$\le a^2+b^2+c^2+d^2+2ab+2ac+2bc+2ad+2bd+2cd=(a+b+c+d)^2\le 1$$

Tenglik sharti $a=b=c=d=\frac{1}{4}$ bo'lganda bajariladi.

268. Ushbu $a=\frac{b+c}{x-2}$, $b=\frac{c+a}{y-2}$, $c=\frac{a+b}{z-2}$ berilganlardan $x-2$, $y-2$ va $z-2$ larni topib olamiz. Oldin ularni qo'shamiz, keyin ko'paytiramiz:

$$x-2+y-2+z-2=\frac{b+c}{a}+\frac{a+c}{b}+\frac{a+b}{c}\ \Rightarrow\ \frac{b+c}{a}+\frac{a+c}{b}+\frac{a+b}{c}=2014$$

$$(x-2)(y-2)(z-2)=\left(\frac{b}{a}+\frac{c}{a}\right)\left(\frac{a}{b}+\frac{c}{b}\right)\left(\frac{a}{c}+\frac{b}{c}\right)$$

$$xyz-2(xy+yz+xz)+4(x+y+z)-8=\frac{b+c}{a}+\frac{a+c}{b}+\frac{a+b}{c}+2$$

$$xyz=2014+2+2\cdot 67-4\cdot 2020+8=-5922$$

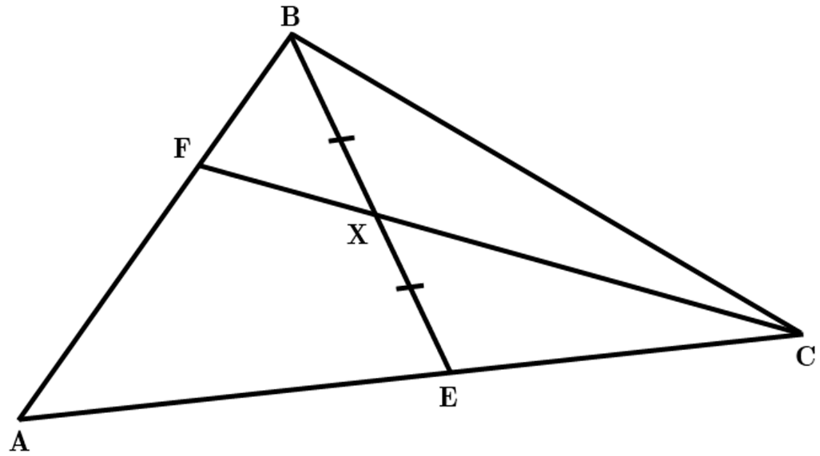
Javob: -5922

269. 24 ning har qanday juft darajasining oxirgi ikki raqami 76, toq darajasining oxirgi ikki raqami 24 bo‘lishidan foydalanamiz.

$$2^{2019} = (2^{10})^{201} \cdot 2^9 = 1024^{201} \cdot 2^9 = \ldots 24 \cdot 512 = \ldots 88$$

Javob: 88

270. Menelay teoremasiga ko‘ra $\frac{AF}{FB} \cdot \frac{BX}{XE} \cdot \frac{EC}{AC} = 1$ tenglik o‘rinli.



Agar $BX = XE$ ekanini hisobga olsak, $\frac{AF}{FB} \cdot \frac{EC}{AC} = 1$ yoki $\frac{AF}{FB} = \frac{AC}{EC} = 1 + \frac{AE}{EC}$ ga ega bo‘lamiz. Masala shartidagi ushbu $\frac{AF}{FB} = \left(\frac{AE}{EC}\right)^2$ tenglikdan foydalanib, $\left(\frac{AE}{EC}\right)^2 = 1 + \frac{AE}{EC}$ ni hosil qilamiz. Bundan $\frac{AE}{EC} = \frac{1+\sqrt{5}}{2}$ nisbatni topamiz.

Yana Menelay teoremasiga ko‘ra ushbu $\frac{AE}{EC} \cdot \frac{CX}{XF} \cdot \frac{FB}{AB} = 1$ tenglikdan va yuqoridagilardan foydalanib so‘ralgan nisbatni topamiz:

$$\frac{CX}{XF} = \frac{AB}{FB} \cdot \frac{EC}{AE} = \left(1 + \frac{AF}{FB}\right) \cdot \frac{EC}{AE} = \left(1 + 1 + \frac{AE}{EC}\right) \cdot \frac{EC}{AE} =$$

$$= \left(2 + \frac{1+\sqrt{5}}{2}\right) \cdot \frac{2}{1+\sqrt{5}} = \sqrt{5}$$

Javob: $\sqrt{5}$

271. *Fermaning kichik teoremasi: Agar* p*-tub son,* $a \in \mathbb{N}$ *va* $EKUB(a;p) = 1$ *bo‘lsa,* $a^p - a \equiv 1(\operatorname{mod} p)$ *munosabat o‘rinli.*

Quyidagi 4 ta holni qaraymiz:

1-hol: $(5^p - 2^p) \vdots pq$ bo‘lsin. U holda Fermaning kichik teoremasiga ko‘ra $(5^p - 5) \vdots p$ va $(2^p - 2) \vdots p$ munosabatlar o‘rinli. Bundan $(5^p - 5) - (2^p - 2) = (5^p - 2^p - 3) \vdots p$ bo‘lib, $(5^p - 2^p) \vdots p$ munosabatdan $3 \vdots p$ ekanligi kelib chiqadi. p tub son ekanini hisobga olib, $p = 3$ ekanini topamiz. U holda $5^3 - 2^3 = 3 \cdot 3 \cdot 13 \vdots q$ bo‘lib, $q = 3$ va $q = 13$ ekanligini topish mumkin.

2-hol: $(5^q - 2^q) \vdots pq$ bo‘lsin. Bu holatda ham xuddi yuqoridagidek $q = 3$ bo‘lib, $p = 3$ va $p = 13$ ekanligi kelib chiqadi.

3-hol: $(5^p - 2^p) \vdots p$ va $(5^q - 2^q) \vdots q$ bo‘lsin. Bunda yana Fermaning kichik teoremasidan $p = q = 3$ ekanligi kelib chiqadi.

4-hol: $(5^p - 2^p) \vdots q$ va $(5^q - 2^q) \vdots p$ bo‘lsin. Agar $p = q$ bo‘lsa, oldingi holatga tushadi. $p > q$ bo‘lsin. U holda $p = qk + r$ deb olamiz($0 < r < q$).

$$5^p - 2^p = 5^{qk+r} - 2^{qk+r} = 5^{qk+r} - 5^r \cdot 2^{qk} + 5^r \cdot 2^{qk} - 2^{qk+r} =$$
$$= 5^r(5^{qk} - 2^{qk}) + 2^{qk}(5^r - 2^r)$$

Bundan $(5^{qk} - 2^{qk}) \vdots (5^q - 2^q) \vdots p \Rightarrow (5^r - 2^r) \vdots p$ munosabatni hosil qilamiz. Bu esa $(5^p - 2^p) \vdots p$ deganidir. Bundan yana 3-holga qaytib qolamiz. $p < q$ bo‘lganda ham xuddi shunga o‘xshash oldingi holatga kelinadi.

Javob: $(p;q) \in \{(3;3),(3;13),(13;3)\}$

272. Ushbu $a^2 + a + 1 = 0$ tenglamani kompleks sonlarda yechib,

$a_1 = -\frac{1}{2} + \frac{3}{2}i = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$ va $a_2 = -\frac{1}{2} - \frac{3}{2}i = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$

yechimlarni topamiz. Bulardan istalgan birini $a^{2020} + \frac{1}{a^{2020}}$ ga qo‘yamiz va Muavr formulasidan foydalanamiz:

$$a^{2020} + \frac{1}{a^{2020}} = a^{2020} + a^{-2020} = \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)^{2020} + \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)^{-2020} =$$
$$= \cos\frac{4040\pi}{3} + i\sin\frac{4040\pi}{3} + \cos\frac{4040\pi}{3} - i\sin\frac{4040\pi}{3} = 2\cos\left(1347\pi - \frac{\pi}{3}\right) =$$
$$= -2\cos\frac{\pi}{3} = -2 \cdot \frac{1}{2} = -1$$

Javob: -1

273. $x \le 1$ yoki $x \ge 7$ ekanini bilgan holda $\sqrt{\sqrt{x^2-8x+9}+\sqrt{x^2-8x+7}} = a$ deb belgilasak, u holda $\sqrt{\sqrt{x^2-8x+9}-\sqrt{x^2-8x+7}} = \frac{\sqrt{2}}{a}$ ekanini topish mumkin($a > 0$). Bundan berilgan tenglama quyidagi ko‘rinishga keladi:

$$a^x + \frac{2^{\frac{x}{2}}}{a^x} = 2^{1+\frac{x}{4}} \Rightarrow a^{2x} - 2^{1+\frac{x}{4}} \cdot a^x + 2^{\frac{x}{2}} = 0 \Rightarrow (a^x - 2^{\frac{x}{4}}) = 0 \Rightarrow a^x = 2^{\frac{x}{4}}$$

Oxirgi tenglikdan $x = 0$ yoki $a = \sqrt[4]{2}$ ekanini topamiz. Belgilashga qaysak, quyidagi tenglamaga ega bo‘lamiz:

$$\sqrt{\sqrt{x^2-8x+9}+\sqrt{x^2-8x+7}} = \sqrt[4]{2}$$

Bunda $x^2 - 8x + 7 = t\,(t \ge 0)$ desak, $\sqrt{t+2}+\sqrt{t} = \sqrt{2}$ tenglamadan $t = 0$ ekanini topamiz. Bundan $x = 1$ va $x = 7$ yechimlarni topish mumkin.

Javob: $x = \{0;1;7\}$

274. Ma’lumki, har qanday natural sonni 3 ga bo‘lganda 0, 1 yoki 2 qoldiq qoladi. Istalgan natural sonning kvadratini 3 ga bo‘lganda esa, 0 yoki 1 qoldiq qoladi. 2021 ni 3 ga bo‘lganda 2 qoldiq qoladi. Demak, N sonini 3 ga bo‘lganda 2 qoldiq qolar ekan. Bu esa N soni hech bir sonning kvadrati bo‘la olmasligini bildiradi.

275. Berilganlarga ko‘ra quyidagilarni yozamiz:

$$a * b = a + b - \frac{2019}{2}$$

$$1 * 2 = 1 + 2 - \frac{2019}{2} \cdot 1$$

$$1 * 2 * 3 = 1 + 2 - \frac{2019}{2} \cdot 1 + 3 - \frac{2019}{2} = 1 + 2 + 3 - \frac{2019}{2} \cdot 2$$

$$1 * 2 * 3 * 4 = 1 + 2 + 3 - \frac{2019}{2} \cdot 2 + 4 - \frac{2019}{2} = 1 + 2 + 3 + 4 - \frac{2019}{2} \cdot 3$$

$$\cdots$$

$$1 * 2 * 3 * \ldots * n = 1 + 2 + 3 + \ldots + n - \frac{2019}{2} \cdot (n-1)$$

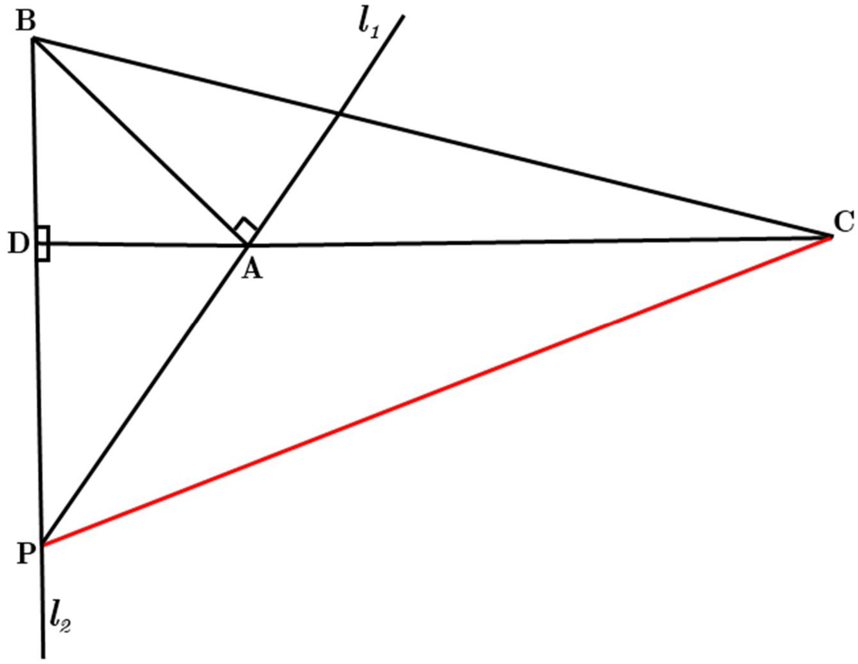
Bundan

$$1 * 2 * 3 * \ldots * n = 1 + 2 + 3 + \ldots + 2019 - \frac{2019}{2} \cdot (2019 - 1) =$$

$$= \frac{1+2019}{2} \cdot 2019 - \frac{2019}{2} \cdot 2018 = 1010 \cdot 2019 - 2019 \cdot 1009 = 2019$$

ekanini topish mumkin.

276. Masala shartiga mos chizma chizib olamiz. Uchburchak o'tmas burchakli ekani ma'lum. l_2 to'g'ri chiziq CA ning davomini D nuqtada kessin.



ABC uchburchakda kosinuslar teoremasini qo'llab, $\angle BAC$ ni topib olamiz.

$$\sqrt{7}^2 = 1^2 + \sqrt{3}^2 - 2 \cdot 1 \cdot \sqrt{3} \cdot \cos\angle BAC \;\Rightarrow\; \angle BAC = 150^0$$

U holda $\angle CAP = 120^0$, $\angle BAD = 30^0$ va $\angle DAP = 60^0$ tengliklar o'rinli. Bundan ADB to'g'ri burchakli uchburchakdan $AD = \frac{\sqrt{3}}{2}$, ADP to'g'ri burchakli uchburchakdan $AP = \sqrt{3}$ ekanini topib olamiz. Endi APC uchburchakda kosinuslar teoremasini qo'llab, $PC = 3$ ekanligini topishimiz mumkin.

Javob: 3

277. Berilganlarga asoslanib quyidagilarni yozishimiz mumkin:

$$a^2 - b^2 + c^2 - d^2 = 2020$$
$$(a-b)(a+b) + (c-d)(c+d) = a+b+c+d$$
$$(a+b)(a-b-1) + (c+d)(c-d-1) = 0$$

$a > b > c > d$ va $a, b, c, d \in \mathbb{N}$ ekanidan $a - b \geq 1$ va $c - d \geq 1$ munosabatlar o'rinli. U holda $(a+b)(a-b-1) + (c+d)(c-d-1) = 0$ tenglikning bajarilishi uchun $a-b=1$ va $c-d=1$ bo'lishi zarur. Bundan quyidagi tenglikka ega bo'lamiz:

$$a+b+c+d=2020 \Rightarrow a+a-1+d+1+d=2020 \Rightarrow a+d=1010 \Rightarrow d=1010-a$$

Ushbu $a>b>c>d\geq 1$ munosabatdan foydalanib,

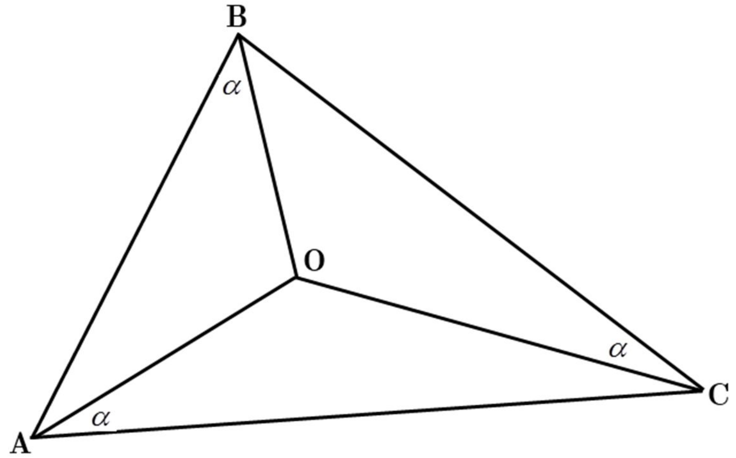
$$d+d+d+d<a+b+c+d \Rightarrow 4d<2020 \Rightarrow 1\leq d<505 \Rightarrow$$

$$\Rightarrow 1\leq 1010-a<505 \Rightarrow 505<a\leq 1009 \;\; 506\leq a\leq 1009$$

ekanligini topamiz.

Javob: $a=\{506,507,...,1009\}$

278. AOB, BOC va COA uchburchaklarga kosinuslar teoremasini qo'llaymiz:



$$+\begin{cases} AO^2=AB^2+BO^2-2\cdot AB\cdot BO\cdot\cos\alpha \\ BO^2=BC^2+CO^2-2\cdot BC\cdot CO\cdot\cos\alpha \\ CO^2=AC^2+AO^2-2\cdot AC\cdot AO\cdot\cos\alpha \end{cases} \Rightarrow$$

$$\Rightarrow AB^2+BC^2+AC^2=2\cos\alpha(AB\cdot BO+BC\cdot CO+AC\cdot AO)$$

Oxirgi tenglikning ikkala tomonini $\frac{\sin\alpha}{2}$ ga ko'paytiramiz:

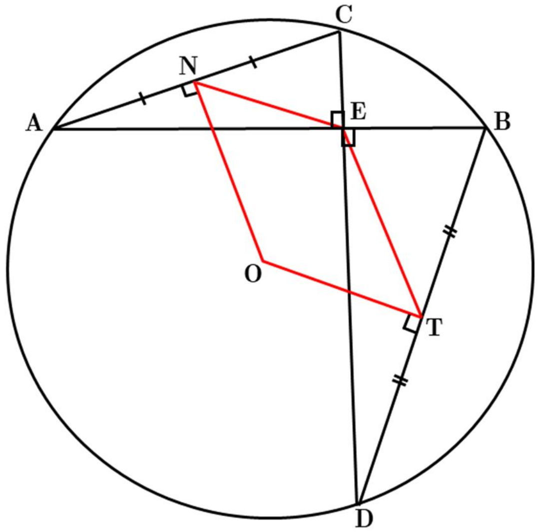
$$m\cdot\frac{\sin\alpha}{2}=2\cos\alpha\left(AB\cdot BO\cdot\frac{\sin\alpha}{2}+BC\cdot CO\cdot\frac{\sin\alpha}{2}+AC\cdot AO\cdot\frac{\sin\alpha}{2}\right)$$

$$m\cdot\frac{\sin\alpha}{2}=2\cos\alpha(S_{AOB}+S_{BOC}+S_{AOC}) \Rightarrow$$

$$\Rightarrow m\cdot\frac{\sin\alpha}{2}=2\cos\alpha\cdot S \Rightarrow ctg\alpha=\frac{m}{4S}$$

Javob: $\frac{m}{4S}$

279. To‘g‘ri burchakli uchburchak medianasining xossasiga ko‘ra $EN = AN = NC$ va $ET = BT = TD$ tengliklar o‘rinli.



AOC va BOD teng yonli uchburchaklarda $ON \perp AC$ va $OT \perp BD$ ekani ma’lum. BC vatarga tiralgan burchaklarni $\angle CAB = \angle CDB = \alpha$ deb olaylik. U holda $\angle NCE = \angle NEC = \angle TBE = \angle TEB = 90^0 - \alpha$ ekanini topamiz. Shunga ko‘ra $\angle CNE = \angle BTE = 2\alpha$ bo‘lib, $\angle ENO = \angle ETO = 90^0 - 2\alpha$ ekani kelib chiqadi. Bundan tashqari $\angle NET = 90^0 + 2\alpha$ ekanidan $ENOT$ to‘rtburchakda $\angle NOT = 90^0 + 2\alpha$ ekanini topish mumkin. Ichki bir tomonli burchaklar yig‘indisi 180^0 ekanligidan $NE \parallel OT$ va $NO \parallel ET$ munosabatlarga ega bo‘lamiz. Demak, $ENOT$ to‘rtburchakning qarama-qarshi burchaklari o‘zaro teng va qarama-qarshi tomonlari o‘zaro parallel bo‘lgani uchun u parallelogrammdir.

280. $0 < a, b, c < 1$ va Koshi tengsizligidan foydalanamiz:

$$+\begin{cases} \sqrt{abc} < \sqrt[3]{abc} \le \dfrac{a+b+c}{3} \\ \sqrt{(1-a)(1-b)(1-c)} < \sqrt[3]{(1-a)(1-b)(1-c)} \le \dfrac{1-a+1-b+1-c}{3} \end{cases} \Rightarrow$$

$$\Rightarrow \sqrt{abc} + \sqrt{(1-a)(1-b)(1-c)} < \frac{a+b+c+1-a+1-b+1-c}{3} = 1$$

Isbot tugadi

281. ***a)*** XYZ uchburchakning mavjudligini ko‘rsatish uchun $x + y > z$ tengsizlikni isbotlaymiz.

ΔABC da uchburchak tengsizligiga ko‘ra $c + b - a > 0$ va $c + a - b > 0$ munosabatlar o‘rinli. Bularni ko‘paytirib, $c^2 - (a-b)^2 > 0$ ekanini topamiz. Bundan quyidagini yoza olamiz:

$$c^2-(a-b)^2+2\sqrt{ab(c^2-(a-b)^2)}>0$$
$$c^2-a^2+2ab-b^2+2\sqrt{ab(c-a+b)(c+a-b)}>0$$
Oxirgi tengsizlikning har ikkala tomoniga $ac+bc$ ni qo‘shib, shakl almashtirishlar bajaramiz:
$$-a^2+ab+ac+ab-b^2+bc+2\sqrt{ab(-a+b+c)(a-b+c)}>ac+bc-c^2$$
$$a(-a+b+c)+2\sqrt{a(-a+b+c)}\cdot\sqrt{b(a-b+c)}+b(a-b+c)>c(a+b-c)$$
$$\left(\sqrt{a(-a+b+c)}+\sqrt{b(a-b+c)}\right)^2>\left(\sqrt{c(a+b-c)}\right)^2$$
$$\sqrt{a(-a+b+c)}+\sqrt{b(a-b+c)}>\sqrt{c(a+b-c)}$$
Bundan $x+y>z$ ekani kelib chiqadi. $y+z>x$ va $x+z>y$ tengsizliklar ham xuddi yuqoridagiga o‘xshash isbotlanadi. Demak, tomonlari $x=\sqrt{a(-a+b+c)}$, $y=\sqrt{b(a-b+c)}$ va $z=\sqrt{c(a+b-c)}$ bo‘lgan XYZ uchburchak mavjud.

b) Endi $x+y+z\le a+b+c$ munosabatni isbot qilamiz. Buning uchun Koshi tengsizligidan foydalanamiz.
$$x+y+z=\sqrt{a(-a+b+c)}+\sqrt{b(a-b+c)}+\sqrt{c(a+b-c)}\le$$
$$\le\frac{a+(-a+b+c)}{2}+\frac{b+(a-b+c)}{2}+\frac{c+(a+b-c)}{2}=a+b+c$$
Tenglik sharti $a=b=c$ bo‘lganda bajariladi.

c) Geron formulasidan foydalanamiz.
$$S_{\Delta XYZ}=\frac{\sqrt{(x+y+z)(-x+y+z)(x-y+z)(x+y-z)}}{4}=$$
$$=\frac{\sqrt{(-x^2+xy+xz-xy+y^2+yz-xz+yz+z^2)}}{2}\times$$
$$\times\frac{\sqrt{(x^2+xy-xz-xy-y^2+yz+xz+yz-z^2)}}{2}=$$
$$=\frac{\sqrt{(2yz-x^2+y^2+z^2)(2yz+x^2-y^2-z^2)}}{4}=\frac{\sqrt{(2yz)^2-(x^2-y^2-z^2)^2}}{4}=$$
$$=\frac{\sqrt{4bc(a-b+c)(a+b-c)-(-a^2+ab+ac-ab+b^2-bc-ac-bc+c^2)^2}}{4}=$$
$$=\frac{\sqrt{4bc(a^2-(b-c)^2)-(a^2-(b-c)^2)^2}}{4}=\frac{\sqrt{(a^2-(b-c)^2)(4bc-a^2+(b-c)^2)}}{4}=$$

$$= \frac{\sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)}}{4} = S_{\Delta ABC} = 2021$$

282. Berilgan tenglamani quyidagi ko‘rinishda yozib olamiz:

$$41x - yz = 2009 \Rightarrow yz = 41x - 41 \cdot 49 \Rightarrow yz = 41(x-49)$$

Oxirgi tenglikdan ko‘rinadiki, tenglama tub sonlarda yechimga ega bo‘lishi uchun $y = 41$ yoki $z = 41$ bo‘lishi kerak.

$y = 41$ bo‘lsin. U holda berilgan tenglama $x - z = 49$ ko‘rinishga keladi. Agar $z = 2$ bo‘lsa, $x = 51$ bo‘lib, masala shartiga zid. Qolgan hollarda x va z lar bir vaqtda toq ekanligidan ularning ayirmasi juft son bo‘ladi ammo, 49 toq son, ziddiyat. $z = 41$ bo‘lganda ham xuddi shunday ziddiyatga kelinadi. Bundan berilgan tenglama tub sonlarda yechimga ega emasligi kelib chiqadi.

Javob: Tub sonlarda yechimga ega emas

283. $P(2020) = 0$ ekanidan $P(x) = Q(x) \cdot (x - 2020)$ deb olishimiz mumkin, bunda $Q(x)$-bosh koeffitsienti 1 ga teng bo‘lgan 2020-darajali ko‘phad. Quyidagi tenglar o‘rinli:

$$P(2019) = 1 \Rightarrow P(2019) = Q(2019) \cdot (2019 - 2020) = 1 \Rightarrow Q(2019) = -1$$

$$P(2018) = 2 \Rightarrow P(2018) = Q(2018) \cdot (2018 - 2020) = 2 \Rightarrow Q(2018) = -1$$

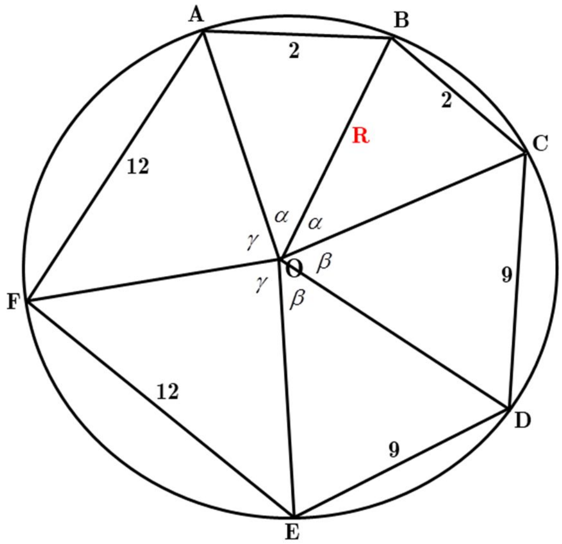
$$\dots$$

$$P(0) = 2020 \Rightarrow P(0) = Q(0) \cdot (0 - 2020) = 2020 \Rightarrow Q(0) = -1$$

Yuqoridagilarga asosan $x = 0, 1, 2, \dots, 2019$ bo‘lganda $Q(x) = x(x-1)(x-2)\dots(x-2019) - 1$ deb olishimiz mumkin. U holda $P(x) = \left(x(x-1)(x-2)\dots(x-2019) - 1\right) \cdot (x - 2020)$ bo‘lib, bundan $P(2021) = 2021! - 1$ ekanligi kelib chiqadi.

Javob: $2021! - 1$

284. Aylana markazini O bilan, radiusini R deb belgilab olaylik. $\angle AOB = \angle BOC = \alpha$, $\angle COD = \angle DOE = \beta$ va $\angle EOF = \angle FOA = \gamma$ bo‘lsin.



AOB, COD va EOF uchburchaklarda mos ravishda $\sin\frac{\alpha}{2}=\frac{1}{R}$, $\sin\frac{\beta}{2}=\frac{9}{2R}$ va $\sin\frac{\gamma}{2}=\frac{6}{R}$ tengliklar o‘rinli. $2\alpha+2\beta+2\gamma=2\pi$ ekanidan $\frac{\alpha}{2}+\frac{\beta}{2}=\frac{\pi}{2}-\frac{\gamma}{2}$ ni topamiz. Bundan quyidagiga ega bo‘lamiz:

$$\cos\left(\frac{\alpha}{2}+\frac{\beta}{2}\right)=\cos\left(\frac{\pi}{2}-\frac{\gamma}{2}\right) \Rightarrow \cos\frac{\alpha}{2}\cos\frac{\beta}{2}-\sin\frac{\alpha}{2}\sin\frac{\beta}{2}=\sin\frac{\gamma}{2}$$

$$\sqrt{1-\frac{1}{R^2}}\cdot\sqrt{1-\frac{81}{4R^2}}-\frac{9}{2R^2}=\frac{6}{R} \Rightarrow \sqrt{(R^2-1)(4R^2-81)}=12R+9$$

$$4R^4-85R^2+81=144R^2+216R+81 \Rightarrow R(4R^3-229R-216)=0$$

$$4R^3-229R-216=0 \Rightarrow 4R^3-256R+27R-216=0$$

$$4R(R-8)(R+8)+27(R-8)=0 \Rightarrow (R-8)(4R^2+32R+27)=0$$

Oxirgi tenglamadan $R=8$ ekanini topish mumkin.

Javob: 8

285. Koshi-Bunyakovskiy tengsizligidan foydalanamiz:

$$1=\frac{x_1}{\sqrt{1}}+\frac{x_2}{\sqrt{2}}+\frac{x_3}{\sqrt{3}}+\ldots+\frac{x_n}{\sqrt{n}}=x_1\cdot\frac{1}{\sqrt{1}}+x_2\cdot\frac{1}{\sqrt{2}}+x_3\cdot\frac{1}{\sqrt{3}}+\ldots+x_n\cdot\frac{1}{\sqrt{n}}\le$$

$$\le\sqrt{x_1^2+x_2^2+x_3^2+\ldots+x_n^2}\cdot\sqrt{1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}}$$

Bundan $x_1^2 + x_2^2 + x_3^2 + \ldots + x_n^2 \geq \dfrac{1}{1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}}$ ekanligi kelib chiqadi.

Tenglik sharti $\dfrac{x_1}{\frac{1}{\sqrt{1}}} = \dfrac{x_2}{\frac{1}{\sqrt{2}}} = \dfrac{x_3}{\frac{1}{\sqrt{3}}} = \ldots = \dfrac{x_n}{\frac{1}{\sqrt{n}}}$ yoki $x_1 = \sqrt{2}x_2 = \sqrt{3}x_3 = \ldots = \sqrt{n}x_n$ bo‘lganda bajariladi.

Javob: $\dfrac{1}{1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}}$

286. $P(x) = 1 + x + x^2 + \ldots + x^{100}$ deb olaylik. Dastlab $P^2(x)$ ni topamiz:

$$P^2(x) = (1 + x + x^2 + x^3 + \ldots + x^{100})(1 + x + x^2 + x^3 + \ldots + x^{100}) =$$
$$= (1 + x + x^2 + x^3 + \ldots + x^{100}) + (x + x^2 + x^3 + x^4 + \ldots + x^{101}) +$$
$$+(x^2 + x^3 + x^4 + x^5 + \ldots + x^{102}) + \ldots + (x^{100} + x^{101} + x^{102} + x^{103} + \ldots + x^{200}) =$$
$$= 1 + 2x + 3x^2 + 4x^3 + \ldots + 100x^{99} + 101x^{100} + Q(x^{101})$$

Bu yerda $Q(x^{101})$ ifoda eng kichik darajasi 101 bo‘lgan ko‘phad.

Endi $P^3(x)$ ni topamiz:

$$P^3(x) = P^2(x) \cdot (1 + x + x^2 + \ldots + x^{100}) =$$
$$= (1 + 2x + 3x^2 + \ldots + 100x^{99} + 101x^{100} + Q(x^{101})) \cdot (1 + x + x^2 + \ldots + x^{100}) =$$
$$= (1 + x + x^2 + \ldots + x^{100}) + (2x + 2x^2 + 2x^3 + \ldots + 2x^{100} + 2x^{101}) +$$
$$+(3x^2 + 3x^3 + 3x^4 + \ldots + 3x^{100} + 3x^{101} + 3x^{102}) + \ldots + (100x^{99} + 100x^{100} + 100x^{101} +$$
$$+\ldots + 100x^{199}) + (101x^{100} + 101x^{101} + 101x^{102} + \ldots + 101x^{200}) + R(x^{101})$$

Bu yerda $R(x^{101})$ ifoda eng kichik darajasi 101 bo‘lgan ko‘phad.

Oxirgi ifodadagi x^{100} lar oldidagi koeffitsientlarni qo‘shamiz:

$$1 + 2 + 3 + \ldots + 100 + 101 = \frac{1 + 101}{2} \cdot 101 = 5151$$

Javob: 5151

287. Berilgan ifodani $n^n - n = ((n-1) + 1)^n - n$ ko‘rinishda yozib olamiz va Nyutonning binomial formulasidan foydalanamiz:

$$n^n - n = ((n-1)+1)^n - n = C_n^0 \cdot (n-1)^n + C_n^1 \cdot (n-1)^{n-1} + \ldots + C_n^{n-2} \cdot (n-1)^2 +$$
$$+C_n^{n-1} \cdot (n-1) + C_n^n - n$$

Bundan ko‘rinadiki, $C_n^0 \cdot (n-1)^n + C_n^1 \cdot (n-1)^{n-1} + \ldots + C_n^{n-2} \cdot (n-1)^2$ ifoda $(n-1)^2$ ga bo‘linadi. Biz $C_n^{n-1} \cdot (n-1) + C_n^n - n$ ning $(n-1)^2$ ga bo‘linishini ko‘rsatishimiz yetarli.

$$C_n^{n-1} \cdot (n-1) + C_n^n - n = \frac{n!}{(n-1)! \cdot 1!} \cdot (n-1) + \frac{n!}{n! \cdot 0!} - n = n(n-1) + 1 - n = (n-1)^2$$

Bundan $C_n^{n-1} \cdot (n-1) + C_n^n - n$ ning ham $(n-1)^2$ ga bo‘linishi kelib chiqadi. Demak, istalgan natural $n > 1$ lar uchun $(n^n - n) \vdots (n-1)^2$. Isbot tugadi.

288. Berilgan tenglamada $\sqrt{n}$ ni tenglikning o‘ng tomoniga o‘tkazib, kvadratga oshiramiz:

$$\sqrt{n + 2020^k} = (\sqrt{2021} + 1)^k - \sqrt{n}$$

$$n + 2020^k = (\sqrt{2021} + 1)^{2k} - 2(\sqrt{2021} + 1)^k \sqrt{n} + n$$

$$\sqrt{n} = \frac{(\sqrt{2021} + 1)^{2k} - 2020^k}{2(\sqrt{2021} + 1)^k} = \frac{((2022 + 2\sqrt{2021})^k - 2020^k) \cdot (\sqrt{2021} - 1)^k}{2(2021 - 1)^k}$$

$$\sqrt{n} = \frac{\left((2022 + 2\sqrt{2021})(\sqrt{2021} - 1)\right)^k - 2020^k \cdot (\sqrt{2021} - 1)^k}{2 \cdot 2020^k}$$

$$\sqrt{n} = \frac{2020^k \cdot (\sqrt{2021} + 1)^k - 2020^k \cdot (\sqrt{2021} - 1)^k}{2 \cdot 2020^k} = \frac{(\sqrt{2021} + 1)^k - (\sqrt{2021} - 1)^k}{2}$$

$$n = \left(\frac{(\sqrt{2021} + 1)^k - (\sqrt{2021} - 1)^k}{2} \right)^2$$

Nyuton binomiga ko‘ra $(\sqrt{2021} + 1)^k = A + B\sqrt{2021}$ va $(\sqrt{2021} - 1)^k = A - B\sqrt{2021}$ ko‘rinishida bo‘lib, bundan $(\sqrt{2021} + 1)^k - (\sqrt{2021} - 1)^k = 2B\sqrt{2021}$ yoki $n = 2021B^2$ ekanligi kelib chiqadi, bu yerda $A, B \in \mathbb{N}$.

Javob: $n=\left(\frac{(\sqrt{2021}+1)^k-(\sqrt{2021}-1)^k}{2}\right)^2$ *bunda* k *istalgan natural son.*

289. Berilgan ko‘phad $(x-1)^2$ ga bo‘linishi uchun oldin $x-1$ ga bo‘linishi kerak. U holda Bezu teoremasiga ko‘ra $x=1$ da $P(1)=0$ bo‘lib, bundan $a=b$ ekanligi kelib chiqadi.

$$P(x)=x^{2019}-ax^{2018}+ax-1=(x^{2019}-1)-ax(x^{2017}-1)=$$
$$=(x-1)(x^{2018}+x^{2017}+\ldots+x+1)-ax(x-1)(x^{2016}+x^{2015}+\ldots+x+1)=$$
$$=(x-1)\Big((x^{2018}+x^{2017}+\ldots+x+1)-ax(x^{2016}+x^{2015}+\ldots+x+1)\Big)$$

Endi $(x^{2018}+x^{2017}+\ldots+x+1)-ax(x^{2016}+x^{2015}+\ldots+x+1)$ ko‘phad $x-1$ ga bo‘linishi kerak. Yana Bezu teoremasiga ko‘ra $x=1$ da

$$(1^{2018}+1^{2017}+\ldots+1+1)-a\cdot 1\cdot(1^{2016}+1^{2015}+\ldots+1+1)=0$$

bo‘lib, bundan $2019-2017a=0 \Rightarrow a=\frac{2019}{2017}$ ekanligi kelib chiqadi

Javob: $a=b=\frac{2019}{2017}$

290. Istalgan natural a,b,c sonlari uchun $x=a(a^2+b^3+c^6)^3$, $y=b(a^2+b^3+c^6)^2$ va $z=a(a^2+b^3+c^6)$ sonlarni tenglamaga qo‘yib, w ni topib olamiz:

$$a^2(a^2+b^3+c^6)^6+b^3(a^2+b^3+c^6)^6+c^6(a^2+b^3+c^6)^6=w^7$$
$$(a^2+b^3+c^6)^6\cdot(a^2+b^3+c^6)=w^7$$
$$w=a^2+b^3+c^6$$

Bundan berilgan tenglamaning natural sonlarda cheksiz ko‘p yechimga ekanligi kelib chiqadi

291. *Javob: ha, mumkin*

$n\geq 2$ ratsional son bo‘lsin. a irratsional son bo‘lsa, u holda $b=\log_a n$ ham irratsional son bo‘ladi. $a^b=a^{\log_a n}=n$ ifoda ratsional son.

292. Koshi-Bunyakovskiy tengsizligidan kelib chiqadigan quyidagi natijadan foydalanamiz:

$\forall a_1, a_2, b_1, b_2 \in \mathbb{R}$ sonlari uchun ushbu $\sqrt{a_1^2 + b_1^2} + \sqrt{a_2^2 + b_2^2} \geq \sqrt{(a_1 + a_2)^2 + (b_1 + b_2)^2}$ tengsizlik o'rinli(Kvadratga oshirilsa, Koshi-Bunyakovskiy tengsizligi hosil bo'ladi).

$$\left|x - y\right| + \sqrt{(x-3)^3 + (y+1)^2} = \sqrt{(x-y)^2 + 0^2} + \sqrt{(y+1)^2 + (x-3)^3} \geq$$
$$\geq \sqrt{(x - y + y + 1)^2 + (0 + x - 3)^2} = \sqrt{(x+1)^2 + (x-3)^2} = \sqrt{2x^2 - 4x + 10} =$$
$$= \sqrt{2(x-1)^2 + 8} \geq \sqrt{8} = 2\sqrt{2}$$

Javob: $2\sqrt{2}$

293. Umumiylikka zarar yetkazmasdan $x \geq y$ deb olaylik. U holda $x^2 \geq y^2 \geq y$ munosabat o'rinli. $\frac{y^2 + x}{x^2 - y} \in \mathbb{N}$ bo'lgani uchun $y^2 + x \geq x^2 - y$ yoki $(x+y)(y - x + 1) \geq 0$ bo'lib, bundan $x \leq y + 1$ munosabatni aniqlaymiz. Biz $y \leq x \leq y + 1$ ga ega bo'ldik. Quyidagi ikki hol bo'lishi mumkin:

1-hol: $x = y$. Bunda $\frac{x^2 + y}{y^2 - x} = \frac{x^2 + x}{x^2 - x} = \frac{x+1}{x-1} = 1 + \frac{2}{x-1} \in \mathbb{Z}$ bo'lishidan $x = y = \{2;3\}$ ekani kelib chiqadi.

2-hol: $x = y + 1$. Bu holatda $\frac{x^2 + y}{y^2 - x} = \frac{y^2 + 3y + 1}{y^2 - y - 1} = 1 + \frac{4y + 2}{y^2 - y - 1} \in \mathbb{Z}$ bo'lishi kerak. U holda $4y + 2 \geq y^2 - y - 1$ bo'lib, bundan $y^2 - 5y - 3 \leq 0$ tengsilikka kelamiz. Bu tengsizlik $y \geq 6$ da o'rinli emas. Demak, $y = \{1, 2, 3, 4, 5\}$ hollarni qarash yetarli. $y = \{1;2\}$ masala shartiniqanoatlantirishini tekshirish qiyin emas. Bundan $x = \{2;3\}$ ekani kelib chiqadi. Masala x va y ga nisbatan simmetrik bo'lgani uchun $(x;y) = \{(1;2), (3;2)\}$ ham yecim bo'ladi.

Javob: $(x;y) = \{(2;2), (3;3), (2;1), (2;3), (1;2), (3;2)\}$

294. Ma'lumki, 2 dan katta istalgan tub sonni 4 ga bo'lganda 1 yoki 3 qoldiq qolishi mumkin. $4k + 3$ ko'rinishidagi tub sonlar chekli deb faraz qilaylik. Ular $p_1, p_2, \ldots, p_n$ bo'lsin. Ushbu $4 \cdot p_1 \cdot p_2 \cdot \ldots \cdot p_n - 1$ sonni qaraylik. Bu sonni 4 ga bo'lganda 3 qoldiq qolgani uchun uning kamida bitta 4 ga bo'lganda 3 qoldiq qoladigan tub bo'lvuchisi mavjud. Lekin biz qarayotgan son $p_1, p_2, \ldots, p_n$ larning har biri bilan o‘zaro tub. Ziddiyat. Demak, $4k+3$ ko‘rinishidagi tub sonlar cheksiz ko‘p, bunda $k \in \mathbb{N}$.

295. Umumiylikka zarar yetkazmasdan $p_1 < p_2$ deb olaylik. U holda berilgan tenglikdan quyidagilarga ega bo‘lamiz:

$$\begin{cases} 2p_1 < p_1 + p_2 = 2q \\ 2p_2 > p_1 + p_2 = 2q \end{cases} \Rightarrow \begin{cases} p_1 < q \\ p_2 > q \end{cases} \Rightarrow p_1 < q < p_2$$

Ko‘rinib turibdiki, q soni ikkita ketma-ket kelgan tub sonlar orasida yotibdi. Demak, q murakkab son. Isbot tugadi

296. Ushbu $f(x)+f(1-x)=\dfrac{4^x}{4^x+2}+\dfrac{4^{1-x}}{4^{1-x}+2}=\dfrac{4^x}{4^x+2}+\dfrac{2}{4^x+2}=1$ tenglikdan foydalanamiz:

$$f(0)+f(\frac{1}{2020})+f(\frac{2}{2020})+\ldots+f(\frac{2019}{2020})+f(1)=$$

$$=\underbrace{f(0)+f(1)}_{1}+\underbrace{f(\frac{1}{2020})+f(\frac{2019}{2020})}_{1}+\ldots+\underbrace{f(\frac{1009}{2020})+f(\frac{1011}{2020})}_{1}+f(\frac{1}{2})=$$

$$=1\cdot 1010+\frac{\sqrt{4}}{\sqrt{4}+2}=1010+\frac{1}{2}=1010,5$$

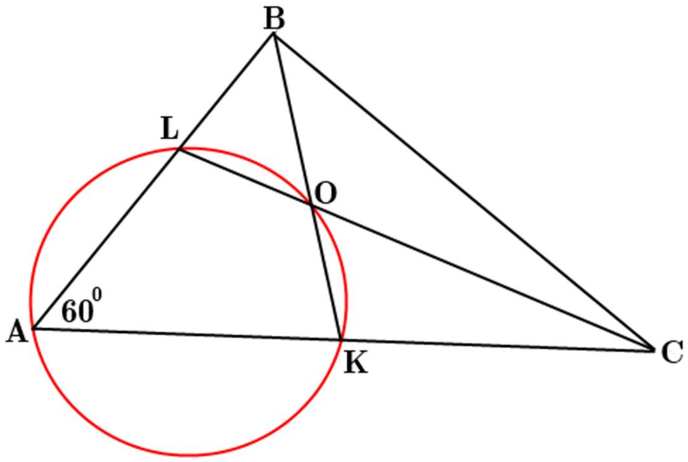
Javob: 1010,5

297. $y=1$ da $f(x)=2f(x)-f(x+1)+1$ bo‘lib, $f(x+1)=f(x)+1$ ekanini topamiz. Bundan $\forall n \in \mathbb{N}$ uchun $f(x+n)=f(x)+n$ ekanini ko‘rish mumkin. $x=1$ da $f(n+1)=n+2$ yoki $f(n)=n+1$ bo‘ladi. Demak, $\forall n \in \mathbb{Z}$ uchun $f(n)=n+1$ ekan. Endi $x=\dfrac{m}{n}$ va $y=n$ bo‘lsin, bunda $m \in \mathbb{Z}$ va $n \in \mathbb{N}$.

$$f(m)=f(\frac{m}{n})f(n)-f(\frac{m}{n})+1$$

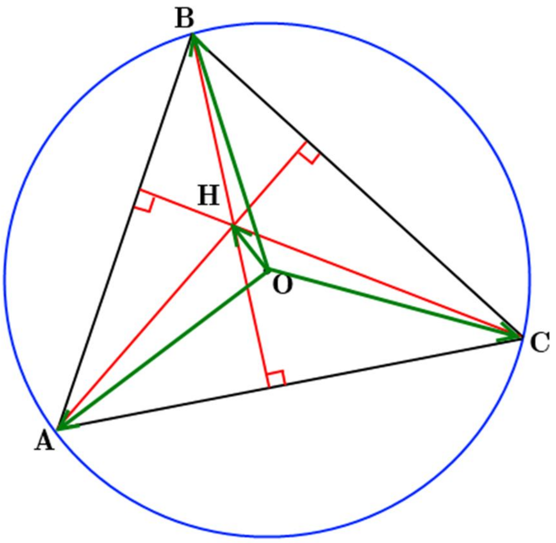
$f(m)=m+1$ va $f(n)=n+1$ ekanidan $f(\dfrac{m}{n})=\dfrac{m}{n}+1$ ekanligi kelib chiqadi. Demak, $\forall x \in \mathbb{Q}$ uchun $f(x)=x+1$.

298. Berilganlarga ko‘ra $\angle BOC=\angle LOK=120^0$ ekanini topamiz. Bundan $AKOL$ to‘rtburchakka tashqi aylana chizish mumkinligi kelib chiqadi.



$\angle OAL = \angle KAO = 30^0$ va bu burchaklar mos ravishda OL va OK vatarlarga tiralgani uchun $OL = OK$ tenglikka ega bo‘lamiz. Shuni isbotlash talab qilingan edi.

299. Ushbu $\overrightarrow{OH} = \overrightarrow{OC} + \overrightarrow{CH}$ tenglik o‘rinli. Demak, $\overrightarrow{CH} = \overrightarrow{OA} + \overrightarrow{OB}$ tenglikni isbotlash kifoya.



$\overrightarrow{CH}$ vektor ham, $\overrightarrow{OA} + \overrightarrow{OB}$ vektor ham $\overrightarrow{AB}$ vektorga perpendikular bo‘lgani uchun ular o‘zaro parallel bo‘ladi. Demak, bu vektorlar kollinear ekan ya’ni, $\overrightarrow{CH} = \lambda \cdot (\overrightarrow{OA} + \overrightarrow{OB})$ tenglik o‘rinli, bunda $\lambda \in \mathbb{R}$. Bu vektorlar yo‘nalishdosh bo‘lgani uchun $\lambda \geq 0$. Biz λ ning qiymatini topish uchun $\overrightarrow{CH}$ va $\overrightarrow{OA} + \overrightarrow{OB}$ vektorlar uzunliklarini topamiz. Ushbu $\left|\overrightarrow{OA} + \overrightarrow{OB}\right|^2 = 4R^2 \cos^2 \gamma$ va $\left|\overrightarrow{CH}\right|^2 = 4R^2 \cos^2 \gamma$ ifodalarni topish qiyin emas. Bu yerda R orqali ABC uchburchakka tashqi chizilgan aylana radiusi va γ orqali AB tomon qarshisidagi burchak belgilangan. Demak, $\lambda = 1$ bo‘lib, ushbu $\overrightarrow{CH} = \overrightarrow{OA} + \overrightarrow{OB}$ tenglik o‘rinli. Bundan $\overrightarrow{OH} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$ ekanligi kelib chiqadi.

300. Bizga ma’lumki, uch xonali sonni $\overline{xyz} = 100x + 10y + z$ ko‘rinishda yozish mumkin. Berilgan tengsizlikni quyidagi ko‘rinishga keltiramiz:

$$(100a + 10b + c)(100b + 10c + a)(100c + 10a + b) \geq 111^3 abc$$

$$(\frac{100a + 10b + c}{a})(\frac{100b + 10c + a}{b})(\frac{100c + 10a + b}{c}) \geq 111^3$$

$$(100 + 10\frac{b}{a} + \frac{c}{a})(100 + 10\frac{c}{b} + \frac{a}{b})(100 + 10\frac{a}{c} + \frac{b}{c}) \geq 111^3$$

Oxirgi tengsizlikni isbotlash kifoya. Buning uchun qavslarni ochib chiqamiz va AM-GM tengsizligining $n = 3$ holidan foydalanamiz:

$$\left(100 + 10 \cdot \frac{b}{a} + \frac{c}{a}\right)\left(100 + 10 \cdot \frac{c}{b} + \frac{a}{b}\right)\left(100 + 10 \cdot \frac{a}{c} + \frac{b}{c}\right) =$$

$$= 10^6 + 10^5 \cdot \left(\frac{a}{c} + \frac{c}{b} + \frac{b}{a}\right) + 2 \cdot 10^4 \cdot \left(\frac{a}{b} + \frac{b}{c} + \frac{c}{a}\right) + 10^3 \cdot \left(\frac{a^2}{bc} + \frac{c^2}{ab} + \frac{b^2}{ac}\right) +$$

$$+2 \cdot 10^2 \cdot \left(\frac{a}{c} + \frac{c}{b} + \frac{b}{a}\right) + 10 \cdot \left(\frac{a}{b} + \frac{b}{c} + \frac{c}{a}\right) + 3 \cdot 10^3 + 1 \geq$$

$$\geq 10^6 + 3 \cdot 10^5 + 6 \cdot 10^4 + 6 \cdot 10^3 + 6 \cdot 10^2 + 3 \cdot 10 + 1 =$$

$$= (100 + 10 + 1)^3 = 111^3$$

Tenglik sharti $a = b = c$ bo‘lganda bajariladi. Isbot tugadi.

TEST KALITLARI

	0	1	2	3	4	5	6	7	8	9
0		D	D	A	B	D	A	C	A	B
1	C	B	A	C	B	B	C	D	D	A
2	C	B	D	C	D	C	B	C	B	A
3	C	D	C	D	B	A	D	A	A	B
4	C	D	D	B	D	D	D	B	B	A
5	A	D	B	A	D	D	A	B	C	C
6	B	A	B	A	A	C	C	B	D	A
7	C	D	D	D	D	B	B	A	B	D
8	C	D	D	C	A	C	D	C	D	A
9	A	C	A	C	C	C	D	C	C	C
10	D	C	C	C	B	C	A	D	A	B
11	A	C	C	A	B	B	D	D	A	C
12	A	A	D	A	A	D	C	A	B	D
13	D	A	B	C	A	B	B	D	A	C
14	B	A	A	C	B	B	C	C	A	C
15	A	B	A	C	A	B	B	A	B	C
16	B	C	C	B	D	D	D	C	A	B
17	D	A	C	D	A	C	D	C	A	B
18	D	C	A	C	C	D	D	B	B	C
19	A	B	B	C	A	D	D	C	A	B
20	A									

Foydalanilgan adabiyotlar

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OLIMBAYEV TO‘LQIN G‘AYRAT O‘G‘LI

MATEMATIKADAN SIRTQI OLIMPIADA MASALALARI

(Uslubiy qo‘llanma)

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