DIFFERENSIAL TENGLAMALAR BO‘YICHA MISOL VA MASALALAR

O‘ZBEKISTON RESPUBLIKASI
OILY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

ALISHER NAVOIY NOMIDAGI
SAMARQAND DAVLAT UNIVERSITETI

DIFFERENSIAL TENGLAMALAR BO‘YICHA MISOL VA MASALALAR

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Differensial tenglamalar bo'yicha micol va masalalar **-**

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Mazkur masalalar to'plami hozirda amalda universitetlar uchun qullanilayotgan differensial tenglamalar fani bo'yicha o'quv rejasiga moslab tuzilgan.

Bu to'plamda amaliy dars jarayonida yechilishi kerak bo'lgan misollardan tashqari mustaqil ishlash uchun 638 ta misollar keltirilgan.Ayrim misollar yechimi Maple tizimida tekshirilan.

Ushbu qo'llanma matematika fakultetlari talabalariga mo'ljallangan.

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M U N D A R I J A

S O‘ Z B O SH I

Mustaqillik sharofati bilan respublikamizda xalq xo‘jaligining barcha sohalari, shu jumladan, ta’lim sohasida ulkan islohatlar amalga oshirilmoqda.

Mustaqillikning dastlabki kunlaridanoq mamlakatimizda ta’lim tizimi, yosh avlod tarbiyasiga jiddiy e’tibor berila boshlandi. “Ta’lim to‘g‘risida”gi Qonun, “Kadrlar tayyorlash Milliy dastur”larining qabul qilishi, Davlat ta’lim standartlarining chop etilishi ushbu jarayonlardagi eng muhim bosqichlardir. Bugungi kun ta’lim va tarbiya berishning yangi shakllari va usullarini izlab topish hamda ularni hayotga tatbiq etish zaruriyatini qo‘ymoqda.

Ijobiy taffakkur sohibini, ya’ni o‘sayotgan, rivojlanayotgan va taraqqiyot sari yuz tutayotgan mamlakatimiz uchun zarur bo‘lgan ijodkor va mustaqil fikrlay oladigan shaxsni tarbiyalab, voyaga yetkazish oily ta’lim oldida turgan eng muhim va dolzarb vazifa hisoblanadi.

Ushbu uslubiy qo‘llanma Davlat ta’lim standartlari talablariga mos ravishda, yangi zamonaviy texnologiyalarga asoslanib, matematika, ama-liy matematika, mexanika, fizika bakalavr yo‘nalishi talabalariga mo‘ljal-langan.

Mazkur uslubiy qo‘llanma universitetlar va pedagogik oily-gohlarning differensial tenglamalar fani dasturidagi materiallarini o‘z ichi-ga oladi.

Uslubiy qo‘llanma §larga bo‘lingan bo‘lib unda nazariy materiallar va tipik misollar yechish usullari keltirilgan. Bo‘limlar mustaqil yechish uchun misollar bilan tugaydi.

Qo‘llanma SamDU matematika fakultetida differensial tenglamalar bo‘yicha amaliy §lar o‘tkazish tajribasi asosida tuzilgan.

Mustaqil echish uchun misollar bir qismi muallif tomonidan tuzilgan, bir qismi esa shu kurs bo‘yicha chop etilgan asosiy darsliklardan olindan.

Shuni ta’kudlash joizki, ushbu qo‘llanma ayrim kamchiliklardan xolibo‘lmasligi shubhasizdir. Shu bois uslubiy qo‘llanmani takomillashtirishga qaratilgan tanqidiy fikr-mulohasalar va takliflarnimualliflar mamnuniyat bilan qabul qiladi.

1 -BOB
HOSILAGA NISBATAN YECHILGAN BIRINCHI TARTIBLI DIFFERENSIAL TENGLAMALAR

$G\subset R^2$ sohada aniqlangan va uzluksiz *f(x,y)* funksiya berilgan bo‘lsin, u holda

$$y' = f(x,y) \tag{1}$$

tenglama hosilaga nisbatan yechilgan birinchi tartibli differensial tenglama deyiladi.

Agar [*a*,*b*] oraliqda o‘z hosilasi bilan aniqlangan *y=y(x)* funksiya shu oraliqda (1) tenglamani ayniyatga aylantirsa, bunday funksiyaga (1) differensial tenglamaning yechimi deyiladi.

Ko‘pincha (1) tengmani to‘la tekshirish uchun shu tenglamaga G sohada, $f(x,y)\neq 0$ bo‘lganda, teng kuchli bo‘lgan

$$\frac{dx}{dy} = \frac{1}{f(x,y)} \tag{2}$$

«to‘ntarilgan» tenglama ko‘riladi; bu tenglama (1) tenglamaning yechimini $\frac{1}{f(x,y)} = 0$ ga aylantiruvchi *(x,y)* nuqtalar atrofida tekshirishga imkon beradi.

Differensial tenglama yechimining grafigi integral chiziq deyiladi.

Differensial tenglamalar nazariyasida boshlang‘ich shart yoki Koshi masalasi muhim rol o‘ynaydi.

(1) differensial tenglamaning barcha yechimlari orasida shunday *y=y(x)* yechiimni topish kerakki, bu yechim boshlang‘ich $y(x_0)=y_0$ shartni qanoatlantirsin, bu yerda x_0 va y_0 berilgan boshlang‘ich qiymatlar va

$$(x_0, y_0)\in G.$$

Koshi masalasi qisqacha qo‘yidagicha yoziladi:

$$y' = f(x,y) \quad y(x_0) = y_0 \tag{3}$$

Birinchi tartibili

$$M(x,y)dx+N(x,y)dy=0 \tag{4}$$

normal shakldagi differensial tenglama deyiladi, bu yerda *M(x,y),N(x,y)* G sohadan olingan uzluksiz funksiyalardir. $(x_0, y_0)\in G$ nuqta maxsus nuqta deyiladi, agar $M(x_0,y_0)=N(x_0,y_0)=0$ bo‘lsa, sohaning boshqa nuqtalari maxsusmas nuqtalar deyiladi.

Differensiallanuvchi $y=y(x), x\in[a,b]$ funksiyaga (yoki $x=x(y),\ y\in[c,d]$ funksiyaga) (4) tenglamaning yechimi deyiladi, agar u tenglamani [*a*,*b*] (yoki [*c*,*d*]) oraliqda ayniyatga aylantirsa.

Agar $(x_0, y_0)\in G$ maxsusmas nuqta bo‘lsa, u holda shu nuqtaning qandaydir atrofida (4) tenglama quyidagi tenglamalarni birortasiga ekvivalent bo‘ladi:

$$\frac{dy}{dx}=-\frac{M(x,y)}{N(x,y)},\ \frac{dx}{dy}=-\frac{N(x,y)}{M(x,y)}$$

(4) tenglama uchun Koshi masalasi (1) tenglamaniqi kabi qo‘yiladi.

Agar (3) masalasining ixtiyoriy ikki $y=y_1(x),\ x\in[a_1,b_1]$ va $y=y_2(x),\ x\in[a_2,b_2]$ yechimi uchun x_0 nuqtaning ε atrofida $|x-x_0|<\varepsilon\ \ (\varepsilon>0)$ bo‘lganda $y_1(x)\equiv y_2(x)$ ayniyat bajarilsa, (3) Koshi masalasi yagona yechimga ega bo‘ladi.

Agar *f* funksiya o‘zining xususiy $\frac{\partial f}{\partial y}$ hosilasi bilan birorta (x_0,y_0) nuqta atrofida uzluksiz bo‘lsa, (3) Koshi masalasi $[x_0-h,x_0+h]$ oraliqda aniqlangan yagona yechimga ega bo‘ladi.

Agar ixtiyoriy $(x_0,y_0)\in G$ nuqta uchun $y_0=\varphi(x_0,c)$ tenglama $c_0=u(x_0,y_0)$ yechimga ega bo‘lsa va $y_0=\varphi(x,c_0)$ funksiya (3) Koshi masalasining yechimi bo‘lsa, $y=\varphi(x,c)$ uzluksiz funksiya (1) tenglamaning *G* sohadagi umumiy yechimi deb aytiladi. Ko‘pincha tenglamaning yechimini $\Phi(x,y,c)=0$ oshkormas shaklda tuzish mumkin.

Agar *G* sohada aniqlangan $u(x,y)=c,\ (x,y)\in G$ funksional munosabat (1) tenglamaning umumiy yechimni aniqlasa, bu munosabatga (1) tenglamaning *G* sohadagi umumiy integrali deb ataladi.

Agar yechimning xar bir nuqtasida Koshi masalasi yagona yechimga ega bo‘lsa, bunday yechim xususiy yechim deyiladi. Aks holda, ya’ni agar yechimning xar bir nuqtasida Koshi masalasining yagonaligi bajarilmasa, bunday yechim maxsus yechim deyiladi.

Differensial tenglamaning yechimini topish differensial tenglamani integrallash deyiladi.

1- §. BIRINCHI TARTIBLI SODDA DIFFERENSIAL TENGLAMALAR

Birinchi tartibli eng oddiy differensial tenglama analizda uchraydi: berilgan $f:(a,b)\to R$ uzluksiz funksiyaning boshlang‘ich funksiyasini topish masalasi

$$y'=f(x) \tag{5}$$

tenglamani qanoatlantiruvchi $y=y(x),\ x\in(a,b)$ funksiyani topish masalasiga teng kuchlidir.(5) tenglamaning $a<x<b,\ |y|<\infty$ sohada aniqlangan umumiy yechimi (Koshi shaklidagi umumiy yechimi) quyidagi formulalar yordamida topiladi:

$$y=\int f(x)dx+c,\ x\in(a,b) \tag{6}$$

Bu yerda *c* - ixtiyoriy o‘zgarmas, $x_0\in(a,b),\ y_0\in R$.

Agar f funksiya $x=\alpha\in(a,b)$ nuqtada uzilishga ega bo‘lib, oraliqning qolgan hamma nuqtalarida uzluksiz funksiya bo‘lsa, u holda (6) formula yordamida (5) tenglamaning umumiy yechimini $a<x<\alpha,\ |y|<\infty$ va $\alpha<x<b,\ |y|<\infty$ sohalarda aniqlash mumkin. $x=\alpha$ esa «to‘ntarilgan» tenglamaning yechimi bo‘ladi.

Bu $x=\alpha$ chiziq (6) integral chiziqlar oilasining $\int f(s)ds$ integralning xususiyatiga qarab, asimptotikasi yoki o‘ramasi bo‘ladi.

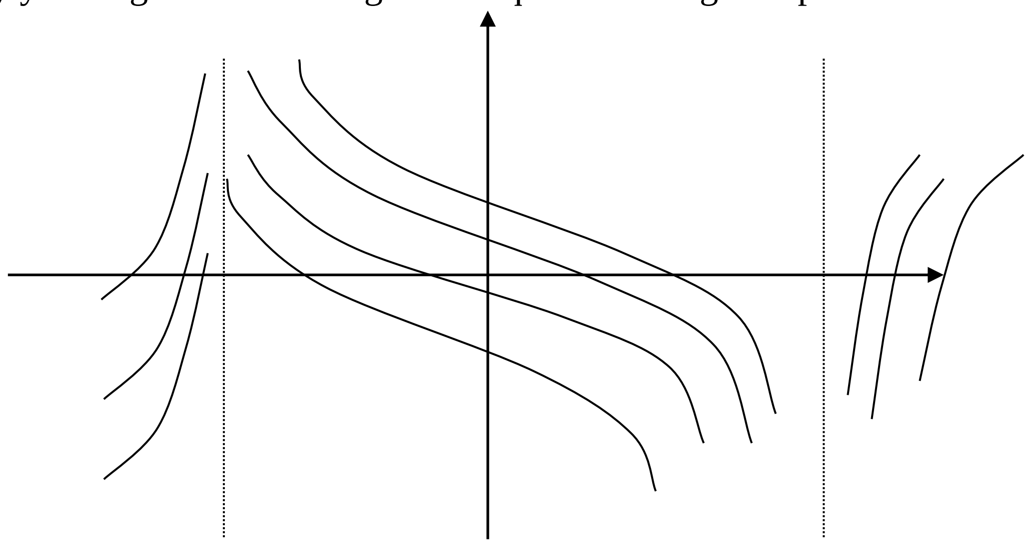
Tenglamalarni integrallang va integral chiziqni yasang.

1-misol. $y'=\dfrac{1}{x^2-4}$; $f(x)=\dfrac{1}{x^2-4}$

f(x) funksiya $]-\infty,-2[,\ ]-2,+2[,\ ]+2,+\infty[$ oraliqlarda aniqlangan va uzluksiz. $x=\pm2$ funksiyaning cheksiz uzilish nuqtalari. Bu tenglamaning aniqlanish sohasidagi umumiy yechimi

$$y=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+c$$

$x=\pm2$ «to‘ntarilgan» tenglamaning integral chizig‘i bo‘ladi va chiziqlar umumiy yechimga kiruvchi integral chiziqlar oilasining asimptotasi bo‘ladi.



Tenglama yechimini **Meple** dasturi yordamida tekshiramiz

```
> d1:=diff(y(x),x)=1/(x^2-4);
```

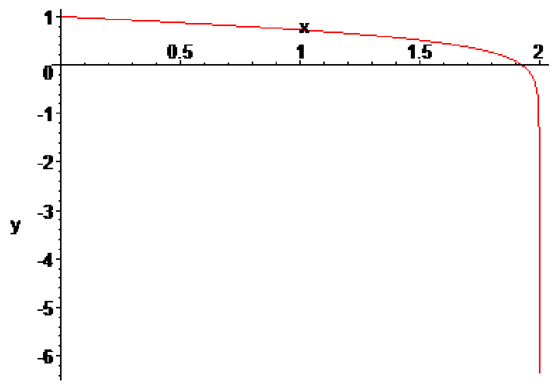
$$d := \frac{\partial}{\partial x}\mathrm{y}(x) = \frac{1}{x^2-4}$$

```
> dsolve(d1,y(x));
```

$$\mathrm{y}(x)=\frac{1}{4}\ln(x-2)-\frac{1}{4}\ln(x+2)+\_C1$$

```
> p1 := dsolve({ diff(y(x),x) = 1/(x^2-
4),y(0)=1},y(x),type=numeric):
> odeplot(p1,[x,y(x)],0..10,labels=[x,y]);
```

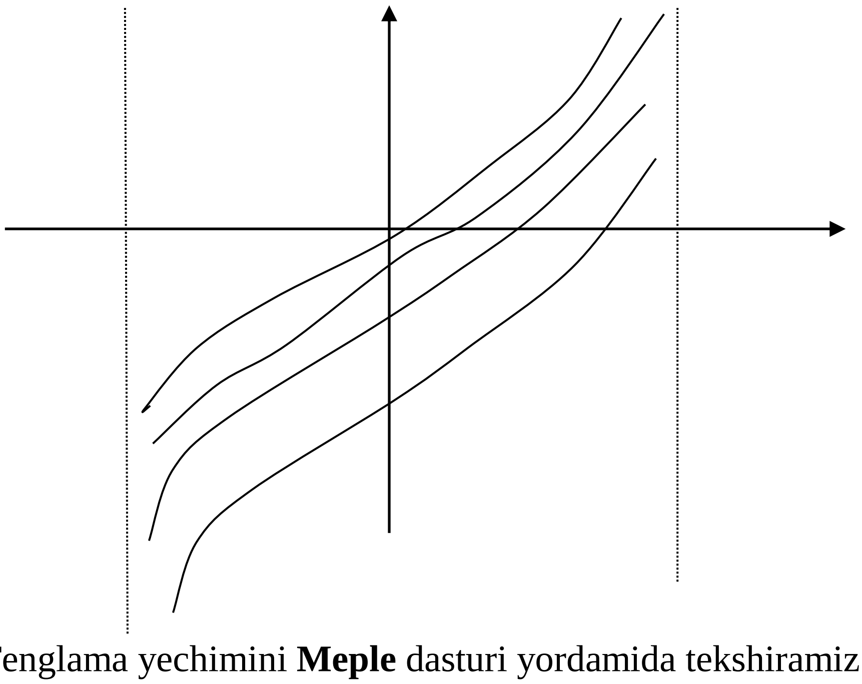
```
Warning, cannot compute solution further right of
1.9999999999937006
```



<u>2-misol</u>. $y' = \dfrac{1}{\sqrt{9-x^2}}$ tenglamaning umumiy yechimi

$$y = \arcsin\frac{x}{3} + c, \ \ |x| < 3.$$

$x = \pm 3$ «to‘ntarilgan» tenglamaning maxsus yechimi va bu yechimlar umumiy yechimga kiruvchi integral chiziqlar oilasining o‘ramasi bo‘ladi.



Tenglama yechimini **Meple** dasturi yordamida tekshiramiz

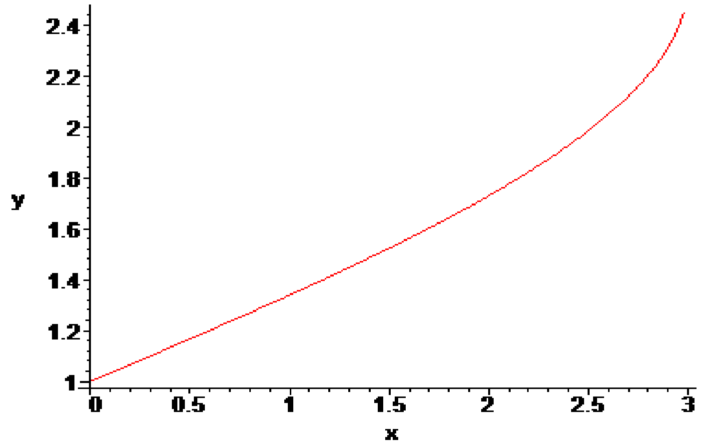
```
> d2:=diff(y(x),x)=1/sqrt(9-x^2);
```

$$d := \frac{\partial}{\partial x}\mathrm{y}(x) = \frac{1}{\sqrt{9-x^2}}$$

```
> dsolve(d2,y(x));
```

$$\mathrm{y}(x) = \arcsin\left(\frac{1}{3}x\right) + \_C1$$

```
> p2 := dsolve({ diff(y(x),x) = 1/sqrt(9-
x^2),y(0)=1},y(x),type=numeric):
> odeplot(p2,[x,y(x)],0..6,labels=[x,y]);
```



Endi, erkli o‘zgaruvchi qatnashmagan

$$y' = f(y) \tag{7}$$

tenglamani qaraymiz.

$f(y) \neq 0$ bo‘lganda, (7) ga teng kuchli bo‘lgan «to‘ntarilgan» tenglamani qaraymiz:

$$\frac{dx}{dy} = \frac{1}{f(y)} \tag{8}$$

bu tenglama uchun yuqorida ko‘rib o‘tilgan usulni qo‘llaymiz.

$f:(c,d) \to R$ uzluksiz va (c,d) da nolga teng emas deb faraz qilamiz. U holda (8) tenglamaning $|x| < +\infty,\ c < x < d$ sohadagi umumiy yechimi (Koshi shaklidagi umumiy yechimi) quyidagicha bo‘ladi:

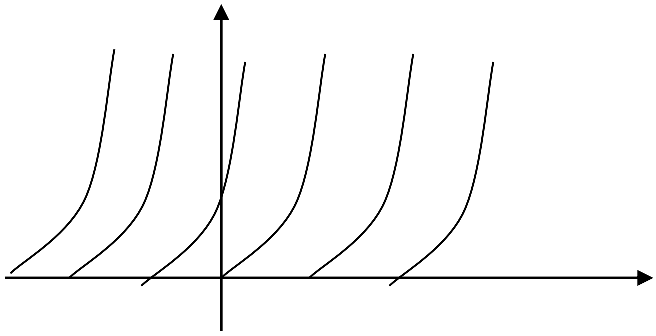
$$x = \int \frac{1}{f(y)} dy + c, \quad \text{yoki} \quad x = \int_{y_0}^{y} \frac{1}{f(t)} dt + x_0$$

Agar $f(y)$ $\beta \in (c,d)$ nuqtalarda nolga aylansa, u holda $y = \beta$ (7) tenglamaning yechimi bo‘ladi.

<u>3-misol</u>. $y' = 4y^{3/4}$, $f(y) = 4y^{3/4}$

f(y) funksiya $y \geq 0$ bo‘lganda aniqlangan va uzluksiz, hamda $f(0)=0$.

«T o‘ntarilgan» tenglama $\dfrac{dx}{dy} = \dfrac{1}{4\sqrt[4]{y^3}}$ bo‘lib, yechimi $(x+c) = \sqrt[4]{y}$ bo‘ladi.



Demak, $y=(x+c)^4,\ x\geq 0$ umumiy yechim, y=0 maxsus yechim.

Differensial tenglamalarni integrallashda almashtirishlar muhim rol o‘ynaydi, masalan

$$y' = f(ax+by) \tag{9}$$

Tenglama $z=ax+by$ almashtirish yordamida (7) tenglamaga keltiriladi. Bu yerda z yangi noma’lum funksiya.

<u>4-misol</u>. $y' = 4\sqrt[4]{(y-x)^3}+1,\ \left[z=y-x,\ y'=z'+1\right] \Rightarrow z' = 4\sqrt[4]{z^3}$

3 misolni yechishda qo‘llangan usuldan foydalanib: $z=(x+c)^4,\ x\geq -c,\ z=0$ ni hosil qilamiz. Eski o‘zgaruvchilarga qaytsak, berilgan tenglamaning $y=x+(x+c)^4,\ x\geq -c,\ y=x$ yechimini hosil qilamiz.

Differensial tenglamalarni tuzish

Bitta parametrga bog‘liq bo‘lgan

$$\Phi(x,y,c)=0 \tag{10}$$

Egri chiziqlar oilasi berilgan bo‘lsin, bu yerda Φ differensiallanuvchi funksiya. Egri chiziqlar oilasining differensial tenglamasini tuzish uchun y ni x ning funksiyasi deb qarab (10) tenglikni differensiallaymiz:

$$\hat{O}'_x + \hat{O}'_y \frac{dy}{dx} = 0.$$

So‘ngra (agar c yo‘qolmasa) hosil bo‘lgan tenglama va (10) tenglamadan parametr c ni yo‘qotsak, berilgan egri chiziqlar oilasining differensial tenglamasi hosil bo‘ladi.

<u>5-misol</u>. $y=(x-c)^3,\ y'=3(x-c)^2 \Rightarrow y'=3y^{2/3}.$

<u>6-misol</u>. $y^2+cx=x^3,\ 2yy'+c=3x^2 \Rightarrow 2xyy'-y^2=2x^3.$

Agar egri chiziqlar oilasi bir necha parametrga bog‘liq bo‘lsa, u holda shu oila differensial tenglamasini tuzish uchun parametrlar nechta bo‘lsa, shuncha marotaba ketma-ket hosila olinadi. So‘ngra olingan hosilalar va egri chiziq oilasi tenglamasidan parametrlarni yo‘qotish kerak.

Mustaqil yechish uchun misollar

Differensial tenglamalarni integrallang va integral chiziqlarni chizing. Yechimni **Meple** dasturi yordamida tekshiring.

1. $y' = \dfrac{1}{\sqrt{x^2-1}}$; 2. $y' = ctgx$; 3. $y' = \dfrac{x}{\ln x}$;

4. $y' = \sin 5x \cos 3x$; 5. $y' = |y|^{\alpha}$; 6. $y' = 2e^x \cos 2x$;

7. $y' = x^2 e^x$; 8. $y' = 4e^x \cos 2x$; 9. $y' = shx$.

Differensial tenglamalarni integrallang va $M(x_0, y_0)$ nuqtadan o‘tuvchi integral chiziqni aniqlang.

10. $y' = -2xe^{-x^2}$, $M(0,3)$; 11. $y' = \dfrac{1}{x^2}$, $M(1,0)$;

12. $y' = \dfrac{1}{2\sqrt{x}}$, $M(1,0)$; 13. $y' = tg\dfrac{x}{2}$, $M(\dfrac{\pi}{2},1)$.

Egri chiziqlar oilasining differensial tenglamasini tuzing.

14. $y = e^{cx}$; 15. $y = (x-c)^3$; 16. $y = cx^3$;

17. $y = \sin(x+c)$; 18. $x^2 + cy^2 = 2y$; 19. $y^2 + cx = x^3$;

20. $y = \tilde{n}(x-c)^2$; 21. $\tilde{n}y = \sin cx$; 22. $y = ax^3 + e^x$;

23. $(x-1)^2 + by^2 = 1$.

24. Markazi $y=2x$ to‘g‘ri chiziqda va radiusi 1 ga teng bo‘lgan egri chiziqlar oilasining differensial tenglamasini tuzing.

25. $y=0$ va $y=x$ to‘g‘ri chiziqlarga bir vaqtda urinuvchi va simmetriya o‘qi *OY* o‘qiga parallel bo‘lgan parabolalar oilasining differensial tenglamasini tuzing.

26. Bir vaqtda koordinata o‘qlariga urinuvchi va I, III choraklarda joylashgan aylanalar oilasining differensial tenglamasini tuzing.

27. Koordinata boshidan o‘tuvchi va simmetriya o‘qi *OY* o‘qiga parallel bo‘lgan parabolalar oilasining differensial tenglamasini tuzing.

2- §. BIRINCHI TARTIBLI HOSILAGA NISBATAN YECHILGAN DIFFERENSIAL TENGLAMANING GEOMETRIK MA’NOSI

Yo‘nalishlar maydoni. Izoklina usuli.

(1) tenglamaning har qanday $(x,y)\in G$ nuqtada $k = \dfrac{dy}{dx} = f(x,y)$ qiymatini aniqlaydi. (1) tenglamaning *G* sohasidagi geometrik ma’nosini yo‘nalishlar maydoni yordamida aniqlash mumkin. Buning uchun har qanday $(x,y)\in G$ nuqta uchun burchak koeffisiyenti $k = f(x,y)$ bo‘lgan kesma

o‘tkazamiz. Bu holda integral chiziq o‘zining har bir nuqtasida yo‘nalishlar maydoniga urinadi.

Yo‘nalishlar maydonini yasashda izoklinlar muhim rol o‘ynaydi. Izoklina-bu yo‘nalishlar maydonida burchak koeffisiyentlar bir xil bo‘lgan nuqtalar to‘plami.

k - izoklinlar tenglamasi $f(x,y)=k$.

Izoklinlar orasida 0 - izoklina $f(x,y)=0$ muhim o‘rin tutadi.

0 - izoklinada integral chiziqning maksimum va minimum nuqtalari joylashadi va demak, bu izoklina differensial tenglamalar berilish sohasini integral chiziqlarning o‘sish va kamayish sohalariga bo‘ladi.

Shaklni aniqrok chizish uchun yechimlar grafiklarining burilish nuqtalarini

$$\frac{\partial f(x,y)}{\partial x}+f(x,y)\frac{\partial f(x,y)}{\partial y}=0$$

tenglamadan topish mumkin.

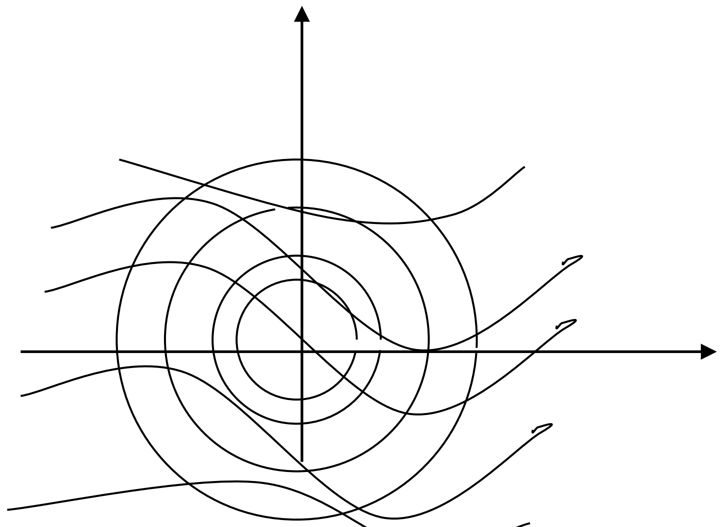
7- misol. $y'=\frac{1}{2}(x^2+y^2)-1$

Izoklina yordamida berilgan tenglamaning integral chiziqlarini chizing. k - izoklina tenglamasi $x+y^2=2(k+1),\ k\geq-1$. Bu chiziqlar radiusi $\sqrt{2(k+1)}$ bo‘lgan va markazi koordinata boshida bo‘lgan konsentrik aylanalardan iborat. k - izoklinaning ixtiyoriy nuqtasida yo‘nalishlar maydoni va x o‘qining musbat yo‘nalishi orasidagi α burchak $\alpha=arctgk$ formula yordamida aniqlanadi. k ga 1,0,-1 qiymatlar berib jadval tuzamiz:

$$k=1,\ x^2+y^2=4,\ \alpha=\frac{\pi}{4}$$

$$k=0,\ x^2+y^2=2,\ \alpha=0$$

$$k=-1,\ x^2+y^2=0,\ x=y=0,\ \alpha=-\frac{\pi}{4}$$



Ekstremumlar chizig‘i: $x^2 + y^2 = 2$.

$x^2 + y^2 < 2$ da $y' < 0$ (yechimlar kamayadi)

$x^2 + y^2 > 2$ da $y' > 0$ (yechimlar o‘sadi)

k ga ixtiyoriy *-1<k<0, k>0* qiymatlar berib izoklinlarni topish mumkin. Tenglama yechimini **Meple** dasturi yordamida tekshiramiz

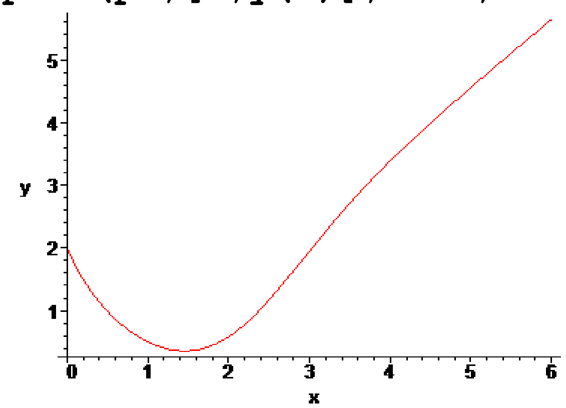
```
> d7:=diff(y(x),x)=(1/2)*(x^2+y(x)^2)-1;
```

$$d2 := \frac{\partial}{\partial x} y(x) = \frac{1}{2} x^2 + \frac{1}{2} y(x)^2 - 1$$

```
> dsolve(d7,y(x));
```

$$y(x) = -\Big(-\text{WhittakerM}\Big(\frac{1}{4} I, \frac{1}{4}, \frac{1}{2} I x^2\Big) + I x^2 \text{ WhittakerM}\Big(\frac{1}{4} I, \frac{1}{4}, \frac{1}{2} I x^2\Big) - I \text{ WhittakerM}\Big(\frac{1}{4} I, \frac{1}{4}, \frac{1}{2} I x^2\Big) + I \text{ WhittakerM}\Big(1 + \frac{1}{4} I, \frac{1}{4}, \frac{1}{2} I x^2\Big) + 3 \text{ WhittakerM}\Big(1 + \frac{1}{4} I, \frac{1}{4}, \frac{1}{2} I x^2\Big) - \_C1 \text{ WhittakerW}\Big(\frac{1}{4} I, \frac{1}{4}, \frac{1}{2} I x^2\Big) + I \_C1\ x^2 \text{ WhittakerW}\Big(\frac{1}{4} I, \frac{1}{4}, \frac{1}{2} I x^2\Big) - I \_C1 \text{ WhittakerW}\Big(\frac{1}{4} I, \frac{1}{4}, \frac{1}{2} I x^2\Big) - 4 \_C1 \text{ WhittakerW}\Big(1 + \frac{1}{4} I, \frac{1}{4}, \frac{1}{2} I x^2\Big)\Big) \Big/ \Big(x \Big(\_C1 \text{ WhittakerW}\Big(\frac{1}{4} I, \frac{1}{4}, \frac{1}{2} I x^2\Big) + \text{WhittakerM}\Big(\frac{1}{4} I, \frac{1}{4}, \frac{1}{2} I x^2\Big)\Big)\Big)$$

```
> p1 := dsolve({ diff(y(x),x) = (1/2)*(x^2-y(x)^2)-
1,y(0)=2},y(x),type=numeric):
> odeplot(p1,[x,y(x)],0..6,labels=[x,y]);
```



Mustaqil yechish uchun misollar

Izoklina yordamida berilgan tenglamaning integral chiziqlarini chizing va **Meple** dasturi yordamida natijani tekshiring.

28. $yy'+x=0;$ 29. $xy'+y=0;$ 30. $y'=x^2-y^2;$

31. $y'=\dfrac{x+y}{x-y};$ 32. $y'=y-x^2;$ 33. $2(y+y')=x+3;$

34. $xy'=2y;$ 35. $y'=x-e^y.$

3- §. O‘ZGARUVCHILARI AJRALADIGAN DIFFERENSIAL TENGLAMALAR

$$M\left(x\right)dx+N\left(y\right)dy=0. \qquad (11)$$

Tenglamaga o‘zgaruvchilai ajralgan tenglama deyiladi. Bu yerda *M(x), N(y)* mos ravishda $x\in[a,b],\ y\in[c,d]$ oraliqlarda aniqlangan uzluksiz funksiyalardir. (11) tenglamaning umumiy integrali

$$\int M(x)dx+\int N(y)dy=c,\ \int_{x_0}^{x}M(t)dt+\int_{y_0}^{y}N(s)ds=0.$$

Bu yerda $x_0\in[a,b],\ y_0\in[c,d]$ ixtiyoriy qiymatlar.

Agar (x_0,y_0) maxsusmas nuqta bo‘lsa (11) tenglamaning boshlang‘ich $y(x_0)=y_0$ shartni qanoatlantiruvchi yagona yechimi bo‘ladi va u oshkormas shaklda

$$\int_{x_0}^{x}M(t)dt+\int_{y_0}^{y}N(s)ds=0$$

tenglama bilan aniqlanadi.

O‘zgaruvchilari ajralgan tenglamalarning ko‘rinishi quyidagicha:

$$M_1\left(x\right)N_1\left(y\right)dx+M_2\left(x\right)N_2\left(y\right)dy=0. \qquad (12)$$

Bu yerda M_1, M_2 va N_1, N_2 funksiyalar mos ravishda [*a*,*b*] va [*c*,*d*] oraliqlarda aniqlangan. M_2 va N_1 funksiyalar $\{\alpha_i\}$ $(M_2(\alpha_i)=0)$ va $\{\beta_j\}$ $(N_1(\beta_j)=0)$ to‘plamlarda nolga aylansin.

$M_2(x)N_1(y)\neq 0$ sohada (12) tenglama $M_2(x)N_1(y)$ ga bo‘lish natijasida o‘zgaruvchilari ajralgan tenglamaga keladi. U vaqtda (12) tenglamaning shu sohada umumiy integrali quyidagicha topiladi:

$$\int \frac{M_1(x)}{M_2(x)}dx + \int \frac{N_2(y)}{N_1(y)}dy = c \quad \int_{x_0}^{x} \frac{M_1(t)}{M_2(t)}dt + \int_{y_0}^{y} \frac{N_2(s)}{N_1(s)}ds = c.$$

Agar $\{\alpha_i\}$ va $\{\beta_j\}$ tenglamalar bo‘sh to‘plam bo‘lmasa, u holda $x=\{\alpha_i\}$ va $y=\{\beta_j\}$; (12) tenglamaning yechimlari bo‘lishi mumkin, bu qiymatlarni berilgan tenglamaga qo‘yib tekshirish bajarish kerak.

Agar $f(x,y)=\varphi(x)g(y)$ bo‘lsa, (1) - tenglama o‘zgaruvchilari ajraladigan tenglama bo‘ladi.

8- misol $xydx+(x+1)dy=0$

O‘zgaruvchilarni ajratamiz

$$\frac{x}{x+1}dx+\frac{dy}{y}=0 \;\; [(x+1)y=0?]$$

$x\neq -1,\; y\neq 0$ sohada integrallab, $x-\ln|x+1|+\ln|y|=c$ umumiy yechimni hosil qilamiz.

$x=-1,\; y=0$ ham yechimdir. Tenglama yechimini **Meple** dasturi yordamida tekshiramiz

```
> d8:=diff(y(x),x)=(-x*y(x)/(x+1));
```

$$d8 := \frac{d}{dx}\mathrm{y}(x) = -\frac{x\,\mathrm{y}(x)}{x+1}$$

```
> dsolve(d8,y(x));
```

$$\mathrm{y}(x) = \_C1\,(\mathbf{e}^{(-x)}\,x+\mathbf{e}^{(-x)})$$

Ko‘rsatma. Agar 8 - misolining umumiy yechimida $\tilde{n}=\ln|c_1|,\; c_1\neq 0$ deb olsak, umumiy yechimni soddaroq holda yozish mumkin:

$$\ln\frac{e^x|y|}{|x+1|}=\ln c_1$$

Bundan $ye^x=c_1(x+1)$

Agar $c_1=0$ bo‘lsa, u holda $y = 0$ va demak, javobni quyidagicha yozish mumkin:

$$ye^x=c_1(x+1),\; x=-1$$

9-misol. $(x^2-1)y'+2xy^2=0,\; y(0)=1.$

O‘zgaruvchilarni ajratamiz:

$$\frac{dy}{y^2}+\frac{2xdx}{x^2-1}=0 \;\; (y^2(x^2-1)=0?)$$

Umumiy yechim

$$y(\ln\left|x^2-1\right|+c)=1,\ \ y\equiv 0.$$

Koshi masalasini yechish uchun berilgan boshlang‘ich qiymatlarni tenglamaga qo‘yamiz va parametr c ning qiymatini topamiz: $c = 1$. Demak Koshi masalasining umumiy yechimi:

$$y(\ln\left|x^2-1\right|+1)=1.$$

Tenglama yechimini **Meple** dasturi yordamida tekshiramiz

```
> d9:=(x^2-1)*diff(y(x),x)+2*x*y(x)^2=0;
```

$$d6 := (x^2-1)\left(\frac{\partial}{\partial x}\mathrm{y}(x)\right)+2\,x\,\mathrm{y}(x)^2=0$$

```
> dsolve(d9,y(x));
```

$$\mathrm{y}(x)=\frac{1}{\ln(x-1)+\ln(x+1)+\_C1}$$

10-misol. Agar egri chiziqga o‘tkazilgan urinmaning burchak koeffisiyenti urinish nuqtasi ordinatasiga teng bo‘lsa, shu egri chiziqlar oilasi tenglamasini toping. *A (0,1)* nuqtadan o‘tuvchi egri chiziqni aniqlang.

Masala shartiga ko‘ra $y'=y^2$ bo‘lib, o‘zgaruvchilarni ajratsak $\frac{dy}{y^2}=dx\ \ (y=0?)$ tenglamaga kelamiz. Integrallash natijasida

$-\frac{1}{y}=x+c$ va $y=-\frac{1}{x+c}$ ni hosil qilamiz. Bu chiziq asimptotalari *OX* o‘qi va *x*=-*c* chiziq bo‘lgan giperbolalar oilasidan iborat. $y = 0$ tenglamaning xususiy yechimi.

Demak masala yechimi $y=-\frac{1}{x+c}$, $y = 0$ dan iborat. *A* nuqtadan o‘tuvchi egri chiziqni topish uchun umumiy yechimda x va y ni *A* nuqtaning koordinatalari bilan almashtiramiz $1=-\frac{1}{0+c}$ va

c= -1. Demak berilgan nuqtadan o‘tuvchi integral chiziq tenglamasi $y=-\frac{1}{x-1}$ yoki $y=\frac{1}{1-x}$ dan iborat.

11-misol. Shunday chiziqlarni topish keraki, *PN* normal osti har doim *p* - ga teng bo‘lsin.

Ma’lumki $PN = yy'$. Demak masala shartiga ko‘ra $PN = p$, yoki $yy'=p$. O‘zgaruvchilarni ajratib $ydy=pdx$ ni hosil qilamiz. Bu tenglamani integrallaymiz

$\frac{y^2}{2} = px + c$ yoki $y^2 = 2px + c$ va berilgan masalaning umumiy yechimini hosil qilamiz.

Mustaqil yechish uchun misollar

Teglamalarni yeching va Koshi sharti qo‘yilgan masalalrning yechimini ham aniqlang. Tenglama yechimini **Meple** dasturi yordamida tekshiring.

36. $y\ln ydx + xdy = 0,\ y(1) = 1;$

37. $3e^x tgydx + (2 - e^x)\sec^2 ydy = 0$

38. $y\sin xdx + x\sin ydy = 0\ \ y(1) = 0;$ 39. $y' = \left|y\right|^{\alpha},\ y(2) = 0;$

40. $xydx + (x+1)dy = 0;$ 41. $\sqrt{y^2+1}dx = xydy;$

42. $(x^2 - 1)y' + 2xy^2 = 0,\ y(0) = 1;$

43. $y'ctgx + y = 2,\ y(0) = -1;$ 44. $y' = 3\sqrt[3]{y^2},\ y(2) = 0;$

45. $xy' + y = y^2,\ y(1) = 0,5;$ 46. $2x^2yy' + y^2 = 2;$

47. $y' - xy^2 = 2xy;$ 48. $e^{-s}(1 + \frac{ds}{dt}) = 1;$

49. $z' = 10^{x+z};$ 50. $x\frac{dx}{dt} + t = 1;$ 51. $y' = \cos(y - x);$

52. $y' - y = 2x - 3;$ 53. $(x + 2y)y' = 1,\ y(0) = -1;$

54. $y' = \sqrt{4x + 2y - 1};$ 55. $(y - x)\sqrt{1 + x^2}dy = (1 + y^2)^{3/2}dx.$

56. Egri chiziqga o‘tkazilgan urinmaning burchak koeffisiyenti urinish nuqtasi ordinatasiga proporsional bo‘lsa, shu chiziqlar oilasi tenglamasini toping.

57. Shunday egri chiziqlarni topingki unda *MN* normal va *PN* normal osti kesmalar yig‘indisi o‘zgarmas a songa teng bo‘lsin.

58. Shunday egri chiziqlarni topingki unga o‘tkazilgan o‘rinmaning koordinata o‘qlari orasiga joylashgan qismi urinish nuqtasida teng ikkiga bo‘linsa. Bunda *M* (2;3) nuqtadan o‘tuvchi chiziqni aniqlang.

4- §. BIRJINSLI VA BIRJINSLIGA KELTIRILADIGAN DIFFERENSIAL TENGLAMALAR.

$$M\left(x,y\right)dx + N\left(x,y\right)dy = 0 \qquad (4)$$

(4)-differensial shakldagi tenglamaga bir jinsli deb aytiladi. Agar *M* va *N* koeffisiyentlar bir xil *ò* - chi darajali birjinsli funksiya bo‘lsa, ya’ni

$M(tx,ty) = t^m M(x,y);\ N(tx,ty) = t^m N(x,y),\ \forall(x,y);\ (xt,yt) \in D.$

Masalan.

$$x+y,\ x^2+y^2-xy,\ \frac{x^2-y^2}{x^2+y^2},\ \varphi\left(\frac{x}{y}\right)$$

Funksiyalar mos ravishda 1,2,0,0 darajali birjinsli funksiyalar. $\sqrt{t^2x^2+t^2y^2}=|t|\sqrt{x^2+y^2}$ birnchi darajali (musbat) birjinsli funksiya.

Bir jinsli differensial tenglamani $y=zx,\ z=z(x)$ (ayrim vaqtlarda $x=zy,\ z=z(y)$ almashtirish olish maqsadga muvofiqdir) almashtirish yordamida o‘zgaruvchilari ajraladigan differensial tenglamaga keltirish mumkin. Agar $f(x,y)\equiv\varphi\left(\frac{y}{x}\right)$ bo‘lsa, (1) - tenglama birjinsli tenglama bo‘ladi. $(0,0)$ - nuqta birjinsli differensial tenglamaning maxsus nuqtasi bo‘ladi.

12-misol. $(x+2y)dx-xdy=0$

$M(x,y)=x+2y,\ N(x,y)=-x$ funksiyalar birinchi darajali birjinsli funksiyalar, demak berilgan tenglama birjinsli differensial tenglamadir. $y=zx$ almashtirish olamiz:

$$(x+2zx)dx-xd(zx)=0$$

Soddalashtiramiz

$$x(1+2z)dx-xzdx-x^2dz=0,\ x(1+z)dx-x^2dz=0$$

o‘zgaruvchilarni ajratamiz:

$$\frac{dx}{x}-\frac{dz}{1+z}=0,\ \left[x^2(1+z)=0?\right].$$

Integrallab, $x=c(1+z)$ yechimni topamiz, *x=0 va z=-1* ham yechim bo‘ladi.

Eski o‘zgaruvchilarga qaytib $x^2=c(y+x)$ berilgan tenglamaning umumiy yechimini hosil qilamiz. $x=0$ yechim umumiy yechimdan $c=0$ bo‘lganda kelib chiqadi. $y=-x$ ham differensial tenglamaning yechimidir. Demak, tenglamaning yechimlari $x^2=c(y+x),\ y=-x.$

Bir jinsliga kelitiriladigan differensial tenglamalar

$$y'=f\left(\frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}\right),\ c_1^2+c_2^2\neq 0 \qquad (13)$$

Tenglama, agar $\Delta=a_1b_2-b_1a_2\neq 0$ bo‘lsa, $x=u+x_0,\ y=\upsilon+y_0$ almashtirish yordamida birjinsli tenglamaga keltiriladi. Bu yerda (x_0,y_0) $a_1x+b_1y+c_1=0$ va $a_2x+b_2y+c_2=0$ to‘g‘ri chiziqlarning kesish nuqtasi.

Agar Δ=0 bo‘lsa, bu holda $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\lambda$ faraz qilinsa $a_1=a_2\lambda,\ b_1=b_2\lambda$ bo‘lib, (13) ning ko‘rinishi

$$y'=f\left(\frac{\lambda(a_2x+b_2y)+c_1}{a_2x+b_2y+c_2}\right)\equiv\varphi(a_2x+b_2y)$$

shaklga keladi (bunday tenglama 1 masalada ko‘rib o‘tilgan).

13-misol. $(y+2)dx+(2x+y-4)dy=0$

$$\Delta=\begin{vmatrix}0 & 1\\ 2 & 1\end{vmatrix}=-2\neq 0\quad \left.\begin{matrix}y+2=0\\ 2x+y-4=0\end{matrix}\right\}$$

Sistemani yechamiz. Demak tenglamani birjinsli tenglamaga keltirish uchun *x=u+3, y=v-2* almashtirish olamiz. U holda *dx=du, dy=dv* bo‘lib, birjinsli differensial tenglamaga keladi, ya’ni

$$vdu=(2u+v)dv$$

Yechish uchun *v=uz, z=z(u)* almashtirish olamiz, u holda

$$uzdu=(2u+uz)(udz+zdu),$$

$$u\left[zdu(2+z)(udz+zdu)\right]=0,$$

$$u\left[-z(z+1)du-(2+z)udz\right]=0.$$

Bu tenglamani yechib, yana eski o‘zgaruvchilar (x,y) ga qaytsak,

$$(x+y-1)=c(y+2)^2,\ y=-2,\ (c=\infty)$$

umumiy yechimni hosil qilamiz.

14-misol. $(x-y-1)dx+(y-x+2)dy=0$

$$\Delta=\begin{vmatrix}1 & -1\\ 1 & -1\end{vmatrix}=0$$

$z=y-x,\ z=z(x),\ [dy=dz+dx]$ almashtirish berilgan tenglamani o‘zgaruvchilari ajraladigan tenglamaga keltiradi, ya’ni $(z+2)dz+dx=0$. Yechimi $(y-x+2)^2+2x=c,$

Agar differensial tenglama $y=z^k,\ (z=z(x))$ yoki $x=z^m\ \ (z=z(y))$ almashtirish yordamida bir jinsli tenglamaga aylansa, bunday tenglama umumlashgan bir jinsli tenglama deyiladi.

k(m) sonni topish uchun tenglamada $y=z^k$ yoki $x=z^m$ almashtirish bajaramiz va *k*(m) sonni tanlash natijasida tenglama bir jinsli bo‘lishini tekshiramiz.

15- misol. $2y'+x=4\sqrt{y}$ $\qquad y=z^k$ almashtirish bajaramiz.

$$2(z^k)' + x = 4\sqrt{z^k} \Rightarrow 2kz^{k-1}z' + x = 4z^{k/2}.$$

Bir had darajalari bir xil bo‘lsa, tenglama birjinsli tenglama bo‘ladi, ya’ni $k-1=1=k/2$. Bu tenglamalarni qanoatlantiruvchi yechim $k=2$.

Demak, tenglama umumlashgan birjinsli va integrallash uchun $y=z^2$ $(z=z(x))$ almashtirish olamiz.

Bu almashtirishga asosan tenglama quyidagicha bo‘ladi:

$$4zz' + x = 4z.$$

$z=ux,(u=u(x))$ almashtirish bajarsak, o‘zgaruvchilari ajraladigan tenglama hosil qilamiz.

Berilgan tenglamaning yechimi

$$(2\sqrt{y}-x)\ln c(2\sqrt{y}-x) = x;\ 2\sqrt{y} = x$$

Tenglama yechimini **Meple** dasturi yordamida tekshiramiz.

```
> d15:=2*diff(y(x),x)+x=4*sqrt(y(x));
```

$$d7 := 2\left(\frac{\partial}{\partial x}\mathrm{y}(x)\right) + x = 4\sqrt{\mathrm{y}(x)}$$

```
> dsolve(d15,y(x));
```

$$-\left(2x^2 - 4\ln\left(\frac{-4\,\mathrm{y}(x)+x^2}{x^2}\right)\mathrm{y}(x) + \ln\left(\frac{-4\,\mathrm{y}(x)+x^2}{x^2}\right)x^2 + 4\,I\sqrt{-\frac{\mathrm{y}(x)}{x^2}}\,x^2 + 8\,I\arctan\left(2\sqrt{-\frac{\mathrm{y}(x)}{x^2}}\right)\mathrm{y}(x) - 2\,I\arctan\left(2\sqrt{-\frac{\mathrm{y}(x)}{x^2}}\right)x^2\right) \Big/ (-4\,\mathrm{y}(x)+x^2) - 2\ln(x) + \_C1 = 0$$

Ko‘rsatma. Yuqorida bayon qilingan usulni qo‘llab $k(m)$ son topilgandan so‘ng berilgan tenglamani $y=zx^k$ $(x=zy^m)$ almashtirish yordamida o‘zgaruvchilari ajraladigan tenglamaga keltirish mumkin.

Mustaqil yechish uchun misollar

Teglamalarni yeching va natijani **Meple** dasturi yordamida tekshiring.

59. $(y+\sqrt{x^2+y^2})dx - xdy = 0;$ 60. $x(x+2y)dx + (x^2-y^2)dy = 0;$

61. $y' = \frac{y}{x}\ln\frac{y}{x};$ 62. $y' = e^{-y/x} + \frac{y}{x};$

63. $(y^2-2xy)dx + x^2dy = 0;$ 64. $2x^3y' = y(2x^2-y^2);$

65. $y^2 + x^2y' = xyy';$ 66. $(x^2+y^2)y' = 2xy;$

67. $xy' - y = xtg\frac{y}{x}$; 68. $xy' = y - xe^{y/x}$;

69. $xy' - y = (x+y)\ln\frac{x+y}{x}$; 70. $(y+\sqrt{xy})dx = xdy$;

71. $(2x - y + 1)dx + (2y - x - 1)dy = 0$;

72. $(x - y)dx + (2y - x + 1)dy = 0$;

73. $(x + y + 1)dx + (2x + 2y - 1)dy = 0$;

74. $(2x + y + 1)dx + (4x + 2y - 3)dy = 0$;

75. $y' = 2\left(\frac{y+2}{x+y-1}\right)$; 76. $y' = \frac{y+2}{x+1} + tg\frac{y-2x}{x+1}$;

77. $(y^4 - 3x^2)dy + xydx = 0$; 78. $y^3dx + 2(x^2 - xy^2)dy = 0$;

79. $(x^2y^2 - y)y' + 2xy^3 = 0$; 80. $(1+\sqrt{\frac{y^2}{x} - 1})dx - 2ydy = 0$;

81. $x^3(y' - x) = y^2$; 82. $2x^2y' = y^3 + xy$.

5- §. CHIZIQLI VA BERNULLI TENGLAMALARI

$$y' + p(x)y = q(x) \qquad (13)$$

Tenglamaga chiziqli tenglama deyiladi, bu yerda *p(x)* va *q(x)* $x \in (a,b)$ oraliqda uzluksiz funksiyalar. (13) tenglamaning ikkala tomonini $x \in (a,b)$ oralig‘ida integrallovchi ko‘paytuvchi $\mu(x) = \exp(\int p(x)dx)$ ga ko‘paytirsak $\frac{d}{dx}(ye^{\int p(x)dx}) = q(x)e^{\int p(x)dx}$ ni hosil qilamiz.Hosil bo‘lgan sodda differensial tenglamani integrallab chiziqli tenglamaning umumiy yechimi topish formulasini keltirib chiqaramiz:

$$y = e^{-\int p(x)dx}[c + \int q(x)e^{\int p(x)dx}dx],$$

(13) differensial tenglamaning Koshi formasidagi yechimi

$$y = e^{-\int_{x_0}^{x} p(t)dt}[y_0 + \int_{x_0}^{x} q(s)e^{\int_{x_0}^{s} p(t)dt} ds$$

formula orqali aniqlanadi.

16 misol. $y' + ytgx = \sec x$

Integrallovchi ko‘paytuvchi, $\mu(x) = e^{\int tgxdx} = \frac{1}{\cos x}$ ga berilgan tenglamani ko‘paytiramiz:

$$\frac{1}{\cos x}y'+y\frac{\sin x}{\cos^2 x}=\frac{1}{\cos^2 x}\Rightarrow\left(\frac{y}{\cos x}\right)'=\frac{1}{\cos^2 x}$$

Ikkala tomonini integrallasak

$$\frac{y}{\cos x}=\int\frac{1}{\cos^2 x}dx+c\Rightarrow\frac{y}{\cos x}=tgx+c,$$

bu yerdan $y=c\cos x+\sin x$ umumiy yechim hosil bo'ladi. Tenglama yechimini **Meple** dasturi yordamida tekshiramiz

```
> d16:=diff(y(x),x)+y(x)*tan(x)=sec(x);
```

$$d5:=\left(\frac{\partial}{\partial x}y(x)\right)+y(x)\tan(x)=\sec(x)$$

```
> dsolve(d16,y(x));
```

$$y(x)=\cos(x)\tan(x)+\cos(x)\_C1$$

17-misol. $(x+y^2)dy=ydx.$

Bu tenglama $x=x(y)$ ga nisbatan chiziqli tenglama bo'ladi.

$$\frac{dx}{dy}-\frac{1}{y}x=y,\ [y=0?]$$

tenglamani $\mu(y)=\exp(-\int\frac{1}{y}dy)=\frac{1}{y}$ ga ko'paytirsak $\frac{d}{dx}\left(\frac{x}{y}\right)=1$ oddiy tenglama hosil qilamiz, bu yerdan

$$x=cy+y^2,\ \ y=0\ \ (c=\infty)$$

Tenglama yechimini **Meple** dasturi yordamida tekshiramiz.

```
> d17:=diff(y(x),x)=y(x)/(x+y(x)^2);
```

$$d17:=\frac{d}{dx}y(x)=\frac{y(x)}{x+y(x)^2}$$

```
> dsolve(d17,y(x));
```

$$y(x)=-\frac{\_C1}{2}+\frac{\sqrt{\_C1^2+4x}}{2},\ y(x)=-\frac{\_C1}{2}-\frac{\sqrt{\_C1^2+4x}}{2}$$

tenglama $x=x(y)$ ga nisbatan garasak

```
> d17:=diff(x(y),y)=(x(y)+y^2)/y;
```

$$d17:=\frac{d}{dy}x(y)=\frac{x(y)+y^2}{y}$$

```
> dsolve(d17,x(y));
```

$$x(y)=(y+\_C1)\,y$$

18-misol. $y(x)=\int\limits_0^x y(t)dt+x+1$

Tenglamaning ikkala tomoni *x* bo‘yicha differensiallab, $y'(x)=y(x)+1$ ni hosil qilamiz.

Integral tenglamani yechimi $y'(x)=y(x)+1,\ y(0)=1$ Koshi masalasiga teng kuchli ekanligini isbotlang?

$$y'+p(x)y=q(x)y^m,\ m\neq\overline{0,1} \tag{14}$$

tenglamaga Bernulli tenglamasi deyiladi. Bu tenglama $y=z^{\frac{1}{1-m}},\ z=z(x)$ almashtirish yordamida chiziqli tenglamaga keltiriladi.

19-misol. $xy'+(-2)x^2\sqrt{y}=4y$. Bernulli tenglamasini standart shaklga yozamiz

$$y'-\frac{4}{x}y=2x\sqrt{y},\ m=\frac{1}{2}$$

$y=z^2$ almashtirish qo‘llaymiz

$$(z^2)'-\frac{4}{x}z^2=2x\sqrt{z^2}\Rightarrow 2zz'-\frac{4}{x}z^2=2xz.$$

Bu yerda $z'-\frac{2}{x}z=x\ \ [z=0\,?]$ chiziqlali tenglamani hosil qilamiz. Yechimi $z=x^2(\ln|x|+c)$ va demak, Bernulli tenglamasining umumiy yechimi

$$y=x^4(\ln|x|+c)^2,\ \ln|x|+c\geq 0.$$

Bundan tashqari *y=0* yechimi bo‘ladi. Bu yechim maxsus yechimdir (tushuntiring?).

Mustaqil yechish uchun misollar.

Teglamalarni yeching va natijani **Meple** dasturi yordamida tekshiring.

83. $y'-y\sin x=\sin x\cos x$;
84. $(1+x^2)y'-2xy=(1+x^2)^2$;
85. $ydx+2(x+y)dy=0$;
86. $(xy+e^x)dx-xdy=0$;
87. $x^2y'+xy+1=0$;
88. $y=x(y'-x\cos x)$;
89. $2x(x^2+y)dx=dy$;
90. $(xy'-1)\ln x=2y$;
91. $(x+y^2)dy=ydx$;
92. $(2e^y-x)y'=1$;
93. $(2x+y)dy=ydx+4\ln ydy$;
94. $y'(3x-y^2)=y$;
95. $y'+2xy=2x^3y^3$;
96. $xy'+y=y^2\ln x,\ x_0=1,\ y_0=1$;

97. $3y^2y'+y^3+x=0;$ 98. $y'-9x^2y=(x^5+x^2)y^{2/3}$ $x_0=y_0=0;$

99. $y'-y=xy^2,\ x_0=y_0=0;$ 100. $y'+2y=y^2e^x;$

101. $(x+1)(y'+y^2)=-y;$ 102. $y'=y^4\cos x+ytgx;$

103. $xy'+2y+x^5y^3e^x=0;$ 104. $2y'-\frac{x}{y}=\frac{xy}{x^2-1};$

105. $y'x^3\sin y=xy'-2y;$ 106. $(2x^2y\ln y-x)y'=y;$

6- §. RIKKATI TENGLAMASI

$$y'=a(x)y^2+b(x)y+c(x) \tag{15}$$

Ko‘rinishdagi tenglamaga Rikkati tenglamasi deyiladi. Bu tenglama umumiy holda kvadraturaga keltirilmaydi.

Agar (15) tenglamaning bitta y_1 xususiy yechimi ma’lum bo‘lsa, $y=y_1+z, y=y_1+\frac{1}{z},\ z=z(x)$ almashtirish yordamida, mos ravishda, Bernulli va chiziqli tenglamalarga keltiriladi. Demak, bu tenglama kvadraturada yechiladi.

Xususiy yechimni topishning umumiy usulii yo‘q.Ba’zi hollarda tenglamadagi $c(x)$ ozod hadning ko‘rinishiga qarab yechimni tanlash taklif qilinadi.

20-misol. $xy'-(2x+1)y+y^2=-x^2$

Ozod hadning ko‘rinishiga ko‘ra xususiy yechimni $y_1=ax+b$ shaklda olamiz. Bu ifodani berilgan tenglamaga qo‘yib, x ning bir xil darajalariga mos koeffisentlarini tenglashtirsak, a va b larni aniqlash uchun quyidagi sistemalarni hosil qilamiz:

$$b^2-b=0,\ 2ab-2b=0,\ a^2-2a=-1$$

Natijada, *a=1, b=0* bo‘lib tenglamamaning xususiy yechimini topamiz: $y_1=x$

Demak, tenglamaga $y=y_1+\frac{1}{z}$ almashtirishni tadbiq qilsak, $xz'+z=1$ chiziqli tenglamani hosil qilamiz va uning umumiy yechimi $zx=x+c$ bo‘ladi

Berilgan tenglamaning umumiy yechimi

$$y=x+\frac{x}{x+c}$$

bo‘ladi

$$y'=Ay^2+\frac{B}{x}y+\frac{C}{x^2}, \tag{16}$$

tenglama Rikkatining maxsus tenglamasi bo‘lib bunda, agar A,B,C o‘zgarmaslar $(B+1)^2 \geq 4AC$ tengsizlikni qanoatlantirsa, (16) - tenglama $y_1 = \frac{a}{x}$ ko‘rinishdagi xususiy yechimga ega bo‘ladi

<u>21-misol</u> $y' = y^2 + \frac{1}{4x^2}$

Tenglamani (16)-bilan solishtirsak, $A=1,\ B=0,\ C=\frac{1}{4}$ bu yerdan

$$(0+1)^2 \geq 4\cdot 1\cdot \frac{1}{4} \Rightarrow 1=1.$$

Demak, tenglamaning xususiy yechimini $y_1 = \frac{a}{x}$ shaklda izlaymiz. a ni topish uchun yechimni tenglamaga qo‘yamiz:

$$-\frac{a}{x^2} = \frac{a^2}{x^2} + \frac{1}{4x^2}$$

bu yerdan

$$4a^2 + 4a + 1 = 0 \Rightarrow a = -\frac{1}{2}$$

Demak, tenglamaning xususiy yechimi: $y_1 = -\frac{1}{2x}$.

Tenglamaga $y_1 = -\frac{1}{2x} + \frac{1}{z}$ almashtirish tadbiq etib, uning umumiy yechimini topish mumkin. Tenglama yechimini **Meple** dasturi yordamida topamiz.

```
> d21:=diff(y(x),x)=y(x)^2+1/(4*x^2);
```

$$d9 := \frac{\partial}{\partial x} y(x) = y(x)^2 + \frac{\frac{1}{4}}{x^2}$$

```
> dsolve(d21,y(x));
```

$$y(x) = -\frac{1}{2}\frac{-\ln(x) + \_C1 - 2}{x(-\ln(x) + \_C1)}$$

Rikkati tenglamasini $y = \alpha(x)z$ almashtirish yordamida noma’lum funksiya kvadratining koeffisiyenti + 1 yoki – 1 teng bo‘lgan Rikkati tenglamasiga keltirish mumkin.

$y = z + \beta(x)$ almashtirish yordamida esa noma’lum funksiya koeffisiyentini nolga tenglashtirib olish mumkin.

Umuman $y = \alpha(x)z + \beta(x)$ almashtirish yordamida Rikkati tenglamasini

$$z' = \pm z^2 + R(x)$$

ko‘rinishga keltirish mumkin.

<u>22-misol</u>. $xy'=x^2y^2-y+4$ tenglamaga $y=\alpha(x)z$ almashtirish tadbiq qilamiz.

$$x(\alpha(x)z)'=x^2\alpha^2(x)z^2-\alpha(x)z+4$$

$$x(\alpha' z+\alpha z')=x^2\alpha^2(x)z^2-\alpha(x)z+4$$

α(x) fuknsiyani shunday tanlaymizki $x^2\alpha^2(x)=1$ bo‘lsin bundan $\alpha(x)=\dfrac{1}{x}$ olish mumkin. Demak

$$x(-\frac{1}{x^2}z+\frac{1}{x}z')=z^2-\frac{1}{x}z+4,\ -\frac{1}{x}z+z'=z^2-\frac{1}{x}z+4$$

yoki $z'=z^2+4$ tenglamani hosil qilamiz.

<u>23-misol</u>. $y'=y^2-2x^2y+x^4+2x+9$ tenglamaga $y=z+\beta(x)$ almashtirish tadbiq qilamiz

$$z'+\beta'=(z+\beta)^2-2x^2(z+\beta)+x^4+2x+9$$

$$z'+\beta'=z^2+2\beta z+\beta^2-2x^2z-2x^2\beta+x^4+2x+9$$

funksiyani shunday tanlaymizki *z* koeffisiyenti nolga teng bo‘lsin, $2\beta-2x^2=0$ bundan $\beta=x^2$.

Demak

$$z'+2x=z^2+x^4-2x^4+x^4+2x+9$$

yoki $z'=z^2+9$ tenglamani hosil qilamiz.

22, 23 misollarda hosil bo‘lgan tenglamalarni yechib eski izlanuvchi funksiya y ga qaytsak berilgan tenglamani umumiy yechimini hosil qilamiz.

Mustaqil yechish uchun misollar.

Xususiy yechimi $y=ax+b$ ko‘rinishda bo‘lgan tenglamalarni umumiy yechimini toping.

107. $y'=y^2-xy-x$

108. $xy'=y^2-(2x+1)y+x^2+2x$

tenglamalar umumiy yechimini toping va natijani **Meple** dasturi yordamida tekshiring.

109. $y'+y^2=-\dfrac{1}{4x^2}$; 110. $x^2y'=x^2y^2+xy+1$;

111. $x^2y'+(xy-2)^2=0$; 112. $3y'-y^2+\dfrac{2}{x^2}=0$;

Noma’lum funksiya kvadrati koeffisiyentini birga keltirib tenglamaning umumiy yechimini toping.

113. $xy'=x^2y^2-(2x+1)y+1;$ 114. $xy'=x^2y^2-y-1.$

Noma'lum funksiya koeffisiyentini nolga keltirib tenglamaning umumiy yechimini toping.

115. $y'=4y^2-4x^2y+x^4+x+4$;

116. $y'=y^2-2x^2y+x^4+2x+4.$

Tenglamani $y'=y^2+a$ ko‘rinishga keltirib umumiy yechimini toping.

117. $xy'=x^2y^2+y+\dfrac{2}{x^2}+2;$ 118. $xy'=y^2-3y+4x^2+2.$

7- §. TO‘LIQ DIFFERENSIALLI TENGLAMALAR

$$M(x,y)dx+N(x,y)dy=0$$

tenglama berilgan bo‘lsin.

Agar differensiallanuvchi U(x,y) funksiya mavjud bo‘lib,

$$dU(x,y)=M(x,y)dx+N(x,y)dy,\ (x,y)\in D$$

tenglik bajarilsa, (4)-tenglamaga to‘liq differensialli tenglama deyiladi.

To‘liq differensiallanuvchi tenglama $dU(x,y)=0$ tenglamaga teng kuchli va uning yechimi $U(x,y)=c.$

D soha bir bog‘lamli soha bo‘lib, bu sohada $\dfrac{\partial M}{\partial y}$ va $\dfrac{\partial N}{\partial x}$ hosilalar mavjud va uzluksiz bo‘lsin. (4)-tenglama to‘liq differensiallanuvchi tenglama bo‘lishi uchun $\dfrac{\partial M}{\partial y}\equiv\dfrac{\partial N}{\partial x}\ \ \forall(x,y)\in D,$ shart bajarilshi yetarli va zarurdir.

Differensial tenglamaning umumiy integralini quyidagi formulalarning birortasi yordamida aniqlash mumkin:

$$\int_{x_0}^{x}M(x,y)dx+\int_{y_0}^{y}N(x_0,y)dy=c,$$

$$\int_{x_0}^{x}M(x,y)dx+\int_{y_0}^{y}N(x,y)dy=c,$$

bu yerda $(x_0,y_0)\in D$ ixtiyoriy nuqta. Agar $(x_0,y_0)\in D$ maxsusmas nuqta bo‘lsa, u holda differensial tenglama yechimining mavjudligi va yagonalik nuqtasi mavjud bo‘ladi.

$y(x_0)=y_0$ Koshi masalasining yechimi esa

$$\int_{x_0}^{x}M(x,y)dx+\int_{y_0}^{y}N(x_0,y)dy=0,\ \int_{x_0}^{x}M(x,y)dx+\int_{y_0}^{y}N(x,y)dy=0$$

formulalarning birortasi yordamida aniqlanadi.

24 misol.

$$(2-9xy^2)xdx+(4y^2-6x^3)ydy=0$$

$$\frac{\partial M}{\partial y}=\frac{\partial}{\partial y}(2x-9x^2y^2)=-18x^2y,\ \frac{\partial N}{\partial x}=\frac{\partial}{\partial x}(4y^3-6x^3y)=-18x^2y$$

bo‘lganligi sababli berilgan tenglama to‘liq differensialli tenglamadir.Shunday $u(x,y)$ funksiyani topish kerakki uning to‘liq differensiali $dU=U'_x dx+U'_y dy$ berilgan tenglamaning chap tomoniga teng bo‘lsin, ya’ni u(x,y) uchun quyidagi shartlar o‘rinli bo‘lsin:

$$\begin{cases} U'_x=2x-9x^2y^2 \\ U'_y=4y^3-6x^3y \end{cases}$$

bu sistemaning birinchi tenglamasini x bo‘yicha integrallaymiz, bu holda y o‘zgarmas deb qaraladi. Integrallash natijasida hosil bo‘lgan o‘zgaruvniga $\varphi(y)$ – ni qo‘shish kerak (integrallashni ikkinchi tenglamadan ham boshlash mumkin, bu holda o‘zgarmas o‘rniga $\varphi(x)$ – ni qo‘shish kerak).

$$U(x,y)=x^2-3x^3y^2+\varphi(y)$$

bu ifodani sistemaning ikkinchi tenglamasiga qo‘yib $\varphi(y)$ – ni topamiz

$$(x^2-3x^3y^2+\varphi(y))'_y=4y^3-6x^3y,\ \varphi'(y)=4y^3$$

yoki

$$\varphi(y)=y^4.$$

Demak $U(x,y)=x^2-3x^3y^2+y^4$ va berilgan tenglamaning umumiy integrali quyidagicha bo‘ladi

$$x^2-3x^3y^2+y^4=c$$

Agar berilgan tenglama to‘liq differensialli tenglama bo‘lsa, uning umumiy integralini yuqorida keltirilgan umumiy integralni topish formulalarining birortasini qo‘llab topish mumkin.

25-misol. $2xydx+(x^2-y^2)dy=0$

Tenglamani to‘liq differensialli bo‘lish shartini tekshiramiz.

$$\frac{\partial M}{\partial y}=\frac{\partial}{\partial y}(2xy)=2x,\ \frac{\partial N}{\partial x}=\frac{\partial}{\partial x}(x^2-y^2)=2x$$

Demak, bu tenglama to‘liq differensialli ekan.

$x_0=0,\ y_0=0$ deb, olamiz va umumiy integralni formula bo‘yicha topamiz:

$$\int_0^x 2xydx+\int_0^y (0-y^2)dy=c$$

yoki bu yerdan tenglamani umumiy integralini aniqlaymiz:. $x^2y-\frac{1}{3}y^3=c$

Tenglama yechimini **Meple** dasturi yordamida tekshiramiz

```
> d25:=diff(y(x),x)=-2*x*y(x)/(x^2-y(x)^2);
```

$$d25 := \frac{d}{dx}\,y(x) = -\frac{2\,x\,y(x)}{x^2 - y(x)^2}$$

```
> dsolve(d25,y(x));
```

$$y(x) = \frac{\frac{\left(4+4\sqrt{-4\,x^6\,\_C1^3+1}\right)^{(1/3)}}{2} + \frac{2\,x^2\,\_C1}{\left(4+4\sqrt{-4\,x^6\,\_C1^3+1}\right)^{(1/3)}}}{\sqrt{\_C1}},\ y(x) = \Bigg(-\frac{\left(4+4\sqrt{-4\,x^6\,\_C1^3+1}\right)^{(1/3)}}{4} - \frac{x^2\,\_C1}{\left(4+4\sqrt{-4\,x^6\,\_C1^3+1}\right)^{(1/3)}} - \frac{1}{2}\,I\,\sqrt{3}\left(\frac{\left(4+4\sqrt{-4\,x^6\,\_C1^3+1}\right)^{(1/3)}}{2} - \frac{2\,x^2\,\_C1}{\left(4+4\sqrt{-4\,x^6\,\_C1^3+1}\right)^{(1/3)}}\right)\Bigg)\Big/\sqrt{\_C1},$$

$$y(x) = \Bigg(-\frac{\left(4+4\sqrt{-4\,x^6\,\_C1^3+1}\right)^{(1/3)}}{4} - \frac{x^2\,\_C1}{\left(4+4\sqrt{-4\,x^6\,\_C1^3+1}\right)^{(1/3)}} + \frac{1}{2}\,I\,\sqrt{3}\left(\frac{\left(4+4\sqrt{-4\,x^6\,\_C1^3+1}\right)^{(1/3)}}{2} - \frac{2\,x^2\,\_C1}{\left(4+4\sqrt{-4\,x^6\,\_C1^3+1}\right)^{(1/3)}}\right)\Bigg)\Big/\sqrt{\_C1}$$

Ko'rsatma. To'liq differensialli tenglamani integrallash uchun gruppalash usulini qo'llab, xar bir gruppada to'liq differensial hosil qilib tenglamaning yechimini topish mumkin. Yuqoridagi misolda

$$(2xydx + x^2dy) - y^2dy = 0$$

yoki

$$d(x^2y) - d(\frac{1}{3}y^3) = 0,$$

$$d(x^2y - \frac{1}{3}y^3) = 0, \quad yoki \quad x^2y - \frac{1}{3}y^3 = c.$$

Mustaqil yechish uchun misollar

Differensial tenglamalarni integrallang va yechimini **Meple** dasturi yordamida tekshiring.

119. $\frac{2x(1-e^y)}{(1+x^2)^2}dx + \frac{e^y}{1+x^2}dy = 0;$ 120. $\frac{2x}{y^3}dx + \frac{y^2-3x^2}{y^4}dy = 0;$

121. $(1+e^{x/y})dx+e^{x/y}(1-\frac{x}{y})dy=0;$ 122. $\frac{y}{x}dx+(y^3+\ln x)dy=0;$

123. $x(2x^2+y^2)+y(x^2+2y^2)y'=0;$

124. $x\sin xdx+\cos^2 ydy=0;$

125. $(x\cos y-\cos x+\frac{1}{y})dy+(\sin y+y\sin x+\frac{1}{x})dx=0;$

126. $\frac{3x^2+y^2}{y^2}dx-\frac{2x^3+5y}{y^3}dy=0$;

127. $2x(1+\sqrt{x^2-y})dx-\sqrt{x^2-y}dy=0;$

128. $(1+y^2\sin 2x)dx-2y\cos^2 xdy=0$;

129. $3x^2(1+\ln y)dx=(2y-\frac{x^3}{y})dy;$

130. $(\frac{x}{\sin y}dx+2)dx+\frac{(x^2+1)\cos y}{\cos^2 y-1}dy=0.$

8- §. INTEGRALLOVCHI KO'PAYTUVCHI

Agar

$$M(x,y)dx+N(x,y)dy=0 \tag{4}$$

tenglamani uzluksiz va uzluksiz hosilaga ega bo'lgan $\mu(x,y)\neq 0$ funksiyaga ko'paytirish natijasida to'liq differensialli tenglama hosil bo'lsa, bunday funksiyaga tenglamaning integrallovchi ko'paytuvchisi deyiladi. Masalan, o'zgaruvchilari ajraladigan (12) tenglama uchun integrallovchi ko'paytuvchi $\mu(x,y)=\frac{1}{M_2(x)N_1(y)}$, chiziqli tenglama (13) uchun

$$\mu(x,y)=\mu(x)=\exp(\int p(x)dx).$$

(4) tenglamaning integrallovchi ko'paytuvchisi $\mu(x,y)$ D sohada mavjud bo'lishi uchun *M* va *N* funksiyalarning uzluksiz xususiy hosilalari mavjud bo'lishi kerak va $\mu(x,y)$ funksiya quyidagi xususiy hosilali differensial tenglamani qanoatlantirishi kerak.

$$M\frac{\partial\mu}{\partial y}-N\frac{\partial\mu}{\partial x}=\mu(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}) \tag{17}$$

Umumiy holda $\mu(x,y)$ funksiyani topish usuli mavjud emas. Xususiy hollarda (17) tenglamaning yechimini topish mumkin.

Agar birorta $\omega = \omega(x, y)$ funksiya uchun

$$\frac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{N\dfrac{\partial \omega}{\partial x} - M\dfrac{\partial \omega}{\partial y}} \equiv \psi(\omega) \tag{18}$$

shart bajarilsa, u holda $\mu(\omega) = \exp(\int \psi(\omega)d\omega)$ shakldagi integrallovchi ko‘paytuvchi mavjud bo‘ladi.

(4) tenglama faqat x ga bog‘liq bo‘lgan integrallovchi $\mu = \mu(x)$ $[\omega = x]$ ko‘paytuvchiga ega bo‘ladi, agar

$$\frac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{N} \equiv \psi(x)$$

shart bajarilsa. Xuddi shunday $\mu = \mu(y)$ $[\omega = y]$ agar,

$$\frac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{-M} \equiv \psi(y).$$

26-misol. $y^2dx - (xy + x^3)dy = 0$ tenglamaning integrallovchi ko‘paytuvchi $\mu = \mu(x)$ yoki $\mu = \mu(y)$ shaklda bo‘lganda integrallaymiz.

$$\frac{\partial M}{\partial y} = 2y, \ \frac{\partial N}{\partial x} = -y - 3x^2$$

$$\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) : (-M) = (3y + 3x^2) : y^2 \not\equiv \psi(y)$$

Demak, integrallovchi ko‘paytuvchi y ga bog‘liq emas

$$\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) : N = -\frac{3}{x}$$

Bu yerdan integrallovchi ko‘paytuvchi faqat x ga bog‘liq: $\mu = \mu(x) = e^{\int(-\frac{3}{x})dx} = \dfrac{1}{x^3}$

Tenglamaning ikkala tomonini $\mu(x)$ ga ko‘paytirsak

$$\frac{y^2}{x^3}dx - \left(\frac{y}{x^2} + 1\right)dy = 0$$

to‘liq differensialli tenglama hosil qilamiz.

Bu tenglamaning umumiy yechimi $\dfrac{1}{2}\dfrac{y^2}{x^3} + y = c, x = 0.$

27-misol. $(x^2y^3+y)dx+(x^3y^2-x)dy=0$ tenglamaning integrallovchi ko‘paytuvchisini $\mu(\omega(x,y))$ shaklda izlaymiz.

$$\frac{\partial M}{\partial y}=3x^2y^2+1,\ \ \frac{\partial N}{\partial x}=3x^2y^2-1$$

Bu yerda $\omega=xy$, (18) ayniyatdan.

$$\frac{\dfrac{\partial M}{\partial y}-\dfrac{\partial N}{\partial x}}{Ny-Mx}=\frac{3x^2y^2+1-3x^2y^2+1}{y(x^3y^2-x)-(x^2y^3+y)x}=-\frac{1}{xy}$$

Demak, $\mu(x,y)=\exp\int(-\frac{1}{xy})d(xy)=\frac{1}{xy}$.

Berilgan tenglamaning ikkala tomonini $\frac{1}{xy}$ ga ko‘paytirsak,

$$(xy^2+\frac{1}{x})dx+(x^2y-\frac{1}{y})dy=0$$

to‘liq differensialli tenglama hosil bo‘ladi.

Yechimi

$$x^2y^2+2\ln\left|\frac{x}{y}\right|=c,\ \ x=0\ \ (c=\infty);\ \ y=0\ \ (c=\infty).$$

Agar (4) tenglamada biror $\varphi(x,y)$ funksiyaning to‘liq differensialini ajratish mumkin bo‘lsa, ba’zi hollarda tenglamani almashtirish bajarib soddalashtirish mumkin. Bu holda *(x,y)* o‘zgar uvchilardan *(x,z)* yoki *(y,z)* o‘zgaruvchilarga o‘tish kerak, bu yerda $z=\varphi(x,y)$.

28- misol. $(x^2+y^2+x)dx+ydy=0$

To‘liq differensial ajratamiz

$$(x^2+y^2)dx+\frac{1}{2}d(x^2+y^2)dy=0$$

$x^2+y^2=z$ deb olsak $zdx+\frac{1}{2}dz=0$ tenglamani hosil qilamiz.

O‘zgaruvchilarni ajratib integrallasak $2x+\ln|z|=c$ yoki $2x+\ln(x^2+y^2)=c$ umumiy integralni hosil qilamiz.

29-misol. $(2x^2y^2+y)dx+(x^3y-x)dy=0$ tenglamani yechamiz.

Qavslarni ochamiz $2x^2y^2dx+ydx+x^3ydy-xdy=0$, to‘liq differensial ajratamiz.

$$xy(2xydx + x^2dy) + (ydx - xdy) = 0$$

$$xyd(x^2y) + y^2d(\frac{x}{y}) = 0$$

xy ga bo‘lib, $x^2y = u,\ \frac{x}{y} = \vartheta$ almashtirish olsak $du + \frac{d\vartheta}{\vartheta} = 0$ tenglamani hosil qilamiz. Integrallab eski o‘zgaruvchilarga qaytsak berigan tenglamani umumiy integrali kelib chiqadi:

$$x^2y + \ln\left|\frac{x}{y}\right| = c,\ x = 0,\ y = 0.$$

Mustaqil yechish uchun misollar

Ko‘rsatilgan argumentga bog‘liq integrallovchi ko‘paytuvchini topib tenglamani integrallang.

131. $(x^2 + y)dx - xdy = 0,\ \mu = \mu(x)\ \ yoki\ \ \mu = \mu(y);$

132. $2xy\ln ydx + (x^2 + y^2\sqrt{y^2+1})dy = 0,\ \mu = \mu(x)\ \ yoki\ \ \mu = \mu(y);$

133. $(x + \sin x + \sin y)dx + \cos ydy = 0,\ \mu = \mu(x)\ \ yoki\ \ \mu = \mu(y);$

134. $(x^2y + y)dx - xdy = 0,\ \mu = \mu(x)\ \ yoki\ \ \mu = \mu(y);$

135. $(x^2 + y^2 + 1)dx - 2xydy = 0,\ \mu = \mu(y^2 - x^2)\ \ yoki\ \ \mu = \mu(x);$

136. $xdx + ydy + x(xdy - ydx) = 0,\ \mu = \mu(x^2 + y^2);$

137. $(x^2y^3 + y)dx + (x^3y^2 - x)dy = 0,\ \mu = \mu(x + y),\ \mu = \mu(xy),$
$\mu = \mu(x^2 - y^2);$

138. $\frac{1}{x}dx + (x - \frac{1}{y})dy = 0,\ \mu = \mu(\frac{y}{x}),\ \mu = \mu(x^2 + y^2);$

139. $(x + y^2)dx + y(1 - x)dy = 0,\ \mu = \mu(x^2 + y^2);$

140. $(x^2 - y^2 + 1)xdx + (x^2 - y^2)ydy = 0,\ \mu = \mu(x^2 - y^2).$

To‘liq differensial ajrating va belgilash kiritib tenglamani integrallang.

141. $(x^2 + y^2 + y)dx - xdy = 0;$ 142. $ydy = (xdy + ydx)\sqrt{1 + y^2};$

143. $xy^2(xy' + y) = 1;$ 144. $y^2dx - (xy + x^3)dy = 0;$

145. $(y - \frac{1}{x})dx + \frac{dy}{y} = 0;$ 146. $(x^2 + 3\ln y)ydx = xdy;$

147. $y^2dx + (xy + tgxy)dy = 0;$ 148. $y(x + y)dx + (xy+)dy = 0;$

149. $y(y^2 + 1)dx + x(y^2 - x + 1)dy = 0;$

150. $(x^2+2x+y)dx=(x-2x^2y)dy$;

151. $ydx-xdy=2x^3 tg\frac{y}{x}dx$; 152. $y^2dx+(e^x-y)dy=0$;

153. $xydx=(y^3+x^2y+x^2)dy$;

154. $x^2y(ydx+xdy)=2ydx+xdy$;

155. $(x^2-y^2+y)dx+x(2y-1)dy=0$;

156. $(2x^2y^3-1)ydx+(4x^2y^3-1)xdy=0$.

2-BOB
MAVJUDLIK VA YAGONALIK TEOREMASI

$$y' = f(x, y) \tag{1}$$

$$y(x_0) = y_0 \tag{2}$$

Koshi masalasini qanoatlaniruvchi tenglama berilgan bo‘lsin.

Teorema(Pikar-Lendelyof). $D = \left\{ (x, y) \in R^2 \mid |x - x_0| \le a, \ |y - y_0| \le b \right\}$ yopiq sohada funksiya uzluksiz va y o‘zgaruvchi bo‘yicha Lipshis shartini qanoatlantirsin:

$$|f(x, y_1) - f(x, y_2)| \le L|y_1 - y_2|, \ L - const.$$

Bu shartlar bajarilganda, $x_0 - h \le x \le x_0 + h$ oraliqda (1)-(2) Koshi masalasini qanoatlantiruvchi yagona $y = y(x)$ yechim mavjud bo‘ladi, bu yerda $h = \min\left\{ a, \frac{b}{M} \right\}$, D sohada (ixtiyoriy M>0 mavjud) $|f(x, y)| \le M$.

Eslatma. Agar D sohada $\frac{\partial f}{\partial y}$ mavjud va uzluksiz bo‘lib, $\left| \frac{\partial f}{\partial y} \right| \le K$ tengsizlik bajarilsa, bu sohada f funksiya y o‘zgaruvchi bo‘yicha Lipshis shartini qanoatlantiradi, ya’ni

$$|f(x, y_1) - f(x, y_2)| \le K|y_1 - y_2|.$$

Ketma-ket yaqinlashishlar (Pikar yaqinlashishilari)

$$y_0(x) = y_0, \ y_n(x) = y_0 + \int_{x_0}^{x} f(t, y_{n-1}(t))dt, \ n = 1, 2, ... \tag{3}$$

formulalar yordamida aniqlanadi. Bu yaqinlashishlar $[x_0 - h, x_0 + h]$ oralig‘ida $y = y(x)$ yechimga tekis yaqinlashadi va

$$|y(x) - y_n(x)| \le ML^n \frac{h^{n+1}}{(n+1)!}$$

tengsizlik o‘rinli.

Agar (1) tengalama uchun Pikar teoremasi shartlari yopiq chegaralangan sohada bajarilsa, u holda tenglamaning barcha yechimini soha chegarasiga davom ettirish mumkin.

9- §. PIKAR TEOREMASINING QO‘LLANISHI. MAXSUS YECHIMLAR.

30-misol. $y' = x - y^2, \ y(0) = 0$ tenglamaning $y = y(x)$ yechimga $y_0, \ y_1, \ y_2$ yaqinlashishlarini toping. Yechim mavjudlik oralig‘ining $|x| \le h$

uzunligini aniqlang, $\left|y(x)-y_2(x)\right|$ ni baholang. Qaysi nomer k dan boshlab $k \geq K$ bo‘lganda, $\left|y(x)-y_k(x)\right| \leq 0,001,\ |x| \leq h$ bajariladi.

Pikar teoremasini qo‘llash uchun *D* soha sifatida markazi (0,0) nuqtada bo‘lgan $\left\{(x,y)\in R^2 \mid |x|\leq 1,\ |y|\leq 1\right\}$ kvadratni olamiz, demak, *a=b=1*.

$f(x,y)=x-y^2,\ \dfrac{\partial f}{\partial y}=-2y$ funksiyalar D sohada uzluksiz va $|f(x,y)|\leq 2,\ \left|\dfrac{\partial f}{\partial y}\right|\leq 2,\ (x,y)\in D$ demak *M=2, L=2*.

Pikar teoremasiga asosan $|x|\leq h$ $h=\min\left\{1,\dfrac{1}{2}\right\}$

Ketma-ket $y_0,\ y_1,\ y_2$ yaqinlashishlarni topamiz.

$$y_0=0,\ y_1(x)=0+\int_0^x [s-0]ds=\frac{x^2}{2},$$

$$y_2(x)=0+\int_0^x [s-\frac{s^4}{4}]ds=\frac{x^2}{2}-\frac{x^5}{20},$$

$$\left|y(x)-y_2(x)\right|\leq 2\cdot\frac{2^2}{3!}\left(\frac{1}{2}\right)^3=\frac{1}{6},\ |x|\leq\frac{1}{2}$$

va $\left|y(x)-y_k(x)\right|\leq 2\cdot\dfrac{2^k}{(k+1)!}\left(\dfrac{1}{2}\right)^{k+1}=\dfrac{1}{(k+1)!},\ |x|\leq\dfrac{1}{2}.$ Kerakli *k* nomer $\dfrac{1}{(k+1)!}\leq 0,001,$ yoki $(k+1)!\leq 0,001,$ tengsizlikdan aniqlanadi. Bu yerdan *k=6.*

31-misol. $y'=2+\sqrt[3]{y-2x}$ tenglamani yechimining yagonalik sohasini va maxsus yechimlarini aniqlang.

$f=2+\sqrt[3]{y-2x}$ funksiya xoy tekislikda aniqlangan va uzluksiz y bo‘yicha xususiy hosilani olamiz. $\dfrac{\partial f}{\partial y}=\dfrac{1}{3\sqrt[3]{(y-2x)^2}}$

Bu funksiya $y\neq 2x$ bo‘lganda aniqlangan va uzluksiz. Bu yerdan $y_0\neq 2x_0$ bo‘lganda, differensial tenglama yechimi yagona bo‘ladi. Yagonalik sharti $y=2x$ to‘g‘ri chiziq nuqtalarida bajarilmaydi. Bu chiziq integral chiziq bo‘ladi, chunki differensial tenglamani qanoatlantiradi. $y=2x$ ning maxsus yechim bo‘lishini ko‘rsatamiz. Tenglamaning umumiy yechimi $27(y-2x)^2=8(x+c)^3$. Faraz qilaylik, (x_0, y_0) nuqta $y = 2x$ to‘g‘ri chiziqning nuqtasi bo‘lsin, u holda $y_0 = 2x_0$.

Tenglamaning $(x_0, 2x_0)$ nuqtadan o‘tuvchi $y = 2x$ yechimdan boshqa yechimini topamiz:

Umumiy yechimga $y = 2x_0,\ x = x_0$ qiymatlarini qo‘yib, $c = -x_0$ ni aniqlaymiz. Demak, $27(y-2x)^2 = 8(x-x_0)^3$ integral chiziq $(x_0, 2x_0)$ nuqtada $y = 2x$ chiziq bilan umumiy nuqtaga ega, ya’ni $y = 2x$ chiziqning barcha nuqtalarida yagonalik sharti buziladi, ya’ni $y = 2x$ maxsus yechimdir.

Mustaqil yechish uchun misollar

Berilgan tengamaga quyilgan boshlang‘ich shartni qanoatlantiruvchi yechimga $y_0,\ y_1,\ y_2$ yaqinlashishlarni toping

157. $y' = x - y^2,\ y(0) = 0;$ 158. $y' = y^2 + 3x^2 - 1,\ y(1) = 1;$

159. $y' = y + e^{y-1},\ y(0) = 1;$ 160. $y' = 1 + x\sin y,\ y(\pi) = 2\pi.$

Berilgan boshlang‘ich shart yechimi mavjud bo‘ladigan biror oraliqni ko‘rsating.

161. $y' = x + y^3,\ y(0) = 0;$ 162. $y' = 2y^2 - x,\ y(1) = 1;$

163. $x' = t + e^x,\ x(1) = 0;$ 164. $y' = x + 2y^2,\ y(0) = 1.$

Berilgan tenglamalar uchun shunday (x,y) soha ajratingki, bu sohaning har bir nuqtasidan tenglamaning yagona yechimi o‘tsin.

165. $y' = 2xy + y^2;$ 166. $y' = 2 + \sqrt[3]{y-2x};$ 167. $(x-2)y' = \sqrt{y} - x;$

168. $y' = 1 + tgy;$ 169. $(y-x)y' = y\ln x;$ 170. $xy' = y + \sqrt{y^2 - x^2}.$

171. $0<x<1,\ 0<y<1$ sohada qaysi funksiya y ga nisbatan Lipshis shartini qanoatlantiradi

$a)\ xy^3 + x^2,\ b)\ \sin(x-y),\ c)\ (x+y)^{-1},\ d)\ \sqrt{y^2+2x},$

$e)\ |y-x|,\ f)\ xy^2\ln y.$

172. Qanday a va n ($n \ge 2,\ n \in N$) sonlar uchun $y' = x + ay^n,\ y(x_0) = y_0$ tenglamaning $x_0 \le x \le \infty$ da yechimi mavjud bo‘ladi.

173. $y' = \dfrac{P(x,y)}{Q(x,y)},\ (P,Q) = 1$ va P,Q ko‘phad bo‘lganda tenglamaning maxsus yechimi yo‘qligini ko‘rsating.

Berilgan tenglamalarning maxsus yechimini toping.

174. $y' = \dfrac{\sqrt{y}}{\sqrt{x}};$ 175. $y' = \dfrac{\sqrt{y-1}}{x};$ 176. $y' = \dfrac{\sqrt{1-y^2}}{x} + a;$ 177. $y' = xy^{2/3}.$

3-BOB

HOSILAGA NISBATAN YECHILMAGAN BIRINCHI TARTIBLI DIFFERENSIAL TENGLAMALAR

10-§. m – DARAJALI BIRINCHI TARTIBLI DIFFERENSIAL TENGLAMALAR. TO‘LIQMAS TENGLAMALAR

Hosilaga nisbatan yechilmagan birinchi tartibli differensial tenglamaning umumiy ko‘rinishi quyidagicha:

$$F(x,y,y')=0 \tag{1}$$

Birinchi tartibli m- darajali differensial tenglama

$$(y')^m + A_1(x,y)(y')^{m-1} + ... + A_{m-1}(x,y)y' + A_m(x,y) = 0 \tag{2}$$

ko‘rinishga ega.

Faraz qilaylik, birorta $D \subset R^2$ sohaning har bir *(x,y)* nuqtasida (2) tenglamaning y' hosilaga nisbatan k ta haqiqiy yechimlari

$$y' = f_i(x,y),\ i = \overline{1,k} \tag{3}$$

mavjud bo‘lsin, u vaqtda $\psi_i(x,y) = c$ (3) tenglamalarning umumiy integrallari bo‘ladi.

$$\psi_1(x,y) = c,\ \psi_2(x,y) = c, \dots,\ \psi_k(x,y) = c \tag{4}$$

funksiyalar to‘plamiga (1) tenglamaning umumiy integrali deyiladi. (4) ni boshqa shaklda ham yozish mumkin:

$$[\psi_1(x,y) - c][\psi_2(x,y) - c]\dots[\psi_k(x,y) - c] = 0 \tag{5}$$

Agar (x,y) nuqtadan (1) tenglamaning umumiy urinuvchiga ega bo‘lgan ikkita har xil integral yoyi o‘tsa, bu nuqtaga tenglamaning tarmoqlanish nuqtasi deyiladi. Tarmoqlanish nuqtalaridan tuzilgan (1) tenglamaning yechimi maxsus yechim deyiladi. Bu nuqtaning koordinatalari quyidagi tengliklarni qanoatlantiradi.

$$\begin{cases} F(x,y,y') = 0 \\ F_{y'}'(x,y,y') = 0 \end{cases} \tag{6}$$

Agar (6) sistemadan y' yo‘qotilsa, hosil bo‘lgan $\psi(x,y)=0$ funksiyaning grafigi diskriminant egri chizig‘i deyiladi. Umuman bu chiziq bir necha tarmoqlarga ajraladi. Agar tarmoq integral chiziq bo‘lsa, (tekshiramiz), u maxsus yechim bo‘lishi mumkin (nuqtalari tarmoqlanish nuqtasi bo‘lishini tekshiramiz).

Agar (1) tenglama uchun $y=\varphi(x,c)$ umumiy yechim bo‘lsa, u holda shu tenglamaning maxsus yechimi $y=\varphi(x,c)$ chiziqlar oilasining o‘ramasi bo‘ladi. Ma’lumki $\Phi(x,y,c)=0$ chiziqlar oilasining o‘ramasi

$$\Phi(x,y,c) = 0,\ \frac{\partial}{\partial c}\Phi(x,y,c) = 0$$

tenglamalar bilan aniqlanadigan s diskriminant chiziqlar turkumiga kiradi. *s* diskriminant chiziqning tarmogi o‘rama bo‘lishi uchun chegaralangan $\frac{\partial \Phi}{\partial x}, \ \frac{\partial \Phi}{\partial y}$ lar mavjud bo‘lishi va

$$\left(\frac{\partial \Phi}{\partial x}\right)^2 + \left(\frac{\partial \Phi}{\partial y}\right)^2 \neq 0$$

shart bajarilishi kerak.

Quyidagi tenglamalarning yechimlarini topamiz:

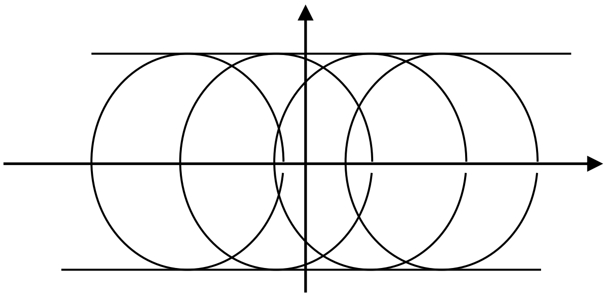
<u>32-misol</u>. $y^2(y'^2+1)=1.$

Tenglamani hosilaga nisbatan yechamiz

$$y' = \pm \frac{\sqrt{1-y^2}}{y}, \ [y=0?]$$

O‘zgaruvchilarga ajratamiz, $dx = \pm \frac{ydy}{\sqrt{1-y^2}}, \ [y=\pm 1?]$va so‘ngra integrallab, $x+c=\mp\sqrt{1-y^2}$ ni hosil qilamiz.

Tekshirib ko‘rsak, *y=0* tenglamaning yechimi bo‘lmaydi. $y=\pm 1$ maxsus yechim, chunki *y=1 (y=-1)* chiziqning har bir nuqtasiga integral chiziq urinadi.Demak, tenglamaning yechimi: $(x+c)^2+y^2=1; \ y=\pm 1.$



<u>33-misol</u>. $y'^3+y^2=yy'(y'+1)$.

y' nisbatan yechsak y'=y, $y'=\pm\sqrt{y}$ tenlamalarni hosil qilamiz. Bu tenglamalar yechimlari mos ravishda

$$y=ce^x; \ 4y=(x+c)^2; \ y=0$$

y=0 birinchi tenglama uchun xususiy yechim, ikkinchisi uchun esa maxsus yechim bo‘ladi.

Demak, *y=0* berilgan tenglama uchun maxsus yechim.

Yechim:

$$y=ce^x; \ 4y=(x+c)^2; \ y=0.$$

1. ***f(y')=0.*** Agar $f(\lambda)=0$ tenglamaning ildizlari yakkalangan bo‘lsa, u holda tenglamaning umumiy yechimi $f(\frac{y-c}{x})=0$ shaklda beriladi.

34-misol. $y'^3+3y'+1=0$ tenglamaning yechimi .

$$(\frac{y-c}{x})^3+3(\frac{y-c}{x})+1=0$$

35-misol. $y'-|y'|=0,\ \lambda-|\lambda|=0$ tenglamaning yechimlari $\lambda\geq 0$ oraliqni butunlay to‘ldiradi. Shuning uchun differensial tenglamaning yechimi ixtiyoriy kamaymaydigan differensiallanuvchi funksiya bo‘ladi.

2. ***f(x,y')=0* va *f(y,y')=0* tenglamalar.** Agar tenglamalar parametrik ko‘rinishda $x=\varphi(p),\ y'=\psi(p);\ y'=\varphi(p),\ y=\psi(p)$ yozilsa, tenglamaning yechimi kvadraturada integrallanadi. Tenglamlarni yechishda

$$dy=y'dx \tag{7}$$

asosiy munosabatdan foydalaniladi.

36-misol. $x=y'^3+y'$.

$y'=p$ deb belgilasak, tenglama $x=p^3+p$ parametrik shakldа ega bo‘ladi. (7) munosabatdan foydalansak, $dy=pd(p^3+p)$ yoki $dy=(3p^3+p)dp$ hosil bo‘ladi. Bu yerdan $y=\frac{3}{4}p^4+\frac{1}{2}p^2+c$. Demak, differensial tenglamaning parametrik shakldagi umumiy yechimini

$$\begin{cases} x=p^3+p \\ y=\frac{3}{4}p^4+\frac{1}{2}p^2+c \end{cases}$$

hosil qilamiz.

Tenglama yechimini **Meple** dasturi yordamida tekshiramiz.

```
> d36:=x=diff(y(x),x)^3+diff(y(x),x);
```

$$d36 := x=\left(\frac{\partial}{\partial x}\mathrm{y}(x)\right)^3+\left(\frac{\partial}{\partial x}\mathrm{y}(x)\right)$$

```
> dsolve(d36,y(x));
```

$$y(x)=\int \frac{1}{6}\frac{(108x+12\sqrt{12+81x^2})^{(2/3)}-12}{(108x+12\sqrt{12+81x^2})^{(1/3)}}dx+\_C1,\ y(x)=\int \frac{1}{12} I\,($$

$$I\,(108x+12\sqrt{12+81x^2})^{(2/3)}-12\,I+\sqrt{3}\,(108x+12\sqrt{12+81x^2})^{(2/3)}$$

$$+12\sqrt{3})\Big/(108x+12\sqrt{12+81x^2})^{(1/3)}dx+\_C1,\ y(x)=\int \frac{-1}{12} I\,($$

$$-I\,(108x+12\sqrt{12+81x^2})^{(2/3)}+12\,I+\sqrt{3}\,(108x+12\sqrt{12+81x^2})^{(2/3)}$$

$$+12\sqrt{3})\Big/(108x+12\sqrt{12+81x^2})^{(1/3)}dx+\_C1$$

37-misol. $y'^2+y'^3=y$

$y'=p$ parametr kiritib, parametrik shakldagi differensial tenglamani hosil qilamiz:

$$y=p^2+p^3,\ y'=p.$$

(7) tenglikka qo‘yib, $dx=\dfrac{1}{p}(2p+6p^2)dp$ ni hosil qilamiz. Bu yerdan $x=2p+3p^2+c$. Demak, differensial tenglamaning umumiy yechimi $x=2p+3p^2+c,\ y=p^2+2p^3$.

$p=0$ bo‘lgan xolatni qaraymiz. $p=0$ ni $y=p^2+p^3$ga qo‘yib, *y=0* yechimni hosil qilamiz, bu yechim maxsus yechim bo‘ladi, chunki har qanday $(0,x_0)$ nuqtadan shu nuqtaga x o‘qiga urinuvchi $x=2p+3p^2+x_0,\ y=p^2+2p^3$ interal chiziq o‘tadi. Demak,

$$\begin{cases} x=2p+3p^2+c \\ y=p^2+2p^3+c,\ y=0 \end{cases}$$

differensial tenglamaning parametrik shakldagi umumiy yechimi bo‘ladi. Tenglama yechimini **Meple** dasturi yordamida tekshiramiz.

```
> d37:=y(x)=diff(y(x),x)^3+diff(y(x),x)^2;
```

$$d37:=y(x)=\left(\frac{d}{dx}y(x)\right)^3+\left(\frac{d}{dx}y(x)\right)^2$$

```
> dsolve(d37,y(x));
```

$$y(x)=0,\ x-\int^{y(x)} 6\,(-8+108\_a+12\sqrt{3}\sqrt{\_a\,(-4+27\_a)})^{(1/3)} \Big/ ($$

$$(-8+108\_a+12\sqrt{3}\sqrt{\_a\,(-4+27\_a)})^{(2/3)}+4$$

$$-2\,(-8+108\_a+12\sqrt{3}\sqrt{\_a\,(-4+27\_a)})^{(1/3)})\,d\_a-\_C1=0,\ x-\int^{y(x)} \text{-}12\,I$$

$$(-8+108\_a+12\sqrt{3}\sqrt{\_a\,(-4+27\_a)})^{(1/3)} \Big/ (($$

$$(-8+108\_a+12\sqrt{3}\sqrt{\_a\,(-4+27\_a)})^{(1/3)}\,I+2\,I$$

$$+\sqrt{3}\,(-8+108\_a+12\sqrt{3}\sqrt{\_a\,(-4+27\_a)})^{(1/3)}-2\sqrt{3})$$

$$((-8+108\_a+12\sqrt{3}\sqrt{\_a\,(-4+27\_a)})^{(1/3)}+2))\,d\_a-\_C1=0,\ x-\int^{y(x)} \text{-}12\,I$$

$$(-8+108\_a+12\sqrt{3}\sqrt{\_a\,(-4+27\_a)})^{(1/3)} \Big/ (($$

$$(-8+108\_a+12\sqrt{3}\sqrt{\_a\,(-4+27\_a)})^{(1/3)}\,I+2\,I$$

$$-\sqrt{3}\,(-8+108\_a+12\sqrt{3}\sqrt{\_a\,(-4+27\_a)})^{(1/3)}+2\sqrt{3})$$

$$((-8+108\_a+12\sqrt{3}\sqrt{\_a\,(-4+27\_a)})^{(1/3)}+2))\,d\_a-\_C1=0$$

Mustaqil yechish uchun misollar

Tenglamalarni y' nisbatan yeching va berilgan nuqtadan o‘tuvchi chiziq tenglamasini aniqlang .**Meple** dasturi yordamida natijani tekshiring.

178. $yy'^2-|xy+1|\,y'+x=0,\ M(1;1);$ 179. $y'^2-4y=0;\ M(1,0);$

180. $y'^2=4|y|;$ 181. $y'^2=\dfrac{1}{4|x|};$ 182. $y^2(1+y'^2)=a^2;$

183. $y'^3-\dfrac{1}{4x}y'=0;$

184. $y'^3-xy'^2-4yy'+4xy=0,\ M(1,0);$

185. $yy'+y'^2=x^2+xy;$ 186. $xy'=\sqrt{1+y'^2};$

187. $x^2y'^2+3xyy'+2y^2=0;$ 188. $y'^2-2xy'=x^2-4y;$

189. $y'^3-7y'+6=0;$

190. $y'^3-(a+b+1)y'^2+(ab+a+b)y'-ab=0;$

191. $y=y'^2-y'x+\dfrac{x^2}{2}.$

Parametr kiritish usuli yordamida to‘liqmas tenglamalarni yeching.

192. $x = ay' + by'^2$; 193. $x(1+y'^2)^{3/2} = a$; 194. $x = y'^3 + 1$;

195. $xy'^3 = 1 + y'$; 196. $x^3 - y'^3 = xy'$; 197. $y = \dfrac{y'^2}{2} + \ln y'$;

198. $y = y'^2 + 2y'^3$; 199. $y^{2/3} + y'^{2/3} = a^{2/3}$; 200. $\dfrac{y}{\sqrt{1+y'^2}} = a$;

201. $xy'^2 = 1 + y'$; 202. $y = \dfrac{y'^2}{2} + \ln y'$; 203. $x = -\dfrac{1}{(1+y'^2)}$;

204. $x = y' + \ln y'$; 205. $x = y'\sin y' + \cos y'$; 206. $\arcsin\dfrac{x}{y'} = y'$;

207. $x = \ln y' + \sin y'$; 208. $y' = arctg\dfrac{x}{y'^2}$; 209. $y'\ln y' - y = 0$;

210 $y\sqrt{1+y'^2} = y'$; 211. $y' = e^{y'/y}$.

11-§. PARAMETR KIRITISHNING UMUMIY USULI. LAGRANJ VA KLERO TENGLAMALARI

Agar (1) tenglama $x = \xi(u,\vartheta)$, $y = \eta(u,\vartheta)$ $y' = \gamma(u,\vartheta)$ (8) parametrik ko‘rinishga bo‘lsa, (8) ni (7) munosabatga qo‘yib, (1) tenglamani hosilaga nisbatan yechilgan tenglama shakliga keltirish mumkin.

Agar (1) tenglamani x yoki y o‘zgaruvchilarga nisbatan yechish mumkin bo‘lsa, u va v parametr sifatida qolgan o‘zgaruvchilarni olish mumkin, Masalan, (1) ni $x = \zeta(y, y')$ shaklda yozish mumkin bo‘lsa, u holda parametr sifatida y va $y' = p$ olinadi va parametrik shakldagi

$$x = \varphi(y, p),\ \ y = y,\ \ y' = p$$

differensial tenglama hosil bo‘ladi.

Lagranj $y = \varphi(y')x + \psi(y')$, $\psi(y') \not\equiv y'$ va Klero $y = y'x + \psi(y')$ tenglamalari yuqorida ko‘rsatilgan usul yordamida integrallash mumkin bo‘lgan tenglamalar turkumiga kiradi.

38-misol. $y = 2xy' - 4y'^3$

Bu tenglama Lagranj tenglamasi. *y'=p* deb olamiz va *x=x*, $y = 2p - 4p^3$, $y' = p$ parametrik shakldagi tenglamani hosil qilamiz. *dy=pdx* munosabatga qo‘yib, $2xdp + 2pdx - 12p^3dp = pdx$ yoki $pdx + 2xdp = 12p^2dp$ tenglama hosil qilamiz. Bu yerdan

$\frac{dx}{dp}+\frac{2}{p}x=12p,\ [p=0?]$ bo‘lib, bu tenglamaning yechimi $x=cp^{-2}+3p^2$.
Demak berilgan differensial tenglamaning umumiy yechimi

$$x=cp^{-2}+3p^2,\ y=cp^{-1}+2p^3$$

p=0 bo‘lgan holni ko‘ramiz.

Bu holda *y=0* ni hosil qilamiz. *y=0* - xususiy yechim (asoslang).

Tenglama yechimini **Meple** dasturi yordamida tekshiramiz.

```
> d38:=y(x)=2*x*diff(y(x),x)-4*diff(y(x),x)^3;
```

$$d38 := y(x) = 2\,x\left(\frac{\partial}{\partial x}y(x)\right)-4\left(\frac{\partial}{\partial x}y(x)\right)^3$$

```
> dsolve(d38,y(x));
```

$$y(x)=\frac{1}{3}x\sqrt{6\,x+6\sqrt{x^2-12\,\_C1}}-\frac{1}{54}\left(6\,x+6\sqrt{x^2-12\,\_C1}\right)^{(3/2)},$$

$$y(x)=-\frac{1}{3}x\sqrt{6\,x+6\sqrt{x^2-12\,\_C1}}+\frac{1}{54}\left(6\,x+6\sqrt{x^2-12\,\_C1}\right)^{(3/2)},$$

$$y(x)=\frac{1}{3}x\sqrt{6\,x-6\sqrt{x^2-12\,\_C1}}-\frac{1}{54}\left(6\,x-6\sqrt{x^2-12\,\_C1}\right)^{(3/2)},$$

$$y(x)=-\frac{1}{3}x\sqrt{6\,x-6\sqrt{x^2-12\,\_C1}}+\frac{1}{54}\left(6\,x-6\sqrt{x^2-12\,\_C1}\right)^{(3/2)}$$

39-misol. $y=xy'-y'^2$

Bu Klero tenglamasini $x=x,\ y=xp-p^2,\ y'=p$ parametrik tenglama shakliga keltirish mumkin. Lekin tenglamaning xossalaridan foydalanish, maqsadga muvofiq bo‘ladi. Klero tenglamasining umumiy yechimini hosil qilish uchun tenglamada *y′=c* deb olish kerak ya’ni $y=cx-c^2$. Maxsus yechim $y=cx-c^2,\ 0=x-2c$ sistemadan topiladi. Bu yerda c parametrni yo‘qotsak , $y=\frac{1}{4}x^2$ maxsus yechim hosil bo‘ladi.

40-misol $y'^3+y^2=yxy'$

Tenglamani *x* ga nisbatan yechamiz: $x=\frac{y'^2}{y}+\frac{y}{y'}$. Bundan differensial tenglamaning parametrik shakldagi $y=y,\ y'=p\ \ x=\frac{y'^2}{y}+\frac{y}{y'}$ ko‘rinishi kelib chiqadi. Bu qiymatlarni (7) asosiy munosabatga qo‘ysak, $\frac{dy}{dp}=\frac{2}{p}y-\frac{y^3}{p^4}$ Bernulli tenglamasini hosil qilamiz. Tenglamani yechib,

$y^2(2p+c)=p^4,\; pxy=y^2+p^3$ ga ega bo‘lamiz. *y=0* ham tenglamaning yechimi bo‘ladi.

41-misol. $y=xy'^2+y'^3$. $y'=p$ deb olamiz. $y=xp^2+p^3$. Berilgan tenglama Lagranj tenglamasi bo‘lib, bunda,

$$\varphi(p)=p^2 \quad \psi(p)=p^3$$

Ko‘rsatilgan usul bo‘yicha berilgan tenglamani x ga nisbatan differensiallaymiz:

$$p=p^2+2xp\frac{dp}{dx}+3p^2\frac{dp}{dx}$$

yoki

$$p-1+(2x+3p)\frac{dp}{dx}=0.$$

yoki

$$\frac{dx}{dp}+\frac{2}{p-1}x=-\frac{3p}{p-1},$$

bu esa chiziqli tenglama bo‘lib, bunda

$$P(p)=\frac{2}{p-1},\; Q(p)=-\frac{3p}{p-1},$$

Bu tenglamani chiziqli tenglamaning umumiy yechimini topish formulasi

$$x=e^{-\int P(p)dp}\left[c+\int Q(p)e^{\int P(p)dp}dp\right]$$

yordamida integrallaymiz:

$$õ=\frac{1}{(ð-1)^2}\left[ñ-ð^3+\frac{3}{2}ð\right]$$

demak

$$\begin{cases} ó=\dfrac{ð^2}{(ð-1)^2}\left[ñ-ð^3+\dfrac{3}{2}ð\right]+ð^3 \\ õ=\dfrac{1}{(ð-1)^2}\left[ñ-ð^3+\dfrac{3}{2}ð\right] \end{cases}$$

tenglamani parametr ko‘rinishidagi umumiy yechimi bo‘ladi.

42-misol. $y'^2-y^2=0$ tenglamaning maxsus yechimi mavjudmi?

Qo‘yilgan savolga javob berish uchun diskriminant chiziqni tuzamiz, buning uchun $y'^2+(-y^2)=0,\; 2y'=0$ tenglamalardan *y'* ni yo‘qotib *y=0* ni hosil qilamiz. Tekshirish natijasida *y=0* differensial tenglamaning yechimi ekanligi aniqlanadi.

43-misol. Agar differensial tenglama chiziqlar oilasining yechimi $y = c(x-c)^2$ ma'lum bo'lsa, differensial tenglamaning maxsus yechimini toping. Agar maxsus yechim ma'lum bo'lsa u chiziqlar oilasining o'ramasi bo'ladi. Avvalo *C* diskriminant chizig'ini tuzamiz:

$$y = c(x-c)^2,\ 0 = (x-c)^2 - 2c(x-c)$$

Bu tenglamlardan *c* parametrni yo'qotsak, *C* diskriminant chiziqning ikkita $y=0$, $y = \frac{4}{27}x^3$ tarmog'ini topamiz.

Birinchi tarmoq uchun $\frac{\partial}{\partial y}[y - c(x-c)^2]|_{y=0} \neq 0$ va yetarli shartga asosan $y=0$ o'rama, demak $y=0$ maxsus yechim.

Xuddi shu usulda $y = \frac{4}{27}x^3$ ning maxsus yechim ekanligini aniqlaymiz.

Mustaqil yechish uchun misollar

Tenglamalarni yeching va **Meple** dasturi yordamida natijani tekshiring.

213. $y = x(1+y') + y'^2$;

214. $y = -xy' + y'^2$;

215. $2y(y'+2) = xy'^2$;

216. $y = xy' - y'^2$;

217. $y = xy' - a\sqrt{1+y'^2}$;

218. $y = xy' + \sqrt{1-y'^2}$;

219. $y = x + y'^2 - y$;

220*. $x = \frac{y}{y'} + \frac{1}{y'^2}$;

221*. $x + \frac{y}{y'} = \frac{4}{\sqrt{y'}}$;

222. $xy'^2 - yy' - y' + 1 = 0$;

223. $y'^3 = 3(xy' - y)$;

Differerensial tenglama maxsus yechimini toping.

224. $y' = \frac{3}{2}y^{1/3}$;

225. $y' = 1 + \frac{3}{2}(y-x)$;

226. $y' = \sqrt{y}$;

227. $y' = x^2 + y^2$;

228. $y' = \cos(xy)$;

229. $y' = \sqrt[3]{x-y} - 1$;

230. $y' = \sqrt[3]{x-y} + 1$;

231. $y' = \sin x + y\cos x$;

232. $y' = y + \sqrt[3]{2y}$;

233. $y' = y + \sqrt{x^2+y^2}$.

Berilgan egri chiziqlar oilasining o'ramasi tenglamasini toping

234. $y = ñx + \frac{1}{c}$;

235. $y^2 = 2cx + c^2$;

236. $(x-c)^2+y^2=\frac{c^2}{2}$;   237. $(x-c)^2+y^2=1$;

238. $y=ce^x+\frac{1}{c}$;   239. $y=x(c-\frac{1}{x})^2,\ x\neq 0$.

12-§. BIRINCHI TARTIBLI HAR XIL DIFFERENSIAL TENGLAMALARNI INTEGRALLASH

Berilgan tenglamalarni tipini aniqlang.

240. 1) $(2xy^2+3x^2+\frac{1}{x^2}+2\frac{x^2}{y^2})dx+(2x^2y+3y^2+\frac{1}{y^2}-2\frac{x^3}{y^3})dy=0$;

2) $xy(1+y^2)dx+(1+x^2)dy=0$;   3) $2xydx-(x^2-y^2)dy=0$;

4) $y'ctgx-y=2\cos^2 xctgx$;   5) $x^2y^2y'+xy^3=a^2,\ a\in R$;

6) $y'=4y^2-4x^2y+x^4+x+4$;   7) $yx'-2x+y^2=0$;

8) $xy(1+y^2)dx-(1+x^2)dy=0$;   9) $\frac{x^2dy-y^2dx}{(x-y)^2}=0$;

10) $(x^2+y^2)dx-2xydy=0$;   11) $(4-x^2)y'+xy=4$;

12) $y'tgx+2ytg^2x=by^2,\ b\in R$;   13) $xy'=x^2y^2-y+1$;

14) $dx+(x+y^2)dy=0$.

Ko‘rsatilgan almashtirishni bajarib tenglamani integrallang

241. $(x-2y^3)dx+3y^2(2x-y^3)dy=0,\ y^3=u(x)$;

242. $xy'+1=xe^{x-y},\ e^y=u(x)$;

243. $y'+\sin y+x\cos x+x=0,\ tg\frac{y}{2}=u(x)$;

244. $y'=\frac{y-x^2\sqrt{x^2-y^2}}{xy\sqrt{x^2-y^2}+x},\ y=xu(x)$;

245. $y'-e^{x-y}+e^x=0\ \ y=\ln u(x)$.

Ko‘rsatilgan amallarni bajaring va integrallash usulini ko‘rsatin

246. $y(x+\ln y)+(x-\ln y)y'=0,\ \ln y=u(x)$;

247. $y'=\cos(ay+bx),\ a\neq 0,\ u(x)=ay+bx$;

248. $y'+\alpha\sin(ay+bx)+\beta=0,\ u(x)=ay+bx$;

249. $y'=\frac{\sqrt{x^2+y^2}-x}{y},\ x=r\cos\varphi,\ y=r\sin\varphi$;

250. $xy^3-(x^2y^2-y^8)y'=0,\ u(x)=y^3;$

251. $xy+1+(x^2-x^3y)y'=0,\ x=\frac{1}{t};$

252. $(ay^3+bx^2+cxy^3)+(a_1x^2y+b_1x^3+c_1x^3y)y'=0,\ x=\frac{1}{t},\ y=\frac{1}{u};$

253. $a\varphi'(y)y'+P(x)\varphi(y)=Q(x),\ u(x)=\varphi(y);$

254. $xy'-(\ln xy-1)=0,\ u(x)=xy;$

255. $xy'-y(x\ln\frac{x^2}{y}+2)=0,\ u=\frac{x^2}{y};$

256. $xy'+\sin(y-x)=0,\ u(x)=xtg\frac{y-x}{2};$

257. $(x^2+1)y'+x\sin y\cos y-x(x^2+1)\cos^2 y=0,\ u(x)=tgy.$

Tenglama tipini aniqlang va integrallang va **Meple** dasturi yordamida natijani tekshiring.

258. $(xy^2+x)dx+(y-x^2y)dy=0;$

259. $y'+\sin\frac{x+y}{2}=\sin\frac{x-y}{2};$

260. $(x^2-y^2)dy=2xydx;$

261. $y'=\frac{x}{y}+\frac{y}{x};$

262. $e^y dx+(xe^y-2y)dy=0;$

263. $x'+x=\cos y;$

264. $(1+x^2)dy-(2xy+(1+x^2)^2)dx=0$;

265. $(x+y)dx+(x+y-1)dy=0;$

266. $y'-2xy=3x^3y^2;$

267. $y'=\frac{2x-1}{x^2}y+1;$

268. $y'=y^2-x^2+1;$

269. $(1+x\sqrt{x^2+y^2})dx+(-1+\sqrt{x^2+y^2})ydy=0;$

270. $(x\cos\frac{y}{x}+y\sin\frac{y}{x})ydx+(x\cos\frac{y}{x}-y\sin\frac{y}{x})xdy=0;$

271. $y'+y=xy^3;$

272. $(xy^4-x)dx+(y+xy)dy=0;$

273. $(\sin x+y)dy+(y\cos x-x^2)dx=0;$

274. $3y'^3-xy'+1=0;$

275. $yy'+y^2ctgx=\cos x;$

276 $(e^y+2xy)dx+(e^y+x)xdy=0$;

277. $xy'^2=y-y';$

278. $x(x+1)(y'-1)=y;$ 279. $y(y-xy')=\sqrt{x^4+y^4};$

280. $xy'+y=\ln y';$ 281. $x^2(dy-dx)=(x+y)ydx;$

282. $y'+x\sqrt[3]{y}=3y;$ 283. $(x\cos y+\sin 2y)y'=1;$

284. $y'^2-yy'+e^x=0;$ 285. $y'=\frac{x}{y}e^{2x}+y;$

286. $(xy'-y)^3=y'^3-1;$ 287. $(4xy-3)y'+y^2+1=0;$

288. $y'\sqrt{x}=\sqrt{y-x}+\sqrt{x};$ 289. $xy'=2\sqrt{y}\cos x-2y;$

290. $3y'^4=y'+y;$ 291. $y^2(y-xy')=x^3y';$

292. $y'=(4x+y-3)^2;$

293. $(\cos x-x\sin x)ydx+(x\cos x-2y)dy=0$;

294. $x^2y'^2-2xyy'=x^2+3y^2;$ 295. $\frac{xy'}{y}+2xy\ln x+1=0;$

296. $xy'=x\sqrt{y-x^2}+2y;$ 297. $(2xe^y+y^4)y'=ye^y;$

298. $xy'(\ln y-\ln x)=y;$ 299. $2y'=x+\ln y';$

300. $yy'=4x+3y-2;$ 301. $y^2y'+x^2\sin^3 x=y^3ctgx;$

302. $2xy'-y=\sin y';$ 303. $y'=\sqrt[3]{2x-y}+2;$

304. $(x-y\cos\frac{y}{x})dx+x\cos\frac{y}{x}dy=0;$

305. $(y'-x\sqrt{y})(x^2-1)=xy;$ 306. $y=y'\sqrt{y'^2+1};$

307. $y^2=(xyy'+1)\ln x.$

13-§. AMALIY MASALALARNING MATEMATIK MODELINI TUZISH

Bunday masalalarning quyidagi turkumlarga bo‘lish mumkin:

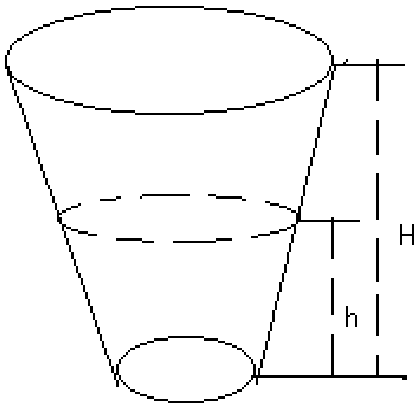
a) idishdan suyuqlikni oqishi;
b) issiqlik tarqalishi;
c) moddiy nuqta harakati;
d) eritmaga doir;
e) biologik populyasiyaning matematik modeli;

Har bir turkumdagi masalalarni alohida ko‘rib o‘tamiz:

a) idishdan suyuqlikni oqishi;

Suyuqlik bilan to‘ldirgan idish tubida tuynuk bo‘lib bu tuynukdan suyuqlik oqib chiqadi. Oqish tezligi $\vartheta = \tilde{n}\sqrt{2gh}$ formula bilan aniqlanadi, bu yerda *c* - suyuqlikning ko‘rinishiga bog‘liq bo‘lgan o‘zgarmas miqdor (masalan suv uchun *c=0,6*), *g* - erkin tushish tezlanishi, *h* - tuynukdan suyuqlik sirtigacha bo‘lgan masofa.

1-masala. Konus ko‘rinishidagi voronka balandligi *H* ga va o‘q kesimi uchidagi burchak α - ga teng bo‘lib, *u* suv bilan to‘ldirilgan. Suv yuzasi σ - ga teng bo‘lgan tuynukdan oqib chiqadi. Qancha vaqtdan so‘ng idishda suv qolmaydi.



a

Yechimi. *t* - vaqt momentda suv yuzasi tuynukdan *h = h (t)* masofada bo‘lsin va *dt* vaqtda suv balandligi *dh* miqdorga kamaysin, u holda yetarlicha kichik *dt* uchun oqib chiqqan suv miqdori balandligi *dh* va radiusi $x = h \cdot tg\frac{\alpha}{2}$ bo‘lgan silindr hajmiga teng bo‘ladi, ya’ni $dV = -\pi h^2 \cdot tg^2\frac{\alpha}{2}$. Boshqa tomondan shu *dt* vaqt ichida tuynukdan oqib chiqqan suv miqdori asos yuzi σ va balandligi ϑdt ga teng bo‘lgan silindr hajmiga teng bo‘ladi, ya’ni $dV = c\sigma\sqrt{2gh}dt$ hosil qilingan hajmlarni tenglashtirsin quyidagi differensial tenglamani hosil qilamiz

$$-\pi h^2 \cdot tg^2\frac{\alpha}{2} = c\sigma\sqrt{2gh}dt, \quad h(0) = H.$$

Bu tenglama qo‘yilgan masalani matematik modelidir. Hosil qilgan masala o‘zgaruvchilari ajraladigan tenglama bo‘lib, uni o‘zgaruvchilarini ajratib integrallaymiz

$$t = \frac{2\pi tg^2(\alpha/2)}{3\sigma\sqrt{2g}}(H^{5/2} - h^{5/2}).$$

Demak, idishda suv qolmasligi uchun *h=0* bo‘lishi kerak ya’ni

$$T = \frac{2}{3}\pi\frac{tg^2(\alpha/2)}{\sigma\sqrt{2g}}H^{5/2}.$$

Mustaqil yechish uchun masalalar

308.Balandligi *H* ga va asos yuzi *S* ga teng bo‘lgan silindr tubidagi teshikdan suv oqib chiqadi. Agar teshik yuzi σ ga teng bo‘lsa, qancha vaqtdan so‘ng idishda suv qolmaydi.

309.Radiusi 1m bo‘lgan yarim shar ko‘rinishdagi qozon suv bilan tuldirilgan. Agar qozon tubida yuzi *0,25* sm^2 bo‘lgan teshik hosil bo‘lsa, qancha vaqtdan so‘ng qozonda suv qolmaydi?

310.Radiusi *2m* bo‘lgan yarim shar ko‘rinishdagi idish suv bilan tuldirilgan. Agar idish tubida radiusi *0,1m* bo‘lgan aylana shaklidagi teshik kesib olinsa, qancha vaqtdan so‘ng idishda suv qolmaydi?

311.Vertikal ukli silindr balandligi *6m* va diametri *4m* bo‘lgan silindr suv bilan tuldirilgan. Silindr ostki asosidan radiusi $\frac{1}{12}m$ bo‘lgan aylana ko‘rinishidagi teshikdan suv qancha vaqtda to‘la oqib chiqadi.

312.Gorizontal o‘qli silindr balandligi 6m va diametri 4m bo‘lgan silindr suv bilan to‘ldirilgan. Silindr ostki asosidan radiusi $\frac{1}{12}$m bo‘lgan aylana ko‘rinishidagi teshikdan suv qancha vaqtda to‘la oqib chiqadi.

313.Suv bilan to‘ldirilgan vertikal turgan sisterna tubida teshik mavjud. Agar birinchi suv oqib chiqish tezligi bosimga proporsional va birinchi sutkada suvning $\frac{1}{10}$ qismi oqib chiqsa, qancha sutka idishdagi suvning yarimi oqib chiqadi.

314.Balandligi 4m va ko‘ndalang kesimi tomoni 6m ga teng bo‘lgan xovuz $10\,ì^{3}/ì\,èí$ ga ega bo‘lgan tezlikda suv oqib tushadi. Agar idish tubida tomoni $\frac{1}{12}$m teng bo‘lgan kvadrat shaklidagi teshikdan suv oqib chiqsa, qancha vaqtdan so‘ng idishni to‘ldirish mumkin.

b) issiqlik tarqalishi;

Agar biror jismni sirtdagi temperatura (harorat) o‘zgarmas bo‘lsa, u holda ma’lum vaqtdan so‘ng jismning har bir nuqtasidagi harorat o‘zgarmas miqdor bo‘ladi, ya’ni harorat vaqtga bog‘liq bo‘lmaydi. Agar *T* harorat birta koordinataga *x*-ga bog‘liq bo‘lsa, u holda Nyutonning issiqlik o‘tkazish qonuniga muvofiq *OX* o‘qiga perpendikulyar *A* yuzali maydondan *1s* da o‘tadigan issiqlik miqdori

$$Q = -kA\frac{dT}{dx}$$

ga teng, bu yerda *k* - jismning issiqlik o‘tkazish koeffisiyenti bo‘lib *u* o‘zgarmas son. Jismning havoda sovush tezligi jism va havo haroratlari ayirmasiga proporsional, ya’ni $\frac{dT}{d\tau} = -\ell(T - t)$ bu yerda *T–τ* vaqtda jismning harorati, *t* - havo harorati, ℓ - musbat proporsionallik koeffisiyenti.

2-masala. Kovak temir ($k = 58,66 \, {}^{Dj}\!/\!_{(m \cdot sek)}$) shar radiuslari mos ravishda 6sm va 10sm bo‘lib stasionar issiqlik xolatida va ichki sirtining harorati *200^0C*, tashqi sirt harorati *20^0C*. Shar markazidan *r (6<r<10) sm* masofadagi haroratni va 1sek. da sharni muhitga tarqatadigan issiqlik miqdorini aniqlang.

Yechimi. Radiusi *r (6<r<10)sm* ga teng bo‘lgan sfera *A* sirtidagi harorat *r* - ga bog‘liq, ya’ni *T=T(r)* . *A* sirt yuzi $4\pi r^2$ ga teng bo‘lganligi sababli Nyuton qonuniga asosan *A* sirtdan o‘tuvchi issiqlik miqdori $Q = -4\pi r^2 R \frac{dT}{dr}$ ga teng. Shar sirtlari orasida issiqlik manbai yo‘qligi sababli *A* sirtdan ixtiyoriy *r* uchun birxil issiqlik miqdori o‘tadi, ya’ni *Q* – o‘zgarmas. O‘zgaruvchilari ajraladigan tenglamani integrallab

$$4\pi kT = \frac{Q}{r} + c$$

ni hosil qilamiz. *Q* va *C* larni topish uchun *$T=20^0C$, r=*$10 \cdot 10^{-2}$ va *$T=200^0C$*, *r=*$6 \cdot 10^{-2}$ *m* larni qo‘yamiz va hosil bo‘lgan chiziqli tenglamalar sistemasini yechib $\tilde{n} = -1000\pi k,\ Q = 10800\pi k$ ni hosil qilamiz. Demak $T = \frac{2700}{r} - 250,\ Q = 108\pi k = 19892,77 \, {}^{Dj}\!/\!_{sek}$.

Mustaqil yechish uchun masalalar

315.Diametri 20 *sm* bo‘lgan par o‘tkazuvchi truba 10sm qalinlikdagi magneziy bilan o‘ralgan. Magneziyni issiqlik o‘tkazishi $k = 0,71 \, {}^{Dj}\!/\!_{(m.sek)}$. Truba harorati 160^0C magneziyning tashqi qobig‘i harorati 30^0S bo‘lsa, o‘rama ichida harorat taqsimlanishini aniqlang va 1metr uzunlikdagi trubani 1 sutkada muhitga chiqargan issiqlik miqdorini aniqlang.

316.G‘ishtdan yasalgan devor qalinligi *30 sm* $k = 0,63 \, {}^{Dj}\!/\!_{(m.sek)}$. Agar devor harorati ichki qismida *20^0C*, tashqi qismidagi esa *0^0C* bo‘lsa, devor haroratini uning ichki nuqtasidan tashqi nuqtasigacha bo‘lgan masofaga qanday bog‘liqligini ko‘rsating. 1m^2 maydonga ega bo‘lgan devor 2 sutkada tashqi muhitga chiqargan issiqlik miqdorini aniqlang.

317. Nyuton qonuniga muvofiq jismning havoda sovish tezligi jism va havo haroratlari ayirmasiga proporsional. Agar havo harorati *20^0C* va jism 20 minutda *100^0S* dan *60^0C* ga sovusa qancha vaqtdan so‘ng uning harorati *30^0C* bo‘ldi.

318.Agar jasad topilagn vaqtda uning temperaturasi *31^0S* va bir soatdan so‘ng *29^0C* bo‘lsa, jinoyat sodir bo‘lgan vaqtni aniqlang, bunda inson temperaturasi *37^0C* va havo harorati *21^0C* deb olish kerak.

319.Tandirdan uzilgan nonning issiqligi 20 minutda 100^0C dan 60^0C ga kamayadi. Havo harorati 25^0C bo‘lganda qancha vaqtdan so‘ng nonning issiqligi 30^0C ni tashqil etadi.

c) moddiy nuqta harakati;

Bu turkumdagi masalalar yechishda Nyutonning ikkinchi qonuni ishlatiladi, ya’ni $\overline{F} = m\overline{a}$.

3-masala. Boshlang‘ich tezligi $\overline{\vartheta}_t$ va massasi m ga teng bo‘lgan moddiy nuqta to‘g‘ri chiziq bo‘ylab tekis harakatlanadi. Harakat yo‘nalishiga qarama-qarshi yo‘nalishda $\overline{F}$ qarshilik kuchi ta’sir etadi, bu kuch moduli $k\sqrt[3]{\vartheta}$ ga teng (k - o‘lcham o‘zgarmas koeffisiyenti). Nuqtaning harakat boshlanishidan to‘xtashgacha saralangan vaqt va bosib utgan yul uzunligini aniqlang.

Yechim. Nuqta OX o‘qi bo‘yicha tekis harakatlanayapti deb, koordinata boshini esa harakat boshi deb olamiz. Nuqtaga birta kuch $\overline{F}$ ta’sir etadi, demak nuqta harakati differensial tenglamasi

$$m\frac{d^2x}{dt^2} = -k\sqrt[3]{\vartheta}$$

bo‘ladi. Nuqta to‘g‘ri chiziq bo‘yicha harakatlanligi sababli $\frac{d^2x}{dt^2} = \frac{d\vartheta}{dt}$ va demak tenglama quyidagi ko‘rinishga keladi

$$m\frac{d\vartheta}{dt} = -k\sqrt[3]{\vartheta}\ .$$

Tenglamani integrallab umumiy yechimini topamiz

$$\frac{3}{2}m\tau^{2/3} = -kt + c$$

$t = 0$ da $\overline{\vartheta} = \overline{\vartheta}_0$ bo‘lganligi sababli $c = \frac{3}{2}m\vartheta_0^{2/3}$ va $t=t_1$ da $\vartheta = 0$ bo‘lganligidan $t_1 = \frac{3m}{2k}\vartheta_0^{2/3}$ ni olamiz.

Harakat qonuni $x(t)$ ni topish uchun topilgan xususiy yechimni quyidagi ko‘rinishda yozib olamiz

$$\frac{dx}{dt} = (\vartheta_0^{2/3} - \frac{2kt}{3m})^{3/2}$$

tenlamani integrallab

$$x = -\frac{3m}{5k}(\vartheta_0^{2/3} - \frac{2kt}{3m})^{5/2} + c_1$$

ni hosil qilamiz. $t=0$ da $x=0$ bo‘lganligi sababli

$$c = \frac{3m}{5m}\vartheta_0^{5/3}$$

Demak

$$x=\frac{3m}{5k}\vartheta_0^{5/3}-\frac{3m}{5k}(\vartheta_0^{2/3}-\frac{2kt}{3m})^{5/2}$$

da bosib o‘tilgan yo‘l $x=\frac{3m}{5k}\vartheta_0^{5/3}$ ga teng.

Mustaqil yechish uchun masalalar

320.Poyezdga uning tezligining chiziqli funksiyasi bo‘lgan qarshilik kuchi ta’sir etsa, uni to‘la to‘xtashi uchun qancha vaqt sarflanishi va u qancha masofa bosib o‘tishi kerak.

321.*m* massali moddiy nuqta boshlang‘ich turtki ta’sirida gorizontal ravishda erkin parvoz qilmoqda. Tashqi muhit ta’sir kuchi *F*, uning moduli esa $F=-k_1\vartheta^{\alpha}-k_2\vartheta$ formula bilan berilsa tushish tezligini toping, bu yerda k_1, k_2, α - o‘lchovga bog‘liq o‘zgarmaslar.

322.*m* massali moddiy zarracha qarshiligi uning tezligini kvadratiga proporsional bo‘lgan muhitga tushadi. Bu zarracha tezligini o‘zgarish qonunini aniqlang va $t\to\infty$ da bu tezlikni $\sqrt{\frac{g}{h}}$ ga tengligini ko‘rsating, bu yerda *g* - erkin tushish tezlanishi.

323.Yer ustidagi moddiy nuqtaga (yer radiusi-*R*) $\vartheta_0=\sqrt{2gR}$ boshlang‘ich vertikal tezlik berilgan (ikkinchi kosmik tezlik). Havo qarshiligini e’tiborga olmagan holda nuqtaning harakat qonunini toping

324. $\vartheta_0=400\, m/sek$ tezlikda harakatlanayotgan o‘q qalinligi *h=20sm* bo‘lgan devorni teshib undan $\vartheta_1=100\, m/sek$ tezlikda otilib chiqadi. agar devor qarshiligi o‘q tezligining kvadratiga proporsional bo‘lsa, uni devorga harakati vaqtini aniqlang.

325.12000t hajmli kema $\vartheta_0=20\, m/sek$ tezlikda to‘g‘ri chiziq buylab tekis harakatlanmoqda. Suv qarshiligi kema tezligini kvadratiga proporsional bo‘lib, u $1\, m/sek$ tezlikda $36000\ gh$ ni tashkil qiladi. Kema motori to‘xtagandan so‘ng tezligi $5\, m/sek$ ga yetgancha qadar kema qancha masofani bosib o‘tadi.

d) eritmaga doir masalalar;

Qattiq modda o‘zgarmas temperaturadagi suyuqlikda erish tezligi berilgan vaqtdagi erimagan modda massasi va boyigan eritma va berilgan vaqtdagi eritmalar konsentrasiyalari ayirmasiga proporsionaldir.

4-masala. Erimaydigan modda tarkibida $x_0 = 10kg$ tuz bor. Bu moddani 90*l* suvga botirilganda 1 soatda tarkibidagi tuz miqdori 2 marta oshsa 1 soatda qancha tuz eriydi? Boyitilgan eritma quyuqlanishi $\frac{1}{3}$ ga teng.

Yechimi. *x=x(t)* - *t* vaqt momentida erimagan tuz miqdori bo‘lsin. Modda erish prosessi quyidagi tenglama yordamida aniqlanadi.

$$\frac{dx}{dt} = kx\left(c - \frac{m-x}{V}\right)$$

bu yerda *k* - proporsionallik koeffitsiyentini, *m* - tuzning boshlang‘ich massasi. Berilgan masala uchun Koshi masalasi quyidagicha bo‘ladi

$$\frac{dx}{dt} = kx\left(\frac{1}{3} - \frac{10-x}{90}\right), \; x(0) = 10$$

Bu tenglama o‘zgaruvchilari ajraladigan tenglama bo‘lib uni integrallasak quyidagini hosil qilamiz

$$\frac{x}{x+20} = \frac{1}{3}e$$

k koeffisiyentini aniqlash uchun *t = 1* da $x = \frac{x_0}{2}$ miqdorda tuz eriganligidan foydalanamiz va $k = \frac{9}{2}\ln\frac{3}{5}$ ni hosil qilamiz. Agar $V = 2V_0$ bo‘lsa, Koshi masalasi quyidagicha bo‘ladi

$$\frac{dx}{dt} = \frac{9}{2}\ln\frac{3}{5}x\left(\frac{1}{3} - \frac{10-x}{180}\right), \; x(0) = 10$$

va bu masala yechimi

$$\frac{x}{x+50} = \frac{1}{6}\left(\frac{3}{5}\right)$$

ko‘rinishda bo‘ladi. *t=1 c* deb olsak *x=5,2kg* tuzni hosil qilamiz.

Mustaqil yechish uchun masalalar

326.Tarkibida *2kg* tuzli egri maydigan modda ni 30 litr suvli idishga qo‘yilganda 5 minutda *1kg* tuz eriganligi ma’lum bo‘ldi. Qancha vaqtdan so‘ng boshlang‘ich tuz miqdorining 99% eriydi. Boyitilgan eritma quyuqlanishi $\frac{1}{3}$ ga teng.

327.Ximik harakatsiz moddadan uni benzolda eritib oltingugurt olindi. Agar moddada *6 gr* oltingugurt bo‘lib, uni *100g* benzolda 6 soat eritishga qo‘yilsa qancha oltingugurt olish mumkin. (Boyitilgan eritmada 11g oltingugurt eriydi). Proporsionallik koeffisiyenti $k = -0,42 \cdot 10^{-4}\, m^3 / (sek \cdot kg)$.

328. *0,3* m^3 hajmli katta idish tubi tuz va erimaydigan modda bilan koplangan. Tuz erish tezligi shu vaqtdagi quyuqlanish va boyitilgan eritma quyuqlanishi $\left(\frac{1}{3}\right)$ orasidagi ayirmaga proporsinal bo‘lsa va olingan toza suv massasi 1 minutda $\frac{1}{3}\, kg$ tuz eritaolsa 1 soatdan so‘ng eritmada qancha tuz miqdori bor.

329.*0,1*m^3 hajmli katta idishda *10kg* tuz eritmasi bor. Bu idishga $3 \cdot 10^{-3}\, m^3/\min$ tezlikda suv kuyiladi va shu tezlikda eritma oqib chiqadi, shu bilan birga quyuqlanish birjinsli saqlanadi (masalan aralashtirish yordamida). 1 soatdan so‘ng idishda qancha tuz koldi?

330.*0,1*m^3 hajmli katta idishda *10kg* tuz eritmasi bor. Bu idishga $3 \cdot 10^{-3}\, m^3/\min$ tezlikda suv qo‘yilib, $2 \cdot 10^{-3}\, m^3/\min$ tezlikda oqib chiqadi. Aralashtirish yordamida quyuqlanish birjinsli saqlanadi . 1 soatdan so‘ng idishda qancha tuz qoldi?

331.Sig‘imi *V=10800* m^3 bo‘lgan binoda havo tarkibida *0,12%* CO_2 bor. Bu tarkibida *0,04%* CO_2 bo‘lgan toza havo tekis kiradi. Agar *10 min*. So‘ng binodagi havo tarkibidagiCO_2 *0,06%* ga tushsa binoga 1 minutga necha kub metr havo kiradi. (birlik vaqt momentida binoga q m^2 havo kiradi deb olish kerak).

e) biologik populyasiyaning matematik modeli;

Populyasiyaning o‘sish tezligi *t* vaqt momentida tug‘ilish va o‘lish orasidagi ayirmaga teng. Chegaralangan fazo va oziq ovqat imkoniyatlarida tug‘ilish zotlar soniga, o‘lishi esa zotlar sonining kvadratiga proporsional. Bu holda populyasiyaning o‘sishini matematik modeli

$$\frac{dx}{dt} = \beta x - \delta x^2$$

tenglama bilan ifodalanidi, bu yerda *x(t)* - *t* vaqt momentida populyasiyadagi zotlar soni, *β,δ* – mos ravishda, o‘rtacha tug‘ilish va o‘lish koeffisiyentlari. Bunday tenglama logistik tenglama deyiladi, *x(t)* funksiya esa populyasiyaning logistik sonini aniqlaydi. Logistik o‘sishda populyasiyadagi zotlar soni vaqt o‘tishi bilan limitik o‘lchovga yaqinlashadi, ya’ni $\lim_{t \to \infty} x(t)$.

Mustaqil yechish uchun masalalar

332.Populyasiyadagi zotlar soni 10 ta. Agar birlik vaqt momentida 1000 zotlardan 100 zot tug‘ilib 1ta zot o‘lsa, populyasiyaning limitik o‘lchovini toping.

333.Bakteriyalar populyasiyasida ular 1 soatda 120 donagacha ko‘payadi. Agar bakteriyalarning boshlang‘ich soni 100 dona bo‘lib limitik o‘lchovi 100000 dona bo‘lsa ularni *t* vaqt momentidagi sonini aniqlang.

334.Logistik tenglamani integrallang

$$\frac{dx}{dt} = x(\beta - \delta x)\left(1 - \frac{m}{x}\right)$$

Bu yerda β*=100, ,*δ*=1, m=10.*

4-BOB
TARTIBINI PASAYTIRISH MUMKIN BO‘LGAN n-chi TARTIBLI DIFFERENSIAL TENGLAMALAR

$$F(x,y,y',y'',...,y^{(n)})=0$$

tenglamaga *n*-chi tartibli differensial tenglama deyiladi, bu erda *x* - erkli o‘zgaruvchi, *y=y(x)* izlanuvci funksiya.

$$y^{(n)}=f(x,y,y',y'',...,y^{(n)})$$

tenglamaga yuqori tartibli hosilaga nisbatan yechilgan *n*-chi tartibli differensial tenglama deyiladi. Koshi masalasi yoki boshlang‘ich masala deb $x=x_0,\ x_0\in[a,b]$ bo‘lganda.

$$y=y_0,\ y'=y_0',...,y^{(n-1)}=y_0^{(n-1)} \tag{2}$$

shartni qanoatlantiruvchi $y=y(x)$ funksiyani topishga aytiladi. Bu yerda $x,\ y_0,\ y_0',...,y_0^{(n-1)}$ berilgan sonlar. Ko‘p hollarda, (1) tenglamani integrallash vaqtida

$$\Phi(x,y,y',...,y^{(n-k)},c_1,...,c_k)=0$$

shakldagi tenglik hosil bo‘lishi mumkin. Bu tenglamaga berilgan tenglamaning *k* -chi tartibli oraliq integrali deyiladi.

14-§. TARTIBINI PASAYTIRISH MUMKIN BO‘LGAN N-CHI TARTIBLI DIFFERENSIAL TENGLAMALAR
(TO‘LIQMAS TENGLAMALAR)

$$\text{I. } F(x,y^{(n)})=0` \tag{3}$$

1. Agar (3) tenglamani $y^{(n)}$ ga nisbatan yechish mumkin bo‘lsa, u holda bitta yoki bir nechta $y^{(n)}=f(x)$ ko‘rinishdagi oddiy tenglama hosil qilamiz. Bu tenglamani ketma-ket *n* marta integrallab umumiy yechimni topish mumkin.

Ko‘rsatmalar. Bu holda

$$\int_{x_0}^{x}...\int_{x_0}^{x}f(x)dx...dx=\frac{1}{(n-1)!}\int_{x_0}^{x}f(t)(x-t)^{n-1}dt$$

formuladan foydalanish mumkin.

2. Agar (3) tenglamani parametrik ko‘rinishda, ya’ni $x=\varphi(t),\ y^{(n)}=\psi(t)$ shaklda yozish mumkin bo‘lsa, u holda $dy^{(n-1)}=y^{(n)}dx$ munosabatdan foydalanib, tenglamaning umumiy yechimi parametrik ko‘rinishda topiladi.

44-misol. $xy^{(4)}=1.$

Tenglamani $y^{(4)}$ ga nisbatan yechsak, $y^{(4)}=\frac{1}{x}$ tenglama hosil bo‘ladi. Ketma-ket to‘rt marta integrallab, $6y=x^3\ln|x|+c_1x^3+c_2x^2+c_3x+c_4$ umumiy yechimni hosil qilamiz.

45-misol. $x=e^{-y''}+y''.$

Bu tenglamada $y''=t$ almashtirish olamiz. $x=e^{-t}+t,\ dy'=y''dx$ ga qo‘ysak,

$$dy'=t(-e^{-t}+1)dt,\ y'=te^{-t}+e^{-t}+\frac{t^2}{2}+c_1$$

$$dy=y'dx$$

ga qo‘yamiz.

$$dy'=(te^{-t}+e^{-t}+\frac{t^2}{2}+c_1)(-e^{-t}+1)dt$$

Bu ifodani integrallab tenglamaning umumiy yechimini topamiz:

$$x=e^{-t}+t;\ y=(\frac{t}{2}+\frac{3}{4})e^{-2t}+(\frac{t^2}{2}-1+c_1)e^{-t}+\frac{t^3}{6}+c_1t+c_2.$$

$$\text{II. } F(x,y^{(k)},...,y^{(n)})=0 \qquad (4)$$

(4) tenglamani $z=y^{(k)},\quad z=z(x)$ almashtirish yordamida n-k tartibli $F(x,z,z',...,z^{(n-k)})=0,$ tenglamaga keltirish mumkin.

46-misol $y''^2+y'=xy''$

Tenglamada noma’lum funksiya y qatnashmagan. $z=y'$ yordamchi funksiyani kiritamiz. U vaqtda $z'=y''$ va tenglama $z=xz'-z'^2$ ko‘rinishga keladi.

Bu tenglama Klero tenglamasi, demak umumiy yechimi $z=c_1x+c_1^2,$ maxsus yechim $z=\frac{x^2}{4}$ bo‘ladi.

Bu yerdan $y'=c_1x+c_1^2,$ va tenglamaning umumiy yechimi $y=\frac{c_1}{2}x^2-c_1^2x+c_2,$ maxsus yechimi $y'=\frac{x^2}{4}$ tenglamadan topiladi va $y=\frac{1}{12}x^3+c$. Tenglama yechimini **Meple** dasturi yordamida tekshiramiz.

```
> restart;
> d46:=(diff(y(x),x$2))^2+diff(y(x),x)=x*diff(y(x),x$2);
```

$$deq := \left(\frac{d^2}{dx^2}y(x)\right)^2 + \left(\frac{d}{dx}y(x)\right) = x\left(\frac{d^2}{dx^2}y(x)\right)$$

```
> dsolve(d46,y(x));
```

$$y(x) = \frac{x^3}{12} + \_C1,\ y(x) = -\_C1^2\,x - \frac{1}{2}\_C1\,x^2 + \_C2,\ y(x) = -\_C1^2\,x + \frac{1}{2}\_C1\,x^2 + \_C2$$

$$\text{III. } F(y, y', ..., y^{(n)}) = 0. \qquad (5)$$

tenglamani $y' = z, z = z(y)$ almashtirish olib (bu yerda erkli o‘zgaruvchi vazifasini y bajaradi) tartibini bitta birlikka pasaytirish mumkin. Bu holda hosilalar quyidagicha topiladi:

$$y'' = \frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx} = z \cdot z'_y$$

$$y''' = \frac{d}{dy}(z \cdot z'_y)\frac{dy}{dx} = (z \cdot z''_y + z'^2_y)z$$

va hokazo.

47-misol. $y'^2 + 2yy'' = 0$.

$z = y'$ almashtirish olamiz, u holda $y'' = \frac{dz}{dy}z$ va tenglama $z + 2yz\frac{dz}{dy} = 0$ shaklga keladi. Bu yerdan $z = \frac{c_1}{\sqrt{y}}$ va demak, $y' = c_1 y^{-1/2}$. Bu tenglamani integrallab, berilgan tenglamaning umumiy yechimini topamiz, $\frac{2}{3}y^{3/2} = c_1 x + c_2$.

Tenglama yechimini **Meple** dasturi yordamida tekshiramiz.

```
> restart;
> d47:=(diff(y(x),x))^2
+2*y(x)*diff(y(x),x$2)=0;
```

$$d47 := \left(\frac{d}{dx}y(x)\right)^2 + 2\,y(x)\left(\frac{d^2}{dx^2}y(x)\right) = 0$$

```
> dsolve(d47,y(x));
```

$$y(x)=0, \frac{2}{3}y(x)^{(3/2)}-\_C1\,x-\_C2=0$$

Mustaqil yechish uchun misollar

Differensial tenglamalarni integrallang va **Meple** dasturi yordamida tekshiramiz.

335. $y'''=-\cos x;$ 336. $y'''=\frac{2}{x^3};$ 337. $x-\sin y''+2y''=0;$

338. $x=e^{-y''}+y'';$ 339. $x=\frac{y''}{\sqrt{1+y''^2}};$ 340. $y''^2-1=0;$

341. $(1+x^2)y''+y'^2+1=0;$ 342. $xy''=y'\ln\frac{y'}{x};$

343. $xy''-y'=0;$

344. $y'(1+y'^2)=\alpha y'';$ 345. $yy''=y'^3;$ 346. $yy''^2=1;$

347. $1+y'^2=2yy'';$ 348. $2yy''+y'^2+y'^4=0;$

349. $xy''+xy'^2=y';$ 350. $y'''-(y'')^3=0;$

351. $y'''-2y''=0;$

352. $y^3y''+1=0;$ 353. $y^4-y^3y''=1.$

Tenglamalarni berilgan boshlang‘ich shartlarni qanoatlantiruvchi yechimini toping.

354. $y''=6x,\ y=0$ *àgar* $x=0$ *bo′lsà*;

355. $y'''=e^{-x},\ y=0,\ y'=0,\ y''=0$ *àgar* $x=0$ *bo′lsà*;

356. $y''=(1+y'^2)^{3/2};\ y=1,\ y'=0$ *àgar* $x=0$ *bo′lsà*;

357. $y''^2=y',\ y=0,\ y'=1$ *àgar* $x=0$ *bo′lsà*;

358. $4y'+y''^2=4xy'',\ y=0,\ y'=-1$ *àgar* $x=0$ *bo′lsà*;

359. $2xy''+y'''=0,\ y=y_0,\ y'=y_0',\ y''=y_0''$ *àgar* $x=0$ *bo′lsà*;

360. $2yy''-3y'^2=4y^2,\ y=1,\ y'=0$ *àgar* $x=0$ *bo′lsà*.

15-§. TARTIBINI PASAYTIRISH MUMKIN BO‘LGAN TENGLAMALAR (birjinsli, umumlashgan birjinsli, to‘liq differensialli)

$$\text{IV. } F(x,y,y',...,y^{(n)})=0$$

tenglama noma’lum funksiya va $y',y'',...,y^{(n)}$ hosilalarga nisbatan birjinsli deyiladi, agar F funksiya ko‘rsatilgan o‘zgaruvchilarga nisbatan birjinsli bo‘lsa, ya’ni

$$F(x,ty,ty',...,ty^{(n)})=t^mF(x,y,y',...,y^{(n)})$$

Bu tenglamaning tartibini $y'=zy,\ z=z(x)$ almashtirish yordamida bittagina pasaytirish mumkin. Bu holda hosilalar quyidagicha topiladi: $y'=zy, y''=y(z^2+z),\ y'''=y(z''+3zz'+z^3)$ va hokazo.

48-misol. $yy''=y'^2+15y^2\sqrt{x}$

$y'\ =zy$ almashtirish olamiz $y^2(z^2+z')=z^2y^2+15y^2\sqrt{x}$, bu yerdan tenglamani integrallab, berilgan tenglamaning umumiy yechimini topamiz.

$$\ln|y|=4x^{3/2}+c_1x+c_2;\ y=0$$

V. $F(x,y,y',...,y^{(n)})=0$

tenglama umulashgan birjinli tenglama deyiladi, agar shunday k son topib, tenglamaning chap tomoni barcha o‘zgaruvchilarga nisbatan m-chi darajali birjinsli funksiya bo‘lsa, bu yerda $x,y,y',...,y^{(n)}$ larni mos ravishda *1, k, k-1, ...,k-n* darajali birjinsli funksiyalar deb olinishi kerak.

Bunday tenglama $x=e^t,\ y=ze^{kt}$ (t yangi erkli o‘zgaruvchi, z yangi izlanuvchi funksiya) almashtirish yordamida oshkor ravishda erkli o‘zgaruvchi qantashmagan tenglamaga keladi.

49-misol. $\dfrac{y^2}{x^2}+(y')^2=3xy''+\dfrac{2yy'}{x}$.

Umumlashgan birjinsli tenglama ekanligini ko‘rsatamiz. Tenglama bir jinsli bo‘lishi uchun tenglamadagi bir hadlar darajalari bir xil bo‘lishi kerak, ya’ni

$$2k-2=2(k-1)=1+k-2=k+k-1-1$$

tenglamalar $k=1$ bo‘lganda o‘rinli. Demak, $x=e^t,\ y=ze^t$ almashtirish olamiz. Hosilalarni topsak,

$$y'=(z_t'e^t+ze^t)e^{-t}=z_t'+z$$

$$y''=(z_t''+z_t')e^{-t}$$

Tenglamaga qo‘yib, soddalashtirsak,

$z'^2=3z''+3z'$ ni hosil qilamiz. Bu tenglama n^0 III ga tegishli, ya’ni $z'=u(z)$ almashtirish olib tartibini bittaga pasaytiriladi.

Tenglama yechimi $y=x[c_3-3\ln\left|\dfrac{1}{x}-c_1\right|];\ y=cx$.

VI. To‘liq differensialli tenglama

Agar differensiallanuvchi $\Phi(x,y,y',...,y^{(n-1)})$ funksiya mavjud bo‘lib,

$$\frac{d}{dx}\Phi(x,y,y',...,y^{(n-1)})=F(x,y,y',...,y^{(n)})$$

tenglik bajarilsa, $F(x,y,y',...,y^{(n)})=0$ tenglama to‘liq differensialli tenglama deyiladi va $\Phi(x,y,y',...,y^{(n-1)})=\tilde{n}_1$ berilgan tenglamaning 1-tartibli oraliq integrali (birinchi integral) bo‘ladi.

50-misol. $yy'''+3y'y''=0$.

Tenglamani $\frac{1}{y''y}$ ga ko‘paytirsak, $\frac{y'''}{y''}+3\frac{y'}{y}=0,\ [y''=0?]$ to‘liq differensialli tenglama hosil qilamiz, ya’ni

$$\frac{d}{dx}(\ln y''+\ln y^3)=\frac{y'''}{y''}+3\frac{y'}{y},$$

bu yerdan $\ln y''+\ln y^3=c_1$ oraliq integralni hosil qilamiz.

Yoki $y''y^3=c_1$ (birinchi integral).

Bu tenglamada $y'=z$ almashtirish olsak $z'zy^3=c_1$ hosil bo‘ladi.

Bu yerdan $z=\pm\sqrt{\frac{c_2y^2-c_1}{y^2}}$ yoki $y'=\pm\frac{\sqrt{c_2y^2-c_1}}{y}$

u holda tenglamaning umumiy yechimi $c_2y^2+c_1=c_2^2(x+c_3)^2$.

Agar $y''=0$ bo‘lsa, $y=c_1x+c_2$ differensial tenglamaning yechimi bo‘ladi.

Mustaqil yechish uchun misollar

361. $xyy''-xy''^2-yy'-\frac{6xy'^2}{\sqrt{a^2-x^2}}=0;$

362. $x^2(yy''-y'^2)+xyy'=y\sqrt{x^2y'^2+y^2};$

363. $x^2yy''=(y-xy')^2;$ 364. $yy''-y'^2=\frac{yy'}{\sqrt{1+x^2}};$

365. $xyy''+yy'-x^2y'^3=0;$

366. $x^4y''-x^3y'^3+3x^2yy'^2-(3xy^2+2x^3)y'+2x^2y+y^3=0;$

367. $x^2y''-3xy'+4y+x^2=0;$ 368. $y''=2yy';$ 369. $y''=y'^2y;$

370. $yy''=y';$ 371. $yy'''-y'y''=0;$ 372. $y''=(1+y'^2)^{3/2};$

373. $(1+y'^2)y'''-3y'y''^2=0;$ 374. $y''-\frac{1}{x}y'+\frac{1}{x^2}y=1;$

375. $y''+y'\cos x-y\sin x=0.$

Tenglamani berilgan boshlang‘ich shartlarni qanoatlantiruvchi yechimini toping.

376. $y'y''' - 3y''^2 = 0;\ y(0) = 0;\ y'(0) = 1.$

377. $2y'^2 = (y-1)y'';\ y(1) = 2;\ y'(1) = 0.$

378. $(y'' - 2x)y - 2(y' - x^2)y' = 0;\ y(1) = \frac{1}{3};\ y'(1) = 1.$

379. $y''' = yy'' + y'^2;\ y(0) = 0;\ y'(0) = \frac{1}{2},\ y''(0) = 0.$

380. $yy'' = 2xy'^2;\ y(2) = 2;\ y'(2) = 0{,}5.$

381. $2y''' - 3y'^2 = 0;\ y(0) = -3,\ y'(0) = 1,\ y''(0) = -1.$

382. $x^2y'' - 3xy' = \frac{6y^2}{x^2} - 4y\quad y(1) = 1;\ y'(1) = 4.$

383. $y''' = 3yy';\ y(0) = -2;\ y'(0) = 0.$

384. $y''\cos y + y'^2 \sin y = y';\ y(-1) = \frac{\pi}{6};\ y'(-1) = 2.$

5-BOB

n-CHI TARTIBLI CHIZIQLI DIFFERENSIAL TENGLAMALAR

$$a_0(x)y^{(n)}+a_1(x)y^{(n-1)}+...+a_{n-1}(x)y'+a_n(x)y=f(x), \qquad (1)$$

ko‘rinishdagi tenglama chiziqli tenglama deyiladi.

Agar qaralayotgan barcha qiymatlarda $f(x)$ nolga teng bo‘lsa, (1) tenglama birjinsli, aks holda bir jinslimas deyiladi.

Tenglama koeffistiyentlari $a_0(x), a_1(x),...a_p(x)$ va ozod had $f(x)$ $(a,\ b)$ intervalda aniqlangan va uzluksiz bo‘lganda

$$y(x_0)=y_0,\ y'(x_0)=y_0',...,\ y^{(n-1)}(x_0)=y_0^{(n-1)}$$

Koshi (boshlang‘ich) masalasi yagona yechimga ega bo‘ladi, bu yerda $x_0\in(a,b)$.

(1) tenglama maxsus yechimga ega emas. Bir jinslimas tenglamani integrallash masalasi shu tenglamaga mos bo‘lgan birjinsli tenglama

$$a_0(x)y^{(n)}+a_1(x)y^{(n-1)}+...+a_n(x)y=0 \qquad (2)$$

ni integrallash masalasiga keltiriladi, buning uchun (1) tenglamani biror xususiy yechimini bilish yetarlidir.

Bir jinsli (2) tenglama $y\equiv 0$ (trivial) yechimga ega va bu yechim $y(x_0)=y'(x_0)=...=y^{(n-1)}(x_0)=0$ Koshi masalasini qanoatlantiradi.

Birjinsli n - chi tartibli tenglamaning umumiy yechimini topish uchun uni (a,b) intervalda chiziqli bog‘lanmagan n - ta xususiy yechimini aniqlash kerak, bu yechimlar fundamental yechimlar sistemasi deyiladi (FES). Agar fundamental yechimlar sistemasi aniqlansa, bu yechimlarning chiziqli kombinasiyasi (2) tenglamaning umumiy yechimini beradi.

16-§. CHIZIQLI BOG‘LANGAN VA BOG‘LANMAGAN FUNKSIYALAR SISTEMASI

$y_1(x),\ y_2(x),...,\ y_n(x)$ funksiyalar (a,b) intervalida aniqlangan bo‘lsin. Agar hyech bo‘lmaganda bittasi noldan farqli bo‘lgan $\alpha_1,\alpha_2,...,\alpha_n$ sonlar topilib, $x\in(a,b)$ uchun

$$\alpha_1 y_1(x)+\alpha_2 y_2(x)+\cdots+\alpha_n y_n(x)=0 \qquad (3)$$

tenglik o‘rinli bo‘lsa, $y_1(x),\ y_2(x),...,\ y_{\grave{o}}(x)$ funksiyalar (a,b) intervalda chiziqli bog‘langan deyiladi.

Agar (3) tenglik faqat va faqat $\alpha_1=\alpha_2=\cdots=\alpha_n=0$ qiymatlardagina o‘rinli bo‘lsa, $y_1(x), y_2(x),\cdots, y_n(x)$ funksiyalar chiziqli bog‘lanmagan deyiladi.

<u>51-misol</u>. $1, x, x^2, x^3, x^4$ funksiyalar sistemasi $(-\infty;\infty)$ intervalda chiziqli bog‘lanmaganligini ko‘rsatamiz. Haqiqatdan ham

$$\alpha_1+\alpha_2 x+\alpha_3 x^2+\alpha_4 x^3+\alpha_5 x^4=0 \qquad (4)$$

tenglik $x\in(-\infty;+\infty)$ ning barcha qiymatlarida, faqat $\alpha_1=\alpha_2=\alpha_3=\alpha_4=\alpha_5=0$ bo‘lganda bajariladi, agar $\alpha_i\,(i=\overline{1,5})$- ning birortasi nolga teng bo‘lmasa, u holda (4) tenglikning chap tomoni ko‘pi bilan 4-chi darajali tenglama bo‘lib, algebraning asosiy teoremasiga asosan u ko‘pi bilan *x* ning 4 ta qiymatida nolga teng bo‘lishi mumkin.

52-misol. $e^{\lambda_1 x}$, $e^{\lambda_2 x}$, $e^{\lambda_3 x}$ $(\lambda_1\neq\lambda_2\neq\lambda_3)$ funksiyalar $(-\infty;+\infty)$ intervalda chiziqli bog‘lanmagan funksiyalar sistemasini tashqil etishini ko‘rsatamiz.

Bu funksiyalarni chiziqli bog‘langan deb faraz qilamiz, u holda $x\in(-\infty;+\infty)$ uchun

$$\alpha_1 e^{\lambda_1 x}+\alpha_2 e^{\lambda_2 x}+\alpha_3 e^{\lambda_3 x}\equiv 0 \qquad (5)$$

va $\alpha_1,\alpha_2,\alpha_3$ sonlarning hyech bo‘lmaganda birortasi noldan farqli, masalan $\alpha_3\neq 0$ bo‘lsin. (5) ni $e^{\lambda_1 x}$ ga bo‘lamiz:

$$\alpha_1+\alpha_2 e^{(\lambda_2-\lambda_1)x}+\alpha_3 e^{(\lambda_3-\lambda_1)x}\equiv 0$$

Bu ayniyatni differensiallaymiz

$$\alpha_2(\lambda_2-\lambda_1)e^{(\lambda_2-\lambda_1)x}+\alpha_3(\lambda_3-\lambda_1)e^{(\lambda_3-\lambda_1)x}\equiv 0 \qquad (6)$$

(6) ni $e^{(\lambda_2-\lambda_1)x}$ ga bo‘lamiz

$$\alpha_2(\lambda_2-\lambda_1)+\alpha_3(\lambda_3-\lambda_1)e^{(\lambda_3-\lambda_2)x}\equiv 0$$

Oxirgi tenglikni differensiallab,

$$\alpha_3(\lambda_3-\lambda_1)e^{(\lambda_3-\lambda_2)x}\equiv 0$$

ayniyatga kelamiz.

Bunda $\alpha_3\neq 0$, $\lambda_1\neq\lambda_2\neq\lambda_3$ va $e^{kx}\neq 0$ bo‘lganligi sababli ziddiyat hosil qildik. Demak, berilgan funksiyalar sistemasi barcha $x\in(-\infty;+\infty)$ larda chiziqli bog‘langandir.

53-misol. $sinx$, $sin\left(x+\frac{\pi}{6}\right)$, $sin\left(x-\frac{\pi}{6}\right)$ $x\in(-\infty;+\infty)$ ga chiziqli bog‘langan funksiyalar sistemasini tashqil etishini ko‘rsating.

Hyech bo‘lmaganda bittasi noldan farqli bo‘lgan $\alpha_1,\alpha_2,\alpha_3$ sonlar uchun $x\in(-\infty;+\infty)$ da

$$\alpha_1\sin x+\alpha_2\sin(x+\frac{\pi}{6})+\alpha_3\sin(x-\frac{\pi}{6})\equiv 0 \qquad (7)$$

ayniyat o‘rinli bo‘lishini ko‘rsatamiz. (7) ayniyatni to‘g‘ri deb faraz qilib, unga $x = 0, \frac{\pi}{6}, -\frac{\pi}{6}$ qiymatlarni berib, quyidagi sistemani hosil qilamiz

$$\begin{cases} \frac{1}{2}\alpha_2 - \frac{1}{2}\alpha_3 = 0 \\ \frac{1}{2}\alpha_1 + \frac{\sqrt{3}}{2}\alpha_2 = 0 \\ -\frac{1}{2}\alpha_1 - \frac{\sqrt{3}}{2}\alpha_3 = 0 \end{cases} . \qquad (8)$$

Bu sistemaning determinanti

$$\begin{vmatrix} 0 & 1 & -1 \\ 1 & \sqrt{3} & 0 \\ 1 & 0 & \sqrt{3} \end{vmatrix} = 0 .$$

Demak, (8) birjinsli chiziqli sistema cheksiz ko‘p yechimga ega.

$\alpha_2 = \alpha_3 = -\frac{\sqrt{3}}{3}\alpha_1$, $\alpha_1 = -\sqrt{3}$ deb olsak, $\alpha_2 = \alpha_3 = 1$ bo‘ladi. Bu qiymatlarda $x \in (-\infty; +\infty)$ lar uchun (7) ayniyat o‘rinli bo‘lishini ko‘rsatamiz.

$$\sqrt{3}\sin x + \sin x(x + \frac{\pi}{6}) + \sin(x - \frac{\pi}{6}) \equiv -\sqrt{3}\sin x + 2\sin x \cos\frac{\pi}{6} \equiv$$

$$\equiv -\sqrt{3}\sin x + \sqrt{3}\sin x \equiv 0$$

Demak, berilgan funksiyalar sistemasi chiziqli bog‘langan.

Vronskiy va Gram determinantlari

Yuqorida qarab chiqilgan misollardan ko‘rinayaptiki, berilgan funksiyalar sistemasining chiziqli bog‘langan yoki bog‘lanmaganligini bevosita aniqlash yetarlicha murakkab ekan. Lekin, agar funksiyalar chiziqli birjinsli tenglamaning yechimlari bo‘lsa, bu masalani Vronskiy determinanti yordamida yechish mumkin.

$y_1(x), y_2(x), \dots, y_n(x)$ funksiyalar (*n–1*) - tartibgacha hosilalarga ega bo‘lsin, u holda Vronskiy determinanti quyidagicha tuziladi:

$$W[y_1, y_2, \dots, y_n] = \begin{vmatrix} y_1 & y_2 & \cdots & y_n \\ y_1' & y_2' & \cdots & y_n' \\ \cdots & \cdots & \cdots & \cdots \\ y_1^{(n-1)} & y_2^{(n-i)} & \cdots & y_n^{(n-1)} \end{vmatrix}$$

Qo‘yidagi teorema o‘rinli.

Teorema. Birjinsli *n* - chi tartibli tenglama koeffisiyentlari uzluksiz bo‘lgan *D* oraliqda uning $y_1(x), y_2(x), \dots, y_n(x)$ yechimlari chiziqli bog‘lanmagan funksiyalar sistemacsini tashqil etishi uchun *D* oraliqning biror nuqtasida Vronskiy determinanti noldan farqli bo‘lishi zarur va yetarlidir. Bu holda Vronskiy determinanti *D* oraliqning barcha nuqtalarida noldan farqli bo‘ladi.

$y_1(x), y_2(x), \ldots, y_n(x)$ funksiyalar $[a,b]$ kesmada berilgan bo‘lsin, u holda bu funksiyalar chiziqli bog‘langan bo‘lishi uchun Gram determinanti

$$G\left(y_{1,}y_2,\ldots,y_n\right) = \begin{vmatrix} (y_1,y_1) & (y_1,y_2) & \cdots & (y_1,y_n) \\ (y_2,y_1) & (y_2,y_2) & \cdots & (y_2,y_n) \\ \cdots & \cdots & \cdots & \cdots \\ (y_n,y_1) & (y_n,y_2) & \cdots & (y_n,y_n) \end{vmatrix}$$

nolga teng bo‘lishi zarur va yetarlidir, bu yerda $(y_i, y_j) = \int_a^b y_i(x) y_j(x) dx$; $i, j = 1, 2, 3, ..., n$

<u>54-misol</u>. $y_1 = e^{\lambda_1 x}$, $y_2 = e^{\lambda_2 x}, y_3 = e^{\lambda_3 x}$ funksiyalar yordamida Vronskiy determinantini tuzing.

$$W\left[y_1, y_2, y_3\right] = \begin{vmatrix} e^{\lambda_1 x} & e^{\lambda_2 x} & e^{\lambda_3 x} \\ \lambda_1 e^{\lambda_1 x} & \lambda_2 e^{\lambda_2 x} & \lambda_3 e^{\lambda_3 x} \\ \lambda_1^2 e^{\lambda_1 x} & \lambda_2^2 e^{\lambda_2 x} & \lambda_3^2 e^{\lambda_3 x} \end{vmatrix} = .$$

$$= e^{(\lambda_1+\lambda_2+\lambda_3)x}(\lambda_3-\lambda_2)(\lambda_3-\lambda_1)(\lambda_2-\lambda_1)$$

<u>55-misol</u>. $y_1 = x, y_2 = 2x$ funksiyalar $[0,1]$ kesmada chiziqli bog‘langanligini ko‘rsating. Gram determinantini tuzamiz

$$(y_1, y_1) = \int_0^1 x^2 dx = \frac{1}{3}$$

$$(y_1, y_2) = (y_2, y_1) = \int_0^1 2x^2 dx = \frac{2}{3}$$

$$(y_2, y_2) = \int_0^1 4x^2 dx = \frac{4}{3}$$

$$G(y_1, y_2) = \begin{vmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{4}{3} \end{vmatrix} = 0$$

Demak, $y_1(x)$ va $y_2(x)$ funksiyalar chiziqli bog‘langan.

Ostrogradskiy – Liuvill formulasi

(2) tenglama yechimlari va koeffisiyentlari orasidagi bog‘lanish Ostrogradskiy-Liuvill formulasi orqali ifodalanadi:

$$W\left[y_1, y_2, \ldots, y_n\right] = W\left[x_0\right]\exp\left[-\int\limits_{x_0}^{x} \frac{a_1(t)}{a_0(t)}dt\right]$$

56-misol. $x(x-1)y'' + (x+1)y' - y = 0$ tenglama xususiy yechimi

$$y = x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n \qquad (8)$$

ko‘rinishida bo‘lsa, uning umumiy yechimini toping.

(8) funksiyani berilgan tenglamaga qo‘yamiz va hosil bo‘lgan ko‘phadning bosh koeffisiyentini nolga tenglashtirib,

$$n(n-1) + (n-1) = 0$$

tenglamani hosil qilamiz. Bundan *n =1, n = - 1*. Agar musbat butun *n* qiymat topilmasa, bu holda berilgan tenglama (8) ko‘rinishdagi yechimga ega bo‘lmas edi.

Demak, berilgan tenglama $y = x + a$ ko‘rinishdagi yechimga ega. Bu yechimni tenglamaga qo‘yib *a =1* ni olamiz. Tenglama yechimini topish uchun Ostrogradskiy-Liuvill formulasidan foydalanamiz

$$\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = ce^{\left[-\int \frac{a_1(x)}{a_o(x)}dx\right]}$$

$$\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = ce^{\left[-\int \frac{x+1}{x(x-1)}dx\right]}$$

$$y_1y_2' - y_1'y_2 = \frac{Cx}{(x-1)^2}$$

Bu yerda y_1 ni o‘rniga $y_1 = x+1$ ni qo‘yib, y_2 ga nisbatan hosil bo‘lgan birinchi tartibli chiziqli tenglamani yechish mumkin, lekin qulayroq usul qo‘llash afzalroq, ya’ni oxirgi tenglamani $y_1^{\;2}$ ga bo‘lamiz:

$$\left[\frac{y_2}{y_1}\right]' = \frac{Cx}{(x-1)^2 y_1^2},$$

$$\frac{y_2}{y_1} = C\int \frac{xdx}{(x-1)^2(x+1)^2} + C_2 \text{ yoki } \frac{y_2}{y_1} = -\frac{1}{2}C\frac{1}{(x^2-1)} + C_2 .$$

$-\frac{1}{2}C = C_1$ deb olsak,

$$\frac{y_2}{y_1} = \frac{C_1}{(x^2-1)} + C_2$$

yoki

$$y_2 = \frac{C_1}{(x-1)} + C_2(x+1).$$

Hosil bo‘lgan funksiya berilgan tenglamaning umumiy yechimini beradi.

Mustaqil yechish uchun misollar

Berilgan funksiyalar sistemasining o‘z aniqlanish sohasida chiziqli bog‘lanmaganligini tekshiring.

385. $4, x$; 386. $x, 2x, x^2$; 387. ye^x, xe^x, x^2ye^x;

388. $\sin x, \cos x, \cos 2x$; 389. $5, \cos^2 x, \sin^2 x$; 390. $1, \sin x, \cos 2x$;

Berilgan funksiyalar sistemasi uchun Vronskiy determinantini tuzing.

391. $ye^{-x}, x^2, \cos x$; 392. $\sin^2 x, \cos^2 x, x$;

393. $ye^{\lambda_1 x}, ye^{\lambda_2 x}, \dots, ye^{\lambda_n x}$ 394. $ye^x, 2ye^x, ye^{-x}$;

395. $2, \cos x, \cos 2x$; 396. $ye^x \sin x, ye^x \cos x$

397. $x, \ln x, 3$ 398. π, $\arcsin x, \arccos x$;

Gram determinanti yordamida funksiyalar sistemasi $[-\pi, \pi]$ kesmada chiziqli bog‘langanligini ko‘rsating.

399. $x, 2x, x^2$; 400. $5, \cos^2 x, \sin^2 x$;

401. $1, \sin 2x, (\sin x - \cos x)^2$;

Xususiy yechimi berilgan chiziqli birjinsli tenglamaning umumiy yechimini toping.

402. $y'' + y = 0,\ y_1 = \sin x$;

403. $y'' + \frac{2}{x} y' + y = 0$, x > 0, $y_1 = \frac{\sin x}{x}$.

404. $y'' + \frac{2}{\sin^2 x} y = 0$, $y_1 = ctgx$, $x \in (\frac{\pi}{10}, \frac{\pi}{6})$.

405. $x(x+1)y'' + (x+2)y' - y = 0,\ y_1 = x+2.$

406. $(2t+1)x + (4t-2)x - 8x = 0$, $x_1(t) = t^2 + \frac{1}{4}$.

407. $(1-x^2)y'' - xy' + \frac{1}{4} y = 0,\ y_1 = \sqrt{1+x}.$

408. $x^2(\ln x - 1)y'' - xy' + y = 0$, $y_1 = x.$

409. $y'' + (tgx - 2ctgx)y' + 2ctg^2 x \cdot y = 0$, $y_1 = \sin x.$

410. $y'' + tgx \cdot y' + \cos^2 x \cdot y = 0$, $y_1 = \cos(\sin x).$

411. $(1+x^2)y'' + xy' - y = 0$, $y_1 = x.$

17-§. CHIZIQLI DIFFERENSIAL TENGLAMA TUZISH

$y_1(x), y_2(x), \ldots, y_n(x)$ funksiyalar (a,b) oraliqda uzluksiz va chiziqli bog‘lanmagan funksiyalar sistemasini tashqil etib, shu bilan birgalikda n - chi tartibgacha uzluksiz hosilalarga ega bo‘lsin.

$$\begin{vmatrix} y_1 & y_2 & \cdots & y_n & y \\ y_1' & y_2' & \cdots & y_n' & y' \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ y_1^{(n)} & y_2^{(n)} & \cdots & y_n^{(n)} & y^{(n)} \end{vmatrix} = 0$$

tenglama n - chi tartibli chiziqli tenglama bo‘lib, bu tenglamaning (*FES*) fundamental yechimlar sistemasini berilgan $y_1(x), y_2(x), \ldots, y_n(x)$ funksiyalar tashqil etadi.

<u>57-misol</u>. Fundamental yechimlar sistemasi $y_1 = \sin x$, $y_2 = tgx$ bo‘lgan chiziqli differensial tenglama tuzing.

Tenglama 2 - chi tartibli bo‘ladi, chunki *FES* ikkita funksiyadan iborat. Uchinchi tartibli determinant tuzamiz.

$$\begin{vmatrix} \sin x & tgx & y \\ \cos x & \dfrac{1}{\cos^2 x} & y' \\ -\sin x & \dfrac{2\sin x}{\cos^3 x} & y'' \end{vmatrix} = 0$$

va uni oxirgi ustun elementlari bo‘yicha yoyamiz.

$$y'' \begin{vmatrix} \sin x & tgx \\ \cos x & \dfrac{1}{\cos^2 x} \end{vmatrix} - y' \begin{vmatrix} \sin x & tgx \\ -\sin x & \dfrac{2\sin x}{\cos^3 x} \end{vmatrix} +$$

$$+y \begin{vmatrix} \cos x & \dfrac{1}{\cos^2 x} \\ -\sin x & \dfrac{2\sin x}{\cos^3 x} \end{vmatrix} = 0.$$

Bundan $\dfrac{\sin^3 x}{\cos^2 x} y'' - \dfrac{\sin^2 x(2+\cos^2 x)}{\cos^3 x} y' + \dfrac{3\sin x}{\cos^2 x} y = 0$ tenglamani olamiz.

Bu tenglamani keltirilgan tenglamaga keltirish uchun uni $\dfrac{\sin^3 x}{\cos^2 x}$ ga bo‘lamiz.

$$y'' - \frac{2+\cos^2 x}{\sin x \cos x} y' + \frac{3}{\sin^2 x} y = 0$$

tenglama maxsus nuqtalarga ega. Bu nuqtalar $sinx = 0$, $cosx = 0$ tenglamaning yechimlaridan iborat, ya'ni

$$x = \frac{\pi n}{2}, n \in Z.$$

O'zgarmasni variasiyalash usuli

Agar (1) bir jinslimas tenglamaga mos bo'lgan bir jinsli tenglamaning umumiy yechimi aniqlangan bo'lsa, u holda (1) tenglama umumiy yechimini kvadraturalar yordamida topish mumkin.

58-misol .

$$y'' + y = \cos ecx \tag{9}$$

Tenglamaning umumiy yechimini toping. Mos birjinsli tenglamaning umumiy yechimi: $y = C_1 \cos x + C_2 \sin x$. (9) tenglamaning yechimini $y = C_1(x)\cos x + C_2(x)\sin x$ ko'rinishda izlaymiz.

Topish kerak bo'lgan $C_1(x)$, $C_2(x)$ funksiyalar ikkita bo'lganligi sababli bitta ko'shimcha shart qo'yish kerak. y' va y'' hosilalarni hisoblaymiz.

$$y' = C_1'(x)\cos x + C_2'(x)\sin x = C_1(x)\sin x + C_2'(x)\cos x$$

$C_1(x)$ va $C_2(x)$ funksiyalarga qo'yiladigan shart

$$C_1'(x)\cos x + C_2'(x)\sin x = 0.$$

Demak,

$$y' = -C_1(x)\sin x + C_2(x)\cos x$$

$$y'' = -C_1'(x)\sin x + C_2'(x)\cos x - C_1(x)\cos x - C_2(x)\sin x$$

Hisoblangan hosilalarni (9) tenglamaga qo'yib, uni soddalashtirgandan so'ng

$$-C_1'(x)\sin x + C_2'(x)\cos x = \cos ecx$$

tenglamani hosil qilamiz.

Demak, $C_1(x)$ va $C_2(x)$ larni topish uchun qo'yidagi sistemani hosil qilamiz

$$\begin{cases} Ñ_1'(x)\cos x + C_2'(x)\sin x = 0 \\ -C_1'(x)\sin x + C_2'(x)\cos x = \cos ecx \end{cases}.$$

Bu sistema $C_1'(x)$ va $C_2'(x)$ larga nisbatan chiziqli algebraik tenglamalar sistemasidir. Uni yechamiz

$$\begin{cases} \tilde{N}_1'(x) = -1 \\ C_2'(x) = \dfrac{\cos x}{\sin x} \end{cases} \quad \text{yoki} \quad \begin{matrix} C_1(x) = -x + C_1 \\ C_2(x) = \ln|\sin x| + C_2 \end{matrix}.$$

Demak, (9) tenglamaning umumiy yechimi

$$y = C_1 \cos x + C_2 \sin x - x\cos x + \sin x \ln|\sin x|.$$

Tenglama yechimini **Meple** dasturi yordamida tekshiramiz.

```
> restart;
> deq:=diff(y(x),x$2)+diff(y(x),x)=1/cos(x);
```

$$deq := \left(\frac{d^2}{dx^2}\mathrm{y}(x)\right) + \left(\frac{d}{dx}\mathrm{y}(x)\right) = \frac{1}{\cos(x)}$$

```
> dsolve(deq,y(x));
```

$$\mathrm{y}(x) = \int \mathbf{e}^{(-x)} \left(\int \frac{\mathbf{e}^x}{\cos(x)}\, dx + \_C1 \right) dx + \_C2$$

Mustaqil yechish uchun misollar

Berilgan fundamental yechimlar sistemasiga ko‘ra differensial tenglama tuzing.

412. $y_1 = x,\ y_2 = cosx,\ y_3 = sinx;$ 413. $y_1 = 2x,\ y_2 = x - 2,\ y_3 = ye^x;$

414. $y_1 = x,\ y_2 = x^2,\ y_3 = x^3;$ 415. $y_1 = cos^2x,\ y_2 = sin^2x;$

416. $y_1 = ye^x,\ y_2 = xe^{x;}$ 417. $y_1 = ye^x,\ y_2 = ye^x sinx,\ y_3 = ye^x cosx.$

Berilgan chiziqli birjinslimas tenglamaga mos bo‘lgan birjinsli tenglamaning fundamental yechimlar sistemasini bilgan holda uning umumiy yechimini toping.

418. $y'' + y = 5,\ \ y_1 = Cosx,\ \ y_2 = Sinx;$

419. $y'' - \dfrac{y'}{x} = x,\ (x > 0)\ \ y_1 = 1,\ \ y_2 = x^2;$

420. $y'' + y = tg^2 x,\ \ x \in (-\dfrac{\pi}{2}; \dfrac{\pi}{2})\ \ y_1 = Cosx,\ \ y_2 = Sinx;$

421. $y'' - y' = \dfrac{1}{e^{x-1}},\ \ y_1 = 1,\ \ y_2 = e^x;$

422. $y''' + y'' = \dfrac{x-1}{x^2}, x \neq 0,\ \ y_1 = 1,\ \ y_2 = x,\ \ y_3 = e^{-x};$

423. $xy'' - (1 + 2x^2)y' = 4x^2 e^{x^2},\ \ y_1 = 1,\ \ y_2 = e^{x^2};$

424. $y'' - 2y' tgx = 1,\ \ y_1 = 1,\ \ y_2 = tgx;$

425. $x \ln xy'' - y' = \ln^2 x,\ \ y_1 = 1,\ \ y_2 = x(\ln x - 1);$

426. $y'' + y'tgx = Cosx \cdot tgx, \ y_1 = 1, \ y_2 = Sinx.$

18-§. n- CHI TARTIBLI O‘ZGARMAS KOEFFISIYENTLI CHIZIQLI DIFFERENSIAL TENGLAMALAR

1. Birjinsli differensial tenglama

$$a_0 y^{(n)} + a_1 y^{(n-1)} + ... + a_n y = 0 \qquad (1)$$

tenglamaga *n* - chi tartibli o‘zgarmas koeffisiyentli birjinsli differensial tenglama deyiladi. Bu yerda $a_0, a_1, ..., a_n$ o‘zgarmas sonlar.

Tenglamaning xususiy yechimi $y = e^{\lambda x}$ ko‘rinishda bo‘lib, u

$$a_0 \lambda^n + a_1 \lambda^{n-1} + ... + a_n = 0 \qquad (2)$$

λ - xarakteristik tenglamaning ildizi bo‘lishi kerak. Yechim ko‘rinishi (2) xarakteristik tenglama ildizlariga bog‘liq:

a) (2) tenglamaning barcha ildizlari haqiqiy va har xil.

Bu holda $y_1 = e^{\lambda_1 x}, y_2 = e^{\lambda_2 x}, ..., y_n = e^{\lambda_1 x}$ yechimlar tenglamaning fundamental yechimlar sistemasini tashqil etadi, chunki ular yordamida tuzilgan Vronskiy determinanti noldan farqli (oldingi bobning 54- misoliga qarang).

<u>59-misol</u>. $y'' - 7y' + 12y = 0$.

Xarakteristik tenglamani tuzamiz

$$\lambda^2 - 7\lambda + 12 = 0.$$

λ=3, λ=4 bu tenglamaning ildizlaridir. Demak, $y_1 = e^{3x}, y_2 = e^{4x}$ tenglamaning hususiy yechimlari va $y = c_1 e^{3x} + c_2 e^{4x}$ berilgan tenglamaning umumiy yechimi bo‘ladi.

b) (2) tenglamaning ildizlari orasida kompleks yechim mavjud.

Xarakteristik tenglama haqiqiy koyeffisiyentli bo‘lganligi sababli ildizga qo‘shma bo‘lgan son ham ildiz bo‘ladi. Bu ildizlar $\lambda_1 = \alpha + \beta i, \ \lambda_2 = \alpha - \beta i,$ bo‘lsin. Bu ildizlarga (1) tenglamaning $y_1 = e^{\lambda x} Cos\beta x, y_2 = e^{\lambda x} Sin\beta x$ ko‘rinishdagi ikkita yechim mos keladi.

<u>60-misol</u>. $y'' + 4y' + 13y = 0$.

Xarakteristik tenglama $\lambda^2 + 4\lambda = 13 = 0$. U $\lambda_{1,2} = -2 \pm 3i$ ildizlarga ega, demak, $y_1 = e^{-2x} Cos3x, y_2 = e^{-2x} Sin3x$ berilgan tenglamaning xususiy yechimlari bo‘lib, ular chiziqli bog‘lanmagan va $y = c_1 e^{-2x} Cos3x + c_2 e^{-2x} Sin3x$ tenglamaning umumiy yechimi bo‘ladi. Tenglama yechimini **Meple** dasturi yordamida tekshiramiz.

```
> restart;
> d60:=diff(y(x),x$2)+4*diff(y(x),x)+13*y(x)=0;
```

$$d60 := \left(\frac{\partial^2}{\partial x^2} y(x)\right) + 4\left(\frac{\partial}{\partial x} y(x)\right) + 13\, y(x) = 0$$

```
> dsolve(d60,y(x));
```

$$y(x) = \_C1\ \mathbf{e}^{(-2\,x)} \sin(3\,x) + \_C2\ \mathbf{e}^{(-2\,x)} \cos(3\,x)$$

61-misol. Xarakteristik tenglama ildizlari $\lambda_{1,2} = 2 \pm 4i,\ \lambda_{3,4} = -3 \pm i,\ \lambda_5 = -4$ bo'lgan differensial tenglamaning umumiy yechimini yozing.

Umumiy yechim

$$y = c_1 e^{2x} Cos4x + c_2 e^{2x} Sin4x + c_3 e^{-3\div} Cosx + c_4 e^{-3x} Sinx + c_5 e^{-4x}$$

ko'rinishda bo'ladi.

c) Xarakteristik tenglamaning ildizlari orasida karrali ildiz mavjud.

Masalan, λ_1 tenglamaning r karrali ildizi bo'lsin, bu holda (1) tenglama r ta

$$y_1 = e^{\lambda_1 x}, y_2 = xe^{\lambda_1 x}, ..., y_r = x^{r-1} e^{\lambda_1 x} \qquad (3)$$

ko'rinishdagi hususiy yechimga ega bo'ladi. Bu yechimlarni chiziqli bog'lanmaganligini bevosita Gram determinantidan foydalanmasdan aniqlash mumkin.

$$(c_1 + c_2 x + ... + c_r x^{r-1}) e^{\lambda_1 x} = 0 \qquad (4)$$

tenglik barcha x lar uchun o'rinli bo'lsin, u holda

$$c_1 + c_2 x + ... + c_r x^{r-1}$$

ko'phad aynan nolga teng bo'ladi, bu esa ko'phadning barcha koeffisiyentlari nol bo'lgandagina bajarilishi mumkin. Demak, (4) tenglik faqat $c_1 = c_2 = ... = c_r = 0$ bo'lganda bajariladi va bundan (3) chiziqli bog'lanmagan funksiyalar sistemasini tashqil etadi.

62-misol. Xarakteristik tenglama ildizlari $\lambda_{1,2,3,4} = 2,\ \lambda_{5,5} = -3$ bo'lgan $L[y] = 0$ differensial tenglamaning umumiy yechimini yozing.

$\lambda = 2$ to'rt karrali ildiz bo'lganligi sababli tenglamaning xususiy yechimlari $e^{2x}, xe^{2x}, x^2 e^{2x}, x^3 e^{2x}$ bo'ladi; $\lambda = -3$ ikki karrali – yechim e^{-3x}, xe^{-3x}

Shunday qilib, tenglamaning umumiy yechimi quyidagi ko'rinishga ega

$$y = (c_1 + c_2 x + c_3 x^2 + c_4 x^3) e^{2x} + (c_5 + c_6 x) e^{-3x};$$

63-misol. *L[y]=0* tenglamaning xarakteristik tenglamasi ildizlari $\lambda_{1,2,3} = -4 + 3i$, $\lambda_{4,5,6} = -4 + 3i$ $\lambda_{7,8} = 2i,\ \lambda_{9.10} = -2i$ bo'lsa. uning umumiy yechimini yozing. $\lambda_{1,2,3} = -4 \pm 3i$ uch karrali va $\lambda_{7.8} = \pm 2i$ ikki karrali ildizlar bo'lganligidan foydalanamiz. Umumiy yechim quyidagi ko'rinishga ega

$$y - e^{-4x}((c_1 + c_2 x + c_3 \quad x^2)\cos 3x + (c_4 + c_5 x + c_6 x \sin 3x) +$$
$$+(c_7 + c_8 x)\sin 2x + (c_9 + c_{10} x)\cos 2x.$$

Mustaqil yechish uchun misollar

Tenglamalarning umumiy yechimini toping va **Meple** dasturi yordamida tekshiring.

427. $y'' - 4y' + 3y = 0;$ 428. $y'' - 7y' + 12y = 0;$

429. $y'' - 2y' + 10y = 0;$ 430. $y'' - 3y' = 0;$

431. $y'' + 4y' + 8y = 0;$ 432. $y'' + 16y = 0;$

433. $y'' - 6y' + 9y = 0;$ 434. $y'' + 6y' + 10y = 0;$

435. $y^{IV} - y = 0;$ 436. $y^{IV} - 5y'' + 10y' - 6y = 0;$

437. $y^{IV} + 2y''' + 4y'' + 6y' + 3y = 0;$

438. $y''' + 5y'' + 7y' + 3 = 0;$ 439. $y^{V} - 10y''' + 9y' = 0;$

440. $y^{VII} + 4y^{VI} - y''' - 4y'' = 0;$

441. $y^{VI} - 8y''' + 26y'' - 40y' + 25y = 0.$

Koshi masalasini qanoatlantiruvchi xususiy yechimni toping

442. $y'' - 5y' + 6y = 0,\ y(0) = \frac{1}{2},\ y'(0) = 1;$

443. $y'' + 4y = 0,\ y(\frac{\pi}{2}) = -4,\ y'(\frac{\pi}{2}) = 2;$

444. $y'' - 6y' + 9y = 0,\ y(0) = 0,\ y'(0) = 2;$

445. $y'' - y = 0,\ y(0) = 7,\ y'(0) = 3.$

Xarakteristik tenglamaga ko‘ra chiziqli bir jinsli tenglamani tuzing.

446. a) $9\lambda^2 - 6\lambda + 1 = 0;$ b) $\lambda(\lambda + 1)(\lambda + 2) = 0;$

c) $(\lambda^2 + 1)^2 = 0;$ d) $\lambda^2(\lambda - 1) = 0.$

Xarakteristik tenglamani ildizlariga ko‘ra birjinsli chiziqli tenglamani tuzing va uning umumiy yechimini yozing.

447. a) λ_1=1, λ_2=2; b) $\lambda_{1,2,3}$=1; c) $\lambda_{1,2}$=3$\pm$2i;

d) λ_1=2, $\lambda_{2,3}$=$\pm$ i.

19-§. BIRJINSLI BO‘LMAGAN CHIZIQLI DIFFERENSIAL TENGLAMA

$$a_0 y^{(n)} + a_1 y^{(n-1)} + \ldots + a_n y = f(x) \tag{5}$$

tenglamani qaraymiz. Bu yerda $a_0, a_1, \ldots, a_n$ - o‘zgarmas sonlar, $f(x)$ $x \in [a,b]$ da aniqlangan va uzluksiz funksiya.

Birjinslimas tenglamaning umumiy yechimi ikkita yechimning algebraik yig'indisidan iborat bo'lib, bunda birinchi qo'shiluvchi berilgan tenglamaga mos bo'lgan birjinsli tenglamaning umumiy yechimidan, ikkinchi qo'shiluvchi esa berilgan tenglamaning bitta hususiy yechimidan iborat.

Umumiy holda, agar mos bir jinsli tenglama umumiy yechimi ma'lum bo'lsa, o'zgarmasni variasiyalash usuli yordamida berilgan tenglamaning umumiy yechimini kvadraturalar yordamida topish mumkin. Lekin amaliyotga tegishli muhim hollarda, ya'ni (5) tenglamaning o'ng tomoni

$$f(x) = e^{\alpha x}(P_m(x)\cos\beta x + P_k(x)\sin\beta x) \quad (6)$$

ko'rinishida bo'lganda, tenglama xususiy yechimini aniqmas koeffisiyentlar usuli yordamida ham aniqlash mumkin.Bu holda xususiy yechim ko'rinishini quyidagi jadvaldan foydalanib topish mumkin

№	O'ng tomon ko'rinishi	Xarakteristik tenglama ildizlari	Xususiy yechim ko'rinishi
	$A_m(x)$ *n*- chi darajali ko'phad	0 son xarakteristik tenglama ildizi emas	$y_1 = P_m(x)$ *m*-chi darajali to'la ko'phad
		0 son xarakteristik tenglamaning *s*-karrali ildizi	$y_1 = x^s P_m(x)$
	$A_m(x)\, e^{\alpha x}$	α son xarakteristik tenglama ildizi emas	$y_1 = P_m(x)e^{\alpha x}$
		α son xarakteristik tenglama ning *s*-karrali ildizi	$y_1 = x^s P_m(x)e^{\alpha x}$
	$A_m(x)\cos\beta x +$ $+B_k(x)\sin\beta x$ $A_m(x)$, $B_k(x)$ mos ravishda *n*-chi va *k*–chi darajali ko'phadlar	$\pm\beta i$ son xarakteristik tenglama ildizi emas	$y_1 = P_r(x)\cos\beta x +$ $+Q_r(x)\sin\beta x$ *r = max(m,k)* $P_r(x), Q_r(x)$ r-chi darajali ko'phadlar
		$\pm\beta i$ son xarakteristik tenglamaning *s*-karrali ildizi	$y_1 = x^s(P_r(x)\cos\beta x +$ $+Q_r(x)\sin\beta x)$
		$\alpha \pm\beta i$ son xarakteristik tenglama ildizi emas	$y_1 = e^{\alpha x}(P_r(x)\cos\beta x +$ $+Q_r(x)\sin\beta x)$

	$e^{\alpha x}(A_m(x)\cos\beta x +$ $+B_k(x)\sin\beta x)$	$\alpha \pm \beta i$ son xarakteristik tenglamaning *s* - karrali ildizi	$y_1 = x^s e^{\alpha x}(P_r(x)\cos\beta x +$ $+Q_r(x)\sin\beta x)$

64-misol. Tenglama umumiy yechimini toping: $y''' + 3y'' = x^2 - 1$.
Mos birjinsli tenglama $y''' + 3y'' = 0$ dan iborat. Xarakteristik tenglama ildizlari $\lambda_{1,2} = 0$, $\lambda_3 = -3$ va umumiy yechimi: $y^* = c_1 + c_2 x + c_3 e^{-3x}$. Berilgan tenglama xususiy yechimini topamiz.

0 – xarakteristik tenglamaning ikki karrali ildizi, demak, *s = 2*. Tenglama o‘ng tomoni 2 - chi darajali ko‘phad, *n = 2*. Jadvalning 1 bandining ikkinchi qismiga ko‘ra

$$y_1 = x^2(ax^2 + bx + c) = ax^4 + bx^3 + cx^2,$$

bu xususiy yechim koeffisiyentlarini topish uchun y_1 ni (7) ga qo‘yamiz.

$$24ax + 6b + 36ax^2 + 18bx + bc = x^2 - 6$$

bir xil darajali koeffisiyentlarni tenglashtirib,

$$\begin{cases} 36a = 1 \\ 24a + 18b = 0 \\ 6c = -6 \end{cases}$$

sistemani yechamiz.

$$a = \frac{1}{36}, b = -\frac{1}{27}, c = -1$$

Bundan $y_1 = \frac{1}{36}x^4 - \frac{1}{27}x^3 - x^2$ ni olamiz va umumiy yechim:

$$y = y^* + y_1 = c_1 + c_1 x + c_3 e^{-3x} + \frac{1}{36}x^4 - \frac{1}{27}x^3 - x^2$$

65-misol. Tenglamani yeching

$$y'' - 4y = xe^{2x} + (5x^2 + 2x)e^{3x}$$

Mos birjinsli tenglamaning xarakteristik tenglamasi ildizlari $\lambda_{1,2} = \pm 2$, bundan uning umumiy yechimi $y^* = c_1 e^{-2x} + c_2 e^{2x}$ bo‘ladi.

Berilgan tenglamaning o‘ng tomoni $f_1(x) = xe^{2x}$, $f_2(x) = (5x^2 + 2x)e^{3x}$ funksiyalar yig‘indisidan iborat bo‘lganligi sababli ikkita yordamchi tenglamalarni qaraymiz:

$$\begin{cases} y'' - 4y = xe^{2x} \\ y'' - 4y = (5x^2 + 2x)e^{3x} \end{cases}.$$

Birinchi tenglamada, $\alpha=2$ xarakteristik tenglamaning bir karrali ildizi bo‘ladi *(s=1)*, ko‘rsatkichli funksiyaning koeffisiyenti esa birinchi darajali ko‘phaddan iborat *(m = 1)*. Jadvalning 2 bandining ikkinchi qismiga ko‘ra xususiy yechim $y_1 = x(ax+b)e^{2x} = (ax^2+bx)e^{2x}$ ko‘rinishda bo‘ladi. Uni tenglamaga qo‘yamiz,

$$2ae^{2x} + 4(2ax+b)e^{2x} + 4(ax^2+bx)e^{2x} - 4(ax^2+bx)e^{2x} = xe^{2x}$$

soddalashtirib mos koeffisiyentlarni tenglashtiramiz va $\begin{cases} 8a=1 \\ 2a+4b=0 \end{cases}$ sistemani hosil qilamiz. Bundan $a=\frac{1}{8}, b=-\frac{1}{16}$, demak, $y_1=(\frac{1}{8}x^2-\frac{1}{16}x)e^{2x}$. Yordamchi tenglamaning ikkinchisida $\alpha=3$ va $m=2$ xarakteristik tenglamaning ildizlari emas. Jadvalning ikkinchi bandining birinchi qismiga ko‘ra

$$y_2=(ax^2+bx+c)e^{3x}.$$

Bu yechimni berilgan tenglamaga qo‘yib,

$$2ae^{3x}+6(2ax+b)e^{3x}+9(ax^2+bx=c)e^{3x} -$$
$$-4(ax^2+bx=c)e^{3x}=(5x^2+2x)e^{3x}.$$

Oxirgi ifodani soddalashtiramiz.

$$5ax^2+(12a+5b)x+2a+6b+5c=5x^2+2x.$$

Ko‘phadlarning mos darajalar koeffisiyentlarini tenglashtirib:

$$\begin{cases} 5a=5 \\ 12a+5b=2 \\ 2a+6b+5c=0 \end{cases}$$

sistemadan $a = 1, b = -2, c = 2$ ni olamiz.

Demak, $y_2=(x^2-2x+2)e^{3x}$. Natijada berilgan tenglamaning yechimi

$y=c_1e^{2x}+c_2e^{-2x}+(\frac{1}{8}x^2-\frac{1}{16}x)e^{2x}+(x^2-2x+2)e^{3x}$ ko‘rinishga ega ekanligini hosil qilamiz. Tenglama yechimini **Meple** dasturi yordamida tekshiramiz.

```
> restart;
> de:=diff(y(x),x$2)-
4*y(x)=x*exp(2*x)+(5*x^2+2*x)*exp(3*x);
```

$$de := \left(\frac{d^2}{dx^2}y(x)\right) - 4\,y(x) = x\,e^{(2x)} + (5x^2+2x)\,e^{(3x)}$$

```
> dsolve(de,y(x));
```

$$y(x) = e^{(2x)}\_C2 + e^{(-2x)}\_C1 + \frac{1}{64}(8x^2-4x+1)e^{(2x)} + e^{(3x)}(x^2-2x+2)$$

66-misol. $y''+25y=x\cos 5x$ tenglamaning umumiy yechimini toping.

Mos birjinsli tenglamaning xarakteristik tenglamasi $\lambda^2+25=0$, bundan $\lambda_{1,2}=\pm 5i$ va umumiy yechimi $y^*=c_1\cos 5x+c_2\sin 5x$ bo‘ladi.

Tenglama o‘ng tomoni jadvalning 3 bandiga mos keladi, bunda $\pm\beta \ \ i=\pm 5i$ va $r=1$, $k=0$, $\pm 5i$ xarakteristik tenglama ildizi, demak, xususiy yechim

$$y_1=x((a_0x+a_1)\cos 5x+(b_0x+b_1)\sin 5x)$$

ko‘rinishda bo‘ladi.

Koeffisiyentlarni topish uchun y_1 - ni tenglamaga qo‘yamiz va soddalashtirib

$$(20b_0x+2a_0+10b_1)\cos 5x+(20a_0x+2b_0-10a_1)\sin 5x=x\cos 5x$$

ni olamiz.

Bu tenglik o‘rinli bo‘lishi uchun mos trigonometrik funksiyalar koeffisiyentlari teng bo‘lishi kerak, ya’ni:

$$\begin{cases}20b_0x+2a_0+10b_1=x\\ 20a_0x+2b_0-10a_1=0\end{cases}$$

bundan

$$\begin{cases}20b_0=1\\ 20a_0+10b_1=0\\ 20a_0=0\\ 2b_0-10a_1=0\end{cases}.$$

Bu sistemaning yechimi $a_0=0$, $b_0=\frac{1}{20}, a_1=\frac{1}{100}, b_1=0$ va xususiy yechim

$$y_1=0{,}01x\cos 5x+\frac{1}{20}x^2\sin 5x.$$

Demak, berilgan tenglamaning umumiy yechimi:

$$y_1=c_1\cos 5x+c_2\sin 5x+0{,}01x\cos 5x+0{,}05x^2\sin 5x$$

bo‘ladi. Tenglama yechimini **Meple** dasturi yordamida tekshiramiz.

```
> restart;
> de:=diff(y(x),x$2)+25*y(x)=x*cos(5*x);
```

$$de := \left(\frac{d^2}{dx^2}\mathrm{y}(x)\right)+25\,\mathrm{y}(x)=x\cos(5\,x)$$

```
> dsolve(de,y(x));
```

$$\mathrm{y}(x)=\sin(5\,x)\_C2+\cos(5\,x)\_C1+\frac{1}{100}x\cos(5\,x)+\frac{1}{20}\sin(5\,x)\,x^2-\frac{1}{500}\sin(5\,x)$$

<u>67-misol</u>. $y''-6y'+13y=8e^{3x}\sin 2x$ tenglamaning xususiy yechimining ko‘rinishini aniqlang. Mos birjinsli tenglamaning xarakteristik tenglamasi $\lambda^2-6\lambda+13=0$, ildizlari $\lambda_{1,2}=3\pm 2i$. Jadvalning 4 bandiga ko‘ra

$\alpha \pm \beta \ i = 3 \pm 2i$ va $s = 1,\ A_m(x) = 0,\quad B_k(x) = 1$ bo‘lib, nolinchi darajali ko‘po‘addan iborat. Demak, xususiy yechim

$$y_1 = xe^{3x}(a\cos 2x + b\sin 2x)$$

ko‘rinishda bo‘ladi.

3. Koshi masalasi

n - chi tartibli chiziqli tenglama uchun Koshi masalasi berilgan tenglamaning

$$y(x_0) = y_0,\ y'(x_0) = y'_0,\ \dots,\ y^{(n-1)}(x_0) = y_0^{(n-1)}$$

shartlarni qanoatlantiruvchi yechimni topishdan iboratdir.

68-misol. $y'' - 4y = 4e^{2x}$ tenglamaning $y(0) = 0,\quad y'(0) = 2$ shartni qanoatlantiruvchi xususiy yechimini toping.

Tenglamaning umumiy yechimini topamiz.

$$y = c_1e^{2x} + c_2e^{-2x} + xe^{2x}.$$

Topilgan yechimni differensiallaymiz

$$y' = 2c_1e^{2x} - 2c_2e^{-2x} + e^{2x} + 2xe^{2x}.$$

Koshi masalasi shartiga ko‘ra $\begin{cases} c_1 + c_2 = 0 \\ 2c_1 - 2c_2 + 1 = 2 \end{cases}$ bundan $c_1 = \frac{1}{4},\ c_2 = -\frac{1}{4},$

demak, xususiy yechim $y = \frac{1}{4}e^{2x} - \frac{1}{4}e^{-2x} + xe^{2x}$ dan iborat.

69-misol. $y'' - 3y' + 2y = 4 + 2e^{-x}\cos x$ tenglamani $x \to +\infty$ *ga* $y \to 2$ shartni qanoatlantiruvchi xususiy yechimini toping.

Tenglama umumiy yechimi $y = c_1e^{x} + c_2e^{2x} + 2 + e^{-x}(\sin x - \cos x)$ dan iborat.

Bir vaqtda nolga teng bo‘lmagan *c1* va *c2* lar uchun $x \to +\infty$ da yechim chegaralanmagan. $c_1 = c_2 = 0$ da $y = 2 + e^{-x}(\sin x - \cos x)$ va $\lim\limits_{x\to+\infty} y = 2$ bo‘ladi.

Demak, masala yechimi: $y = 2 + e^{-x}(\sin x - \cos x)$.

Mustaqil yechish uchun misollar

Tenglamalarning umumiy yechimini toping va **Meple** dasturi yordamida tekshiring.

448. $y'' + 4y' + y = 4$;

449. $y'' + 6y' + 9y = 12e^{-3x}$;

450. $y'' - 6y' + 9y = x^2$;

451. $y'' + 4y' = 4xe^{-4x}$;

452. $y'' + 6y' - 3y = 12\cos 3x$;

453. $y'' - y = 2x - 1 + e^{5x}$;

454. $y'' - 3y' = 1 + e^x + \cos x + \sin x$;

455. $y'' + y' + y + 1 = \sin x + x + x^2$;

456. $y'' + y = 2\sin x \sin 2x$; 457. $y''' - 2y'' + y' = 2x + e^x$;

Koshi masalasini qanoatlantiruvchi xususiy yechimni toping

458. $y'' - 2y' = e^x(x^2 + x - 3),\ y(0) = 2,\ y'(0) = 2$;

459. $y'' - 5y' + 6y = e^{-x}(3x - 2),$ y(0) = $y'(0) = 0$;

460. $y'' - y = 3x,\ y(1) = -1,\ y'(1) = 0$;

461. $y'' + 6y' + 9y = 10\sin x,$ y(0) = $y'(0) = 0$;

462. $y'' - y' = -5e^{-x}(\sin x + \cos x),\ y(0) = -4,\ y'(0) = 5$.

Cheksizlikda berilgan shartlarni qanoatlantiruvchi chiziqli differensial tenglamaning xususiy yechimini toping.

463. $y'' - y = 1,\ x \to \infty$ da y – chegaralangan;

464. $y'' - y = 3 - 2\cos x,\ x \to \infty$ da y – chegaralangan;

465. $y'' - 2y' + y = 4e^{-x},\ y \to 0$, agar $x \to \infty$;

466. $y'' + 4y' + 3y = 8e^x + 9,\ y \to 3$, agar $x \to -\infty$;

467. $y'' - y' - 5y = 1,\ y \to -\frac{1}{5}$, agar $x \to \infty$.

Tenglamalarning xususi yechimining ko‘rinishini aniqlang

468. $y'' + k^2 y = k\sin(kx + \alpha)$.

469. $y^{IV} + 4y'' + 4y = x\sin 2x.$

470. $y^{IV} + 2n^2 y'' + n^4 y = a\sin(nx + \alpha).$

471. $y^{IV} - 2n^2 y'' + n^4 y = x\cos(nx + \alpha).$

472. $y'' - 2y' + 2y = e^x + x\cos x.$

473. $y'' + 6y' + 10y = 3xe^{-3x} - 2e^{3x}\cos x.$

474. $y'' - 8y' + 20y = 5xe^{4x}\sin^2 x.$

475. $y'' + 7y' + 10y = xe^{-2x}\cos 5x.$

476. $y'' - 2y + 5y = 2xe^x + e^x\sin 2x.$

477. $y'' - 8y' + 17x = (x^2 - 3x\sin x)e^{4x}.$

478. $y''' + y' = \sin x + x\cos x + e^{-x}\cos 2x + x^2.$

479. $y''' - 2y'' + 4y' - 8y = e^{2x}\sin 2x + 2x^2.$

480. $y'' - 9y = (x^2 + \sin 3x)e^{-3x}.$

481. $y'' + 4y = \cos x \cos 3x.$

482. $y^{IV}+5y''+4y=\sin x\cos 2x.$

483. $y''-4y+5y=e^{2x}\sin^2 x.$

484. $y''-3y'+2y=2^x.$

485. $y''+2y'+2y=chx\sin x.$

486. $y''-8y'+17y=\left(x-3\sin 2x+x^2\cos^2 x+\sin x\right)e^{3x}.$

487. $y'''+3y''+3y'+y=xe^{-x}+x^2e^x+\sin x+ch2x.$

488. $y''-2y'+y=2xe^x+xe^x\cos x.$

489. $y''-y=4x^2shx.$

490. $y^{(4)}-a^4y=x^2+3+\sin x+x\cos x.$

$\mathrm{L}[y]=f(x)$ tenglamani xarakteristik tenglamasi ildizlari va o‘ng tomoni $f(x)$-larga
ko‘ra xususi yechimi ko‘rinishini aniqlang

491. $\lambda_1=\lambda_2=\lambda_3=-1,\ f(x)=(3x^2-7)e^{-x}+4e^{3x};$

492. $\lambda_1=\lambda_2=-ki,\lambda_3=\lambda_4=ki,\ f(x)=3\sin kx+5x\cos kx;$

493. $\lambda_1=\lambda_2=3,\lambda_3=i,\lambda_4=-i,\ f(x)=x\sin x+\left(x^2+2\right)\cos x-x^2e^{-3x};$

494. $\lambda_{1,2}=3\pm 2i,\lambda_3=\lambda_4=0,\ f(x)=3xe^{3x}\sin x+x^2-3;$

495. $\lambda_1=\lambda_2=\lambda_3=k,\ f(x)=2x\sin kx+\left(3x^2-4x+5\right)e^{kx};$

496. $\lambda_1=\lambda_2=\lambda_3=1,\lambda_4=0,\ f(x)=2x^3-6x+4+2xe^{-x};$

497. $\lambda_1=\lambda_2=\lambda_3=0,\lambda_4=\lambda_5=3,,\ f(x)=5x^2+7-\left(x^3+2x\right)e^{-3x}.$

20-§. O‘ZGARMAS KOEFFITSIYENTLI CHIZIQLI DIFFERENSIAL TENGLAMAGA KELTIRILADIGAN TENGLAMALAR

1. Eyler tenglamasi.

$$a_0x^ny^{(n)}+a_1x^{n-1}y^{(n-1)}+\ldots+a_{n-1}xy'+a_ny=f(x)$$

ko‘rinishdagi tenglama Eyler tenglamasi deyiladi. Bu tenglama $x=e^t$ almashtirish yordamida o‘zgarmas koyeffisiyentli bir jinslimas chiziqli tenglamaga keltiriladi. Yoki mos birjinsli tenglamani xususiy yechimini $y=x^{\lambda}$ ko‘rinishda olish ham mumkin.

$$a_0(ax+b)^n y^{(n)} + a_1(ax+b)^{n-1} y^{(n-1)} + ... + a_n y = f(x)$$

tenglama umumlashgan Eyler tenglamasi bo‘lib bu tenglamani o‘zgarmas koeffisiyentli chiziqli tenglamaga keltirish uchun $ax + b = e^t$ almashtirish qo‘llash kerak.

70-misol . $x^2 y'' - xy' + 2y = x\ln x$ tenglamaning umumiy yechimini toping.

Bu tenglama Eyler tenglamasi, chunki birhadlardagi hosila tartibi, argument x ning darajasiga teng.

1 usul.

$x = e^t$ almashtirish bajaramiz.

Bunda

$$y' = \frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = e^{-t}\frac{dy}{dt}$$

$$y'' = \frac{dy'}{dx} = \frac{dy'}{dt}\cdot\frac{dt}{dx} = e^{-t}\frac{d}{dt}(e^{-t}\frac{dy}{dt}) = e^{-2t}(\frac{d^2y}{dt^2} - \frac{dy}{dt})$$

larni tenglamaga qo‘yib

$$y'' - 2y' + 2y = te^t \qquad (1)$$

o‘zgarmas koeffisiyentli tenglamani hosil qilamiz. Mos bir jinsli tenglama xarakteristik tenglamasi $\lambda^2 - 2\lambda + 2 = 0$ ildizlari $\lambda_{1,2} = 1 \pm i$

Demak, mos bir jinsli tenglama umumiy yechimi

$$y^* = (c_1\cos t + c_2\sin t)e^t.$$

Uning xususiy yechimi $y_1 = (at+b)e^t$ ko‘rinishda bo‘lib, bu yechimni berilgan tenglamaga qo‘yib,

$$(at + b + 2a)\,e^t - 2\,(at + b + a)e^t + 2(at + b)e^t = te^t$$

ni hosil qilamiz. Bundan $a = 1, b = 0$.

(1) tenglamaning umumiy yechimi

$$y = (c_1\cos t + c_2\sin t)e^t + te^t.$$

Demak , berilgan tenglamaning umumiy yechimi

$$y = (c_1\cos\ln|x| + c_2\sin\ln|x|)x + x\ln|x|.$$

2 usul.

Berilgan tenglamaga mos bir jinsli tenglamaning yechimi $y = x^\lambda$ ko‘rinishda izlaymiz (λ - noma’lum son)

$$y' = \lambda x^{\lambda-1},$$

$$y'' = \lambda(\lambda-1)x^{\lambda-2}$$

tenglamaga qo‘yib

$$x^2\lambda(\lambda-1)x^{\lambda-2} - x^{\lambda-1} + 2x^\lambda = 0$$

yoki

$$x^\lambda(\lambda(\lambda-1) - \lambda + 2) = 0$$

$x^{\lambda} \neq 0$ bo‘lganligi sababli $\lambda^2 - 2\lambda + 2 = 0$ tenglama hosil qilamiz. Bu tenglama 1 usulda qaralgan xarakteristik tenglamadir.

2. Erkli o‘zgaruvchi va izlanuvchi funksiyani almashtirish

Ma’lumki, chiziqli differensial tenglama erkli o‘zgaruvchini xosmas almashtirish yoki izlanayotgan funksiyani chiziqli almashtirish bajarganda tenglama shakli o‘zgarmaydi. N.P. Yerugin

$$y^{(n)} + p_1(x)y^{(n-1)} + ... + p_{n-1}(x)y' + p_n(x)y = 0$$

tenglama erkli o‘zgaruvchini faqat $t = c\int \sqrt[n]{p_n(x)}dx$ shakldagi almashtirish yordamida o‘zgarmas koeffisiyentli tenglamaga keltirilishni isbotlagan.

$y = \alpha(x)z, z = z(x)$ almashtirishda $\alpha(x)$ ni shunday tanlash mumkinki, hosil bo‘lgan tenglamada (*n* - *1*) tartibli hosila qatnashmaydi.

Masalan, $y'' + p(x)y' + q(x)y = 0$ tenglamaga

$$y = e^{-\int \frac{p(x)}{2}dx} z$$

almashtirish tadbiq etilsa, tenglama $z'' + F(x)z = 0$ shakilga keladi, bu yerda $F(x) = -\frac{p'(x)}{2} - \frac{p^2(x)}{2} + q(x)$.

Mustaqil yechish uchun misollar.

Eyler tenglamalarining umumiy yechimini toping.

498. $x^2y'' + y = 0$;
499. $x^2y''' - 2y' = 0$;
500. $xy''' + y'' = 0$;
501. $(x+1)^2y'' - 2(x+1)y' + 2y = 0$;
502. $x^2y'' - xy' + y = 6x\ln x$;
503. $x^2y'' - xy' = -x + \frac{3}{x}$;
504. $x^2y'' + xy' + y = 2\sin(\ln x)$.

Erkli o‘zgaruvchini almashtirish yordamida tenglamalarni o‘zgarmas koeffisiyentli chiziqli tenglamaga keltiring va uning yechimini toping.

505. $x^4y'' + 2x^3y' + n^2y = 0$;
506. $2xy'' + y' - 2y = 0$;
507. $xy'' + \frac{1}{2}y' + y = 0$;
508. $(1+x^2)^2y'' + 2x(1+x^2)y' + y = 0$;
509. $(1-x^2)y'' - 2xy' + n^2y = 0$ (Chebishev tenglamasi);
510. $y''\sin x\cos x - y' + m^2 ytgx\sin^2 x = 0$.

Izlanayotgan funksiyani almashtirish yordamida tenglamalarda y' koeffisiyentini nolga keltiring va uni integrallang.

511. $x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0$ (Bessel' tenglamasi);

512. $xy'' + 2y' - xy = e^x$; 513. $y'' + \frac{2}{x}y' - a^2y = 2.$

21-§. DIFFERENSIAL TENGLAMALARNI DARAJALI QATORLAR YORDAMIDA YECHIMINI TOPISH

$$y'' + p(x)y' + q(x)y = 0 \qquad (1)$$

tenglama berilgan bo'lib *p(x)* va *q(x)* koeffisiyentlar *x* ning butun musbat darajalari bo'yicha qatorga yoyish mumkin.

Bu holda (1) tenglama yechimini $y = \sum_{k=0}^{\infty} c_k x^k$ ko'rinishda izlaymiz. Bu yechimni (1) tenglamaga qo'yamiz

$$\sum_{k=0}^{\infty} k(k-1)c_k x^{k-2} + \sum_{k=0}^{\infty} a_k x^k \sum_{k=1}^{\infty} kc_k x^{k-1} + \sum_{k=0}^{\infty} b_k x^k \sum_{k=0}^{\infty} c_k x^k = 0$$

Soddalashtirishdan so'ng ko'phad koeffisiyentlarini nolga tenglashtiramiz.

$$\begin{array}{l|c} x^0 & 2\cdot 1c_2 + a_0c_1 + b_0c_0 = 0 \\ x^1 & 3\cdot 2c_2 + 2a_0c_2 + a_1c_1 + b_0c_1 + b_1c_0 = 0 \\ x^2 & 4\cdot 3\cdot c_4 + 3a_0c_3 + 2a_1c_2 + a_2c_1 + b_0c_2 + b_1c_1 + b_2c_0 = 0 \\ & \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots \end{array} \qquad (2)$$

c_0, c_1, c_2 , … larga nisbatan chiziqli tenglamalar sistemasi bo'lib, har bir tenglamada undan oldingi tenglamadan bitta ko'p noma'lum c qatnashgan.

c_0 va c_1 koeffisiyentlar ixtiyoriy bo'lib, c_2, c_3 … ular orqali ifodalanadi.

Amaliyotda quyidagi usuldan foydalanish afzalroq.

Yuqorida ko'rsatilgan usul yordamida (1) tenglamaning 2 ta yechimini topamiz. Bunda $y_1(x)$ uchun $c_0 = 1,\ c_2 = 0;$

$y_2(x)$ uchun $c_0 = 0,\ \ c_1 = 1$ olinadi, ya'ni $y_1(x)$ uchun boshlang'ich shart $y_1(0) = 1,\ y_1'(0) = 0.$

$y_2(x)$ uchun esa $y_2(0) = 0,\ \ y_2'(0) = 1.$

Agar (1) tenglama uchun $y(0) = A,\ \ y'(0) = B$ shartni qanoatlantiruvchi yechim topish talab qilingan bo'lsa, u holda bu yechim

$y = Ay_1(x) + By_2(x)$ ko'rinishda bo'ladi.

71-misol. $y'' - xy' - 2y = 0$ tenglama yechimini darajali qator shaklida toping.

Tenglamani yechimini $y_1 = \sum_{k=0}^{\infty} c_k x^k$ qator ko‘rinishda izlaymiz. Bu funksiyani berilgan tenglamaga qo‘yamiz.

$$\sum_{k=0}^{\infty} k(k-1)c_k x^{k-2} - \sum_{k=1}^{\infty} kc_k x^k - 2\sum_{k=0}^{\infty} c_k x^k = 0 \cdot$$

$y_1(0) = 1,\ y_1'(0) = 0$ deb olamiz va oxirgi tenglamadan x ning barcha darajalari koeffisiyentlarini nolga tenglashtiramiz $(c_0 = 1,\ \ c_1 = 0)$

$$\begin{array}{l|c}
x^0 & 2c_2 - 2c_0 = 0 \\
x^1 & 3\cdot 2c_3 - 1\cdot c_1 - 2c_1 = 0 \\
x^2 & 4\cdot 3\cdot c_4 - 2c_2 - 2c_2 = 0 \\
x^3 & 5\cdot 4c_5 - 3c_3 - 2c_3 = 0 \\
x^4 & 6\cdot 5c_6 - 4c_4 - 2c_4 = 0 \\
\dots & \dots\ \dots\ \dots\ \dots\ \dots
\end{array}$$

Bu tenglamalarni yechib $c_2 = 1,\ \ c_3 = 0,$ $c_4 = \frac{1}{3}$, $c_5 = 0$, $c_6 = \frac{1}{3\cdot 5}$, … larni olamiz.

Demak, $y_1(x) = 1 + x^2 + \frac{1}{3}x^4 + \frac{1}{15}x^6 + ...$

Shu tartibda $y_2(x) = \sum_{k=0}^{\infty} a_k x^k$ va $y_2(0) = 0,\ y_2'(0) = 1$ boshlang‘ich shartlarni olib, berilgan tenglamadan $\sum_{k=2}^{\infty} k(k-1)a_k x^{k-2} - \sum_{k=1}^{\infty}(k+2)a_k x^k = 0$ ni hosil qilamiz. Bu tenglamadan $(a_0 = 0,\ a_1 = 1)$

$$\begin{array}{l|c}
x^0 & 2_2 = 0 \\
x^1 & 3\cdot 2a_3 - 3a_1 = 0 \\
x^2 & 4\cdot 3\cdot a_4 - 4a_2 = 0 \\
x^3 & 5\cdot 4a_5 - 5a_3 = 0 \\
x^4 & 6\cdot 5a_6 - 6a_4 = 0 \\
x^5 & 7\cdot 6a_7 - 7a_5 = 0 \\
\dots & \dots
\end{array}$$

Bundan $a_2 = 0,\ a_3 = \frac{1}{2}$, $a_4 = 0,\ a_5 = \frac{1}{2\cdot 4}$, $a_6 = 0$, $a_7 = \frac{1}{2\cdot 4\cdot 6}$, …

Demak, $a_{2k} = 0,\ a_{2k+1} = \frac{1}{2\cdot 4\cdot 6\dots(2k)}$, $k = 1,\ 2,\ 3,\ \dots$

va $y_2 = x + \frac{x^3}{3} + \frac{x^5}{2\cdot 4} + \frac{x^7}{2\cdot 4\cdot 6} + ... = x\sum_{k=0}^{\infty}\frac{1}{k!}(\frac{x^2}{2})^k = xe^{\frac{x^2}{2}}$

Berilgan tenglama umumiy yechimi:

$y = Ay_1(x) + By_2(x)$ bo‘ladi.

72-misol. $y'' = e^{xy}$ tenglamaning $y_1(0) = 1$, $y_1'(0) = 0$ shartlarni qanoatlantiruvchi yechimini Teylor qatori yoyilmasining dastlabki to‘rtta hadini toping.

Ma’lumki e^{xy} funksiya (0, 0) nuqta atrofida $-\infty < x < \infty$, $-\infty < y < \infty$ sohada yaqinlashuvchi darajali qatorga yoyiladi, ya’ni golomorfdir.

Tenglamaning yechimini

$y(x) = y(0) + \sum_{k=1}^{\infty} \frac{1}{k!} y^k(o) x^k$ shaklda izlaymiz.

Tenglamani differensiallab va uni $x = 0$ dagi qiymatini hisoblaymiz.

$$y'''(0) = (y + xy')\, y e^{xy}\big|_{x=0} = 1$$

$$y^{IV}(0) = \left[2y' + xy'' + (y + xy')^2\right] e^{xy}\big|_{x=0} = 1$$

topilgan qiymatlarni yechim ko‘rinishiga qo‘yib, berilgan masala yechimini topamiz.

$$y(x) = 1 + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$$

Mustaqil yechish uchun misollar

Qo‘yidagi tenglamalarni qatorlar yordamida yechimini toping.

514. $y' - 2xy = 0,\ y(0) = 1$

515. $y'' + xy' + y = 0,$

516. $y'' - xy' + y - 1 = 0,\ y(0) = 1,\ y'(0) = 0$.

Qo‘yidagi misollarda qo‘yilgan boshlang‘ich shartlarda yechimning darajali qatorga yoyilmasining dastlabki uchta hadini toping.

517. $y' = 1 - xy$, $y(0) = 0;$ 518. $y' = \frac{y - x}{y + x}$, $y(0) = 1;$

519. $y' = \sin xy$, $y(0) = 1;$ 520. $y'' + xy = 0, y(0) = 0, y'(0) = 1;$

521. $y'' - \sin xy' = 0, y(0) = 0, y'(0) = 1;$

522. $xy'' + y \cdot \sin x = x,\ y(\pi) = 1, y'(\pi) = 0;$

523. $y'' \ln x \cdot \sin(xy) = 0,\ y(e) = e^{-1}, y'(e) = 0;$

524. $y''' + x \sin y = 0,\ y(0) = \frac{\pi}{2}, y'(0) = 0, y''(0) = 0.$

6-BOB
O‘ZGARMAS KOEFFISIYENTLI CHIZIQLI DIFFERENSIAL TENGLAMALAR SISTEMASI

1. O‘zgarmas koeffisiyentli chiziqli bir jinsli tenglamalar sistemasi qo‘yidagicha ko‘rinishga ega.

$$\begin{cases} \frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \\ \frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \\ ... \quad ... \quad ... \quad ... \quad ... \quad ... \quad ... \\ \frac{dx_n}{dt} = a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n \end{cases}, \qquad (1)$$

bu yerda a_{ij} - o‘zgarmas sonlar $i,\ j = (1, n)$.

Bu sistemani matrisaviy ko‘rinishda yozish mumkin:

$$\frac{dx}{dt} = Ax, \qquad (2)$$

bu yerda

$$A = \begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{n1} & a_{n2} & ... & a_{nn} \end{pmatrix}, \quad x = \begin{pmatrix} x_1(t) \\ x_2(t) \\ ... \\ x_n(t) \end{pmatrix}, \quad \frac{dx}{dt} = \begin{pmatrix} \frac{dx_1}{dt} \\ ... \\ \frac{dx_n}{dt} \end{pmatrix}$$

$y = \begin{pmatrix} y_1(t) \\ y_2(t) \\ ... \\ y_n(t) \end{pmatrix}$ vektor ustun $t \in (a,b)$ da (2) sistemaning yechimi bo‘ladi, agar barcha $a < t < b$ lar uchun $\frac{dy}{dt} = Ay$ ayniyat bajarilsa.

$x = \begin{pmatrix} x_k^{(1)}(t) \\ x_k^{(2)}(t) \\ ... \\ x_k^{(n)}(t) \end{pmatrix} (k = \overline{1, n})$ sistemasining xususiy yechimlari bo‘lsin, bunda yuqori indeks yechimdagi funksiya nomerini, qo‘yi indeks esa yechim nomerini bildiradi. Bu yechimlar (a,b) oraliqda fundamental yechimlar sistemasi deyiladi, agar shu oraliqda uning Vronskiy determinanti uchun

$$W(t) \equiv W[x_1, ..., x_n] = \begin{vmatrix} x_1^{(1)} & x_2^{(1)} & \dots & x_n^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \dots & x_n^{(2)} \\ \dots & \dots & \dots & \dots \\ x_1^{(n)} & x_2^{(n)} & \dots & x_n^{(n)} \end{vmatrix} \neq 0$$

shart bajarilsa.

22-§. EYLER USULI

(1) differensial tenglamalar sistemasi yechimini

$$x_i = \gamma_i e^{\lambda t}, \quad (i = \overline{1, n}) \tag{3}$$

shakldа izlaymiz. (3) ni (1) ga qo‘yib, soddalashtirgandan so‘ng γ_i va λ larga nisbatan sistemani hosil qilamiz:

$$\begin{cases} (a_{11} - \lambda)\gamma_1 + a_{12}\gamma_2 + ... + a_{1n}\gamma_n = 0 \\ a_{21}\gamma_1 + (a_{22} - \lambda)\gamma_2 + ... + a_{2n}\gamma_n = 0. \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ a_{n1}\gamma_1 + a_{n2}\gamma_2 + ... + (a_{nn} - \lambda)\gamma_n = 0 \end{cases} \tag{4}$$

Bu sistema nolmas yechimga ega, agar uning asosiy determinanti nolga teng bo‘lsa, ya’ni

$$\begin{vmatrix} a_{11} - \lambda & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} - \lambda & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} - \lambda \end{vmatrix} = 0 \tag{5}$$

Bu tenglama xarakteristik tenglama deyiladi. (1) tenglama yechimining ko‘rinishi (5) xarakteristik tenglama ildizlarining ko‘rinishiga bog‘liq. Bu hollarga doir misollar qaraymiz.

a) xarakteristik tenglama ildizlari haqiqiy va har xil.

73-misol.

$$\frac{dx}{dt} = 3x - y + z$$

$$\frac{dy}{dt} = -x + 5y - z$$

$$\frac{dz}{dt} = x - y + 3z$$

Sistema xususiy yechimini $x = \gamma_1 e^{\lambda t}, y = \gamma_2 e^{\lambda t}, z = \gamma_3 e^{\lambda t}$ shaklda izlaymiz.

Bu yechimni sistemaga qo‘yish natijasida

$$\begin{cases}(3-\lambda)\gamma_1-\gamma_2+\gamma_3=0\\ -\gamma_1+(5-\lambda)\gamma_2-\gamma_3=0\\ \gamma_1-\gamma_2+(3-\lambda)\gamma_3=0\end{cases}$$

bir jinsli tenglamalar sistemasini hosil qilamiz.

Bu yerdan

$$\begin{vmatrix}3-\lambda & -1 & 1\\ -1 & 5-\lambda & -1\\ 1 & -1 & 3-\lambda\end{vmatrix}=0$$

xarakteristik tenglamani hosil qilamiz, yoki $\lambda^3-11\lambda^2+36\lambda-36=0.$ Uning yechimlari $\lambda_1=2,\lambda_2=3,\lambda_3=6$ bo‘ladi. Bu ildizlarni ketma – ket (6) sistemaga qo‘yamiz va hosil bo‘lgan sistemalarni yechib, λ_i larga mos $\gamma_i(i=1,2,3)$ qiymatlarni topamiz:

$$\begin{array}{ll}\lambda=2 & \gamma_1=1,\gamma_2=0,\gamma_3=-1\\ \lambda=3 & \gamma_1=1,\gamma_2=1,\gamma_3=1\\ \lambda=6 & \gamma_1=1,\gamma_2=-2,\gamma_3=1\end{array}.$$

Demak, sistemaning xususiy yechimlari quyidagicha bo‘ladi:

$$\begin{array}{lll}x_1=e^{2t} & y_1=0 & z_1=-e^{2t}\\ x_2=e^{3t} & y_2=e^{3t} & z_2=e^{3t}\\ x_3=e^{6t} & y_3=-2e^{6t} & z_3=e^{6t}\end{array}.$$

Sistemaning umumiy yechimi:

$$\begin{cases}x=c_1e^{2t}+c_2e^{3t}+c_3e^{6t}\\ y=c_2e^{3t}-2c_3e^{6t}\\ z=-c_1e^{2t}+c_2e^{3t}+c_3e^{6t}\end{cases}.$$

xarakteristik tenglama ildizlari kompleks son.

74-misol.

$$\begin{cases}\dfrac{dx}{dt}=x-5y\\ \dfrac{dy}{dt}=2x-y\end{cases}$$

γ_1 va γ_2 ni aniqlash uchun sistema

$$\begin{cases}(1-\lambda)\gamma_1-5\gamma_2=0\\ 2\gamma_1-(1+\lambda)\gamma_2=0\end{cases}. \quad (7)$$

Bu yerdan xarakteristik tenglama $\begin{vmatrix} 1-\lambda & -5 \\ 2 & -1-\lambda \end{vmatrix}=0$ ni olamiz, yoki $\lambda^2+9=0, \lambda=\pm 3i$. (7) sistemaga $\lambda=3i$ ni qo'yamiz.

$$\begin{cases} (1-3i)\gamma_1-5\gamma_2=0 \\ 2\gamma_1-(1+3i)\gamma_2=0 \end{cases}$$

bu sistema cheksiz ko'p yechimga ega, yechimlardan bittasi: $\gamma_1=5, \gamma_2=1-3i$.

Bu qiymatlarni yechim ko'rinishiga qo'yamiz: $x=5e^{3it}, y=(1-3i)e^{3it}$, bu yechimlarni $e^{i\varphi}=\cos\varphi+i\sin\varphi$ Eyler formulasidan foydalanib haqiqiy va mavhum qismlarini aniqlaymiz va sistemani 2 ta yechimini topamiz:

$$x_1=\operatorname{Re}x=5\cos 3t \qquad x_2=\operatorname{Im}x=5\sin 3t$$

$$y_1=\operatorname{Re}y=\cos 3t+3\sin 3t \quad y_2=\operatorname{Im}y=\sin 3t-3\cos 3t\cdot$$

Demak, sistemaning umumiy
yechimi $y=c_1y_1+cy_2=c_1(\cos 3t+3\sin 3t)+c_2(\sin 3t-3\cos 3t)$

$$\begin{cases} x=c_1x_1+c_2x_2=5c_1\cos 3t+5c_2\sin 3t \\ y=c_1y_1+cy_2=c_1(\cos 3t+3\sin 3t)+c_2(\sin 3t-3\cos 3t) \end{cases}$$

Tenglama yechimini **Meple** dasturi yordamida tekshiramiz.

```
> sys:=diff(x(t),t)=x(t)-5*y(t),
diff(y(t),t)=2*x(t)-y(t):

> dsolve({sys},{x(t),y(t)});
```

$$\{x(t)=\_C1\sin(3t)+\_C2\cos(3t),$$
$$y(t)=-\frac{3}{5}\_C1\cos(3t)+\frac{3}{5}\_C2\sin(3t)+\frac{1}{5}\_C1\sin(3t)+\frac{1}{5}\_C2\cos(3t)\}$$

b) xarakteristik tenglama ildizlari karrali.

Agar k karrali λ_0 ildiz uchun chiziqli bog'lanmagan k ta xos vektorlar $g^1,...,g^k$ mavjud bo'lsa, bu holda shu ildizga mos yechim $c_1g^1e^{\lambda_0 t}+...+c_kg^ke^{\lambda_0 t}$ bo'ladi.

Agar k karrali λ_0 ildizning m – ta chiziqli bog'lanmagan xos vektorlari bo'lsa, va $m<k$ bo'lsa, u holda bu ildizga mos yechimlarni qo'yidagi ko'rinishda izlaymiz:

$$\begin{cases} x_1=(a_1+b_1t+...+e_1t^{k-m})e^{\lambda_0 t} \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ x_n=(a_n+b_nt+..+e_nt^{k-m})e^{\lambda_0 t} \end{cases} \qquad (8)$$

$a_i, b_i, ..., e_i$ koeffisiyentlarni topish uchun (8) ni (1) ga qo‘yib hosil bo‘lgan chiziqli bir jinsli algebraik tenglamalar sistemasini yechish kerak.

75-misol.

$$\begin{cases} \dot{x} = 2x - y - z \\ \dot{y} = 3x - 2y - 3z \\ \dot{z} = 2z - x + y \end{cases}$$

Sistema yechimini topamiz. Yechimni $x = \gamma_1 e^{\lambda t}, y = \gamma_2 e^{\lambda t}, z = \gamma_3 e^{\lambda t}$ ko‘rinishda izlaymiz. Bularni sistemaga qo‘yib, soddalashtirgandan so‘ng

$$\begin{cases} (2-\lambda)\gamma_1 - \gamma_2 - \gamma_3 = 0 \\ 3\gamma_1 - (2+\lambda)\gamma_2 - 3\gamma_3 = 0 \\ -\gamma_1 + \gamma_2 + (2-\lambda)\gamma_3 = 0 \end{cases} \tag{9}$$

sistemani hosil qilamiz. Bu sistemaning xarakteristik tenglamasi

$$\begin{vmatrix} 2-\lambda & -1 & -1 \\ 3 & -(2+\lambda) & -3 \\ -1 & -1 & 2-\lambda \end{vmatrix} = 0 \quad .$$

Yoki $\lambda^3 - 2\lambda^2 + \lambda = 0$, bunda $\lambda_1 = 0, \lambda_2 = \lambda_3 = 1$. $\lambda = 0$ oddiy ildiz, uni (9) ga qo‘yamiz:

$$\begin{cases} 2\gamma_1 - \gamma_2 - \gamma_3 = 0 \\ 3\gamma_1 - 2\gamma_2 - 3\gamma_3 = 0 \\ -\gamma_1 + \gamma_2 + 2\gamma_3 = 0 \end{cases}$$

$$\gamma_1 = -\gamma_3$$

$$\gamma_2 = -3\gamma_3$$

Bundan $\gamma_1 = 1, \gamma_2 = 3, \gamma_3 = -1$ va yechim $x = 1$, $y = 3$, $z = -1$ bo‘ladi. $\lambda = 1$ ikki karrali ildiz, uni (9)ga qo‘yamiz

$$\begin{cases} \gamma_1 - \gamma_2 - \gamma_3 = 0 \\ 3\gamma_1 - 3\gamma_2 - 3\gamma_3 = 0 \\ -\gamma_1 + \gamma_2 + \gamma_3 = 0 \end{cases} \Rightarrow \quad \gamma_1 = \gamma_2 + \gamma_3 .$$

$\gamma_2 = c_2, \gamma_3 = c_3$ deb olsak, $\lambda = 1$ ga mos yechim qo‘yidagicha bo‘ladi:

$$x = c_2 + c_3,\ y = c_2,\ z = c_3.$$

Natijada berilgan sistemaning umumiy yechimi:

$$x = c_1 + (c_2 + c_3)e^t,\ y = 3c_1 + c_2 e^t,\ z = -c_1 + c_3 e^t \ .$$

76–misol.

$$\begin{cases} \dot{x} = x - y + z \\ \dot{y} = x + y - z \\ \dot{z} = 2z - y \end{cases} \tag{10}$$

sistemaning umumiy yechimini topamiz.

Yuqorida qarab chiqilgan misol kabi xususiy yechimlarni olamiz va ularni sistemaga qo‘yib, $\gamma_1, \gamma_2, \gamma_3$ larga nisbatan chiziqli bir jinsli sistemani hosil qilamiz:

$$\begin{cases} (1-\lambda)\gamma_1 - \gamma_2 + \gamma_3 = 0 \\ \gamma_1 + (1-\lambda)\gamma_2 - \gamma_3 = 0 \\ -\gamma_2 + (2-\lambda)\gamma_3 = 0 \end{cases} \tag{11}$$

Bundan,

$$\begin{vmatrix} 1-\lambda & -1 & 1 \\ 1 & (1-\lambda) & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0.$$

Yoki $(1-\lambda)^2(2-\lambda) = 0$ xarakteristik tenglamani hosil qilamiz. Bundan $\lambda_1 = 2, \lambda_{2,3} = 1$, $\lambda = 2$ uchun (11) sistemadan $\gamma_2 = 0, \gamma_1 = \gamma_3 = 1$ va mos yechim $x = e^{2t}$, $y = 0$, $z = e^{2t}$ bo‘ladi. $\lambda = 1$ karrali ildizga mos yechimni topishni batafsil qarab chiqamiz: (11) sistemaga $\lambda = 1$ ni qo‘yamiz:

$$\begin{cases} -\gamma_2 + \gamma_3 = 0 \\ \gamma_1 - 3\gamma_3 = 0 \\ -\gamma_2 + \gamma_3 = 0 \end{cases}.$$

Bu sistemaning matrisasi $\begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{pmatrix}$ bo‘ladi. Uning rangi 2 ga teng. Chiziqli bog‘lanmagan erkli o‘zgaruvchilar *m = 1* va *k – m = 1* bo‘ladi. Demak, bu holda sistemaning yechimi

$$\begin{aligned} x &= (a_1 + b_1 t)e^t \\ y &= (a_2 + b_2 t)e^t \\ z &= (a_3 + b_3 t)e^t \end{aligned} \tag{12}$$

ko‘rinishda bo‘ladi.

$a_1, b_1, a_2, b_2, a_3, b_3$ koeffisiyentlarni topish uchun (12) ni (10) ga o‘yamiz:

$$\begin{cases} b_1e^t + (a_1 + b_1t)e^t = (a_1 + b_1t)e^t - (a_2 + b_2t)e^t + (a_3 + b_3t)e^t \\ b_2e^t + (a_2 + b_2t)e^t = (a_1 + b_1t)e^t + (a_2 + b_2t)e^t - (a_3 + b_3t)e^t \\ b_3e^t + (a_3 + b_1t)e^t = 2(a_3 + b_3t)e^t - (a_2 + b_2t)e^t \end{cases}.$$

Bu sistemani soddalashtirib va noma'lumning mos darajalari koeffisiyentlarini tenglashtiramiz.

$b_1 = b_1 - b_2 + b_3,\ b_2 = b_1 + b_2 - b_3,\ b_3 = 2b_3 - b_2,$

$b_1 + a_1 = a_1 - a_2 + a_3,\ b_2 + a_2 = a_1 + a_2 - a_3,\ b_3 + a_3 = 2a_3 - a_2,$

$b_3 = b_2 = b_1 = c_1,\ a_3 = a_1 - b_2 = c_1 - c_2,\ a_1 = c_1,$

$a_2 = a_1 - 2b_2 = c_1 - 2c_2$

topilgan qiymatlarni (12) qo'yamiz va sistemaning umumiy yechimini topamiz:

$$\begin{aligned} x &= (c_1 + c_2t)e^t + c_3e^{2t} \\ y &= (c_1 - 2c_2 + c_2t)e^t \\ z &= (c_1 - c_2 + c_2t)e^t + c_3e^{2t} \end{aligned}.$$

Mustaqil yechish uchun misollar

Eyler usuli yordamida berilgan A matrisaga ko'ra, $\frac{dx}{dt} = Ax$ tenglamalar sistemasining umumiy yechimini toping. **Meple** dasturi yordamida natijani tekshiring.

525.

$$\begin{pmatrix} 7 & -12 & -2 \\ 3 & -4 & 0 \\ -2 & 0 & -2 \end{pmatrix}$$

$\lambda_1 = 2, \lambda_2 = -1, \lambda_3 = 0$

526.

$$\begin{pmatrix} 4 & 6 & 0 \\ -3 & -5 & 0 \\ -3 & -6 & 1 \end{pmatrix}$$

$\lambda_{1,2} = 1, \lambda_3 = -2$

527.

$$\begin{pmatrix} 2 & 6 & -15 \\ 1 & 1 & -5 \\ 1 & 2 & -6 \end{pmatrix}$$

$\lambda_{1,2,3} = -1$

528.

$$\begin{pmatrix} 3 & -1 & 0 \\ 6 & -3 & 2 \\ 8 & -6 & 5 \end{pmatrix}$$

$\lambda_1 = 1, \lambda_{2,3} = 2 \pm i$

529.
$$\begin{pmatrix} 4 & -5 & 7 \\ 1 & -4 & 9 \\ -4 & 0 & 5 \end{pmatrix}$$
$\lambda_1 = 1, \lambda_{2,3} = 2 \pm 3i$

530.
$$\begin{pmatrix} 2 & 1 \\ -1 & 4 \end{pmatrix}$$

23-§. SISTEMANI YUQORI TARTIBLI TENGLAMAGA KELTIRISH USULI. DALAMBER USULI

1. Sistemani yuqori tartibli tenglamaga keltirish usuli.

Bu usulni $\begin{cases} \dot{x} = ax + by \\ \dot{y} = cx + ey \end{cases}$ sistemaning yechimini topish uchun qo‘llaymiz. Sistemaning birinchi tenglamasini t bo‘yicha differensiallaymiz $\ddot{x} = a\dot{x} + b\dot{y}$.

$\dot{y}$ o‘rniga ikkinchi tenglamani qo‘yamiz: $\ddot{x} = a\dot{x} + cx + ey$, agar $b \neq 0$ bo‘lsa, sistemaning birinchi tenglamasidan y ni topib, uning ikkinchi tenglamasiga qo‘ysak, ikkinchi tartibli bir jinsli chiziqli tenglamani hosil qolamiz:

$$\ddot{x} - (a+e)\dot{x} + (ae - bc)x = 0.$$

Shunday qilib, sistemaning yechimini topishni qo‘yidagi sistema yechimini topishga keltirildi:

$$\begin{cases} \ddot{x} - (a+e)\dot{x} + (ae - bc)x = 0 \\ y = \dfrac{1}{b}(\dot{x} - ax) \end{cases}.$$

Agar berilgan sistemada $b = 0$ bo‘lsa, $c \neq 0$ bo‘lganda uni y ga nisbatan ikkinchi tartibli tenglamaga keltirish mumkin.

Agar $c = b = 0$ bo‘lsa, ajralgan tenglamalar sistemasi bo‘lib, uni ikkinchi tartibli tenglamaga keltirib bo‘lmaydi.

2. **Dalamber usuli**

Bu usul bo‘yicha integrallanuvchi kombinasiyalar tuzish yordamida chiziqli tenglamalar sistemasining yechimi topiladi.

$$\begin{cases} \dot{x} = ax + by + f_1(t) \\ \dot{y} = cx + ey + f_2(t) \end{cases}$$

sistema uchun integrallanuvchi kombinasiyani tuzamiz.

Ikkinchi tenglamani k ga ko‘paytirib, birinchisiga qo‘shamiz:

$$(\dot{x}+k\dot{y})=(a+kc)(x+\frac{b+ke}{a+kc}y)+f_1(t)+f_2(t)\cdot$$

Agar $\frac{b+ke}{a+kc}=k$ shart bajarilsa, ya'ni, $ck^2+(a-e)k-b=0$ kvadrat tenglama haqiqiy ildizga ega bo'lsa, integrallanuvchi kombinasiya mavjud bo'ladi.

Agar $k_1 \neq k_2$ bo'lsa, ikkita integrallanuvchi konbinasiya mavjud bo'ladi va sistemaning umumiy yechimini topish mumkin bo'ladi.

Agar $k_1 = k_2$ bo'lsa, bitta birinchi integral topiladi va bu holda sistemani bitta tenglamaga keltirish mumkin.

<u>77–misol</u>. $\begin{cases} \dot{x}=5x+4y+e^t \\ \dot{y}=4x+5y+1 \end{cases}$

$\frac{4+5k}{5+4k}=k$ tenglamadan $k_{1,2}=\pm 1$ ni topamiz. $k=1$ da

$$\frac{d(x+y)}{dt}=9(x+y)+e^t+1,$$

$k=-1$ da

$$\frac{d(x+y)}{dt}=(x-y)+e^t-1$$

tenglamani hosil qilamiz. Bu holda mos ravishda $x+y$ va $x-y$ larga nisbatan chiziqli tenglamalarni integrallab, berilgan sistemaning umumiy yechimini topamiz:

$$\begin{cases} x+y=c_1e^{9t}-\frac{1}{8}e^t-\frac{1}{9} \\ x-y=c_2e^t+te^t+1 \end{cases}$$

Tenglama yechimini **Meple** dasturi yordamida tekshiramiz.

```
> sys:=diff(x(t),t)=5*x(t)+4*y(t)+exp(t),
diff(y(t),t)=4*x(t)+5*y(t)+1:

> dsolve({sys},{x(t),y(t)});
```

$$\{x(t)=-\mathbf{e}^t\,\_C2+\mathbf{e}^{(9\,t)}\,\_C1-\frac{1}{16}\mathbf{e}^t+\frac{1}{2}t\,\mathbf{e}^t+\frac{4}{9},$$

$$y(t)=\mathbf{e}^t\,\_C2+\mathbf{e}^{(9\,t)}\,\_C1-\frac{5}{9}-\frac{1}{2}t\,\mathbf{e}^t-\frac{1}{16}\mathbf{e}^t\}$$

Mustaqil yechish uchun misollar

Quyidagi sistemalarni yuqori tartibli tenglamaga keltirib yechimini toping va **Meple** dasturi yordamida natijani tekshiring.

531. $\begin{cases} \dot{x} = y \\ \dot{y} = -x \end{cases}$

532. $\begin{cases} \dot{x} = 3y - x \\ \dot{y} = y + x + e^t \end{cases}$

533. $\begin{cases} \dot{x} = -x + y + e^t \\ \dot{y} = x - y + e^t \end{cases}$

534. $\begin{cases} \dot{x} = -x + y + \sin t \\ \dot{y} = -4x + 3y + e^t \end{cases}$

535. $\begin{cases} \dot{x} = -x + y \\ \dot{y} = -4x + 3y \end{cases}$

536. $\begin{cases} \dot{x} = z + t \\ \dot{y} = 2y + \sin t \\ \dot{z} = -x \end{cases}$

537. $\begin{cases} \dot{x} = 2x + y + z \\ \dot{y} = -2x - z \\ \dot{z} = 2x + y + 2z \end{cases}$

538. $\begin{cases} \dot{x} = 3x - 2y \\ \dot{y} = -3x + 2y \end{cases}$

$x(0) = y(0) = 1$

539. $\begin{cases} \dot{x} = y + z \\ \dot{y} = 3x + z \\ \dot{z} = 3x + y \end{cases}$

$x(0) = 0, y(0) = z(0) = 1$

Quyidagi sistemalarning yechimini Dalamber usuli yordamida toping va **Meple** dasturi yordamida natijani tekshiring.

540. $\begin{cases} \dot{x} = -x + 2y \\ \dot{y} = 3x + 4y \end{cases}$

541. $\begin{cases} \dot{x} = 2x - y \\ \dot{y} = x + 2y \end{cases}$

542. $\begin{cases} \dot{x} = 3x + 5y \\ \dot{y} = -2x - 8y \end{cases}$

543. $\begin{cases} \dot{x} = 2x + 3y + 4z \\ \dot{y} = 3x + 2y + 4z \\ \dot{z} = 5x + 5y + 2z \end{cases}$

544. $\begin{cases} \dot{x} = 5x + 4y + e^t \\ \dot{y} = 4x + 5y + 1 \end{cases}$

545. $\begin{cases} \dot{x} = 2x + 4y + \cos t \\ \dot{y} = -x - 2y + \sin t \end{cases}$

546.

$\begin{cases} \dot{x} = y + z \\ \dot{y} = x + z \\ \dot{z} = x + y \end{cases}$

547.

$\begin{cases} \dot{x} = y + z + 10\cos t \\ \dot{y} = x + z + 2e^t \\ \dot{z} = x + y - 10\sin t \end{cases}$

24-§.BIRJINSLI CHIZIQLI TENGLAMALARNI MATRISAVIY USULDA INTEGRALLASH

Ma'lumki, $\frac{dx}{dt} = Ax$ chiziqli tenglamalar sistemasining xususiy yechimlari $x_k(t) = \left(x_{1k}(t), x_{2k}(t), ..., x_{nk}(t)\right)^T \; (k = \overline{1,n})$ yechimlar fazosining bazisini tashqil etadi. Bu yechimlar *n*-chi tartibli matrisa tashqil etadi

$$\Phi(t) = \begin{pmatrix} x_{11}(t) & x_{12}(t) & ... & x_{1n}(t) \\ x_{21}(t) & x_{22}(t) & ... & x_{2n}(t) \\ ... & ... & ... & ... \\ x_{n1}(t) & x_{n2}(t) & ... & x_{nn}(t) \end{pmatrix}.$$

Bir jinsi tenglamaning yechimidan $\Phi(t) = e^{At} = \exp(At)$ kelib chiqadi, bu yerda

$$\exp(At) = \sum_{m=0}^{\infty} \frac{1}{m!} A^m t^m, \, t \in R, \Phi(0) = E .$$

Demak, sistemaning umumiy yechimi $x(t) = e^{At} \cdot c$ bo'ladi. Agar $x\big|_{t=t_0} = x_0$ bo'lsa, Koshi masalasining yechimi $x(t) = e^{A(t-t_0)} \cdot x_0$ shaklda topiladi.

<u>78 – misol</u> $\frac{dx}{dt} = \begin{pmatrix} -1 & -2 \\ 3 & 4 \end{pmatrix} x$ tenglamaning umumiy yechimini topamiz.

A^0 matrisaning darajalarini topamiz:

$$A^0 = E, \, A^1 = A = \begin{pmatrix} -1 & -2 \\ 3 & 4 \end{pmatrix},$$

$$A^2 = \begin{pmatrix} -5 & -6 \\ 9 & 10 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} -13 & -14 \\ 21 & 22 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} -29 & -30 \\ 45 & 46 \end{pmatrix},...$$

Bu yerdan $e^{At} \cdot c = (E + At + \frac{t^2}{2!} A^2 + \frac{t^3}{3!} A^3 + \frac{1}{4!} A^4 + ...) \cdot c =$

$$\begin{pmatrix} 1 - t - \frac{5}{2!} t^2 - \frac{13}{3!} t^3 - \frac{29}{4!} t^4 - ... & 2t - \frac{6}{2!} t^2 - \frac{14}{3!} t^3 - \frac{30}{4!} t^4 - ... \\ 3t + \frac{9}{2!} t^2 + \frac{21}{3!} t^3 + \frac{45}{4!} t^4 + ... & 1 + 4t + \frac{10}{2!} t^2 + \frac{22}{3!} t^3 + \frac{46}{4!} t^4 - ... \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

tenglamaning umumiy yechimi bo'ladi.

e^A matrisani hisoblashda *A* matrisani Jordan kataklariga keltirib hisoblash mumkin.

$$I_k(\lambda)=\begin{pmatrix} \lambda & 1 & 0 & ... & 0 & 0 \\ 0 & \lambda & 0 & ... & 0 & 0 \\ ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & \lambda & 1 \\ 0 & 0 & 0 & ... & 0 & \lambda \end{pmatrix}$$

k o‘lchamli Jordan katagi bo‘lsa, u holda bu matrisadaga mos eksponensial matrisa quyidagi ko‘rinishda bo‘ladi

$$\exp(I_k(\lambda)t)=e^{\lambda t}I_k(\lambda)=\begin{pmatrix} 1 & t & \frac{t^2}{2!} & ... & \frac{t^{k-1}}{(k-1)!} \\ 0 & 1 & t & ... & \frac{t^{k-2}}{(k-2)!} \\ ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & 1 \end{pmatrix}$$

chiziqli bir jinsli tenglamaning umumiy yechimi $x(t)=Se^{It}S^{-1}c$ shaklda bo‘ladi, bu yerda S matrisaning ustunlari *A* matrisaning xos vektorlari va unga biriktirilgan vektorlaridan iborat.

79- misol. $\frac{dx}{dt}=Ax$ tenglamaning umumiy yechimini topamiz, bu yerdan

$$A=\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 2 \\ 3 & 0 & 1 \end{pmatrix}.$$

A matrisaning Jordan matrisasini topamiz. Buning uchun xarakteristik tenglama $\det(A-\lambda E)=0$, ya’ni $(1-\lambda)^3=0$ ni ildizini topamiz. $\lambda=1$ uch karrali ildiz *(A-Ye)* matrisaning rangi 2-ga teng, bu xos qiymatga xos vektor mos keladi, demak,

$$I=\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix},\ e^{\lambda t}=e^t\begin{pmatrix} 1 & t & \frac{t^2}{2} \\ 0 & 1 & t \\ 0 & 0 & 1 \end{pmatrix}.$$

S matrisani topamiz $a_0=(\gamma_1,\gamma_2,\gamma_3)^T$ xos vektor

$$\begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 2 \\ 3 & 0 & 0 \end{pmatrix}\cdot\begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

tenglikdan aniqlanadi, bu yerdan $a_0 = \alpha(0,1,0)^T$, α - ixtiyoriy noldan farqli son, biriktirilgan a_1 *ва* a_2 vektorlar kuidagicha topiladi

$$\begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 2 \\ 3 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix} = \begin{pmatrix} 0 \\ \alpha \\ 0 \end{pmatrix} \text{ va } a_1 = \begin{pmatrix} 0 \\ \beta \\ \dfrac{\alpha}{2} \end{pmatrix};$$

$$\begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 2 \\ 3 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix} = \begin{pmatrix} 0 \\ \beta \\ \alpha/2 \end{pmatrix} \text{ va } a_2 = \begin{pmatrix} \alpha/6 \\ \delta \\ \beta/2 + \alpha/12 \end{pmatrix}$$

bu yerda β, δ - ixtiyoriy sonlar, $\alpha = 12$, $\beta = \delta = 0$ deb olsak $a_0 = (0,12,0)^T$, $a_1 = (0,0,6)^T$, $a_0 = (2,0,1)^T$ bo‘ladi.Demak:

$$S = \begin{pmatrix} 0 & 0 & 2 \\ 12 & 0 & 0 \\ 0 & 6 & 1 \end{pmatrix}, \; S^{-1} = \frac{1}{12}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -2 \\ -6 & 0 & 0 \end{pmatrix},$$

bu yerdan yechimning umumiy formulasiga ko‘ra,

$$x(t) = \frac{e^t}{12}\begin{pmatrix} 0 & 0 & 2 \\ 12 & 0 & 0 \\ 0 & 6 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & t & \dfrac{t^2}{2} \\ 0 & 1 & t \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -2 \\ -6 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} -c_1e^t \\ (t-3t^2)c_1e^t - c_2e^t - 2c_3e^t \\ -c_3e^t \end{pmatrix}$$

yechimni hosil qilamiz.

<u>80 – misol.</u>

$$\frac{dx}{dt} = Ax, \; A = \begin{pmatrix} 5 & -1 & -4 \\ -12 & 5 & 12 \\ 10 & -3 & -9 \end{pmatrix}$$

tenglamaning yechimini topamiz.

A matrisaning xos qiymatlari $\lambda_1 = \lambda_2 = 1, \lambda_3 = -1$ lardan iborat.Xarakteristik tenglama ildizlari 2 va 1 karrali bo‘lganligi sababli Jordan matrisasi va unga mos eksponensal matrisa

$$I = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix}, \; e^{It} = \begin{pmatrix} e^t & te^t & 0 \\ 0 & e^t & 0 \\ 0 & 0 & e^{-t} \end{pmatrix}$$

bo‘ladi.

$\lambda = 1$ xos qiymatga bitta xos vektor $a_0 = (1, 0, 1)^T$ mos keladi, unga biriktirilgan vektor $a_1 = (1, 3, 0)^T$ bo‘ladi. $\lambda = -1$ xos qiymat uchun $a_2 = (1, -2, 2)^T$.

Demak, $S = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & -2 \\ 1 & 0 & 2 \end{pmatrix}$, $S^{-1} = \begin{pmatrix} 6 & -2 & -5 \\ -2 & 1 & 2 \\ -3 & 1 & 3 \end{pmatrix}$

Bu yerdan qo‘yilgan masala yechimini topamiz.

$$x(t) = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & -2 \\ 1 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} e^t & te^t & 0 \\ 0 & e^t & 0 \\ 0 & 0 & e^{-t} \end{pmatrix} \cdot \begin{pmatrix} 6 & -2 & -5 \\ -2 & 1 & 2 \\ 3 & 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

Bir jinsimas tenglamalar sistemasini yechimini topish uchun o‘zgarmaslarni variatsiyalash usulini qo‘llash mumkin.

<u>81 – misol.</u>

$$\begin{cases} \dfrac{dx}{dt} + 2x + 4y = 1 + 4t \\ \dfrac{dy}{dt} + x - y = \dfrac{3}{2}t^2 \end{cases} \qquad (1)$$

bu sistemaga mos bir jinsli sistemaning umumiy yechimi

$$\begin{cases} x = -c_1 e^{2t} + 4c_2 e^{-3t} \\ y = c_1 e^{2t} + c_2 e^{-3t} \end{cases}.$$

Bir jinslimas sistemaning yechimini

$$\begin{cases} x = -c_1(t) e^{2t} + 4c_2(t) e^{-3t} \\ y = c_1(t) e^{2t} + c_2(t) e^{-3t} \end{cases} \qquad (2)$$

ko‘rinishda izlaymiz. Bu funksiyalarni (1) ga qo‘yib va uni soddalashtirib,

$$\begin{cases} -c_1' e^{2t} + 4c_2' e^{-3t} = 1 + 4t \\ c_1' e^{2t} + c_2' e^{-3t} = \dfrac{3}{2}t^2 \end{cases}$$

sistemani hosil qilamiz. Sistemani yechamiz va hosil bo‘lgan sodda tenglamalarni integrallaymiz:

$$c_1(t) = -\frac{1}{5}(6t^2 - 4t - 1)e^{-2t} + c_1$$

$$c_2(t) = -\frac{1}{10}(t^2 + 2t)e^{3t} + c_2$$

topilgan $c_1(t)$ va $c_2(t)$ larni (2) ga qo‘yib (1) ning umumiy yechimini topamiz.

$$\begin{cases} x = -c_1e^{2t} + 4c_2e^{-3t} + t + t^2 \\ y = c_1e^{2t} + c_2e^{-3t} - \dfrac{1}{2}t^2 \end{cases}.$$

Mustaqil yechish uchun misollar

$\exp At$ ni darajali qatorga yoyilmasidan foydalanib sistemalarning umumiy yechimini toping.

548. $\begin{cases} \dot{x} = 2x \\ \dot{y} = 3y \end{cases}$

549. $\begin{cases} \dot{x} = -y \\ \dot{y} = x \end{cases}$

550. $\begin{cases} \dot{x} = x \\ \dot{y} = x + y \end{cases}$

551. $\begin{cases} \dot{x} = -x \\ \dot{y} = x - y \\ \dot{z} = 2z \end{cases}$

552. $\begin{cases} \dot{x} = x \\ \dot{y} = -x + y + 2z \\ \dot{z} = 3x + z \end{cases}$

553. $\begin{cases} \dot{x} = x + z \\ \dot{y} = y + z \\ \dot{z} = x - y + z \end{cases}$

Tenglamalar sistemasining yechimini matrisaviy usul (Jordan matrisasiga keltirish) yordamida toping.

554. $\begin{cases} \dot{x} = x + 2y \\ \dot{y} = 2y \\ \dot{z} = -2x - 2y - z \end{cases}$

$\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = -1$

555. $\begin{cases} \dot{x} = 4x + 6y \\ \dot{y} = -3x - 5y \\ \dot{z} = -3x - 6y + z \end{cases}$

$\lambda_1 = -2, \lambda_{2,3} = 1$

556. $\begin{cases} \dot{x} = 3x + 8z \\ \dot{y} = 3x - y + 6z \\ \dot{z} = -2x - 5z \end{cases}$

$\lambda_{1,2,3} = -1$

557. $\begin{cases} \dot{x} = -4x + 2y + 10z \\ \dot{y} = -4x + 3y + 7z \\ \dot{z} = -3x + y + 7z \end{cases}$

$\lambda_{1,2,3} = 2$

558. $\begin{cases} \dot{x} = 3y + 3z \\ \dot{y} = -x + 8y + 6z \\ \dot{z} = 2x - 14y - 10z \end{cases}$

$\lambda_{1,2} = -1, \lambda_3 = 0$

559. $\begin{cases} \dot{x} = x - y \\ \dot{y} = x + 3y \end{cases}$

25-§. O‘ZGARMASLARNI VARIATSIYALASH USULI.

Bir jinsimas tenglamalar sistemasini yechimini topish uchun o‘zgarmaslarni variatsiyalash usulini qo‘llash mumkin.

81 – misol..

$$\begin{cases} \dfrac{dx}{dt} + 2x + 4y = 1 + 4t \\ \dfrac{dy}{dt} + x - y = \dfrac{3}{2}t^2 \end{cases} \qquad (1)$$

bu sistemaga mos bir jinsli sistemaning umumiy yechimi

$$\begin{cases} x = -c_1e^{2t} + 4c_2e^{-3t} \\ y = c_1e^{2t} + c_2e^{-3t} \end{cases} .$$

Bir jinslimas sistemaning yechimini

$$\begin{cases} x = -c_1(t)e^{2t} + 4c_2(t)e^{-3t} \\ y = c_1(t)e^{2t} + c_2(t)e^{-3t} \end{cases} \qquad (2)$$

ko‘rinishda izlaymiz. Bu funksiyalarni (1) ga qo‘yib va uni soddalashtirib,

$$\begin{cases} -c_1'e^{2t} + 4c_2'e^{-3t} = 1 + 4t \\ c_1'e^{2t} + c_2'e^{-3t} = \dfrac{3}{2}t^2 \end{cases}$$

sistemani hosil qilamiz. Sistemani yechamiz va hosil bo‘lgan sodda tenglamalarni integrallaymiz:

$$c_1(t) = -\frac{1}{5}(6t^2 - 4t - 1)e^{-2t} + c_1$$

$$c_2(t) = -\frac{1}{10}(t^2 + 2t)e^{3t} + c_2$$

topilgan $c_1(t)$ va $c_2(t)$ larni (2) ga qo‘yib (1) ning umumiy yechimini topamiz.

$$\begin{cases} x = -c_1e^{2t} + 4c_2e^{-3t} + t + t^2 \\ y = c_1e^{2t} + c_2e^{-3t} - \dfrac{1}{2}t^2 \end{cases} .$$

Tenglama yechimini **Meple** dasturi yordamida tekshiramiz.

```
> sys:=diff(x(t),t)+2*x(t)+4*y(t)=1+4*t,
diff(y(t),t)+x(t)-y(t)=(3/2)*t^2:
>  dsolve({sys},{x(t),y(t)});
```

$$\{ y(t) = \frac{1}{4}\mathbf{e}^{(-3\,t)}\,\_C2 - \mathbf{e}^{(2\,t)}\,\_C1 - \frac{t^2}{2},\ x(t) = \mathbf{e}^{(-3\,t)}\,\_C2 + \mathbf{e}^{(2\,t)}\,\_C1 + t + t^2 \}$$

Mustaqil yechish uchun misollar

O‘zgarmaslarni variasiyalash usulini qo‘llab quyidagi sistemalarning umumiy yechimini toping va **Meple** dasturi yordamida natijani tekshiring.

560. $\begin{cases} \dot{x} = y + tg^2 t - 1 \\ \dot{y} = -x + tgt \end{cases}$

$-\frac{\pi}{2} < t < \frac{\pi}{2}$

561. $\begin{cases} \dot{x} = -2x + 2y - e^{2t} \\ \dot{y} = -x + y + 6e^{2t} \end{cases}$

$x \in R$

562. $\begin{cases} \dot{x} = y \\ \dot{y} = -x + \dfrac{1}{\cos t} \end{cases}$

$I \in (\frac{\pi}{2}, \frac{3\pi}{2})$

563. $\begin{cases} \dot{x} = y + \dfrac{1}{t} \\ \dot{y} = -x \end{cases}$

564. $\begin{cases} \dot{x} = x + y - e^t \\ \dot{y} = y + te^t \\ \dot{z} = z + e^t \end{cases}$

$\lambda_{1,2,3} = 1$

7 - BOB.
MAXSUS NUQTALAR

26 - §. CHIZIQLI BIR JINSI SISTEMANING MAXSUS NUQTASI

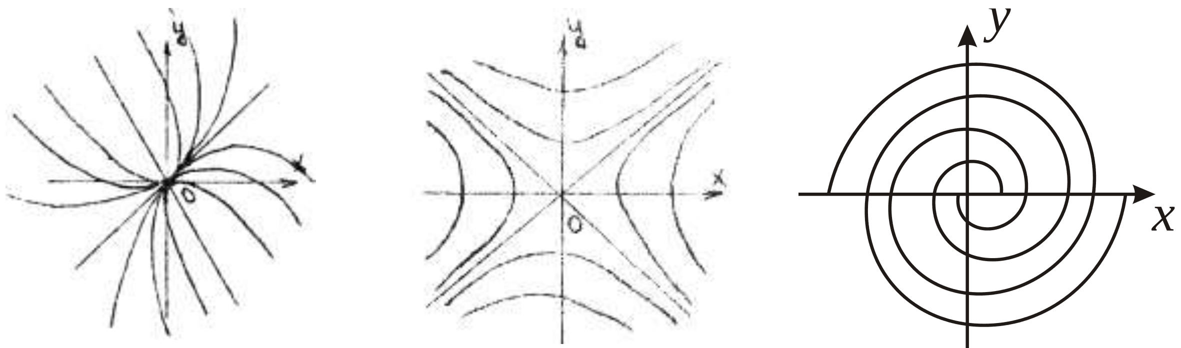
Chiziqli birjinsli tenglamalar sistemasi

$$\begin{cases} \dfrac{dx}{dt} = ax + by \\ \dfrac{dy}{dt} = cx + ey \end{cases} \qquad (1)$$

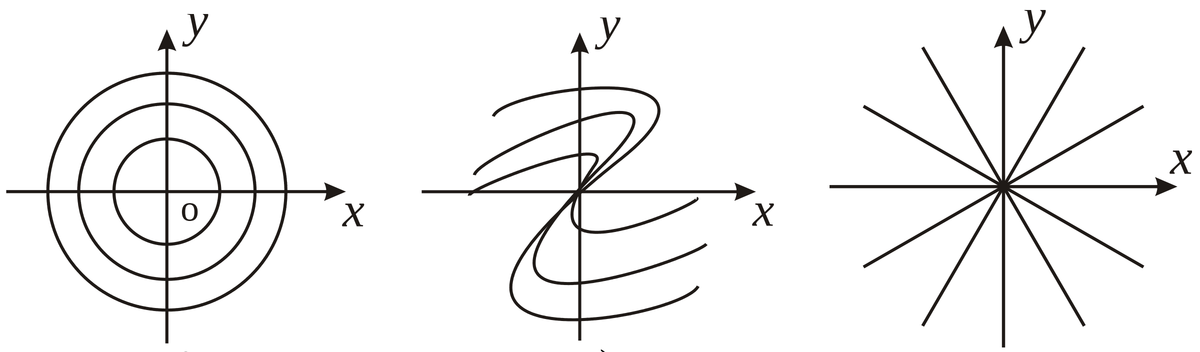
da yoki $\dfrac{dy}{dx} = \dfrac{cx+ey}{ax+by}$ tenglamada $\det A = \begin{vmatrix} a & b \\ c & e \end{vmatrix} \neq 0$ bo‘lsa, (0,0) nuqta yakkalangan maxsus nuqta bo‘ladi. Bu nuqtaning tipi $A = \begin{pmatrix} a & b \\ c & e \end{pmatrix}$ matrisaning λ_1, λ_2 xos qiymatlariga bog‘liq:

1) λ_1, λ_2 haqiqiy va har xil ishorali $\Leftrightarrow$ egar (b);
2) λ_1, λ_2 haqiqiy va bir xil ishorali ($\lambda_1 \neq \lambda_2$)$\Leftrightarrow$ tugun (a);
3) $\lambda_1 = \lambda_2$ va *A* diagonal matrisa emas $\Leftrightarrow$ tug‘ma tugun (c);
4) $\lambda_1 = \lambda_2$ va A diagonal matrisa $\Leftrightarrow$ dikrektik tugun (d);
5) $\lambda_{1,2} = \alpha \pm \beta i \ (\alpha \neq 0)$ $\Leftrightarrow$ fokus (e);
6) $\lambda_{1,2} = \pm \beta i$ $\Leftrightarrow$ markaz (f);

Agar $\det A = 0$ bo‘lsa, (1) sistema $y = kx$ ko‘rinishdagi maxsus yechimga ega bo‘ladi.



a) *b*) *e*)



f) *c*) *d*)

<u>82-misol.</u> $\frac{dy}{dx}=\frac{-2x+y}{-x+2y}$ tenglamaning (0,0) maxsus nuqtasi tipini aniqlang.

Tenglama mos sistema matrisasi $A=\begin{pmatrix}-1 & 2\\ -2 & 1\end{pmatrix}$ bo‘ladi, uning xos qiymatlari $A-\lambda I=0$, ya’ni $\begin{vmatrix}-1-\lambda & 2\\ -2 & 1-\lambda\end{vmatrix}=0$ tenglamadan topiladi.

$$\lambda^2+3=0$$

$$\lambda_{1,2}=\pm i\sqrt{3}\,.$$

Demak, (0,0) nuqta markaz tipidagi maxsus nuqta

Chiziqli bo‘lmagan tenglamalar sistemasini maxsus nuqtalari

$$\begin{cases}\dfrac{dx}{dt}=P(x,y)\\[2mm] \dfrac{dy}{dt}=Q(x,y)\end{cases} \qquad (2)$$

sistemaning maxsus nuqtalari

$$\begin{cases}P(x,y)=0\\ Q(x,y)=0\end{cases}$$

Algebraik tenglamalar sistemasining yechimlari bo‘ladi. Bu nuqtalarni tipini aniqlash uchun

$$\begin{pmatrix}\dfrac{\partial P}{\partial x} & \dfrac{\partial P}{\partial y}\\[2mm] \dfrac{\partial Q}{\partial x} & \dfrac{\partial Q}{\partial y}\end{pmatrix}$$

matrisani tuzamiz va tekshirilayotgan nuqta koordinatalarini matrisaga qo‘yib, sonli matrisa hosil qilamiz.Bu matrisa (2) sistemaning tekshirilayotgan maxsus nuqta atrofidagi chiziqlashtirilgan sistemaning matrisasi bo‘ladi. Tekshirilayotgan maxsus nuqta tipi chiziqlashtirilgan sistemaning markaz tipidagi maxsus nuqtadan farqli nuqtaning tipi bilan bir hil bo‘ladi. Chiziqlashtirilgan sistemaning markaz tipidagi nuqta bo‘lganda, tekshirilayotgan maxsus nuqta markaz yoki fokus tipida bo‘lishi mumkin va masalani oydinlashtirish uchun qo‘shimcha izlanishlar o‘tkazish kerak.

83 – misol. $\begin{cases} \dot{x} = x^2 - y = 0 \\ \dot{y} = \ln(1 - x + x^2) - \ln 3 = 0 \end{cases}$

sistemaning maxsus nuqtalarni aniqlaymiz.

$$\begin{cases} x^2 - y = 0 \\ \ln(1 - x + x^2) - \ln 3 = 0 \end{cases}$$

bundan $1 - x + x^2 = 3,\ x^2 - x - 2 = 0\ x_1 = -1, x_2 = 2\ y_1 = 1,\ y_2 = 4$.

Bu sistema ikkita $(-1,1),(2;4)$ maxsus nuqtaga ega.

$$\begin{pmatrix} \frac{\partial P}{\partial x} & \frac{\partial P}{\partial y} \\ \frac{\partial Q}{\partial x} & \frac{\partial Q}{\partial y} \end{pmatrix} = \begin{pmatrix} 2x & -1 \\ \frac{-1+2x}{1-x+x^2} & 0 \end{pmatrix}$$

matrisa yordamida nuqtalar tipini aniqlaymiz (-1,1) – nuqta uchun $A = \begin{pmatrix} -2 & -1 \\ -1 & 0 \end{pmatrix}$, bu matrisaning xos quymatlari $\begin{vmatrix} -2-\lambda & -1 \\ -1 & -\lambda \end{vmatrix} = 0$ tenglamadan aniqlanadi: $\lambda^2 + 2\lambda - 1 = 0, \lambda_{1,2} = -1 \pm \sqrt{2}$ va $\lambda_1 \cdot \lambda_2 < 0,$ demak, (-1,1) – egar tipidagi maxsus nuqta.

(2,4) – nuqta uchun $\begin{pmatrix} 4 & -1 \\ 1 & 0 \end{pmatrix}$ matrisaning xos quymatlari $\lambda^2 - 4\lambda + 1 = 0$ tenglamaning ildizlaridir. Bundan $\lambda_{1,2} = 2 \pm \sqrt{3}$ va $\lambda_1 \cdot \lambda_2 < 0,$ demak, (2,4) – tugun tipidagi maxsus nuqta.

Mustaqil yechish uchun misollar

Tenglama va tenglamalar sistemasining *(0,0)* maxsus nuqtasining tipini aniqlang.

565. $y' = \frac{-x+y}{-2x+2y}$

566. $y' = \frac{x+y}{x}$

567. $y' = \frac{-x+y}{-4x-2y}$

568. $\begin{cases} \dot{x} = 2x \\ \dot{y} = 2y \end{cases}$

569. $\begin{cases} \dot{x} = y \\ \dot{y} = 2x \end{cases}$

570. $\begin{cases} \dot{x} = -2x + y \\ \dot{y} = -x + 2y \end{cases}$

571. $\begin{cases} \dot{x} = -2x + y \\ \dot{y} = -x - y \end{cases}$

572. $\begin{cases} \dot{x} = 2x - 2y \\ \dot{y} = x - y \end{cases}$

573. $\begin{cases} \dot{x} = 2x + y \\ \dot{y} = 3x + 4y \end{cases}$

Tenglama va tenglamalar sistemasining maxsus nuqtalarini toping va ularning tipini aniqlang

574. $y' = \dfrac{4y^2 - x^2}{2xy - 4y - 8}$

575. $y' = \dfrac{2y}{x^2 - y^2 - 1}$

576. $y' = \dfrac{x^2 + y^2 - 2}{x - y}$

577. $y' = \dfrac{y + \sqrt{1 + 2x^2}}{x + y + 1}$

578. $\begin{cases} \dot{x} = (2x - y)(x - 2) \\ \dot{y} = xy - 2 \end{cases}$

579. $\begin{cases} \dot{x} = x^2 - y \\ \dot{y} = x^2 - (y-2)^2 \end{cases}$

580. $\begin{cases} \dot{x} = (x + y)^2 - 1 \\ \dot{y} = -y^2 - x + 1 \end{cases}$

581. $\begin{cases} \dot{x} = (2x - y)^2 - 9 \\ \dot{y} = (x - 2y)^2 - 9 \end{cases}$

27-§. TENGLAMALAR SISTEMASINING YECHIMINI TURG‘UNLIGINI TEKSHIRISH

$$\frac{dx_i}{dt} = f_i(x_1, x_2, ..., x_n, t) \ (i = \overline{1,n}) \tag{3}$$

Sistema uchun $\varphi(t_0) = \varphi_{i0} (i = \overline{1,n})$ boshlang‘ich shartni qanoatlantiruvchi $\varphi_i(t) \ (i = \overline{1,n})$ yechim Lyapunov bo‘yicha turg‘un deyiladi, agar ixtiyoriy $\varepsilon > 0$ uchun $\delta(\varepsilon) > 0$ mavjud bo‘lib, (3) sistemaning barcha $x_i(t) \ (i = \overline{1,n})$ yechimlari uchun $|x_i(t_0) - \varphi_{i0}| < \delta \ (i = \overline{1,n})$ bo‘lganda barcha $t \geq t_0$ larda

$$|x_i(t) - \varphi_i(t)| < \varepsilon \ \ (i = \overline{1,n}) \tag{4}$$

o‘rinli bo‘lsa.

Agar yetarlicha kichik $\delta > 0$ uchun birorta $x_i(t) \ (i = \overline{1,n})$ yechim uchun (4) shart bajarilmasa $\varphi_i(t)$ turg‘unmas deyiladi.

Agar turg‘un yechim uchun $\lim\limits_{t \to \infty} |x_i(t) - \varphi_i(t)| = 0 \ (i = \overline{1,n})$ shart bajarilsa, bunday yechim asimptotik turg‘un deyiladi.

Umuman tenglamalar sistemasi yechimining turg‘unligini tekshirish masalasini uning nol yechimi turg‘unligini tekshirish masalasiga keltirish mumkin. Nol yechimini tekshirish Lyapunov usullari yordamida bajariladi.

A) Lyapunovning I-chi usuli.

Bu usul bo‘yicha agar chiziqli sistema matrisasining barcha xos qiymatlarining haqiqiy qismi manfiy bo‘lsa, sistema nol yechimi asimptotik turg‘un bo‘ladi.

Agar birorta xos qiymatning haqiqiy qismi musbat bo‘lsa, nol yechim turg‘unmas bo‘ladi.

84-misol. vektor tenglamaning nol yechimini turg‘unligini tekshiring, bunda

$$A=\begin{pmatrix} 3 & 0 & 8 \\ 3 & -1 & 6 \\ -2 & 0 & -5 \end{pmatrix}.$$

Matrisaning xos quymatlarini $\det(A-\lambda E)=0$ tenglamadan topamiz.

$$\begin{vmatrix} 8-\lambda & 0 & 8 \\ 3 & -1-\lambda & 6 \\ -2 & 0 & -5-\lambda \end{vmatrix}=0,\ (\lambda^3+3)^2+3\lambda+1=0 \text{ bundan } \lambda_{1,2,3}=-1.$$

Xos qiymatlar manfiy, demak, nol yechim asimptotik turg‘un.

85-misol. α,β parametlarning qanday qiymatlarida $\frac{dx}{dt}=Ax$ vektor tenglamaning nol yechimi asimptotik turg‘un bo‘ladi, bunda

$$A=\begin{pmatrix} -1 & \alpha & 0 \\ \beta & -1 & \alpha \\ 0 & \beta & -1 \end{pmatrix}.$$

A matrisaning xarakteristik tenglamasi $(1+\lambda)^3-2\alpha\beta(1+\lambda)=0$.

Bundan $\lambda_1=-1,\ va\ \lambda_2,\lambda_3 lar \begin{cases} \lambda_2+\lambda_3=-2 \\ \lambda_2\cdot\lambda_3=1-2\alpha\beta \end{cases}$ shartlarni qanoatlantiradi.

Nol yechim asimptotik turg‘un bo‘lishi uchun $\lambda_2,\lambda_3>0$ shart bajarilishi kifoya. Demak, $\alpha\beta<\frac{1}{2}$ bo‘lganda nol yechim asimptotik turg‘un bo‘ladi.

Ko‘phadning barcha nollarini haqiqiy qismini manfiyligini Raus – Gurbis shartlari yordamida aniqlash mumkin.

$$\lambda^n+a_{n-1}\lambda^{n-1}+...+a_1\lambda+a_0$$

ko‘phad koeffisiyentlari yordamida Gurvis matrisasini tuzamiz

$$\begin{pmatrix} a_{n-1} & 1 & 0 & 0 & \dots & 0 \\ a_{n-3} & a_{n-2} & a_{n-1} & 1 & \dots & 0 \\ a_{n-5} & a_{n-4} & a_{n-3} & a_{n-2} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & a_0 \end{pmatrix}.$$

Ko‘phadning barcha nollarining haqiqiy qismi manfiy bo‘lishi uchun Gurvis matrisasining barcha bosh minorlari musbat bo‘lishi zarur va yetarlidir.

Izoh: Chiziqli bo‘lmagan tenglamalar sistemasini koordinata boshi atrofida chiziqlashtirilgan sistemasini tuzish uchun sistemaning o‘ng tomonidagi funksiyalarni Makloren qatoriga yoyish kerak.

86–misol. $\frac{dx}{dt} = Ax$ vektor tenglama nol yechimining asimptotik turg‘unligi tekshiring, bunda

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & 2 \end{pmatrix}.$$

A matrisaning xarakteristik tenglamasi

$$\lambda^3 + 2\lambda^2 + 2\lambda + 3 = 0 .$$

Gurvis matrisasini tuzamiz:

$$\begin{pmatrix} 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 2 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 2 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 2 & 2 & 1 \end{pmatrix}.$$

Bu matrisaningg bosh minorlari $M_1 = 2 > 0$, $M_2 = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 1 > 0$

$$M_3 = \begin{vmatrix} 2 & 1 & 0 \\ 3 & 2 & 2 \\ 0 & 0 & 3 \end{vmatrix} = 3 \cdot M_1 > 0 .$$

Demak, nol yechim asimptotik turg‘un.

B) Lyapunovning 2-chi usuli.

Agar sistema matrisasining birorta xos qiymatining haqiqiy qismi nolga teng bo‘lsa, Lyapunovning 1-chi usuli yordamida yechimni turg‘unligini aniqlab bo‘lmaydi. Bu holda Lyapunovning 2-chi usulini qo‘llash qulay.

Bu usul bo‘yicha berilgan sistema uchun shunday funksiya topilib, u *V(0)=0* va musbat (manfiy) aniqlangan va sistema bo‘yicha olingan to‘liq differensiali manfiy (musbat) aniqlangan bo‘lsa, u holda sistemaning nol yechimi asimptotik turg‘un bo‘ladi.

Bunday *V* funksiyaga Lyapunov funksiyasi deyiladi.

Agar *V* funksiya musbat (manfiy) ishorali yoki uning sistema bo‘yicha olingan to‘liq differensiali manfiy (musbat) ishorali bo‘lsa, u holda sistemaning nol yechimi turg‘un bo‘ladi.

<u>87 – misol.</u>

$$\begin{cases} \dfrac{dx}{dt} = y \\ \dfrac{dy}{dt} = -3(x - \dfrac{x^3}{3} + ...) \end{cases}$$

demak, koordinata boshi atrofida chiziqlashtirilgan sistema

$$\begin{cases} \dfrac{dx}{dt} = y \\ \dfrac{dy}{dt} = -3x \end{cases}$$

bo‘lib, uning matrisasi $\begin{pmatrix} 0 & 1 \\ -3 & 0 \end{pmatrix}$ va xos qiymatlari $\lambda_{1,2} = \pm i\sqrt{3}$, ya’ni, haqiqiy qismi nolga teng. Demak, Lyapunovning 1-chi usuli yordamida turg‘unlikni aniqlab bo‘lmaydi. Lyapunovning 2-chi usulini qo‘llaymiz. Lyapunov funksiyasini $V(x,y) = \dfrac{y^2}{2} + 3(1-\cos x)$ shaklida olamiz. *V (0,0)=0* va nol nuqta atrofida musbat.

$$\frac{dv}{dt} = \frac{\partial v}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial v}{\partial y}\cdot\frac{dy}{dt} = 3y\sin x - 3y\sin x \equiv 0,$$

demak, sistemaning nol yechimi turg‘un bo‘ladi.

Mustaqil yechish uchun misollar

Lyapunovning 1-chi usuli yordamida sistemaning nol yechimini asimptotik turg‘unligini tekshiring.

582. $\begin{cases} \dot{x} = y + x^2 y + y^3 \\ \dot{y} = x - 4y^5 \end{cases}$ 583. $\begin{cases} \dot{x} = -y + x^2\cos y \\ \dot{y} = 3x - 4y \end{cases}$

584. $\begin{cases} \dot{x} = -y + \sin x^2 \\ \dot{y} = 4x - 4y + \sin^2 y \end{cases}$ 585. $\begin{cases} \dot{x} = -\sin(y+z) \\ \dot{y} = -x - z + z^3 \\ \dot{z} = -x - y - 2yx \end{cases}$

586. $\begin{cases} \dot{x} = 3x + 8z = y^2 - z^3 \\ \dot{y} = 3x - y + 6z \\ \dot{z} = -2x - 5z + x^5 \end{cases}$ 587. $\begin{cases} \dot{x} = -\sin x + z \\ \dot{y} = 2y \\ \dot{z} = -x - 3z \end{cases}$

Paramatrlarining qanday qiymatlarida sistemaning nol yechimi asimptotik turg‘un bo‘ladi.

588. $\begin{cases} \dot{x} = ax + y + 5y^2 \\ \dot{y} = -e^x + e^{ax} \end{cases}$ 589. $\begin{cases} \dot{x} = ax \\ \dot{y} = bx - 3tgy \end{cases}$ 590. $\begin{cases} \dot{x} = -tgx - tgy \\ \dot{y} = ax - a^2 y \end{cases}$

591. $\begin{cases} \dot{x} = y - 7y^2x^3 \\ \dot{y} = z + y^2 + 3x^3 \\ \dot{z} = -2x - by - az \end{cases}$

592. $\begin{cases} \dot{x} = ax + z \\ \dot{y} = \sin ay \\ \dot{z} = ax + az \end{cases}$ 593. $\begin{cases} \dot{x} = -x + ay + bz \\ \dot{y} = -ax - y - az - \cos^2 x + \cos z \\ \dot{z} = -bx - ay - z \end{cases}$

Raus – Gurvis sharti yordamida tenglamalarning nol yechimini asimptotik turg‘unligini tekshiring.

594. $y''' + 2y'' + 2y' + 3y = 0$ 595. $y^{IV} + 2y''' + 4y'' + 7y' + 2y = 0$

596. $y^{IV} + 2y''' + 6y'' - y = 0$ 597. $y^{IV} + 2y''' + 3y'' + 7y' + 2y = 0$

598. $y''' - 2y'' + 2y' - 3y = 0$ 599. $y^{IV} + 2y''' + 6y'' + 5y' + 6y = 0$

a va b paramatrning qanday qiymatlarida tenglamalarning nol yechimi asimptotik turg‘un bo‘ladi.

600. $y''' + ay'' + by' + 2y = 0$ 601. $y''' + 3y'' + ay' + by = 0$

602. $y^{IV} + ay''' + y'' + 2y' + y = 0$ 603. $y^{IV} + y''' + ay'' + y' + by = 0$

604. $y^{IV} + 2y''' + 3y'' + 2y' + ay = 0$

605. $y^{IV} + ay''' + 4y'' + 2y' + by = 0$

Ko‘rsatilgal Lyapunov funksiyasi yordamida sistemaning nol yechimini asimptotik turg‘unligini tekshiring.

606. $\begin{cases} \dot{x} = -xe^x - y \\ \dot{y} = x^3 \end{cases}$ 607.

$V = x^4 + 2y^4$

$\begin{cases} \dot{x} = -f(x) - y^3 \\ \dot{y} = x^3 - \varphi(y) \end{cases}$ $f(0) = \varphi(0) = 0$

$V = x^4 + y^4$ $zf(z) > 0, z\varphi(z) > 0$

608. $\begin{cases} \dot{x} = f(x) + y^5 \\ \dot{y} = -x^5 - \varphi(y) \end{cases}$ $f(0) = \varphi(0) = 0$

$V = x^6 + y^6$ $zf(z) > 0, z\varphi(z) > 0$

609. $\begin{cases} \dot{x} = xy - x^3 + y \\ \dot{y} = x^4 - x^2 y - x^3 \end{cases}$

$V = x^6 + 2y^2$

610. $\begin{cases} \dot{x} = -x^3 - y^3 \\ \dot{y} = x^3 - y^3 \end{cases}$

$V = x^4 + y^4$

611. $\begin{cases} \dot{x} = -xe^x - y^3 \\ \dot{y} = x^3 - y \end{cases}$

$V = x^4 + y^4$

612. $\begin{cases} \dot{x} = -xe^x + y^5 \\ \dot{y} = -x^5 - y^3 \end{cases}$

$V = x^6 + y^6$

613. $\begin{cases} \dot{x} = -x^3 - y^3 \\ \dot{y} = x^3 - y^5 \end{cases}$

$V = x^4 + y^4$

8 - BOB.
CHIZIQLI BO‘LMAGAN TENGLAMALAR SISTEMASI. XUSUSIY HOSILALI DIFFERENSIAL TENGLAMALAR

28-§. SIMMETRIK SISTEMALAR

Chiziqli bo‘lmagan tenglamalar sistemasini noma’lumlarini yo‘qotish usuli yordamida yuqori tartibli bir o‘zgaruvchili tenglamaga keltirib yechish mumkin. Bu usulni murrakab bo‘lmagan sistemalarni yechish uchun qo‘llash mumkin.

Umumiy holda sistemalarni integrallanuvchi kombinasiyalarini izlash yordamida yechimini topish qulay. Integrallanuvchi kombinasiya – bu berilgan sistemaning tenglamalaridan tuzilgan ikki o‘zgaruvchiga bog‘liq bo‘lgan va ikkala tomoni to‘liq differensialdan iborat tenglamadir.

Integrallanuvchi kombinasiyalarni aniqlash uchun sistemani simmetrik shaklda yozish qulay, ya’ni

$$\frac{dx_1}{f_1(t,x_1,...,x_n)} = \frac{dx_2}{f_2(t,x_1,...,x_n)} = ... = \frac{dx_n}{f_n(t,x_1,...,x_n)} = \frac{dt}{f\left(t,x_1,...,x_n\right)} \quad (1)$$

Agar $f(t,x_1,...,x_n) \neq 0$ bo‘lsa, bu sistema

$$\frac{dx_1}{dt} = \frac{f_1(t,x_1,...,x_n)}{f(t,x_1,...,x_n)}, ..., \frac{dx_n}{dt} = \frac{f_n(t,x_1,...,x_n)}{f(t,x_1,...,x_n)}$$

sistemaga teng kuchli bo‘ladi.

88-misol. $\frac{dx}{xy} = \frac{dy}{x^2z^2} = \frac{dz}{yz}$ tenglamalar sistemasini yeching.

Bitta integrallanuvchi kombinasiya:

$$\frac{dx}{xy} = \frac{dz}{yz},\ \frac{dx}{x} = \frac{dz}{z} \quad \Rightarrow \quad x = c_1 z.$$

Bu tenglikdan foydalanib,

$$\frac{dy}{x^2z^2} = \frac{dz}{yz}$$

tenglamadan *x* ni yo‘qotish mumkin, ya’ni,

$$\frac{dy}{c_1{}^2 z^4} = \frac{dz}{yz},\ ydy = c_1^2 z^3 dz$$

Oxirgi tenglamani integrallab, $\frac{y^2}{2} = c_1^2 \frac{z^4}{4} + c_2$ ni olamiz.

Demak, sistemaning umumiy yechimi $x = c_1 z$, $y = \pm\sqrt{c_1^2 \frac{z^4}{2} + 2c_2}$ bo‘ladi.

Bu misolni yechishda x - ni yo‘qotmasdan boshqa integrallanuvchi kombinasiya tuzish mumkin, berilgan simmetrik sistemaning birinchi tenglamasini z ga uchinchisini esa x - ga ko‘paytirib, proporsiya xossasidan foydalansak,

$$\frac{zdx+xdz}{zxy+xyz}=\frac{dy}{(xz)^2}$$

tenglamaga kelamiz, bundan

$$\frac{d(xz)}{2y}=\frac{dy}{(xz)},$$

$$xzd(xz)=2ydy,$$

$$\frac{(xz)^2}{2}=y^2+c_2.$$

Demak, bu holda sistemaning $\frac{x}{z}=c_1$, $\frac{x^2z^2}{2}-y^2=c_2$ birinchi integrallarini hosil qilamiz.

Mustaqil yechish uchun misollar

Tenglamalar sistemasining yechimini toping,

614. $y'=\frac{x}{z},\ z'=-\frac{x}{y}$

615. $y'=\frac{y^2}{z-x},\ z'=y+1$

616. $\frac{dx}{2y-z}=\frac{dx}{y}=\frac{dz}{z}$

617. $\frac{dx}{y}=\frac{dx}{y}=\frac{dz}{z}$

618. $\frac{dx}{y+z}=\frac{dy}{x+z}=\frac{dz}{x+y}$

619. $\frac{dx}{z}=\frac{dy}{u}=\frac{dz}{x}=\frac{du}{y}$

620. $\frac{dx}{z^2-x^2}=\frac{dy}{z}=-\frac{dz}{y}$

621. $\frac{dx}{x}=\frac{dy}{y}=\frac{dz}{xy+z}$

622. $\frac{dx}{xz}=\frac{dy}{yz}=\frac{dz}{xy\sqrt{z^2+1}}$

623. $\frac{dx}{x+y^2+z^2}=\frac{dy}{y}=\frac{dz}{z}$

29-§. BIRINCHI TARTIBLI XUSUSIY HOSILALI CHIZIQLI DIFFERENSIAL TENGLAMALAR

$$f_1\frac{\partial u}{\partial x_1}+\ldots+f_n\frac{\partial u}{\partial x_n}=\Phi \qquad (1)$$

ko‘rinishdagi tenglamaga xususiy hosilali birinchi tartibli tenglama deyiladi. Bu yerda $f_1,\ldots,f_n,\Phi$ lar $x_1,\ldots,x_n,u$ ga bog‘liq funksiyalar, $u=u(x_1,\ldots,x_n)$ izlanayotgan funksiya.

Bu tenglamani yechish uchun simmetrik ko‘rinishdagi

$$\frac{dx_1}{f_1}=...=\frac{dx_n}{f_n}=\frac{du}{\Phi} \qquad (2)$$

sistemani yozib, uni n – ta o‘zaro bog‘lanmagan birinchi integrallarini topish kerak. Bu integrallar $\varphi_i=(x_1,...,x_n,z)=c_i \quad (i=\overline{1,n})$ bo‘lsa, u holda (1) tenglamani umumiy

yechimi $F(\varphi_1,...,\varphi_n)=0$ ko‘rinishda bo‘lib, bu yerda F ixtiyoriy differensiallanuvchi funksiya.

(1) tenglamaning

$$u(x_1,...,x_{i-1},a,x_{i+1},...,x_n)=\varphi(x_1,...,x_{i-1},x_{i+1},...,x_n)$$

shartlarni qanoatlantiruvchi yechimini topishni masalasi Koshi masalasi deyiladi, bu yerda a – berilgan o‘zgarmas va $\varphi(x_1,...,x_{i-1},x_{i+1},...,x_n)$ berilgan funksiya.

<u>89– misol.</u> $u\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0$ tenglamaning umumiy yechimini toping.

Mos simmetrik sistema tuzamiz

$$\frac{dx}{u}=\frac{dy}{1}=\frac{du}{0}.$$

Bundan $u=c_1, x-uy=c_2$ birinchi integrallarni topamiz. Demak, $F(u,x-uy)=0$ berilgan tenglamaning umumiy yechimi bo‘ladi.

<u>90–misol.</u> $(y+u)^2\frac{\partial u}{\partial x}-x(y+2u)\frac{\partial u}{\partial y}=xu$ tenglamani $u(x,y)\big|_{y=0}=x^2$ shartni qanoatlantiruvchi yechimini toping.

Mos simmetrik sistema

$$\frac{dx}{(y+u)^2}=\frac{dy}{-x(y+2u)}=\frac{du}{xu}.$$

Bundan

$$\frac{dy}{-x(y+2u)}=\frac{du}{xu}$$

va

$$\frac{d(y+u)}{-x(y+2u)+xu}=\frac{dx}{(y+u)^2}$$

integrallanuvchi kombinasiyalar olib,

$$\begin{cases}(y+2u)u=c_1\\(y+u)^2+x^2=c_2\end{cases}$$

birinchi integrallarni topamiz, shartga asosan $y=0$

$$\begin{cases} 2u^2 = c_1 \\ u^2 + x^2 = c_2 \end{cases},$$

bundan

$$\begin{cases} x^2 = c_2 - \frac{1}{2}c_1 \\ u = \sqrt{\frac{1}{2}c_1} \end{cases}$$

$a = x^2$ ga asosan $\sqrt{\frac{1}{2}c_1} = c_2 - \frac{1}{2}c_1$ bog‘lanishini topamiz.

Demak, qo‘yilgan masalaning yechimi

$\sqrt{\frac{1}{2}(y+u)u} = (y+u)^2 + x^2 - \frac{1}{2}(y+u)u$ bo‘ladi.

Mustaqil yechish uchun misollar

Tenglamalarni umumiy yechimini toping.

624. $y\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} = 0$

625. $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$

626. $yz\frac{\partial u}{\partial x} + xz\frac{\partial u}{\partial y} + xy\frac{\partial u}{\partial z} = 0$

627. $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = x - y$

628. $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = xy + u$

629. $y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = u$

630. $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u - x^2 - y^2$

631. $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = u + \frac{xy}{z}$

632. $y\frac{\partial u}{\partial x} = u$

Koshi masalasini yeching.

633. $(4y - z)\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$, $u\big|_{y=0} = y^2 + z^2$

634. $xz\frac{\partial u}{\partial x} + yz\frac{\partial u}{\partial y} + xy\frac{\partial u}{\partial z} = 0$, $u\big|_{z=0} = xy$

635. $x(z-y)\frac{\partial u}{\partial x} + y(y-x)\frac{\partial u}{\partial y} + (y^2 - xz)\frac{\partial u}{\partial z} = 0$ $u\big|_{x=1} = \frac{z}{y}$

636. $x\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial y} = 0$, $u\big|_{x=1} = -y$

637. $2x^3\frac{\partial u}{\partial x} + (3x^2 + y^3)\frac{\partial u}{\partial y} = 2x^2u$ $u\big|_{x=1} = y^2$

638. $2x^3\frac{\partial u}{\partial x} + (3x^2 + y^3)\frac{\partial u}{\partial y} = 2x^2u$ $u\big|_{x=1} = 1 + \frac{1}{y^2}$