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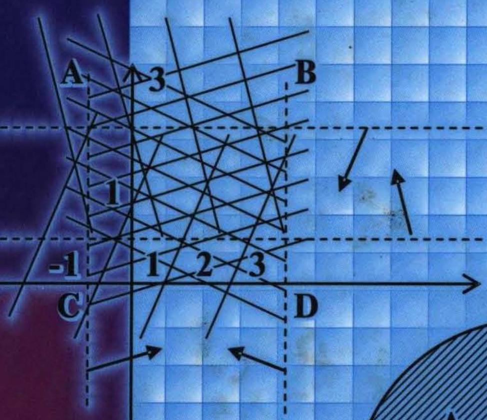

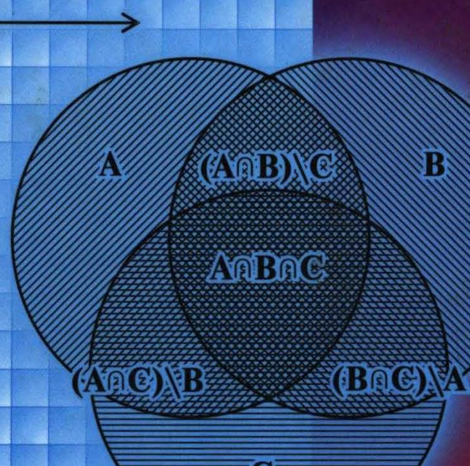

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OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

D.I. YUNUSOVA, A.S. YUNUSOV

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MODUL TEXNOLOGIYASI ASOSIDA TAYYORLANGAN MISOL VA MASHQLAR TO‘PLAMI

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O'quv qo'llanma «Algebra va sonlar nazariyasi» fani Davlat ta'lim standarti hamda dasturiga to'la mos keladi. Tavsiya etilayotgan nazariy savollar va amaliy topshiriqlar modullarda jamlangan bo'lib, ularda keltirilgan metodik tavsiyalar ushbu o'quv qo'llanmadan foydalanuvchilarga keng imkoniyatlar yaratadi.

Keltirilgan misol va mashqlar nazariy bilimlarni chuqurlashtirish, tadbiqini kengaytirishga qaratilgan bo'lib, talabalarni mustaqil, ijodiy izlanishga yo'naltiradi.

Umumiy o'rta, o'rta maxsus matematika ta'limiga ushbu fanning keng tatbiqlarini e'tiborga olsak, o'quv qo'llanmadan akademik litsey, kasb-hunar kollejlari o'qituvchilari, o'quvchilari; o'qituvchilar malakasini oshirish va qayta tayyorlash kurslari tinglovchilari foydalanishlari mumkin.

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SO'ZBOSHI

«Algebra va sonlar nazariyasi» fani pedagogika oliy o'quv yurtlarining «Matematika va informatika» yo'nalishi o'quv rejasiga kiritilgan asosiy mutaxassislik fanlaridan biri bo'lib, I–V semestrlar davomida o'qitiladi.

Mazkur o'quv qo'llanma Oliy va o'rta maxsus ta'lim vazirligi tomonidan tasdiqlangan o'quv dasturiga mos ravishda tayyorlangan bo'lib, u talabalarning egallagan nazariy bilimlari asosida amaliy ko'nikma va malakalar hosil qilishlariga qaratilgan.

O'quv qo'llanma matematik mantiq elementlari, to'plamlar va munosabatlar, algebra va algebraik sistemalar, asosiy sonli sistemalar, arifmetik vektor fazo, chiziqli tenglamalar sistemasi, matritsalar, determinantlar, vektor fazolar, chiziqli akslantirishlar, chiziqli tengsizliklar sistemasi, butun sonlar halqasida bo'linish munosabati, taqqoslamalar, ko'phadlar mavzusidagi modullardan tashkil topgan bo'lib, I–VII modullar I kursga, VIII–XII modullar II kursga va XIII modul III kursga mo'ljallangan.

O'quv qo'llanmada fan bo'limlarining modullar bo'yicha jamlanganligi talabalarning bir bo'lim yuzasidan nazariy va amaliy bilimlari orasidagi o'zaro bog'lanishlarni o'rnatish, yaxlitligini ta'minlash, ko'nikma va malakalarni takomillashtirish, umumlashtirishga imkoniyat yaratadi. Mavzular bo'yicha keltirilgan amaliy topshiriqlarni bajarishda talabalarga metodik yordam sifatida paragraflar boshida zaruriy nazariy ma'lumotlar hamda misol va masalalarning yechilish namunalari keltirilgan.

O'quv qo'llanmadan nafaqat pedagogika oliy o'quv yurtlari, balki «Algebra», «Sonlar nazariyasi», «Algebra va sonlar nazariyasi», «Algebra va geometriya» fanlari o'qitiladigan ta'lim muassasalari matematika ta'limi jarayonida foydalanish mumkin.

I MODUL. MATEMATIK MANTIQ ELEMENTLARI

1-§. Mulohaza. Mulohazalar ustida mantiq amallari

✓ **Asosiy tushunchalar:** mulohaza, rost mulohaza, yolg'on mulohaza, konyunksiya, dizyunksiya, implikatsiya, ekvivalensiya, inkor, rostlik jadvali.

Mulohaza matematik mantiqning asosiy tushunchalaridan bo'lib, u rost yoki yolg'onligi bir qiymatli aniqlanadigan darak gapdir. Masalan, «Kvadrat to'g'ri to'rtburchakdir», «2>5» kabi tasdiqlar mulohazalar bo'lib, birinchi mulohaza rost, ikkinchi mulohaza esa yolg'on mulohazadir.

Berilgan A mulohaza rost bo'lganda yolg'on, A mulohaza yolg'on bo'lganda rost bo'ladigan mulohaza A mulohazaning *inkori* deyiladi va $\rceil A$ yoki $\overline{A}$ orqali belgilanadi.

A va B mulohazalar rost bo'lgandagina rost bo'lib, qolgan hollarda yolg'on bo'ladigan mulohaza A va B mulohazalarning *konyunksiyasi* deyiladi va $A \wedge B$ yoki $A \,\&\, B$ ko'rinishda belgilanadi.

A va B mulohazalar *dizyunksiyasi* deb, A va B mulohazalarning ikkalasi ham yolg'on bo'lgandagina yolg'on, qolgan hollarda rost bo'ladigan $A \vee B$ mulohazaga aytiladi.

A va B mulohazalar *implikatsiyasi* deb, A mulohaza rost va B mulohaza yolg'on bo'lgandagina yolg'on, qolgan hollarda rost bo'ladigan $A \Rightarrow B$ mulohazaga aytiladi.

A va B mulohazalar *ekvivalensiyasi* deb, A va B mulohazalarning ikkalasi ham yolg'on yoki ikkalasi ham rost bo'lganda rost, qolgan hollarda yolg'on bo'ladigan $A \Leftrightarrow B$ mulohazaga aytiladi.

Yuqorida ta'riflangan amallar rostlik jadvali quyidagi ko'rinishda bo'ladi.

Misol. $\forall (x, y, z \in Z)(x \vdots y \wedge y \vdots z \Rightarrow x \vdots z)$ mulohazaning rost yoki yolg'onligini aniqlang.

Yechish. Berilgan mulohaza konyunksiya hamda implikatsiya amallari yordamida hosil qilingan. Bu mantiq amallarining ta'rif-

A	B	$\rceil A$	$A \wedge B$	$A \vee B$	$A \Rightarrow B$	$A \Leftrightarrow B$
1	1	0	1	1	1	1
1	0	0	0	1	0	0
0	1	1	0	1	1	0
0	0	1	0	0	1	1

lariga ko'ra qaralayotgan $x \vdots y \wedge y \vdots z \Rightarrow x \vdots z$ mulohaza $x \vdots y \wedge y \vdots z$ rost va $x \vdots z$ yolg'on bo'lganda yolg'on, boshqa hollarda rost. Har bir mulohazaning rostlik qiymatini aniqlaymiz.

$x \vdots y$ predikat butun sonlar to'plamida olingan har qanday (x, y) juftlikda rost mulohaza bo'lmaydi. Masalan, $x = 1$, $y = 2$.

$y \vdots z$ predikat butun sonlar to'plamida olingan har qanday (y, z) juftlikda rost mulohaza bo'lmaydi. Masalan, $y = 2$, $z = 3$.

$x \vdots z$ predikat butun sonlar to'plamida olingan har qanday (x, z) juftlikda rost mulohaza bo'lmaydi. Masalan, $x = 1$, $z = 3$.

Quyidagi holatlarni qarab chiqamiz:

1) $\forall(x,y,z \in N)(x \vdots y \wedge y \vdots z \Rightarrow x \vdots z)$ mulohazadagi $\forall(x,y,z \in N)(x \vdots y)$ mulohaza yolg'on. U holda konyunksiya va implikatsiya amallari ta'rifiga ko'ra $\forall(x,y,z \in N)(x \vdots y \wedge y \vdots z \Rightarrow x \vdots z)$ mulohaza rost.

2) $\forall(x,y,z \in N)(x \vdots y \wedge y \vdots z \Rightarrow x \vdots z)$ mulohazadagi $\forall(x,y,z \in N)(y \vdots z)$ mulohaza yolg'on. U holda konyunksiya va implikatsiya amallari ta'rifiga ko'ra $\forall(x,y,z \in N)(x \vdots y \wedge y \vdots z \Rightarrow x \vdots z)$ mulohaza rost.

3) $\forall(x,y,z \in N)(x \vdots y \wedge y \vdots z \Rightarrow x \vdots z)$ mulohazadagi $\forall(x,y,z \in N)(x \vdots z)$, $\forall(x,y,z \in N)(y \vdots z)$ yolg'on. U holda konyunksiya va implikatsiya amallari ta'rifiga ko'ra $\forall(x,y,z \in N)(x \vdots y \wedge y \vdots z \Rightarrow x \vdots z)$ mulohaza rost.

4) $\forall(x,y,z \in N)(x \vdots y \wedge y \vdots z)$ rost bo'lsa, $\forall(x,y,z \in N)(x \vdots y)$ va $\forall(x,y,z \in N)(y \vdots z)$ lar bir vaqtda rost. $x \vdots y$ va $y \vdots z$ bo'lsa, u holda shunday $k, 1 \in N$ sonlar topiladiki, $x = y \cdot k$ va $y = z \cdot 1$, Bundan $x = y \cdot k = (z \cdot 1) \cdot k = z \cdot (1 \cdot k)$. Demak, $x \vdots z$ implikatsiya ta'rifiga ko'ra, bu holda ham berilgan $\forall(x,y,z \in N)(x \vdots y \wedge y \vdots z \Rightarrow x \vdots z)$ mulohaza rost.

Demak, berilgan mulohaza rost mulohaza.

Misol va mashqlar

1. Quyidagi gaplarning qaysilari mulohaza bo'ladi?

1.1. $ABCD$ to'rtburchakning yuzi $A'B'C'D'$ to'rtburchakning yuziga teng.

1.2. Tomonlari teng parallelogramm rombdir.

1.3. Berilgan uchburchaklar o'xshash.

1.4. Har qanday tub son toq.

1.5. $\sqrt{3}$ – irratsional son.

1.6. Yashasin O'zbekiston yoshlari!

1.7. 2 ga qarama-qarshi son mavjud emas.

1.8. 5 ning butun bo'luvchilari 4 ta.

1.9. -1 kompleks son.

1.10. 6 soni 3 ga karrali son.

1.11. Oyda hayot mavjud.

1.12. Ertaga qor yog'adi.

1.13. Guruhdagi talabalar soni 20 nafar.

1.14. Sirdaryo Orol dengiziga quyiladi.

1.15. Siz qaysi oliygohda o'qiysiz?

1.16. O'zbekiston Mustaqilligining 15 yilligi muborak bo'lsin!

1.17. Har qanday son musbat.

1.18. 0 har qanday haqiqiy songa bo'linadi.

1.19. 2, 3, 5 sonlari tub sonlar.

1.20. Barcha insonlar yoshi 20 da.

1.21. Galaktikamizda shunday sayyora borki, unda hayot mavjud.

1.22. 5 soni 25 va 70 sonlarining eng katta umumiy bo'luvchisidir.

1.23. $3x^3 - 5y + 9$.

2. Mulhazaning rost yoki yolg'onligini aniqlang:

2.1. $2 \in \{x \mid 2x^3 - 3x^2 + 1 = 0,\ x \in R\}$.

2.2. 1966-yil Toshkentda yer qimirlagan.

2.3. 8-mart dam olish kuni.

2.4. $3 \in \left\{ n \,\middle|\, \frac{2n+1}{3n-2},\ n \in N \right\}$.

2.5. $\{1;\ 1{,}2\} \subset \{x \mid x^3 + x^2 - x - 1 = 0,\ x \in Z\}$.

2.6. $2 \leq 3$.

2.7. 10 ning natural bo‘luvchilari 2 ta.

2.8. $2 \cdot 2 \leq 4$.

2.9. [4, 12, 24]=24.

2.10. Gipotenuza to‘g‘ri burchakli uchburchakning eng uzun tomoni.

3. Quyidagi mulohazalarning inkorini ifodalang:

3.1. 15 soni 5 songa bo‘linadi.

3.2. Oy Yerning yo‘ldoshi.

3.3. $2 > 3$.

3.4. $5 + 3 < 10$.

3.5. i – mavhum son.

3.6. $ABCD$ to‘rtburchak – romb.

3.7. n – juft natural son.

3.8. Shunday haqiqiy son mavjudki, u juft son.

3.9. Barcha natural sonlar musbat.

3.10. Barcha natural sonlar birdan katta.

4. Biri ikkinchisining inkori bo‘lgan mulohazalar juftligini aniqlang:

4.1. $2 < 3$; $3 < 2$.

4.2. $5 \leq 4$; $5 > 4$.

4.3. «4 – murakkab son», «4 – tub son».

4.4. «Shunday natural son mavjudki, u tub son», «Barcha natural sonlar murakkab sonlardir».

4.5. «6 ning barcha natural bo‘luvchilari tub sonlardir», «6 ning kamida bitta natural bo‘luvchisi murakkab son».

4.6. «ABC – to‘g‘ri burchakli uchburchak», «ABC – o‘tmas burchakli uchburchak».

4.7. «f – toq funksiya» , «f – juft funksiya».

4.8. «Barcha tub sonlar toq» , «Shunday tub son mavjudki, u juft».

4.9. «Irratsional sonlar mavjud», «Barcha sonlar ratsional sonlardir».

5. Quyidagi mulohazalarning rostlik shartlarini mulohazalar dizyunksiyasi yoki konyunksiyasi orqali ifodalang:

5.1. $x \cdot y \neq 0$.

5.2. $x \cdot y = 0$.

5.3. $x^2 + y^2 \neq 0$.

5.4. $\frac{x}{y} = 0$.

5.5. $|x| < 6$.

5.6. $|x| = 4$.

6. Quyidagi mulohazalarning rostlik qiymatlarini aniqlang:

6.1. Har qanday natural son yo tub, yo murakkab.

6.2. Shunday natural son mavjudki u ham tub, ham juft son.

6.3. 2 ga teng bo'lmagan son yoki 2 dan katta, yoki 2 dan kichik bo'ladi.

6.4. Agar uchburchak teng tomonli bo'lsa, u teng yonli bo'ladi.

6.5. Agar to'rtburchak romb bo'lsa, u kvadrat bo'ladi.

6.6. Agar $15 \vdots 5$, u holda $15 \vdots 4$.

6.7. Agar $x^2 = 4$ bo'lsa, u holda $x = 2$ va $x = -2$.

6.8. Natural son 6 ga bo'linadi, faqat va faqat shu holdaki, agar u 2 ga va 3ga bo'linsa.

6.9. Ikkita uchburchak teng bo'ladi, faqat va faqat shu holdaki, agar ularning mos tomonlari teng bo'lsa yoki ularning mos burchaklari teng bo'lsa.

6.10. Berilgan butun sonning butun bo'luvchilari kamida to'rtta bo'lsa, u murakkab son bo'ladi.

7. *A* orqali «10 soni 5 soniga bo'linadi», *B* orqali «10 soni 3 soniga bo'linadi» mulohazalar belgilangan bo'lsa, u holda quyidagi mulohazalarni o'qing va ularning rostlik qiymatlarini aniqlang:

7.1. $A \wedge B$.

7.2. $A \vee B$.

7.3. $\urcorner A \wedge B$.

7.4. $A \wedge \urcorner B$.

7.5. $\urcorner A \wedge \urcorner B$.

7.6. $A \Rightarrow B$.

7.7. $B \Rightarrow A$.

7.8. $\urcorner A \Rightarrow B$.

7.9. $\urcorner B \Rightarrow A$.

7.10. $A \Leftrightarrow B$.

7.11. $\neg A \Leftrightarrow \neg B$.

7.12. $\neg B \Leftrightarrow \neg A$.

8. Quyidagi mulohazalarni sodda mulohazalarga ajrating. Sodda mulohazalarni harflar yordamida belgilab, berilgan mulohazalarni ular yordamida ifodalang.

8.1. Agar berilgan funksiya juft ham emas, toq ham bo'lmasa, u holda u yoki juft funksiya, yoki toq funksiya bo'ladi.

8.2. Agar berilgan son 3 ga bo'linsa va 5 ga bo'linmasa, u holda bu son 15 ga bo'linmaydi.

8.3. Ketma-ket kelgan uchta natural sonning kamida bittasi toq son bo'ladi.

8.4. Har qanday natural sonni 3 ga bo'lganda yoki 0, yoki 1, yoki 2 qoldiq qoladi.

9. Bir vaqtda $A \wedge B$ – rost, $A \wedge C$ – yolg'on, $(A \wedge B) \wedge \neg C$ – yolg'on bo'luvchi A, B, C mulohazalar mavjudmi?

10. Berilgan shartlar asosida quyidagi mulohazalarning rostlik qiymatini aniqlash mumkinmi? Agar mumkin bo'lsa, mulohazaning rostlik qiymatini aniqlang.

10.1. $(A \Rightarrow B) \Rightarrow C$, C – rost mulohaza.

10.2. $A \wedge (B \Rightarrow C)$, $(B \Rightarrow C)$ – yolg'on mulohaza.

10.3. $A \vee (B \Rightarrow C)$, B – yolg'on mulohaza.

10.4. $\neg(A \vee B) \Leftrightarrow (\neg A \vee \neg B)$, A – rost mulohaza.

10.5. $(A \Rightarrow B) \Rightarrow (\neg B \vee \neg A)$, B – rost mulohaza.

10.6. $(A \wedge B) \Rightarrow (A \vee C)$, A – yolg'on mulohaza.

Takrorlash uchun savollar

1. Mulohaza deb qanday gapga aytiladi? Har qanday o'tgan zamon darak gapi mulohaza bo'la oladimi? Kelasi zamon darak gaplari-chi?

2. Mulohazalar konyunksiyasi nima? U qanday o'qiladi? Rost konyunksiyaga, yolg'on konyunksiyaga misollar keltiring.

3. Mulohazalar dizyunksiyasi nima? Qanday o'qiladi? Rost dizyunksiyaga, yolg'on dizyunksiyaga misollar keltiring.

4. Mulohazalar implikatsiyasi nima? U qanday o'qiladi? Rost implikatsiya, yolg'on implikatsiyaga misollar keltiring.

5. Mulohazalar ekvivalensiyasi nima? U qanday o'qiladi? Rost ekvivalensiyaga, yolg'on ekvivalensiyaga misollar keltiring.

6. Mulohaza inkori nima? Qanday o'qiladi? Rost inkorga, yolg'on inkorga misollar keltiring.

7. Mantiqiy amallarning bajarilish tartibini ayting.

8. Rostlik jadvali nima?

2-§. Formula. Teng kuchli formulalar. Mantiq qonunlari

✓ **Asosiy tushunchalar**: mulohazaviy formula, rostlik qiymatlar tizimi, formulaning rostlik jadvali, teng kuchli formulalar, aynan rost formula, tavtologiya, mantiq qonuni, aynan yolg'on formula, ziddiyat, bajariluvchi formula.

1) Har qanday mulohaza formuladir.

2) Agar A, B lar formula bo'lsa, u holda

($\rceil A$), ($A \wedge B$), ($A \vee B$), ($A \Rightarrow B$), ($A \Leftrightarrow B$) lar ham formuladir.

A formula faqat A_1, ..., A_n mulohazalardan hosil qilingan bo'lsin, u holda A formulani A (A_1, ..., A_n) ko'rinishida yozib olamiz va A_1, ..., A_n mulohazalarni elementar mulohazalar deymiz. Har bir A_k ($k=\overline{1, n}$) mulohaza 0 yoki 1 qiymatlarni qabul qilishi mumkin. A_k mulohazaning qabul qiladigan qiymati i_k bo'lsin, u holda (i_1, ..., i_n) – n lik A_1, ..., A_n – mulohazalarning *qabul qiladigan qiymatlari tizimi* deyiladi.

A va B formulalar tarkibiga kirgan barcha mulohazalar A_1, ..., A_n lardan iborat bo'lsin. Agar A_1, ..., A_n mulohazalarning barcha (i_1, ..., i_n) qiymatlari tizimida A va B formulalar bir xil qiymatlar qabul qilsa, u holda bu formulalar *teng kuchli formulalar* deyiladi va $A \equiv B$ ko'rinishida belgilanadi.

Formulada qatnashgan mantiq amallari soni formulaning *rangi* deyiladi.

1. A formula – A mulohazadan iborat bo'lsa, uning formulaosti faqat uning o'zidan iborat.

2. Agar formulaning ko'rinishi $A * B$ dan iborat bo'lsa, u holda uning formulaostilari A, B, $A * B$ lar hamda A va B larning barcha formulaostilaridan iborat bo'ladi. Bu yerda $*$ – $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$ amallaridan biri.

Agar formulaning ko'rinishi $\rceil A$ bo'lsa, uning formulaostilari A formula, A formulaning barcha formulaostilari va $\rceil A$ ning o'zidan iborat.

A formula, shu formula tarkibiga kirgan barcha mulohazalarning qabul qilishi mumkin bo'lgan barcha qiymatlari tizimida rost bo'lsa, bu formula *aynan rost formula* yoki *mantiq qonuni,* yoki *tovtologiya*; mulohazalarning kamida bitta qiymatlari tizimida rost qiymat qabul qilsa, bajariluvchi formula; barcha qiymatlari tizimida yolg'on qiymat qabul qilsa, *aynan yolg'on formula* yoki *ziddiyat* deyiladi.

1-misol. $(A \wedge B \Rightarrow A \wedge C)$ formulaning turini aniqlang.

Yechish. Berilgan formulada uchta A, B, C mulohazalar qatnashganligi sababli, ularning qiymatlar tizimlari $2^3 = 8$ ta bo'ladi. Formulaning rostlik jadvaliga 8 ta tizimni tartib bilan joylashtiramiz. Mantiq amallarining bajarilish tartibiga ko'ra avval $A \wedge B$ konyunksiyani, keyin $A \vee C$ dizyunksiyani va nihoyat hosil qilingan formulalarning implikatsiyasini bajaramiz. Ya'ni amallarning ta'riflariga ko'ra mos ustunlarni to'ldiramiz. Natijada quyidagi rostlik jadvali hosil bo'ladi:

A	B	C	$A \wedge B$	$A \vee C$	$A \wedge B \to A \vee C$
1	1	1	1	1	1
1	1	0	1	1	1
1	0	1	0	1	1
1	0	0	0	1	1
0	1	1	0	1	1
0	1	0	0	0	1
0	0	1	0	1	1
0	0	0	0	0	1

Formulaning rostlik jadvalidagi oxirgi ustun – formulaning rostlik qiymatlar ustuni faqat rost qiymatlardan iborat bo'lganligi uchun berilgan formula aynan rost (tavtologiya, mantiq qonuni), degan xulosaga kelamiz.

2-misol. Berilgan $\rceil(A \wedge B)$, $\rceil A \vee \rceil B$ formulalar teng kuchli ekanligini isbotlang.

Yechish. Berilgan formulalar teng kuchli ekanligini isbotlash uchun rostlik jadvallari tuzamiz:

A	B	$A \wedge B$	$\neg(A \wedge B)$
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	1

A	B	$\neg A$	$\neg B$	$\neg A \vee \neg B$
1	1	0	0	0
1	0	0	1	1
0	1	1	0	1
0	0	1	1	1

Formulalarning rostlik jadvallaridagi formulalar rostlik qiymatlari ustunlari mos tizimlarda bir xil ekanligidan berilgan formulalarning teng kuchli ekanligi kelib chiqadi.

Formulalarning teng kuchli ekanligini isbotlash uchun bitta rostlik jadvalini tuzish ham mumkin:

A	B	$A \wedge B$	$\neg(A \wedge B)$	$\neg A$	$\neg B$	$\neg A \vee \neg B$
1	1	1	0	0	0	0
1	0	0	1	0	1	1
0	1	0	1	1	0	1
0	0	0	1	1	1	1

Hosil bo'lgan 4- va 7- ustunlardagi rostlik qiymatlarini solishtirib, berilgan formulalarning teng kuchli ekanligiga ishonch hosil qilamiz.

3-misol. $A \Leftrightarrow B \equiv A \wedge B \vee A \wedge B$ tengkuchlilikni asosiy teng kuchliliklar yordamida isbotlang.

Yechish. Asosiy teng kuchliliklardan foydalanib, quyidagi teng kuchli formulalar ketma-ketligini hosil qilamiz:

$$A \Leftrightarrow B \equiv (A \Rightarrow B) \wedge (B \Rightarrow A) \equiv (\neg A \vee B) \wedge (\neg B \vee A) \equiv$$
$$\equiv ((\neg A \vee B) \wedge \neg B) \vee ((\neg A \vee B) \wedge A) \equiv (\neg A \wedge \neg B) \vee (B \wedge \neg B) \vee$$
$$\vee (\neg A \wedge A) \vee (B \wedge A) \equiv (\neg A \wedge \neg B) \vee 0 \vee 0 \vee (B \wedge A) \equiv$$
$$\equiv (\neg A \wedge \neg B) \vee (B \wedge A).$$

Misol va mashqlar

1. Quyidagi ifodalarning qaysilari mulohazaviy formula bo'ladi?

1.1. $A(B \Rightarrow C)$.

1.2. $((A \vee (B \wedge C)) \Leftrightarrow (\neg D))$.
1.3. $A \Rightarrow (B \wedge D \Rightarrow C)$.
1.4. $(A \vee (B \wedge C)) \Rightarrow D$.
1.5. $(A \Leftrightarrow D \vee (B \wedge C)) \Rightarrow D \wedge \neg A$.
1.6. $(((A \Leftrightarrow (\neg D)) \vee (B \wedge C)) \Rightarrow (D \wedge (\neg A)))$.

2. Quyidagi ifodalarga qavslarni turli xil joylashtirish yordamida mulohazaviy formulalar hosil qiling:

2.1. $A \wedge B \Rightarrow C$.

2.2. $A \Rightarrow B \wedge C \Rightarrow \neg C$.

2.3. $\neg A \Leftrightarrow \neg B \vee C \wedge B$.

2.4. $\neg A \wedge B \Rightarrow C$.

3. Berilgan formulalarning barcha qismformulalarini aniqlang:

3.1. $(((A \Leftrightarrow B) \wedge (\neg C)) \Rightarrow (((A \vee B) \Rightarrow A) \Rightarrow (\neg C)))$.
3.2. $(((A \vee B) \vee (\neg C)) \wedge ((\neg A) \vee ((\neg B) \vee C)))$.
3.3. $(((\neg A) \Leftrightarrow (\neg B)) \vee (C \wedge B))$.
3.4. $((\neg(A \Leftrightarrow (\neg(B \vee C)))) \wedge B)$.
3.5. $(((A \Rightarrow B) \wedge (C \Rightarrow A)) \vee (B \wedge (\neg C)))$.
3.6. $((\neg((\neg A) \Leftrightarrow C) \wedge B)) \vee ((A \vee C) \Leftrightarrow C))$.
3.7. $(((A \Leftrightarrow C) \Rightarrow B) \vee ((\neg A) \wedge (\neg C)))$.
3.8. $(((B \Rightarrow (((\neg B) \vee A) \wedge C)) \Leftrightarrow (\neg A)))$.

4. Quyidagi formulalarning turini aniqlang (formulalarning tashqi qavslari tushirib qoldirilgan):

4.1. $(\neg (X \vee Y) \Rightarrow \neg (X \wedge Y))$.
4.2. $(X \Rightarrow Y) \Rightarrow (\neg Y \Rightarrow \neg X)$.
4.3. $\neg (X \Rightarrow (Y \Rightarrow X)) \wedge Z$.
4.4. $\neg X \Rightarrow (X \Rightarrow Y) \vee Z$.
4.5. $\neg (X \Rightarrow Y) \Rightarrow ((X \wedge Z) \Rightarrow (Y \wedge Z))$.
4.6. $(X \wedge Y) \Rightarrow Z \Leftrightarrow X \Rightarrow (Y \Rightarrow Z)$.
4.7. $(X \wedge Y) \Rightarrow Z \Leftrightarrow (X \wedge \neg Z) \Rightarrow \neg Y$.
4.8. $\neg (X \Rightarrow Y) \Leftrightarrow X \wedge \neg Y$.
4.9. $(X \Rightarrow Y) \wedge \neg Y \Rightarrow \neg X$.
4.10. $(X \Rightarrow Y) \Rightarrow (X \wedge Z \Rightarrow Y \wedge Z)$.
4.11. $(X \Rightarrow Y) \wedge (Z \Rightarrow T) \Rightarrow (X \wedge Z \Rightarrow Y \wedge T)$.
4.12. $\neg (X \Leftrightarrow Y) \Leftrightarrow (\neg (X \Rightarrow Y) \vee \neg (Y \Rightarrow X))$.

4.13. $(X \wedge Y) \Rightarrow (Z \wedge \rceil Z \Rightarrow X \vee Z)$.

4.14. $(X \Leftrightarrow Y) \Leftrightarrow (X \Rightarrow Y) \wedge (Y \Rightarrow X)$.

5. 4-misolda keltirilgan formulalar ranglarini aniqlang.

6. Quyidagi formulalarning aynan rost ekanligini isbotlang:

6.1. $(A \Leftrightarrow B) \Leftrightarrow (\rceil A \Leftrightarrow \rceil B)$.

6.2. $(A \Rightarrow B) \Rightarrow ((A \Rightarrow (B \Rightarrow C)) \Rightarrow (A \Rightarrow C))$.

6.3. $(A \Rightarrow B) \Rightarrow ((B \Rightarrow A) \Rightarrow (A \Leftrightarrow B))$.

6.4. $(A \Rightarrow C) \Rightarrow ((A \vee B) \Rightarrow (C \vee B))$.

7. Quyidagi formulalarning aynan yolg'on ekanligini isbotlang:

7.1. $A \wedge (B \wedge (\rceil A \vee \rceil B))$.

7.2. $\rceil(\rceil(A \vee B) \Rightarrow \rceil(A \wedge B))$.

7.3. $\rceil(A \Rightarrow (B \Rightarrow A))$.

7.4. $\rceil(A \Rightarrow C) \Rightarrow ((B \Rightarrow C) \Rightarrow (A \vee B \Rightarrow C))$.

7.5. $\rceil(A \Rightarrow B) \Rightarrow ((A \wedge C) \Rightarrow (B \wedge A))$.

8. Quyidagi formulalarning qaysilari bajariluvchi ekanligini aniqlang:

8.1. $\rceil(A \Rightarrow \rceil A)$.

8.2. $(A \Rightarrow B) \Rightarrow (B \Rightarrow A)$.

8.3. $(B \Rightarrow (A \wedge C)) \wedge \rceil((A \vee C) \Rightarrow B)$;

8.4. $\rceil((A \Leftrightarrow \rceil B) \vee C) \wedge B$;

8.5. $(A \wedge B) \Rightarrow ((C \vee B) \Rightarrow (B \wedge \rceil B))$.

9. Rostlik jadvali yordamida quyidagi formulalar tavtologiya ekanligini isbotlang:

9.1. $(A \vee (\rceil A))$ (uchinchisini inkor qilish qonuni).

9.2. $(\rceil(A \wedge (\rceil A)))$ (ziddiyatni inkor qilish qonuni).

9.3. $((\rceil(\rceil A)) \Leftrightarrow A)$ (qo'sh inkor qonuni).

9.4. $(A \Rightarrow A)$ (ayniyat qonuni).

9.5. $((A \wedge A) \Leftrightarrow A)$ (konyunksiyaning idempotentlik qonuni).

9.6. $((A \vee A) \Leftrightarrow A)$ (dizyunksiyaning idempotentlik qonuni).

9.7. $((A \Rightarrow B) \Leftrightarrow ((\rceil B) \Rightarrow (\rceil A)))$ (kontrapozitsiya qonuni).

9.8. $((A \Rightarrow B) \Leftrightarrow ((\rceil A) \vee B))$.

9.9. $((A \Leftrightarrow B) \Leftrightarrow ((A \Rightarrow B) \wedge (B \Rightarrow A)))$.

9.10. $((A \wedge (B \vee A)) \Leftrightarrow A)$ (yutilish qonuni).

9.11. $((A \wedge (B \vee A)) \Leftrightarrow A)$ (yutilish qonuni).

9.12. $((\rceil(A \wedge B)) \Leftrightarrow ((\rceil A) \vee (\rceil B)))$ (de Morgan qonuni).

9.13. $(\rceil(A \vee B)) \Leftrightarrow ((\rceil A) \wedge (\rceil B))$ (de Morgan qonuni).

9.14. $((A \vee B) \Leftrightarrow ((\rceil A) \Rightarrow B))$.

9.15. $(((A \Rightarrow B) \wedge (B \Rightarrow C)) \Rightarrow (A \Rightarrow C))$ (tranzitiv xulosa qoidasi).

9.16. $((A \Leftrightarrow B) \Leftrightarrow ((\rceil A) \Leftrightarrow (\rceil B)))$ (qarama-qarshilik qonuni).

9.17. $((A \wedge B) \Leftrightarrow (B \wedge A))$ (konyunksiyaning kommutativlik qonuni).

9.18. $((A \vee B) \Leftrightarrow (B \vee A))$ (dizyunksiyaning kommutativlik qonuni).

9.19. $(((A \wedge B) \wedge C) \Leftrightarrow (A \wedge (B \wedge C)))$ (konyunksiyaning assotsiativlik qonuni).

9.20. $(((A \vee B) \vee C) \Leftrightarrow (A \vee (B \vee C)))$ (dizyunksiyaning assotsiativlik qonuni).

9.21. $((A \wedge (B \vee C)) \Leftrightarrow ((A \wedge B) \vee (A \wedge C)))$ (konyunksiyaning dizyunksiyaga nisbatan distributivlik qonuni).

9.22. $((A \vee (B \wedge C)) \Leftrightarrow ((A \vee B) \wedge (A \vee C)))$ (dizyunksiyaning konyunksiyaga nisbatan distributivlik qonuni).

10. Quyidagi tengkuchliliklarni isbotlang (mulohazaviy formulalarning tashqi qavslari tashlab yuborilgan):

10.1. $A \wedge A \equiv A$.

10.2. $A \vee A \equiv A$.

10.3. $A \vee \rceil A \equiv 1$.

10.4. $A \wedge \rceil A \equiv 0$.

10.5. $A \vee 0 \equiv A$.

10.6. $A \vee 1 \equiv 1$.

10.7. $A \wedge 0 \equiv 0$.

10.8. $A \wedge 1 \equiv A$.

10.9. $\rceil\rceil A \equiv A$.

10.10. $A \wedge B \equiv B \wedge A$.

10.11. $A \vee B \equiv B \vee A$.

10.12. $A \wedge (B \vee A) \equiv A$.

10.13. $A \vee (B \wedge A) \equiv A$.

10.14. $A \Rightarrow B \equiv \urcorner A \vee B$.

10.15. $A \Leftrightarrow B \equiv (A \Rightarrow B) \wedge (B \Rightarrow A)$.

10.16. $\urcorner(A \vee B) \equiv \urcorner A \wedge \urcorner B$.

10.17. $\urcorner(A \wedge B) \equiv \urcorner A \vee \urcorner B$.

10.18. $(A \wedge B) \wedge C \equiv A \wedge (B \wedge C)$.

10.19. $(A \vee B) \vee C \equiv A \vee (B \vee C)$.

10.20. $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$.

10.21. $A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$.

11. 10-misoldagi tengkuchliliklar yordamida quyidagi formulalarni soddalashtiring (mulohazaviy formulalarning tashqi qavslari tashlab yuborilgan):

11.1. $\urcorner(\urcorner A \vee B) \Rightarrow ((A \vee B) \Rightarrow A)$.

11.2. $\urcorner(\urcorner A \wedge \urcorner B) \vee ((A \Rightarrow B) \wedge A)$.

11.3. $(A \Rightarrow B) \wedge (B \Rightarrow A) \wedge (A \vee B)$.

11.4. $(A \Rightarrow B) \wedge (B \Rightarrow \urcorner A) \wedge (C \Rightarrow A)$.

11.5. $(A \wedge C) \vee (A \wedge \urcorner C) \vee (B \wedge C) \vee (\urcorner A \wedge B \wedge C)$.

11.6. $\urcorner((A \Rightarrow B) \wedge (B \Rightarrow \urcorner A))$.

12. Teng kuchli almashtirishlar yordamida quyidagi formulalarni shunday almashtiringki, natijada hosil bo‘lgan formulalarda faqat $\urcorner$ va $\wedge$ amallari qatnashsin:

12.1. $(A \vee B) \Rightarrow (\urcorner A \Rightarrow C)$.

12.2. $(\urcorner A \Rightarrow B) \vee \urcorner(A \Rightarrow B)$.

12.3. $((A \vee B \vee C) \Rightarrow A) \vee C$.

12.4. $((A \Rightarrow B) \Rightarrow C) \Rightarrow \urcorner A$.

12.5. $(A \vee (B \Rightarrow C)) \Rightarrow A$.

13. Teng kuchli almashtirishlar yordamida quyidagi formulalarni shunday almashtiringki, natijada hosil bo‘lgan formulalarda faqat $\urcorner$ va $\vee$ amallari qatnashsin:

13.1. $(A \Rightarrow B) \Rightarrow (B \wedge C)$.

13.2. $(\urcorner A \wedge \urcorner B) \Rightarrow (A \wedge B)$.

13.3. $((\urcorner A \wedge \urcorner B) \vee C) \Rightarrow (C \wedge \urcorner B)$;

13.4. $((A \Rightarrow (B \wedge C)) \Rightarrow (\urcorner B \Rightarrow \urcorner A)) \Rightarrow \urcorner B$;

13.5. $((A \Rightarrow B) \wedge (B \Rightarrow C)) \Rightarrow (A \Rightarrow C)$.

14. Quyidagi formulalarning inkorini toping:

14.1. $(A \wedge (B \vee \neg C)) \vee (\neg A \wedge B)$.

14.2. $((\neg A \wedge \neg B \wedge \neg C) \vee D) \wedge \neg Q \wedge \neg R \wedge \neg P$.

14.3. $(((\neg A \wedge (\neg B \vee C)) \vee D) \wedge \neg Q) \vee (\neg R \wedge (P \vee \neg F))$.

14.4. $((A \wedge (\neg B \vee (\neg C \wedge D))) \vee \neg Q) \wedge R$.

15. Teng kuchli almashtirishlar yordamida quyidagi formulalarning ziddiyat ekanligini isbotlang:

15.1. $(A \Rightarrow B) \wedge (B \Rightarrow A) \wedge ((A \wedge \neg B) \vee (\neg A \wedge B))$.

15.2. $((A \wedge \neg B) \Rightarrow (\neg A \vee (A \wedge B))) \wedge ((\neg B \vee (A \wedge B)) \Rightarrow (A \wedge \neg B))$.

15.3. $((A \Rightarrow B) \wedge (B \Rightarrow C)) \Rightarrow \neg (A \Rightarrow C)$.

15.4. $(A \Rightarrow B) \wedge (A \Rightarrow \neg B) \wedge A$.

15.5. $(A \wedge \neg B) \vee (A \wedge \neg C)) \Leftrightarrow ((A \Rightarrow B) \wedge (A \Rightarrow C))$.

Takrorlash uchun savollar

1. Mulohazaviy formula ta'rifini ayting va misol keltiring.
2. Mantiqiy amallarni bajarilish tartibi qanday?
3. Mulohazalarning qabul qiladigan qiymatlar tizimi nima? Ularning soni nimaga bog'liq?
4. Formulaning rostlik jadvali qanday tuziladi?
5. Teng kuchli formulalarga ta'rif bering.
6. Formulalarning teng kuchli ekanligi qanday isbotlanadi?
7. Aynan rost, aynan yolg'on, bajariluvchi formulalar ta'riflarini ayting.
8. Tavtologiya, ziddiyat, mantiq qonuni ta'rifini ayting.
9. Asosiy tengkuchliliklardan qaysilarini eslab qoldingiz?
10. Teng kuchli formula bilan mantiq qonuni orasida qanday bog'lanish bor?

3-§. Predikatlar. Kvantorlar

Asosiy tushunchalar: predikat, bir o'zgaruvchili predikatning qiymatlar sohasi, predikatning rostlik sohasi, predikatlar konyunksiyasi, dizyunksiyasi, implikatsiyasi, ekvivalensiyasi, predikat inkori, umumiylik kvantori, mavjudlik kvantori, predikatli formula.

M to'plamning a elementi haqida aytilgan tasdiqqa a ning o'rniga M ning aniq bitta elementini qo'ysak mulohaza hosil bo'lsa, bunday tasdiqlarni *bir o'zgaruvchili mulohazaviy formula* yoki *bir o'zgaruvchili predikat* deb ataymiz. n ta $x_1, ..., x_n$ o'zgaruvchilarga bog'liq $R(x_1, ..., x_n)$ tasdiq berilgan bo'lsin. U holda $x_1, ..., x_n$ o'zgaruvchilarning mazmunga ega bo'ladigan qiymatlar to'plami, shu o'zgaruvchilarning *yo'l qo'yiladigan qiymatlari sohasi* deyiladi. Agar $R(x_1, ..., x_n)$ tasdiq $x_1, ..., x_n$ o'zgaruvchilarning yo'l qo'yilishi mumkin bo'lgan har qanday qiymatlarida mulohazaga aylansa, *n o'zgaruvchili predikat* yoki *n o'zgaruvchili mulohazaviy formula* deyiladi. Bu yerda $n = 0, 1, 2$ va hokazo manfiy bo'lmagan butun qiymatlar qabul qiladi. 0 o'rinli predikat sifatida mulohaza tushuniladi.

$M \neq \varnothing$ to'plamda aniqlangan bir o'rinli $R(x)$ predikat berilgan bo'lsin, u holda $R(x)$ *predikatning inkori* deb har qanday $x \in M$ element uchun $R(x)$ predikat rost bo'lganda yolg'on bo'ladigan; $R(x)$ yolg'on bo'lganda rost bo'ladigan $\urcorner R(x)$ predikatga aytiladi. Ya'ni M ning ixtiyoriy elementi uchun

$(\urcorner R)(x) = \urcorner(R(x))$ tenglik o'rinli bo'ladi.

Xuddi shunday $M \neq \varnothing$ to'plamda aniqlangan $P(x)$ va $Q(x)$ bir o'rinli predikatlar uchun $\wedge, \vee, \Rightarrow, \Leftrightarrow$ amallari quyidagi tengliklar yordamida aniqlanadi:

$$(R \wedge Q)(x) = R(x) \wedge Q(x);$$
$$(R \vee Q)(x) = R(x) \vee Q(x);$$
$$(R \Rightarrow Q)(x) = R(x) \Rightarrow Q(x);$$
$$(R \Leftrightarrow Q)(x) = R(x) \Leftrightarrow Q(x).$$

$M \neq \varnothing$ to'plamda aniqlangan $R(x)$ predikat berilgan bo'lsin, u holda $R(x)$ predikatni rost mulohazaga aylantiradigan x ning M to'plamga tegishli barcha elementlarini E_r orqali belgilaymiz. E_r ni $R(x)$ predikatning *rostlik sohasi* deyiladi.

$\forall x R(x)$ ifoda, M to'plamning barcha elementlari uchun $R(x)$ rost bo'lganda rost, M to'plamning kamida bitta x_0 elementi uchun $R(x_0)$ yolg'on bo'lganda yolg'on bo'ladigan mulohazadir. Bu yerdagi $\forall$ belgi umumiylik kvantorini bildiradi.

$\exists x R(x)$ mulohaza bo'lib, M to'plamning kamida bitta x_0 elementi uchun $R(x_0)$ rost bo'lganda rost, qolgan hollarda, ya'ni

M to'plamning barcha elementlari uchun *R*(*x*) yolg'on bo'lganda yolg'on bo'ladigan mulohazadir.

R(*x*, *y*) butun sonlar to'plami *Z* da aniqlangan «$x+y>0$» mazmunidagi predikat bo'lsin, u holda:

$\forall x \forall y R(x, y)$ – «ixtiyoriy ikkita butun son yig'inidisi musbat bo'ladi» – yolg'on mulohaza;

$\forall x \exists y R(x, y)$ – «har qanday butun son *x* uchun shunday *y* butun son mavjud bo'lib, ulraning yig'indisi musbat» – rost mulohaza;

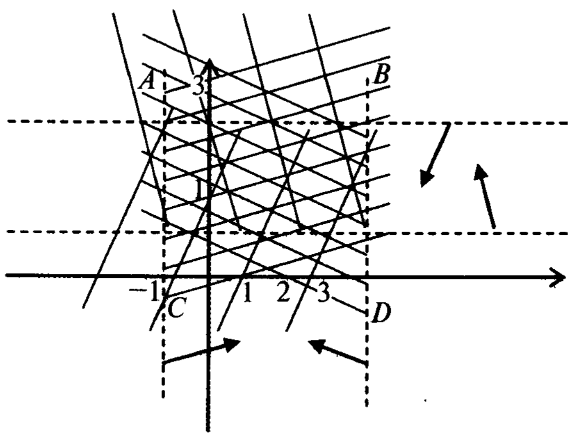
$\exists x \forall y R(x, y)$ – «shunday *x* butun son mavjud bo'lib, uning ixtiyoriy *y* butun son bilan yig'idisi musbat» – yolg'on mulohaza;

$\exists x \exists y R(x, y)$ – «shunday *x* va *y* butun sonlar mavjudki, ularning yig'indisi musbat» – rost mulohaza bo'ladi.

1-misol. Dekart koordinatalar tekisligida $x < 3 \wedge x > -1 \wedge \wedge y < 3 \wedge y > 1$ predikatning rostlik sohasini tasvirlang.

Yechish. Berilgan ikki o'rinli predikat to'rtta bir o'rinli predikatlarning konyunksiyasidan tashkil topgan. Konyunksiya amalining ta'rifidan, predikatlardagi ikkala o'zgaruvchi o'rniga qiymatlar berganimizda, ularning barchasini rost mulohazaga aylantiruvchi *x* va *y* larning qiymatlari berilgan predikatning rostlik sohasi bo'ladi. Buning uchun har bir predikatning rostlik sohalarini aniqlab, ularning kesishmasini topamiz (chizmaga q.)

Hosil bo'lgan chizmadagi *ABCD* to'rtburchakning ichki nuqtalari berilgan predikatning rostlik sohasi bo'ladi.



2-misol. $M = \{1, 2, ..., 20\}$ to'plamda quyidagi predikatlar berilgan: $A(x)$: «(x : 5)»; $B(x)$: «x – juft son»; $C(x)$: «x – tub son»; $D(x)$: «x 3 ga karrali». $A(x) \wedge B(x) \Rightarrow C(x) \vee D(x)$ predikatning rostlik sohasini toping.

Yechish. K orqali M to'plamning $A(x) \wedge B(x)$ predikatni rost, $C(x) \vee D(x)$ predikatni yolg'on mulohazaga aylantiradigan elementlarini belgilab olamiz. Mantiq amallarining ta'rifiga ko'ra berilgan predikatning rostlik sohasi M to'plamdan K to'plamni ayirishdan hosil bo'lgan to'plamdan iborat. K to'plamni aniqlaymiz:

1) $A(x) \wedge B(x)$ predikat rost mulohazaga aylanadigan qiymatlar to'plami $A(x)$ va $B(x)$ predikatlarni bir vaqtda rost mulohazaga aylantiradigan M to'plamning elementlari, ya'ni $A_1 = \{5, 10, 15, 20\}$ va $B_1 = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$ to'plamlarning kesishmasidan iborat. Bu to'plamni M_1 orqali belgilaymiz:

$$M_1 = A_1 \cap B_1 = \{10, 20\}.$$

2) $C(x) \vee D(x)$ predikat $C(x)$ va $D(x)$ predikatlar bir vaqtda yolg'on mulohazaga aylanadigan M to'plamning qiymatlarida yolg'on mulohaza bo'ladi. U $C_1 = \{1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\}$ va $D_1 = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20\}$ to'plamlarning kesishmasidan hosil bo'lgan $M_2 = \{1, 4, 8, 10, 14, 16, 20\}$ to'plamdan iborat.

Demak, $K = M_1 \cap M_2 = \{10, 20\}$ to'plamdan iborat. U holda $M \setminus K$ berilgan $A(x) \wedge B(x) \Rightarrow C(x) \vee D(x)$ predikatning rostlik sohasi.

Misol va mashqlar

1. Quyidagi gaplardan qaysilari predikat ekanligini aniqlang:

1.1. Natural sonning natural bo'luvchilari faqat ikkita bo'lsa, u tub son bo'ladi.

1.2. «x – mevali daraxt» (x daraxtlar to'plamining elementi).

1.3. «Nargizaning yoshi 18 da».

1.4. «$x + y = 5$», ($x, y \in N$).

1.5. «$x + yi$ –mavhum son», ($x, y \in R$).

1.6. «$x^3 + 3x - 4$», ($x \in Z$).

1.7. «x va y bir sinfda o'qiydi», (x, y o'quvchilar to'plamining elementlari).

1.8. «x va y o‘xshash» (x, y geometrik shakllar to‘plamining elementlari).

1.9. «5=2+4».

1.10. «Nodir x ning akasi» (x insonlar to‘plamining elementi).

1.11. «x 5 ga bo‘linadi» ($x \in N$).

1.12. «$x^2 + 2x + 4$» ($x \in R$).

1.13. «ctg 45° = 1».

1.14. « x va y lar z ning turli tomonlarida yotadi» (x va y lar tekislikdagi nuqtalar to‘plamiga, z esa tekislikdagi to‘g‘ri chiziqlar to‘plamiga tegishli).

2. Quyidagi mulohazalar uchun shunday predikatlar tuzingki, ulardagi o‘zgaruvchilar o‘rniga qiymat berganda berilgan mulohaza hosil bo‘lsin:

2.1. $2+3 > 7$.

2.2. Feruzaning farzandlari 3 nafar.

2.3. A.Navoiy ko‘chasi Toshkent shahridagi markaziy ko‘chalardan biri.

2.4. $\log_2 2 = 1$.

2.5. Nargiza Namangan viloyatida tug‘ilgan.

2.6. $(5^2 - 1) = (5 - 1)(5 + 1)$.

2.7. 16 – murakkab son.

2.8. (4, 12, 20)=4.

2.9. Palov – o‘zbek milliy taomlaridan.

2.10. Berilgan ABC uchburchak $A'B'C'$ uchburchakka teng.

3. Sonlar o‘qida quyidagi bir o‘rinli predikatlarning rostlik sohasini tasvirlang:

3.1. $\frac{x^2+3x+2}{x^2+4x+3} < 0$.

3.2. $\sqrt{x^2-1} = -3$.

3.3. $2x^2 + x - 30 > 0$.

3.4. $(\sin x \geq 0)$.

3.5. $(|x + 2| < 0)$.

3.6. $\frac{x}{x-1} < 0$.

3.7. $3x^2 - 2x + 4 > 0$.

3.8. $|x - 1| < |x + 3|$.

3.9. $|x + 5| \leq 3$.

3.10. $|x| \leq 1$.

4. Dekart koordinatalar tekisligida quyidagi ikki o‘rinli predikatlarning rostlik sohasini tasvirlang:

4.1. $((x > 2) \wedge (y \geq 1)) \wedge ((x < -1) \wedge (y < -2))$.

4.2. $x + 3y = 3$.

4.3. $x - y \geq 0$.

4.4. $(x - 2)^2 + (y + 3)^2 = 4$.

4.5. $\lg x = \lg y$.

4.6. $\rceil(x > 2) \wedge (y < 2)$.

4.7. $(x = y) \vee (|x| \leq 1)$.

4.8. $(x \geq 3) \Rightarrow (y < 5)$.

4.9. $((x - 1)^2 + y^2 = 4) \wedge (y = x)$.

4.10. $(x^2 + 2x + 1 = 0) \wedge (y = 2x + 3)$.

5. $M = \{1, 2, ..., 20\}$ to‘plamda quyidagi predikatlar berilgan: $A(x)$: «$\rceil(x \vdots 5)$»; $B(x)$: «x – juft son»; $C(x)$: «x – tub son»; $D(x)$: «x 3 ga karrali». Quyidagi predikatlarning rostlik sohasini toping:

5.1. $A(x) \wedge D(x) \Rightarrow \rceil C(x)$.

5.2. $A(x) \wedge C(x) \Rightarrow \rceil D(x)$.

5.3. $A(x) \Rightarrow B(x)$.

5.4. $D(x) \Rightarrow \rceil C(x)$.

5.5. $C(x) \Rightarrow A(x)$.

5.6. $A(x) \vee B(x) \vee D(x)$.

5.7. $\rceil B(x) \vee \rceil D(x)$.

5.8. $\rceil C(x) \vee D(x) \wedge B(x)$.

5.9. $\rceil B(x) \vee C(x) \Rightarrow D(x)$.

5.10. $A(x) \wedge B(x) \wedge D(x)$.

5.11. $\rceil A(x) \wedge \rceil C(x) \vee B(x)$.

5.12. $\rceil B(x) \wedge \rceil C(x) \wedge D(x)$.

5.13. $A(x) \vee \rceil B(x) \wedge D(x)$.

5.14. $A(x) \wedge C(x) \vee \rceil D(x)$.

5.15. $B(x) \vee C(x) \wedge A(x)$.

5.16. $A(x) \Leftrightarrow \rceil B(x) \wedge D(x)$.

6. Quyidagi mulohazalarni o‘qing va ularning rostlik qiymatini aniqlang:

6.1. $\forall x A(x)$, $A(x)$: «x – natural son», $x \in R$.
6.2. $\exists x A(x)$, $A(x)$: «x – butun son», $x \in R$.
6.3. $\forall x(x + 3 = 5)$, $x \in R$.
6.4. $\exists x(4 + x = 10)$, $x \in R$.
6.5. $\forall x \forall y(x + y < 4)$, $x, y \in R$.
6.6. $\forall x \forall y(x + y > 4)$, $x, y \in R$.
6.7. $\exists x \forall y(x + y = 14)$, $x, y \in R$.
6.8. $\exists x \exists y(x \leq y)$, $x, y \in R$.
6.9. $\forall x \forall y \exists z([x, y] = z)$, $x, y \in R$.
6.10. $\forall x \forall y \exists z([x, y] = z)$, $x, y, z \in R$.
6.11. $\forall x(x < 0 \Rightarrow x > 0)$, $x \in \{0, 1, 2\}$.
6.12. $(x \in T)\ (a^2 + b^2 = c^2)$, T – uchburchaklar to'plami va a, b, c – uchburchak tomonlari.
6.13. $\forall x \forall y(\frac{x}{y} \Rightarrow \frac{y}{x})$, $x, y \in N$.
6.14. $\forall x(f(x) > 0)$, $f(x) = x^2 - 4x + 3$, $x \in R$.
6.15. $\forall x\ (\frac{2x-5}{x} \in R)$, $x \in R$.
6.16. $\forall x\ (x < 10)$, $x \in \{1, ..., 10\}$.
6.17. $\forall x(x + 5 \leq 15)$, $x \in \{1, ..., 10\}$.
6.18. $\forall x \forall y(x - y < 10)$, $x, y \in \{1, ..., 10\}$.
6.19. $\forall x \exists y\ (\frac{x}{y} \in A)$, $x, y \in \{1, ..., 10\}$.
6.20. $\forall x \forall y(x < y)$, $x \in \{1, ..., 5\}$, $y \in \{5, ..., 10\}$.
6.21. $\forall x \forall y\ (x \vdots y)$, $x \in \{4k \mid k \in Z\}$, $y \in \{1, 2, 4\}$.

7. Quyidagi predikatlardan kvantorlar yordamida mulohazalar hosil qiling va ularning qiymatlar, rostlik sohalarini toping:

7.1. $A(x)$: «x – talaba».
7.2. $A(x)$: «x – butun son».
7.3. $A(x)$: «x – to'g'ri chiziq».
7.4. $A(x)$: «$x \vdots 5$».
7.5. $A(x, y)$: «$x + y = 4$».
7.6. $A(x, y)$: «$x < y$».
7.7. $A(x, y)$: «$x \vdots y$».
7.8. $A(x, y)$: «$x \| y$».
7.9. $A(x, y, z)$: «$\frac{x}{y} = z$».
7.10. $A(x, y, z)$: «$[x, y] = z$».

8. Quyidagi formulalardagi erkli va bog'liq o'zgaruvchilarni aniqlang:

8.1. $\forall x A(x)$.

8.2. $A(y) \Rightarrow \exists x B(\mathrm{x})$.

8.3. $\exists x \forall \mathrm{y}(A(x) \wedge B(y)) \Rightarrow \forall y C(t, y)$.

8.4. $\forall x \exists y(A(x,y)) \Rightarrow B(t, z)$.

Takrorlash uchun savollar

1. Predikatga ta'rif bering.
2. Predikatning qiymatlar sohasi, rostlik sohasi nima? Misollar yordamida tushuntiring
3. Predikatlar dizyunksiyasi, konyunksiyasi, implikatsiyasi, ekvivalensiyasiga misollar keltiring.
4. Mantiq amallarini qo'llash natijasida hosil bo'ladigan predikat o'zgaruvchilarining soni haqida nima deyish mumkin?
5. Umumiylik va mavjudlik kvantorlarini qo'llashga misollar keltiring.
6. Predikatli formula qanday hosil qilinadi?
7. Predikatli formulaning qanday turlarini bilasiz?

4-§. Matematik mantiqning tatbiqlari

✔ **Asosiy tushunchalar**: teorema, to'g'ri teorema, to'g'riga teskari teorema, to'g'riga qarama-qarshi teorema, teskariga qarama-qarshi teorema, teorema isboti. Rele kontakt sxemasi.

Matematik mantiq elementlari mavzuning o'qitilishidan qo'yilgan asosiy maqsad o'quvchilarni matematik analiz fanining algebra, geometriya, matematik analiz kabi bir qancha matematik fanlarga tatbiqining eng sodda ko'rinishlaridan biri – matematik jumla (aksioma, teorema, ta'rif, ...)larni mulohazalar va predikatlar algebralari tili orqali ifodalashga o'rgatishdir.

Natural sonlar to'plamida qaralgan tub son tushunchasi uchun quyidagi formulani keltirish mumkin:

$(\forall n \in N)((n - \text{tub son}) \Leftrightarrow (n \neq 1 \wedge n \,\vdots\, p \Rightarrow p = 1 \vee p = n))$.

Yoki quyidagi belgilashlarni kiritsak:

$A(x)$ – «x – tub son», $B(x)$ – «$x \neq 1$», $C(x)$ – «$x \vdots p$», $D(x)$ – «$x=1$», $P(x)$ – «$x=p$» , u holda yuqoridagi formulani quyidagicha ifodalash mumkin:

$(\forall x \in N)\ (A(x) \Leftrightarrow B(x) \wedge C(x) \Rightarrow D(x) \vee P(x))$.

Teorema va uning turlari. Har qanday teorema shart va natijadan iborat. Agar A teoremaning sharti, B esa uning xulosasi bo'lsa, u holda teoremani

$A \Rightarrow B$ (1) ko'rinishda yozishimiz mumkin.

$B \Rightarrow A$ (2) teoremaga (1) teoremaga *teskari teorema* deyiladi.

$\rceil A \Rightarrow \rceil B$ (3) teoremaga (1) teoremaga *qarama-qarshi teorema* deyiladi.

$\rceil B \Rightarrow \rceil A$ (4) teoremaga berilgan (1) teoremaning *teskarisiga qarama-qarshi* (yoki berilgan (1) teoremaning *qarama-qarshisiga teskari*) teorema deyiladi.

Rostlik jadvallari orqali $A \Rightarrow B \equiv \rceil B \Rightarrow \rceil A$ va $B \Rightarrow A \equiv \rceil A \Rightarrow \rceil B$ tengkuchliliklarni isbot qilib, quyidagi xulosani chiqaramiz:

$A \Rightarrow B$ teorema o'rniga $\rceil B \Rightarrow \rceil A$ teoremani isbot qilib, $A \Rightarrow B$ rost, ya'ni to'g'ri deb aytishimiz mumkin.

Isbot tushunchasi. $A_1, A_2, \ldots, A_n$ (1) mulohazalar berilgan bo'lib, quyidagi shartlar bajarilsa:

A_1 – aksioma yoki avval isbot qilingan mulohaza bo'lsin;

har bir A_i, $i \geq 2$ yoki o'zidan oldingi mulohazadan keltirib chiqarilsin, yoki avval isbot qilingan mulohaza bo'lsin.

U holda (1) ketma-ketlikni biz A_n *mulohazaning isboti* deymiz.

Isbot qilish usullari. Teorema shartining rostligidan, xulosaning rostligini to'g'ridan to'g'ri keltirib chiqarishni *bevosita isbot qilish*, deb tushunamiz. Mantiq qonunlari orqali isbot qilishga, *teskarisidan isbot qilish, uchinchisini inkor qilish qonuni orqali isbot qilish, induksiya yordamida isbot qilish* va h.k.lar kiradi.

Avtomatik boshqarish qurilmalari va elektron hisoblash mashinalarida ko'plab rele-kontakt sxemalar uchraydi. Har qanday sxemaga mulohazalar algebrasining biror-bir formulasini mos qo'yish mumkin va aksincha. RKS bilan mulohazalar algebrasining formulalari orasidagi bunday munosabat murakkab RKS larni mulohazalar algebrasining formulalari yordamida soddalashtirish imkoniyatini beradi. Kontaktni shartli ravishda

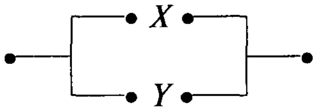
yoki —•—, yoki — —, yoki —• •—

ko‘rinishda belgilaymiz. Kontakt yopiq (tok o‘tkazadigan) yoki ochiq (tok o‘tkazmaydigan) holatda bo‘lishi mumkin. Kontaktning yopiq holatiga 1 ni, ochiq holatiga 0 ni mos qo‘yamiz.

Barcha kontaktlar orasida doimo tok o‘tkazadigan (doimo yopiq) hamda butunlay tok o‘tkazmaydigan (doimo ochiq) kontaktlar mavjuddir. Ularni ham, mos ravishda, 1 va 0 bilan belgilaymiz va hamda —•—, — — ko‘rinishda ifodalaymiz.

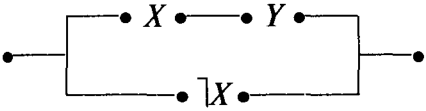
Biz o‘zgaruvchi kontaktlar bilan ish ko‘rganimiz uchun ularni X, Y, Z, ... harflar bilan belgilaymiz. U holda ikkita X va Y mulohazalarning konyunksiyasiga kontaktlarni ketma-ket ulash natijasida hosil bo‘ladigan

—• X •—• Y •— sxemani, X va Y mulohazalarning dizyunksiyasiga kontaktlarni parallel ulash natijasida hosil bo‘ladigan quyidagi

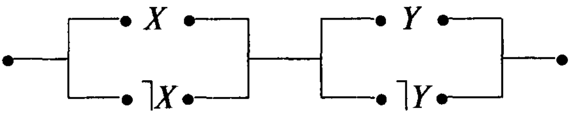


sxemani mos qo‘yamiz. Har qanday mulohazviy formulani faqat $\neg$, $\wedge$, $\vee$ amallar qatnashgan formulaga keltirish mumkin bo‘lganligidan har bir formulani RKS orqali ifoda qilish va aksincha, har qanday RKS ni mulohazaviy formula orqali ifodalash mumkin.

1-misol. $(X \wedge Y) \vee \neg X$ formulaga quyidagi rele-kontakt sxemasi mos keladi:



2-misol.



sxemaga $(X \vee \neg X) \wedge (Y \vee \neg Y)$ formula mos keladi.

Misol va mashqlar

1. Quyidagi teoremalarga teskari, to‘g‘riga qarama-qarshi, teskariga qarama-qarshi teoremalarni ifodalang:

1.1. Agar berilgan to‘rtburchak kvadrat bo‘lsa, u romb bo‘ladi.

1.2. Agar berilgan ikki to‘g‘ri chiziqning har biri uchinchi to‘g‘ri chiziqqa parallel bo‘lsa, u holda berilgan to‘g‘ri chiziqlar parallel bo‘ladi.

1.3. Agar parallelogramm romb bo‘lsa, uning diagonallari perpendikular bo‘ladi.

1.4. Agar ketma-ketlik monoton va chegaralangan bo‘lsa, u holda u limitga ega bo‘ladi.

1.5. Agar berilgan natural sonning raqamlar yig‘indisi 3 ga bo‘linsa, berilgan son 3 ga bo‘linadi.

1.6. Agar ratsional sonlar ketma-ketligi yaqinlashuvchi bo‘lsa, u holda u fundamental ketma-ketlik bo‘ladi.

1.7. Agar 3 ta sonning ko‘paytmasi nolga teng bo‘lsa, u holda ko‘paytuvchilarning kamida bittasi nolga teng bo‘ladi.

1.8. Uchburchakning ichki burchaklari yig‘indisi 180° ga teng.

2. Quyidagi mulohazalarni predikatlar algebrasi tilida ifodalang:

2.1. «Barcha ratsional sonlar haqiqiy».

2.2. «Ayrim ratsional sonlar haqiqiy emas».

2.3. «12 ga bo‘linuvchi har qanday natural son 2, 4 va 6 ga bo‘linadi».

2.4. «Ayrim ilonlar zaharli».

2.5. «Bir to‘g‘ri chiziqda yotmagan 3 ta nuqta orqali yagona tekislik o‘tkazish mumkin».

2.6. «Yagona x mavjudki, $R(x)$».

3. $A(x)$: «x – tub son», $B(x)$: «x – juft son», $C(x)$: «x – toq son», $D(x)$: «x y ni bo‘ladi» kabi xossalarni bildirsa quyidagilarni o‘qing:

3.1. $A(7)$.

3.2. $B(2) \wedge A(2)$.

3.3. $\forall x(B(x) \Rightarrow \forall y(D(x, y) \Rightarrow B(y))$.

3.4. $\forall x(C(x) \Rightarrow \forall y(A(y) \Rightarrow \rceil D(x, y)))$.

4. Quyidagi tasdiqlar va ularning inkorlarini predikatlar tilida ifodalang:

4.1. Tartiblangan to‘plam chiziqli tartiblangan deyiladi, agar to‘plamning ixtiyoriy x, y elementlari uchun yoki $x = y$, yoki $x < y$, yoki $x > y$ bo‘lsa.

4.2. $f(x)$ funksiya M to‘plamda chegaralangan deyiladi, agar shunday manfiymas L soni mavjud bo‘lib, har qanday $x \in M$ uchun $|f(x)| \le L$ bo‘lsa.

4.3. $f(x)$ funksiya M to‘plamda o‘suvchi deyiladi, agar to‘plamning ixtiyoriy x_1, x_2 elementlari uchun, $x_1 < x_2$ ekanligidan $f(x_1) < f(x_2)$ kelib chiqsa.

4.4. $f(x)$ funksiya davriy deyiladi, agar shunday $T \neq 0$ soni mavjud bo‘lib, funksiyaning aniqlanish sohasidan olingan har qanday x uchun $x - T$ va $x + T$ lar ham shu sohaga tegishli bo‘lib, $f(x \pm T) = f(x_2)$ shart bajarilsa.

5. Quyidagi formulalar uchun rele-kontakt sxemalarini tuzing:

5.1. $(X \wedge Y \wedge Z) \vee \urcorner(X \wedge Y \wedge Z) \vee (X \wedge Y)$.

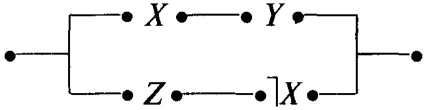
5.2. $(X \Rightarrow Y) \wedge (Y \Rightarrow Z)$.

5.3. $((X \Rightarrow Y) \wedge (Y \Rightarrow Z)) \Rightarrow (X \Rightarrow Z)$.

5.4. $(X \Rightarrow (Y \Rightarrow Z)) \Rightarrow (Y \Rightarrow X)$.

6. Quyidagi rele-kontakt sxemalariga mos keluvchi mulohazalar algebrasining formulasini aniqlang:

6.1.



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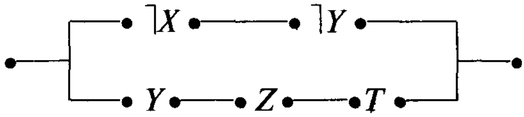
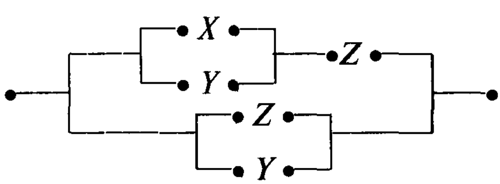
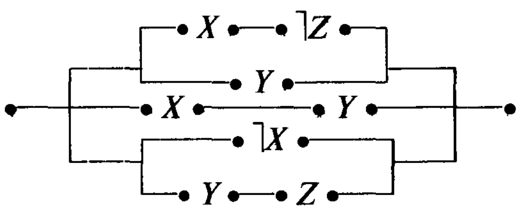


6.3.

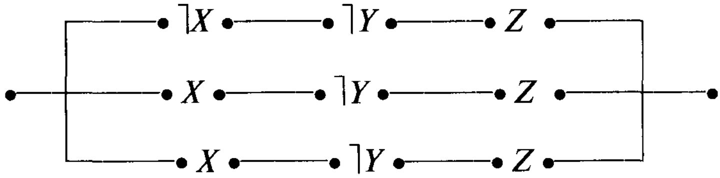


6.4.

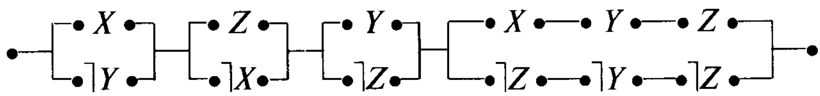


7. Quyidagi rele-kontakt sxemalarini soddalashtiring:

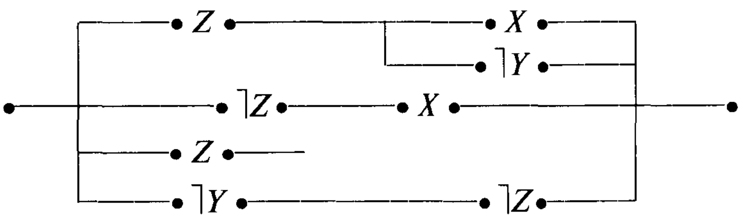
7.1.



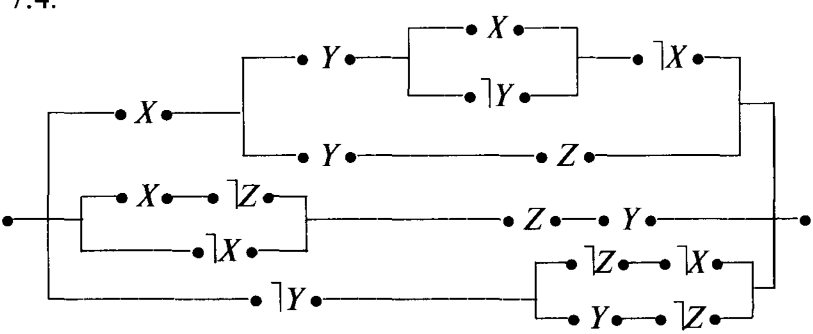
7.2.



7.3.



7.4.



Takrorlash uchun savollar

1. Teoremaning qanday turlarini bilasiz?
2. Teoremalarni isbotlash usullari qanday?
3. Matematik tasdiqlarni predikatlar tilida ifodalashga misol keltiring.
4. Rele-kontakt sxemalarini soddalashtirishda mulohazaviy formulalarning ahamiyati nimada?

II MODUL. TO'PLAMLAR VA MUNOSABATLAR

5-§. To'plam. To'plamlar ustida amallar. Eyler–Venn diagrammalari

Asosiy tushunchalar: to'plam, to'plam elementi, to'plamlarning tengligi, qismto'plam, bo'sh to'plam, universal to'plam, to'plamlar birlashmasi, to'plamlar kesishmasi, to'plamlar ayirmasi, to'plamlar simmetrik ayirmasi, idempotentlik xossasi, kommutativ amal, assotsiativ amal, distributivlik xossasi, to'plam to'ldiruvchisi, Eyler–Venn diagrammalari.

To'plam tushunchasi matematikaning asosiy tushunchalaridan biri bo'lib, misollar yordamida tushuntiriladi. To'plamni tashkil qiluvchi obyektlar to'plamining *elementlari* deyiladi.

Agar A to'plamning har bir elementi B to'plamning ham elementi bo'lsa, u $A \subset B$ orqali belgilanadi va A to'plam B to'plamning *to'plamostisi* deyiladi.

Bir xil elementlardan tashkil topgan to'plamlar teng deyiladi. A va B to'plamlarning teng bo'lishi uchun $A \subset B$ va $B \subset A$ bo'lishi zarur va yetarli ekanligini ko'rish qiyin emas. Bitta ham elementi yo'q to'plamni *bo'sh to'plam* deb ataymiz va uni $\varnothing$ orqali belgilaymiz.

A va B to'plamlarning kamida biriga tegishli bo'lgan barcha elementlardan tashkil topgan $A \cup B$ to'plam A va B to'plamlarning *birlashmasi* yoki *yig'indisi* deyiladi.

A va B to'plamlarning *kesishmasi* yoki *ko'paytmasi* deb, A va B to'plamlarning barcha umumiy, ya'ni A ga ham, B ga ham tegishli elementlardan tashkil topgan $A \cap B$ to'plamga aytiladi.

A va B to'plamlarning *ayirmasi* deb, A to'plamning B to'plamga kirmagan barcha elementlardan tashkil topgan to'plamga aytiladi. A va B to'plamlarning ayirmasi $A \backslash B$ ko'rinishida belgilanadi.

$(A \backslash B) \cup (B \backslash A)$ to'plam A va B to'plamlarning *simmetrik ayirmasi* deyilidi va $A \Delta B$ orqali belgilanadi.

Agar $A \subset B$ bo'lsa, $B \setminus A$ to'plam A to'plamning B to'plamgacha *to'ldiruvchi to'plam* deyiladi va $\subset A$ yoki A' orqali belgilanadi.

Misol. $A, B \subset M = \{1, ..., 20\}$ to'plamlar uchun quyidagilarni aniqlang:

$A \setminus B$, $B \setminus A$, $A \cup B$, $A \cap B$, A', B'. $A=\{1,3,5,7,9\}$, $B=\{2,4,7,8\}$.

Yechish. Berilgan to'plamlar uchun to'plamlar ustida bajariladigan amallarning ta'riflarini qo'llab, quyidagi to'plamlarni hosil qilamiz:

$A \setminus B = \{1,3,5,9\}$; $B \setminus A = \{2,4,8\}$; $A \cup B=\{1,2,3,4,5,7,8,9\}$;
$A \cap B = \{7\}$; $A'=\{2,4,6,8,10,11,12,13,14,15,16,17,18,19,20\}$;
$B'=\{1,3,5,6,9,10,11,12,13,14,15,16,17,18,19,20\}$.

Misol. $(A \cup B) \setminus C = (A \setminus B) \cup (B \setminus C)$ tenglikni isbotlang.

Yechish. To'plamlarning tengligini isbotlash uchun $M=N \Leftrightarrow \Leftrightarrow M \subset N \wedge N \subset M$ tasdiqdan foydalanamiz.

1) $\forall x \in ((A \cup B) \setminus C) \Rightarrow x \in (A \cup B) \wedge x \notin C \Rightarrow$
$\Rightarrow x \in A \vee x \in B \wedge x \notin C \Rightarrow (x \in A \wedge x \notin C) \vee (x \in B \wedge x \notin C) \Rightarrow$
$\Rightarrow x \in (A \setminus C) \vee x \in (B \setminus C) \Rightarrow [x \in ((A \setminus C) \cup (B \setminus C))$.

Bundan $(A \cup B) \setminus C \subset (A \setminus C) \cup (B \setminus C)$ ekanligi kelib chiqadi.

2) $\forall x \in ((A \setminus C) \cup (B \setminus C)) \Rightarrow x \in (A \setminus C) \vee x \in (B \setminus C) \Rightarrow$
$\Rightarrow (x \in A \wedge x \notin C) \vee (x \in B \wedge x \notin C) \Rightarrow (x \in A \vee x \in B \wedge x \notin C) \Rightarrow$
$\Rightarrow x \in (A \cup B) \wedge x \notin C \Rightarrow x \in ((A \cup B) \setminus C)$.

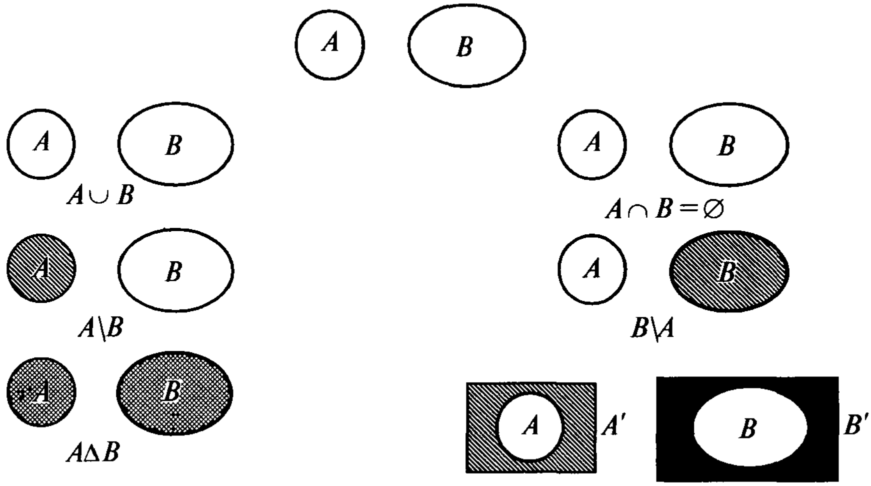
Bundan $(A \setminus C) \cup (B \setminus C) \subset (A \cup B) \setminus C$ ekanligi kelib chiqadi.

Demak, $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$.

To'plamlar ustida amallarni *Eyler—Venn diagrammalari* deb ataladigan quyidagi shakllar yordamida ifoda qilish amallarning xossalarini isbot qilishni ancha yengillashtiradi.

Universal to'plam to'g'ri to'rtburchak shaklida, uning to'plamostilari to'g'ri to'rtburchak ichidagi doiralar orqali ifoda qilinadi. U holda, ikki to'plam birlashmasi, kesishmasi, ayirmasi, to'lduruvchi to'plamlar, ikki to'plamning simmetrik ayirmasi, mos ravishda, quyidagicha ifodalanadi.

$(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$ tenglikni Eyler—Venn diagrammalarida tasvirlaymiz. Buning uchun tenglikda qatnashgan uchta to'plam uchun biror-bir vaziyatni aniqlab, tenglikning ikkala tomonini ikkita diagrammada tasvirlaymiz:



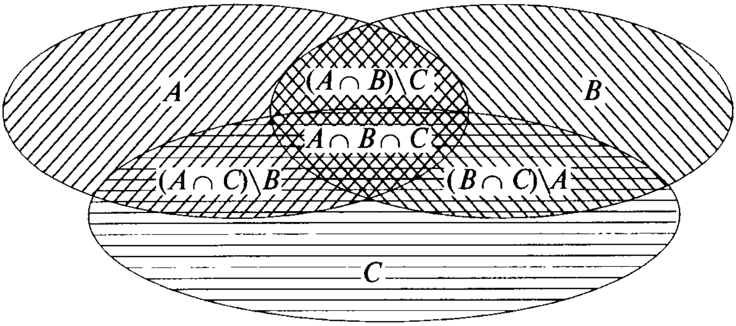
$A \cup B$ ni ; $(A \cup B) \setminus C$ ni ;

$A \setminus C$ ni ; $B \setminus C$ ni ;

$(A \setminus C) \cup (B \setminus C)$ ni orqali belgilab, quyidagilarni hosil qilamiz:

Masala. 58 nafar mexanizatorlar haydovchi, traktorchi, kombaynchi mutaxassisligiga ega. Ulardan 11 nafari ham haydovchi, ham traktorchi; 6 nafari ham traktorchi ham kombaynchi; 5 nafari – haydovchi va kombaynchi; 3 nafari – uchala mutaxassislikka ega. Agar kombaynchi, haydovchi va traktorchilar soni ayirmasi 7 ga teng arifmetik progressiyani tashkil etsa, u holda mexanizatorlardan necha nafari faqat bitta mutaxassislikka ega?

Yechish. A, B, C to'plamlar – haydovchi, traktorchi va kombaynchilar to'plamlari bo'lsin. Masala shartidan kelib chiqqan holda quyidagi Eyler–Venn diagrammasini tuzamiz:



Masala shartiga ko‘ra, $A \cap B \cap C = 3$, $A \cap B = 11$, $A \cap C = 5$, $B \cap C = 6$. U holda $(A \cap B)\backslash C = 8$, $(A \cap C)\backslash B = 2$, $(B \cap C)\backslash A = 3$ bo‘ladi. Faqat haydovchilar to‘plami $(A\backslash B)\backslash C$ ni x, faqat traktorchilar to‘plami $(B\backslash A)\backslash C$ ni y, faqat kombaynchilar to‘plami $(C\backslash A)\backslash B$ ni z orqali belgilasak, u holda $x + y + z + 3 + 8 + 2 + 3 = 58$ bo‘lib, bundan $x + y + z = 42$ kelib chiqadi. z, x, y lar ayirmasi 7 ga teng arifmetik progressiyani tashkil etganligi uchun $x = 14$, $y = 7$, $z = 21$.

J a v o b. Faqat haydovchilar — 14 nafar, faqat kombaynchilar — 7 nafar, faqat traktorchilar — 21 nafar.

Misol va mashqlar

1. A, $B \subset M = \{1, ..., 20\}$ to‘plamlar uchun quyidagilarni aniqlang:

$A\backslash B$, $B\backslash A$, $A \cup B$, $A \cap B$, A', B', $A \Delta B$:

1.1. $A = \{1, 3, 5\}$, $B = \{11, 13, 15\}$.

1.2. $A = \{3, 5, 7\}$, $B = \{8, ..., 15\}$.

1.3. $A = \{1, ..., 5\}$, $B = \{1, ..., 13\}$.

1.4. $A = \{5, ..., 12\}$, $B = \{12, ..., 15\}$.

2. Shunday A, B, C to‘plamlarni topingki, ular uchun quyidagi shartlar bajarilsin: $A \subset B$, $A \not\subset C$, $A \subset B \subset C$, $A \subset B \wedge A \subset C \wedge B \not\subset C$.

3. $M = \{\{\varnothing\}, \{1\}, \{1,2\}\}$ to‘plamning barcha to‘plamostilarini toping.

4. Agar $n \in N$ uchun $M_n=\{1,2, ..., n\}$ bo‘lsa, M_1, M_2, M_3, M_4, ..., M_n larning barcha to‘plamostilari sonini aniqlang.

5. Quyidagilarni isbotlang va Eyler—Venn diagrammalarini tuzing:

5.1. $(A \backslash B) \backslash C = (A \backslash C) \backslash (B \backslash C)$.

5.2. $A \backslash (B \backslash C) \subset A \cup C$.

5.3. $(A \backslash C) \backslash (B \backslash A) \subset A \backslash C$.

5.4. $A \backslash C \subset (A \backslash B) \cup (B \backslash C)$.

5.5. $((A \cup B)' \cap (A' \cup B'))' = A \cup B$.

5.6. $A \subset B \subset C \equiv A \cup B = B \cap C$.

5.7. $A \subset B \Rightarrow A \backslash C \subset B \backslash C$.

5.8. $A \subset B \Rightarrow A \cap C \subset B \cap C$.

5.9. $A \subset B \Rightarrow A \cup C \subset B \cup C$.

5.10. $B \subset A \wedge C = A \backslash B \Rightarrow A = B \cup C$.

5.11. $A \not\subset B \wedge B \cap C = \varnothing \Rightarrow A \cup C \not\subset B \cup C$.

5.12. $C = A \setminus B \Rightarrow B \cap C = \varnothing$.

5.13. $B \cap C = \varnothing \wedge A \cap C \neq \varnothing \Rightarrow A \backslash B \neq \varnothing$.

5.14. $A \subset C \Rightarrow A \cup (B \cap C) = (A \cup B) \cap C$.

5.15. $A \backslash (B \cap C) = (A \backslash B) \cup (A \backslash B)$.

6. Lyuis Kerrol masalasi. Shiddatli jangda 100 nafar qaroqchidan 70 nafari bitta ko'zidan, 75 nafari bitta qulog'idan, 80 nafari bitta qo'lidan va 85 nafari bitta oyog'idan ayrildi. Bir vaqtning o'zida ham ko'zi, ham qulog'i, ham qo'li va oyog'idan ayrilgan qaroqchilarning eng kam sonini aniqlang.

7. To'plamlar ustida bajariladigan algebraik amallarning quyidagi xossalarini isbotlang:

7.1. $A \cap A = A$ kesishmaning idempotentligi.

7.2. $A \cup A = A$ birlashmaning idempotentligi.

7.3. $A \cap B = B \cap A$ kesishmaning kommutativligi.

7.4. $A \cup B = B \cup A$ birlashmaning kommutativligi;

7.5. $(A \cap B) \cap C = A \cap (B \cap C)$ kesishmaning assotsiativligi.

7.6. $(A \cup B) \cup C = A \cup (B \cup C)$ birlashmaning assotsiativligi.

7.7. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ kesishmaning birlashmaga nisbatan distributivligi.

7.8. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ birlashmaning kesishmaga nisbatan distributivligi.

7.9. $(A \backslash B) \cap C = (A \cap C) \backslash B = (A \cap C) \backslash (B \cap C)$.

7.10. $X \setminus \bigcup_{i=1}^{\infty} A_i = \bigcap_{i=1}^{\infty} (X \setminus A_i)$.

7.11. $X \setminus \bigcap_{i=1}^{\infty} A_i = \bigcup_{i=1}^{\infty} (X \setminus A_i)$.

Takrorlash uchun savollar

1. To'plam tushunchasiga misollar keltiring.
2. To'plam elementi deb nimaga aytiladi?
3. Qismto'plam ta'rifini ayting.
4. Teng to'plamlar tushunchasiga ta'rif bering.
5. Bo'sh to'plam, universal to'plamlar ta'riflarini ayting. Misollar keltiring.
6. To'plamlar birlashmasi, kesishmasiga ta'rif bering.
7. To'plamlar ayirmasi, simmetrik ayirmasiga ta'rif bering.
8. To'plamlar birlashmasining qanday xossalarini bilasiz?
9. To'plamlar kesishmasining qanday xossalarini bilasiz?
10. To'plamlar ustida bajariladigan amallarning xossalari qanday tushunchalar yordamida isbotlanadi?
11. Eyler—Venn diagrammalarini tushuntiring.
12. Eyler—Venn diagrammalari yordamida to'plamlarning tengligini isbotlash mumkinmi?

6- §. Dekart ko'paytma. Binar munosabatlar. Ekvivalentlik munosabati

Asosiy tushunchalar: tartiblangan juftlik, kortej, to'plamlarning to'g'ri (Dekart) ko'paytmasi, binar, n-ar munosabatlar, binar munosabatning aniqlanish va qiymatlar sohasi, binar munosabat inversiyasi, binar munosabatlar kompozitsiyasi, refleksiv, antirefleksiv, simmetrik, antirefleksiv, tranzitiv binar munosabatlar, ekvivalentlik munosabati, faktor-to'plam.

$A_1, ..., A_n$ — bo'sh bo'lmgan to'plamlar $\forall a_1 \in A_1, ..., \forall a_n \in A_n$ — elementlardan tuzilgan barcha $(a_1, ..., a_n)$ n-liklar to'plami $A_1, ..., A_n$ to'plamlarning *dekart ko'paytmasi* deyiladi. $A_1, ..., A_n$ to'plamlarning dekart ko'paytmasi $A_1 \times ... \times A_n$ ko'rinishida belgilanadi.

$A = \varnothing$ to‘plam berilgan bo‘lsin. A^n ning ixtiyoriy ρ to‘plamostisi A to‘plamda aniqlangan *n-ar munosabat* deyiladi. A^2 ning ixtiyoriy to‘plamostisi A to‘plamida berilgan *binar munosabat* deyiladi.

Agar R — binar munosabatga tegishli barcha juftliklarning barcha birinchi koordinatalaridan tuzilgan to‘plam Dom$R$ *aniqlanish*, barcha ikkinchi koordinatalaridan tuzilgan to‘plam esa ImR *o‘zgarish sohalari* deyiladi.

$R^{\cup}=\{(b,\ a)|((b,\ a)\in R)\}$ munosabat R munosabatning *inversiyasi* deyiladi.

P va Q binar munosabatlar bo‘sh bo‘lmagan A to‘plamda berilgan bo‘lsin. U holda $P\circ Q = \{(a,c)|\ \exists\ b\in A,\ (a,b)\in Q\wedge\ (b,c)\in P\}$ to‘plam P va Q binar munosabatlarning *kompozitsiyasi* deyiladi.

A to‘plamida R binar munosabat berilgan bo‘lsin.

a) Agar $\forall a\in A$ uchun $(a,a)\in R$ bo‘lsa, R binar munosabat *refleksiv munosabat* deyiladi.

b) Agar $(a,b)\in R$ bo‘lishidan $(b,a)\in R$ bo‘lishi kelib chiqsa, ya’ni $R^{-1}=R$ shart bajarilsa, R *simmetrik munosabat* deyiladi.

d) Agar $\forall(a,b)\in R$ va $(b,a)\in R$ bo‘lishidan $(a,c)\in R$ bo‘lishi kelib chiqsa, ya’ni $R\circ R\subset R$ shart bajarilsa, R *tranzitiv munosabat* deyiladi.

e) Refleksiv, simmetrik va tranzitiv bo‘lgan binar munosabat *ekvivalentlik munosabati* deyiladi.

$\forall a\in A$ uchun a orqali A to‘plamning a ga ekvivalent bo‘lgan barcha elementlarini belgilaymiz va bu to‘plamni a element yaratgan ekvivalentlik sinfi deb ataymiz. R ekvivalentlik munosabati bo‘yicha aniqlangan barcha ekvivalentlik sinflari to‘plami A to‘plamning R ekvivalentlik munosabati bo‘yicha *faktor-to‘plami* deyiladi.

1-misol. $R,\ S,\ T\subset A\times A$ — binar munosabatlar uchun $R\circ(S\backslash T) = (R\circ S)\backslash(R\circ T)$ tenglikni isbotlang.

Yechish. Binar munosabatlar tartiblangan juftliklardan iborat to‘plamlar ekanligini bilgan holda to‘plamlar ayirmasi, to‘plamlar tengligi hamda binar munosabatlar kompozitsiyasining ta’riflaridan foydalanib, berilgan tenglikni isbotlaymiz:

1) $\forall(x,y) \in (R \circ (S\backslash T)) \Rightarrow \exists z \in A,\ (x,z) \in (S\backslash T) \wedge (z,y) \in R \Rightarrow$
$\Rightarrow (x,z) \in S \wedge (x,z) \notin T \wedge (z,y) \in R \Rightarrow (x,z) \in S \wedge (z,y) \in R \wedge$
$\wedge (x,z) \notin T \wedge (z,y) \in R \Rightarrow (x,y) \in (R \circ S) \wedge (x,y) \notin (R \circ T) \Rightarrow$
$\Rightarrow (x,y) \in ((R \circ S)\backslash(R \circ T))$. Demak, $R \circ (S\backslash T) \subset (R \circ S)\backslash(R \circ T)$;
2) $\forall(x,y) \in ((R \circ S)\backslash(R \circ T)) \Rightarrow (x,y) \in (R \circ S) \wedge (x,y) \in (R \circ T) \Rightarrow$
$\Rightarrow \exists\ z \in A,\ ((x,z) \in S \wedge (z,y) \in R) \wedge ((x,z) \notin T \wedge (z,y) \in R) \Rightarrow$
$\Rightarrow (x,z) \in S \wedge (x,z) \notin T \wedge (z,y) \in R \Rightarrow (x,z) \in (S\backslash T) \wedge (z,y) \in R \Rightarrow$
$\Rightarrow (x,y) \in (R \circ (S\backslash T))$. Demak, $(R \circ S)\backslash(R \circ T) \subset R \circ (S\backslash T)$.

Natijada $R \circ (S\backslash T) = (R \circ S)\backslash(R \circ T)$ tenglik isbotlandi.

2-misol. $M = \{1, 2, ..., 10\}$ to'plamda berilgan.

$R = \{<x,y>|\ x,\ y \in M \wedge x = y - 1\}$ binar munosabatning xossalarini tekshiring va grafini chizing.

Yechish. Berilgan binar munosabat qanday xossalarga bo'ysunishini tekshiramiz:

1) refleksivlik xossasi: $\forall(x \in M)\ (x = x - 1)$ yolg'on, chunki, masalan, M to'plamning 2 elementi uchun $2 \neq 2 - 1$. Demak, R – refleksiv emas;

2) antirefleksivlik xossasi: $\forall(x \in M),\ \neg(x = x - 1)$ rost. Demak, R – antirefleksiv;

3) simmetriklik xossasi: $\forall(x,y \in M)\ (x = y - 1 \Rightarrow y = x - 1)$ yolg'on. Chunki, masalan, $3, 4 \in M$ uchun $3 = 4 - 1 \Rightarrow 4 = 3 - 1$ da birinchi mulohaza rost va ikkinchi mulohaza yolg'on bo'lganligi uchun implikatsiya yolg'on. Demak, R – simmetrik emas;

4) antisimmetriklik xossasi: $\forall(x,y \in M)\ (x = y - 1 \wedge y = x - 1 \Rightarrow x = y)$ rost. Chunki, M to'plamning har qanday x, y elementlari uchun $x = y - 1$ va $y = x - 1$ mulohazalar bir vaqtda rost bo'la olmaydi. Bundan ularning konyunksiyasi berilgan to'plam elementlari uchun yolg'on. Birinchi mulohaza yolg'on bo'lgan implikatsiya rost ekanligini e'tiborga olsak, R antisimmetrik binar munosabat ekanligi kelib chiqadi;

5) tranzitivlik xossasi: $\forall(x,y,z \in M)\ (x = y - 1 \wedge y = z - 1 \Rightarrow x = z - 1)$ yolg'on mulohaza. Chunki, masalan, M to'plamning 3, 4, 5 elementlari uchun $(3 = 4 - 1) \wedge (4 = 5 - 1) \Rightarrow (3 = 5-1)$ implikatsiyada konyunksiya rost, lekin implikatsiya natijasi yol-

g'on mulohaza. Implikatsiya ta'rifiga ko'ra, $(3 = 4-1) \wedge (4 = 5-1) \Rightarrow$ $\Rightarrow (3 = 5-1)$ mulohaza yolg'on. Demak, R – tranzitiv emas;

6) R ekvivalentlik munosabati bo'la olmaydi, chunki refleksivlik, simmetriklik, tranzitivlik xossalariga ega emas;

7) R tartib munosabati bo'la olmaydi, chunki R antisimmetrik bo'lgani bilan tranzitiv emas.

3-misol. $A = \{1,2\}$, $B = \{2,5\}$ to'plamlar uchun $R = A \times B$, $S = B \times A$ binar munosabatlarni topib, $R \circ S$, $S \circ R$, R^2, S^2 larni aniqlang.

Yechish. To'plamlarning to'g'ri ko'paytmasi, binar munosabatlar kompozitsiyasi ta'riflaridan foydalanib, quyidagi to'plamlarni hosil qilamiz:

$R = A \times B = \{(1,2), (1,5), (2,2), (2,5)\}$;

$S = B \times A = \{(2,1), (2,2), (5,1), (5,2)\}$;

$R \circ S = \{(2,2), (2,5), (5,2), (5,5)\}$;

$S \circ R = \{(1,1), (1,2), (2,1), 2,2)\}$;

$R^2 = R \circ R = \{(1,2), (1,5), (2,2), (2,5)\}$;

$S^2 = S \circ S = \{(2,1), (2,2), (5,1), (5,2)\}$.

4-misol. Berilgan A = {lola, shoda, olomon, osmon, olma, boshoq} so'zlaridan iborat to'plam va undagi S binar munosabat:

«$x\ S\ y$» $\rightleftarrows$ «x va y so'zlarda o harfi bir xil sonda qatnashgan» berilgan. A/S faktor-to'plamni aniqlang.

Yechish. Faktor to'plam – bo'sh bo'lmagan to'plamda aniqlangan ekvivalentlik munosabati yordamida hosil qilingan ekvivalentlik sinflaridan tuzilgan to'plam. Berilgan to'plam 6 ta so'zdan iborat to'plam va undagi har qanday ikkita x,y so'zlar berilgan binar munosabatda bo'ladi, agar bu so'zlar tarkibida o harfi bir xil sonda qatnashgan bo'lsa.

To'plamda berilgan S binar munosabat ekvivalentlik munosabati ekanligini isbotlaymiz:

1. S – refleksivlik munosabati, chunki A to'plamning har bir so'zini o'zi bilan solishtirsak, ularda o harfi bir xil sonda qatnashgan.

2. S – simmetriklik munosabati, chunki A to'plamning har qanday x, y so'zlari uchun agar x so'z bilan y so'zda o harfi bir xil sonda qatnashgan bo'lsa, u holda y so'z bilan x so'zlarda ham o harfi bir xil sonda qatnashadi.

3. S – tranzitivlik munosabati, chunki A to'plamning har qanday x, y, z so'zlari uchun agar x so'z bilan y so'zda va y so'z bilan z so'zda o harfi bir xil sonda qatnashgan bo'lsa, u holda x so'z bilan z so'zlarda ham o harfi bir xil sonda qatnashadi.

Endi S ekvivalentlik munosabati yordamida ekvivalentlik sinflarini tuzamiz. Buning uchun «lola» so'zi bilan ekvivalentlik munosabatida bo'lgan so'zlarni bir to'plamga yig'amiz:

$S/_{\text{lola}}$ = {lola, shoda, olma}. Xuddi shunday yo'l bilan qolgan ekvivalentlik sinflarini tuzamiz:

$S/_{\text{osmon}}$ = {osmon, boshoq}, $S/_{\text{olomon}}$ = {olomon}.

U holda $A/_S = \{S/_{\text{lola}}, S/_{\text{osmon}}, S/_{\text{olomon}}\}$.

Misol va mashqlar

1. Quyidagi tengliklarni isbotlang

1.1. $(A \backslash B) \times C = (A \times C) \backslash (B \times C)$.

1.2. $A \times (B \backslash C) = (A \times B) \backslash (A \times C)$.

1.3. $(A \cup B) \times C = (A \times C) \cup (B \times C)$.

1.4. $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

1.5. $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$.

1.6. $A \subset B \Rightarrow A \times C \subset B \times C$.

1.7. $A \cup B \subset C \Rightarrow A \times B = (A \times B) \cap (C \times B)$.

1.5. $(A \times B) \cup (B \times A) = C \times C \Rightarrow A = B = C$.

2. R, S, T – binar munosabatlar uchun quyidagilarni isbotlang:

2.1. $(R \cap S)^{\cup} = R^{\cup} \cap S^{\cup}$.

2.2. $(R \cup S)^{\cup} = R^{\cup} \cup S^{\cup}$.

2.3. $R \circ (S \circ T) = (R \circ S) \circ T$.

2.4. $(R \circ S)^{\cup} = S^{\cup} \circ R^{\cup}$.

2.5. $(R \cup S) \circ T = R \circ T \cup S \circ T$.

2.6. $R \circ (S \cup T) = (R \circ S) \cup (R \circ T)$.

2.7. $(R \cap S) \circ T \subset R \circ T \cap S \circ T$.

2.8. $R \circ (S \cap T) \subset R \circ S \cap R \circ T$.

2.9. $\mathrm{Dom}(R^{\cup}) = \mathrm{Im}\, R$.

2.10. $\mathrm{Im}\,(R^{\cup}) = \mathrm{Dom} R$.

2.11. Dom $(R \circ S) \subset \mathrm{Dom} S$.

2.12. $\mathrm{Im}(R \circ S) \subset \mathrm{Im} R$.

2.13. $(R \backslash S)^{\cup} = R^{\cup} \backslash S^{\cup}$.

2.14. R, S − tranzitiv $\Rightarrow R \cup S$ − tranzitiv.

2.15. R , S − refleksiv $\Rightarrow R \cup S$, $R^{\cup}$, $S^{\cup}$ − refleksiv.

2.16. R, S − simmetrik $\Rightarrow R \cup S$, $R^{\cup}$, $S^{\cup}$ − simmetrik.

2.17. R, S − ekvivalent $\Rightarrow R \cup S$, $R^{\cup}$, $S^{\cup}$ − ekvivalent.

2.18. R, S − antirefleksiv $\Rightarrow R \cup S$, $R^{\cup}$, $S^{\cup}$ − antirefleksiv.

2.19. R, S − antisimmetrik $\Rightarrow R \cup S$, $R^{\cup}$, $S^{\cup}$ − antisimmetrik.

3. $M_3=\{1,2,3\}$ to'plamda nechta turli ekvivalentlik munosabatlari mavjud? M_4 da-chi?

4. $M = \{1, 2, ..., 20\}$ to'plamda berilgan quyidagi binar munosabatlarning xossalarini tekshiring:

4.1. $R = \{<x,y> \mid x,y \in M \wedge x \leq y + 1\}$.

4.2. $R = \{<x,y> \mid x,y \in M \wedge |x| = |y| \}$.

4.3. $R = \{<x,y> \mid x,y \in M \wedge x \vdots y\}$.

4.4. $R = \{<x,y> \mid x,y \in M \wedge x < y\}$.

4.5. $R = \{<x,y> \mid x,y \in M \wedge x \leq y\}$.

4.6. $R = \{<x,y> \mid x,y \in M \wedge x \neq y\}$.

4.7. $R = \{<x,y> \mid x,y \in M \wedge x^2 + x = y^2 + y\}$.

4.8. $R = \{<x,y> \mid x,y \in M \wedge x \vdots y \vee x < y\}$.

4.9. $R = \{ <x,y> \mid x,y \in M \wedge (x - y) \vdots 2\}$.

4.10. $R = \{<x,y> \mid x,y \in M \wedge x + y = 12\}$.

4.11. $R = \{<x,y> \mid x,y \in M \wedge x + y \leq 7\}$.

4.12. $R = \{<x,y> \mid x,y \in M \wedge (x > y \wedge x \vdots 3)\}$.

4.13. $R = \{<x,y> \mid x,y \in M \wedge x + y \geq 10\}$.

4.14. $R = \{<x,y> \mid x,y \in M \wedge x - y = -2\}$.

5. $R = A \times B$, $S = B \times A$ binar munosabatlar uchun $R \circ S$, $S \circ R$, R^2, S^2 larni aniqlang:

5.1. $A = \{1, 3, 5\}$, $B = \{1, 3, 15\}$.

5.2. $A = \{3, 6, 9\}$, $B = \{4, 8, 16\}$.

5.3. $A = B = \{1, 2, 3, 4\}$.

5.4. $A = \{0, 2, 4\}$, $B = \{\alpha, \beta, \gamma\}$.

5.5. $A = \{\square, \Diamond\}$, $B = \{\clubsuit, \blacklozenge, \heartsuit, \spadesuit\}$.

5.6. $A = \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$, $B = \{\cap, \cup, \in, \subset\}$.

6. Berilgan A to'plam va undagi S binar munosabat yordamida A/S faktor-to'plamni aniqlang:

6.1. A – tekislikdagi to'g'ri chiziqlar to'plami, S – parallellik munosabati.

6.2. A – tekislikdagi romblar to'plami, S – o'xshashlik munosabati.

6.3. A – tekislikdagi to'rtburchaklar to'plami, S – o'xshashlik munosabati.

6.4. $A = \{ax + by + c = 0 \mid a, b, c \in R\}$, S – parallellik munosabati.

6.5. A – tekislikdagi muntazam ko'pburchaklar to'plami, S – o'xshashlik munosabati.

6.6. A – bir ko'chada joylashgan binolar to'plami, S – «qavatlar soni teng» munosabati.

6.7. A – tekislikdagi aylanalar to'plami, S – «radiuslari teng» munosabati.

6.8. A – maktabdagi sinflar to'plami, S – «o'quvchilar soni teng» munosabati.

6.9. A – maktabdagi sinflar to'plami, S – «qizlar soni teng» munosabati.

6.10. A – sinfdagi o'quvchilar to'plami, S – «ismlari bir xil harfdan boshlanadi» munosabati.

6.11. A – sinfdagi o'quvchilar to'plami, S – «ismlarda a harfi bir xil marta qatnashgan» munosabati.

6.12. A – tekislikdagi kesmalar to'plami, S – parallellik munosabati.

6.13. A – tekislikdagi vektorlar to'plami, S – tenglik munosabati.

6.14. $A = Z$, S – «p tub songa bo'lgandagi qoldiqlari teng» munosabati.

Takrorlash uchun savollar

1. Tartiblangan juftlik nima?
2. Tartiblangan juftliklar qachon teng bo'ladi?

3. To'plamlarning to'g'ri (Dekart) ko'paytmasi nima?
4. Tartiblangan *n* lik qanday hosil qilinadi?
5. Binar munosabatga ta'rif bering. Misollar keltiring.
6. Binar munosabatning aniqlanish sohasiga misol keltiring.
7. Binar munosabatning o'zgarish sohasiga ta'rif bering.
8. Binar munosabat inversiyasi qanday hosil qilinadi?
9. Binar munosabatlar kompozitsiyasini misol yordamida tushuntiring.
10. Refleksiv binar munosabatni ta'riflang va misol keltiring.
11. Simmetrik binar munosabatni ta'riflang va misol keltiring.
12. Tranzitiv binar munosabatni ta'riflang va misol keltiring.
13. Ekvivalentlik binar munosabatini ta'riflang va misol keltiring.
14. To'plamni bo'laklash deganda nimani tushunasiz?
15. Faktor-to'plamni tushuntiring.

7-§. Akslantirish (funksiya). Tartib munosabati. Graflar

Asosiy tushunchalar: akslantirish (funksiya), akslantirishning aniqlanish sohasi, akslantirishning qiymatlar to'plami, akslantirishlar kompozitsiyasi, inyektiv syuryektiv, biyektiv, teskarilanuvchi akslantirish, tartib munosabati, qisman tartib, qat'iy tartib, chiziqli tartib, tartiblangan to'plam, to'la tartiblangan to'plam, binar munosabat grafi.

f — A to'plamda berilgan binar munosabat bo'lsin. Agar $\forall x, y, z \in A$ lar uchun $(x, y) \in f$ va $(x, z) \in f$ bo'lishidan $y=z$ kelib chiqsa, u holda f binar munosabat *akslantirish* (funksiya) deyiladi.

$\mathrm{Dom} f = \{x / \exists y (x, y) \in f\}$ to'plam funksiyaning *aniqlanish sohasi*, $\mathrm{Im} f = \{y / \exists x (x, y) \in f\}$ to'plam funksiyaning *o'zgarish sohasi* deyiladi.

f va g funksiyalar berilgan bo'lsin, u holda $f \circ g = \{(x, z) / \exists t (x, t) \in g$ va $(t, z) \in f\}$ to'plam f va g funksiyalarning *kompozitsiyasi* deyiladi.

Agar $\forall x_1, x_2 \in A$ va $x_1 \in x_2$ elementlar uchun $f(x_1) = f(x_2)$ bo'lsa, $f: A \to B$ — inyektiv, $\mathrm{Im} f = B$ bo'lsa, *syuryektiv akslantirish* deyiladi. Agar f ham syuryektiv, ham inyektiv akslantirish bo'lsa, u holda *biyektiv akslantirish* deyiladi.

$f: A \to B$, $g: B \to A$ akslantirishlar berilgan bo'lsin, agar $f \circ g = E_B$ bo'lsa, f akslantirish g akslantirishga *chapdan teskari* deyiladi.

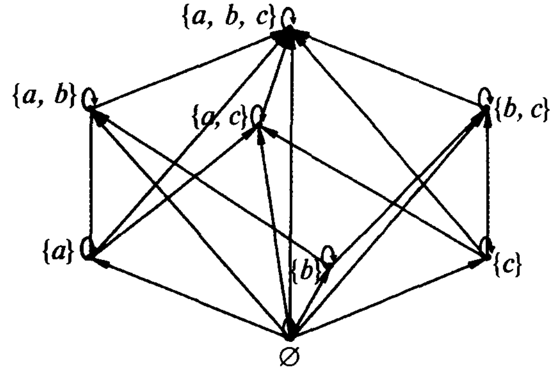
A to'plamda berilgan $R \subset A \times A$ antisimmetrik va tranzitiv munosabat A to'plamdagi *tartib munosabati* deyiladi. Tartib munosabati refleksiv munosabat bo'lsa, *noqat'iy*; antirefleksiv munosabat bo'lsa, *qat'iy tartib munosabat* deyiladi.

A to'plamda R – tartib munosabat berilgan bo'lsin. U holda, agar $\forall a, b \in A$ elementlar uchun xRy yoki $x=y$ yoki yRx munosabatlardan kamida bittasi albatta bajarilsa, bunday munosabat A to'plamdagi *chiziqli tartib munosabat* deyiladi.

Chiziqli bo'lmagan tartib munosabat, *qisman tartib munosabat* deyiladi.

Tekislikda chekli sondagi nuqtalardan va shu nuqtalarning ba'zilarini tutashtiruvchi chiziqlardan iborat geometrik shakl *graf* deyiladi. Nuqtalar grafning *uchlari*, chiziqlar esa grafning *qirralari* deyiladi.

Misol. $A = \{a, b, c\}$ to'plam va $\mathcal{B}(A)$ uning barcha to'plamostilari bo'lsin. U holda to'plamosti bo'lish munosabatini quyidagi graf yordamida ifoda qilish mumkin:



Misol va mashqlar

1. Quyidagi munosabatlardan qaysilari akslantirish bo'ladi? Akslantirishlarning aniqlanish va qiymatlar sohalarini aniqlang:

1.1. $R = \{(1, 1), (2, 1), (3, 1)\}$.

1.2. $R = \{(1, 1), (1, 4), (3, 2), (4, 4)\}$.

1.3. $R = \{(n, n+1) \mid n \in N\}$.

1.4. $R = \{(1, z) \mid z \in Z\}$.

2. Maktab matematikasidan inyektiv, syuryektiv, biyektiv, teskari funksiyalarga misollar keltiring.

3. $R = \{(1, 1), (2, 1), (3, 2), (1, 5), (4, 4)\}$ binar munosabatni M_5 dagi qisman tartib munosabatgacha to'ldiring.

4. M_4 dagi barcha qisman tartib munosabatlarni graflar yordamida keltiring.

5. R, S – binar munosabatlar uchun quyidagilarni isbotlang:

5.1. R, S – qat'iy tartib munosabat $\Rightarrow R \cup S$, $S^{\cup}$ – qat'iy tartib munosabat.

5.2. R, S – qisman tartib munosabat $\Rightarrow R \cup S$, $R^{\cup}$ – qisman tartib munosabat.

5.3. R, S – chiziqli tartib munosabat $\Rightarrow R \cup S$ – chiziqli tartib munosabat.

6. $M = \{1, 2, ..., 20\}$ to'plamda berilgan quyidagi binar munosabatlarning xossalarini tekshiring va grafini chizing:

6.1. $R = \{<x, y> \mid x,y \in M \wedge x^2 = y^2\}$.

6.2. $R = \{<x, y> \mid x,y \in M \wedge x < y\}$.

6.3. $R = \{<x, y> \mid x,y \in M \wedge x + y \geq 20\}$.

6.4. $R = \{<x, y> \mid x,y \in M \wedge (x + y) \vdots 5\}$.

6.5. $R = \{<x, y> \mid x,y \in M \wedge (x > y \wedge x \vdots 3)\}$.

6.6. $R = \{<x, y> \mid x,y \in M \wedge x - y = 2\}$.

6.7. $R = \{<x, y> \mid x,y \in M \wedge (x - y) \vdots 4\}$.

6.8. $R = \{<x, y> \mid x,y \in M \wedge x - y = 6\}$.

Takrorlash uchun savollar

1. Akslantirishning aniqlanish, qiymatlar sohasiga misol keltiring.
2. Akslantirishlar kompozitsiyasini tushuntiring.
3. Akslantirishlar kompozitsiyasi xossalarini ayting.
4. Inyektiv akslantirishga maktab matematikasidan misol keltiring.

5. Syuryektiv akslantirishga maktab matematikasidan misol keltiring.

6. Biyektiv akslantirish maktabda qanday nomlangan? Misol keltiring.

7. Tartib munosabatiga misollar keltiring.

8. Tartib munosabat turlarini maktab matematikasidan olingan misollar yordamida tushuntiring.

9. Tartiblangan to'plamlarga misollar keltiring.

10. Butun sonlar to'plami to'la tartiblangan to'plam bo'ladimi?

11. Qanday binar munosabatni graf yordamida ifodalash mumkin?

III MODUL. ALGEBRA VA ALGEBRAIK SISTEMALAR

8-§. Algebra. Faktor-algebra

✔ **Asosiy tushunchalar**: binar algebraik amal, n-ar algebraik amal, kommutativ binar amal, assotsiativ binar amal, neytral element, regular element, simmetrik element, kongruensiya, algebra, algebra turi, gomomorfizm, izomorfizm, faktor-algebra, algebraik sistema.

A^n to'plamni A ga akslantiradigan har qanday akslantirish A to'plamda berilgan *n-ar* yoki *n-o'rinli algebraik amal* deyiladi. Bu yerda n – manfiy bo'lmagan butun son bo'lib, algebraik amalning rangi deyiladi. $A \neq \varnothing$ to'plam va A da bajariladigan algebraik amallar to'plami Ω berilgan bo'lsin. (A, Ω) juftlik *algebra* deyiladi.

a) Agar $\forall a, b \in A$ uchun $a * b = b * a$ bo'lsa, u holda $*$ amali A to'plamida *kommutativ algebraik amal* deyiladi;

b) Agar $\forall a, b, c \in A$ uchun $a * (b * c) = (a * b) * c$ shart bajarilsa, $*$ amali A to'plamida *assotsiativ algebraik amal* deyiladi;

d) Agar $\forall a \in A$ uchun shunday $e \in A$ topilib, $e * a = a$ shart bajarilsa, e element $*$ amalga nisbatan *chap neytral element*, agar $a * e = a$ shart bajarilsa, *o'ng neytral element*, agar ikkala shart ham bajarilsa *neytral element* deyiladi.

$a \in A$ element va $\forall b, c \in A$ elementlar uchun $a * b = a * c$ tenglikdan $b = c$ kelib chiqsa, u holda a element *chap regular element* deyiladi.

$a \in A$ element uchun shunday $a' \in A$ element topilib, $a' * a = e$ bo'lsa, a' element a elementga *chap simmetrik element* deyiladi.

A to'plamdagi ekvivalentlik munosabati bo'lsin. Agar a_1, a_2, b_1, $b_2 \in A$ elementlar uchun $a_1 R b_1$ va $a_2 R b_2$ shartlardan $(a_1 * a_2) R (b_1 * b_2)$ kelib chiqsa, u holda R ekvivalentlik munosabati *kongruensiya* deyiladi.

Agar (A, Ω) algebra berilgan bo'lsa, Ω to'plamdagi amallarning ranglaridan iborat to'plam *algebraning turi* deyiladi.

$A \neq \varnothing$ to'plam uchun Ω to'plam A da aniqlangan amallar to'plami, Ω' to'plam A da aniqlangan munosabatlar to'plami bo'lsin. U holda (A, Ω, Ω'') tartiblangan uchlik *algebraik sistema* deyiladi.

(A, Ω), (B, Ω') algebralar berilgan bo'lsin. Ω dagi barcha amallarni saqlaydigan $\varphi: A \to B$ akslantirish (A, Ω) algebraning (B, Ω) algebraga *gomomorfizmi* deyiladi.

$\varphi: A \to B$ akslantirish (A, Ω) algebraning (B, Ω') algebraga gomomorfizmi bo'lsin. U holda agar φ inyektiv akslantirish bo'lsa, *monomorfizm*, φ syuryektiv akslantirish bo'lsa, *epimorfizm*, φ biyektiv akslantirish bo'lsa, *izomorfizm* deyiladi.

Algebrani o'zini o'ziga gomomorf akslantirish *endomorfizm*, algebrani o'zini o'ziga izomorf akslantirish esa *avtomorfizm* deyiladi.

(A, Ω_1) va (A, Ω_2) bir xil tipli algebralar berilgan bo'lib, $B \subset A$ bo'lsin. Agar $\forall \omega_1 \in \Omega$ n-ar algebraik amalga Ω_2 dan mos keladigan n-ar algebraik amalni ω_2 orqali belgilaymiz. Agar $\forall b_1, ..., b_n \in B$ uchun $\omega_2(b_1, ..., b_n) = \omega_1(b_1, ..., b_n)$ tenglik bajarilsa, u holda ω_2 n-ar algebraik amal ω_1 n-ar algebraik amalning B to'plami bo'yicha *cheklangani*, (B, Ω_1) algebra esa (A, Ω_2) algebraning *qism algebrasi* yoki *algebraosti* deyiladi.

(A, Ω) algebra va $\sim\Omega$ dagi har bir amalga nisbatan kongruensiya bo'lsin. Ω^* to'plam esa $A/\sim$ faktor-to'plamda aniqlangan va Ω dagi amallar bilan assotsirlangan barcha amallar to'plami bo'lsin. U holda $(A/\sim, \Omega^*)$ algebra (A, Ω) algebraning $\sim$ kongruensiya bo'yicha *faktor-algebrasi* deyiladi.

Misol. Z – butun sonlar to'plami bo'lsin. Z da $a \sim b$ deymiz va a ga $a - b$ juft son bo'lsa, $\sim$ munosabat kongruensiya bo'lishi ravshan. Bu munosabat bo'yicha ekvivalentlik sinflari faqat ikkita bo'lib, ular $[0]$; $[1]$ sinflardan iborat. Bu sinflar to'plamini $Z/\sim$ orqali belgilaylik, $\forall [a], [b] \in Z/\sim$ uchun $\oplus$, $\odot$ amallarini $[a] \oplus [b] = [a+b]$, $[a] \odot [b] = [a \cdot b]$ tengliklar orqali aniqlasak, $(\{[0], [1]\}, \oplus, \odot, [0], [1])$ algebra $(Z+, \cdot, 0, 1)$ algebraning faktor algebrasi bo'ladi.

Misol va mashqlar

1. Berilgan munosabatlarning qaysilari A to‘plamda aniqlangan amal bo‘lishini tekshiring va uning rangini aniqlang:

1.1. $\circ = \{(a,b,c) \mid a,b,c \in R \wedge a = bc\}, A = R.$

1.2. $\circ = \{(a,b) \mid a,b \in R_+ \wedge b^3 = a\}, A = R_+.$

1.3. $\circ = \{(a,b,c) \mid a,b,c \in R \wedge c = a^b\}, A = R.$

1.4. $\circ = \{(a,b) \mid a,b \in Z \wedge ab = 1\}, A = Z.$

1.5. $\circ = \{(a,b,c,d) \mid a,b,c,d \in Z \wedge d = [a,b,c]\}, A = Z.$

2. A to‘plamda amal quyidagi Keli jadvali yordamida berilgan:

$\circ$	a	b	c	d
a	a	a	a	a
b	b	b	b	b
c	c	c	c	c
d	d	d	d	d

Uning assotsiativligini, kommutativ emasligini isbotlang. Neytral element mavjudmi?

3. Natural sonlar to‘plamida har qanday n, m natural sonlar uchun quyidagi amallar kommutativ, assotsiativ ekanligini isbotlang. Bu amallarga nisbatan neytral element mavjudmi?

3.1. $n * m = l,\ l = (n, m).$

3.2. $n * m = l,\ l = [n, m].$

4. Quyidagi shartlar asosida algebraga misol keltiring:

4.1. Ikkita binar amal va bitta unar amal aniqlangan.

4.2. Ikkita binar amal va ikkita unar amal aniqlangan.

4.3. Uchta binar amal va bitta binar amal aniqlangan.

5. R^2 to‘plamda quyidagi amallarning kommutativ, assotsiativ, distributiv ekanligini tekshiring:

5.1. $(a,b)+(c,d)=(a+c,b+d),(a,b)\cdot(c,d)=(ac,ad+bc).$

5.2. $(a,b)+(c,d)=(a+c,b+d),(a,b)\cdot(c,d)=(ac+bd,ad+bc).$

6. Quyidagi algebralar orasida gomomorfizm o‘rnating va uning turini aniqlang:

6.1. $< \{3^z \mid z \in Z\}; \cdot, -1, 1 > \wedge < Z; +, -, 0 >$.

6.2. $< Z; +, -, 0 > \wedge < 2Z; +, -, 0 >$.

6.3. $< \{a + bi \mid a, b \in R \wedge i^2 = -1\}; +, -, 0 > \wedge < R^2; +, -, 0 >$.

6.4. $< Z; +, -, 0 > \wedge < Z^2, +, -, 0 >$.

6.5. $< Z^-; +, > \wedge < Z^+; + >$.

6.6. $< \{2^z \mid z \in Z\}; \cdot, -1, 1 > \wedge < \{3z \mid z \in Z\}; \cdot, -1, 1 >$.

6.7. $< 2Z; +, -, 0 > \wedge < 5Z; +, -, 0 >$.

6.8. $< \{a + b\sqrt{p} \mid a, b \in Q\}; +, \cdot > \wedge < \{a - b\sqrt{p} \mid a, b \in Q\}; +, \cdot >$.

7. Quyida berilgan A to'plam, undagi S binar munosabat tashkil qilgan A/S faktor-to'plamni faktor-algebragacha to'ldirish mumkinmi:

7.1. $A = \{ax + by + c = 0 \mid a, b, c \in R\}$, S – parallellik munosabati.

7.2. $A = \{ax + by = 0 \mid a, b \in R\}$, S – tenglik munosabati.

7.3. A – tekislikdagi kesmalar to'plami, S – parallellik munosabati.

7.4. A – tekislikdagi kesmalar to'plami, S – tenglik munosabati.

7.5. A – tekislikdagi vektorlar to'plami, S – parallellik munosabati.

7.6. A – tekislikdagi vektorlar to'plami, S – tenglik munosabati.

7.7. $A = Z$, S – «p tub songa bo'lgandagi qoldiqlari teng» munosabati.

8. Algebraosti bo'lish munosabati noqat'iy tartib munosabat bo'lishini isbotlang.

9. Agar φ– (A, Ω_1) algebraning (B, Ω_2) algebraga izomorfizmi bo'lsa, u holda φ ga teskari bo'lgan φ^{-1} akslantirish (B, Ω_2) algebraning (A, Ω_1) algebraga izomorfizmi ekanligini isbotlang.

10. (G_1, Ω_1), (G_2, Ω_2), (G, Ω) – bir xil turdagi algebralar berilgan bo'lib, $G_1 \cong G_2$, (G_2, Ω_2) algebra (G, Ω) algebraning algebraostisi bo'lsin. U holda (G_1, Ω_1) qism algebradan iborat qism algebraga ega bo'lgan (G, Ω) algebraga izomorf (G_3, Ω_3) algebra mavjudligini isbotlang.

11. $\varphi: A \to B$ akslantirish (A, Ω_1) algebraning (B, Ω_2) algebraga epimorfizmi, $R = \{(x', x'')\} \mid \forall x', x'' \in A$, $\varphi(x') = \varphi(x'')$ esa A da aniqlangan ekvivalentlik munosabati bo'lsin. U holda $(A/R, \Omega^*)$ faktor algebra (B, Ω_1) algebraga izomorfligini isbotlang.

Takrorlash uchun savollar

1. Algebra tushunchasiga maktab matematikasidan misollar keltiring.
2. Algebraning turi qanday aniqlanadi?
3. Algebralar gomomorfizmini tushuntiring.
4. Monomorfizm, epimorfizmga misollar keltiring.
5. Izomorfizm, avtomorfizm ta'rifidagi umumiy, farqli shartlarni aniqlang.
6. Gomomorfizmlar kompozitsiyasi yana gomomorfizm ekanligini isbotlang.
7. Biyektiv akslantirishlar izomorfizm bo'la oladimi?
8. Algebralar izomorfizmi ekvivalentlik munosabati ekanligini asoslang.
9. Algebraostilar kesishmasi yana algebra bo'lishini isbotlang.
10. Faktor-algebra tushunchasini misol yordamida tushuntiring.
11. Akademik litsey, maktab matematikasidan algebraik sistemaga doir misollar keltiring.

9-§. Gruppa. Halqa. Maydon

✓ **Asosiy tushunchalar**: gruppoid, yarimgruppa, monoid, gruppa, gruppalar gomomorfizmi, gruppaosti, halqa, kommutativ halqa, butunlik sohasi, halqalar gomomorfizmi, qismhalqa.

$A \neq 0$ to'plam va unda aniqlangan $*$ binar algebraik amal berilgan bo'lsin. U holda $(A, *)$ juftlik *gruppoid* deb ataladi.

$(A, *)$ gruppoidda $*$ assotsiativ amal bo'lsa, bunday gruppoid *yarimgruppa* deyiladi.

Neytral elementga ega bo'lgan yarimgruppa *monoid* deyiladi.

Bizga (2, 1) turli $(G, *, 1)$ algebra berigan bo'lib, quyidagi shartlar bajarilsin:

1) $*$ – binar algebraik amal assotsiativ, ya'ni $\forall a, b, c \in G$ uchun $(a * b) * c = a * (b * c)$ bo'lsin;

2) G da neytral element mavjud, ya'ni $\forall a \in G$ uchun shunday $e \in G$ topilib, $e * a = a$ shart bajarilsin.

3) har qanday $a \in G$ uchun $a' * a = e$ bo'lsin.

U holda $(G, *, ')$ algebra *gruppa* deyiladi.

Gruppadagi amal kommutativ, ya'ni $\forall a, b \in G$ uchun $(a * b) = b * a$ shart bajarilsa, bunday gruppa *abel gruppasi* deyiladi.

Gruppadagi elementlar soni uning *tartibi* deyiladi. Agar gruppa tartibi natural sondan iborat bo'lsa, bunday gruppa *chekli tartibli gruppa*, aks holda *cheksiz tartibli gruppa* deyiladi.

Gruppadagi binar algebraik amal « · » bo'lsa, bunday gruppani *multiplikativ gruppa*, «+» bo'lsa, *additiv gruppa* deymiz.

Gruppalar nazariyasida gomomorfizm, izomorfizm, gruppaosti tushunchalari algebradagi mos tushunchalarning xususiy hollaridir.

Agar $(K, +, -, \cdot)$ turli algebra uchun quyidagi shartlar bajarilsa:

(1) $(K, +, -)$ – abel gruppasi;

(2) $(K, \cdot)$ – yarimgruppa;

(3) $\forall a, b, c \in K$ uchun $a \cdot (b + c) = a \cdot b + a \cdot c$ va $(b + c) \cdot a = b \cdot a + c \cdot a$, u holda $(K, +, -, \cdot)$ algebra *halqa* deyiladi.

$(K; +, -)$ additiv gruppaning neytral elementi halqaning noli deyiladi va 0 orqali belgilanadi.

Agar ko'paytirish amali assotsiativ bo'lsa, halqa *assotsiativ halqa*, ko'paytirish amaliga nisbatan *birlik element* mavjud bo'lsa, halqa birlik elementli halqa deyiladi.

Nolning bo'luvchilariga ega bo'lmagan assotsiativ, kommutativ halqada $1 \neq 0$ shart bajarilsa, bunday halqa *butunlik sohasi* deyiladi.

$(K; +, -, \cdot)$ halqa berilgan bo'lsin. L esa K ning bo'sh bo'lmagan to'plamostisi bo'lsin. Agar L to'plam K dagi $+, -, \cdot$ amallariga nisbatan algebraik yopiq bo'lsa, ya'ni $\forall a, b \in L$ uchun

$a+b\in L$, $a\cdot b\in L$, $-a\in L$ shartlar bajarilsa, (L, +, −, ·) algebra (K; +, −, ·) halqaning *halqaostisi* deyiladi.

1-misol. $K=\{a+b\sqrt{p}\mid a,b\in R\}$ to'plam maydon tashkil etishini isbotlang.

Yechish. Maydon ta'rifiga ko'ra berilgan to'plamda quyidagi shartlar bajarilishini tekshiramiz:

1) $\forall(z_1,z_2\in K),\exists!(z\in K)\ (z_1+z_2=z)$;

2) $\forall(z_1,z_2\in K)\ (z_1+z_2=z_2+z_1)$;

3) $\forall(z,z_1,z_2\in K),\ ((z+z_1)+z_2)=z+(z_1+z_2))$;

4) $\forall(z\in K),\exists!(e\in K)\ (z+e=z)$;

5) $\forall(z\in K),\exists\ (z'\in K)\ (z+z'=e)$;

6) $\forall(z_1,z_2\in K),\exists!(z\in K)\ (z_1\cdot z_2=z)$;

7) $\forall(z_1,z_2\in K)\ (z_1\cdot z_2=z_2\cdot z_1)$;

8) $\forall(z,z_1,z_2\in K),\ ((z\cdot z_1)\cdot z_2)=z\cdot(z_1\cdot z_2))$;

9) $\forall(z\in K),\exists!(e\in K)\ (z\cdot e=z)$;

10) $\forall(z\in K),\exists\ (z'\in K)\ (z\cdot z'=e)$.

1. To'plamning ixtiyoriy $z_1=a_1+b_1\sqrt{p}$, $z_2=a_2+b_2\sqrt{p}$ elementlari uchun $z_1+z_2=(a_1+b_1\sqrt{p})+(a_2+b_2\sqrt{p})=(a_1+a_2)+$ $+(b_1+b_2)\sqrt{p}$ tenglik bilan aniqlanuvchi shu to'plamning $a+b\sqrt{p}=z$ elementi mavjud. Demak, K to'plamda qo'shish amali aniqlangan. <K; +> − additiv gruppoid.

2. To'plamning ixtiyoriy $z_1=a_1+b_1\sqrt{p}$, $z_2=a_2+b_2\sqrt{p}$ elementlari uchun $z_1+z_2=((a_1+a_2)+(b_1+b_2)\sqrt{p})=(a_2+a_1)+$ $+(b_2+b_1)\sqrt{p}=z_2+z_1$. Demak, qo'shish amali kommutativ va <K; +> − additiv abel gruppoid.

3. To'plamning ixtiyoriy $z=a+b\sqrt{p}$, $z_1=a_1+b_1\sqrt{p}$, $z_2=a_2+b_2\sqrt{p}$ elementlari uchun $(z+z_1)+z_2=((a+a_1)+$ $+(b+b_1)\sqrt{p})+(a_2+b_2\sqrt{p})=((a+a_1)+a_2+((b+b_1)+b_2)\sqrt{p})=$ $=a+(a_1+a_2)+(b+(b_1+b_2))\sqrt{p}=z+(z_1+z_2)$. Demak, qo'shish amali assotsiativ va <K; +> − additiv abel yarimgruppa.

4. To'plamning ixtiyoriy $z=a+b\sqrt{p}$ elementi uchun $z+e=z$ tenglikni qanoatlantiruvchi elementni aniqlaymiz: $(a+b\sqrt{p})+$ $+(x+y\sqrt{p})=a+b\sqrt{p}$ tenglikdan $(a+x)+(b+y)\sqrt{p}=a+b\sqrt{p}$

tenglikni va undan $\begin{cases} a+x=a, \\ b+y=b \end{cases}$ tenglamalar sistemasini hosil qilamiz. Uning yechimi $\begin{cases} x=0, \\ y=0 \end{cases}$ bo‘lib, bundan $e=0+0\sqrt{p}=0$ hosil bo‘ladi. Demak, K to‘plamda qo‘shish amaliga nisbatan neytral element mavjud ekan. $<K; +>$ additiv abel monoidni tashkil etdi.

5. To‘plamning ixtiyoriy $z=a+b\sqrt{p}$ elementi uchun $z+z'=e$ tenglikni qanoatlantiruvchi elementni aniqlaymiz: $(a+b\sqrt{p})+$ $+(x+y\sqrt{p})=0+0\sqrt{p}$ tenglikdan $(a+x)+(b+y)\sqrt{p}=0+0\sqrt{p}$ tenglikni va undan $\begin{cases} a+x=0, \\ b+y=0 \end{cases}$ tenglamalar sistemasini hosil qilamiz. Uning yechimi $\begin{cases} x=-a, \\ y=-b \end{cases}$ bo‘lib, bundan $z'=-a+(-b)\sqrt{p}=$ $=-(a+b\sqrt{p})$ hosil bo‘ladi. Demak, K to‘plamda qo‘shish amaliga nisbatan simmetrik element mavjud ekan. $<K; +>$ additiv abel gruppani tashkil etdi.

6. To‘plamning ixtiyoriy $z_1=a_1+b_1\sqrt{p}$, $z_2=a_2+b_2\sqrt{p}$ elementlari uchun $z_1\cdot z_2=(a_1+b_1\sqrt{p})\cdot(a_2+b_2\sqrt{p})=(a_1\cdot a_2+pb_1b_2)+$ $+(a_1b_2+b_1a_2)\sqrt{p}$ tenglik bilan aniqlanuvchi shu to‘plamning $a+b\sqrt{p}=z$ elementi mavjud. Demak, K to‘plamda ko‘paytirish amali aniqlangan. $<K; \cdot>$ — multiplikativ gruppoid.

7. To‘plamning ixtiyoriy $z_1=a_1+b_1\sqrt{p}$, $z_2=a_2+b_2\sqrt{p}$ elementlari uchun $z_1\cdot z_2=(a_1a_2+pb_1b_2)+(a_1b_2+b_1a_2)\sqrt{p})=$ $=(a_2a_1+pb_2b_1)+(a_2b_1+b_2a_1)\sqrt{p}=z_2\cdot z_1$. Demak, ko‘paytirish amali kommutativ va $<K; \cdot>$ — multiplikativ abel gruppoid.

8. To‘plamning ixtiyoriy $z=a+b\sqrt{p}$, $z_1=a_1+b_1\sqrt{p}$, $z_2=a_2+b_2\sqrt{p}$ elementlari uchun $(z\cdot z_1)\cdot z_2=((aa_1+pbb_1)+$ $+(ab_1+ba_1)\sqrt{p})\cdot(a_2+b_2\sqrt{p})=((aa_1)a_2+p(bb_1)a_2+p(ab_1)b_2+$ $+p(ba_1)b_2)+((aa_1)b_2+p(bb_1)b_2+(ab_1)a_2+(ba_1)a_2)\sqrt{p}=(a(a_1b_2)+$

$+pb(b_1b_2)+pa(b_1b_2)+pb(a_1b_2))+(a(a_1b_2)+pb(b_1b_2)+a(b_1a_2)+$
$+b(a_1a_2))\sqrt{p}=(a+b\sqrt{p})\cdot((a_1a_2+pb_1b_2)+(a_1b_2+b_1a_2)\sqrt{p})=$
$=(a+b\sqrt{p})((a_1+b_1\sqrt{p})\cdot(a_2+b_2\sqrt{p}))=z\cdot(z_1\cdot z_2)$. Demak, ko'paytirish amali assotsiativ va $<K;\cdot>$ — multiplikativ abel yarimgruppa.

9. To'plamning ixtiyoriy $z=a+b\sqrt{p}$ elementi uchun $z\cdot e=z$ tenglikni qanoatlantiruvchi elementni aniqlaymiz: $(a+b\sqrt{p})\times$ $\times(x+y\sqrt{p})=a+b\sqrt{p}$ tenglikdan $(ax+pby)+(ay+bx)\sqrt{p}=$ $=a+b\sqrt{p}$ tenglikni va undan $\begin{cases} ax+pby=a, \\ ay+bx=b \end{cases}$ tenglamalar sistemasini hosil qilamiz. Uning yechimi $\begin{cases} x=1, \\ y=0 \end{cases}$ bo'lib, bundan $e=1+0\sqrt{p}=1$ hosil bo'ladi. Demak, K to'plamda ko'paytirish amaliga nisbatan neytral element mavjud ekan. $<K;\cdot>$ multiplikativ abel monoidni tashkil etdi..

10. To'plamning ixtiyoriy noldan farqli $z=a+b\sqrt{p}$ elementi uchun $z\cdot z'=e$ tenglikni qanoatlantiruvchi elementni aniqlaymiz: $(a+b\sqrt{p})(x+y\sqrt{p})=1+0\sqrt{p}$ tenglikdan $(ax+pby)+$ $+(ay+bx)\sqrt{p}=1+0\sqrt{p}$ tenglikni va undan $\begin{cases} ax+pby=1, \\ ay+bx=0 \end{cases}$ tenglamalar sistemasini hosil qilamiz. Uning yechimi $\begin{cases} x=\dfrac{a}{a^2-pb^2}, \\ y=\dfrac{-b}{a^2-pb^2} \end{cases}$ bo'lib, bundan $z'=\dfrac{a}{a^2-pb^2}+\dfrac{-b}{a^2-pb^2}\sqrt{p}$ hosil bo'ladi. Demak, K to'plamda ko'paytirish amaliga nisbatan simmetrik element mavjud ekan.

$<K;\cdot,{}^{-1},1>$ multiplikativ abel gruppani tashkil etdi.

K to'plam qo'shish va ko'paytirish amallariga nisbatan abel gruppa shartlariga bo'ysunganligi uchun $<K;+,\cdot,-,{}^{-1},0,1>$ maydon bo'ladi.

2-misol. $K=\{a+b\sqrt{p}\mid a,b\in R\}$ va $P=\{a+b\sqrt{q}\mid a,b\in R\}$ to'plamlar tashkil etgan maydonlar orasida izomorfizm o'rnating.

Yechish. Algebralar izomorfizmi ta'rifiga ko'ra berilgan $<K;+,\cdot,-,{}^{-1},0,1>$ maydon elementlarini $<R,+,\cdot,-,{}^{-1},0,1>$ maydon elementlariga akslantiradigan akslantirish asosiy amallarni saqlashi, inyektiv va syuryektiv bo'lishi kerak.

$f: K \to P$ akslantirishni $f(a+b\sqrt{p}) = a+b\sqrt{q}$ ko'rinishda olamiz.

K to'plamning ixtiyoriy $z_1 = a_1 + b_1\sqrt{p}$, $z_2 = a_2 + b_2\sqrt{p}$ elementlariga R to'plamning $t_1 = a_1 + b_1\sqrt{q}$, $t_2 = a_2 + b_2\sqrt{q}$ elementlari mos keladi. Tanlab olingan akslantirish izomorfizm ekanligini isbotlaymiz:

1) $\forall z_1, z_2 \in K$ uchun $f(z_1+z_2) = f((a_1+b_1\sqrt{p})+(a_2+b_2\sqrt{p})) =$
$= f((a_1+a_2)+(b_1+b_2)\sqrt{p})) = (a_1+a_2)+(b_1+b_2)\sqrt{q} = (a_1+b_1\sqrt{q})+$
$+(a_2+b_2\sqrt{q}) = f(a_1+b_1\sqrt{p}) + f(a_2+b_2\sqrt{p})$.

Demak, qo'shish amali akslantirish natijasida saqlanadi.

2) $\forall z_1, z_2 \in K$ uchun $f(z_1 \cdot z_2) = f((a_1+b_1\sqrt{p})\cdot(a_2+b_2\sqrt{p})) =$
$= f((a_1a_2+pb_1b_2)+(a_1b_2+b_1a_2)\sqrt{p})) = (a_1a_2+qb_1b_2)+(a_1b_2+b_1a_2)\sqrt{q} =$
$= (a_1+b_1\sqrt{q})+(a_2+b_2\sqrt{q}) = f(a_1+b_1\sqrt{p})\cdot f(a_2+b_2\sqrt{p})$.

Demak, ko'paytirish amali akslantirish natijasida saqlanadi.

3) $f(0) = f(z+(-z)) = f((a+b\sqrt{p})+(-a-b\sqrt{p})) =$
$= f((a-a)+(b-b)\sqrt{p})) = (a-a)+(b-b)\sqrt{q} = (a+b\sqrt{q})$ $+$
$(-a-b\sqrt{q}) = f(a+b\sqrt{p})+(-f(a+b\sqrt{p})) = 0$.

Demak, qo'shish amaliga nisbatan neytral element neytral elementga, simmetrik element simmetrik elementga o'tdi.

4) $f(1) = f(z\cdot z^{-1}) = f((a+b\sqrt{p})\cdot(\frac{a}{a^2-pb^2}+\frac{-b}{a^2-pb^2}\sqrt{p})) =$
$= 1 = (a+b\sqrt{q})\cdot(\frac{a}{a^2-qb^2}+\frac{-b}{a^2-qb^2}\sqrt{q}) = (a+b\sqrt{q})\cdot(a+b\sqrt{q})^{-1} =$
$= f(z)\cdot f^{-1}(z) = 1$. Demak, ko'paytirish amaliga nisbatan neytral element neytral elementga, simmetrik element simmetrik elementga o'tdi. Aniqlangan akslantirishning gomomorfizm ekanligini isbotladik.

5) $\forall z_1, z_2 \in K$ lar uchun $f(z_1) = f(z_2)$ ekanligidan $a_1 + b_1\sqrt{q} = a_2 + b_2\sqrt{q}$ kelib chiqadi. Bu shart $a_1 = a_2$, $b_1 = b_2$

shartlar bajarilganda to‘g‘ri. Bundan esa $a_1 + b_1\sqrt{p} = a_2 + b_2\sqrt{p}$ ni hosil qilamiz. Demak, bir-biriga teng tasvirlarga bir-biriga teng asllar mos keldi. Tekshirilayotgan akslantirish inyektiv akslantirish ekan.

6) R To‘plamdan olingan har qanday $f(z) = a + b\sqrt{q}$ elementga $a + b\sqrt{p} \in K$ element mos keladi. Demak, akslantirish syuryektiv ekan.

Tekshirilgan xossalarga ko‘ra, $f :< K; +, \cdot, -, ^{-1}, 0, 1 > \to$ $\to < P; +, \cdot, -, ^{-1}, 0, 1 >$ akslantirish izomorfizm.

Misol va mashqlar

1. Gruppadagi ixtiyoriy elementga chap teskari element, shu elementga o‘ngdan ham teskari bo‘lishini isbotlang.

2. Gruppada o‘ng birlik element, chap birlik element bo‘lishini isbotlang.

3. Gruppaning ixtiyoriy a va b elementlari uchun $ax = b$ va $ya = b$ tenglamalarning har biri yagona yechimga ega bo‘lishini isbotlang.

4. Quyidagi to‘plamlarni multiplikativ gruppa tashkil etishini isbotlang:

4.1. $G = \{a + b\sqrt{2} \mid a, b \in Q, a^2 + b^2 > 0\}$.

4.2. $G = \{p^z \mid p$ – tub son , $z \in Z\}$.

5. Quyidagi to‘plamlarni additiv gruppa tashkil etishini isbotlang:

5.1. $G = \{a + b\sqrt{3} \mid a, b \in Z\}$.

5.2. $G = \{\frac{a}{7^k} \mid a \in Z, k \in N\}$.

5.3. $G = \{a - b\sqrt{p} \mid a, b \in Z; p$ – tub son$\}$.

6. $(G; \cdot, ^{-1})$, $(H; \cdot, ^{-1})$ gruppalar berilgan bo‘lsin. $\varphi : G \to H$ akslantirish gomomorf akslantirish bo‘lishi uchun $\forall a, b \in G$, $\varphi(a \cdot b) = \varphi(a) \cdot \varphi(b)$ bo‘lishi yetarliligini isbotlang.

7. $(G; \cdot, ^{-1})$ gruppa berilgan bo‘lsin. $H \neq Q, H \subset G$ to‘plamosti gruppaosti bo‘lishi uchun $\forall a, b \in H$ elementlari uchun $a \cdot b^{-1} \in H$ bo‘lishi zarur va yetarli. Isbotlang.

8. a, b, c lar $(K; +, -, \cdot)$ halqaning ixtiyoriy elementlari bo'lsin, quyidagi xosssalarni isbotlang:

1) agar $a + b = a$ bo'lsa, $b = 0$;

2) agar $a + b = 0$ bo'lsa, $a = -b$;

3) $-(-a) = a$;

4) $0 \cdot a = a \cdot 0 = 0$;

5) $(-a)(-b) = a \cdot b$;

6) $(a - b) \cdot c = a \cdot c - b \cdot c$;

7) $c \cdot (a - b) = c \cdot a - c \cdot b$.

9. Quyidagi to'plamlarning halqa tashkil etishini isbotlang:

9.1. $G = \{a + b\sqrt{2} \mid a, b \in Q\}$.

9.2. $G = \{a - b\sqrt{p} \mid a, b \in Z\,;\, p$ – tub son$\}$.

9.3. $< Z_{11}; +, \cdot >$.

10. $(Z; +, -, \cdot)$ butun sonlar halqasi, $(K; \oplus, \ominus, \odot)$, $K = \{0, e, a, b\}$ to'plamida amallari quyidagi jadvallar orqali berilgan halqa bo'lsin:

$\odot$	0	e	a	b
0	0	0	0	0
e	0	e	a	b
a	0	a	0	a
b	0	b	a	e

$\oplus$	0	e	a	b
0	0	e	0	b
e	e	a	a	a
a	a	b	0	e
b	b	a	a	a

Bu halqalar orasida gomomorfizm o'rnating.

11. Butunlik sohasi bo'lmagan kommutativ halqaga misol keltiring.

12. Halqaning barcha qismhalqalari kesishmasi yana shu halqaga qismhalqa bo'lishini isbotlang.

13. Jismga tashkil etadigan algebraga misol tuzing.

14. Quyidagi algebralarning maydon tashkil etishini isbotlang:

14.1. $< \{a - b\sqrt{3} \mid a, b \in Q\}; +, \cdot >$.

14.2. $< Z_7; +, \cdot >$.

15. $<\{a+b\sqrt{p} \mid a,b \in Q\}; +, \cdot>$ va $< \{a-b\sqrt{p} \mid a,b \in Q\}; +, \cdot >$ maydonlar orasida izomorfizm o'rnating.

Takrorlash uchun savollar

1. Yarimgruppa deb nimaga aytiladi?
2. Monoidga ta'rif bering va misol keltiring.
3. Gruppa ta'rifini keltiring. Uning asosiy xossalarini ayting.
4. Additiv, multiplikativ gruppalarga algebra, geometriya kursidan misollar keltiring.
5. Gruppalar gomomorfizmining qanday turlarini bilasiz?
6. Har qanday gomomorfizm izomorfizm bo'la oladimi yoki aksincha?
7. Gruppalar avtomorfizmi nima?
8. Gruppaosti tushunchasiga misollar keltiring.
9. Halqaning qanday turlarini bilasiz?
10. Halqalar gomomorfizmi, izomorfizmiga misollar keltiring.
11. Halqalar avtomorfizmi ta'rifini bayon qiling.
12. Halqaostilar kesishmasi yana halqaosti bo'lishini isbotlang.

IV MODUL. ASOSIY SONLI SISTEMALAR

10-§. Natural sonlar sistemasi. Matematik induksiya prinsipi

✔ **Asosiy tushunchalar**: natural sonlar sistemasi aksiomalari, matematik induksiya prinsipi, induksiya qadami, natural sonlar yarimhalqasi, tartib munosabati.

N to'plami unda bajariladigan $+$, $\cdot$ binar algebraik amallar, N to'plamining ajratilgan elementlari 0 va 1 lardan iborat (N, $\cdot$, $+$, 0, 1) algebra uchun quyidagi aksiomalar (shartlar) bajarilsin:

I. $\forall a, b \in N$ uchun $a + b \neq 1$.

II. $\forall a \in N$ uchun yagona shunday a' element mavjud bo'lib, $a+1=a'$.

III. $\forall a, b \in N$ uchun $a+1=b+1$ bo'lsa, $a=b$.

IV. $\forall a, b \in N$ uchun $a+(b+1) = (a+b)+1$.

V. $\forall a \in N$ uchun $a \cdot 1 = a$.

VI. $\forall a, b \in N$ uchun $a(b+1) = ab + a$.

VII. Induksiya aksiomasi. N to'plamning M ixtiyoriy to'plamostisi uchun:

1) $1 \in M$;

2) $\forall a \in N$ uchun $a + 1 \in M$ bo'lsa, $M = N$ bo'ladi.

Agar (N, $\cdot$, $+$, 0, 1) algebra uchun yuqorida sanab chiqilgan I—VII shartlar bajrilsa, u holda bu algebra *natural sonlar sistemasi* deyiladi.

$A(n)$ natural sonlar to'plamida aniqlangan bir o'rinli predikat bo'lib, quyidagi shartlar bajarilsin:

1. $A(1)$ — rost mulohaza.

2. $\forall k \in N$ uchun $A(k)$ rost mulohaza bo'lishidan $A(k+1)$ mulohazaning rost mulohaza bo'lishi kelib chiqsin. U holda $\forall n \in N$ uchun $A(n)$ mulohaza rost mulohaza bo'ladi.

$A(1)$ ni *induksiya bazisi*, $A(k)$ ni *induksiya farazi*, deb ataymiz.

$A(k) = 1$ dan $A(k+1) = 1$ kelib chiqishi *induksiya qadami* deyiladi.

$\forall a,\ b \in N$ natural sonlar uchun shunday $n \in N$ topilib, $a=b+n$ bo‘lsa, a natural son b natural sondan katta deymiz va $a > b$ deb belgilaymiz.

$(A;\ +,\ \cdot)$ algebra uchun quyidagi shartlar bajarilsin:

1) $\forall a, b, c$ uchun $(a + b) + c = a + (b + c)$;

2) $\forall a, b \in A$ uchun $a + b = b + a$;

3) $\forall a, b, x \in A$ uchun $((a + x = b + x) \Rightarrow (a = b) \wedge$ $\wedge((x+a=x+b) \Rightarrow (a=b)$;

4) $\forall a, b, c \in A$ uchun $(a \cdot b) \cdot c = a \cdot (b \cdot c)$;

5) $\forall a, b, c \in A$ uchun $((a + b) \cdot c = ac + bc) \wedge (c(a+b)=$ $=ca+cb)$.

U holda $(A;\ +,\ \cdot)$ algebra *yarimhalqa* deyiladi.

Misol. $\frac{4^n}{n+1} < \frac{(2n)!}{(n!)^2}, (n > 1)$ ekanligini isbotlang.

Isbot. 1. $n=2$ da $\frac{4^2}{2+1} < \frac{(2\cdot 2)!}{(2!)^2} \Rightarrow \frac{16}{3} < 6$ kelib chiqadi, ya’ni tengsizlik o‘rinli.

2. Har qanday $k>0$ uchun $\frac{4(k+1)}{k+2} < \frac{(2k+1)(2k+2)}{(k+1)^2}$ ekanligidan $\frac{4^k}{k+1} \cdot \frac{4(k+1)}{k+2} < \frac{(2k)!}{(k!)^2} \cdot \frac{(2k+1)(2k+2)}{(k+1)^2}$. Bundan $\frac{4^{k+1}}{k+2} < \frac{(2k+2)!}{((k+1)!)^2}$ kelib chiqadi.

Demak, har qanday $n > 1$ natural son uchun $\frac{4^n}{n+1} < \frac{(2n)!}{(n!)^2}$.

Misol va mashqlar

1. Quyidagi teoremalarni isbotlang:

1.1. $(\forall a, b \in N) \Rightarrow (\exists! c \in N) \wedge (a + b = c)$.

1.2. $\forall a, b, c \in N \Rightarrow (a + b) + c = a + (b + c)$.

1.3. $(\forall a \in N) \Rightarrow (a + 1 = 1 + a)$.

1.4. $(\forall a, b \in N) \Rightarrow (a + b = b + a)$.

1.5. $(\forall a, b, c \in N) \wedge (a + c = b + c) \Rightarrow (a = b)$.

1.6. $(\forall a, b \in N) \Rightarrow \exists! p(a + b = p)$.

1.7. $(\forall a, b, c \in N) \Rightarrow ((a + b) \cdot c = a \cdot c + b \cdot c)$.

1.8. $(\forall a \in N) \Rightarrow a \cdot 1 = 1 \cdot a$.

1.9. $(\forall a, b \in N) \Rightarrow (a \cdot b = b \cdot a)$.

1.10. $(\forall a, b, c \in N) \Rightarrow c \cdot (a + b) = c \cdot a + c \cdot b$.

1.11. $(\forall a, b, c \in N) \Rightarrow (a \cdot b) \cdot c = a \cdot (b \cdot c)$.

1.12. $(\forall a \in N) \wedge (a \neq 1) \Rightarrow (\exists n \in N) \wedge (a = 1 + n)$.

1.13. $(\forall a, b \in N) \Rightarrow a \neq a + b$.

2. $\forall a, b \in N$ uchun quyidagi tengsizliklardan faqat biri o'rinli ekanligini isbotlang:

1) $a = b$; 2) $\exists n \in N,\ b = a + n$; 3) $\exists l \in N,\ a = b + l$.

3. Natural sonlar to'plamidagi tengsizlik munosabatining quyidagi xossalarini isbotlang:

1°. $(\forall a, b \in N) \wedge (a \neq b) \Rightarrow (a > b) \vee (b > a)$.

2°. $(\forall a \in N)$ uchun $(\overline{a > a})$.

3°. $\forall a, b \in N$ uchun $a > b \Rightarrow (\overline{b > a})$.

4°. $\forall a, b, c \in N$ uchun $(a > b) \wedge (b > c) \Rightarrow (a > c)$.

5°. $\forall a, b, c \in N$ uchun $((a > b) \Rightarrow (a + c) > (b + c))$.

6°. $\forall a, b \in N$ uchun $a + b > a$.

7°. $\forall a, b, c \in N$ uchun $(a > b) \Rightarrow ac > bc$.

4. Tekislikda berilgan bir nuqtadan o'tuvchi n ta turli to'g'ri chiziqlar tekislikni $2n$ bo'lakka bo'lishini isbotlang.

5. (x_n) ketma-ketlik uchun $x_1 = 1,\ x_{n+1} = x_n - \dfrac{1}{n(n+1)}$ berilgan bo'lsa, x_n ni ifodalang.

6. Quyidagi tasdiqlarni isbotlang:

6.1. $(4^n + 15n - 1) \vdots 9$.

6.2. $(10^n + 18n - 1) \vdots 27$.

6.3. $(3^{2n+3} + 40n - 27) \vdots 64$.

6.4. $(6^{2n+3n+2} + 3n) \vdots 11$.

6.5. $(5^n + 2 \cdot 3^{n-1} + 1) \vdots 8$.

6.6. $(11^{n+2} + 12^{2n+1}) \vdots 133$.

6.7. $(9^{n+1} - 8n - 9) \vdots 16$.

6.8. $(3^{6n} - 2^{6n}) \vdots 35$.

7. Quyidagi tengliklarni isbotlang:

7.1. $1^3 + 2^3 + \ldots + n^3 = \dfrac{n^2(n+1)^2}{4}$.

7.2. $1\cdot 1! + 2\cdot 2! + ... + n\cdot n! = (n+1)! - 1.$

7.3. $\left(1-\frac{4}{1}\right)\left(1-\frac{4}{9}\right)\left(1-\frac{4}{25}\right)...\left(1-\frac{4}{(2n-1)^2}\right) = -\frac{2n+1}{2n-2}.$

7.4. $\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+...+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}.$

7.5. $1 - 2^2 + 3^2 - 4^2 + ... + (-1)^{n-1}n^2 = (-1)^{n-1}\frac{n(n+1)}{2}.$

7.6. $\frac{1^2}{1\cdot 3}+\frac{2^2}{3\cdot 5}+...+\frac{n^2}{(2n-1)(2n+1)}=\frac{n(n+1)}{2(2n+1)}.$

7.7. $(x_1+...+x_n)^2 = x_1^2 +...+x_n^2 +2(x_1x_2 +...+x_{n-1}x_n).$

7.8. $1 + x + x^2 + ... + x^n = \frac{x^{n+1}-1}{x-1}$ $(x \neq 1).$

7.9. $\frac{1}{2!}+\frac{2}{3!}+...+\frac{n}{(n+1)!}=1-\frac{1}{(n+1)!}.$

7.10. $\frac{1}{a\cdot(a+1)}+\frac{1}{(a+1)(a+2)}+...+\frac{1}{(a+n-1)(a+n)}=\frac{n}{a(a+n)}.$

8. Quyidagi tengsizliklarni isbotlang:

8.1. $\frac{1}{n+2}+\frac{1}{n+3}+...+\frac{1}{2n+2}>\frac{13}{24}.$

8.2. $\frac{1}{n+1}+\frac{1}{n+2}+...+\frac{1}{3n+1}>1.$

8.3. $\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot...\cdot\frac{2n-1}{2n}\le\frac{1}{\sqrt{3n+1}}.$

8.4. $|x_1+x_2+...+x_n|\le|x_1+|x_2|+...+|x_n|.$

8.5. $\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+...+\frac{1}{n!}<2.$

9. Agar $n\ge 2$, $a>-1$ bo‘lsa, n natural son uchun $(1+a)^n \ge 1+na$ o‘rinli ekanligini isbotlang.

10. Har qanday n natural son uchun $\frac{n^3}{6}+\frac{n^2}{2}+\frac{n}{3}$ natural son bo‘lishini isbotlang.

Takrorlash uchun savollar

1. Natural sonlar sistemasi deb nimaga aytiladi?
2. Matematik induksiya metodini ifodalovchi teoremani ayting.
3. Induksiya bazisi, induksiya farazi, induksiya qadami nima?
4. Natural sonlarni qo'shishning qanday xossalarini bilasiz?
5. Natural sonlarni ko'paytirishning xossalarini ayting.
6. Natural sonlar to'plamida tartib munosabatini aniqlang.
7. Natural sonlar to'plamidagi tengsizlik munosabatining xossalarini bayon qiling.
8. Natural sonlar to'plami kommutativ yarimhalqa bo'lishini isbotlang.

11-§. Butun sonlar halqasi. Ratsional sonlar maydoni. Haqiqiy sonlar sistemasi

✓ **Asosiy tushunchalar**: butun sonlar halqasi, bo'linish munosabati, maydon, ratsional sonlar maydoni, haqiqiy sonlar sistemasi.

$\forall z \in Z$ element ikkita natural son ayirmasi ko'rnishida ifoda qilinadi. $(Z; +, \cdot)$ halqani *butun sonlar halqasi*, deb ataymiz.

Agar $a, b, b \neq 0$ butun sonlar uchun shunday q butun son topilib, $a=bq$ tenglik o'rinli bo'lsa, a butun son butun songa bo'linadi yoki b butun son a butun sonni bo'ladi deyiladi va, mos ravishda, $a \vdots b$ yoki $b \backslash a$ ko'rinishda belgilanadi.

$\forall a, b \in Z$ sonlar uchun $a \vdots b$ va $b \vdots a$ shartlar bajarilsa, a va b sonlar *assotsirlangan* deyiladi.

Assotsiativ, kommutativ, birlik elementga ega, noldan farqli har bir element teskarilanuvchi bo'lgan va $1 \neq 0$ shart bajariladigan halqa *maydon* deyiladi.

Maydonning noldan farqli har qanday elementi teskarilanuvchi bo'lgan halqaosti *maydonosti* deyiladi.

K butunlik sohasi P maydonning halqaostisi bo'lsin. P maydonning ixtiyoriy P elementi uchun R butunlik sohasining m, n

elementlari topilib, $P=mn^{-1}$ tenglik o'rinli bo'lsa, P maydon K butunlik sohasining *nisbatlar maydoni* deyiladi. Butun sonlar halqasining nisbatlar maydoni *ratsional sonlar maydoni* deyiladi.

Tartiblangan maydonning ixtiyoriy musbat a, b elementlari uchun shunday n natural son mavjud bo'lib, $n \cdot a > b$ shart bajarilsa, bu maydon *Arximedcha tartiblangan maydon* deyiladi.

F tartiblangan maydondagi har qanday fundamental ketma-ketlik shu maydonda yaqinlashuvchi bo'lsa, bunday maydon *to'liq* deyiladi.

Arximedcha turtiblangan ratsional sonlar sistemasini o'z ichiga olgan eng kichik to'liq maydon *haqiqiy sonlar sistemasi* deyiladi.

Misol va mashqlar

1. 5 ga bo'linuvchi butun sonlar to'plami halqa tashkil etishini isbotlang.

2. Z_6 to'plam halqa tashkil etishini isbotlang.

3. Q^2 to'plam $(a,b)+(c,d)=(a+c,b+d)$, $(a,b)\cdot(c,d)=$ $=(ac+2bd,\ ad+bc)$ amallarga nisbatan maydon tashkil etishini isbotlang.

4. Butun sonlar halqasidagi bo'linish munosabatining quyidagi xossalarini isbotlang:

1°. $\forall a \in Z$, $a \neq 0$ uchun $a \vdots a$.

2°. $\forall a \in Z$, $a \neq 0$ uchun $0 \vdots a$.

3°. $\forall a \in Z$, $a \vdots 0$ va $a \vdots (-1)$.

4°. $\vdots$ munosabati tranzitiv munosabatdir, ya'ni $\forall a, b, c \in Z$ uchun $a \vdots b$ va $b \vdots c$ bo'lsa, $a \vdots c$.

5°. $\forall a, b, c \in Z$ uchun $a \vdots c$ bo'lsa, $a \cdot b \vdots c$.

6°. $\forall a, b, c \in Z$ uchun $a \vdots c$ va $b \vdots c$ bo'lsa, $(a+b) \vdots c$.

7°. $\forall a, b, c \in Z$ va $c \neq 0$ uchun $bc \vdots ac$ bo'lsa, $b \vdots a$.

8°. $\forall a, b, c \in Z$ uchun $a \vdots c$ va $b \vdots d$ bo'lsa, $(a \cdot b) \vdots (c \cdot d)$.

9°. $\forall a, b, c \in Z$ va $a \vdots b$ bo'lsa, $a \cdot c \vdots b \cdot c$.

10°. $\forall a, b, c, m, n \in Z$ va $a \vdots c$ va $b \vdots c$ bo'lsa, $(ma+nb) \vdots c$.

5. $(P;\ +,\ -,\ \cdot,\ 1)$ maydonning ixtiyoriy $a,\ b,\ c$ elementlari uchun quyidagi munosabatlar o‘rinli ekanligini isbotlang:

1°. $0 \cdot a = a \cdot 0 = 0$.

2°. Agar $ab = 1$ bo‘lsa, u holda $b = a^{-1}$.

3°. $c \neq 0$ bo‘lib, $ac = bc$ bo‘lsa, $a = b$.

4°. $ab = 0$ bo‘lsa, $a = 0$ yoki $b = 0$.

5°. $a \neq 0$ va $b \neq 0$ bo‘lsa, $ab \neq a$.

6°. $\frac{a}{b} = \frac{c}{d}$ bo‘lsa, $ad = bc$.

7°. $\frac{a}{b} + \frac{c}{b} = \frac{ad+bc}{bd}$.

8°. $\frac{a}{b} \cdot \frac{c}{b} = \frac{ac}{bd}$.

9°. $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$.

10°. Noldan farqli $a,\ b$ elementlar uchun $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$.

11°. $c \neq 0$ element uchun $\frac{a}{b} = \frac{ac}{bc} = \frac{ac^{-1}}{bc^{-1}}$.

6. $(F;\ +,\ -,\ \cdot,\ 1,\ >)$ tartiblangan maydon uchun quyidagi xossalarni isbotlang:

1°. $\forall a,\ b \in F$ uchun $a<b$ bo‘lishi uchun $b - a > 0$ bo‘lishi zarur va yetarli.

2. $\forall a \in F$ uchun $a<0$, $0<a$, $0=a$ shartlardan bir vaqtda faqat biri o‘rinli.

3°. Agar $a \geq 0$ va $b \geq 0$ bo‘lsa, u holda $ab \geq 0$ va $a+b \geq 0$ bo‘ladi.

4°. $a \leq b$ va $c \leq d$ bo‘lsa, $a + c < b + d$.

5°. Agar $a < b$ va $c < 0$ bo‘lsa, $ac > bc$.

6°. $\forall a \in F$ element uchun $a^2 \geq 0$. Xususan, $a \neq 0$ bo‘lsa, $a^2 > 0$.

7°. $\forall n \in N$ uchun $n \cdot 1 > 0$, xususan, $1 > 0$.

8°. Tartiblangan maydon butunlik sohasidir.

7. $(F;\ +,\ -,\ \cdot,\ 1,\ >)$ tartiblangan maydonning $\forall a,\ b$ elementlari uchun quyidagi munosabatlarni isbotlang:

1°. $|a| = |-a|$.

2°. $|a| \geq a$ va $|a| \geq -a$.

3°. $|a + b| \leq |a| + |b|$.

4°. $|a \cdot b| \leq |a| \cdot |b|$.

5°. $|b| > 0$.

6°. $\forall a \in F$, $a^2 \geq 0$ element uchun $|b| < a$ bo‘lishi uchun $-a < b < a$ bo‘lishi zarur va yetarli.

7°. $\forall a, b, c, F$, $a > 0$ elementlar uchun $|b| > a$ bo‘lishi uchun $b > a$ yoki $b < -a$ bo‘lishi zarur va yetarli.

8. Har qanday $a + b\sqrt{7}$ $(a, b \in Q)$ ko‘rinishdagi haqiqiy son uchun aniqlangan $f(a + b\sqrt{7}) = a - b\sqrt{7}$ akslantirish haqiqiy sonlar maydonining avtomorfizmi bo‘lishini isbotlang.

Takrorlash uchun savollar

1. Natural sonlar yarimhalqasini qamrab olgan eng kichik kommutativ halqani quring.
2. Butun sonlar to‘plami halqa tashkil etishini isbotlang.
3. Butun sonlar halqasida tartib munosabatini aniqlang.
4. Butun sonlar halqasida bo‘linish munosabatining xossalarini ayting.
5. Maydon tushunchasiga ta’rif bering.
6. Maydonning sodda xossalarini ayting.
7. Maydonlar izomorfizmiga misol keltiring.
8. Ratsional sonlar maydonida tartib munosabatini aniqlang.

12-§. Kompleks sonlar maydoni

✔ **Asosiy tushunchalar**: kompleks son, kompleks sonlar maydoni, sonli maydon, o‘zaro qo‘shma kompleks sonlar, kompleks son moduli, kompleks tekislik, mavhum o‘q, kompleks son argumenti, kompleks sonning trigonometrik shakli, Muavr formulalari, *n*- darajali ildizlar.

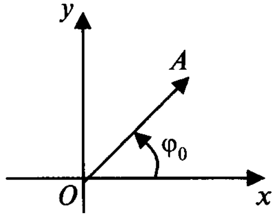
$C = \{a+bi / a,b \in R\}$ to‘plamni qaraylik. C da $(a+bi) + (c+di) = (a+c) + (b+d)i$; $(a+bi) \cdot c + di = (ac - bd) + (ad + bc)i$; $-(a+bi) = (-a)+(-b)i$ tengliklar orqali qo‘shish, ko‘paytirish, qarama-qarshisini olish amallarini aniqlaymiz. Ixtiyoriy $a+bi \neq 0$ element uchun teskari element $(a+bi)^{-1} = \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2}i$ formula bilan aniqlanadi.

Kompleks sonlar maydonining har qanday maydonostisi *sonli maydon* deyiladi.

$z = a+bi$ kompleks son uchun $z = a-bi$ *qo‘shma kompleks son* deyiladi.

$|z| = \sqrt{a^2 + b^2}$ son $a+bi$ $(a,b \in R)$ kompleks sonining *moduli* deyiladi.

Har bir $a+bi$ kompleks songa tekislikda (a,b) nuqtani mos qo‘ysak, bu nuqta kompleks sonning *geometrik tasviri* deyiladi. Bu nuqtani koordinatalar boshi bilan tutashtirsak, boshi koordinatalar boshida, uchi esa (a,b) koordinatali nuqtada bo‘lgan $\overline{OA}$ vektor hosil bo‘ladi. Bu vektorning uzunligi esa $a+bi$ kompleks sonning moduliga tengligi ayon.



Har bir bi kompleks songa Oy o‘qida $(0, b)$ nuqta mos keladi. Bu o‘qni *mavhum o‘q* deb ataymiz. Ox o‘qni *haqiqiy o‘q* deymiz.

OA vektorning Ox o‘qi musbat yo‘nalishining soat mili qarama-qarshi yo‘nalishida hosil qilgan φ_0 burchagi $a+bi$ kompleks sonning *boshlang‘ich argumenti* deyiladi.

$a+bi$ kompleks son berilgan bo‘lib, r uning moduli φ esa argumenti bo‘lsin, u holda $b = r\sin\varphi$, $a = r\cos\varphi$ tengliklarni ko‘rsatish qiyin emas. Demak, $a+bi = r(\cos\varphi + i\sin\varphi)$ tenglik o‘rnili. Bu esa kompleks sonning *trigonometrik ko‘rinishi* deyiladi.

$z_1 = r_1(\cos\varphi_1 + i\sin\varphi_1)$, $z_2 = r_2(\cos\varphi_2 + i\sin\varphi_2)$ kompleks sonlar berilgan bo‘lsin. U holda quyidagilar o‘rinli:

1°. $z_1 \cdot z_2 = r_1 \cdot r_2(\cos(\varphi_1+\varphi_2) + i\sin(\varphi_1+\varphi_2))$.

2°. $\frac{z_1}{z_2}=\frac{r_1}{r_2}(\cos(\varphi_1-\varphi_2)+i\sin(\varphi_1-\varphi_2))$.

$z^n=(r(\cos\varphi+i\sin\varphi))^n=r^n(\cos n\varphi+i\sin n\varphi)$, bu formula *Muavr formulasi* deyiladi.

$z=r(\cos\varphi+i\sin\varphi)$ kompleks son berilgan bo'lsin. Bunda r kompleks sonning moduli, $\varphi=\varphi_0+2k\pi$ kompleks sonning argumenti, φ_0 — boshlang'ich argumenti bo'lsin. U holda z kompleks son n ta har xil n- darajali kompleks ildizlarga ega bo'ladi va bu ildizlar quyidagi formula yordamida topiladi:

$$U_k=\sqrt[n]{r}\left(\cos\frac{\varphi_0+2k\pi}{n}+i\sin\frac{\varphi_0+2k\pi}{n}\right),\ k=1,\ \ldots,\ n-1.$$

1-misol. $|z+2|=3$ tenglamani yeching.

Yechish. Kompleks sonning moduli, kompleks sonlarning tengligi ta'riflaridan quyidagi ifodani hosil qilamiz:

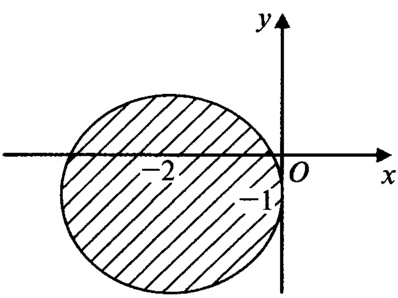
$$|z+2|=|x+yi+2|=|(x+2)+yi|=\sqrt{(x+2)^2+y^2}$$

$$\text{va}\quad \sqrt{(x+2)^2+y^2}=3.$$

Bundan $(x+2)^2+y^2=9$ tenglamani hosil qilamiz. Hosil bo'lgan tenglamaning yechimlari tekislikdagi markazi $(-2;\ 0)$ nuqtada bo'lgan, radiusi 3 ga teng aylananing nuqtalaridan iborat.

2-misol. $|z+2+i|\le 3$ tengsizliklarni yeching va yechimlar to'plamini Dekart koordinatalar tekisligida ifodalang.

Yechish. $|z+2+i|\le 2$ tengsizlikka kompleks sonning moduli ta'rifini qo'llasak, $|z+2+i|=|x+yi+2+i|=|(x+2)+(y+1)i|=$ $=\sqrt{(x+2)^2+(y+1)^2}$ ni hosil qilamiz. Bundan $(x+2)^2+(y+1)^2\le 4$ tengsizlikni hosil qilamiz. Bu tengsizlikning yechimlari markazi $(-2;\ -1)$ nuqtada bo'lgan, radiusi 2 ga teng doiradan iborat. Doirani Dekart koordinatalar tekisligida chizamiz.



3-misol. $\sqrt{3+4i}$ ni hisoblang.

Yechish. Kompleks sondan kvadrat ildiz chiqarish formulalari

$$1)\ \underset{b>0}{\sqrt{a+bi}} = \pm\left(\sqrt{\frac{a+\sqrt{a^2+b^2}}{2}} + i\sqrt{\frac{-a+\sqrt{a^2+b^2}}{2}}\right) \text{ va}$$

$$2)\ \underset{b<0}{\sqrt{a+bi}} = \pm\left(\sqrt{\frac{a+\sqrt{a^2+b^2}}{2}} - i\sqrt{\frac{-a+\sqrt{a^2+b^2}}{2}}\right) \text{ lardan foydala-}$$

namiz. Berilgan misolda $b > 0$ bo‘lganligi uchun birinchi formulani qo‘llaymiz:

$$\underset{4>0}{\sqrt{3+4i}} = \pm\left(\sqrt{\frac{3+\sqrt{3^2+4^2}}{2}} + i\sqrt{\frac{-3+\sqrt{3^2+4^2}}{2}}\right) = \pm\left(\sqrt{\frac{3+5}{2}} + i\sqrt{\frac{-3+5}{2}}\right) =$$

$$= \pm(2+2i) = \pm 2(1+i).$$

4-misol. $\sqrt[3]{2+3i}$ ildizlarni hisoblang.

Yechish. Ixtiyoriy kompleks sondan n- darajali ildizlarni topish formulasi

$$u_k = |c|^{\frac{1}{n}}\left(\cos\frac{\varphi+2\pi k}{n} + i\sin\frac{\varphi+2\pi k}{n}\right),\ k = 0,\dots,n-1 \qquad (1)$$

dan foydalanamiz. Buning uchun avval berilgan z=2+3i kompleks sonni trigonometrik ko‘rinishga keltiramiz: kompleks sonning moduli $|z| = \sqrt{a^2+b^2} = \sqrt{2^2+3^3} = \sqrt{13}$; argumenti $\varphi = \operatorname{arctg}\frac{b}{a} = \operatorname{arctg}\frac{3}{2}$ dan iborat. U holda $z = 2+3i = \sqrt{13}\left(\cos\left(\operatorname{arctg}\frac{3}{2}\right) + i\sin\left(\operatorname{arctg}\frac{3}{2}\right)\right)$. Topilgan modul va argumentni (1) formulaga

qo‘yamiz: $u_k = \sqrt{13}^{\frac{1}{3}}\left(\cos\frac{\operatorname{arctg}\frac{3}{2}+2\pi k}{3} + i\sin\frac{\operatorname{arctg}\frac{3}{2}+2\pi k}{3}\right),\ k = 0,1,2.$

Bundan $u_0 = \sqrt[6]{13}\left(\cos\frac{\operatorname{arctg}\frac{3}{2}}{3} + i\sin\frac{\operatorname{arctg}\frac{3}{2}}{3}\right)$,

$$u_1=\sqrt[6]{13}\left(\cos\frac{\operatorname{arctg}\frac{3}{2}+2\pi}{3}+i\sin\frac{\operatorname{arctg}\frac{3}{2}+2\pi}{3}\right),$$

$$u_2=\sqrt[6]{13}\left(\cos\frac{\operatorname{arctg}\frac{3}{2}+4\pi}{3}+i\sin\frac{\operatorname{arctg}\frac{3}{2}+4\pi}{3}\right)$$

ildizlarni hosil qilamiz.

5-misol. $\sin x+\sin 2x+\ldots+\sin nx=\dfrac{\sin\frac{n+1}{2}}{\sin\frac{x}{2}}\sin\dfrac{nx}{2}$ tenglikni isbotlang.

Isbot. 1. $n=1$ bo'lsin. U holda $\sin x=\dfrac{\sin\frac{1+1}{2}}{\sin\frac{x}{2}}\sin\dfrac{x}{2}\Rightarrow\sin x=\sin x$ tenglik o'rinli.

2. $n=k$ uchun $\sin x+\sin 2x+\ldots+\sin kx=\dfrac{\sin\frac{k+1}{2}}{\sin\frac{x}{2}}\sin\dfrac{kx}{2}$ bo'lsin. U holda $n=k+1$ da

$$\sin x+\sin 2x+\ldots+\sin kx+\sin(k+1)x=$$

$$=\frac{\sin\frac{k+1}{2}}{\sin\frac{x}{2}}\sin\frac{kx}{2}+\sin(k+1)x=\frac{\sin\frac{k+1}{2}}{\sin\frac{x}{2}}\sin\frac{kx}{2}+2\sin\frac{k+1}{2}x\cos\frac{k+1}{2}x=$$

$$=2\cos\frac{k+1}{2}x\sin\frac{x}{2}=\sin\frac{k+2}{2}x-\sin\frac{kx}{2}=\frac{\sin\frac{k+2}{2}}{\sin\frac{x}{2}}\sin\frac{k+1}{2}x.$$

Demak, har qanday $n\in N$ uchun $\sin x+\sin 2x+\ldots+\sin nx=$ $=\dfrac{\sin\frac{n+1}{2}}{\sin\frac{x}{2}}\sin\dfrac{nx}{2}$ tenglik o'rinli.

Misol va mashqlar

1. Berilgan kompleks sonlarning haqiqiy va mavhum qismlarini toping:

1.1. $\frac{(1+i)(2+i)}{2-i} - \frac{(1-i)(2-i)}{2+i}$.

1.2. $\left(\frac{i^7+2}{1+i^{11}}\right)^2$.

1.3. $\left(\frac{1-i}{1+i}\right)^{2007}$.

1.4. $\frac{(1+2i)^3-(1+3i)^2}{(3-i)^3+(1+5i)^2}$.

2. Ixtiyoriy z_1, z_2 kompleks sonlar uchun quyidagi xossalar o'rinli ekanligini isbotlang:

1°. $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$.

2°. $\overline{(-z_1)} = -\overline{z_1}$.

3°. $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$.

4°. $\overline{(\overline{z})} = z$.

5°. $z_2 \neq 0, \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$.

6°. $z = \overline{z}$ bo'lishi uchun $z \in R$ bo'lishi zarur va yetarli.

7°. Agar $z = a + bi$ bo'lsa, u holda $z \cdot \overline{z} = a^2 + b^2$.

3. Ixtiyoriy z_1, z_2 kompleks sonlar uchun quyidagi xossalar o'rinli ekanligini isbotlang:

1°. $|z|^2 = z \cdot \overline{z}$.

2°. $|z| = 0$ faqat va faqat shu holdaki, agar $z = 0$ bo'lsa.

3°. $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$.

4°. $|z^{-1}| = |z|^{-1}$ $(z \neq 0)$.

5°. $|z_1 + z_2| \leq |z_1| + |z_2|$.

6°. $|z_1| - |z_2| \leq |z_1 + z_2|$.

7°. $||z_1| - |z_2|| \leq |z_1 + z_2|$.

4. Tenglamani yeching:

4.1. $\bar{z} = 5 - z$.

4.2. $\bar{z} = -3z - 1 + 2i$.

4.3. $z^2 + \bar{z} = 1$.

4.4. $z^2 - 2z - 3 = 3i$.

4.5. $z^2 + 5z + 5 - 3i = 0$.

4.6. $z^2(1+i) - z + 1 + 2i = 0$.

4.7. $z^2 + (2 + i)z - 1 + 7i = 0$.

4.8. $(1 + i)z^2 + iz + 2 + 4i = 0$.

5. Quyidagi tenglamalarni yeching:

5.1. $z + |z + 1| + i = 0$.

5.2. $z|z| - 2z + i = 0$.

5.3. $z|z| - 2iz^2 + 2i = 0$.

6. Quyidagi tenglamalar sistemasini yeching:

6.1. $\begin{cases} |z - 2i| = |z|, \\ |z - i| = |z - 1|. \end{cases}$

6.2. $\begin{cases} \left|\dfrac{z-12}{z-8i}\right| = \dfrac{5}{3}, \\ \left|\dfrac{z-4}{z-8}\right| = 1. \end{cases}$

6.3. $\begin{cases} z_1 + 2z_2 = 1 + i, \\ 3z_1 + iz_2 = 2 - 3i. \end{cases}$

6.4. $\begin{cases} 4iz_1 - 5z_2 = -4 + 14i, \\ 3z_1 + 2iz_2 = 7 + 3i. \end{cases}$

6.5. $\begin{cases} |z + 1 - i| = |z + i|, \\ |3 + 2i - z| = |z + 1|. \end{cases}$

6.6. $\begin{cases} |z + 1| = |z + 2|, \\ |3z + 9| = |5z + 10i|. \end{cases}$

7. Quyidagi tengsizliklarning yechimlar to‘plamini koordinatalar tekisligida ifodalang:

7.1. $|z + 2| \geq |z|$.
7.2. $|z - 5 + i| < 4$.
7.3. $|z + 6i| > |z - 3|$.
7.4. $|z + 2 + 2i| \leq \sqrt{2}$.
7.5. $|z - 1 - 3i| \leq |z - 1|$.
7.6. $|z + 5 + 6i| > 0$.
7.7. $|z + i| \geq |z - 2|$.
7.8. $|z - 3 - i| < |z + 2i|$.

8. Nyuton binomi va Muavr formulalari yordamida quyidagilarni hisoblang:

8.1. $1 - 3C_n^2 + 9C_n^4 - 27C_n^6 + \dots$.

8.2. $C_n^1 - 3C_n^3 + 9C_n^5 - 27C_n^7 + \dots$.

8.3. $\sqrt{3^n} - \sqrt{3^{n-2}}C_n^2 + \sqrt{3^{n-4}}C_n^4 - \sqrt{3^{n-6}}C_n^6 + \dots$.

8.4. $C_n^1 - \frac{1}{3}C_n^3 + \frac{1}{9}C_n^5 - \frac{1}{27}C_n^7 + \dots$.

9. Har qanday k butun sonlar uchun quyidagilarni isbotlang:

9.1. $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^{3k} = 1$.

9.2. $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^{6k} = 1$.

10. Quyidagi kompleks sonlarni trigonometrik shaklga keltiring:

10.1. 5.

10.2. $-2i$.

10.3. $-\frac{1}{2}-\frac{1}{2}i$.

10.4. $(\sqrt{12}-2i)4i^{13}$.

10.5. $\sin\alpha + i\cos\alpha$.

11. Quyidagilarni hisoblang:

11.1. $\frac{(\cos\theta - i\sin\theta)(\mathrm{ctg}\beta + i)(-\cos\alpha + i\sin\alpha)}{1 - i\,\mathrm{tg}(\alpha+\beta)}$.

11.2. $\left(-2+i\sqrt{12}\right)^{2008}$.

11.3. $(1-\cos\alpha + i\sin\alpha)^n, n \in Z$.

11.4. $\frac{(\mathrm{tg}\,30° - i)^{15}}{(-1+i)^{2000}}(-\sin 30° + i\cos 30°)$.

12. Ildizlarni toping:

12.1. $\sqrt[3]{2-i}$.

12.2. $\sqrt[6]{64}$.

12.3. $\sqrt[4]{2+3i}$.

12.4. $\sqrt[4]{\frac{1}{8}+\frac{i\sin 60°}{4}}$.

12.5. $\sqrt[5]{\frac{-2i}{1-\sqrt{3}i}}$.

12.6. $\sqrt[4]{\frac{1}{2}((\sqrt{3}+1)+(\sqrt{3}-1)i)}$.

12.7. $\sqrt[6]{\frac{1-i}{-\sqrt{3}+i}}$.

12.8. $\sqrt[8]{\left(\frac{-5+7i}{i}\right)^3}$.

13. Birning 6-darajali ildizlari to'plami multiplikativ gruppa tashkil etishini isbotlang.

14. Agar *n* va *m* o'zaro tub bo'lsa, u holda birning *nm*-darajali ildizlarini birning *n*- va *m*- darajali ildizlari yordamida ifodalash mumkinligini isbotlang.

Takrorlash uchun savollar

1. Haqiqiy sonlar maydonining kompleks kengaytmasini quring.
2. Kompleks sonlar ustida arifmetik amallarni aniqlang.
3. Kompleks sonning geometrik tasviri nimadan iborat?
4. Geometrik ko'rinishdagi kompleks sonlarni qo'shish qanday bajariladi?
5. Kompleks sonning argumenti qanday aniqlanadi?
6. Trigonometrik ko'rinishda berilgan kompleks sonlarni ko'paytirish, bo'lish amallari qanday bajariladi?
7. Kompleks sonning trigonometrik shaklga keltirish qanday amalga oshiriladi?
8. Birning *n*-darajali ildiziga ta'rif bering.
9. Birning *n*-darajali ildizlari soni nechta? Javobingizni asoslang.
10. Ixtiyoriy kompleks sondan *n*-darajali ildiz topish formulasini ifodalang.

V MODUL. ARIFMETIK VEKTOR FAZO. CHIZIQLI TENGLAMALAR SISTEMASI

13-§. Arifmetik vektor fazo

✓ **Asosiy tushunchalar**: n-o'lchovli arifmetik vektor, vektorlar yig'indisi, skalarni vektorga ko'paytirish, n-o'lchovli arifmetik vektor fazo, chiziqli vektor fazo, fazoosti, vektorlar sistemasi, chiziqli kombinatsiya, chiziqli bog'liq sistema, chiziqli bog'lanmagan sistema, vektorlarning ekvivalent sistemalari, vektorlar sistemasini elementar almashtirishlar, vektorlar sistemasining bazisi, vektorlar sistemasining rangi, vektorlar sistemasining chiziqli qobig'i, chiziqli ko'pxillik.

$F =< F;+,\cdot,-,^{-1},0,1 >$ ixtiyoriy maydon bo'lib, F uning asosiy to'plami bo'lsin. F^n to'g'ri ko'paytmaning ixtiyoriy $\vec{a} = (a_1, a_2, ..., a_n)$ elementi *n-o'lchovli arifmetik vektor* deyiladi.

F^n ning ixtiyoriy ikkita $\vec{a} = (a_1, a_2, ..., a_n)$ va $\vec{b} = (b_1, b_2, ..., b_n)$ vektorlari uchun $a_1 = b_1$, $a_2 = b_2$, ..., $a_n = b_n$ bo'lsa, berilgan vektorlar teng deyiladi.

F^n ning ixtiyoriy ikkita $\vec{a} = (a_1, a_2, ..., a_n)$ va $\vec{b} = (b_1, b_2, ..., b_n)$ vektorlarining yig'indisi deb $\vec{a} + \vec{b} = (a_1 + b_1, a_2 + b_2, ..., a_n + b_n)$ vektorga aytiladi.

$\forall\lambda \in F$ skalarning $\forall\vec{a} \in F^n$ vektorga ko'paytirish deb, $\lambda\vec{a} = (\lambda a_1, \lambda a_2, ..., \lambda a_n)$ vektorga aytiladi.

F^n to'plam, unda aniqlangan qo'shish binar amali va skalarni vektorga ko'paytirish unar amallari yordamida hosil qilingan $F^n = < F^n;+,\{\omega_\lambda \mid \lambda \in F\} >$ algebra maydon ustida qurilgan *n o'lchovli arifmetik vektor fazo* deyiladi.

$F^n = < F^n;+,\{\omega_\lambda \mid \lambda \in F\} >$ n-o'lchovli arifmetik vektor fazo berilgan bo'lsin. F^n ning ixtiyoriy bo'sh bo'lmagan qism to'plami F^k $(k \le n)$ arifmetik vektor fazo tashkil qilsa, F^k arifmetik vektor fazo F^n arifmetik vektor fazoning *fazoostisi* (qismfazosi) deyiladi.

F^n vektor fazoning vektorlaridan iborat $\vec{a}_1, \vec{a}_2, ..., \vec{a}_n, ...$ sistemaga *vektorlarning cheksiz sistemasi*; $\vec{a}_1, \vec{a}_2, ..., \vec{a}_n$ sistemaga *vektorlarning chekli sistemasi* deyiladi.

F^n vektor fazoning $\vec{a}_1, \vec{a}_2, ..., \vec{a}_n, ...$ sistemasi va F maydonning $\lambda_1, \lambda_2, ..., \lambda_n, ...$ skalarlari berilgan bo'lsin. $\lambda_1\vec{a}_1 + \lambda_2\vec{a}_2 + ... + \lambda_n\vec{a}_n + ...$ ifoda $\vec{a}_1, \vec{a}_2, ..., \vec{a}_n, ...$ vektorlar sistemasining *chiziqli kombinatsiyasi* deyiladi. Agar kamida bittasi noldan farqli shunday $\lambda_1, \lambda_2, ..., \lambda_n$ skalarlar topilib, $\lambda_1\vec{a}_1 + \lambda_2\vec{a}_2 + ... + \lambda_n\vec{a}_n = \vec{0}$ tenglik bajarilsa, u holda $\vec{a}_1, \vec{a}_2, ..., \vec{a}_n$ sistema vektorlarning *chiziqli bog'langan sistemasi* deyiladi; tenglik $\lambda_1 = 0, \lambda_2 = 0, ..., \lambda_n = 0$ bo'lganda bajarilsa, u holda $\vec{a}_1, \vec{a}_2, ..., \vec{a}_n$ vektorlarning *chiziqli bog'lanmagan (chiziqli erkli) sistemasi* deyiladi.

Agar S va T sistemalarning ixtiyoriy biridan olingan har qanday noldan farqli vektorni ikkinchi sistema vektorlarining chiziqli kombinatsiyasi sifatida ifodalash mumkin bo'lsa, bunday sistemalar *ekvivalent sistemalar* deyiladi va $R \sim T$ ko'rinishda belgilanadi.

Vektorlar chekli sistemasini *elementar almashtirishlar* deb quyidagi almashtirishlarga aytiladi:

1) sistemaning qandaydir bir vektorini noldan farqli skalarga ko'paytirish;

2) sistemaning skalarga ko'paytirilgan bir vektorini ikkinchi vektoriga qo'shish yoki ayirish;

3) nol vektorni sistemadan chiqarish yoki sistemaga kiritish.

Vektorlar chekli sistemasining chiziqli erkli, bo'sh bo'lmagan qism sistemasi yordamida sistemaning har qanday vektorini chiziqli ifodalash mumkin bo'lsa, bunday qism sistema berilgan sistemaning *bazisi* deyiladi.

Vektorlar chekli sistemasining ixtiyoriy bazisidagi vektorlar soni uning *rangi* deyiladi.

$\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\ldots+\alpha_n\vec{a}_n(\alpha_i\in F)$ ko‘rinishdagi barcha chiziqli kombinatsiyalar to‘plami $\vec{a}_1,\vec{a}_2,\ldots,\vec{a}_n$ vektorlarning chiziqli qobig‘i deyiladi va u $L(\vec{a}_1,\vec{a}_2,\ldots,\vec{a}_n)$ ko‘rinishda belgilanadi.

$\vec{x}_0+W=\{\vec{x}_0+\vec{y}\mid \vec{x}_0\in F^n\}$ to‘plam W qism fazoning $\vec{x}_0$ vektorga siljitishdan hosil bo‘lgan *chiziqli ko‘pxillik* deyiladi va u $N=\vec{x}_0+W$ orqali belgilanadi.

1-misol. $V=\{ax^2+by+c\mid a,b,c,x,y\in R\}$ to‘plam R maydon ustida chiziqli fazo tashkil etishini isbotlang.

Yechish. Chiziqli fazo ta’rifiga ko‘ra, berilgan V to‘plamda qo‘shish binar amalini, skalarni vektorga ko‘paytirish unar amallarini aniqlab, ular uchun quyidagi xossalar tekshiriladi:

1°. $\forall(\vec{a},\vec{b}\in V)\ (\vec{a}+\vec{b}=\vec{b}+\vec{a})$.

2°. $\forall(\vec{a},\vec{b},\vec{c}\in V)\ (\vec{a}+(\vec{b}+\vec{c})=(\vec{a}+\vec{b})+\vec{c})$.

3°. $\forall(\vec{a}\in V)\wedge\exists(\vec{e}\in V)\ (\vec{a}+\vec{e}=\vec{a})$.

4°. $\forall(\vec{a}\in V)\wedge\exists(\vec{a'}\in V)\ (\vec{a}+\vec{a'}=\vec{0})$.

5°. $\forall(\alpha\in R)\wedge\forall(\vec{a},\vec{b}\in V)\ (\alpha(\vec{a}+\vec{b})=\alpha\vec{a}+\alpha\vec{b})$.

6°. $\forall(\alpha,\beta\in R)\wedge\forall(\vec{a}\in V)\ ((\alpha\beta)\vec{a}=\alpha(\beta\vec{a}))$.

7°. $\forall(\alpha,\beta\in R)\wedge\forall(\vec{a}\in V)\ ((\alpha+\beta)\vec{a}=\alpha\vec{a}+\beta\vec{a})$.

8°. $\forall(\vec{a}\in V)\ (1\cdot\vec{a}=\vec{a})$.

V to‘plamning ixtiyoriy $\vec{z}_1=a_1x^2+b_1y+c_1$ va $\vec{z}_2=a_2x^2+b_2y+c_2$ elementlari berilgan bo‘lsin.

$$\vec{z}_1+\vec{z}_2=a_1x^2+b_1y+c_1+a_2x^2+b_2y+c_2=$$
$$=(a_1+a_2)x^2+(b_1+b_2)y+(c_1+c_2)\in V.$$

Demak, qo‘shish binar amali V to‘plamda aniqlangan va $<V;+>$ additiv gruppoid bo‘ladi.

1-xossaning isboti. V to‘plamning ixtiyoriy $\vec{z}_1,\vec{z}_2$ elementlari berilgan bo‘lsin:

$$\vec{z}_1+\vec{z}_2=a_1x^2+b_1y+c_1+a_2x^2+b_2y+c_2=(a_1+a_2)x^2+(b_1+b_2)y+$$
$$+(c_1+c_2)=|R \text{ da qo‘shish amali kommutativ bo‘lganligi uchun}|=$$
$$=(a_2+a_1)x^2+(b_2+b_1)y+(c_2+c_1)=a_2x^2+b_2y+c_2+$$
$$+a_1x^2+b_1y+c_1=\vec{z}_2+\vec{z}_1.$$

Demak, V da qo'shish amali kommutativ va $<V; +>$ additiv abel gruppoid.

2-x o s s a n i n g i s b o t i. V to'plamning ixtiyoriy $\vec{z}_1 = a_1x^2 + b_1y + c_1$, $\vec{z}_2 = a_2x^2 + b_2y + c_2$, $\vec{z}_3 = a_3x^2 + b_3y + c_3$ elementlari berilgan bo'lsin. $(\vec{z}_1 + \vec{z}_2) + \vec{z}_3 = |1° \text{ ga ko'ra}| = (a_1 + a_2)x^2 + (b_1 + b_2)y + (c_1 + c_2) + a_3x^2 + b_3y + c_3 =$ | haqiqiy sonlar to'plamida qo'shish amali kommutativ, ko'paytirish qo'shishga nisbatan distributivligidan | $= ((a_1 + a_2) + a_3)x^2 + ((b_1 + b_2) + b_3)y + (c_1 + c_2) + c_3 =$ | haqiqiy sonlar maydonida qo'shish amali assotsiativ bo'lganligi uchun | =

$$= (a_1 + (a_2 + a_3))x^2 + (b_1 + (b_2 + b_3))y + c_1 + (c_2 + c_3) =$$
$$= a_1x^2 + b_1y + c_1 + (a_2 + a_3)x^2 + (b_2 + b_3)y + (c_2 + c_3) =$$
$$= a_1x^2 + b_1y + c_1 + (a_2x^2 + b_2y + c_2 + a_3x^2 + b_3y + c_3) = \vec{z}_1 + (\vec{z}_2 + \vec{z}_3).$$

Demak, V to'plamda qo'shish amali assotsiativ va $<V; +>$ – additiv abel gruppa.

3-x o s s a n i n g i s b o t i. V to'plamning ixtiyoriy $\vec{z} = ax^2 + by + c$ va shunday $\vec{e} = e_1x^2 + e_2y + e_3$ elementlari uchun $\vec{z} + \vec{e} = \vec{z}$ ekanligini aniqlaymiz: $\vec{z} + \vec{e} = \vec{z}$ dan $ax^2 + by + c + e_1x^2 + e_2y + e_3 = ax^2 + by + c$ tenglamani, bundan $\begin{cases} a + e_1 = a, \\ b + e_2 = b, \\ c + e_3 = c \end{cases}$

tenglamalar sistemasini hosil qilamiz. R da tenglamalar sistemasi yagona $e_1=0$, $e_2=0$, $e_3=0$ yechimga ega.

Demak, $\vec{e} = 0 \cdot x^2 + 0 \cdot y + 0 = \vec{0} \in V$ va $<V; +, 0>$ – additiv abel monoid.

4-x o s s a n i n g i s b o t i. V to'plamning ixtiyoriy $\vec{z} = ax^2 + by + c$ va shunday $\overrightarrow{z'} = a'x^2 + b'y + c'$ elementlari uchun $\vec{z} + \overrightarrow{z'} = \vec{0}$ ekanligini keltirib chiqaramiz:

$\vec{z}+\overrightarrow{z'}=\vec{0}$ dan $ax^2+by+c+a'x^2+b'y+c'=0\cdot x^2+0\cdot y+0$

tenglamani, bundan $\begin{cases} a+a'=0, \\ b+b'=0, \\ c+c'=0 \end{cases}$ tenglamalar sistemasini hosil qilamiz.

Tenglamalar sistemasi R maydonda yagona $a'=-a$, $b'=-b$, $c'=-c$ yechimga ega. Demak, $\overrightarrow{z'}=-ax^2-by-c=-(ax^2+by+c)\in V$ va $<V;+,-,\vec{0}>$ additiv abel gruppa.

V to'plamda skalarni ko'paytirish unar amalini aniqlaymiz:

ixtiyoriy $\alpha\in R$ skalar va ixtiyoriy $\vec{z}=ax^2+by+c\in V$ berilgan bo'lsin. $\omega_\alpha(\vec{z})=\alpha\cdot z=\alpha\cdot(ax^2+by+c)=\alpha\cdot(ax^2)+\alpha\cdot(by)+\alpha\cdot c=$ $=|$ R da ko'paytirishning assotsiativligidan$|$ $=(\alpha\cdot a)x^2+(\alpha\cdot b)y+$ $+\alpha\cdot c\in V$ ni hosil qilamiz. Demak, V da skalarni V ning elementiga ko'paytirish unar amallari aniqlangan.

5-x o s s a n i n g i s b o t i. Ixtiyoriy $\alpha\in R$ skalar va V ning $\vec{z}_1,\vec{z}_2$ elementlari berilgan bo'lsin.

$\alpha(\vec{z}_1+\vec{z}_2)=\alpha((a_1+a_2)x^2+(b_1+b_2)y+(c_1+c_2))=$

$=\alpha(a_1+a_2)x^2+\alpha(b_1+b_2)y+\alpha(c_1+c_2)=$

$=|$ R da ko'paytirishning qo'shishga nisbatan distributivligidan$|=$

$=\alpha\cdot a_1x^2+\alpha\cdot a_2x^2+\alpha\cdot b_1y+\alpha\cdot b_2y+\alpha\cdot c_1+\alpha\cdot c_2=$

$=|$ R da qo'shish amalining kommutativligidan$|=$

$=\alpha\cdot a_1x^2+\alpha\cdot b_1y+\alpha\cdot c_1+\alpha\cdot a_2x^2+\alpha\cdot b_2y+\alpha\cdot c_2=$

$=|$ R da ko'paytirishning qo'shishga nisbatan distributivligidan$|=$

$=\alpha(a_1x^2+b_1y+c_1)+\alpha(a_2x^2+b_2y+c_2)=\alpha\cdot\vec{z}_1+\alpha\cdot\vec{z}_2.$

Demak, skalarni vektorlar yig'indisiga ko'paytirish distributiv.

6-x o s s a n i n g i s b o t i. Ixtiyoriy α, $\beta\in R$ skalarlar va ixtiyoriy $\vec{z}\in V$ element berilgan bo'lsin.

$(\alpha\cdot\beta)\vec{z}=(\alpha\cdot\beta)(ax^2+by+c)=$ $|$ R da ko'paytirishning qo'shishga nisbatan distributivligidan $|$ $=(\alpha\cdot\beta)ax^2+(\alpha\cdot\beta)by+(\alpha\cdot\beta)c=$ $=|$ ko'paytirishning R da assotsiativligi va ko'paytirishni qo'shish-

ga nisbatan distributivligidan | $= \alpha(\beta \cdot ax^2) + \alpha(\beta \cdot by) + \alpha(\beta \cdot c) =$
$= \alpha(\beta(ax^2) + (\beta(by) + (\beta c))) = \alpha(\beta(ax^2 + by + c)) = \alpha(\beta \cdot \vec{z})$.

Demak, skalarlar ko'paytmasini V elementiga ko'paytirish assotsiativ.

7-x o s s a n i n g   i s b o t i. Ixtiyoriy α, $\beta \in R$ skalarlar va ixtiyoriy $\vec{z} \in V$ element berilgan bo'lsin.

$(\alpha + \beta)\vec{z} = (\alpha + \beta)(ax^2 + by + c) =$ | R da ko'paytirishning qo'shishga nisbatan distributivligidan| $= (\alpha + \beta)ax^2 + (\alpha + \beta)by + (\alpha + \beta)c =$
=| R da yig'indini o'ngdan ko'paytirishning distributivligidan | =
$= \alpha \cdot ax^2 + \beta \cdot ax^2 + \alpha \cdot by + \beta \cdot by + \alpha \cdot c + \beta \cdot c =$| R da qo'shishning komutativligidan | $= \alpha \cdot ax^2 + \alpha \cdot by + \alpha \cdot c + \beta \cdot ax^2 + \beta \cdot by + \beta \cdot c =$
= | R da ko'paytirishning qo'shishga nisbatan distributivligidan|=
$\alpha(ax^2 + by + c) + \beta(ax^2 + by + c) = \alpha \cdot \vec{z} + \beta \cdot \vec{z}$.

Demak, skalarlar yig'indisini V elementiga ko'paytirish distributiv.

8-x o s s a n i n g   i s b o t i. V to'plamning ixtiyoriy $\vec{z}$ elementi berilgan bo'lsin. Skalarlar to'plami maydon tashkil etishi va har qanday maydonda 1 mavjud ekanligidan

$$1 \cdot \vec{z} = 1 \cdot (ax^2 + by + c) = ax^2 + by + c = \vec{z}.$$

Demak, $< V; +, \{\omega_\alpha \mid a \in R\} >$ algebra chiziqli fazo bo'ladi.

2-misol. $\overrightarrow{a_1} = (1,2,3,4)$, $\overrightarrow{a_2} = (0,1,2,1)$, $\overrightarrow{a_3} = (1,-1,2,1)$ vektorlar sistemasining chiziqli bog'liq yoki chiziqli bog'liqmasligini aniqlang. Uning bazisi va rangini toping.

Yechish. *1-usul.* Chiziqli tenglamalar sistemasi yordamida. Ixtiyoriy α, β, γ skalarlar berilgan bo'lsin. Berilgan vektorlar sistemasining chiziqli kombinatsiyasini tuzamiz: $\alpha\overrightarrow{a_1} + \beta\overrightarrow{a_2} + \gamma\overrightarrow{a_3} = \vec{0}$, ya'ni $\alpha(1,2,3,4) + \beta(0,1,2,1) + \gamma(1,-1,2,1) = \vec{0}$. Hosil bo'lgan tenglikdan skalarlar qiymatini topamiz. Buning uchun chiziqli tenglamalar sistemasini tuzamiz:

$$\begin{cases} \alpha + \gamma = 0, \\ 2\alpha + \beta - \lambda = 0, \\ 3\alpha + 2\beta + 2\gamma = 0, \\ 4\alpha + \beta + \gamma = 0. \end{cases}$$

Hosil bo'lgan chiziqli tenglamalar sistemasini Gauss usulida yechamiz:

$$\begin{cases} \alpha+\gamma=0, \\ \beta-3\gamma=0, \\ 2\beta-\gamma=0, \\ \beta-3\gamma=0 \end{cases} \Rightarrow \begin{cases} \alpha+\gamma=0, \\ \beta-3\gamma=0, \\ 2\beta-\gamma=0, \end{cases} \Rightarrow \begin{cases} \alpha+\gamma=0, \\ \beta-3\gamma=0, \\ 5\gamma=0 \end{cases} \Rightarrow \begin{cases} \alpha=0, \\ \beta=0, \\ \gamma=0. \end{cases}$$

$\alpha\overline{a_1}+\beta\overline{a_2}+\gamma\overline{a_3}=\bar{0}$ tenglikdan skalarlarning barchasi nolga tengligi kelib chiqdi. Demak, $\overline{a_1}$, $\overline{a_2}$, $\overline{a_3}$ vektorlar sistemasi chiziqli bog'liq emas. Shuning uchun sistema o'ziga bazis, rangi 3 ga teng.

3-misol. $(a):\overline{a_1}=(1,-1,3);\overline{a_2}=(-1,1,1)$ va $(b):\overline{b_1}=(-2,2,-6)$; $\overline{b_2}=(-1,0,1)$ sistemalar ekvivalentligini tekshiring.

Yechish. Vektorlarning chekli sistemalari ekvivalentligining ta'rifiga ko'ra (*a*) sistemaning har bir vektori (*b*) sistema orqali va (*b*) sistemaning har bir vektori (*a*) sistema orqali chiziqli ifodalanishini tekshiramiz. Buning uchun

$$\overline{a_1}=\beta_{11}\overline{b_1}+\beta_{12}\overline{b_2},$$

$$\overline{a_2}=\beta_{21}\overline{b_1}+\beta_{22}\overline{b_2},$$

$$\overline{b_1}=\alpha_{11}\overline{a_1}+\alpha_{12}\overline{a_2},$$

$$\overline{b_2}=\alpha_{21}\overline{a_1}+\alpha_{22}\overline{a_2}$$

tenglamalardagi skalarlarning qiymatlarini topamiz. Ya'ni

1) $(1,-1,3)=\beta_{11}(-2,2,-6)+\beta_{12}(-1,0,1)$;

2) $(-1,1,1)=\beta_{21}(-2,2,-6)+\beta_{22}(-1,0,1)$;

3) $(-2,2,-6)=\alpha_{11}(1,-1,3)+\alpha_{12}(-1,1,1)$;

4) $(-1,0,1)=\alpha_{21}(1,-1,3)+\alpha_{22}(-1,1,1)$.

Tenglamalarning har biridan quyidagi tenglamalar sistemalarini hosil qilamiz:

$$1)\ \begin{cases} 1 = -2\beta_{11} - \beta_{12}, \\ -1 = 2\beta_{11}, \\ 3 = -6\beta_{11} + \beta_{12}; \end{cases} \qquad 2)\ \begin{cases} -1 = -2\beta_{21} - \beta_{22}, \\ 1 = 2\beta_{21}, \\ 1 = -6\beta_{21} + \beta_{22}; \end{cases}$$

$$3)\ \begin{cases} -2 = \alpha_{11} - \alpha_{12}, \\ 2 = -\alpha_{11} + \alpha_{12}, \\ -6 = 3\alpha_{11} + \alpha_{12}; \end{cases} \qquad 4)\ \begin{cases} -1 = \alpha_{21} - \alpha_{22}, \\ 0 = -\alpha_{21} + \alpha_{22}, \\ 1 = 3\alpha_{21} + \alpha_{22}. \end{cases}$$

Tenglamalar sistemalarini yechamiz:

$$1)\ \begin{cases} 1 = -2\beta_{11} - \beta_{12}, \\ -1 = 2\beta_{11}, \\ 3 = -6\beta_{11} + \beta_{12} \end{cases} \Rightarrow \begin{cases} \beta_{11} = -\frac{1}{2}, \\ \beta_{12} = 0; \end{cases}$$

$$2)\ \begin{cases} -1 = -2\beta_{21} - \beta_{22}, \\ 1 = 2\beta_{21}, \\ 1 = -6\beta_{21} + \beta_{22}, \end{cases} \Rightarrow \begin{cases} \beta_{21} = \frac{1}{2}, \\ \beta_{22} = 0; \end{cases}$$

$$3)\ \begin{cases} -2 = \alpha_{11} - \alpha_{12}, \\ 2 = -\alpha_{11} + \alpha_{12}, \\ -6 = 3\alpha_{11} + \alpha_{12}, \end{cases} \Rightarrow \begin{cases} -2 = \alpha_{11} - \alpha_{12}, \\ -6 = 3\alpha_{11} + \alpha_{12}, \end{cases} \Rightarrow \begin{cases} -2 = \alpha_{11} - \alpha_{12}, \\ 0 = 4\alpha_{12}, \end{cases} \Rightarrow \begin{cases} \alpha_{12} = 0, \\ \beta\alpha_{11} = -2; \end{cases}$$

$$4)\ \begin{cases} -1 = \alpha_{21} - \alpha_{22}, \\ 0 = -\alpha_{21} + \alpha_{22}, \\ 1 = 3\alpha_{21} + \alpha_{22}, \end{cases} \Rightarrow \begin{cases} -1 = \alpha_{21} - \alpha_{22}, \\ 0 = -\alpha_{21} + \alpha_{22}, \\ 0 = 4\alpha_{21}, \end{cases} \Rightarrow \begin{cases} \alpha_{21} = 0, \\ \alpha_{22} = 0, \\ -1 \neq 0 - 0 \end{cases} \Rightarrow \text{yechim}$$

mavjud emas.

Demak, $\overrightarrow{a_1} = -\frac{1}{2}\overrightarrow{b_1}$, $\overrightarrow{a_2} = \frac{1}{2}\overrightarrow{b_1}$, $\overrightarrow{b_1} = -2\overrightarrow{a_1}$.

Ya'ni $\overrightarrow{b_2}$ vektor (*a*) sistema yordamida chiziqli ifodalanmaydi. Shu sababli (*a*), (*b*) sistemalar ekvivalent emas.

4-misol. $\vec{x}=(1,2,1,1)$ vektorni $(a): \vec{a_1}=(0,1,1,0)$, $\vec{a_2}=(-1,0,0,1)$, $\vec{a_3}=(2,1,1,4)$ vektorlar orqali chiziqli ifodalang.

Yechish. Haqiqiy sonlar maydonidan shunday α, β, γ skalarlarni aniqlashimiz kerakki, ular $\vec{x}=\alpha\vec{a_1}+\beta\vec{a_2}+\gamma\vec{a_3}$ tenglikni qanoatlantirsin.

Buning uchun tenglamalar sistemasini tuzamiz:

(1, 2, 1, 1)=α(0, 1, 1, 0)+β(−1, 0, 0, 1)+γ(2, 1, 1, 4) tenglikdan quyidagi sistema kelib chiqadi:

$$\begin{cases} 1=-\beta+2\alpha, \\ 2=\alpha+\gamma, \\ 1=\alpha+\gamma, \\ 1=\beta+4\gamma. \end{cases}$$

Hosil bo'lgan sistema hamjoyli bo'lsa, $\vec{x}$ vektor $\vec{a_1}, \vec{a_2}, \vec{a_3}$ vektorlar orqali chiziqli ifodalanadi. Lekin sistemaning 2- va 3-tenglamalari birgalikda yechimga ega emas.

Demak, $\vec{x}$ vektor (a) sistema orqali chiziqli ifodalanmaydi.

5-misol. $\vec{x}=(1,3,0)$ vektorni $(a): \vec{a_1}=(1,0,2), \vec{a_2}=(2,3,2)$ vektorlar orqali chiziqli ifodalang.

Yechish. $\vec{x}=\alpha\vec{a_1}+\beta\vec{a_2}$ tenglikni qanoatlantiruvchi α, β haqiqiy sonlarni aniqlaymiz:

(1, 3, 0)=α(1, 0, 2)+β(2, 3, 2) tenglamadan $\begin{cases} 1=\alpha+2\beta, \\ 3=3\beta, \\ 0=2\alpha+2\beta \end{cases}$

tenglamalar sistemasini hosil qilamiz.

Chiziqli tenglamalar sistemasining $\begin{cases} \beta=1, \\ \alpha=-1 \end{cases}$ yechimi yordamida $\vec{x}=(-1)\cdot\vec{a_1}+1\cdot\vec{a_2}$, ya'ni $\vec{x}=-\vec{a_1}+\vec{a_2}$ ifodani yozish mumkin. Demak, $\vec{x}$ vektorning $\vec{a_1}, \vec{a_2}$ vektorlar orqali chiziqli ifodasi mavjud.

6-misol. $(a): \overrightarrow{a_1} = (0,1,1,0)$, $\overrightarrow{a_2} = (-1,0,0,1)$, $\overrightarrow{a_3} = (2,1,1,4)$ vektorlar sistemasining chiziqli qobig‘i chiziqli fazo tashkil etishini isbotlang.

Yechish. Ta’rifga ko‘ra $(a): \overrightarrow{a_1} = (0,1,1,0)$, $\overrightarrow{a_2} = (-1,0,0,1)$, $\overrightarrow{a_3} = (2,1,1,4)$ sistemaning chiziqli qobig‘i

$$L(a_1, a_2, a_3) = \{\alpha_1 a_1 + \alpha_2 a_2 + \alpha_3 a_3 \; a_1, a_2, a_3 \in R\}$$

to‘plamdan iborat. Uning chiziqli fazo tashkil etishini tekshiramiz:

1. $L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ to‘plamning ixtiyoriy $\vec{z}_1 = \alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3$ va $\vec{z}_2 = \beta_1\vec{a}_1 + \beta_2\vec{a}_2 + \beta_3\vec{a}_3$ elementlari berilgan bo‘lsin.

$$\vec{z}_1 + \vec{z}_2 = \alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3 + \beta_1\vec{a}_1 + \beta_2\vec{a}_2 + \beta_3\vec{a}_3 =$$
$$= (\alpha_1 + \beta_1)\vec{a}_1 + (\alpha_2 + \beta_2)\vec{a}_2 + (\alpha_3 + \beta_3)\vec{a}_3 \in L(\vec{a}_1, \vec{a}_2, \vec{a}_3).$$

Demak, qo‘shish binar amali $L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ to‘plamda aniqlangan.

2. $L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ to‘plamning ixtiyoriy $\vec{z}_1, \vec{z}_2$ elementlari berilgan bo‘lsin.

$$\vec{z}_1 + \vec{z}_2 = \alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3 + \beta_1\vec{a}_1 + \beta_2\vec{a}_2 + \beta_3\vec{a}_3 =$$
$$= (\alpha_1 + \beta_1)\vec{a}_1 + (\alpha_2 + \beta_2)\vec{a}_2 + (\alpha_3 + \beta_3)\vec{a}_3 =$$
$$= (\beta_1 + \alpha_1)\vec{a}_1 + (\beta_2 + \alpha_2)\vec{a}_2 + (\beta_3 + \alpha_3)\vec{a}_3 =$$
$$= \beta_1\vec{a}_1 + \beta_2\vec{a}_2 + \beta_3\vec{a}_3 + \alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3 = \vec{z}_2 + \vec{z}_1.$$

Demak, $L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ da qo‘shish amali kommutativ.

3. $L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ to‘plamning ixtiyoriy $\vec{z}_1 = \alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3$, $\vec{z}_2 = \beta_1\vec{a}_1 + \beta_2\vec{a}_2 + \beta_3\vec{a}_3$, $\vec{z}_3 = \gamma_1\vec{a}_1 + \gamma_2\vec{a}_2 + \gamma_3\vec{a}_3$ elementlari berilgan bo‘lsin.

$(\vec{z}_1 + \vec{z}_2) + \vec{z}_3 = (\alpha_1 + \beta_1)\vec{a}_1 + (\alpha_2 + \beta_2)\vec{a}_2 + (\alpha_3 + \beta_3)\vec{a}_3 +$
$+\gamma_1\vec{a}_1 + \gamma_2\vec{a}_2 + y_3\vec{a}_3 =|$ haqiqiy sonlar to‘plamida qo‘shish amali kommutativ, ko‘paytirish qo‘shishga nisbatan distributivligidan $|=$
$= ((\alpha_1 + \beta_1) + \gamma_1)\vec{a}_1 + ((\alpha_2 + \beta_2) + \gamma_2)\vec{a}_2 + ((\alpha_3 + \beta_3) + \gamma_3)\vec{a}_3 =$

= | haqiqiy sonlar maydonida qo'shish amali assotsiativ bo'lganligi uchun | =

$$(\alpha_1 + (\beta_1 + \gamma_1))\vec{a}_1 + (\alpha_2 + (\beta_2 + \gamma_2))\vec{a}_2 + (\alpha_3 + (\beta_3 + \gamma_3))\vec{a}_3 =$$
$$= (\alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3) + ((\beta_1 + \gamma_1)\vec{a}_1 + (\beta_2 + \gamma_2)\vec{a}_2 + (\beta_3 + \gamma_3)\vec{a}_3) =$$
$$= (\alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3) + ((\beta_1\vec{a}_1 + \beta_2\vec{a}_2 + \beta_3\vec{a}_3) + (\gamma_1\vec{a}_1 + \gamma_2\vec{a}_2 + \gamma_3\vec{a}_3)) =$$
$$= \vec{z}_1 + (\vec{z}_2 + \vec{z}_3).$$

Demak, $L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ to'plamda qo'shish amali assotsiativ.

4. $L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ to'plamning ixtiyoriy $\vec{z} = \alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3$ elementi $\vec{z} + \vec{e} = \vec{z}$ uchun tenglikni qanoatlantiruvchi shunday $\vec{e} = e_1\vec{a}_1 + e_2\vec{a}_2 + e_3\vec{a}_3$ mavjudligini aniqlaymiz.

$\vec{z} + \vec{e} = \vec{z}$ tenglikdan $(\alpha_1 + e_1)\vec{a}_1 + (\alpha_2 + e_2)\vec{a}_2 + (\alpha_3 + e_3)\vec{a}_3 =$

$= \alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3$ tenglamani, bundan $\begin{cases} \alpha_1 + e_1 = \alpha_1, \\ \alpha_2 + e_2 = \alpha_2, \\ \alpha_3 + e_3 = \alpha_3 \end{cases}$ tenglamalar sistemasini hosil qilamiz. R da tenglamalar sistemasi yagona e_1=0, e_2=0, e_3=0 yechimga ega.

Demak, $\vec{e} = 0 \cdot \vec{a}_1 + 0 \cdot \vec{a}_2 + 0 \cdot \vec{a}_3 = \vec{0} \in L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$.

5. $L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ to'plamning ixtiyoriy $\vec{z} = \alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3$ va shunday $\vec{z}' = \alpha_1'\vec{a}_1 + \alpha_2'\vec{a}_2 + \alpha_3'\vec{a}_3$ elementlari uchun $\vec{z} + \vec{z}' = \vec{0}$ ekanligini keltirib chiqaramiz:

$\vec{z} + \vec{z}' = \vec{0}$ dan

$(\alpha_1 + \alpha'_1)\vec{a}_1 + (\alpha_2 + \alpha'_2)\vec{a}_2 + (\alpha_3 + \alpha'_3)\vec{a}_3 = 0 \cdot \vec{a}_1 + 0 \cdot \vec{a}_2 + 0 \cdot \vec{a}_3$

tenglamani, bundan $\begin{cases} \alpha_1 + \alpha_1' = 0, \\ \alpha_2 + \alpha_2' = 0, \\ \alpha_3 + \alpha_3' = 0 \end{cases}$ tenglamalar sistemasini hosil qilamiz.

Tenglamalar sistemasi R maydonda yagona $\alpha_1 = -\alpha_1$, $\alpha_2 = -\alpha_2$, $\alpha_3 = -\alpha_3$ yechimga ega. Demak,

$$\vec{z}' = -\alpha_1\vec{a}_1 - \alpha_2\vec{a}_2 - \alpha_3\vec{a}_3 = -(\alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3) \in L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$$
va $< L(\vec{a}_1, \vec{a}_2, \vec{a}_3); +, -, \vec{0} >$ – additiv abel gruppa.

6. $L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ to'plamda λ skalarni $\vec{z} = \alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3$ vektorga ko'paytirish unar amalini aniqlaymiz.

Ixtiyoriy $\lambda \in R$ skalar va ixtiyoriy $\vec{z} = \alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3$ berilgan bo'lsin.

$$\omega_\alpha(\vec{z}) = \lambda \cdot \vec{z} = \lambda \cdot (\alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3) = \lambda \cdot (\alpha_1\vec{a}_1) + \lambda \cdot (\alpha_2\vec{a}_2) + \lambda \cdot (\alpha_3\vec{a}_3) =$$
$$= (\lambda \cdot \alpha_1)\vec{a}_1 + (\lambda \cdot \alpha_2)\vec{a}_2 + (\lambda \cdot \alpha_3)\vec{a}_3 \in L(\vec{a}_1, \vec{a}_2, \vec{a}_3).$$

Demak, $L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ da skalarni $L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ ning elementiga ko'paytirish unar amallari aniqlangan.

7. Ixtiyoriy $\lambda \in R$ skalar va $L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ ning $\vec{z}_1$, $\vec{z}_2$ elementlari berilgan bo'lsin.

$$\lambda(\vec{z}_1 + \vec{z}_2) = \lambda((\alpha_1 + \beta_1)\vec{a}_1 + (\alpha_2 + \beta_2)\vec{a}_2 + (\alpha_3 + \beta_3)\vec{a}_3) =$$
$$= \lambda(\alpha_1 + \beta_1)\vec{a}_1 + \lambda(\alpha_2 + \beta_2)\vec{a}_2 + \lambda(\alpha_3 + \beta_3)\vec{a}_3 =$$
$$= \lambda(\alpha_1\vec{a}_1) + \lambda(\alpha_2\vec{a}_2) + \lambda(\alpha_3\vec{a}_3) + \lambda(\beta_1\vec{a}_1) + \lambda(\beta_2\vec{a}_2) + \lambda(\beta_3\vec{a}_3) =$$
$$= \lambda(\alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3) + \lambda(\beta_1\vec{a}_1 + \beta_2\vec{a}_2 + \beta_3\vec{a}_3) = \lambda \cdot \vec{z}_1 + \lambda \cdot \vec{z}_2.$$

Demak, skalarni vektorlar yig'indisiga ko'paytirish distributiv.

8. Ixtiyoriy λ, $\delta \in R$ skalarlar va ixtiyoriy $\vec{z} = \alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3$ element berilgan bo'lsin.

$$(\lambda \cdot \delta) \cdot \vec{z} = (\lambda \cdot \delta)(\alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3) =$$
$$= (\lambda \cdot \delta)(\alpha_1\vec{a}_1) + (\lambda \cdot \delta)(\alpha_2\vec{a}_2) + (\lambda \cdot \delta)(\alpha_3\vec{a}_3) =$$
$$= ((\lambda \cdot \delta)\alpha_1)\vec{a}_1 + ((\lambda \cdot \delta)\alpha_2)\vec{a}_2 + ((\lambda \cdot \delta)\alpha_3)\vec{a}_3 =$$
$$= \lambda(\delta \cdot \alpha_1)\vec{a}_1 + \lambda(\delta \cdot \alpha_2)\vec{a}_2 + \lambda(\delta \cdot \alpha_3)\vec{a}_3 =$$
$$= \lambda((\delta \cdot \alpha_1)\vec{a}_1 + (\delta \cdot \alpha_2)\vec{a}_2 + (\delta \cdot \alpha_3)\vec{a}_3 =$$
$$= \lambda(\delta(\alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3) = \lambda(\delta \cdot \vec{z}).$$

Demak, skalarlar ko'paytmasini $L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ elementga ko'paytirish assotsiativdir.

9. Ixtiyoriy λ, $\delta \in R$ skalarlar va ixtiyoriy $\vec{z} = \alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3$ element berilgan bo'lsin.

$$(\lambda+\delta)\cdot\vec{z}=(\lambda+\delta)(\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3)=(\lambda+\delta)\alpha_1\vec{a}_1+(\lambda+\delta)\alpha_2\vec{a}_2+$$
$$+(\lambda+\delta)\alpha_3\vec{a}_3=\lambda(\alpha_1\vec{a}_1)+\delta(\alpha_1\vec{a}_1)+\lambda(\alpha_2\vec{a}_2)+\delta(\alpha_2\vec{a}_2)+\lambda(\alpha_3\vec{a}_3)+$$
$$+\beta(\alpha_3\vec{a}_3)=\lambda(\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3)+\delta(\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3)=$$
$$=\lambda\cdot\vec{z}+\delta\cdot\vec{z}.$$

Demak, skalarlar yig'indisini $L(\vec{a}_1,\vec{a}_2,\vec{a}_3)$ elementga ko'paytirish distributivdir.

10. $L(\vec{a}_1,\vec{a}_2,\vec{a}_3)$ to'plamning ixtiyoriy $\vec{z}=\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3$ elementi berilgan bo'lsin. Skalarlar to'plami maydon tashkil etishi va har qanday maydonda 1 mavjud ekanligidan

$$1\cdot\vec{z}=1\cdot(\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3)=1\cdot(\alpha_1\vec{a}_1)+1\cdot(\alpha_2\vec{a}_2)+1\cdot(\alpha_3\vec{a}_3)=$$
$$=\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3=\vec{z}.$$

Demak, $< L(\vec{a}_1,\vec{a}_2,\vec{a}_3);+,\omega_\lambda \mid \lambda\in R >$ algebra chiziqli fazo bo'ladi.

$\vec{a}_1,\vec{a}_2,\vec{a}_3$ vektorlar sistemasi chiziqli qobig'i tashkil etgan chiziqli fazo bazisini topish uchun $\vec{a}_1,\vec{a}_2,\vec{a}_3$ vektorlar sistemasining bazisini topamiz.

$\vec{a}_1=(0,1,1,0)$, $\vec{a}_2=(-1,0,0,1)$, $\vec{a}_3=(2,1,1,4)$ sistema vektorlarini matritsaning satrlari sifatida olamiz va matritsaning bazisini topamiz:

$$\begin{matrix}\vec{a}_1\\ \vec{a}_2\\ \vec{a}_3\end{matrix}\begin{pmatrix}0&1&1&0\\-1&0&0&1\\2&1&1&4\end{pmatrix}\sim\begin{matrix}\vec{a}_2\\ \vec{a}_1\\ \vec{a}_3+\vec{a}_2\end{matrix}\begin{pmatrix}-1&0&0&1\\0&1&1&0\\0&1&1&6\end{pmatrix}\sim\begin{matrix}\vec{a}_2\\ \vec{a}_1\\ (\vec{a}_3+\vec{a}_2)-\vec{a}_1\end{matrix}\begin{pmatrix}-1&0&0&1\\0&1&1&0\\0&0&0&6\end{pmatrix}$$

Matritsaning satr vektorlari chiziqli erkli bo'lganligi uchun, $\vec{a}_1=(0,1,1,0)$, $\vec{a}_2=(-1,0,0,1)$, $\vec{a}_3=(2,1,1,4)$ vektorlar sistemasining bazisi sistemaning o'zidan iborat. Demak, $L=$ $=< L(\vec{a}_1,\vec{a}_2,\vec{a}_3);+,\omega_\lambda \mid \lambda\in R >$ chiziqli fazoning bazisi berilgan vektorlar sistemasidan iborat.

Misol va mashqlar

1. $\vec{x}=\vec{a}+\vec{b}-\vec{c}$ vektorni toping:

1.1. $\vec{a}=(4,2,3)$, $\vec{b}=(2,3,7)$, $\vec{c}=(1,7,11)$.

1.2. $\vec{a} = (2,4-,2,0), \vec{b} = (-1,3,17,3), \vec{c} = (0,-7,1,4)$.

1.3. $\vec{a} = (4,2,3) + (1,1,1), \vec{b} = (2,3,7), \vec{c} = 2(1,7,11)$.

1.4. $\vec{a} = (-1)(4,2,3), \vec{b} = \vec{c}, \vec{c} = (1,7,11)$.

2. $\vec{x}$ vektorni toping:

2.1. $\vec{x} = 2\vec{a} - 3\vec{b} + \vec{c}, \vec{a} = (1,2,3,0), \vec{b} = (-2,1,5,-1), \vec{c} = (\sqrt{2},-1,0,1)$.

2.2. $-3\vec{a} - \vec{x} = 2\vec{b}, \vec{a} = (0,-2,1), \vec{b} = (1,-3,7)$.

2.3. $2\vec{x} + 3\vec{a} - 4\vec{b} = \vec{0}, \vec{a} = (\sin\alpha, 0, -\cos\alpha), \vec{b} = (\sin^3\alpha, \frac{1}{4}, -\cos^3\alpha)$.

2.4. $\frac{1}{2}\vec{a} + 3\vec{b} - \vec{x} = 6\vec{c}, \vec{a} = (1,-3,2), \vec{b} = (\frac{1}{9}, \frac{11}{3}, -2), \vec{c} = (1, \frac{2}{3}, -2)$.

3. Vektorlarning quyidagi sistemalari chiziqli bog‘liq yoki chiziqli erkliligini aniqlang hamda uning bazisi va rangini toping:

3.1. $\vec{a}_1=(-1,2,-3,4)$; $\vec{a}_2=(-1,1,-1,1)$.

3.2. $\vec{a}_1=(0,2,0,4)$; $\vec{a}_2=(0,-2,-3,0)$; $\vec{a}_3=(-1,1,-1,1)$.

3.3. $\vec{a}_1=(1,2,3)$; $\vec{a}_2=(2,-3,4)$; $\vec{a}_3=(-1,-1,1)$; $\vec{a}_4=(3,4,2)$.

3.4. $\vec{a}_1=(1,2,3,4,-3)$; $\vec{a}_2=(-1,-2,3,4,1)$; $\vec{a}_3=(-5,1,-7,1,2)$; $\vec{a}_4=(0,4,1,2,0)$.

3.5. $\vec{a}_1=(2,3)$; $\vec{a}_2=(-1,-3)$; $\vec{a}_3=(1,-1)$; $\vec{a}_4=(3,1)$.

3.6. $\vec{a}_1=(1,1,-1)$; $\vec{a}_2=(4,1,2)$; $\vec{a}_3=(-2,4,7)$.

3.7. $\vec{a}_1=(0,2,3,4)$; $\vec{a}_2=(5,-2,-3,-4)$; $\vec{a}_3=(3,1,2,-3)$.

3.8. $\vec{a}_1=(0,2,0)$; $\vec{a}_2=(0,-2,-3)$; $a_3=(-1,1,-1)$.

3.9. $\vec{a}_1=(-4,2,3)$; $\vec{a}_2=(2,0,4)$; $\vec{a}_3=(-1,-1,-1)$.

3.10. $\vec{a}_1=(-4,2,3,0)$; $\vec{a}_2=(2,0,0,4)$; $\vec{a}_3=(-1,-1,0,-1)$.

4. Vektorlarning (*a*) va (*b*) sistemalari ekvivalent ekanligini tekshiring:

4.1. $\vec{a}_1=(2,3)$; $\vec{a}_2=(-1,-3)$; $\vec{a}_3=(1,-1)$; $\vec{a}_4=(3,1)$;
$\vec{b}_1=(-2,-6)$; $\vec{b}_2=(1,-1)$; $\vec{b}_3=(4,0)$.

4.2. $\vec{a}_1=(1,-3,4)$; $\vec{a}_2=(-1,-2,-3)$; $\vec{a}_3=(8,1,-1)$; $\vec{a}_4=(-3,-4,-1)$;
$\vec{b}_1=(-1, 3, -4)$; $\vec{b}_2=(0, -5, 1)$; $\vec{b}_3=(5, -3, -2)$.

4.3. $\vec{a}_1=(0,1,2,3,4)$; $\vec{a}_2=(-2,-1,2,-3,4)$; $\vec{a}_3=(3,-1,1,-1,1)$;
$\vec{a}_4=(9,3,4,1,2)$; $\vec{b}_1=(0, -1, -2, -3, -4)$; $\vec{b}_2=(-2, 0, 4, 0, 8)$.

4.4. $\vec{a}_1=(1,2,3,4)$; $\vec{a}_2=(-1,2,-3,4)$; $\vec{a}_3=(-1,1,-1,1)$;
$\vec{a}_4=(3,4,1,2)$; $\vec{b}_1=(5, 6, 7, 8)$; $\vec{b}_2=(0,4,0,8)$; $\vec{b}_3=(2,3,2,1)$.

4.5. $\vec{a}_1=(0,2,0,4)$; $\vec{a}_2=(0,-2,-3,0)$; $\vec{a}_3=(-1,1,-1,1)$;
$\vec{b}_1=(0,0,-3,4)$; $\vec{b}_2=(6,-8,3,-6)$; $\vec{b}_3=(-1,3,-1,5)$.

4.6. $\vec{a}_1=(0,1,4)$; $\vec{a}_2=(2,-3,4)$; $\vec{a}_3=(-1,1,-1)$;
$\vec{b}_1=(6,-3,8)$; $\vec{b}_2=(3,-4,5)$; $\vec{b}_3=(-2, 2, -2)$.

4.7. $\vec{a}_1=(-1,2,-3,4)$; $\vec{a}_2=(1,2,3,-4)$; $\vec{a}_3=(1,1,-1,1)$;
$\vec{b}_1=(-1, 6, -3, 4)$; $\vec{b}_2=(0, 4, 0, 0)$; $\vec{a}_3=(2, 3, 2, -3)$.

4.8. $\vec{a}_1=(1,-2,3,-4)$; $\vec{a}_2=(-1,-1,-1,-1)$; $\vec{a}_3=(-3,4,1,2)$;
$\vec{b}_1=(0, -3, 2, -5)$; $\vec{b}_2=(-4,3,0,1)$.

4.9. $\vec{a}_1=(0,2,3,4)$; $\vec{a}_2=(-1,0,-1,1)$; $\vec{a}_3=(3,4,1,0)$;
$\vec{b}_1=(-1,2,2,5)$; $\vec{b}_2=(2,4,0,-1)$; $\vec{b}_3=(-6,-8,-2,0)$.

4.10. $\vec{a}_1=(-1,1,-3,3)$; $\vec{a}_2=(-4,1,-3,0)$;
$\vec{b}_1=(-3,0,0,-3)$; $\vec{b}_2=(4,3,2,2)$.

5. $\vec{x}$ vektorning (*a*) sistemadagi chiziqli ifodasini toping :

5.1. $\vec{x}=(1,1,1)$; $\vec{a}_1=(1,2,3)$; $\vec{a}_2=(2,-3,4)$; $\vec{a}_3=(-1,-1,1)$; $\vec{a}_4=(3,4,2)$.

5.2. $\vec{x}=(4,7,1,-1)$; $\vec{a}_1=(-1,7,-3,9)$; $\vec{a}_2=(-1,6,-1,1)$; $\vec{a}_3=(3,-4,-1,2)$.

5.3. $\vec{x}=(-4,9)$; $\vec{a}_1=(3,4)$; $\vec{a}_2=(-2,-3)$; $\vec{a}_3=(-1,6)$.

5.4. $\vec{x}=(1,1,1,1,1)$; $\vec{a}_1=(0,1,2,3,4)$; $\vec{a}_2=(-2,-1,2,-3,4)$; $\vec{a}_3=(3,-1,1,-1,1)$.

5.5. $\vec{x}=(1,-3,0)$; $\vec{a}_1=(0,1,4)$; $\vec{a}_2=(2,-3,4)$; $\vec{a}_3=(-1,1,-1)$.

5.6. $\vec{x}=(-6,-1,0,0,1)$; $\vec{a}_1=(1,2,3,4,-3)$; $\vec{a}_2=(-1,-2,3,4,1)$; $\vec{a}_3=(-5,1,-7,1,2)$.

5.7. $\vec{x}=(-9,1)$; $\vec{a}_1=(2,3)$; $\vec{a}_2=(-1,-3)$; $\vec{a}_3=(1,-1)$; $\vec{a}_4=(3,1)$.

5.8. $\vec{x}=(-2,-1,0)$; $\vec{a}_1=(1,-3,4)$; $\vec{a}_2=(-1,-2,-3)$; $\vec{a}_3=(8,1,-1)$; $\vec{a}_4=(-3,-4,-1)$.

5.9. $\vec{x}=(4,-1,1)$; $\vec{a}_1=(5,3,-4)$; $\vec{a}_2=(0,2,4)$; $\vec{a}_3=(1,5,2)$.

6. 5-misoldagi (*a*) sistema chiziqli qobig‘ining chiziqli fazo tashkil etishini tekshiring.

7. F^n da aniqlangan qo'shish va skalarni vektorga ko'paytirish amallarining quyidagi xossalarini isbotlang:

1°. $\forall(\vec{a},\vec{b} \in F^n)(\vec{a}+\vec{b}=\vec{b}+\vec{a})$ – qo'shishning kommutativlik xossasi.

2°. $\forall(\vec{a},\vec{b},\vec{c} \in F^n)((\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c}))$ – qo'shishning assotsiativlik xossasi.

3°. $\forall(\vec{a} \in F^n)(\vec{a}+\vec{0}=\vec{a})$ (qo'shish amaliga nisbatan neytral element mavjud).

4°. $\forall(\vec{a} \in F^n)(\vec{a}+(-\vec{a})=\vec{0})$ (qo'shish amaliga nisbatan simmetrik element mavjud).

5°. $\forall(\lambda \in F) \wedge \forall(\vec{a} \in F^n)(\lambda(\vec{a}+\vec{b})=\lambda\vec{a}+\lambda\vec{b})$ (skalarni vektorlar yig'indisiga ko'paytirish distributiv).

6°. $\forall(\lambda,\mu \in F) \wedge \forall(\vec{a} \in F^n)((\lambda\cdot\mu)\vec{a}=\lambda(\mu(\vec{a})))$ (skalarlar ko'paytmasini vektorga ko'paytirish assotsiativ).

7°. $\forall(\lambda,\mu \in F) \wedge \forall(\vec{a} \in F^n)((\lambda+\mu)(\vec{a})=(\lambda\vec{a}+\mu\vec{a}))$ (skalarlar yig'indisini vektorga ko'paytirish distributiv).

8°. $\forall(\vec{a} \in F^n)(1\cdot\vec{a}=\vec{a})$.

8. Quyidagi to'plamlar R maydon ustida chiziqli fazo tashkil etishini isbotlang:

8.1. $V=\{\frac{a}{b} \mid a,b \in Z \wedge b \neq 0\}$.

8.2. $V=\{(a,\ 0) \mid a \in R\}$.

8.3. $V=\{(a,\ 0,\ 0\ ) \mid a \in R\}$.

8.4. $V=\{(a,\ 0,\ c) \mid a,\ c \in R\}$.

8.5. $V=\{(a,\ 0,\ b,\ 0) \mid a,\ b \in R\}$.

8.6. $V=\{(0,\ a,\ b,\ 0) \mid a,\ b \in R\}$.

8.7. $V=\{(ax+by) \mid a,\ b \in R\}$.

8.8. $V=\{(y=ax+b) \mid a\ ,\ b \in R\ \}$.

8.9. $V=\{(ax+b\sqrt{2}\ ) \mid a,\ b \in Q\}$.

8.10. $V=\{(ax-by+cz) \mid a,\ b,\ c \in R\}$.

9. Har qanday α_{ij} sonlar uchun quyidagi vektorlar sistemasi chiziqli erkli bo'lishini isbotlang:

$$\vec{a}_1 = (1,0,0,...,0,0,\alpha_{11},\alpha_{12},...,\alpha_{1k}),$$
$$\vec{a}_2 = (0,2,0,...,0,0,\alpha_{21},\alpha_{22},...,\alpha_{2k}),$$
$$...$$
$$\vec{a}_m = (0,0,0,...,0,1,\alpha_{m1},\alpha_{m2},...,\alpha_{mk}).$$

10. Har qanday α_i $(i = 1,2,...,n)$ sonlar uchun quyidagi n-o'l-chovli vektorlar sistemasi chiziqli bog'liq bo'lishini isbotlang:

$$\vec{e}_1 = (1,0,0,...,0,0),$$
$$\vec{e}_2 = (0,1,0,...,0,0),$$
$$..............................$$
$$\vec{e}_n = (0,0,0,...,0,1),$$
$$\vec{a} = (\alpha_1,\alpha_2,...,\alpha_n).$$

11. Kamida bitta nol vektorga ega vektorlarning $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ chekli sistemasi chiziqli bog'langan sistema bo'lishini isbotlang.

12. Chekli vektorlar sistemasining biror-bir qismi chiziqli bog'langan bo'lsa, sistemaning o'zi ham chiziqli bog'langan bo'lishini isbotlang.

13. Vektorlarning chiziqli bog'lanmagan sistemasining har qanday qism sistemasi chiziqli bog'lanmagan sistema bo'lishini isbotlang.

14. Agar $\vec{a}_2,...,\vec{a}_n$ vektorlardan kamida bittasi o'zidan oldingi vektorlarning chiziqli kombinatsiyasidan iborat bo'lsa, u holda $\vec{a}_1 = \vec{0}$ bo'lgan $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ vektorlardan iborat sistema chiziqli bog'langan bo'lishini isbotlang.

15. Agar $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ vektorlarning sistemasi chiziqli bog'lanmagan bo'lib, $\vec{a}_1,\vec{a}_2,...,\vec{a}_n\vec{b}$ sistema chiziqli bog'langan bo'lsa, u holda $\vec{b}$ vektor $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ vektorlar sistemasi orqali yagona usulda chiziqli ifodalanishini isbotlang.

16. Agar $\vec{a}$ vektor $\vec{b}_1,\vec{b}_2,...,\vec{b}_n$ orqali va $\vec{b}_i$ $(i=\overline{1,n})$ vektorlar $\vec{c}_1,\vec{c}_2,...,\vec{c}_m$ vektorlar orqali chiziqli ifodalansa, u holda $\vec{a}$ vektor $\vec{c}_1,\vec{c}_2,...,\vec{c}_m$ vektorlar orqali chiziqli ifodalanishini isbotlang.

17. Agar $\vec{a}_1,...,\vec{a}_{n+1}$ vektorlar $\vec{b}_1,\vec{b}_2,...,\vec{b}_n$ vektorlar orqali chiziqli ifodalansa, u holda $\vec{a}_1,...,\vec{a}_{n+1}$ sistema chiziqli bog'langan bo'lishini isbotlang.

18. Agar $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ vektorlar $\vec{b}_1,\vec{b}_2,...,\vec{b}_m$ sistema orqali chiziqli ifodalansa va $n>m$ bo'lsa, u holda $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ sistema chiziqli bog'langan bo'lishini isbotlang.

19. Agar $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ vektorlar $\vec{b}_1,\vec{b}_2,...,\vec{b}_m$ sistema orqali chiziqli ifodalansa va $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ sistema chiziqli bog'lanmagan bo'lsa, u holda $n \le m$ bo'lishini isbotlang.

20. Agar vektorlarning har qanday chiziqli erkli ikkita chekli sistemasi ekvivalent bo'lsa, ulardagi vektorlar soni teng bo'lishini isbotlang.

21. Agar vektorlarning bir chekli sistemasi ikkinchi sistemani elementar almashtirishlar natijasida hosil qilingan bo'lsa, bunday sistemalar ekvivalent bo'lishini isbotlang.

22. Kamida bitta noldan farqli vektorga ega bo'lgan har qanday chekli sistema bazisga ega. Vektorlar chekli sistemasining har qanday ikkita bazisi bir xil sondagi vektorlardan iborat bo'lishini isbotlang.

23. $\vec{a}_2,...,\vec{a}_n$ vektorlar $\vec{b}_1,\vec{b}_2,...,\vec{b}_m$ sistemasi vektorlar sistemasi orqali chiziqli ifodalansa, u holda $\vec{a}_2,...,\vec{a}_n$ sistemaning rangi $\vec{b}_1,\vec{b}_2,...,\vec{b}_m$ sistemaning rangidan katta emasligini ko'rsating.

24. Vektorlar chekli sistemasining har qanday qism sistemasining rangi sistema rangidan katta emasligini isbotlang.

25. Vektorlar ekvivalent chekli sistemalarining ranglari teng bo'lishini isbotlang.

26. n-o'lchovli arifmetik vektor fazo har qanday chekli sistemasining rangi n dan katta emasligini ko'rsating.

27. Agar vektorlar chekli sistemasining rangi n ga teng bo'lsa, u holda uning k ta vektordan iborat har qanday qism sistemasi $k>n$ bo'lganda chiziqli bog'langan bo'lishini isbotlang.

28. Agar $\vec{a}_2,...,\vec{a}_n$ vektorlar sistemasining rangi $\vec{a}_2,...,\vec{a}_n,\vec{b}$ vektorlar sistemasining rangiga teng bo'lsa, u holda $\vec{b}$ vektorni

$\vec{a}_2, ..., \vec{a}_n$ vektorlar sistemasining chiziqli kombinatsiyasi ko‘rinishida ifodalash mumkinligini ko‘rsating.

29. $L(\vec{a}_1, \vec{a}_2, ..., \vec{a}_n)$ chiziqli qobiq vektor fazo tashkil etishini isbotlang.

30. Agar $\vec{b}_1, \vec{b}_2, ..., \vec{b}_m$ sistemaning har bir vektori $\vec{a}_1, \vec{a}_2, ..., \vec{a}_n$ sistema orqali chiziqli ifodalansa, u holda $L(\vec{b}_1, \vec{b}_2, ..., \vec{b}_m) \subset$ $\subset L(\vec{a}_1, \vec{a}_2, ..., \vec{a}_n)$ bo‘lishini isbotlang.

31. Agar $\vec{a}_1, \vec{a}_2, ..., \vec{a}_n$ sistemaning rangi k bo‘lsa, u holda $L(\vec{a}_1, \vec{a}_2, ..., \vec{a}_n)$ chiziqli qobiq k o‘lchovli bo‘lishini ko‘rsating.

32. N chiziqli ko‘pxillik F^n fazoning qismfazosini ifodalashi uchun $\vec{x}_0 \in W$, ya’ni $N=W$ munosabat bajarilishi zarur va yetarli ekanligini isbotlang.

33. Ixtiyoriy ikkita $\vec{x}_0 + W$ va $\vec{y}_0 + W$ chiziqli ko‘pxilliklar umumiy elementga ega bo‘lmaydi yoki ular ustma-ust tushadi. Isbotlang.

34. F^n vektor fazoning qismfazolari W va W' berilgan bo‘lsin. U holda $N_1 = \vec{x}_1 + W$, $N_2 = \vec{x}_2 + W'$ ko‘pxilliklarning ustma-ust tushishi va $\vec{x}_1 - \vec{x}_2 \in W$ bo‘lishi zarur va yetarli. Isbotlang.

Takrorlash uchun savollar

1. n-o‘lchovli vektor deb nimaga aytiladi?
2. n-o‘lchovli vektorlarning yig‘indisi va skalarni vektorga ko‘paytmasi deb nimaga aytiladi?
3. n-o‘lchovli arifmetik vektor fazo deb nimaga aytiladi?
4. Vektorlarning chiziqli bog‘liq sistemasi deb nimaga aytiladi?
5. Vektorlarning chiziqli bog‘liq bo‘lmagan sistemasi ta’rifini ayting.
6. Vektorlarning ekvivalent sistemalari deb nimaga aytiladi?
7. Vektorlar sistemasida qanday elementar almashtirishlar bajariladi?
8. Elementar almashtirishlar natijasida qanday sistema hosil bo‘ladi?

9. Vektorlar chekli sistemasining bazisi va rangiga ta'rif bering.

10. Vektorlar sistemasining chiziqli qobig'i deb nimaga aytiladi?

11. Chiziqli qobiqning asosiy xossalarini bayon eting.

12. Chiziqli ko'pxillikka ta'rif bering.

13. Chiziqli ko'pxillikning asosiy xossalarini ayting.

14. Chiziqli ko'pxillikka maktab matematikasidan misol keltiring.

14-§. Matritsa va uning rangi

✔ **Asosiy tushunchalar:** matritsa, nomdosh matritsa, teng matritsalar, matritsaning satr rangi, matritsaning ustun rangi, transponirlangan matritsa, matritsani elementar almashtirishlar, pog'onasimon matritsa.

$F = < F; +, \cdot, -, ^{-1}, 0, 1 >$ maydon berilgan bo'lsin.

F maydonning mn ta a_{ij} $(i=\overline{1,m}, j=\overline{1,n})$ elementlaridan tuzilgan ushbu

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

ko'rinishdagi jadval F maydon ustidagi $m \times n$ *tartibli matritsa* deyiladi.

A va B matritsalar berilgan bo'lib, ularning, mos ravishda, satrlari va ustunlari soni teng bo'lsa, u holda A va B matritsalar *nomdosh matritsalar* deb yuritiladi.

A matritsaning har bir a_{ij} elementi V matritsaning unga mos b_{ij} elementiga teng bo'lsa, u holda A va B nomdosh matritsalar *teng* (aks holda *teng emas) matritsalar* deyiladi.

Bitta satrli matritsalarni satr vektorlar, bitta ustunli matritsalarni ustun vektorlar deb qarash mumkin.

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m3} & \dots & a_{mn} \end{pmatrix}$$ matritsada $\overline{A}_1,\dots,\overline{A}_m$ satr vektorlar

va $\overline{A}^1,\dots,\overline{A}^n$ ustun vektorlar mavjud.

Matritsadagi satr vektorlar sistemasining rangi matritsaning *satr rangi*, ustun vektorlar sistemasining rangi uning *ustun rangi* deyiladi. A matritsaning satr rangini $r(A)$, ustun rangini $\rho(A)$ ko'rinishda belgilaymiz.

Matritsa rangini aniqlash uchun matritsa ustida elementar almashtirishlar bajariladi. Ular quyidagilar:

1. Matritsadagi ixtiyoriy ikkita satr yoki ustun o'rinlarini almashtirish.

2. Matritsadagi ixtiyoriy satr yoki ustun elementlarini noldan farqli songa ko'paytirish.

3. Matritsaning satr yoki ustun elementlarini noldan farqli songa ko'paytirib, boshqa satr yoki ustunning mos elementlariga qo'shish.

4. Barcha elementlari nollardan iborat bulgan satr yoki ustunni matritsadan chiqarish.

Matritsa satrining *boshlovchi elementi* deb uning birinchi (chapdan o'ngga qaraganda) noldan farqli elementiga aytiladi.

Matritsa *pog'onasimon* deyiladi, agar uning nol qatorlari barcha nolmas qatorlardan keyin joylashgan va $\alpha_{1k_1},\alpha_{2k_2},\dots,\alpha_{rk_r}$ boshlovchi elementlari uchun $k_1<k_2<\dots<k_r$ bo'lsa.

A^t matritsa A matritsaning *transponirlangani* deyiladi, agar A^t matritsa A matritsa satrlarini ustunlar orqali yozishdan hosil bo'lgan bo'lsa, ya'ni

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m3} & \dots & a_{mn} \end{pmatrix}; \qquad A^t=\begin{pmatrix} a_{11} & a_{21} & \dots & a_{m1} \\ a_{12} & a_{22} & \dots & a_{m2} \\ \dots & \dots & \dots & \dots \\ a_{1n} & a_{2n} & \dots & a_{mn} \end{pmatrix}$$

Misol. $A = \begin{pmatrix} 2 & 1 & 3 & 1 \\ -1 & 0 & 2 & 2 \\ 5 & 4 & 3 & 4 \end{pmatrix}$ matritsaning ustun va satr ranglarini toping.

Yechish. Matritsaning satr rangini topish uchun satr elementar almashtirishlar bajarilib, matritsaning satr vektorlari sistemasi rangi aniqlanadi:

$$A = \begin{pmatrix} 2 & 1 & 3 & 1 \\ -1 & 0 & 2 & 2 \\ 5 & 4 & 3 & 4 \end{pmatrix} \sim \begin{pmatrix} -1 & 0 & 2 & 2 \\ 0 & 1 & 7 & 5 \\ 0 & 4 & 13 & 14 \end{pmatrix} \sim \begin{pmatrix} -1 & 0 & 2 & 2 \\ 0 & 1 & 7 & 5 \\ 0 & 0 & -15 & -6 \end{pmatrix}.$$

Hosil bo'lgan pog'onasimon matritsada noldan farqli satrlar 3 ta, demak $r(A)=3$.

Endi matritsaning ustun rangini aniqlash uchun unda ustun elementar almashtirishlar bajaramiz:

$$A = \begin{pmatrix} 2 & 1 & 3 & 1 \\ -1 & 0 & 2 & 2 \\ 5 & 4 & 3 & 4 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 0 & 1 \\ -5 & -2 & -4 & 2 \\ -3 & 0 & -9 & 4 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ -3 & -9 & 0 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 4 & 0 & -3 & 0 \end{pmatrix}$$

Hosil bo'lgan ustunli pog'onasimon matritsada 3 ta nolmas ustun mavjud, ya'ni $\rho(A)=3$.

Misol va mashqlar

1. Matritsa rangini aniqlang:

1.1. $\begin{pmatrix} 1 & 2 \\ 1 & 2 \\ 0 & 1 \end{pmatrix}$.

1.2. $\begin{pmatrix} 3 & 0 & 2 \\ 6 & 0 & 4 \\ 9 & 0 & 6 \end{pmatrix}$.

1.3. $\begin{pmatrix} 3 & 0 & 2 \\ 1 & -1 & 3 \\ 4 & -1 & 6 \end{pmatrix}$.

1.4. $(3 \quad 0 \quad 2)$.

2. Matritsaning ustun va satr ranglarining tengligini tekshiring:

2.1. $\begin{pmatrix} 6 & 0 & 9 \\ -4 & 5 & 3 \\ 7 & 1 & 2 \end{pmatrix}$. 2.2. $\begin{pmatrix} -1 & 2 & 3 \\ -6 & 5 & 3 \\ 7 & -1 & 4 \end{pmatrix}$.

2.3. $\begin{pmatrix} 4 & 0 & 7 \\ -8 & 5 & 3 \\ 4 & 11 & 4 \end{pmatrix}$. 2.4. $\begin{pmatrix} 13 & 10 & 3 \\ -6 & 15 & 3 \\ 27 & 1 & 14 \end{pmatrix}$.

2.5. $\begin{pmatrix} 2 & 4 & -7 & 0 \\ -2 & 3 & 5 & -1 \\ 0 & 6 & 7 & 9 \\ 11 & 2 & 5 & 7 \end{pmatrix}$. 2.6. $\begin{pmatrix} 3 & 11 & -7 & 4 \\ -1 & 13 & -5 & 8 \\ 10 & 9 & 11 & 2 \\ 7 & 4 & 0 & 11 \end{pmatrix}$.

2.7. $\begin{pmatrix} -3 & 1 & 7 & 4 \\ -1 & 3 & 5 & 8 \\ 0 & 9 & 11 & 2 \\ 7 & -4 & 0 & 1 \end{pmatrix}$. 2.8. $\begin{pmatrix} 1 & 1 & 1 \\ \sin\alpha & \cos\alpha & \mathrm{tg}\alpha \\ \sin^2\alpha & \cos^2\alpha & \mathrm{tg}^2\alpha \end{pmatrix}$.

3. λ ning turli qiymatlarida $\begin{pmatrix} \lambda & 1 & 3 & 4 & 1 \\ 1 & \lambda & -1 & 1 & 5 \\ \lambda & \lambda & 4 & 3 & -4 \\ 1 & 1 & 7 & 7 & -3 \end{pmatrix}$ matritsa rangini toping.

4. λ ning qanday qiymatida $\begin{pmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{pmatrix}$ matritsa rangi eng kichik bo'ladi?

Takrorlash uchun savollar

1. Matritsa deb nimaga aytiladi?
2. Nomdosh matritsalarga ta'rif bering.

3. Qanday matritsalar teng deyiladi?

4. Matritsaning satr (ustun) vektorlari sistemasi nima?

5. Matritsaning satr (ustun) rangi deb nimaga aytiladi?

6. Matritsani elementar almashtirishlar deb qanday almashtirishlarga aytiladi?

15-§. Chiziqli tenglamalar sistemasi

✔ **Asosiy tushunchalar**: chiziqli tenglamalar sistemasi, CHTSning yechimi, hamjoyli CHTS, hamjoyli bo'lmagan CHTS, CHTSning natijasi, CHTSning chiziqli kombinatsiyasi, teng kuchli CHTSlari, CHTSni elementar almashtirishlar, bir jinsli CHTS, BCHTSning fundamental yechimlar sistemasi.

$F = < F; +, \cdot, -, ^{-1}, 0, 1 >$ maydon berilgan bo'lsin.

Barcha noma'lumlarining darajasi birdan katta bo'lmagan tenglama *chiziqli tenglama* deyiladi. $a_1 x_1 + \ldots + a_n x_n = b$ tenglamani to'g'ri sonli tenglikka aylantiruvchi $\vec{\xi} = (\xi_1, \ldots, \xi_n), \xi_i \in F, i = \overline{1, n}$ vektor berilgan tenglamaning yechimi deyiladi. Ushbu

$$\begin{cases} a_{11} x_1 + a_{12} x_2 + \ldots + a_{1n} x_n = b_1 \\ a_{21} x_1 + a_{22} x_2 + \ldots + a_{2n} x_n = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1} x_1 + a_{m2} x_2 + \ldots + a_{mn} x_n = b_m \end{cases} \qquad (1)$$

sistema F maydon ustida berilgan n ta noma'lumli m ta chiziqli *tenglamalar sistemasi* deyiladi, bunda $a_{ij}, b_i \in F \; (i = \overline{1, m}; \; j = \overline{1, n})$ sistemaning koeffitsiyentlari, a_{ij} no'malumlar koeffitsiyentlari, b_j ozod hadlar bo'lib, x_i lar esa no'malumlardan iborat.

n ta noma'lumli m ta chiziqli *tenglamalar sistemasining yechimi* deb shunday $\vec{\xi} = (\xi_1, \ldots, \xi_n), \xi_i \in F, i = \overline{1, n}$ vektorga aytiladiki, u sistemaning barcha tenglamalarini to'g'ri tenglikka aylantiradi.

CHTS kamida bitta yechimga ega bo'lsa, u *hamjoyli*, yechimga ega bo'lmasa, *hamjoyli bo'lmagan CHTS* deyiladi.

Yagona yechimga ega bo'lgan sistema *aniq sistema*, cheksiz ko'p yechimga ega bo'lgan sistema *aniqmas sistema* deyiladi.

Berilgan ikkita CHTS uchun birinchisining har bir yechimi ikkinchisi uchun ham yechim bo'lsa, ikkinchi CHTS birinchi CHTSning *natijasi* deyiladi.

Ikkita CHTS *teng kuchli* deyiladi, agar birinchisining har bir yechimi ikkinchisiga yechim bo'lsa va aksincha.

CHTSning noma'lumlari oldidagi koeffitsiyentlardan tuzilgan

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

matritsa (1)ning *asosiy matritsasi*, noma'lumlar oldidagi koeffitsiyentlar va ozod hadlardan iborat

$$B=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & a_{2n} & b_2 \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} & b_m \end{pmatrix}$$

matritsa (1)ning *kengaytirilgan matritsasi* deyiladi.

Kroneker–Kapelli teoremasi. *Chiziqli tenglamalar sistemasi hamjoyli bo'lishi uchun uning asosiy va kengaytirilgan matritsalari ranglarining teng bo'lishi zarur va yetarli.*

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = 0, \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = 0, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = 0 \end{cases} \qquad (1^*)$$

chiziqli tenglamalar sistemasi *bir jinsli chiziqli tenglamalar sistemasi* (BCHTS) deyiladi.

F^n arifmetik vektor fazoning W qism fazosining bazisini tashkil etuvchi istalgan vektorlar sistemasi (1*) sistemaning *fundamental (asosiy) yechimlari sistemasi* deyiladi.

1-misol. Tenglamalar sistemasini Kroneker–Kapelli teoremasi asosida tekshiring va yechimlarini toping:

$$\begin{cases} 2x_1 + 3x_2 - x_3 + x_4 = 1, \\ -x_1 - x_2 + 3x_3 + 2x_4 = 3, \\ 3x_1 + 4x_3 - 4x_4 = 5, \\ 5x_1 + x_2 + 2x_3 = 6. \end{cases}$$

Yechish. Kroneker–Kapelli teoremasiga ko'ra, bir jinsli bo'lmagan chiziqli tenglamalar sistemasi hamjoyli bo'lishi uchun uning asosiy A va kengaytirilgan B matritsalarining satr ranglari teng bo'lishi kerak.

Berilgan chiziqli tenglamalar sistemasining asosiy va kengaytirilgan matritsalari ranglarini topamiz. Buning uchun chiziqli tenglamalar sistemasining no'malumlari oldidagi koeffitsiyentlardan A matritsani, unga ozod hadlar ustunini qo'shib, matritsani hosil qilamiz:

$$A = \begin{pmatrix} 2 & 3 & -1 & 1 \\ -1 & -1 & 3 & 2 \\ 3 & 0 & 4 & -4 \\ 5 & 1 & 2 & 0 \end{pmatrix} \qquad B = \left(\begin{array}{cccc|c} 2 & 3 & -1 & 1 & 1 \\ -1 & -1 & 3 & 2 & 3 \\ 3 & 0 & 4 & -4 & 5 \\ 5 & 1 & 2 & 0 & 6 \end{array}\right)$$

Matritsaning satr rangini topish uchun satr elementar almashtirishlar bajarib, uni pog'onasimon matritsa ko'rinishiga keltiramiz. Elementar almashtirishlar natijasida berilgan matritsaga ekvivalent matritsa hosil bo'ladi:

$$\left(\begin{array}{cccc|c} 2 & 3 & -1 & 1 & 1 \\ -1 & -1 & 3 & 2 & 3 \\ 3 & 0 & 4 & -4 & 5 \\ 5 & 1 & 2 & 0 & 6 \end{array}\right) \sim$$

birinchi va ikkinchi satrlar o'rnini almashtiramiz:

$$\sim \left(\begin{array}{cccc|c} -1 & -1 & 3 & 2 & 3 \\ 2 & 3 & -1 & 1 & 1 \\ 3 & 0 & 4 & -4 & 5 \\ 5 & 1 & 2 & 0 & 6 \end{array}\right) \sim$$

birinchi ustun birinchi qator elementi -1 ni qoldirib, birinchi ustun boshqa elementlarini 0 ga aylantiramiz:

$$\sim\left(\begin{array}{cccc|c} -1 & -1 & 3 & 2 & 3 \\ 0 & 1 & 5 & 5 & 7 \\ 0 & -3 & 13 & 2 & 14 \\ 0 & -4 & 17 & 10 & 21 \end{array}\right)\sim$$

birinchi va ikkinchi satrlarni o'zgartirmaymiz, ikkinchi satr yordamida uchinchi, to'rtinchi satrlarning ikkinchi ustunidagi elementlarni nolga aylantiramiz:

$$\sim\left(\begin{array}{cccc|c} -1 & -1 & 3 & 2 & 3 \\ 0 & 1 & 5 & 5 & 7 \\ 0 & 0 & 28 & 17 & 35 \\ 0 & 0 & 37 & 30 & 49 \end{array}\right)\sim$$

1-, 2-, 3- satrlarni qoldirib, 4-satrning 3-ustunidagi elementini nolga aylantiramiz:

$$\sim\left(\begin{array}{cccc|c} -1 & -1 & 3 & 2 & 3 \\ 0 & 1 & 5 & 5 & 7 \\ 0 & 0 & 28 & 17 & 35 \\ 0 & 0 & 0 & -211 & -77 \end{array}\right)$$

Hosil bo'lgan pog'onasimon matritsadan asosiy matritsaning rangi $r(A)=4$ va kengaytirilgan matritsaning rangi $r(B)=4$ ekanligini aniqlaymiz (noldan farqli satrlar soni).

Demak, chiziqli tenglamalar sistemasi asosiy va kengaytirilgan matritsalarining ranglari teng, ya'ni teoremaga asosan berilgan chiziqli tenglamalar sistemasi hamjoyli. Endi chiziqli tenglamalar sistemasining yechimlarini topamiz. Buning uchun to'g'ridan to'g'ri pog'onasimon matritsa yordamida berilgan chiziqli tenglamalar sistemasiga teng kuchli chiziqli tenglamalar sistemasini tuzamiz:

$$\begin{cases} -x_1 - x_2 + 3x_3 + 2x_4 = 3, \\ x_2 + 5x_3 + 5x_4 = 7, \\ 28x_3 + 17x_4 = 35, \\ -211x_4 = -77. \end{cases}$$

Bundan $x_1 = \frac{165}{211}, x_2 = \frac{7}{211}, x_3 = \frac{217}{211}, x_4 = \frac{77}{211}$ yechimni topamiz.

2-misol. Berilgan tenglamalar sistemasini Gauss usulida yeching:

$$\begin{cases} 3x_1 + x_2 - 2x_3 + x_4 = 2, \\ 2x_1 + x_3 - 2x_4 = 3, \\ 4x_1 + 8x_2 - 12x_3 = 4. \end{cases}$$

Yechish. Chiziqli tenglamalar sistemasini Gauss usuli bilan yechish deganda, sistemadagi noma'lumlarni ketma-ket yo'qotish tushuniladi. Ya'ni tenglamalar sistemasida elementar almashtirishlar bajarib, tanlab olingan tenglama yordamida qolganlaridagi noma'lumlardan biri oldidagi koeffitsiyentini nolga aylantiramiz. Bu jarayonni davom ettirib, berilgan chiziqli tenglamalar sistemasiga teng kuchli chiziqli tenglamalar sistemasini hosil qilamiz. Noma'lumlar soni eng kam bo'lgan tenglamadan boshlab, noma'lumlar topiladi.

Berilgan chiziqli tenglamalar sistemasidagi 3-tenglamani 4 ga bo'lib, birinchi o'ringa joylashtiramiz va uning yordamida qolgan tenglamalardan noma'lumni yo'qotamiz:

$$\begin{cases} 3x_1 + x_2 - 2x_3 + x_4 = 2, \\ 2x_1 + x_3 - 2x_4 = 3, \\ 4x_1 + 8x_2 - 12x_3 = 4 \end{cases} \Leftrightarrow \begin{cases} x_1 + 2x_2 - 3x_3 = 1, \\ -5x_2 + 7x_3 + x_4 = -1, \\ -4x_2 + 7x_3 - 2x_4 = 1 \end{cases} \Leftrightarrow$$

Hosil bo'lgan chiziqli tenglamalar sistemasidagi 1-, 2- tenglamalarni o'z o'rnida o'zgarishsiz qoldirib, 3-tenglamaning x_2 noma'lumini 2-tenglama yordamida yo'qotamiz:

$$\Leftrightarrow \begin{cases} x_1 + 2x_2 - 3x_3 = 1, \\ -5x_2 + 7x_3 + x_4 = -1, \\ 7x_3 - 12x_4 = 9. \end{cases}$$

Chiziqli tenglamalar sistemasidagi uchinchi tenglama ikki noma'lumli bitta tenglama bo'lib, uning cheksiz ko'p yechimlari mavjud, ya'ni x_3 yoki x_4 noma'lumni ixtiyoriy haqiqiy son qabul qiladi deb olib, ikkinchisini u orqali chiziqli ifodalaymiz.

Tenglamalar sistemasining yechimlari x_4 noma'lum orqali ifodalanuvchi to'rt o'lchovli arifmetik vektorlardan iborat cheksiz ko'p elementga ega bo'lgan to'plam bo'ladi. Uni topamiz. Buning uchun $x_4 \in R$ deb olib, x_3 noma'lumni x_4 yordamida ifodalaymiz:

$$7x_3 - 12x_4 = 9 \Leftrightarrow 7x_3 = 12x_4 + 9 \Leftrightarrow x_3 = \frac{12}{7}x_4 + \frac{9}{7}.$$

Oxirgi chiziqli tenglamalar sistemasining ikkinchi tenglamasidagi x_3 noma'lum o'rniga uning topilgan ifodasini qo'yamiz:

$$-5x_2 + 7\left(\frac{12}{7}x_4 + \frac{9}{7}\right) + x_4 = -1 \Leftrightarrow -5x_2 + 84x_4 + 63 + x_4 = -1 \Leftrightarrow$$
$$\Leftrightarrow -5x_2 + 85x_4 + 63 = -1.$$

Hosil bo'lgan tenglamadagi x_3 noma'lumning x_4 orqali ifodasini topib, birinchi tenglamaga qo'yamiz:

$$-5x_2 = -85x_4 - 64 \Leftrightarrow x_2 = 17x_4 - \frac{64}{5}.$$

Birinchi tenglamadagi x_1 ni topamiz:

$$x_1 - 2x_2 - 3x_3 = 1 \Leftrightarrow x_1 = 2x_2 + 3x_3 + 1 =$$
$$= 34x_4 - \frac{128}{5} + \frac{36}{7}x_4 + \frac{27}{7} = \frac{274}{7}x_4 - \frac{761}{35}.$$

Demak, berilgan chiziqli tenglamalar sistemasining yechimlar to'plami $\left\{\left(\frac{274}{7}x_4 - \frac{761}{35}; 17x_4 - \frac{64}{5}; \frac{12}{7}x_4 + \frac{9}{7}; x_4\right) \middle| x_4 \in R\right\}$ to'plamdan iborat.

3-misol. Berilgan $\begin{cases} 2x_1 + 3x_2 - x_3 + x_4 = 1, \\ -x_1 - x_2 + 3x_3 + 2x_4 = 3, \\ 3x_1 + 4x_3 - 4x_4 = 5 \end{cases}$ chiziqli tenglamalar sistemasi yordamida ko'pxillik tuzing.

Yechish. 1) Berilgan bir jinsli bo'lmagan chiziqli tenglamalar sistemasi hamjoyli bo'lsa, bitta a_0 yechimini topamiz.

2) Chiziqli tenglamalar sistemasiga assotsirlangan bir jinsli chiziqli tenglamalar sistemasining yechimlar fazosi W aniqlanadi.

3) Chiziqli ko'pxillik ta'rifiga ko'ra $\overrightarrow{a_0}+W$ bir jinsli bo'lmagan hamjoyli chiziqli tenglamalar sistemasi va unga assotsirlangan bir jinsli chiziqli tenglamalar sistemalari yordamida hosil qilingan chiziqli ko'pxillik bo'ladi.

$$1)\ \begin{cases} 2x_1+3x_2-x_3+x_4=1, \\ -x_1-x_2+3x_3+2x_4=3, \\ 3x_1+4x_3-4x_4=5 \end{cases} \Leftrightarrow \begin{cases} x_1+x_2-3x_3-2x_4=-3, \\ x_2+5x_3+5x_4=7, \\ -3x_2+13x_3+2x_4=14 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x_1+x_2-3x_3-2x_4=-3, \\ x_2+5x_3+5x_4=7, \\ 28x_3+17x_4=35. \end{cases}$$

Noma'lumlarni ketma-ket yo'qotish natijasida 4 ta noma'lumli 3 ta tenglamadan iborat sistema hosil bo'ldi.

Demak, chiziqli tenglamalar sistemasi cheksiz ko'p yechimga ega. Umumiy yechim $\left(\frac{115}{14}x_4-\frac{25}{4};-\frac{225}{28}x_4+7;-\frac{17}{28}x_4+\frac{5}{4};x_4\right), x_4\in R$ ko'rinishda bo'lib, uning bitta $\vec{a}_0$ yechimini x_4=0 qiymatni qo'yib topamiz:

$$\overrightarrow{a_0}=\left(-\frac{25}{4};\,7;\,\frac{5}{4};\,0\right).$$

2) Berilgan chiziqli tenglamalar sistemasiga assotsirlangan berilgan chiziqli tenglamalar sistemasi

$$\begin{cases} 2x_1+3x_2-x_3+x_4=0, \\ -x_1-x_2+3x_3+2x_4=0, \\ 3x_1+4x_3-4x_4=0 \end{cases}$$

ko'rinishda bo'lib, uning yechimlari cheksiz ko'p. Chunki

$$\begin{cases}2x_1+3x_2-x_3+x_4=0,\\ -x_1-x_2+3x_3+2x_4=0,\\ 3x_1+4x_3-4x_4=0\end{cases}\Leftrightarrow\begin{cases}x_1+x_2-3x_3-2x_4=0,\\ x_2+5x_3+5x_4=0,\\ 28x_3+17x_4=0.\end{cases}$$

Bundan, $x_4\in R$ desak, yechimlar to'plami $W=\left\{\left(\frac{115}{14}x_4;-\frac{225}{28}x_4;-\frac{17}{28}x_4;x_4\right)\middle|x_4\in R\right\}$ ko'rinishda bo'ladi. Bir jinsli chiziqli tenglamalar sistemasining yechimlar to'plami chiziqli fazo tashkil etadi $(W=<W;+,\{\omega_\lambda|\lambda\in R\}>)$.

3) Hosil bo'lgan $\overrightarrow{a_0}$ vektor va W to'plam yordamida $H=\overrightarrow{a_0}+W$ chiziqli ko'pxillikni hosil qilamiz. Chiziqli ko'pxillik H ning elementlari berilgan bir jinsli bo'lmagan chiziqli tenglamalar sistemasining yechimlar to'plamini tashkil etadi.

4-misol. Berilgan

$$1)\begin{cases}x_1+3x_2-5x_3+x_4=1,\\ x_1-2x_2+3x_3-2x_4=3\end{cases}\text{ va }2)\begin{cases}2x_1+x_2-2x_3-x_4=4,\\ 3x_1-x_2+x_3-3x_4=7\end{cases}$$

chiziqli tenglamalar sistemalari uchun 2-sistema 1-sistemaning natijasi ekanligini tekshiring.

Yechish. 2-chiziqli tenglamalar sistemasi 1-chiziqli tenglamalar sistemasining natijasi bo'lishi uchun ta'rifga ko'ra 1-chiziqli tenglamalar sistemasining har bir yechimi 2-chiziqli tenglamalar sistemasining ham yechimi bo'lishi kerak.

1-chiziqli tenglamalar sistemasining yechimlarini Gauss usulidan foydalanib topamiz:

$$\begin{cases}x_1+3x_2-5x_3+x_4=1,\\ x_1-2x_2+3x_3-2x_4=3\end{cases}\Leftrightarrow\begin{cases}x_1+3x_2-5x_3+x_4=1,\\ -5x_2+8x_3-3x_4=2.\end{cases}$$

Hosil bo'lgan teng kuchli chiziqli tenglamalar sistemasidagi 2-tenglamada x_3, $x_4\in R$ deb olib, qolgan noma'lumlarni aniqlaymiz:

$$\begin{cases}x_1=\frac{1}{5}x_3+\frac{4}{5}x_4+\frac{11}{5};\\ x_2=\frac{8}{5}x_3-\frac{3}{5}x_4-\frac{2}{5};\\ x_3,x_4\in R.\end{cases}$$

Demak, berilgan 1-chiziqli tenglamalar sistemasining cheksiz ko'p yechimlari mavjud bo'lib, umumiy yechim quyidagi ko'rinishda bo'ladi:

$$\left(\frac{1}{5}x_3+\frac{4}{5}x_4+\frac{11}{5};\frac{8}{5}x_3-\frac{3}{5}x_4-\frac{2}{5};x_3;x_4\right), x_3, x_4\in R.$$

Topilgan umumiy yechimni 2-chiziqli tenglamalar sistemasiga qo'yamiz:

$$\begin{cases}2\left(\frac{1}{5}x_3+\frac{4}{5}x_4+\frac{11}{5}\right)+\frac{8}{5}x_3-\frac{3}{5}x_4-\frac{2}{5}-2x_3-x_4=4,\\ 3\left(\frac{1}{5}x_3+\frac{4}{5}x_4+\frac{11}{5}\right)-\left(\frac{8}{5}x_3-\frac{3}{5}x_4-\frac{2}{5}\right)+x_3-3x_4=7\end{cases}\Leftrightarrow\begin{cases}4=4,\\7=7.\end{cases}$$

Demak, 1-chiziqli tenglamalar sistemasining har bir yechimi 2-chiziqli tenglamalar sistemasining ham yechimi bo'ladi. Ta'rifga ko'ra 2-sistema 1-sistemaga natija ekan.

5-misol. Berilgan bir jinsli chiziqli tenglamalar sistemasi $\begin{cases}x_1+3x_2-5x_3+x_4=0,\\x_1-2x_2+3x_3-2x_4=0\end{cases}$ ning yechimlar to'plami fundamental sistemasini toping.

Yechish. Har qanday bir jinsli chiziqli tenglamalar sistemasi hamjoyli hamda yechimlar to'plami chiziqli vektor fazo tashkil etadi.

Agar bir jinsli chiziqli tenglamalar sistemasi yagona nol yechimga ega bo'lsa, u holda yechimlar fazosi nol o'lchovli chiziqli vektor fazo bo'lib, uning fundamental sistemasi mavjud emas.

Agar bir jinsli chiziqli tenglamalar sistemasi cheksiz ko'p yechimlarga ega bo'lsa, u holda yechimlar to'plami tashkil etgan chiziqli vektor fazoning bazisi fundamental sistema bo'ladi.

$$\begin{cases}x_1+3x_2-5x_3+x_4=0,\\x_1-2x_2+3x_3-2x_4=0\end{cases}\Leftrightarrow\begin{cases}x_1+3x_2-5x_3+x_4=0,\\-5x_2+8x_3-3x_4=0.\end{cases}$$

Hosil bo'lgan teng kuchli bir jinsli chiziqli tenglamalar sistemasidagi 2-tenglamada x_3, $x_4\in R$ deb olib, qolgan noma'lumlarni aniqlaymiz:

$$\begin{cases} x_1 = \frac{1}{5}x_3 + \frac{4}{5}x_4; \\ x_2 = \frac{8}{5}x_3 - \frac{3}{5}x_4; \\ x_3, x_4 \in R. \end{cases}$$

Demak, berilgan bir jinsli chiziqli tenglamalar sistemasining cheksiz ko‘p yechimlari mavjud bo‘lib, umumiy yechim quyidagi ko‘rinishda bo‘ladi: $\left(\frac{1}{5}x_3 + \frac{4}{5}x_4; \frac{8}{5}x_3 - \frac{3}{5}x_4; x_3; x_4\right), x_3, x_4 \in R.$

Umumiy yechimdagi x_3, $x_4 \in R$ erkli o‘zgaruvchilarga kamida bittasi noldan farqli qiymatlar beramiz. Masalan, $x_3=1$, $x_4=0$; $x_3=0$, $x_4=1$.

Hosil bo‘lgan $\vec{a}_1 = \left(\frac{1}{5}; \frac{8}{5}; 1; 0\right)$, $\vec{a}_2 = \left(\frac{4}{5}; -\frac{3}{5}; 0; 1\right)$ yechimlar yechimlar to‘plamining ixtiyoriy yechimini chiziqli ifodalaydi. Demak, berilgan bir jinsli chiziqli tenglamalar sistemasi yechimlar to‘plamining fundamental sistemasi $\vec{a}_1 = \left(\frac{1}{5}; \frac{8}{5}; 1; 0\right)$, $\vec{a}_2 = \left(\frac{4}{5}; -\frac{3}{5}; 0; 1\right)$ vektorlardan iborat.

Misol va mashqlar

1. 2-sistema 1-sistema uchun natija bo‘lishini tekshiring:

1.1. 1) $\begin{cases} x_1 - 3x_2 + 4x_3 + 2x_4 = 1, \\ 2x_1 + 4x_2 - 3x_3 + 3x_4 = -1; \end{cases}$

2) $3x_1 + x_2 + x_3 + 5x_4 = 0.$

1.2. 1) $\begin{cases} 2x_1 + 4x_2 - 3x_3 + 3x_4 = -1, \\ 3x_1 + x_2 + 2x_3 - x_4 = 0 \end{cases}$

2) $-x_1 + 3x_2 - 5_3 + 4x_4 = -1.$

1.3. 1) $\begin{cases} 3x_1 + x_2 + 2x_3 - x_4 = 0, \\ 12x_1 + 4x_2 + 7x_3 + 2x_4 = 2; \end{cases}$

2) $-8x_1 - 3x_2 - 5x_3 - 3x_4 = -2.$

1.4. 1) $\begin{cases} 3x_1 - 3x_2 - 6x_3 - x_4 = 1, \\ x_1 - 6x_2 + 5x_3 - 12x_4 = 2, \\ x_1 - 7x_2 + x_3 + 4x_4 = 23, \\ x_2 + 23x_4 = 23, \\ -2x_1 - 7x_2 - 2x_3 + 2x_4 = 14; \end{cases}$

2) $\begin{cases} 4x_1 - 9x_2 - x_3 - 13x_4 = 3, \\ -x_1 - 14x_2 - x_3 + 6x_4 = 37. \end{cases}$

1.5. 1) $\begin{cases} 6x_1 - 5x_3 + 2x_4 = 41, \\ 3x_1 + 5x_2 - x_3 - 3x_4 = 11, \\ x_1 + 2x_2 + 2x_3 + 13x_4 = 10, \\ 2x_1 + 4x_2 + x_3 - x_4 = 3; \end{cases}$

2) $\begin{cases} 3x_1 - 5x_2 - 4x_3 + 5x_4 = 30, \\ -x_1 - 2x_2 + x_3 + 14x_4 = 7. \end{cases}$

2. Quyidagi elementar almashtirishlar yordamida berilgan CHTSga teng kuchli CHTS hosil bo‘lishini isbotlang:

1) sistemadagi tenglamalar o‘rnini almashtirish;

2) sistemani qandaydir tenglamasining ikkala qismini noldan farqli skalarga ko‘paytirish;

3) bir tenglamaning ikkala qismiga skalarga ko‘paytirilgan boshqa tenglamaning mos qismlarini qo‘shish yoki ayirish.

3. Kroneker–Kapelli teoremasi yordamida quyidagi CHTS-larini tekshiring va yechimlar to‘plamini aniqlang:

3.1. $\begin{cases} 5x_1 + 4x_2 + 3x_3 = 1, \\ 2x_1 + x_2 + 4x_3 = 1, \\ -3x_1 - 2x_2 - x_3 = -1, \\ x_1 + 3x_2 + 2x_3 = -2. \end{cases}$

3.2. $\begin{cases} x_1 + 2x_2 + 3x_3 - 2x_4 = 6, \\ 2x_1 - x_2 - 2x_3 - 3x_4 = 8, \\ 3x_1 + 2x_2 - x_3 + 2x_4 = 4, \\ 2x_1 - 3x_2 + 2x_3 + x_4 = -8. \end{cases}$

3.3. $\begin{cases} 6x_1 + 5x_2 + 2x_3 + 4x_4 = -4, \\ 9x_1 + x_2 + 4x_3 - x_4 = -1, \\ 3x_1 + 4x_2 + 2x_3 - 2x_4 = -5, \\ 3x_1 - 9x_2 + 2x_4 = 11. \end{cases}$

3.4. $\begin{cases} x_1 + x_2 + 3x_3 - 2x_4 + 3x_5 = 1, \\ 2x_1 + 2x_2 + 4x_3 - x_4 + 3x_5 = 2, \\ 3x_1 + 3x_2 + 5x_3 - 2x_4 + 3x_5 = 1, \\ 2x_1 + 2x_2 + 8x_3 - 3x_4 + 9x_5 = 2. \end{cases}$

4. λ ning qanday qiymatlarida CHTS hamjoyli bo'lishini aniqlang:

4.1. $\begin{cases} 2x_1 - x_2 + 3x_3 + 4x_4 = 5, \\ 4x_1 - 2x_2 + 5x_3 + 6x_4 = 7, \\ 6x_1 - 3x_2 + 7x_3 + 8x_4 = 9, \\ \lambda x_1 - 4x_2 + 9x_3 + 10x_4 = 11. \end{cases}$

4.2. $\begin{cases} (\lambda + 3)x_1 + 2x_2 - x_3 + 4x_4 = \lambda, \\ \lambda x_1 + (\lambda - 1)x_2 + 2x_3 - x_4 = 2, \\ x_1 + 3x_2 - x_3 + 11x_4 = -10, \\ x_1 + 4x_2 - x_3 + 18x_4 = -18. \end{cases}$

4.3. $\begin{cases} \lambda x_1 + x_2 + x_3 + x_4 = 1, \\ x_1 + \lambda x_2 + x_3 + x_4 = 1, \\ x_1 + x_2 + \lambda x_3 + x_4 = 1, \\ x_1 + x_2 + x_3 + \lambda x_4 = 1. \end{cases}$

4.4. $\begin{cases} -x_1 + (1 + \lambda)x_2 + (2 - \lambda)x_3 + \lambda x_4 = 3, \\ \lambda x_1 - x_2 + (2 - \lambda)x_3 + \lambda x_4 = 2, \\ \lambda x_1 + \lambda x_2 + (2 - \lambda)x_3 + \lambda x_4 = 2, \\ \lambda x_1 + \lambda x_2 + (2 - \lambda)x_3 - x_4 = 2. \end{cases}$

5. Quyidagi CHTS larini mos usul tanlab yeching:

5.1. $\begin{cases} x + y + z + t = a, \\ x - y + z + t = b, \\ x + y - z + t = c, \\ x + y + z - t = d. \end{cases}$

5.2. $\begin{cases} ax + by + cz + dt = p, \\ -bx + ay + dz - ct = q, \\ -cx - dy + az + bt = r, \\ -dx + cy - bz + at = s. \end{cases}$

5.3. $\begin{cases} x_1 + a_1 x_2 + \ldots + a_1^{n-1} x_n = b_1, \\ x_1 + a_2 x_2 + \ldots + a_2^{n-1} x_n = b_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_1 + a_n x_2 + \ldots + a_n^{n-1} x_n = b_n, \end{cases}$ $\quad a_1 \neq a_2 \neq \ldots \neq a_n.$

5.4. $\begin{cases} x_1 + x_2 + \ldots + x_n = 1, \\ a_1 x_1 + a_2 x_2 + \ldots + a_n x_n = b, \\ a_1^2 x_1 + a_2^2 x_2 + \ldots + a_n^2 x_n = b^2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_1^{n-1} x_1 + a_2^{n-1} x_2 + \ldots + a_n^{n-1} x_n = b^{n-1}, \end{cases}$ $\quad a_1 \neq a_2 \neq \ldots \neq a_n.$

6. CHTS ning umumiy yechimi va bitta xususiy yechimini toping:

6.1. $\begin{cases} 2x_1 + 7x_2 + 3x_3 + x_4 = 6, \\ 3x_1 + 5x_2 + 2x_3 + 2x_4 = 4, \\ 9x_1 + 4x_2 + x_3 + 7x_4 = 2. \end{cases}$

6.2. $\begin{cases} 2x_1 - 3x_2 + 5x_3 + 7x_4 = 1, \\ 4x_1 - 6x_2 + 2x_3 + 3x_4 = 2, \\ 2x_1 - 3x_2 - 11x_3 - 15x_4 = 1. \end{cases}$

6.3. $\begin{cases} 2x_1 + 5x_2 - 8x_3 = 8, \\ 4x_1 + 3x_2 - 9x_3 = 9, \\ 2x_1 + 3x_2 - 5x_3 = 7, \\ x_1 + 8x_2 - 7x_3 = 12. \end{cases}$

6.4. $\begin{cases} 3x_1 + 2x_2 + 2x_3 + 2x_4 = 2, \\ 2x_1 + 3x_2 + 2x_3 + 5x_4 = 3, \\ 9x_1 + x_2 + 4x_3 - 5x_4 = 1, \\ 2x_1 + 2x_2 + 3x_3 + 4x_4 = 5, \\ 7x_1 + x_2 + 6x_3 - x_4 = 7. \end{cases}$

6.5. $\begin{cases} 6x_1 + 4x_2 + 5x_3 + 2x_4 + 3x_5 = 1, \\ 3x_1 + 2x_2 + 4x_3 + x_4 + 2x_5 = 3, \\ 3x_1 + 2x_2 - 2x_3 + x_4 = 1, \\ 9x_1 + 6x_2 + x_3 + 3x_4 + 2x_5 = 2. \end{cases}$

6.6. $\begin{cases} 6x_1 + 3x_2 + 2x_3 + 3x_4 + 4x_5 = 5, \\ 4x_1 + 2x_2 + x_3 + 2x_4 + 3x_5 = 4, \\ 4x_1 + 2x_2 + 3x_3 + 2x_4 + x_5 = 0, \\ 2x_1 + x_2 + 7x_3 + 3x_4 + 2x_5 = 1. \end{cases}$

7. Tenglamalar sistemasining yechimlarini Gauss usulida toping:

7.1. $\begin{cases} 7x_1 - 3x_2 - 2x_4 = -1, \\ -x_1 + 3x_2 - 2x_3 + x_4 = -6, \\ 2x_2 + 2x_3 + 13x_4 = 14, \\ 2x_1 - 4x_2 - 3x_3 + 2x_4 = -3. \end{cases}$

7.2. $\begin{cases} 3x_1 + 4x_2 + 2x_3 - x_4 = -1, \\ -x_1 + 4x_2 - x_3 + 3x_4 = -13, \\ -3x_1 + 2x_2 + 3x_3 + x_4 = 10, \\ 22x_1 - 5x_2 + 7x_3 - 2x_4 = 20. \end{cases}$

7.3. $\begin{cases} 3x_1 - 8x_2 + x_3 + 4x_4 = -5, \\ x_1 - 7x_2 + 2x_3 + 14x_4 = 3, \\ -x_1 - x_2 - 4x_3 + 5x_4 = -13. \end{cases}$

7.4. $\begin{cases} -4x_1 + 3x_2 + x_3 - 2x_4 = 0, \\ 2x_1 + x_2 - 3x_3 + 3x_4 = 6, \\ x_1 - 3x_2 + 2x_3 + 5x_4 = 10, \\ x_1 - x_2 - 6x_3 + 2x_4 = 1. \end{cases}$

7.5. $\begin{cases} x_1 - 13x_2 + x_3 - 3x_4 = 0, \\ x_1 - x_2 - 5x_3 + 3x_4 = 1, \\ 3x_1 + x_2 + 2x_3 - 11x_4 = 10, \\ x_1 - 4x_2 - 3x_3 + 2x_4 = 0. \end{cases}$

$$7.6.\ \begin{cases} 3x_1 + 3x_2 - 6x_3 - 2x_4 = -1, \\ 6x_1 + x_2 - 2x_4 = -2, \\ 6x_1 - 7x_2 + 21x_3 + 4x_4 = 3, \\ 9x_1 + 4x_2 - x_3 + 4x_4 = 3. \end{cases}$$

$$7.7.\ \begin{cases} 11x_1 - 5x_3 + 2x_4 = 5, \\ 3x_1 - 4x_2 - x_3 = -1, \\ x_1 + 2x_2 - 12x_3 + 13x_4 = 7, \\ 5x_1 - 4x_2 + 3x_3 - 2x_4 = 20. \end{cases}$$

$$7.8.\ \begin{cases} 5x_1 - 13x_2 + x_3 + 23x_4 = 11, \\ x_1 + 3x_2 - 5x_3 + 3x_4 = 1, \\ 13x_1 + x_2 + 2x_3 - 11x_4 = 0, \\ 12x_1 + 4x_2 - 17x_3 + 2x_4 = 20. \end{cases}$$

8. Quyidagi CHTSning umumiy yechimini va yechimlar fundamental sistemasini toping:

$$8.1.\ \begin{cases} 3x_1 - 3x_2 + 17x_3 - 25x_4 + 7x_5 = 0, \\ x_1 + 2x_2 - 7x_3 + x_4 - 11x_5 = 0. \end{cases}$$

$$8.2.\ \begin{cases} x_1 + 4x_2 + 2x_3 - 3x_5 = 0, \\ 2x_1 + 9x_2 + 5x_3 + 2x_4 + x_5 = 0, \\ x_1 + 3x_2 + x_3 - 2x_4 - 9x_5 = 0. \end{cases}$$

$$8.3.\ \begin{cases} x_1 - 3x_2 + 2x_3 + x_4 = 0, \\ -x_1 - x_2 + 2x_3 + 4x_4 = 0, \\ 4x_1 + 3x_2 - 4x_3 + x_4 = 0. \end{cases}$$

$$8.4.\ \begin{cases} 2x_1 - 5x_2 + 4x_3 + 3x_4 = 0, \\ 3x_1 - 4x_2 + 7x_3 + 5x_4 = 0, \\ 4x_1 - 9x_2 + 8x_3 + 5x_4 = 0, \\ -3x_1 + 2x_2 - 5x_3 + 3x_4 = 0. \end{cases}$$

$$8.5.\ \begin{cases} -5x_1 + 13x_2 - 3x_3 - 25x_4 + 6x_5 = 0, \\ 2x_1 - 7x_2 + x_3 - 11x_5 = 0. \end{cases}$$

8.6. $\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 = 0, \\ 3x_1 + 2x_2 + x_3 + x_4 - 3x_5 = 0, \\ x_2 + 2x_3 + 2x_4 + 6x_5 = 0, \\ 5x_1 + 4x_2 + 3x_3 + 3x_4 - x_5 = 0. \end{cases}$

8.7. $\begin{cases} -9x_1 + 3x_2 - x_3 + 4x_4 = 0, \\ 4x_1 - x_2 + 2x_3 - 5x_4 = 0, \\ -8x_1 + 2x_2 - 3x_3 + 15x_4 = 0. \end{cases}$

8.8. $\begin{cases} \lambda x_1 + \lambda x_2 + \lambda x_3 + \lambda x_4 = 0, \\ 2\lambda x_1 + 3\lambda x_2 + 4\lambda x_3 + 5\lambda x_4 = 0. \end{cases}$

8.9. $\begin{cases} 25x_1 + 13x_2 - x_3 + 3x_4 = 0, \\ -13x_1 - 4x_2 + x_3 - 5x_4 = 0, \\ 2x_1 + 32x_2 - 3x_3 - 15x_4 = 0. \end{cases}$

8.10. $\begin{cases} -4x_1 + (2 + 2\lambda)x_2 + 2\lambda x_3 + 2\lambda x_4 = 0, \\ \lambda x_1 + (1 + \lambda)x_2 + \lambda x_3 + \lambda x_4 = 0, \\ \lambda x_1 + (1 + \lambda)x_2 - 2x_3 + \lambda x_4 = 0, \\ -\lambda x_1 - (1 + \lambda)x_2 - \lambda x_3 - (2 - 2\lambda)x_4 = 0. \end{cases}$

Takrorlash uchun savollar

1. *n* ta noma'lumli *m* ta chiziqli tenglamalar sistemasi deb nimaga aytiladi?
2. CHTSning yechimi deb nimaga aytiladi?
3. Hamjoyli, hamjoyli bo'lmagan CHTSga ta'rif bering.
4. CHTSning natijasiga ta'rif bering.
5. CHTSning chiziqli kombinatsiyasi nima?
6. Teng kuchli CHTSlariga ta'rif bering.
7. CHTSni elementar almashtirishlar deganda qanday almashtirishlar tushuniladi?
8. Kroneker–Kapelli teoremasini bayon eting.
9. Bir jinsli CHTS deb qanday sistemaga aytiladi?

10. CHTS va unga assotsirlangan BCHTS yechimlar yig'indisi, ayirmasi qanday sistemaga yechim bo'ladi?

11. BCHTS yechimlar to'plami vektor fazo tashkil etishini tushuntiring.

12. BCHTSning fundamental yechimlari sistemasiga ta'rif bering.

13. CHTSni yechishning Gauss usulini tushuntiring.

VI MODUL. MATRITSALAR

16-§. Matritsalar va ular ustida amallar

✓ **Asosiy tushunchalar:** kvadrat matritsa, matritsalarni qo'shish, skalarni matritsaga ko'paytirish, matritsalar ko'paytmasi, teskarilanuvchi matritsa, elementar matritsa, matritsali tenglama.

$F = <F;+,\cdot,-,^{-1},0,1>$ maydon va maydon ustida matritsalar to'plami berilgan bo'lsin. Quyidagi munosabatlarni aniqlaymiz:

$\forall A,\ B \in F^{m\times n} \Rightarrow A=B \Leftrightarrow a_{ij}=b_{ij}i = 1, ..., m; j=1, ..., n.$

$\forall A,B \in F^{m\times n},\ A+B=C,\ C \in F^{m\times n}.$

$\forall A \in F^{m\times n} \wedge \forall \alpha \in F \Rightarrow \omega\alpha(A) = \alpha A = B \in F^{m\times n}.$

$\forall A \in F^{m\times n},\ \forall B \in F^{n\times k} \Rightarrow A \cdot B=C,\ C \in F^{m\times k}.$

$A_i \cdot B_j = a_{i1} b_{1j} + a_{i2} b_{2j} + ... + a_{in} b_{nj} = c_{ij},\ i=1, ..., m; j=1, ..., k.$

Shunday X va A n-tartibli kvadrat matritsalar berilgan bo'lib, ular uchun $XA = AX = E$ (E – n-tartibli birlik matritsa) shart bajarilsa, u holda X matritsaga A matritsaga *teskari matritsa* deyiladi va A^{-1} ko'rinishda belgilanadi.

Teskari matritsaga ega matritsa *teskarilanuvchi matritsa* deyiladi.

Birlik matritsadan quyidagi elementar almashtirishlarning biri yordamida hosil qilingan matritsa *elementar matritsa* deyiladi:

1) birlik matritsa satri (ustuni)ni noldan farqli skalarga ko'paytirish;

2) birlik matritsa biror-bir satri (ustuni) ga noldan farqli skalarga ko'paytirilgan satr (ustun)ni qo'shish yoki ayirish.

E birlik matritsada bajarilgan φ satr elementar almashtirish 1) yoki 2) ko'rinishdagi elementar almashtirish bo'lsa, u holda hosil bo'lgan elementar matritsani E_φ ko'rinishda belgilaymiz.

Agar A kvadrat matritsani elementar almashtirishlar zanjiri (ketma-ket bajarilgan elementar almashtirishlar) birlik matritsaga o'tkazsa, u holda A matritsa teskarilanuvchi va bajarilgan elementar almashtirishlar zanjiri E matritsani A^{-1} matritsaga keltiradi. Ya'ni $A \in F^{n\times n}$ matritsaga teskari matritsani topish uchun

tartibi $n\times 2n$ bo'lgan $A \mid E = \left(\begin{array}{cccc|cccc} a_{11} & a_{12} & \cdots & a_{1n} & 1 & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & a_{2n} & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} & 0 & 0 & \cdots & 1 \end{array}\right)$

matritsani elementar almashtirishlar zanjiri yordamida

$\left(\begin{array}{cccc|cccc} 1 & 0 & \cdots & 0 & b_{11} & b_{12} & \cdots & b_{1n} \\ 0 & 1 & \cdots & 0 & b_{21} & b_{22} & \cdots & b_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 & b_{n1} & b_{n2} & \cdots & b_{nn} \end{array}\right) = E \mid B$ ko'rinishga keltiramiz. Hosil bo'lgan B matritsa berilgan A matritsaga teskari matritsa.

R maydon ustida n ta noma'lumli n ta chiziqli tenglamalar sistemasi

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n \end{cases}$$

ko'rinishda berilgan bo'lsin.

Quyidagi belgilashlarni kiritamiz:

$$A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{pmatrix}, \quad B = \begin{pmatrix} b_1 \\ b_2 \\ \ldots \\ b_n \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{pmatrix}.$$

U holda berilgan CHTSni matritsali tenglama, ya'ni $AX=B$ ko'rinishda yozish mumkin.

Agar A matritsaning satrlari chiziqli erkli bo'lsa, u holda $A^{-1}B$ vektor $AX = B$ tenglamaning yagona yechimi bo'ladi.

Lekin matritsali tenglama faqat CHTS yordamida hosil qilinmaydi. Balki A, B matritsalar berilgan bo‘lsa $A \cdot X = B$ yoki $X \cdot Z = B$ ko‘rinishdagi matritsali tenglamalarni; A, B, C matritsalar berilgan bo‘lsa, $A \cdot X \cdot B = C$ ko‘rinishdagi matritsali tenglamalarni tuzish mumkin bo‘lsa, ularni yechish uchun o‘zgaruvchining chap yoki o‘ng tomonidagi A matritsa teskarilanuvchi bo‘lsa, uning yechimi $A^{-1} \cdot B$ yoki $B \cdot A^{-1}$ ko‘rinishda; o‘zgaruvchining o‘ng va chap tomonidagi A va B lar teskarilanuvchi bo‘lsa, $X = A^{-1} \cdot C \cdot B^{-1}$ ko‘rinishda bo‘ladi.

1-misol. $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 1 & -1 \\ -2 & 1 & 0 \end{pmatrix}$ va $f(x) = 3x - x^2 + 4$ berilgan bo‘lsa, $f(A)$ ni hisoblang.

Yechish. $f(A) = 3 \cdot A - A^2 + 4$ ni hisoblash uchun $3 \cdot A$, A^2 va $4 \cdot E$ matritsalarni aniqlaymiz:

$$1)\ 3 \cdot A = 3 \cdot \begin{pmatrix} 1 & 2 & 3 \\ 4 & 1 & -1 \\ -2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 6 & 9 \\ 12 & 3 & -3 \\ -6 & 3 & 0 \end{pmatrix};$$

$$2)\ A^2 = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 1 & -1 \\ -2 & 1 & 0 \end{pmatrix}^2 = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 1 & -1 \\ -2 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ 4 & 1 & -1 \\ -2 & 1 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 1+8-6 & 2+2+3 & 3-2 \\ 4+4+2 & 4+1-1 & 12-1 \\ -2+4 & -4+1 & -6-1 \end{pmatrix} = \begin{pmatrix} 3 & 7 & 1 \\ 10 & 4 & 11 \\ 2 & -3 & -7 \end{pmatrix};$$

$$3)\ 4 \cdot E = 4 \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix};$$

4) $f(A)=3\cdot A-A^2+4=\begin{pmatrix}3 & 6 & 9\\ 12 & 3 & -3\\ -6 & 3 & 0\end{pmatrix}-\begin{pmatrix}3 & 7 & 1\\ 10 & 4 & 11\\ 2 & -3 & -7\end{pmatrix}+$

$$+\begin{pmatrix}4 & -1 & 8\\ 2 & 3 & -14\\ -8 & 6 & 11\end{pmatrix}=\begin{pmatrix}4 & -1 & 8\\ 2 & 3 & -14\\ -8 & 6 & 11\end{pmatrix}.$$

2-misol. $A=\begin{pmatrix}1 & 1 & 0\\ 2 & -1 & 1\\ 4 & 3 & 2\end{pmatrix}$ matritsaga teskari A^{-1} matritsani toping.

Yechish. 1) Berilgan matritsaning teskarilanuvchi ekanligini, ya'ni $r(A)=3$ ekanligini tekshirib olamiz:

$$\begin{pmatrix}1 & 1 & 0\\ 2 & -1 & 1\\ 4 & 3 & 2\end{pmatrix}\sim\begin{pmatrix}1 & 1 & 0\\ 0 & -3 & 1\\ 0 & -1 & 2\end{pmatrix}\sim\begin{pmatrix}1 & 1 & 0\\ 0 & -1 & 2\\ 0 & 0 & -5\end{pmatrix}.$$

Demak, $r(A)=3$ ekan, ya'ni matritsa chiziqli erkli va shu sababli teskarilanuvchi.

2) Teskari matritsani elementar matritsalar yordamida topamiz:

$$\left(\begin{array}{ccc|ccc}1 & 1 & 0 & 1 & 0 & 0\\ 2 & -1 & 1 & 0 & 1 & 0\\ 4 & 3 & 2 & 0 & 0 & 1\end{array}\right)\sim\left(\begin{array}{ccc|ccc}1 & 1 & 0 & 1 & 0 & 0\\ 0 & -3 & 1 & -2 & 1 & 0\\ 0 & -1 & 2 & -4 & 0 & 1\end{array}\right)\sim$$

$$\sim\left(\begin{array}{ccc|ccc}1 & 1 & 0 & 1 & 0 & 0\\ 0 & -1 & 2 & -4 & 0 & 1\\ 0 & 0 & -5 & 10 & 1 & -3\end{array}\right)\sim\left(\begin{array}{ccc|ccc}1 & 1 & 0 & 1 & 0 & 0\\ 0 & 1 & -2 & 4 & 0 & -1\\ 0 & 0 & 1 & -2 & -\frac{1}{5} & \frac{3}{5}\end{array}\right)\sim$$

$$\sim\left(\begin{array}{ccc|ccc}1 & 1 & 0 & 1 & 0 & 0\\ 0 & 1 & 0 & 0 & -\frac{2}{5} & \frac{1}{5}\\ 0 & 0 & 1 & -2 & -\frac{1}{5} & \frac{3}{5}\end{array}\right)\sim\left(\begin{array}{ccc|ccc}1 & 0 & 0 & 1 & \frac{2}{5} & -\frac{1}{5}\\ 0 & 1 & 0 & 0 & -\frac{2}{5} & \frac{1}{5}\\ 0 & 0 & 1 & -2 & -\frac{1}{5} & \frac{3}{5}\end{array}\right).$$

Teskari matritsa to‘g‘ri topilganligiga tekshirish natijasida ishonch hosil qilamiz.

Tekshirish: $\begin{pmatrix} 1 & 1 & 0 \\ 2 & -1 & 1 \\ 4 & 3 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & \frac{2}{5} & -\frac{1}{5} \\ 0 & -\frac{2}{5} & \frac{1}{5} \\ -2 & -\frac{1}{5} & \frac{3}{5} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$

Demak, $A^{-1} = \begin{pmatrix} 1 & \frac{2}{5} & -\frac{1}{5} \\ 0 & -\frac{2}{5} & \frac{1}{5} \\ -2 & -\frac{1}{5} & \frac{3}{5} \end{pmatrix}.$

Misol va mashqlar

1. Matritsalarni qo‘shish amalining quyidagi xossalarini isbotlang:

1.1. $\forall A, B \in F^{m\times n} \Rightarrow A + B = B + A$ (kommutativlik).

1.2. $\forall A, B, C \in F^{m\times n} \Rightarrow (A+B)+C = A+(B+C)$ (assotsiativlik).

1.3. $\forall A \in F^{m\times n}, \exists X \in F^{m\times n} \Rightarrow A + X = A$ ($X=0$ – neytral).

1.4. $\forall A \in F^{m\times n}, \exists A' \in F^{m\times n} \Rightarrow A+A' = 0$ ($A' = -A$ – simmetrik).

2. Skalarni matritsaga ko‘paytirishning quyidagi xossalarini isbotlang:

2.1. $\forall A \in F^{m\times n} \wedge \forall \alpha, \beta \in F \Rightarrow (\alpha+\beta)A = \alpha A + \beta A$.

2.2. $\forall A \in F^{m\times n} \wedge \forall \alpha, \beta \in F \Rightarrow (\alpha \cdot \beta)A = \alpha(\beta A)$.

2.3. $\forall A, B \in F^{m\times n} \wedge \forall \alpha \in F \Rightarrow \alpha(A+B) = \alpha \cdot A + \alpha \cdot B$.

2.4. $\forall A \in F^{m\times n} \wedge \leftrightarrow \alpha \in F \Rightarrow \alpha \cdot A = A \cdot \alpha$.

3. $f(A)$ ni hisoblang:

3.1. $f(x) = x^3 + 4x; \quad A = \begin{pmatrix} -1 & 9 \\ 5 & 2 \end{pmatrix}.$

3.2. $f(x) = x^2 + 24x + 7;$ $\quad A = \begin{pmatrix} -2 & 20 & 1 \\ 6 & -4 & 3 \\ 7 & 5 & 0 \end{pmatrix}.$

3.3. $f(x) = -3x^3 + 15x^2 - 2x + 1;$ $\quad A = \begin{pmatrix} -2 & 3 \\ -1 & 0 \end{pmatrix}.$

3.4. $f(x) = -4x^3 + 25x + 9;$ $\quad A = \begin{pmatrix} 7 & -1 & 1 \\ 2 & 6 & 3 \\ -1 & 2 & 5 \end{pmatrix}.$

4. Matritsalarni ko'paytirish amalining quyidagi xossalarini isbotlang:

1. $\exists A \cdot B \in F^{m\times k} \wedge \exists B \cdot C \in F^{k\times s} \Rightarrow (A \cdot B) \cdot C = A \cdot (B \cdot C)$ (assotsiativlik).

2. $A \in F^{m\times n} \wedge \forall B, C \in F^{n\times k} \Rightarrow A \cdot (B + C) = A \cdot B + A \cdot C$ (yig'indini chapdan ko'paytirish).

3. $\forall A,\ B \in F^{m\times n} \wedge \forall C \in F^{n\times k} \Rightarrow (A + B) \cdot C = A \cdot C + B \cdot C$ (yig'indini o'ngdan ko'paytirish).

4. $\forall \alpha \in F,\ \forall A \in F^{m\times n},\ \forall B \in F^{n\times k} \Rightarrow \alpha \cdot (A \cdot B) = (\alpha \cdot A) \cdot B.$

5. Quyidagi matritsalar ko'paytmasini toping:

5.1. $A = (0 \ \ 1 \ \ 0 \ \ 0),\ B = \begin{pmatrix} 5 & 2 & -7 & 3 \\ 0 & 1 & 2 & -1 \\ 9 & -3 & 1 & 5 \\ 2 & -1 & 3 & 4 \end{pmatrix},\quad AB = ?$

5.2. $A = \begin{pmatrix} 21 & 22 & 23 & 24 \\ 25 & 26 & 27 & 28 \\ 29 & 30 & 31 & 32 \\ 33 & 34 & 35 & 37 \end{pmatrix},\ B = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix},\quad AB = ?$

5.3. $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix},\ B = \begin{pmatrix} \cos\beta & -\cos\beta \\ \sin\beta & \cos\beta \end{pmatrix},\ AB = ?\ BA = ?$

5.4. $A=\begin{pmatrix}-2 & 3 & 0 & 1\\ 1 & 1 & 2 & -1\end{pmatrix}, B=\begin{pmatrix}2 & 0\\ 1 & -1\\ -1 & 2\\ 1 & 3\end{pmatrix}, AB=?\quad BA=?$

5.5. $A=\begin{pmatrix}2 & 3 & 0 & -1\\ 0 & 1 & -1 & 3\\ 9 & -11 & -3 & 0\end{pmatrix}, B=\begin{pmatrix}12 & -6 & 2\\ 18 & -9 & 3\\ 24 & -12 & 4\end{pmatrix}, C=\begin{pmatrix}11\\ 12\\ -30\end{pmatrix}, (AB)C=?$

5.6. $A=\begin{pmatrix}1\\ 6\\ -2\end{pmatrix}, B=\begin{pmatrix}53 & 22 & -35\end{pmatrix}, C=\begin{pmatrix}3\\ -4\\ 2\end{pmatrix}, D=\begin{pmatrix}2 & -3 & -1\end{pmatrix}, ABCD=?$

6. Hisoblang:

6.1. $\begin{pmatrix}2 & -1\\ 3 & -2\end{pmatrix}^n.$ 6.2. $\begin{pmatrix}2 & 1\\ 0 & 2\end{pmatrix}^n.$

6.3. $\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}^n.$ 6.4. $\begin{pmatrix}a & 1\\ 0 & a\end{pmatrix}^n.$

6.5. $\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 2 & 0 & 0\\ 0 & 0 & 3 & 0\\ 0 & 0 & 0 & 4\end{pmatrix}^n.$ 6.6. $\begin{pmatrix}0 & 0 & 0 & \sin\alpha\\ 0 & 0 & \sin\alpha & 0\\ 0 & \cos\alpha & 0 & 0\\ \cos\alpha & 0 & 0 & 0\end{pmatrix}^{2n}.$

7. Berilgan matritsa bilan kommutativ bo‘lgan matritsalarni aniqlang:

7.1. $\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}.$ 7.2. $\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}.$

7.3. $\begin{pmatrix}7 & -3\\ 5 & -2\end{pmatrix}.$ 7.4. $\begin{pmatrix}3 & 1 & 0\\ 0 & 3 & 1\\ 0 & 0 & 3\end{pmatrix}.$

7.5. $\begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{pmatrix}.$ 7.6. $\begin{pmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{pmatrix}.$

7.7. $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$. 7.8. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

8. R maydonda $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ ko'rinishdagi matritsalar to'plami multiplikativ gruppa tashkil etishini va uning haqiqiy sonlar additiv gruppasiga izomorfligini isbotlang.

9. R maydonda $\begin{pmatrix} a+bi & c+di \\ c-di & a-bi \end{pmatrix}$ ko'rinishdagi matritsalar to'plami nolning bo'luvchilariga ega bo'lmagan halqa tashkil etishini isbotlang.

10. Q maydonda $\begin{pmatrix} a & b \\ -b & a-b \end{pmatrix}$ ko'rinishdagi matritsalar to'plami kommutativ halqa tashkil etishini isbotlang.

11. Teskari matritsalarning quyidagi xossalarini isbotlang:

1) $(A^{-1})^{-1} = A$;
2) $(AB)^{-1} = B^{-1}A^{-1}$;
3) $(A^{\mathrm{T}})^{-1} = (A^{-1})^{\mathrm{T}}$.

12. Quyidagi matritsalarning teskarisini toping:

12.1. $\begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix}$. 12.2. $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

12.3. $\begin{pmatrix} -1 & -1 & 1 \\ 2 & 0 & -3 \\ 1 & 1 & 0 \end{pmatrix}$. 12.4. $\begin{pmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{pmatrix}$.

12.5. $\begin{pmatrix} 2 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ 1 & 1 & 3 & 4 \\ 2 & -1 & 2 & 3 \end{pmatrix}$. 12.6. $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$.

12.7. $\begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 1 & \dots & 1 \\ 0 & 0 & 1 & \dots & 1 \\ . & . & . & . & . \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}$. 12.8. $\begin{pmatrix} 1 & a & a^2 & a^3 & \dots & a^n \\ 0 & 1 & a & a^2 & \dots & a^{n-1} \\ 0 & 0 & 1 & a & \dots & a^{n-2} \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & \dots & 1 \end{pmatrix}$.

12.9. $\begin{pmatrix} 2 & -1 & 0 & 0 & \dots & 0 \\ -1 & 2 & -1 & 0 & \dots & 0 \\ 0 & -1 & 2 & -1 & \dots & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & \dots & 2 \end{pmatrix}$.

12.10. $\begin{pmatrix} 1+a_1 & 1 & 1 & \dots & 1 \\ 0 & 1+a_2 & 1 & \dots & 1 \\ 0 & 0 & 1+a_3 & \dots & 1 \\ . & . & . & . & . \\ 0 & 0 & 0 & \dots & 1+a_n \end{pmatrix}$

13. Ixtiyoriy A kvadrat matritsa uchun quyidagi shartlar teng kuchli ekanligini isbotlang:

1) A matritsa teskarilanuvchi;

2) A matritsaning satrlari (ustunlari) chiziqli erkli.

14. Quyidagi matritsali tenglamalarni yeching:

14.1. $\begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix} X = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$.

14.2. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} X = \begin{pmatrix} 3 & 5 \\ 3 & 9 \end{pmatrix}$.

14.3. $X \begin{pmatrix} 1 & -3 \\ 2 & -6 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$.

14.4. $X \begin{pmatrix} 3 & 2 \\ 6 & 4 \end{pmatrix} = \begin{pmatrix} -3 & -2 \\ -9 & -6 \end{pmatrix}$.

14.5. $\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} X \begin{pmatrix} -5 & 1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & -3 \end{pmatrix}.$

14.6. $\begin{pmatrix} 3 & -1 \\ 5 & -2 \end{pmatrix} X \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 14 & 16 \\ 9 & 10 \end{pmatrix}.$

14.7. $\begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{pmatrix} X = \begin{pmatrix} 1 & -3 & 0 \\ 10 & 2 & 7 \\ 10 & 7 & 8 \end{pmatrix}.$

14.8. $\begin{pmatrix} 0 & 2 & 1 \\ 1 & 3 & 0 \\ -3 & 0 & 4 \end{pmatrix} X = \begin{pmatrix} 2 & 0 & 1 \\ -3 & 0 & 4 \\ 1 & 1 & 2 \end{pmatrix}^2.$

14.9. $X \begin{pmatrix} 5 & 3 & 1 \\ 1 & -3 & -2 \\ -5 & 2 & 1 \end{pmatrix} = \begin{pmatrix} -8 & 3 & 0 \\ -5 & 9 & 0 \\ -2 & 15 & 0 \end{pmatrix}.$

14.10. $X \begin{pmatrix} 4 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{pmatrix}.$

14.11. $\begin{pmatrix} 2 & -3 & 1 \\ 4 & -5 & 2 \\ 5 & -7 & 3 \end{pmatrix} X \begin{pmatrix} 9 & 7 & 6 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & -2 \\ 18 & 12 & 9 \\ 23 & 15 & 11 \end{pmatrix}.$

14.12. $\begin{pmatrix} 1 & 3 & 2 & -5 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} X \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 1 & -1 \\ 1 & 2 & 0 & 3 \\ 0 & -2 & 1 & 0 \\ 1 & -1 & 2 & -2 \end{pmatrix}.$

15. Quyidagi tenglamalar sistemasini matritsali tenglamaga keltirib yeching:

15.1. $\begin{cases} x_1 - x_2 + x_3 = 6, \\ 2x_1 + x_2 + x_3 = 3, \\ x_1 + x_2 + 2x_3 = 5. \end{cases}$

15.2. $\begin{cases} 5x_1 - 4x_2 + 2x_3 = -9, \\ 3x_1 - 2x_2 + x_3 = 3, \\ 10x_1 - 9x_2 + 2x_3 = 7. \end{cases}$

15.3. $\begin{cases} 2x_1 + 5x_2 + 4x_3 + x_4 = 20, \\ x_1 + 3x_2 + 2x_3 + x_4 = 11, \\ 2x_1 + 10x_2 + 9x_3 + 7x_4 = 40, \\ 3x_1 + 8x_2 + 9x_3 + 2x_4 = 37. \end{cases}$

15.4. $\begin{cases} 3x_1 + 5x_2 - 3x_3 + 2x_4 = 12, \\ 4x_1 - 2x_2 + 5x_3 + 3x_4 = 27, \\ 7x_1 + 8x_2 - x_3 + 5x_4 = 40, \\ 6x_1 + 4x_2 + 5x_3 + 3x_4 = 41. \end{cases}$

15.5. $\begin{cases} 2x_1 + 2x_2 - x_3 + x_4 = 4, \\ 3x_1 + 4x_2 - x_3 + 2x_4 = 6, \\ 5x_1 + 8x_2 - 3x_3 + 4x_4 = 12, \\ 3x_1 + 3x_2 - 2x_3 + 2x_4 = 6. \end{cases}$

15.6. $\begin{cases} 6x_1 + x_2 + 4x_3 - x_4 = 6, \\ 7x_1 + x_2 + 3x_3 + x_4 = -3, \\ 5x_1 + x_2 + 2x_3 - x_4 = 0, \\ 9x_1 + 2x_2 + 5x_3 - 2x_4 = 0. \end{cases}$

15.7. $\begin{cases} 3x_1 - x_2 + x_3 - 4x_4 = 3, \\ 3x_1 - x_2 + 2x_3 - 2x_4 = 3, \\ 5x_1 - x_2 + x_3 - 4x_4 = -1, \\ 11x_1 - 3x_2 + 2x_3 - 5x_4 = -2. \end{cases}$

15.8. $\begin{cases} 5x_1 + 2x_2 - x_3 + 3x_4 + 2x_5 = 0, \\ 4x_1 - 7x_3 = 0, \\ 2x_1 + 3x_2 - 7x_3 + 5x_4 + 3x_5 = 2, \\ 2x_1 + 3x_2 - 6x_3 + 4x_4 + 5x_5 = 0, \\ 3x_1 - 4x_3 = 0. \end{cases}$

16. Quyidagi matritsali tenglamalar sistemasini yeching:

16.1. $X + Y = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \; 2X + 3Y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$

16.2. $2X - Y = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad -4X + 2Y = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}.$

Takrorlash uchun savollar

1. Kvadrat matritsa va uning turlari.
2. Matritsalarni qo'shish va uning xossalari.
3. Skalarni matritsaga ko'paytirish va uning xossalari.
4. Matritsalarni ko'paytirish va uning xossalari.
5. Teskarilanuvchi matritsa deb qanday matritsaga aytiladi?
6. Elementar matritsalar xossalarini ayting.
7. Teskari matritsani ta'rif asosida topish jarayonini bayon qiling.
8. Matritsaning teskarilanish shartlarini ayting.
9. Teskari matritsani elementar matritsalardan foydalanib topish jarayonini tushuntiring.
10. CHTSning matritsali ifodasi qanday hosil qilinadi?
11. Matritsali tenglamalarning qanday ko'rinishlarini bilasiz?
12. Matritsali tenglamani yechish jarayonini bayon qiling.

VII MODUL. DETERMINANTLAR

17-§. O'rniga qo'yishlar

Asosiy tushunchalar: n-darajali o'rniga qo'yish, n-darajali simmetrik gruppa, inversiya, juft o'rniga qo'yish, toq o'rniga qo'yish, transpozitsiya, o'rniga qo'yishning ishorasi.

$A=\{1,2,3,...,n\}$ to'plamni o'ziga biyektiv akslantirish n-darajali o'rniga qo'yish deyiladi.

A to'plamda aniqlangan φ o'rniga qo'yish

$$\varphi=\begin{pmatrix} 1 & 2 & ... & n \\ \varphi(1) & \varphi(2) & ... & \varphi(n) \end{pmatrix}$$

ko'rinishda belgilanadi. Agar φ va ψ o'rniga qo'yishlarda $i_k=j_k$ $(k=\overline{1,n})$ bo'lsa, u holda φ va ψ o'rniga qo'yishlar *o'zaro teng* deyiladi.

φ va ψ *o'rniga qo'yishlar ko'paytmasi* deb φ va ψ akslantirishlar kompozitsiyasi $\varphi\psi(i)=\varphi(\psi(i)),\ i=1,...,n$ ga aytiladi, ya'ni

$$\varphi\cdot\psi=\varphi\cdot\begin{pmatrix} 1 & 2 & ... & n \\ \psi(1) & \psi(2) & ... & \psi(n) \end{pmatrix}=\begin{pmatrix} \psi(1) & \psi(2) & ... & \psi(n) \\ \varphi(\psi(1)) & \varphi(\psi(2)) & ... & \varphi(\psi(n)) \end{pmatrix}$$

A to'plamdan olingan φ o'rniga qo'yishga *teskari o'rniga qo'yish* deb

$$\varphi^{-1}=\begin{pmatrix} \varphi(1) & \varphi(2) & ... & \varphi(n) \\ 1 & 2 & ... & n \end{pmatrix}=\begin{pmatrix} 1 & 2 & ... & n \\ \varphi^{-1}(1) & \varphi^{-1}(2) & ... & \varphi^{-1}(n) \end{pmatrix}$$

o'rniga qo'yishga aytiladi.

A to'plamning har bir elementini shu elementning o'ziga o'tkazuvchi ε akslantirish *ayniy o'rniga qo'yish* deyiladi va u

$\varepsilon=\begin{pmatrix} 1 & 2 & ... & i & ... & n \\ 1 & 2 & ... & i & ... & n \end{pmatrix}$ ko'rinishda belgilanadi.

$<S_n; \cdot, {}^{-1}>$ gruppa *n-darajali simmetrik gruppa* deyiladi va u S_n orqali belgilanadi.

$\varphi = \begin{pmatrix} 1 & 2 & \ldots & n \\ \varphi(1) & \varphi(2) & \ldots & \varphi(n) \end{pmatrix}$ o'rniga qo'yishda $A=\{1,2,3,\ldots,n\}$ to'plamning ixtiyoriy i, j elementlaridan tuzilgan juftlik uchun $i - j$ va $\varphi(i) - \varphi(j)$ ayirmalar bir xil ishoraga ega bo'lsa, bu juftlik *to'g'ri*, bir xil ishoraga ega bo'lmasa, to'g'ri emas yoki *inversiya tashkil etadi* deyiladi. O'rniga qo'yishda inversiyalar soni juft (toq) bo'lsa, o'rniga qo'yish *juft (toq) o'rniga qo'yish* deyiladi.

O'rniga qo'yishda shunday i, j elementlar mavjud bo'lib, ular uchun $\varphi(i) = j$, $\varphi(j) = i$, $\varphi(s) = s$, $s \in A \backslash \{i, j\}$ shartlar bajarilsa, bunday o'rniga qo'yish *transpozitsiya* deyiladi.

$\varphi = \begin{pmatrix} 1 & 2 & \ldots & n \\ \varphi(1) & \varphi(2) & \ldots & \varphi(n) \end{pmatrix}$ o'rniga qo'yishning *ishorasi* deb

$\operatorname{sgn} \varphi = \begin{pmatrix} 1, \text{ agar } \varphi - \text{juft}, \\ -1, \text{ agar } \varphi - \text{toq} \end{pmatrix}$ qiymatga aytiladi.

1-misol. Berilgan o'rniga qo'yishlar va ular kompozitsiyasining juft-toqligi, ishorasi, inversiyalar sonini aniqlang:

$$\varphi_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 6 & 5 & 4 & 3 \end{pmatrix}; \quad \varphi_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 5 & 6 & 4 & 3 \end{pmatrix}.$$

Yechish.

$$\varphi_1 \circ \varphi_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 3 & 5 & 6 \end{pmatrix};$$

$$\varphi_2 \circ \varphi_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 4 & 6 & 5 \end{pmatrix}.$$

φ_1 dagi inversiyalar sonini aniqlaymiz:

$\varphi_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 6 & 5 & 4 & 3 \end{pmatrix}$ o'rniga qo'yishda birinchi qatordagi $i - j$ ayirmalar manfiy. 2-qatordagi ularga mos ayirmalardan

$\varphi_1(1)-\varphi_1(2)=2-1=1$ musbat, qolganlari manfiy, demak, φ_1 dagi inversiyalar soni 1 ta. Shuning uchun ishorasi $\operatorname{sgn}\varphi_1=-1$; φ_1 — toq o‘rniga qo‘yish;

$\varphi_2=\begin{pmatrix}1&2&3&4&5&6\\1&2&5&6&4&3\end{pmatrix}$ dagi inversiyalar soni 3 ta; φ_2 — toq o‘rniga qo‘yish va $\operatorname{sgn}\varphi_2=-1$;

$\varphi_1\circ\varphi_2=\begin{pmatrix}1&2&3&4&5&6\\2&1&4&3&5&6\end{pmatrix}$ dagi inversiyalar soni 2 ta, u juft o‘rniga qo‘yish va $\operatorname{sgn}(\varphi_1\circ\varphi_2)=1$;

$\varphi_2\circ\varphi_1=\begin{pmatrix}1&2&3&4&5&6\\2&1&3&4&6&5\end{pmatrix}$ o‘rniga qo‘yishda inversiyalar soni 2 ta, $\varphi_2\circ\varphi_1$ juft o‘rniga qo‘yish va $\operatorname{sgn}(\varphi_2\circ\varphi_1)=1$.

2-misol. $\begin{pmatrix}1&2&3&4&5&6\\2&1&3&4&6&5\end{pmatrix}$ o‘rniga qo‘yishni siklli o‘rniga qo‘yishlar kompozitsiyasi ko‘rinishida ifodalang.

Yechish. $\begin{pmatrix}1&2&3&4&5&6\\2&1&3&4&6&5\end{pmatrix}=(12)\ \ (3)\ \ (4)\ \ (56).$

Misol va mashqlar

1. O‘rniga qo‘yishlarni ko‘paytiring:

1.1. $\begin{pmatrix}1&2&3&4&5\\3&4&1&5&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\5&3&1&2&4\end{pmatrix}.$

1.2. $\begin{pmatrix}1&2&3&4&5&6\\3&6&4&5&2&1\end{pmatrix}\begin{pmatrix}1&2&3&4&5&6\\2&4&1&5&6&3\end{pmatrix}.$

1.3. $\begin{pmatrix}1&2&3&4&5&6&7\\4&3&1&2&7&5&6\end{pmatrix}\begin{pmatrix}1&2&3&4&5&6&7\\5&3&1&4&2&7&6\end{pmatrix}.$

1.4. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 5 & 3 & 9 & 1 & 2 & 4 & 6 & 8 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 9 & 5 & 7 & 2 & 1 & 3 & 4 & 6 & 8 \end{pmatrix}$.

2. O‘rniga qo‘yishni sikllarga yoying:

2.1. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 4 & 1 & 7 & 3 & 6 & 2 \end{pmatrix}$.

2.2. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 3 & 2 & 7 & 4 & 5 & 8 & 1 \end{pmatrix}$.

2.3. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 4 & 6 & 8 & 1 & 3 & 5 & 7 & 9 \end{pmatrix}$.

2.4. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 10 & 5 & 4 & 7 & 6 & 9 & 8 & 3 & 2 & 1 \end{pmatrix}$.

2.5. $\begin{pmatrix} 1 & 2 & 3 & 4 & \ldots & 2n-1 & 2n \\ 2 & 1 & 4 & 3 & \ldots & 2n & 2n-1 \end{pmatrix}$.

2.6. $\begin{pmatrix} 1 & 2 & \ldots & n & n+1 & n+2 & \ldots & 2n \\ n+1 & n+2 & \ldots & 2n & 1 & 2 & \ldots & n \end{pmatrix}$.

3. S_4 dagi barcha toq o‘rniga qo‘yishlarni aniqlang.

4. Quyidagi shartlar asosida α, β larni toping:

4.1. $(\alpha\ \ 6\ \ 7\ \ 1\ \ \beta\ \ 5\ \ 3)$ – toq.

4.2. $(11\ \ 5\ \ 7\ \ \alpha\ \ 1\ \ 2\ \ 9\ \ 8\ \ 4\ \ 3\ \ \beta)$ – juft.

5. Qaysi 10 ta sondan iborat o‘rniga qo‘yish eng ko‘p inversiyaga ega? Inversiyalar sonini aniqlang.

6. S_5 dan quyidagi shartlar bo‘yicha o‘rniga qo‘yishlarni aniqlang:

6.1. 4 ta inversiyaga ega.

6.2. 7 ta inversiyaga ega.

6.3. 9 ta inversiyaga ega.

6.4. 11 ta inversiyaga ega.

7. O‘rniga qo‘yishlar ishorasining quyidagi xossalarini isbotlang:

1) sgn funksiya multiplikativ, ya'ni har qanday $\varphi, \psi \in S_n$ lar uchun $\mathrm{sgn}(\varphi\psi) = \mathrm{sgn}\varphi \cdot \mathrm{sgn}\psi$ o'rinli;

2) transpozitsiya ishorasi (-1) ga teng;

3) o'zaro teskari o'rniga qo'yishlar ishorasi bir xil;

4) agar τ – transpozitsiya va φ – ixtiyoriy o'rniga qo'yish bo'lsa, u holda $\mathrm{sgn}(\tau\varphi) = \mathrm{sgn}(\varphi\tau) = -\mathrm{sgn}\varphi$ bo'ladi.

8. O'rniga qo'yishlardagi inversiyalar sonini aniqlang:

8.1. $(1\ 4\ 7\ \dots\ 3n{-}2\ 2\ 5\ 8\ \dots\ 3n{-}1\ 3\ 6\ 9\ \dots\ 3n)$.

8.2. $(3\ 6\ 9\ \dots\ 3n\ 2\ 5\ 8\ \dots\ 3n{-}1\ 1\ 4\ 7\ \dots\ 3n{-}2)$.

8.3. $(2\ 5\ 8\ \dots\ 3n{-}1\ 3\ 6\ 9\ \dots\ 3n\ 1\ 4\ 7\ \dots\ 3n{-}2)$.

8.4. $(2\ 5\ 8\ \dots\ 3n{-}1\ 1\ 4\ 7\ \dots\ 3n{-}2\ 3\ 6\ 9\ \dots\ 3n)$.

Takrorlash uchun savollar

1. n-darajali o'rniga qo'yishga ta'rif bering.
2. O'rniga qo'yishlar gruppa tashkil etishini tekshiring.
3. n-darajali simmetrik gruppaga misol keltiring.
4. Inversiyaga ta'rif bering.
5. Juft, toq o'rniga qo'yishlarni ta'riflang.
6. Transpozitsiya nima?
7. O'rniga qo'yishning ishorasi qanday aniqlanadi?

18-§. Determinantlar

✓ **Asosiy tushunchalar:** determinant, matritsaosti (qismmatritsa), n-tartibli minor, algebraik to'ldiruvchi, Laplas teoremasi, algebraik to'ldiruvchi, Kramer formulalari.

Kvadrat matritsaning har bir satr va har bir ustunidan bittadan elementlar olib tuzilgan ko'paytmalarning algebraik yig'indisi *berilgan kvadrat matritsaning determinanti* deyiladi.

n-tartibli kvadrat matritsa $\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$ ning determinanti deb $|A| = \sum_{\tau\in S_n} \text{sgn}(\tau) a_{1\tau(1)} \cdot \ldots \cdot a_{n\tau(n)}$ ($n!$ qo'shiluvchilardan iborat) yig'indiga aytiladi.

$F = \langle F;\ +,\ \cdot,\ -,\ ^{-1},\ 0,\ 1\rangle$ maydon va maydon ustida $F^{m\times n}$ matritsalar to'plami berilgan bo'lsin.

A matritsaning *matritsaosti* deb, uning qandaydir satr va ustunlarini o'chirishdan hosil bo'lgan matritsaga aytiladi.

k-tartibli matritsaosti determinanti A matritsaning *k-tartibli minori* deyiladi.

Kvadrat matritsaning i- qatori j- ustunini o'chirishdan hosil bo'lgan matritsaosti determinanti a_{ij} *elementning minori* deyiladi va M_{ij} ko'rinishda belgilanadi.

$A_{ij} = (-1)^{i+j} \cdot M_{ij}$ ko'paytma a_{ij} elementning *algebraik to'ldiruvchisi* deyiladi.

Laplas teoremasi. $A=\begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{pmatrix}$ *kvadrat matritsaning determinanti biror-bir satr (ustun) elementlari bilan ularning algebraik to'ldiruvchilari ko'paytmalarining yig'indisiga, ya'ni*

$$|A| = a_{1j}A_{1j} + \ldots + a_{nj}A_{nj} \quad (|A| = a_{i1}A_{i1} + \ldots + a_{in}A_{in}),\ i, j\in\{1,\ldots,n\}$$

ga teng.

A matritsa a_{ij} elementining A_{ij} $(i, j \in \{1,\ldots,n\}$ algebraik to'ldiruvchilaridan iborat $A^* = \begin{pmatrix} A_{11} & A_{21} & \ldots & A_{n1} \\ A_{12} & A_{22} & \ldots & A_{n2} \\ \ldots & \ldots & \ldots & \ldots \\ A_{1n} & A_{2n} & \ldots & A_{nn} \end{pmatrix}$ matritsa A matritsaga *biriktirilgan matritsa* deyiladi.

Agar $|A| \neq 0$ bo'lsa, u holda A matritsa teskarilanuvchi va $A^{-1} = |A|^{-1} \cdot A^*$.

$F = \langle F;\ +,\ \cdot,\ -,\ ^{-1},\ 0,\ 1\rangle$ maydon ustida quyidagi

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}$$

CHTS berilgan va uning asosiy matritsasi A bo'lsin.

Agar $|A| \neq 0$ bo'lsa, u holda CHTS yagona yechimga ega va u quyidagi formulalar orqali ifodalanadi: $x_1 = \frac{|A(1)|}{|A|}, \dots, x_n = \frac{|A(n)|}{|A|}$.

1-misol. Determinantni elementar almashtirishlar yordamida hisoblang.

Yechish.

$$\begin{vmatrix} 1 & -1 & 0 & 1 \\ 2 & 1 & 3 & 1 \\ -1 & 4 & 1 & 2 \\ 3 & 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 0 & 1 \\ 0 & 3 & 3 & -1 \\ 0 & 3 & 1 & 3 \\ 0 & 4 & 0 & -2 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 0 & 1 \\ 0 & 3 & 3 & -1 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & -12 & -2 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 0 & 1 \\ 0 & 3 & 3 & -1 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & 0 & -26 \end{vmatrix} =$$

$$= 1 \cdot 3 \cdot (-2) \cdot (-26) = 156.$$

2-misol. Дeterminantni 1-qator hamda 2-ustun elementlari yoyilmasi orqali hisoblang.

Yechish. 1-qator bo'yicha yoyamiz:

$$\begin{vmatrix} 1 & -1 & 0 & 1 \\ 2 & 1 & 3 & 1 \\ -1 & 4 & 1 & 2 \\ 3 & 1 & 0 & 1 \end{vmatrix} = 1\cdot(-1)^{1+1}\begin{vmatrix} 1 & 3 & 1 \\ 4 & 1 & 2 \\ 1 & 0 & 1 \end{vmatrix} + (-1)(-1)^{1+2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & 1 & 2 \\ 3 & 0 & 1 \end{vmatrix} +$$

$$+0\cdot(-1)^{1+3}\begin{vmatrix} 2 & 1 & 1 \\ -1 & 4 & 2 \\ 3 & 1 & 1 \end{vmatrix} + 1\cdot(-1)^{1+4}\begin{vmatrix} 2 & 1 & 3 \\ -1 & 4 & 1 \\ 3 & 1 & 0 \end{vmatrix} = (1+6-1-12)+$$

$$+(2+18-3+3)+0+(-1)(-3+3-36-2) = -6 + 20 + 38 = 52.$$

2-ustun bo‘yicha hisoblaymiz:

$$\begin{vmatrix} 1 & -1 & 0 & 1 \\ 2 & 1 & 3 & 1 \\ -1 & 4 & 1 & 2 \\ 3 & 1 & 0 & 1 \end{vmatrix} = (-1)\cdot(-1)^{1+2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & 1 & 2 \\ 3 & 0 & 1 \end{vmatrix} + 1\cdot(-1)^{2+2}\begin{vmatrix} 1 & 0 & 1 \\ -1 & 1 & 2 \\ 3 & 0 & 1 \end{vmatrix} +$$

$$+4\cdot(-1)^{3+2}\begin{vmatrix} 1 & 0 & 1 \\ 2 & 3 & 1 \\ 3 & 0 & 1 \end{vmatrix} + 1\cdot(-1)^{4+2}\begin{vmatrix} 1 & 0 & 1 \\ 2 & 3 & 1 \\ -1 & 1 & 2 \end{vmatrix} = (2+18-3+3)+$$

$$+4(3-9)+(6+2+3-1)=20-2-4(-6)+10=18+24+10=52.$$

3-misol. $\begin{cases} x_1 - 2x_2 + 3x_3 = 1, \\ -x_1 + x_2 - x_3 = 2, \\ 2x_1 + 4x_2 + x_3 = 3 \end{cases}$ tenglamalar sistemasini Kramer formulalari yordamida yeching.

Yechish. Kramer formulalari:

$$x_1 = \frac{\Delta(1)}{\Delta}; \quad x_2 = \frac{\Delta(2)}{\Delta}; \quad x_3 = \frac{\Delta(3)}{\Delta}.$$

Дemak, Δ, $\Delta(1)$, $\Delta(2)$, $\Delta(3)$ larni hisoblaymiz:

$$\Delta = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 1 & -1 \\ 2 & 4 & 1 \end{vmatrix} = 1 - 12 + 4 - 6 + 4 - 2 = 9 - 20 = -11;$$

$$\Delta(1) = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ 3 & 4 & 1 \end{vmatrix} = 1 + 24 + 6 - 9 + 4 + 4 = 30;$$

$$\Delta(2) = \begin{vmatrix} 1 & 1 & 3 \\ -1 & 2 & -1 \\ 2 & 3 & 1 \end{vmatrix} = 2 - 9 - 2 - 12 + 3 + 1 = 6 - 23 = -17.$$

$$\Delta(3)=\begin{vmatrix}1 & -2 & 1\\ -1 & 1 & 2\\ 2 & 4 & 3\end{vmatrix}=3-4-8-2-8-6=-25.$$

Bundan, $x_1=-\frac{30}{11}$; $x_2=\frac{17}{11}$; $x_3=\frac{25}{11}$.

Misol va mashqlar

1. Determinantni hisoblang:

1.1. $\begin{vmatrix}-2 & 5\\ -4 & 7\end{vmatrix}$.

1.2. $\begin{vmatrix}n+1 & n\\ n & n-1\end{vmatrix}$.

1.3. $\begin{vmatrix}1-3i & 2i\\ 4i^3 & 1+3i\end{vmatrix}$.

1.4. $\begin{vmatrix}\log_b a & 1\\ 1 & \log_a b\end{vmatrix}$.

1.5. $\begin{vmatrix}\mathrm{tg}\alpha & \sin\alpha\\ 2 & \cos\alpha\end{vmatrix}$.

1.6. $\begin{vmatrix}\frac{1-a^2}{1+a^2} & \frac{2a}{1+a^2}\\ \frac{-2a}{1+a^2} & \frac{1-a^2}{1+a^2}\end{vmatrix}$.

1.7. $\begin{vmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 1 & 3 & 6\end{vmatrix}$.

1.8. $\begin{vmatrix}2 & -2 & 4\\ -3 & 3 & -6\\ 5 & 1 & 0\end{vmatrix}$.

1.9. $\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}$.

1.10. $\begin{vmatrix}a+x & x & x\\ x & b+x & x\\ x & x & c+x\end{vmatrix}$.

1.11. $\begin{vmatrix}\sin\alpha & \sin 2\alpha & 1\\ \cos\alpha & 1+\cos\alpha & 1\\ \mathrm{tg}\alpha & 2\sin\alpha & 1\end{vmatrix}$.

1.12. $\begin{vmatrix}1 & -1 & \varepsilon\\ \varepsilon & \varepsilon^2 & 1\\ -1 & \varepsilon & 1\end{vmatrix}$, $\varepsilon=\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}$.

1.13. $\begin{vmatrix} \alpha^2+1 & \alpha\beta & \alpha\gamma \\ \alpha\beta & \beta^2+1 & \beta\gamma \\ \alpha\gamma & \beta\gamma & \gamma^2+1 \end{vmatrix}$.

1.14. $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$, $\alpha, \beta, \gamma \in \{x \mid x^3 + px + q = 0\}$.

2. Determinantning quyidagi xossalarini isbotlang:

2.1. Nol satr yoki ustunga ega kvadrat matritsaning determinanti nolga teng.

2.2. Diagonal matritsaning determinanti asosiy diagonal elementlari ko'paytmasiga teng.

2.3. Uchburchak matritsaning determinanti asosiy diagonal elementlari ko'paytmasiga teng.

2.4. Kvadrat matritsa va unga transponirlangan matritsalar determinantlari teng.

2.5. Kvadrat matritsaning ikkita satr (ustun)lari o'rnini almashtirish natijasida determinant ishorasi o'zgaradi.

2.6. Ikkita bir xil satr (ustun)ga ega kvadrat matritsa determinanti nolga teng.

2.7. A kvadrat matritsaning biror-bir satr (ustun) elementlarini noldan farqli λ skalarga ko'paytirilsa, u holda A matritsaning determinanti λ skalarga ko'paytiriladi.

2.8. Qandaydir ikkita satr (ustun)lari proporsional bo'lgan kvadrat matritsaning determinanti nolga teng.

2.9. Kvadrat matritsa i- qatori (ustuni)ning har bir elementi m ta qo'shiluvchilardan iborat bo'lsa, bunday kvadrat matritsaning determinanti m ta determinantlar yig'indisidan iborat bo'lib, birinchi determinant i- qatori (ustuni)da birinchi, ikkinchi determinantda ikkinchi qo'shiluvchilar va h.k. boshqa qatorlar A matritsanikidek bo'ladi.

2.10. Kvadrat matritsaning biror-bir satr (ustun)iga noldan farqli skalarga ko'paytirilgan boshqa satr (ustun)ni qo'shish natijasida determinant o'zgarmaydi.

2.11. Kvadrat matritsaning biror-bir satr (ustun)iga qolgan satr (ustun)lar chiziqli kombinatsiyasini qo‘shish natijasida determinant o‘zgarmaydi.

2.12. Kvadrat matritsaning oxirgisidan boshqa barcha qatorlariga undan keyingi qator qo‘shilsa, uning determinanti o‘zgarmaydi.

2.13. Kvadrat matritsaning ikkinchi ustunidan boshlab har bir ustuniga undan oldingi ustun qo‘shilsa, uning determinanti o‘zgarmaydi.

2.14. Kvadrat matritsaning biror-bir satri (ustuni) qolganlarining chiziqli kombinatsiyasidan iborat bo‘lsa, uning determinanti nolga teng.

2.15. Har qanday elementar matritsaning determinanti noldan farqli.

2.16. Kvadrat matritsalar ko‘paytmasining determinanti berilgan matritsalar determinantlari ko‘paytmasiga teng.

2.17. Kvadrat matritsaning determinanti nolga teng bo‘lishi uchun uning satr (ustun)lari chiziqli bog‘langan bo‘lishi zarur va yetarli.

3. Determinant xossalaridan foydalanib quyidagilarni isbotlang:

3.1. $$\begin{vmatrix} (a+b)^2 & c^2 & c^2 \\ a^2 & (b+c)^2 & a^2 \\ b^2 & b^2 & (c+a)^2 \end{vmatrix} = 2abc(a+b+c)^3.$$

3.2. $$\begin{vmatrix} \frac{1}{a+x} & \frac{1}{a+y} & 1 \\ \frac{1}{b+x} & \frac{1}{b+y} & 1 \\ \frac{1}{c+x} & \frac{1}{c+y} & 1 \end{vmatrix} = \frac{(a-b)(a-c)(b-c)(x-y)}{(a+x)(b+x)(c+x)(a+y)(b+y)(c+y)}.$$

3.3. $\begin{vmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{vmatrix} = (a^2 + b^2 + c^2 + d^2).$

3.4. $\begin{vmatrix} -1 & 1 & 1 & 1 & x \\ 1 & -1 & 1 & 1 & y \\ 1 & 1 & -1 & 1 & z \\ 1 & 1 & 1 & -1 & t \\ x & y & z & t & 0 \end{vmatrix} =$

$= -4(x^2 + y^2 + z^2 + t^2 - 2(xy + xz + xt + yz + yt + zt)).$

4. Quyidagi determinantlarni hisoblang:

4.1. $\begin{vmatrix} 1 & 2 & 3 & \dots & n \\ -1 & 0 & 3 & \dots & n \\ -1 & -2 & 0 & \dots & n \\ . & . & . & . & . \\ -1 & -2 & -3 & \dots & 0 \end{vmatrix}.$ 4.2. $\begin{vmatrix} 1 & n & n & \dots & n \\ n & 2 & n & \dots & n \\ n & n & 3 & \dots & n \\ . & . & . & . & . \\ n & n & n & \dots & n \end{vmatrix}.$

4.3. $\begin{vmatrix} 1+x_1 & 1 & 1 & \dots & 1 \\ 1 & 1+x_2 & 1 & \dots & 1 \\ 1 & 1 & 1+x_3 & \dots & 1 \\ . & . & . & . & . \\ 1 & 1 & 1 & \dots & 1+x_n \end{vmatrix}.$

4.4. $\begin{vmatrix} x_1 & a_{12} & a_{13} & \dots & a_{1n} \\ x_1 & x_2 & a_{23} & \dots & a_{2n} \\ x_1 & x_2 & x_3 & \dots & a_{3n} \\ . & . & . & . & . \\ x_1 & x_2 & x_3 & \dots & x_n \end{vmatrix}.$

4.5. $\begin{vmatrix} 1 & 2 & 3 & \dots & n \\ 1 & x+1 & 3 & \dots & n \\ 1 & 2 & x+1 & \dots & n \\ . & . & . & . & . \\ 1 & 2 & 3 & \dots & x+1 \end{vmatrix}.$

4.6. $\begin{vmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & 2-a & 1 & \dots & 1 \\ 1 & 1 & 3-a & \dots & 1 \\ . & . & . & . & . \\ 1 & 1 & 1 & \dots & n+1-a \end{vmatrix}.$

4.7. $\begin{vmatrix} 2 & 1 & 0 & \dots & 0 \\ 1 & 2 & 1 & \dots & 0 \\ 0 & 1 & 2 & \dots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \dots & 2 \end{vmatrix}.$

4.8. $\begin{vmatrix} 3 & 2 & 0 & \dots & 0 \\ 1 & 3 & 2 & \dots & 0 \\ 0 & 1 & 3 & \dots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \dots & 3 \end{vmatrix}.$

4.9. $\begin{vmatrix} x+a_1 & a_2 & a_3 & \dots & a_n \\ a_1 & x+a_2 & a_3 & \dots & a_n \\ a_1 & a_2 & x+a_3 & \dots & a_n \\ . & . & . & . & . \\ a_1 & a_2 & a_3 & \dots & x+a_n \end{vmatrix}.$

4.10. $\begin{vmatrix} 1 & x & x^2 & \dots & x^n \\ a_{11} & 1 & x & \dots & x^{n-1} \\ a_{21} & a_{22} & 1 & \dots & x^{n-2} \\ . & . & . & . & . \\ a_{n1} & a_{n2} & a_{n3} & \dots & 1 \end{vmatrix}.$

5. CHTSni Kramer formulalari yordamida yeching:

5.1. $\begin{cases} x_1 + x_2 - x_3 = 2, \\ -2x_1 + x_2 + x_3 = 3, \\ x_1 - x_2 + x_3 = 6. \end{cases}$

5.2. $\begin{cases} 3x_1 - x_2 = 5, \\ -2x_1 + x_2 + x_3 = 0, \\ 2x_1 - x_2 + 4x_3 = 15. \end{cases}$

5.3. $\begin{cases} 2x_1 + 2x_2 - x_3 = 4, \\ 3x_1 + x_2 - x_3 = 7, \\ x_1 + x_2 - 2x_3 = 3. \end{cases}$

5.4. $\begin{cases} 5x_1 + 4x_3 = 1, \\ x_1 - x_2 + 2x_3 = 0, \\ 4x_1 + x_2 + 2x_3 = 1. \end{cases}$

5.5. $\begin{cases} x_1 + x_2 + 2x_3 + 3x_4 = 1, \\ 3x_1 - x_2 - x_3 - 2x_4 = -4, \\ 2x_1 + 3x_2 - x_3 - x_4 = -6, \\ x_1 + 2x_2 + 3x_3 - x_4 = -4. \end{cases}$

5.6. $\begin{cases} 2x_1 + x_2 + x_3 - x_4 = 1, \\ 3x_1 - x_2 + x_3 + x_4 = 2, \\ 4x_1 + 2x_2 + x_3 - x_4 = 1, \\ x_4 = 2. \end{cases}$

5.7. $\begin{cases} 2x_1 - x_2 - 6x_3 + 3x_4 = -1, \\ 7x_1 - 4x_2 + 2x_3 - 15x_4 = -32, \\ x_1 - 2x_2 - 4x_3 + 9x_4 = 5, \\ x_1 - x_2 + 2x_3 - 6x_4 = -8. \end{cases}$

5.8. $\begin{cases} 2x_1 - 3x_2 + 5x_3 + x_4 = -7, \\ -x_1 + x_2 + 4x_3 - 2x_4 = 1, \\ 5x_1 + 2x_2 - 3x_3 + 6x_4 = 5, \\ -2x_1 + 5x_2 + x_3 - 7x_4 = 5. \end{cases}$

5.9. $\begin{cases} x_1 + 2x_2 - 3x_3 - x_4 = -8, \\ 2x_1 - x_2 + 2x_3 - x_4 = 2, \\ 4x_1 + 3x_2 - x_3 - x_4 = 3, \\ x_1 + 2x_2 + x_3 + x_4 = 12. \end{cases}$

5.10. $\begin{cases} 5x_1 + 2x_2 + x_3 + 4x_4 = 4, \\ 4x_1 + 6x_2 + 3x_3 + 7x_4 = 1, \\ 3x_1 + x_2 + 2x_3 + 4x_4 = 1, \\ 2x_1 + 3x_2 + 4x_3 + 5x_4 = -2. \end{cases}$

6. Bir jinsli n noma'lumli n ta chiziqli tenglamalar sistemasining nolmas yechimga ega bo'lishi uchun, uning determinanti nolga teng bo'lishi zarur va yetarli ekanligini isbotlang.

7. Har qanday kvadrat matritsa uchun quyidagi shartlar teng kuchli ekanligini isbotlang:

7.1. $|A| \neq 0$.

7.2. Matritsaning satr (ustun)lari chiziqli erkli.

7.3. A matritsa teskarilanuvchi.

7.4. A matritsa elementar matritsalar yordamida ifodalanadi.

8. Matritsa rangini minorlar yordamida toping:

8.1. $\begin{pmatrix} 2 & -1 & 3 & -2 & 4 \\ 4 & -2 & 5 & 1 & 7 \\ 2 & -1 & 1 & 8 & 2 \end{pmatrix}$. 8.2. $\begin{pmatrix} 8 & 2 & 2 & -1 & 1 \\ 1 & 7 & 4 & -2 & 5 \\ -2 & 4 & 2 & -1 & 3 \end{pmatrix}$.

8.3. $\begin{pmatrix} 1 & 7 & 7 & 9 \\ 7 & 5 & 1 & -1 \\ 4 & 2 & -1 & -3 \\ -1 & 1 & 3 & 5 \end{pmatrix}$. 8.4. $\begin{pmatrix} 1 & 3 & 5 & -1 \\ 2 & -1 & -3 & 4 \\ 5 & 1 & -1 & 7 \\ 7 & 7 & 9 & 1 \end{pmatrix}$.

8.5. $\begin{pmatrix} 4 & 1 & 7 & -5 & 1 \\ 0 & -7 & 1 & -3 & -5 \\ 3 & 4 & 5 & -3 & 2 \\ 2 & 5 & 3 & -1 & 3 \end{pmatrix}$. 8.6. $\begin{pmatrix} -6 & 4 & 8 & -1 & 6 \\ -5 & 2 & 4 & 1 & 3 \\ 7 & 2 & 4 & 1 & 3 \\ 2 & 4 & 8 & -7 & 6 \\ 3 & 2 & 4 & -5 & 3 \end{pmatrix}$.

8.7. $\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$. 8.8. $\begin{pmatrix} 1 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 1 & 0 & \dots & 0 & 0 \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & \dots & 1 & 1 \\ 1 & 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix}$.

9. Matritsaga teskari matritsani biriktirilgan matritsa yordamida toping:

9.1. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. 9.2. $\begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix}$.

9.3. $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$. 9.4. $\begin{pmatrix} 2-3i & 1-i \\ 3+i & 5+4i \end{pmatrix}$.

9.5. $\begin{pmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 5 & -2 & -3 \end{pmatrix}$. 9.6. $\begin{pmatrix} 7 & 9 & 2 \\ 2 & -2 & 6 \\ 5 & -6 & 3 \end{pmatrix}$.

9.7. $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$. 9.8. $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}$.

10. Berilgan matritsalarni elementar matritsalar ko'paytmasi ko'rinishida ifodalang:

10.1. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. 10.2. $\begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix}$.

10.3. $\begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix}$. 10.4. $\begin{pmatrix} -1 & -1 \\ 2 & 3 \end{pmatrix}$.

10.5. $\begin{pmatrix} -3 & -5 \\ 2 & 4 \end{pmatrix}$. 10.6. $\begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix}$.

Takrorlash uchun savollar

1. Determinantning asosiy xossalarini ayting.
2. n-tartibli minor deb nimaga aytiladi?.
3. Determinantni algebraik to'ldiruvchi yordamida aniqlash jarayonini tushuntiring.
4. Determinant nolga teng bo'lishining zarur va yetarli shartini ayting.
5. Algebraik to'ldiruvchilar yordamida teskari matritsani topish jarayonini tushuntiring.
6. CHTSni Kramer qoidasi bilan yechish usulini tushuntiring.
7. n noma'lumli n ta tenglamadan iborat bir jinsli chiziqli tenglamalar sistemasi qachon yagona yechimga ega?

VIII MODUL. VEKTOR FAZOLAR

19-§. Vektor fazo. Fazoostilar kesishmasi, yig'indisi.

✓ **Asosiy tushunchalar**: vektor fazo, fazoosti, fazoostilar kesishmasi, fazoostilar yig'indisi, fazoostilar to'g'ri yig'indisi, vektor fazo bazisi, vektor fazo o'lchovi.

Bo'sh bo'lmagan $V = \{\bar{x}, \bar{y}, \bar{z}, ...\}$ to'plam va $\mathcal{F} = \{\alpha, \beta, \gamma, ...\}$ maydon berilgan bo'lib, quyidagi aksiomalar bajarilsa, u holda V to'plam $\mathcal{F}$ sonlar maydoni ustiga qurilgan *vektor fazo* deyiladi:

V – additiv abel gruppa;

$(\alpha \cdot \beta)\bar{x} = \alpha(\beta\bar{x}) \quad (\forall \bar{x} \in V, \forall \alpha, \beta \in F);$

$\alpha(\bar{x} + \bar{y}) = \alpha\bar{x} + \alpha\bar{y} \quad (\forall \bar{x}, \bar{y} \in V, \forall \alpha \in F);$

$(\alpha + \beta)\bar{x} = \alpha\bar{x} + \beta\bar{x} \quad (\forall \bar{x} \in V, \forall \alpha, \beta \in F);$

$1 \cdot \bar{x} = \bar{x} \quad (\forall \bar{x} \in V, 1 \in F).$

$\mathcal{F}$ maydon ustida aniqlangan V vektor fazoning biror L to'plamostisi V da aniqlangan algebraik amallarga nisbatan vektor fazosini tashkil etsa, u holda L ga V fazoning *fazoostisi* deyiladi.

V vektor fazoning biror L to'plamostisi shu vektor fazoning fazoostisi bo'lishi uchun quyidagi ikkita shartning bajarilishi zarur va yetarli:

a) $(\forall \bar{x}, \bar{y} \in L) \ (\bar{x} - \bar{y}) \in L;$

b) $(\forall \bar{x} \in L, \forall \alpha \in F) \ \alpha\bar{x} \in L.$

Agar U_1, ..., U_n lar V vektor fazoning fazoostilari bo'lsa, u holda $U = U_1 \cap U_2 \cap ... \cap U_n$ ga U_1, ..., U_n *fazoostilarning kesishmasi* deyiladi.

$\bar{x}_1 \in U_1, \bar{x}_2 \in U_2, ..., \bar{x}_n \in U_n$ bo'lganda $\bar{x}_1 + \bar{x}_2 + ... + \bar{x}_n$ ko'rinishdagi barcha yig'indilar to'plamiga U_1, ..., U_n *fazoostilar yig'indisi* deyiladi va u $U_1 + U_2 + ... + U_n$ ko'rinishda belgilanadi.

Agar V vektor fazoning chiziqli bog'lanmagan

$$\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_n \qquad (1)$$

vektorlar sistemasi mavjud bo'lsaki, V ning qolgan barcha vektorlari (1) sistema orqali chiziqli ifodalansa, u holda (1) vektorlar sistemasi *V vektor fazoning bazisi* deyiladi.

V vektor fazoning bazislaridagi vektorlar soni *V vektor fazoning o'lchovi* deyiladi.

V fazoning o'lchovi $\dim V$ orqali belgilanadi.

1-misol. $V=\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a,b,c,d \in R\right.$ to'plamning R maydon ustida chiziqli fazo tashkil etishini va uni bazisi, o'lchovini aniqlang.

Yechish. Berilgan $V \neq \varnothing$ to'plamda qo'shish va skalarni to'plam elementiga ko'paytirish amallarini aniqlaymiz:

1) $\forall A_1 A_2 \in V$, $A_1 = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, $A_2 = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$ larga yagona

$A_1 + A_2 = \begin{pmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{pmatrix}$ ni mos qo'yamiz. Bu yerda

$a_1, a_2, b_1, b_2, c_1, c_2, d_1, d_2 \in R$ va haqiqiy sonlar to'plamida qo'shish amali aniqlanganligi sababli $a_1 + a_2$, $b_1 + b_2$, $c_1 + c_2$, $d_1 + d_2 \in R$, ya'ni A_1+A_2 matritsa V ning elementi. Bundan tashqari ikkita berilgan haqiqiy sonning yig'indisi yagona uchinchi haqiqiy son ekanligidan berilgan ikkita kvadrat matritsalar yig'indisi bo'lgan uchinchi kvadrat matritsaning yagonaligi kelib chiqadi.

2) V to'plamning ixtiyoriy $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ elementi va ixtiyoriy

$\alpha \in R$ uchun $\omega_\alpha(A) = \alpha A = \alpha \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \alpha a & \alpha b \\ \alpha c & \alpha d \end{pmatrix}$ matritsani hosil qilamiz. Haqiqiy sonlar to'plamida ko'paytirish amali aniqlanganligi uchun $\alpha a, \alpha b, \alpha c, \alpha d \in R$, ya'ni $\alpha A \in V$. Hosil qilingan $V = < V; +, \{\omega_\alpha \,|\alpha \in R\} >$ algebra chiziqli vektor fazo tashkil etishini isbotlaymiz. Buning uchun qo'shish, skalarni matritsaga ko'paytirish amallarining quyidagi xossalari bajarilishini isbotlaymiz:

1) $\forall A_1, A_2 \in V,$ $\quad A_1 + A_2 = A_2 + A_1;$

2) $\forall A_1, A_2, A_3 \in V,$ $\quad A_1 + (A_2 + A_3) = (A_1 + A_2) + A_3;$

3) $\forall A \in V \wedge \exists 0 \in V,$ $\quad A + 0 = A;$

4) $\forall A \in V \wedge \exists A' \in V,$ $\quad A + A' = 0;$

5) $\forall \alpha, \beta \in R \wedge \forall A \in V,$ $\quad (\alpha + \beta) A = \alpha A + \beta A;$

6) $\forall \alpha, \beta \in R \wedge \forall A \in V,$ $\quad (\alpha\beta) A = \alpha(\beta A);$

7) $\forall \alpha \in R \wedge \forall A_1, A_2 \in V,$ $\quad \alpha(A_1 + A_2) = \alpha A_1 + \alpha A_2;$

8) $\forall A \in V,$ $\quad 1 \cdot A = A.$

Isbot. **1.** V to'plamning ixtiyoriy $A_1 = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}, A_2 = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$ elementlari uchun $A_1 + A_2 =$ | matritsalarni qo'shish amali ta'rifiga ko'ra | $= \begin{pmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{pmatrix} =$ | R da qo'shish amali kommutativ ekanligidan | $=$

$= \begin{pmatrix} a_2 + a_1 & b_2 + b_1 \\ c_2 + c_1 & d_2 + d_1 \end{pmatrix} =$ | V da qo'shish amali ta'rifiga ko'ra | $=$

$$= \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} + \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} = B + A.$$

Demak, V to'plamda aniqlangan qo'shish amali kommutativ va $< V; + >$ additiv abel gruppoid.

2. V to'plamning ixtiyoriy $A_1 = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}, A_2 = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix},$

$A_3 = \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$ elementlari uchun

$$A_1+(A_2+A_3)=\begin{pmatrix}a_1 & b_1\\ c_1 & d_1\end{pmatrix}+\left(\begin{pmatrix}a_2 & b_2\\ c_2 & d_2\end{pmatrix}+\begin{pmatrix}a_3 & b_3\\ c_3 & d_3\end{pmatrix}\right)=\left|\begin{matrix}V \text{ da qo‘shish}\\ \text{amali ta’rifiga}\\ \text{ko‘ra}\end{matrix}\right|=$$

$$=\begin{pmatrix}a_1 & b_1\\ c_1 & d_1\end{pmatrix}+\begin{pmatrix}a_2+a_3 & b_2+b_3\\ c_2+c_3 & d_2+d_3\end{pmatrix}=\left|\begin{matrix}V \text{ da qo‘shish amali}\\ \text{ta’rifiga ko‘ra}\end{matrix}\right|=$$

$$=\begin{pmatrix}a_1+(a_2+a_3) & b_1+(b_2+b_3)\\ c_1+(c_2+c_3) & d_1+(d_2+d_3)\end{pmatrix}=\left|\begin{matrix}V \text{ da qo‘shish amalining}\\ \text{assotsiativligidan}\end{matrix}\right|=$$

$$=\begin{pmatrix}(a_1+a_2)+a_3 & (b_1+b_2)+b_3\\ (c_1+c_2)+c_3 & (d_1+d_2)+d_3\end{pmatrix}=\left|\begin{matrix}V \text{ da qo‘shish amali}\\ \text{ta’rifiga ko‘ra}\end{matrix}\right|=$$

$$=\begin{pmatrix}a_1+a_2 & b_1+b_2\\ c_1+c_2 & d_1+d_2\end{pmatrix}+\begin{pmatrix}a_3 & b_3\\ c_3 & d_3\end{pmatrix}=\left|\begin{matrix}V \text{ da qo‘shish amali}\\ \text{ta’rifiga ko‘ra}\end{matrix}\right|=(A+B)+C.$$

Demak, $<V;+>$ additiv abel yarimgruppa.

3. V to‘plamdan olingan ixtiyoriy $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ element uchun shu to‘plamda yagona $0=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$ element mavjudki, $A+0=0+A=0$.

Demak, $<V;+,0>$ – additiv abel manoid.

4. V to‘plamdan olingan har qanday $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ element uchun shunday $A'\in V$ element mavjudki, $A+A'=0$. Bu yerda $A'=\begin{pmatrix}-a & -b\\ -c & -d\end{pmatrix}$ ekanligi har qanday haqiqiy son uchun qarama-qarshisi mavjudligidan kelib chiqadi. Yoki bo‘lmasa, $(-1)A=-A$ ni A ga qarama-qarshi element sifatida hosil qilish mumkin.

Demak, $<V;+,-,0>$ – additiv abel gruppa ekan.

5. $\forall\alpha,\beta\in R$ va $\forall A\in V$ uchun

$$(\alpha+\beta)A=(\alpha+\beta)\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\left|\begin{matrix}V \text{ da skalarni matritsaga ko‘pay-}\\ \text{tirish amali ta’rifiga ko‘ra}\end{matrix}\right|=$$

$$=\begin{pmatrix}(\alpha+\beta)a & (\alpha+\beta)b\\ (\alpha+\beta)c & (\alpha+\beta)d\end{pmatrix}=\left|\begin{matrix}R \text{ da ko‘paytirishning qo‘shishga}\\ \text{nisbatan distributivligidan}\end{matrix}\right|=$$

$$=\begin{pmatrix}\alpha a+\beta a & \alpha b+\beta b\\ \alpha c+\beta c & \alpha d+\beta d\end{pmatrix}=\left|\begin{matrix}V \text{ da qo‘shish}\\ \text{amali ta’rifiga ko‘ra}\end{matrix}\right|=$$

$$=\begin{pmatrix}\alpha a & \alpha b\\ \alpha c & \alpha d\end{pmatrix}+\begin{pmatrix}\beta a & \beta b\\ \beta c & \beta d\end{pmatrix}=\left|\begin{matrix}V \text{ da skalarni matritsaga ko‘paytirish}\\ \text{amali ta’rifiga ko‘ra}\end{matrix}\right|=$$

$$=\alpha\begin{pmatrix}a & b\\ c & d\end{pmatrix}+\beta\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\alpha A+\beta A.$$

6. $\forall\alpha,\beta\in R$ va $\forall A\in V$ lar uchun

$$(\alpha\beta)A=(\alpha\beta)\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\left|\begin{matrix}V \text{ da skalarni matritsaga ko‘paytirish}\\ \text{amali ta’rifiga ko‘ra}\end{matrix}\right|=$$

$$=\begin{pmatrix}(\alpha\beta)a & (\alpha\beta)b\\ (\alpha\beta)c & (\alpha\beta)d\end{pmatrix}=\left|\begin{matrix}R \text{ da ko‘paytirish}\\ \text{amali assotsiativligidan}\end{matrix}\right|=$$

$$=\begin{pmatrix}\alpha(\beta a) & \alpha(\beta b)\\ \alpha(\beta c) & \alpha(\beta d)\end{pmatrix}=\left|\begin{matrix}V \text{ da skalarni matritsaga ko‘paytirish}\\ \text{amali ta’rifiga ko‘ra}\end{matrix}\right|=$$

$$=\alpha\begin{pmatrix}\beta a & \beta b\\ \beta c & \beta d\end{pmatrix}=\left|\begin{matrix}V \text{ da skalarni matritsaga ko‘paytirish}\\ \text{amali ta’rifiga ko‘ra}\end{matrix}\right|=$$

$$=\alpha\left(\beta\begin{pmatrix}a & b\\ c & d\end{pmatrix}\right)=\alpha(\beta A).$$

7. $\forall\alpha\in R\wedge\forall A_1, A_2\in V$ lar uchun

$$\alpha(A_1+A_2)=\alpha\left(\begin{pmatrix}a_1 & b_1\\ c_1 & d_1\end{pmatrix}+\begin{pmatrix}a_2 & b_2\\ c_2 & d_2\end{pmatrix}\right)=\left|\begin{matrix}V \text{ da qo‘shish va skalarni}\\ \text{ko‘paytirish amallarining}\\ \text{ta’rifiga ko‘ra}\end{matrix}\right|=$$

$$=\begin{pmatrix}\alpha(a_1+a_2) & \alpha(b_1+b_2)\\ \alpha(c_1+c_2) & \alpha(d_1+d_2)\end{pmatrix}=\left|\begin{matrix}R \text{ da ko‘paytirishning qo‘shishga}\\ \text{nisbatan distributivligidan}\end{matrix}\right|=$$

$$= \begin{pmatrix} \alpha a_1 + \alpha a_2 & \alpha b_1 + \alpha b_2 \\ \alpha c_1 + \alpha c_2 & \alpha d_1 + \alpha d_2 \end{pmatrix} = \left| \begin{array}{l} V \text{ da qo‘shish amalining} \\ \text{ta’rifiga ko‘ra} \end{array} \right| =$$

$$= \begin{pmatrix} \alpha a_1 & \alpha b_1 \\ \alpha c_1 & \alpha d_1 \end{pmatrix} + \begin{pmatrix} \alpha a_2 & \alpha b_2 \\ \alpha c_2 & \alpha d_2 \end{pmatrix} = \left| \begin{array}{l} V \text{ da skalarni matritsaga ko‘paytirish} \\ \text{amali ta’rifiga ko‘ra} \end{array} \right| =$$

$$= \alpha A_1 + \alpha A_2 .$$

8. V to‘plamning har qanday $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ elementi va $1 \in R$ skalarlar uchun $1 \cdot A = A$ ekanligi V da skalarni ko‘paytirish amali ta’rifi va R to‘plamda $1 \cdot a = a$ $(\forall a \in R)$ ekanligidan kelib chiqadi.

Demak, $V = \langle V; +, \{\omega_\lambda \mid \lambda \in R\} \rangle$ chiziqli vektor fazo ekan.

9. Bu fazoning bazisini aniqlaymiz. Chiziqli vektor fazo bazisi ta’rifiga ko‘ra V to‘plamning ixtiyoriy $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ elementini chiziqli ifodalovchi chiziqli erkli sistemani topamiz. Bunday sistema sifatida $A_a = \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$, $A_b = \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}$, $A_c = \begin{pmatrix} 0 & 0 \\ c & 0 \end{pmatrix}$, $A_d = \begin{pmatrix} 0 & 0 \\ 0 & d \end{pmatrix}$ sistemani olsak, u holda $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = A_a + A_b + A_c + A_d$ ekanligi ravshan. Bazis vektor sifatida A_a, A_b, A_c, A_d bazisdagi a, b, c, d haqiqiy sonlar o‘rniga 0 dan farqli ixtiyoriy haqiqiy sonni qo‘yish natijasida V fazoning boshqa bazislarini hosil qilish mumkin.

Demak, $V = \langle V; +, \{\omega_\lambda \mid \lambda \in R\} \rangle$ vektor fazoning bazisi cheksiz ko‘p.

10. Chiziqli vektor fazoning o‘lchovi ta’rifiga ko‘ra $V = \langle V; +, \{\omega_\lambda \mid \lambda \in R\} \rangle$ fazoning ixtiyoriy bazisidagi vektorlar soni uning o‘lchovidir, ya’ni $\dim V = 4$.

2-misol. $(a):\vec{a}_1=(1,\ 1,\ 2), \vec{a}_2=(0,\ 1,\ 1)$; $(b):\vec{b}_1=(2,\ 1,\ 3)$, $\vec{b}_2=(1,\ 1,\ 3)$, $\vec{b}_3=(0,\ 1,\ 1)$ vektorlar sistemalari tashkil etgan chiziqli fazolar, ularning yig'indisi, kesishmasining bazisi va o'lchovini aniqlang.

Yechish. 1) $(a):\vec{a}_1=(1,\ 1,\ 2), \vec{a}_2=(0,\ 1,\ 1)$ vektorlar sistemasi chiziqli erkli, rang(a)=2. Shuning uchun $L((a))$ chiziqli fazoning o'lchovi dim$L((a))$=2.

2) $(b):\vec{b}_1=(2,\ 1,\ 3)$, $\vec{b}_2=(1,\ 1,\ 3)$, $\vec{b}_3=(0,\ 1,\ 1)$ sistemani tekshiramiz:

$$\begin{matrix}\vec{b}_1\\ \vec{b}_2\\ \vec{b}_3\end{matrix}\begin{pmatrix}2 & 1 & 3\\ 1 & 1 & 3\\ 0 & 1 & 1\end{pmatrix}\sim\begin{matrix}\vec{b}_2\\ \vec{b}_3\\ \vec{b}_1-2\vec{b}_2\end{matrix}\begin{pmatrix}1 & 1 & 3\\ 0 & 1 & 1\\ 0 & -1 & -3\end{pmatrix}\sim\begin{matrix}\vec{b}_2\\ \vec{b}_3\\ \vec{b}_3+\vec{b}_1-2\vec{b}_2\end{matrix}\begin{pmatrix}1 & 1 & 3\\ 0 & 1 & 1\\ 0 & 0 & -2\end{pmatrix}.$$

Demak, (b) sistema chiziqli erkli. Bundan dim$L((b))$=3.

3) $L((a))$, $L((b))$ qism fazolar yig'indisining bazisini topish uchun ularning bazis vektorlaridan $\vec{a}_1=(1,\ 1,\ 2)$, $\vec{a}_2=(0,\ 1,\ 1)$, $\vec{b}_1=(2,\ 1,\ 3)$, $\vec{b}_2=(1,\ 1,\ 3)$, $\vec{b}_3=(0,\ 1,\ 1)$ sistemani hosil qilamiz va bu sistemaning bazis vektorlarini topamiz. Qaralayotgan misolda $L((b))=R^3$ bo'lganligi uchun $L((a))$, $L((b))$ qism fazolar yig'indisi ham R^3 dan iborat bo'ladi. U holda dim$(L((a))+L((b)))$=3. Agar

$$\dim(L((a))+L((b)))+\dim(L((a))\cap L((b)))=\dim L((a))+\dim L((b))$$

tenglikni e'tiborga olsak, u holda dim$L((a))\cap$dim$L((b))$ =2 ekanligi kelib chiqadi.

Demak, dim$L((a))$=2, dim$L((b))$=3, dim$L((a))$+dim $L((b))$=3,

$$\dim(L((a))\cap L((b)))=2.$$

Misol va mashqlar

1. Berilgan to'plamlarning chiziqli fazo tashkil etishini isbotlang va uning bazisi, o'lchovini aniqlang:

1.1. $V=\left\{\begin{pmatrix}a & 0\\ 0 & 0\end{pmatrix} \mid a\in R\right\}.$

1.2. $V = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a,b,c,d \in R \right\}.$

1.3. $V = \left\{ \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \mid a_{ij} \in R \wedge i = \overline{1,3} \wedge j = \overline{1,3} \right\}.$

1.4. $V = \left\{ \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \mid a_{ij} \in R \wedge i = \overline{1,3} \wedge j = \overline{1,3} \right\}.$

1.5. $V = \left\{ \begin{pmatrix} a_{11} & 0 & a_{13} \\ a_{21} & 0 & a_{23} \\ a_{31} & 0 & a_{33} \end{pmatrix} \mid a_{ij} \in R \wedge i = \overline{1,3} \wedge j = 1,3 \right\}.$

1.6. $V = \left\{ \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \mid a_{ij} \in R \wedge i = 1,3 \wedge j = \overline{1,3} \right\}.$

1.7. $V = \left\{ \begin{pmatrix} 0 & a_{12} & a_{13} \\ a_{21} & 0 & a_{23} \\ a_{31} & a_{32} & 0 \end{pmatrix} \mid a_{ij} \in R \wedge i = \overline{1,3} \wedge j = \overline{1,3} \right\}.$

2. Kompleks sonlar maydoni tashkil etgan vektor fazoning bazisi va o‘lchovini aniqlang.

3. V_n arifmetik vektor fazoning quyidagi vektorlar sistemasi tashkil etgan fazoostilari bazisi va o‘lchovini aniqlang:

3.1. Birinchi va keyingi koordinatalari teng bo‘lgan barcha vektorlar.

3.2. Koordinatalari yig‘indisi nolga teng bo‘lgan barcha vektorlar.

3.3. Juft o‘rindagi koordinatalari nolga teng bo‘lgan barcha vektorlar.

3.4. Juft o‘rindagi koordinatalari teng bo‘lgan barcha vektorlar.

3.5. Koordinatalari teng bo‘lgan barcha vektorlar.

3.6. Har bir koordinatasi o'zidan oldingi koordinataning qarama-qarshisiga teng bo'lgan barcha vektorlar.

4. Quyidagi to'plamlarning qaysilari vektor fazo tashkil etadi:

4.1. R_n vektor fazoning butun koeffitsiyentli vektorlar to'plami.

4.2. Tekislikning koordinatalar o'qlaridan birida joylashgan barcha vektorlar to'plami.

4.3. Tekislikning boshi koordinatalar boshida oxiri berilgan to'g'ri chiziqda yotuvchi barcha vektorlari to'plami.

4.4. Tekislikning boshi va oxiri bir to'g'ri chiziqda yotuvchi vektorlari to'plami.

4.5. R_n vektor fazoning barcha koordinatalari yig'indisi 1 ga teng bo'lgan barcha vektorlar to'plami.

4.6. R_n vektor fazoning barcha koordinatalari yig'indisi 0 ga teng bo'lgan barcha vektorlar to'plami.

5. Fazoostining quyidagi xossalarini isbotlang:

5.1. Agar V fazo $\mathcal{F}$ maydon ustida vektor fazo bo'lsa, u holda uning ixtiyoriy fazoostisi $\mathcal{F}$ maydon ustidagi vektor fazo bo'ladi.

5.2. Agar U fazo V vektor fazoning fazoosti va V fazo W vektor fazoning fazoosti bo'lsa, u holda U fazo W vektor fazoning fazoosti bo'ladi.

5.3. V vektor fazoning ixtiyoriy fazoostilarining kesishmasi V vektor fazoning qism fazosi bo'ladi.

6. Quyidagi vektorlar sistemasi tashkil etgan fazoostilar bazisi va o'lchovini aniqlang:

6.1. $\vec{a}_1(1,0,0,-1)$, $\vec{a}_2(2,1,1,0)$, $\vec{a}_3(1,1,1,1)$, $\vec{a}_4(0,1,2,3)$.

6.2. $\vec{a}_1(0,-1,0,-1)$, $\vec{a}_2(1,2,1,2)$, $\vec{a}_3(1,1,1,1)$, $\vec{a}_4(2,3,1,0)$, $\vec{a}_5(4,5,3,2)$.

6.3. $\vec{a}_1(1,1,1,1,0)$, $\vec{a}_2(1,1,-1,-1,-1)$, $\vec{a}_3(2,2,0,0,-1)$, $\vec{a}_4(1,1,5,5,2)$, $\vec{a}_5(1,-1,-1,0,0)$.

6.4. $\vec{a}_1(1,1,0,0)$, $\vec{a}_2(0,1,1,0)$, $\vec{a}_3(0,0,1,1)$.

6.5. $\vec{a}_1(1,-1,-1)$, $\vec{a}_2(-2,2,2)$, $\vec{a}_3(1,0,1)$, $\vec{a}_4(2,3,1)$, $\vec{a}_5(5,3,2)$.

6.6. $\vec{a}_1(-4,3,-2,1)$, $\vec{a}_2(2,2,2,2)$, $\vec{a}_3(-0,1,-1,1)$, $\vec{a}_4(3,3,1,1)$, $\vec{a}_5(0,1,2,3)$.

7. Fazoostilar yig'indisi va to'g'ri yig'indisining quyidagi xossalarini isbotlang:

7.1. Agar L va U lar V vektor fazoning fazoostilari bo'lsa, u holda $L+U = U+L$ bo'ladi.

7.2. Agar L, U, W lar V vektor fazoning fazoostilari bo'lsa, u holda $L+(U+W) = (L+U)+W$ bo'ladi.

7.3. Agar L fazoosti V vektor fazoning fazoostisi bo'lsa, u holda $L+V= V$ bo'ladi.

7.4. L va U lar V fazoning fazoostilari bo'lsa, u holda $L+U$ yig'indi to'g'ri yig'indi bo'lishi uchun $L\cap U=\{\bar{0}\}$ bo'lishi zarur va yetarli.

8. Bektorlarning (a) va (b) sistemalari tashkil etgan chiziqli fazolar, ularning yig'indisi, kesishmasining bazisi va o'lchovini aniqlang:

8.1. (a): $\vec{a}_1(1, 2, 3)$, $\vec{a}_2(0, 1, 1)$;
(b): $\vec{b}_1(1, 0, 1)$, $\vec{b}_2(2, 1, 1)$.

8.2. (a): $\vec{a}_1(1, 2, 3, 3)$, $\vec{a}_2(0, 1, 1, 5)$;
(b): $\vec{b}_1(1, 0, 7, 1)$, $\vec{b}_2(2, 0, 1, 1)$.

8.3. (a): $\vec{a}_1(1, 2, 3)$, $\vec{a}_2(0, 1, 1)$, $\vec{a}_3(-1, 4, 3)$;
(b): $\vec{b}_1(1, 0, 1)$, $\vec{b}_2(2, 1, 1)$, $\vec{b}_3(-3, -2, -1)$.

8.4. (a): $\vec{a}_1(1, 2, 3, 6)$, $\vec{a}_2(0, 1, 1, 7)$, $\vec{a}_3(-1, 4, 3, 8)$;
(b): $\vec{b}_1(1, 0, 1, -4)$, $\vec{b}_2(2, 1, 1, -3)$, $\vec{b}_3(-3, -2, -1, -2)$.

8.5. (a): $\vec{a}_1(1, -2, 3)$, $\vec{a}_2(0, 1, -1)$, $\vec{a}_3(-1, 4, 3)$, $\vec{a}_4(-1, 0, -3)$;
(b): $\vec{b}_1(9, 0, 1)$, $\vec{b}_2(-5, 1, 1)$, $\vec{b}_3(-3, 2, -1)$.

8.6. (a): $\vec{a}_1(0, 2, 3)$, $\vec{a}_2(0, 1, 0)$, $\vec{a}_3(-1, -4, 3)$;
(b): $\vec{b}_1(1, 0, 1)$, $\vec{b}_2(2, 1, 1)$, $\vec{b}_3(-3, -2, -1)$, $\vec{b}_4(-5, 4, -3)$.

8.7. (a): $\vec{a}_1(-5, 1, 2, 3)$, $\vec{a}_2(-6, 0, 1, 1)$, $\vec{a}_3(-1, -1, 4, 3)$;
(b): $\vec{b}_1(-3, 1, 0, 1)$, $\vec{b}_2(-4, 2, 1, 1)$, $\vec{b}_3(-2, -3, -2, -1)$.

8.8. (*a*): $\vec{a}_1(-3, 1, 2, 3)$, $\vec{a}_2(-4, 0, 1, 1)$, $\vec{a}_3(-8, -1, 4, 3)$;

(*b*): $\vec{b}_1(-5, 1, 0, 1)$, $\vec{b}_2(-6, 2, 1, 1)$, $\vec{b}_3(-2, -3, -2, -1)$.

Takrorlash uchun savollar

1. Maydon ustida vektor fazo deb nimaga aytiladi?
2. Vektor fazoning asosiy xossalarini bayon eting.
3. Vektor fazoga misollar keltiring.
4. Vektor fazoning fazoostisi deb nimaga aytiladi?
5. Fazoostilar kesishmasi deb nimaga aytiladi?
6. Fazoostilar yig'indisi, to'g'ri yig'indisi ta'rifini ayting.
7. Fazoostilar yig'indisi va to'g'ri yig'indisining qanday xossalarini bilasiz?
8. Vektor fazoning bazisi deb nimaga aytiladi?
9. Vektor fazoning o'lchovi deb nimaga aytiladi?

20-§. Skalar ko'paytmali vektor fazolar. Evklid vektor fazolar. Vektor fazolar izomorfizmi

✔ **Asosiy tushunchalar**: skalar ko'paytma, xosmas, nol skalar ko'paytmalar, unitar vektor fazo, ortogonal vektorlar, ortogonal bazis, ortonormal bazis, Evklid vektor fazo.

Agar *V* fazoning har bir juft $\bar{x}$ va $\bar{y}$ elementlariga ularning skalar ko'paytmasi deb ataluvchi yagona $(\bar{x}, \bar{y})$ haqiqiy son mos qo'yilib, bu moslik uchun

1) $(\bar{x}, \bar{y}) = (\bar{y}, \bar{x})$;

2) $(\bar{x}+\bar{y}, \bar{z}) = (\bar{x}, \bar{z}) + (\bar{y}, \bar{z})$;

3) $(\lambda\bar{x}, \bar{y}) = \lambda(\bar{x}, \bar{y})$, $\forall\lambda \in R$;

4) $(\bar{x}, \bar{x}) \geq 0$

shartlar bajarilsa, u holda *V* vektorlar fazosi *skalar ko'paytmali fazo* deyiladi.

Agar *V* fazoning istalgan $\bar{x} \neq \bar{0}$ vektori uchun $(\bar{x}, \bar{x}) \neq \bar{0}$ bo'lsa, *V* fazoda aniqlangan skalar ko'paytma *xosmas skalar ko'paytma* deyiladi..

Agar V fazoning istalgan $\bar{x}$ va $\bar{y}$ vektorlari uchun $(\bar{x}, \bar{y}) = 0$ bo‘lsa, V fazoda aniqlangan skalar ko‘paytma *nol skalar ko‘paytma* deyiladi.

Agar V fazoning istalgan $\bar{x} \neq \bar{0}$ vektori uchun $(\bar{x}, \bar{x}) > \bar{0}$ bo‘lsa, bunday fazoga *unitar fazo* deyiladi.

Agar unitar fazoning ikkita $\bar{x}$ va $\bar{y}$ vektorlari uchun $(\bar{x}, \bar{y}) = 0$ bo‘lsa, u holda $\bar{x}$ va $\bar{y}$ vektorlar *ortogonal vektorlar* deyiladi.

Agar V fazoning

$$\bar{a}_1, \bar{a}_2, ..., \bar{a}_n \tag{1}$$

vektorlar sistemasining istalgan ikkita elementi o‘zaro ortogonal bo‘lsa, u holda (1) sistema *ortogonal vektorlar sistemasi* deyiladi.

Agar ortogonal vektorlar sistemasi qaralayotgan fazoning bazisi bo‘lsa, bunday sistema *ortogonal bazis* deyiladi.

R maydon ustida aniqlangan V_n fazoning ixtiyoriy

$$\bar{a}_1, \bar{a}_2, ..., \bar{a}_n \tag{1}$$

bazisini

$$\bar{e}_1, \bar{e}_2, ..., \bar{e}_n \tag{2}$$

ortogonal bazisga aylantirish jarayoni bilan tanishamiz. Bu yerda (1) dan (2) ni hosil qilish ortogonallash jarayoni deyilib, u quyidagicha amalga oshiriladi: $\bar{e}_1 = \bar{a}_1$ deb olamiz, $\bar{a}_1 \neq \bar{0}$ bo‘lgani uchun $\bar{e}_1 \neq \bar{0}$ bo‘ladi. Endi $\bar{e}_2$ ni $\bar{e}_2 = \bar{a}_2 + \alpha\bar{a}_1 = \bar{a}_2 + \alpha\bar{e}_1$ shaklda olib, α sonni shunday aniqlaymizki, natijada $(\bar{e}_1, \bar{e}_2) = 0$, ya’ni

$$(\bar{e}_1, \bar{e}_2) = (\bar{e}_1, \bar{a}_2 + \alpha\bar{e}_1) = (\bar{e}_1, \bar{a}_2) + \alpha(\bar{e}_1, \bar{e}_1) \tag{3}$$

bo‘lsin. $\bar{a}_1 = \bar{e}_1 \neq \bar{0}$ va $\bar{a}_2 \neq \bar{0}$ bo‘lgani uchun $\bar{e}_2 \neq \bar{0}$ bo‘ladi. (3) tenglikdan $\alpha = \frac{(\bar{e}_1, \bar{a}_2)}{(\bar{e}_1, \bar{e}_1)}$ topiladi.

Endi $\bar{e}_3$ ni $\bar{e}_3 = \bar{a}_3 + \gamma\bar{e}_2 + \beta\bar{e}_1$ shaklda yozib olib, β va γ larni shunday tanlaymizki, natijada $(\bar{e}_1, \bar{e}_3) = 0$ va $(\bar{e}_2, \bar{e}_3) = 0$ bo‘lsin, ya’ni

$$(\bar{e}_1, \bar{a}_3 + \gamma\bar{e}_2 + \beta\bar{e}_1) = 0, \tag{4}$$

$$(\bar{e}_2, \bar{a}_3 + \gamma\bar{e}_2 + \beta\bar{e}_1) = 0 \tag{5}$$

tengliklar bajarilsin. (4) va (5) tengliklardan

$$(\bar{e}_1, \bar{a}_3) + \gamma(\bar{e}_1, \bar{e}_2) + \beta(\bar{e}_1, \bar{e}_1) = 0,$$

$$(\bar{e}_2, \bar{a}_3) + \gamma(\bar{e}_2, \bar{e}_2) + \beta(\bar{e}_2, \bar{e}_1) = 0$$

hosil bo'lib, bunda $(\bar{e}_1, \bar{e}_2) = (\bar{e}_2, \bar{e}_1) = 0$ ekanligini e'tiborga olsak,

$$\beta = \frac{(\bar{e}_1, \bar{a}_3)}{(\bar{e}_1, \bar{e}_1)} \quad \text{va} \quad \gamma = \frac{(\bar{e}_2, \bar{a}_3)}{(\bar{e}_2, \bar{e}_2)}$$

lar kelib chiqadi.

Bu jarayonni oxirigacha davom ettirib, (2) ortogonal bazisni hosil qilamiz.

Haqiqiy sonlar maydoni ustida aniqlangan V unitar fazo *Evklid fazosi* deyiladi.

$+\sqrt{(\bar{a}, \bar{a})}$ miqdor $\bar{a} \in V$ vektorning *normasi (uzunligi)* deyiladi va $\|\bar{a}\|$ orqali belgilanadi. Agar $\|\bar{a}\| = 1$ bo'lsa, $\bar{a}$ *normallangan vektor* deyiladi.

Evklid fazosining har biri normallangan

$$\bar{b}_1, \bar{b}_2, \ldots, \bar{b}_n \tag{6}$$

ortogonal vektorlar sistemasi *ortonormallangan vektorlar sistemasi* deyiladi.

Agar (6) sistema bazis tashkil etsa, u *Evklid fazoning ortonormallangan bazisi* deyiladi.

$\mathcal{F}$ maydonda berilgan V_n va V_n' chiziqli fazolar orasida shunday φ akslantirish mavjud bo'lib, u V_n ning har bir $\bar{x}$ vektorini V_n' ning yagona bitta $\bar{x}'$ vektoriga o'zaro bir qiymatli akslantirsa va quyidagi shartlar bajarilsa, V_n va V_n' fazolar *o'zaro izomorf chiziqli fazolar* deyiladi:

1) $\forall \vec{x}, \vec{y} \in V_n (\varphi(\vec{x} + \vec{y}) = \varphi(\vec{x}) + \varphi(\vec{y}))$;

2) $\forall \vec{x} \in V_n \wedge \forall \alpha \in F (\varphi(\alpha \vec{x}) = \alpha \varphi(\vec{x}))$.

1-misol. Berilgan (a): $\vec{a}_1 = (1, 2, 1)$, $\vec{a}_2 = (1, 1, 1)$ vektorlar sistemasini 3 usulda bazisgacha to'ldiring.

Yechish. **1-usul**. Vektorlar sistemasini bazisgacha to'ldirishning birinchi usuli – sistemani fazoning maksimal sondagi chiziqli erkli sistemasiga aylantirish uchun vektorlarni sistemaga bittalab qo'shish va har gal sistemaning chiziqli erkliligini tekshirishdan iborat.

Berilgan vektorlar sistemasining chiziqli bog'liq yoki chiziqli erkli ekanligini tekshirib olamiz:

$$\begin{matrix}\vec{a}_1\\ \vec{a}_2\end{matrix}\begin{pmatrix}1 & 2 & 1\\ 1 & 1 & 1\end{pmatrix}\sim\begin{matrix}\vec{a}_1\\ \vec{a}_2-\vec{a}_1\end{matrix}\begin{pmatrix}1 & 2 & 1\\ 0 & -1 & 0\end{pmatrix}.$$

Demak, $\vec{a}_1, \vec{a}_2$ vektorlar sistemasi chiziqli erkli.

$\vec{a}_1, \vec{a}_2$ vektorlar R^3 fazo vektorlari bo'lganligi uchun $\vec{a}_1, \vec{a}_2$ sistemani R^3 ning bazisigacha to'ldiramiz, ya'ni shunday $\vec{a}_3$ vektorni R^3 dan topamizki, $\vec{a}_1, \vec{a}_2$ vektorlar bilan $\vec{a}_3$ chiziqli ifodalanmasin. Masalan, $\vec{a}_3=(0,0,1)$ bo'lsa, u holda $\vec{a}_1, \vec{a}_2, \vec{a}_3$ sistema chiziqli erkli sistema bo'lishini tekshiramiz:

$$\begin{matrix}\vec{a}_1\\ \vec{a}_2\\ \vec{a}_3\end{matrix}\begin{pmatrix}1 & 2 & 1\\ 1 & 1 & 1\\ 0 & 0 & 1\end{pmatrix}\sim\begin{matrix}\vec{a}_1\\ \vec{a}_2-\vec{a}_1\\ \vec{a}_3\end{matrix}\begin{pmatrix}1 & 2 & 1\\ 0 & -1 & 0\\ 0 & 0 & 1\end{pmatrix}.$$

Demak, $\vec{a}_1, \vec{a}_2, \vec{a}_3$ sistema R^3 uchun bazis.

2-usul. Berilgan vektorlar sistemasiga fazoning biror-bir bazisini qo'shish yordamida vektorlar sistemasini bazisgacha to'ldiramiz. Buning uchun R^3 fazoning $\vec{e}_1\,(1,0,0)$, $\vec{e}_2\,(0,1,0)$, $\vec{e}_3\,(0,0,1)$ bazisini olamiz. U holda hosil bo'lgan $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{e}_1, \vec{e}_2, \vec{e}_3$ vektorlar sistemasi bazis xossalariga ko'ra chiziqli bog'liq. Bu sistemadagi $\vec{a}_1, \vec{a}_2$ vektorlarni saqlab qolgan holda uni chiziqli erkli vektorlar sistemasiga keltiramiz:

$$\begin{matrix}\vec{a}_1\\ \vec{a}_2\\ \vec{e}_1\\ \vec{e}_2\\ \vec{e}_3\end{matrix}\begin{pmatrix}1 & 2 & 1\\ 1 & 1 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\sim\begin{pmatrix}1 & 2 & 1\\ 0 & -1 & 0\\ 0 & -2 & -1\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{matrix}\vec{a}_1\\ \vec{a}_2-\vec{a}_1\\ \vec{e}_1-\vec{a}_1\\ \vec{e}_2\\ \vec{e}_3\end{matrix}\sim\begin{pmatrix}1 & 2 & 1\\ 0 & -1 & 0\\ 0 & 0 & -1\\ 0 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{matrix}\vec{a}_1\\ \vec{a}_2-\vec{a}_1\\ \vec{e}_1-\vec{a}_1-2(\vec{a}_2-\vec{a}_1)\\ \vec{e}_2+\vec{a}_2-\vec{a}_1\\ \vec{e}_3\end{matrix}\sim$$

$$\sim\left(\begin{array}{ccc|c} 1 & 2 & 1 & \vec{a}_1 \\ 0 & -1 & 0 & \vec{a}_2-\vec{a}_1 \\ 0 & 0 & 1 & \vec{e}_3 \\ 0 & 0 & -1 & \vec{e}_1-3\vec{a}_1-2\vec{a}_2 \\ 0 & 0 & 0 & \vec{e}_2+\vec{a}_2-\vec{a}_1 \end{array}\right)$$

Demak, $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{e}_1$ sistema bazis bo'ladi.

3-usul. Berilgan sistema asosida ortogonal bazis hosil qilish.

Buning uchun berilgan $\vec{a}_1, \vec{a}_2$ sistemani ortogonallash jarayoni asosida ortogonal sistemaga keltiramiz:

$\vec{b}_1=\vec{a}_1$ va $\vec{b}_2=\vec{a}_2-\alpha\vec{b}_1$ belgilashlar kiritsak, u holda $\vec{b}_1, \vec{b}_2$ vektorlar ortogonal bo'lishi uchun $(\vec{b}_1, \vec{b}_2)=0$ shartni qo'llaymiz, ya'ni $0=(\vec{b}_1, \vec{b}_2)=(\vec{b}_1, \vec{a}_2)-\alpha(\vec{b}_1, \vec{b}_1)$. Bu tenglamadan

$\alpha=\frac{(\vec{b}_1, \vec{a}_2)}{(\vec{b}_1, \vec{b}_1)}=\frac{((1,2,1),(1,1,1))}{6}=\frac{4}{6}=\frac{2}{3}$ ni hosil qilamiz.

U holda $\vec{b}_2=\vec{a}_2-\alpha\vec{b}_1=(1,1,1)-\frac{2}{3}(1,2,1)=\left(\frac{1}{3},-\frac{1}{3},\frac{1}{3}\right)$ hosil bo'lgan $\vec{b}_2=\left(\frac{1}{3},-\frac{1}{3},\frac{1}{3}\right)$ vektor $\vec{b}_1$ vektorga ortogonal va $\vec{b}_1, \vec{b}_2$ ortogonal sistema chiziqli erkli.

$\vec{b}_1, \vec{b}_2$ sistemani ortogonal bazisgacha to'ldirish uchun shunday $\vec{b}_3$ vektorni topamizki, u $(\vec{b}_1, \vec{b}_3)=0 \wedge (\vec{b}_2, \vec{b}_3)=0$ shartlarni qanoatlantirsin. $\vec{b}_3=(x,y,z)$ deb olib, $(\vec{b}_1, \vec{b}_3)=0 \wedge (\vec{b}_2, \vec{b}_3)=0$ shartlardan $\begin{cases} x+2y+z=0, \\ \frac{x}{3}-\frac{y}{3}+\frac{z}{3}=0 \end{cases}$ tenglamalar sistemasini tuzamiz.

Keltirib chiqarilgan chiziqli tenglamalar sistemasining yechimlarini Gauss usulida topamiz:

$$\begin{cases} x+2y+z=0, & (-1) \\ \frac{x}{3}-\frac{y}{3}+\frac{z}{3}=0 & 3 \end{cases} \Leftrightarrow \begin{cases} x+2y+z=0, \\ -4y=0 \end{cases} \Leftrightarrow \begin{cases} x=-z, \\ y=0, \\ z\in R. \end{cases}$$

Demak, chiziqli tenglamalar sistemasining cheksiz ko‘p yechimlari mavjud. Uning noldan farqli biror-bir yechimini, masalan, (−1, 0, 1) ni $\vec{b}_3$ vektor sifatida olish natijasida $\vec{b}_1, \vec{b}_2, \vec{b}_3$ ortoganal bazisni hosil qilamiz.

2-misol. $\vec{a}_1 = (1, 2, 3)$, $\vec{a}_2 = (1, 1, 1)$ vektorlar berilgan bo‘lsa, $L(\vec{a}_1, \vec{a}_2)$ qismfazoning ortogonal to‘ldiruvchisini toping.

Yechish. Qismfazo ortogonal to‘ldiruvchisi ta’rifiga ko‘ra $\vec{a}_1, \vec{a}_2$ vektorlarga ortogonal vektorlar tashkil etgan qismfazoni topamiz. Buning uchun $(\vec{a}_1, \vec{x}) = 0 \wedge (\vec{a}_2, \vec{x}) = 0$ shartlarni qanoatlantiruvchi $\vec{x}$ vektorlarni aniqlaymiz. Berilgan $\vec{a}_1, \vec{a}_2$ vektorlar yordamida va $\vec{x} = (x, y, z)$ belgilashdan quyidagi chiziqli tenglamalar sistemasini tuzamiz: $\begin{cases} x + 2y + 3z = 0, \\ x + y + z = 0. \end{cases}$ Tenglamalar sistemasi cheksiz ko‘p yechimga ega, chunki sistema 3 noma’lumli 2 ta tenglamadan iborat. Tenglamalar sistemasining umumiy yechimini aniqlaymiz:

$$\begin{cases} x + y + z = 0, \\ x + 2y + 3z = 0 \end{cases} \Leftrightarrow \begin{cases} x + y + z = 0, \\ y + 2z = 0 \end{cases} \Leftrightarrow \begin{cases} x = y, \\ y = -2z, \\ z \in R. \end{cases}$$

Bundan $\vec{a}_1, \vec{a}_2$ sistemaga ortogonal vektorlar $A = \{(z; -2z; z) | z \in R\}$ to‘plamdan iborat. Bu to‘plam z ning qiymatiga bog‘liq bo‘lgan vektorlardan iborat. Shuning uchun A to‘plam tashkil etgan $A = < A; +, \{\omega_\lambda | \lambda \in R\} >$ chiziqli fazo o‘lchovi $\dim A = 1$ va A chiziqli fazo $L(\vec{a}_1, \vec{a}_2)$ chiziqli fazoning ortogonal to‘ldiruvchisi bo‘ladi.

3-misol. Berilgan to‘plamlar tashkil etgan chiziqli fazolar orasida izomorfizm o‘rnating:

$$V = \left\{ \begin{pmatrix} a & b & c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \middle| a, b, c \in R \right\}, \quad V' = \left\{ \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} \middle| a, b, c \in R \right\}.$$

Yechish. Berilgan to'plamlar tashkil etgan chiziqli vektor fazolar 3 o'lchovli fazolar. Berilgan Vning bazislaridan biri

$$A=\begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix}, B=\begin{pmatrix}0&1&0\\0&0&0\\0&0&0\end{pmatrix}, C=\begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix}$$

va V' ning bazislaridan biri

$$A'=\begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix},\quad B'=\begin{pmatrix}0&1&0\\0&0&0\\0&0&0\end{pmatrix},\quad C'=\begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix}$$

bo'lishi ayon. Bu fazolar orasida $f(V)=V'$ akslantirishni quyidagicha aniqlaymiz:

$$f\left(\begin{pmatrix}a&b&c\\0&0&0\\0&0&0\end{pmatrix}\right)=\begin{pmatrix}a&0&0\\0&b&0\\0&0&c\end{pmatrix}.$$

f akslantirish izomorfizm bo'lishini isbotlaymiz:

1) $\forall A_1, A_2 \in V,\ A_1=\begin{pmatrix}a_1&b_1&c_1\\0&0&0\\0&0&0\end{pmatrix}, A_2=\begin{pmatrix}a_2&b_2&c_2\\0&0&0\\0&0&0\end{pmatrix}$ lar uchun

$$f(A_1+A_2)=f\left(\begin{pmatrix}a_1&b_1&c_1\\0&0&0\\0&0&0\end{pmatrix}+\begin{pmatrix}a_2&b_2&c_2\\0&0&0\\0&0&0\end{pmatrix}\right)=\left|\begin{matrix}\text{matritsalarni qo'shish}\\\text{amali ta'rifidan}\end{matrix}\right|=$$

$$=\left(\begin{pmatrix}a_1+a_2&b_1+b_2&c_1+c_2\\0&0&0\\0&0&0\end{pmatrix}\right)=\left|\begin{matrix}\text{akslantirish}\\\text{ta'rifidan}\end{matrix}\right|=$$

$$=\begin{pmatrix}a_1+a_2&0&0\\0&b_1+b_2&0\\0&0&c_1+c_2\end{pmatrix}=\left|\begin{matrix}\text{matritsalarni qo'shish}\\\text{amali ta'rifidan}\end{matrix}\right|=$$

$$=\begin{pmatrix} a_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & c_1 \end{pmatrix}+\begin{pmatrix} a_2 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_2 \end{pmatrix}=f(A_1)+f(A_2).$$

Demak, f akslantirish qo'shish binar amalini saqlaydi.

2) $\forall A \in V \wedge \forall \alpha \in R$ lar uchun

$$f(\alpha \cdot A)=f\left(\alpha\begin{pmatrix} a & b & c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right)=\left|\begin{matrix}\text{skalarni matritsaga ko'paytirish} \\ \text{amali ta'rifidan}\end{matrix}\right|=$$

$$=f\left(\begin{pmatrix} \alpha a & \alpha b & \alpha c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right)=\left|\begin{matrix}\text{akslantirish} \\ \text{ta'rifidan}\end{matrix}\right|=\left(\begin{pmatrix} \alpha a & 0 & 0 \\ 0 & \alpha b & 0 \\ 0 & 0 & \alpha c \end{pmatrix}\right)=$$

$$=\left|\begin{matrix}\text{skalarni matritsaga ko'paytirish} \\ \text{amali ta'rifidan}\end{matrix}\right|=\alpha\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}=\alpha \cdot f(A).$$

Demak, f akslantirish skalarni matritsaga ko'paytirish amalini saqlaydi.

V, V' chiziqli fazolar orasida aniqlangan f akslantirish gomomorfizm ekan.

3) V' dan olingan ixtiyoriy $f(A_1)$, $f(A_2)$ lar uchun $f(A_1) \neq f(A_2)$ bo'lsin. U holda $f(A_1)=\begin{pmatrix} a_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & c_1 \end{pmatrix}$ ning asli

$$A_1=\begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ va } f(A_2)=\begin{pmatrix} a_2 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_2 \end{pmatrix} \text{ ning asli } A_2=\begin{pmatrix} a_2 & b_2 & c_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Agar $\begin{pmatrix} a_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & c_1 \end{pmatrix} \neq \begin{pmatrix} a_2 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_2 \end{pmatrix}$ bo'lsa, u holda $a_1 \neq a_2 \vee b_1 \neq$ $\neq b_2 \vee c_1 \neq c_2$ bo'ladi. Bundan $\begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \neq \begin{pmatrix} a_2 & b_2 & c_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ ekanligi kelib chiqadi. Ya'ni $f(V) = V'$ inyektiv akslantirish.

4) V' to'plamning ixtiyoriy $f(A) = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$ elementi uchun V to'plamda $A = \begin{pmatrix} a & b & c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ element mavjud. Ya'ni f suryektiv akslantirish.

Demak, $f(V) = V'$ akslantirish izomorfizm ekan.

Misol va mashqlar

1. Skalar ko'paytmaning quyidagi xossalarini isbotlang:

1.1. $(\bar{x}, \bar{y} + \bar{z}) = (\bar{y} + \bar{z}, \bar{x}) = (\bar{y}, \bar{x}) + (\bar{z}, \bar{x}) = (\bar{x}, \bar{y}) + (\bar{x}, \bar{z})$.

1.2. $(\bar{x}, \lambda\bar{y}) = (\lambda\bar{y}, \bar{x}) = \lambda(\bar{y}, \bar{x}) = \lambda(\bar{x}, \bar{y})$.

2. Agar V skalar ko'paytmaga ega bo'lgan fazo bo'lsa, u holda $\forall\bar{x} \in V$ uchun $(\bar{x}, 0) = 0$ bo'lishini isbotlang.

3. Biror V fazo Evklid fazosi bo'lishi uchun uning elementlari ustida quyidagi shartlar bajarilishi lozimligini isbotlang:

3.1. $(\bar{x}, \bar{y}) = (\bar{y}, \bar{x})$ $(\forall\bar{x}, \bar{y} \in V)$.

3.2. $(\bar{x}, \bar{y} + \bar{z}) = (\bar{x}, \bar{y}) + (\bar{x}, \bar{z})$ $(\forall\bar{x}, \bar{y}, \bar{z} \in V)$.

3.3. $(\lambda\bar{x}, \bar{y}) = \lambda(\bar{x}, \bar{y})$ $(\forall\bar{x}, \bar{y} \in V, \forall\lambda \in R)$.

3.4. $(\bar{x}, \bar{x}) > 0$ $(\forall\bar{x} \in V, \bar{x} \neq \bar{0})$, $(\bar{x}, \bar{x}) > 0$ $(\forall\bar{x} \in V, \bar{x} = \bar{0})$.

4. $\overline{a}, \overline{b}$ – Evklid fazosining ixtiyoriy vektorlari va $\lambda \in R$ uchun quyidagi xossalar o'rinli ekanligini isbotlang:

4.1. $\|\overline{a}\| \geq 0$, $(\|\overline{a}\| = 0 \Leftrightarrow \overline{a} = \overline{0})$.

4.2. $\|\lambda \overline{a}\| \geq |\lambda| \, \|\overline{a}\|$.

4.3. $\|(\overline{a}, \overline{b})\| \leq \|\overline{a}\| \cdot \|\overline{b}\|$ (Koshi–Bunyakovskiy tengsizligi).

4.4. $\|\overline{a} + \overline{b}\| \leq \|\overline{a}\| + \|\overline{b}\|$ (uchburchak tengsizligi).

5. Quyidagi vektorlar sistemasini ortogonallang:

5.1. $\vec{a}_1(-1,1,1)$, $\vec{a}_2(0,1,2)$, $\vec{a}_3(1,2,3)$.

5.2. $\vec{a}_1(1,2,1,0)$, $\vec{a}_2(-1,1,2,0)$, $\vec{a}_3(1,1,1,1)$, $\vec{a}_4(1,4,4,1)$.

5.3. $\vec{a}_1(-1,3,2,1)$, $\vec{a}_2(0,0,5,5)$, $\vec{a}_3(2,1,-1,1)$.

5.4. $\vec{a}_1(1,0,1)$, $\vec{a}_2(-1,1,3)$, $\vec{a}_3(13,34,5)$.

5.5. $\vec{a}_1(1,0,1,1)$, $\vec{a}_2(1,1,1,1)$, $\vec{a}_3(0,0,1,2)$, $\vec{a}_4(-1,0,3,1)$.

5.6. $\vec{a}_1(1,0,1,1)$, $\vec{a}_2(3,1,-3,0)$, $\vec{a}_3(5,7,1,1)$.

6. Chekli o'lchovli Evklid fazosining istalgan bazisini ortonormallash mumkinligini isbotlang.

7. Agar V xosmas skalar ko'paytmali vektor fazo bo'lsa, u holda V fazoning nolmas vektorlaridan tuzilgan ortogonal vektorlar sistemasi chiziqli erkli bo'lishini isbotlang.

8. Berilgan vektorlar sistemasini 3 usulda bazisgacha to'ldiring:

8.1. $\vec{a}_1(1,0,3)$, $\vec{a}_2(0,-4,-1)$.

8.2. $\vec{a}_1(-1,1,0,3)$, $\vec{a}_2(2,0,-4,-1)$, $\vec{a}_3(3,-1,0,-3)$.

8.3. $\vec{a}_1(0,2,3)$, $\vec{a}_2(3,1,2)$, $\vec{a}_3(6,2,4)$.

8.4. $\vec{a}_1(1,-2,1,0,1)$, $\vec{a}_2(2,0,-5,1,1)$, $\vec{a}_3(3,2,-3,-2,-1)$.

8.5. $\vec{a}_1(-1,1,0,3)$, $\vec{a}_2(2,0,-4,-1)$, $\vec{a}_3(1,1,-4,2)$.

8.6. $\vec{a}_1(5,-1,-4,-4)$, $\vec{a}_2(2,0,-4,-1)$.

8.7. $\vec{a}_1(1,2,1,2,3)$, $\vec{a}_2(3,4,0,1,1)$.

8.8. $\vec{a}_1(7,0,5,1)$, $\vec{a}_2(3,5,5,0)$, $\vec{a}_3(-3,2,-7,4)$.

9. Evklid fazo fazoostisi ortogonal to'ldiruvchisining quyidagi xossalarini isbotlang:

9.1. $(U^{\perp})^{\perp} = U$.

9.2. $(U \cap V)^{\perp} = U^{\perp} + V^{\perp}$.

9.3. $(E)^{\perp} = \{\vec{0}\}$.

9.4. $\{\vec{0}\}^{\perp} = E$.

9.5. $(U^{\perp} \cap V^{\perp})^{\perp} = U + V$.

9.6. $(U^{\perp} + V^{\perp})^{\perp} = U \cap V$.

9.7. $(U \cap V \cap T)^{\perp} = U^{\perp} + V^{\perp} + T^{\perp}$.

9.8. $(U + V + T)^{\perp} = U^{\perp} \cap V^{\perp} \cap T^{\perp}$.

10. Berilgan $L(\vec{a}_1, ..., \vec{a}_n)$ fazoostining ortogonal to'ldiruvchisini toping:

10.1. $\vec{a}_1(3,-2)$.

10.2. $\vec{a}_1(1,-2,0)$.

10.3. $\vec{a}_1(1,-2,0,1)$.

10.4. $\vec{a}_1(-1,2,3)$, $\vec{a}_2(3,-4,1)$.

10.5. $\vec{a}_1(1,1,1,0)$, $\vec{a}_2(1,2,-4,0)$.

10.6. $\vec{a}_1(1,-2,0,1)$, $\vec{a}_2(2,-4,0,2)$, $\vec{a}_3(1,1,1,1)$.

10.7. $\vec{a}_1(4,1,0,1,1)$, $\vec{a}_2(3,1,0,1,0)$, $\vec{a}_3(1,0,0,0,2)$.

10.8. $\vec{a}_1(2,1,2,1,3)$, $\vec{a}_2(1,2,1,2,1)$, $\vec{a}_3(3,3,3,3,3)$, $\vec{a}_4(-1,1,-1,1,0)$.

11. Berilgan $L(\vec{a}_1, ..., \vec{a}_n)$ fazoostining ortogonal to'ldiruvchisi bazisini toping:

11.1. $\vec{a}_1(1,-2,3)$.

11.2. $\vec{a}_1(1,-1,2)$, $\vec{a}_2(1,0,1)$, $\vec{a}_3(2,-1,4)$.

11.3. $\vec{a}_1(1,1,0,-2)$, $\vec{a}_2(2,1,-1,1)$, $\vec{a}_3(3,2,-1,-1)$.

11.4. $\vec{a}_1(1,-1,2,1)$, $\vec{a}_2(1,0,1,-1)$, $\vec{a}_3(2,-1,3,0)$.

11.5. $\vec{a}_1(1,1,-1,2)$, $\vec{a}_2(2,0,1,3)$, $\vec{a}_3(4,2,-1,7)$, $\vec{a}_4(3,1,0,5)$.

11.6. $\vec{a}_1(2,-1,1,-3,1,1)$, $\vec{a}_2(1,-1,1,-1,1)$, $\vec{a}_3(0,2,1,2,0)$, $\vec{a}_4(-1,1,-1,1,-1)$.

12. Quyidagi tenglamalar sistemasi yechimlar to'plami tashkil etgan fazoostining ortogonal to'ldiruvchisini toping:

12.1. $\begin{cases} x_1 + x_2 + x_3 + x_4 = 0, \\ -x_1 + x_2 - x_3 + x_4 = 0. \end{cases}$

12.2. $\begin{cases} x_1 - 2x_2 + x_3 = 0, \\ 2x_1 - x_2 - x_3 = 0, \\ x_1 + x_2 - 2x_3 = 0. \end{cases}$

12.3. $\begin{cases} 2x_1 + x_2 + 3x_3 - x_4 = 0, \\ 3x_1 + 2x_2 - 2x_4 = 0, \\ 3x_1 + x_2 + 9x_3 - x_4 = 0. \end{cases}$

12.4. $\begin{cases} x_1 + x_2 + x_3 + x_4 = 0, \\ x_1 - x_2 - x_3 - 3x_4 = 0, \\ 3x_1 + x_2 + x_3 - x_4 = 0. \end{cases}$

13. $\mathcal{F}$ maydon ustidagi n o'lchovli har qanday ikkita V_n va V_n' chiziqli fazolar izomorfligini isbotlang.

14. Berilgan to'plamlar tashkil etgan chiziqli fazolar orasida izomorfizm o'rnating:

14.1. $V = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \mid a \in R \right\}$ va $V = \left\{ \begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix} \mid a \in R \right\}$.

14.2. $V = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \mid a, b \in R \right\}$ va $V = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \mid a, b \in R \right\}$.

14.3. $V = \left\{ \begin{pmatrix} a & b \\ c & 0 \end{pmatrix} \mid a, b, c \in R \right\}$ va $V = \left\{ \begin{pmatrix} 0 & c \\ b & a \end{pmatrix} \mid a \in R \right\}$.

14.4. $V = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in R \right\}$ va $V = \left\{ \begin{pmatrix} a & 2b \\ 3c & 4d \end{pmatrix} \mid a, b, c, d \in R \right\}$.

14.5. $V=\left\{\begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix} \mid a,b,c\in R\right\}$ va

$$V=\left\{\begin{pmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{pmatrix} \mid a,b,c\in R\right\}.$$

14.6. $V=\left\{\begin{pmatrix} a & b & 0 \\ c & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mid a,b,c\in R\right\}$ va $V=\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & c \\ 0 & b & a \end{pmatrix} \mid a,b,c\in R\right\}$.

14.7. $V=\left\{\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} \mid a,b,c\in R\right\}$ va $V=\left\{\begin{pmatrix} 0 & 0 & c \\ 0 & b & 0 \\ a & 0 & 0 \end{pmatrix} \mid a,b,c\in R\right\}$.

14.8. $V=\left\{\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ c & 0 & 0 \end{pmatrix} \mid a,b,c\in R\right\}$ va $V=\left\{\begin{pmatrix} 0 & 0 & a \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} \mid a,b,c\in R\right\}$.

14.9. $V=\left\{\begin{pmatrix} a & 1 & 0 \\ b & 2 & 0 \\ c & 3 & 0 \end{pmatrix} \mid a,b,c\in R\right\}$ va

$$V=\left\{\begin{pmatrix} a & b & c \\ \alpha & \beta & \gamma \\ 0 & 0 & 0 \end{pmatrix} \mid a,b,c,\alpha,\beta,\gamma\in R\right\}.$$

14.10. $V=\{a+bi \mid a,\ b\in R\}$ va $V=\{a-bi \mid a,\ b\in R\}$.

14.11. $V=\{a+bi \mid a,\ b\in R\}$ va $V=R^2$.

14.12. $V=\{a+b\sqrt{q} \mid a,b\in Q \wedge q$ – tub son$\}$ va

$V=\{a+b\sqrt{p} \mid a,b\in Q \wedge p$ – tub son$\}$.

Takrorlash uchun savollar

1. Vektor fazolar izomorfizmini tushuntiring.
2. Skalar ko‘paytmaning xossalarini bayon eting.
3. Skalar ko‘paytmali vektor fazo deb nimaga aytiladi?
4. Xosmas skalar ko‘paytma ta’rifini ayting.
5. Unitar fazo deb nimaga aytiladi?
6. Ortogonal vektorlar deb nimaga aytiladi?
7. Ortogonal vektorlar sistemasi deb nimaga aytiladi?
8. Ortogonal bazis deb nimaga aytiladi?
9. Ortogonallash jarayonini bayon qiling.
10. Evklid fazo deb nimaga aytiladi?
11. Vektorning normasi deb nimaga aytiladi?
12. Vektor normasining xossalarini bayon qiling.
13. Normallangan vektor deb nimaga aytiladi?
14. Ortonormallangan bazis deb nimaga aytiladi?

IX MODUL. CHIZIQLI AKSLANTIRISHLAR

21-§. Chiziqli akslantirish. Chiziqli operator yadrosi va obrazi. Chiziqli operator matritsasi

✔ **Asosiy tushunchalar**: chiziqli akslantirish, chiziqli operator, operator yadrosi, tasviri, defekti, rangi, matritsasi.

$\mathcal{F}$ sonlar maydoni ustida aniqlangan U vektor fazoni V vektor fazoga akslantiruvchi akslantirish uchun ushbu

1) $\varphi(\overline{x}_1 + \overline{x}_2) = \varphi(\overline{x}_1) + \varphi(\overline{x}_2)$;

2) $\varphi(\lambda\overline{x}_1) = \lambda\varphi(\overline{x}_1)$ $(\lambda \in F)$ shartlar bajarilsa, u holda U vektor fazo V vektor fazoga *chiziqli akslanadi* deyiladi.

U fazoni V fazoga chiziqli akslantirishlar to'plami Hom(U, V) orqali belgilanadi.

U vektor fazoni o'zini o'ziga chiziqli akslantirish U fazoda aniqlangan *chiziqli operator* deyiladi.

U vektor fazoning ixtiyoriy $\overline{x}_1$ va $\overline{x}_2$ elementlari va U da aniqlangan φ operator uchun $\varphi(\overline{x}_1 + \overline{x}_2) = \varphi(\overline{x}_1) + \varphi(\overline{x}_2)$ tenglik bajarilsa, u holda φ ga U da aniqlangan *additiv operator* deyiladi.

Agar λ ixtiyoriy son bo'lganda U fazoning ixtiyoriy $\overline{x}$ elementi uchun $\varphi(\lambda\overline{x}) = \lambda\varphi(\overline{x})$ tenglik o'rinli bo'lsa, u holda φ ga U da aniqlangan *bir jinsli operator* deyiladi.

Agar $\forall\overline{x} \in U$ uchun $\varphi(\overline{x}) = 0$ tenglik bajarilsa, u holda φ operator *nol operator* deyiladi.

Agar $\forall\overline{x} \in U$ uchun $e(\overline{x}) = \overline{x}$ tenglik bajarilsa, u holda e ga *ayniy (birlik) operator* deyiladi.

Agar $\forall\overline{x} \in U$, $\lambda \in P$ uchun $\varphi(\overline{x}) = \lambda\overline{x}$ tenglik bajarilsa, u holda φ ga *o'xshashlik operatori* deyiladi.

Agar U_n fazoning ixtiyoriy $\overline{x}$ vektori uchun $f(\overline{x}) = \varphi(\overline{x}) + \psi(\overline{x})$ tenglik bajarilsa, u holda f ga φ va ψ operatorlarning *yig'indisi* deyiladi va u $\varphi + \psi = f$ orqali yoziladi.

$\alpha \in F$, $\forall \overline{x} \in U_n$ uchun $(\alpha\varphi)\overline{x} = \alpha\varphi(\overline{x})$ tenglik bajarilsa, u holda $\alpha\varphi$ ga α skalarning φ operatorga *ko'paytmasi* deyiladi.

U_n fazoning φ operator yordamida nolga akslanuvchi barcha elementlari to'plami φ operatorning *yadrosi* deyiladi va u $\operatorname{Ker}\varphi$ orqali belgilanadi. φ chiziqli operator yadrosining o'lchovi shu operatorning *defekti* deyiladi.

φU_n fazoosti operatorning *obrazi* deyiladi. φU_n obrazning o'lchovi φ operatorning *rangi* deyiladi.

Agar $\overline{x} = (x_1, x_2, ..., x_n) \in U$ bo'lib, $\varphi(x) = \varphi(x_1, x_2, ..., x_n) = (x_1, x_2, ..., x_k)$ $(1 \le k < n)$ bo'lsa, ya'ni φ operator n o'lchovli fazodagi vektorni k o'lchovli fazodagi vektorga o'tkazuvchi operator bo'lsa, u holda φ *proyeksiyalovchi operator* deyiladi.

$\mathcal{F}$ maydon ustida V_n vektor fazo berilgan bo'lib,

$$\overline{e}_1, \overline{e}_2, ..., \overline{e}_n \tag{1}$$

uning bazisi bo'lsin. Agar φ operator V_n fazoda aniqlangan chiziqli operator bo'lsa, u holda $\varphi(\overline{e}_1), \varphi(\overline{e}_2), ..., \varphi(\overline{e}_n) \in V_n$ vektorlar (1) bazis orqali chiziqli ifodalanadi, ya'ni

$$\begin{cases} \varphi(\overline{e}_1) = \alpha_{11}\overline{e}_1 + \alpha_{21}\overline{e}_2 + ... + \alpha_{n1}\overline{e}_n, \\ \varphi(\overline{e}_2) = \alpha_{12}\overline{e}_1 + \alpha_{22}\overline{e}_2 + ... + \alpha_{n2}\overline{e}_n, \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ \varphi(\overline{e}_n) = \alpha_{1n}\overline{e}_1 + \alpha_{2n}\overline{e}_2 + ... + \alpha_{nn}\overline{e}_n. \end{cases}$$

Ushbu $M(\varphi) = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \alpha_{n1} & \alpha_{n2} & \dots & \alpha_{nn} \end{pmatrix}$ matritsa chiziqli operatorning (1) *bazisdagi matritsasi* deyiladi.

$\overline{x}$ va $\varphi(\overline{x})$ vektorlar (1) bazis orqali $x = \beta_1\overline{e}_1 + ... + \beta_n\overline{e}_n$, $\varphi(\overline{x}) = \gamma_1\overline{e}_1 + ... + \gamma_n\overline{e}_n$ ko'rinishda ifodalansin. $\overline{x}$ va $\varphi(\overline{x})$ vektorlarning (1) bazisga nisbatan ustun koordinatalarini mos ravishda ushbu

$$M(\overline{x})=\begin{bmatrix}\beta_1\\ \beta_2\\ \dots\\ \beta_n\end{bmatrix},\quad M(\varphi(\overline{x}))=\begin{bmatrix}\gamma_1\\ \gamma_2\\ \dots\\ \gamma_n\end{bmatrix}$$

ko'rinishlarda belgilaymiz. U holda $\forall\,\overline{x}\in V_n$ uchun $M(\varphi(\overline{x})=$ $=M(\varphi)M(\overline{x})$ tenglik bajariladi.

*A*gar $\mathcal{F}$ maydon ustida $A,B\in F^{n\times n}$ matritsalar uchun teskarilanuvchi $T\in F^{n\times n}$ matritsa mavjud bo'lib, ular uchun $B=T^{-1}AT$ tenglik o'rinli bo'lsa, u holda A va V matritsalar *o'xshash matritsalar* deyiladi.

$\mathcal{F}$ maydon ustida V_n vektor fazoning (1) dan boshqa

$$\overline{e}_1', \overline{e}_2', \dots, \overline{e}_n' \tag{2}$$

bazisi berilgan bo'lsin. (2) bazisning vektorlarini (1) orqali chiziqli ifodalaymiz:

$$\begin{cases}\overline{e}_1'=\beta_{11}\overline{e}_1+\beta_{21}\overline{e}_2+\dots+\beta_{n1}\overline{e}_n,\\ \overline{e}_2'=\beta_{12}\overline{e}_1+\beta_{22}\overline{e}_2+\dots+\beta_{n2}\overline{e}_n,\\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\ \overline{e}_n'=\beta_{1n}\overline{e}_1+\beta_{2n}\overline{e}_2+\dots+\beta_{nn}\overline{e}_n.\end{cases}$$

U holda $T=\begin{pmatrix}\beta_{11} & \beta_{12} & \dots & \beta_{1n}\\ \beta_{21} & \beta_{22} & \dots & \beta_{2n}\\ \dots & \dots & \dots & \dots\\ \beta_{n1} & \beta_{n2} & \dots & \beta_{nn}\end{pmatrix}$ matritsa (2) bazisdan (1) bazisga *o'tish matritsasi* deyiladi.

$\overline{x}$ vektorning birinchi va ikkinchi bazislardagi ustun koordinatalarini, mos ravishda, $M(\overline{x})$ va $M'(\overline{x})$ deb belgilasak, u holda $\forall\,\overline{x}\in V_n$ vektor uchun $M(\overline{x})=TM'(\overline{x})$ va $M'(\overline{x})=T^{-1}M(\overline{x})$ tengliklar o'rinli bo'ladi.

V_n fazoda aniqlangan φ chiziqli operator uchun $M(\varphi)$ va $M'(\varphi)$ lar φ chiziqli operatorning birinchi va ikkinchi bazislarga nisbatan mos matritsalari bo‘lsa, u holda $M'(\varphi)=T^{-1}M(\varphi)T$ tenglik o‘rinli bo‘ladi.

1-misol. Berilgan $f(x)=(x_1-x_2+x_3;\ x_1;x_2)$; akslantirish chiziqli operator ekanligini isbotlang va uning rangi, defektini aniqlang.

Yechish. Chiziqli operator ta’rifiga ko‘ra f akslantirish berilgan fazoni o‘ziga akslantirishi va $f(\bar{a}+\bar{b})=f(\bar{a})+f(\bar{b}), f(\lambda\bar{a})=\lambda f(\bar{a})$ shartlarga bo‘ysunishi kerak.

1) $\forall\bar{x},\bar{y}\in R^3,\ \bar{x}=(x_1,x_2,x_3),\ \bar{y}(y_1,y_2,y_3)$ lar uchun $f(\bar{x}+\bar{y})=$
$=f\left((x_1+y_1;x_2+y_2;x_3+y_3)\right)=(x_1+y_1-(x_2+y_2)+x_3+y_3;$
$x_1+y_1;x_2+y_2)=|$vektorlarni qo‘shish ta’rifiga ko‘ra$|=$
$=\left(x_1-x_2+x_3;x_1;x_2\right)+\left(y_1-y_2+y_3;y_1;y_2\right)=f(\bar{x})+f(\bar{y})$.

2) $\forall\bar{x}\in R^3$ va $\forall\alpha\in R$ lar uchun

$$f(\alpha\bar{x})=f(\alpha(x_1,x_2,x_3))=f\left((\alpha x_1,\alpha x_2,\alpha x_3)\right)=$$

$$=\left(\alpha x_1-\alpha x_2+\alpha x_3;\alpha x_1;\alpha x_2\right)=\alpha\left(x_1-x_2+x_3;x_1;x_2\right)=\alpha\cdot f(\bar{x}).$$

Demak, f akslantirish chiziqli akslantirish va u R^3 ni o‘ziga akslantirganligi uchun $f-$ chiziqli operator.

3) defect f ni topish uchun Ker f ni aniqlaymiz. $\mathrm{Ker} f=\{\bar{x}\mid f(\bar{x})=\bar{0}\}$ ta’rifdan $f(\bar{x})=\bar{0}$ shartni qanoatlantiruvchi vektorlarni topamiz:

$f(\bar{x})=(x_1-x_2+x_3;x_1,x_2)=\bar{0}$. Bundan

$$\begin{cases}x_1-x_2+x_3=0,\\ x_1=0,\\ x_2=0,\end{cases}\Leftrightarrow\begin{cases}x_1=0,\\ x_2=0,\\ x_3=0.\end{cases}$$ Demak, $\mathrm{Ker} f=\{\bar{0}\}$.

Nol chiziqli fazoning o‘lchovi 0 ga teng. Bundan defect $f=0$.

$\dim V=\mathrm{defect} f+\mathrm{rang} f$ tenglikdan $\mathrm{rang} f=\dim V-\mathrm{defect} f$ ni, bundan, $f=3-0=3$ ni hosil qilamiz.

Demak, 1) $f-$ chiziqli operator;
2) defect $f=0$;
3) rang $f=3$.

2-misol. Agar M — $\overline{e}_1, \overline{e}_2, \overline{e}_3$ bazisdagi, M' — $\overline{e}'_1, \overline{e}'_2, \overline{e}'_3$ bazisdagi $f(\overline{x}) = (x_1, x_2, x_3)$ operator matritsalari bo'lsa, u holda

a) $M(f(\overline{x})) = M(f)M(\overline{x})$;

b) $M'(\overline{x}) = T^{-1}M(\overline{x}) \wedge M(\overline{x}) = TM'(\overline{x})$;

d) $M'(f) = T^{-1}M(f)T \wedge M(f) = TM'(f)T^{-1}$ shartlar bajarilishini tekshiring.

$\overline{x} = (1,3,1)$; $\overline{e}'_1 = (1,0,2)$, $\overline{e}'_2 = (2,1,1)$, $\overline{e}'_3 = (1,3,0)$.

Yechish. 1) Berilgan $f(\overline{x}) = (x_1, x_2, x_3)$ operatorning $\overline{e}_1, \overline{e}_2, \overline{e}_3$ bazisdagi *M(f)* matritsasini topamiz. Buning uchun $f(\overline{e}_1)$, $f(\overline{e}_2)$, $f(\overline{e}_3)$ vektorlarning $\overline{e}_1, \overline{e}_2, \overline{e}_3$ bazisdagi chiziqli kombinatsiyalarini aniqlaymiz:

$f(\overline{e}_1) = f((1,0,0)) = (1,0,0) = 1 \cdot \overline{e}_1 + 0 \cdot \overline{e}_2 + 0 \cdot \overline{e}_3$;

$f(\overline{e}_2) = f((0,1,0)) = (0,0,1) = 0 \cdot \overline{e}_1 + 0 \cdot \overline{e}_2 + 1 \cdot \overline{e}_3$;

$f(\overline{e}_3) = f((0,0,1)) = (0,1,0) = 0 \cdot \overline{e}_1 + 1 \cdot \overline{e}_2 + 0 \cdot \overline{e}_3$.

Chiziqli kombinatsiyalar koeffitsiyentlaridan matritsa hosil qilamiz. Bunda $f(\overline{e}_i)$ $(i = 1,\overline{3})$ ning chiziqli kombinatsiyasida qatnashgan koeffitsiyentlar ustun qilib yoziladi:

$$M(f) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

2) *M(f)* matritsa yordamida $f(\overline{x})$ vektorning ustun koordinatalarini topamiz: $M(f(\overline{x})) = M(f)M(\overline{x})$ dan

$$M(f(\overline{x})) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$$ ni hosil qilamiz.

Olingan natijani tekshirish uchun operator talabini $\overline{x} = (1, 3, 1)$ vektorga qo'llaymiz. Bundan $f(\overline{x}) = f((1,3,1)) = (1,1,3)$ kelib chiqadi.

3) Berilgan birinchi bazisdan ikkinchi bazisga o'tish matritsasini topamiz:

$$f(\overline{e}_1') = \alpha_1\overline{e}_1 + \alpha_2\overline{e}_2 + \alpha_3\overline{e}_3,$$
$$f(\overline{e}_2') = \beta_1\overline{e}_1 + \beta_2\overline{e}_2 + \beta_3\overline{e}_3,$$
$$f(\overline{e}_3') = \gamma_1\overline{e}_1 + \gamma_2\overline{e}_2 + \gamma_3\overline{e}_3.$$

Bundan

$$f\big((1,0,2)\big) = (1,2,0) = 1\cdot\overline{e}_1 + 2\cdot\overline{e}_2 + 0\cdot\overline{e}_3;$$
$$f\big((2,1,1)\big) = (2,1,1) = 2\cdot\overline{e}_1 + 1\cdot\overline{e}_2 + 1\cdot\overline{e}_3;$$
$$f\big((1,3,0)\big) = (1,0,3) = 1\cdot\overline{e}_1 + 0\cdot\overline{e}_2 + 3\cdot\overline{e}_3$$

kelib chiqadi.

Birinchi bazisdan ikkinchi bazisga o‘tish matritsasi $T = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$ dan iborat bo‘ladi. Uning teskarisini topamiz:

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -3 & -2 & -2 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \\ 0 & 0 & 7 & -2 & 1 & 3 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & \frac{2}{7} & -\frac{1}{7} & -\frac{3}{7} \\ 0 & 1 & 0 & \frac{6}{7} & -\frac{3}{7} & -\frac{2}{7} \\ 0 & 0 & 1 & -\frac{2}{7} & \frac{1}{7} & \frac{3}{7} \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{3}{7} & \frac{5}{7} & \frac{1}{7} \\ 0 & 1 & 0 & \frac{6}{7} & -\frac{3}{7} & -\frac{2}{7} \\ 0 & 0 & 1 & -\frac{2}{7} & \frac{1}{7} & \frac{3}{7} \end{array}\right).$$

Demak, $T^{-1} = \frac{1}{7}\begin{pmatrix} -3 & 5 & 1 \\ 6 & -3 & -2 \\ -2 & 1 & 3 \end{pmatrix}.$

U holda

$$M'(\overline{x}) = T^{-1}M(\overline{x}) = \frac{1}{7}\begin{pmatrix} -3 & 5 & 1 \\ 6 & -3 & -2 \\ -2 & 1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 13 \\ -5 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{13}{7} \\ -\frac{5}{7} \\ \frac{4}{7} \end{pmatrix}.$$

Tekshirish maqsadida $M(\overline{x}) = TM'(\overline{x})$ tenglikni tuzamiz:

$$\begin{pmatrix}1\\3\\1\end{pmatrix}=\begin{pmatrix}1&2&1\\2&1&0\\0&1&3\end{pmatrix}\begin{pmatrix}\frac{13}{7}\\-\frac{5}{7}\\\frac{4}{7}\end{pmatrix}\Rightarrow\begin{pmatrix}1\\3\\1\end{pmatrix}=\begin{pmatrix}1\\3\\1\end{pmatrix}.$$

Demak, T, T^{-1} to'g'ri topilgan.

4) f operatorning 1- va 2-bazisdagi matritsalari orasidagi bog'lanishni o'rnatamiz:

$$M'(f)=T^{-1}M(f)T=\frac{1}{7}\begin{pmatrix}-3&5&1\\6&-3&-2\\-2&1&3\end{pmatrix}\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}\begin{pmatrix}1&2&1\\2&1&0\\0&1&3\end{pmatrix}=$$

$$=\frac{1}{7}\begin{pmatrix}-3&5&1\\6&-2&-3\\-2&1&3\end{pmatrix}\begin{pmatrix}1&2&1\\2&1&0\\0&1&3\end{pmatrix}=\frac{1}{7}\begin{pmatrix}-1&0&12\\2&7&-3\\4&0&1\end{pmatrix}.$$

Tekshirish uchun $M(f)=TM'(f)T^{-1}$ tenglikka topilgan qiymatlarni qo'yamiz:

$$\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}=\begin{pmatrix}1&2&1\\2&1&0\\0&1&3\end{pmatrix}\cdot\frac{1}{7}\begin{pmatrix}-1&0&12\\2&7&-3\\4&0&1\end{pmatrix}\cdot\frac{1}{7}\begin{pmatrix}-3&5&1\\6&-3&-2\\-2&1&3\end{pmatrix},$$

$$\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}=\frac{1}{49}\begin{pmatrix}7&14&7\\0&7&21\\14&7&0\end{pmatrix}\begin{pmatrix}-3&5&1\\6&-3&-2\\-2&1&3\end{pmatrix}=\frac{1}{7}\cdot\begin{pmatrix}1&2&1\\0&1&3\\2&1&0\end{pmatrix}\begin{pmatrix}-3&5&1\\6&-3&-2\\-2&1&3\end{pmatrix}=$$

$$=\frac{1}{7}\begin{pmatrix}7&0&0\\0&0&7\\0&7&0\end{pmatrix}=\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}.$$

Demak, berilgan misol to'g'ri yechilgan.

Misol va mashqlar

1. $\mathcal{F}$ sonlar maydoni ustida aniqlangan U vektor fazoda aniqlangan additiv operatorning quyidagi xossalarini isbotlang:

1.1. $\varphi(\vec{0}) = \vec{0}$.

1.2. $\varphi(-\bar{x}) = -\varphi(\bar{x})$ $(\forall \bar{x} \in U)$.

1.3. $\varphi(r\bar{x}) = r\varphi(\bar{x})$ $(\forall r \in Q)$.

1.4. $\varphi(\bar{x}_1 - \bar{x}_2) = \varphi(\bar{x}_1) - \varphi(\bar{x}_2)$ $(\forall \bar{x}_1, \bar{x}_2 \in U)$.

2. φ operator chiziqli operator bo'lishi uchun U fazoning ixtiyoriy $\bar{x}_1$ va $\bar{x}_2$ elementlari va $\lambda_1, \lambda_2 \in F$ berilganda $\varphi(\lambda_1\bar{x}_1 + \lambda_2\bar{x}_2) = \lambda_1\varphi(\bar{x}_1) + \lambda_2\varphi(\bar{x}_2)$ tenglikning bajarilishi zarur va yetarli ekanligini isbotlang.

3. Agar φ chiziqli operator bo'lsa, u holda $\forall x_i \in U$, $\lambda_i \in P$ $(i = 1,n)$ uchun ushbu $\varphi(\lambda_1\bar{x}_1 + \lambda_2\bar{x}_2 + ... + \lambda_n\bar{x}_n) = \lambda_1\varphi(\bar{x}_1) + \lambda_2\varphi(\bar{x}_2) + ... + \lambda_n\varphi(\bar{x}_n)$ tenglik o'rinli bo'lishini isbotlang.

4. Nol operator ham chiziqli operator bo'lishini isbotlang.

5. $\vec{a}_1, \vec{a}_2, \vec{a}_3$ vektorlarni $\vec{b}_1, \vec{b}_2, \vec{b}_3$ vektorlarga o'tkazuvchi yagona chiziqli akslantirish mavjudligini isbotlang va uning matritsasini toping:

5.1. $\vec{a}_1=(2, 3, 5)$, $\vec{a}_2=(-0, 1, 2)$, $\vec{a}_3=(1, 0, 0)$;
$\vec{b}_1=(1, 1, 1)$, $\vec{b}_2=(1, 1, -1)$, $\vec{b}_3=(2, 1, 2)$.

5.2. $\vec{a}_1=(2, 0, 3)$, $\vec{a}_2=(4, 1, 5)$, $\vec{a}_3=(3, 1, 2)$;
$\vec{b}_1=(1, 2, -1)$, $\vec{b}_2=(4, 5, -2)$, $\vec{b}_3=(1, -1, 1)$.

6. Berilgan akslantirishlar chiziqli operator bo'lishini tekshiring:

6.1. $f(x) = (x_2 + x_3;\ 2x_1 + x_3;\ 3x_1 - x_2 + x_3)$.

6.2. $f(x) = (x_1 + x_2;\ 4x_3;\ x_1 + x_3 + 3)$.

6.3. $f(x) = (x_1 - x_2;\ x_2 + x_3;\ x_3)$.

6.4. $f(x) = (x_1;\ x_2 + 2x_3;\ -x_3)$.

6.5. $f(x) = (-3(x_1 + x_2);\ x_2 + x_3;\ x_1)$.

6.6. $f(x) = (0;\ 3(x_2 + x_3);\ x_1)$.

6.7. $f(x) = (x_1 - x_2;\ 3x_2 - x_3;\ 0\ )$.

6.8. $f(x) = (x_2;\ x_3;\ 2)$.

6.9. $f(x) = (\ x_2;\ x_3;\ x_1)$.

6.10. $f(x) = (-x_2;\ x_2 + x_3;\ x_3\ )$.

6.11. $f(x) = (x_2 + x_3;\ 0;\ x_1 - 2x_2 + x_3)$.

6.12. $f(x) = (0;\ x_1;\ -2x_2 + x_3)$.

6.13. $f(x) = (x_1 - x_2 + x_3;\ x_3;\ x_1)$.

6.14. $f(x) = (1 - x_2 + x_3;\ x_3;\ x_1 - 2x_2)$.

7. 6-misoldagi chiziqli operatorlarning rangi defektini aniqlang.

8. Nol bo'lmagan chekli o'lchovli vektor fazodagi φ chiziqli operatorning rangi φ chiziqli operator matritsasining rangiga teng bo'lishini isbotlang.

9. Berilgan chiziqli operatorlarning rangi r va defekti d ni toping:

9.1. $\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$.   9.2. $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$.

9.3. $\begin{pmatrix} 1 & 2 & 3 \\ -2 & -4 & -6 \\ 4 & 8 & 12 \end{pmatrix}$.   9.4. $\begin{pmatrix} 1 & 2 & 3 \\ -2 & -1 & -1 \\ 0 & 3 & -1 \end{pmatrix}$.

9.5. $\begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & 1 \\ -1 & 0 & 3 \end{pmatrix}$.   9.6. $\begin{pmatrix} 1 & 3 & 1 & 2 \\ -1 & -3 & -1 & -1 \\ 1 & 3 & 1 & 2 \\ 2 & 6 & 2 & 4 \end{pmatrix}$.

9.7. $\begin{pmatrix} 1 & -2 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 3 & -1 & 1 & 3 \\ 4 & -3 & 2 & 5 \end{pmatrix}$.   9.8. $\begin{pmatrix} 1 & 1 & 0 & 1 \\ -1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 3 \\ 1 & 2 & 1 & 3 \end{pmatrix}$.

10. φ chiziqli operatorlar yadrosi shu operator qaralayotgan fazoning fazoosti bo'lishini isbotlang.

11. 6-topshiriqdagi chiziqli operatorlar uchun quyidagi shartlar bajarilishini tekshiring:

a) $M(f(\vec{x})) = M(f) \cdot M(\vec{x})$;

b) $M'(\vec{x}) = T^{-1} \cdot M(\vec{x}) \wedge M(x) = T \cdot M'(\vec{x})$;

d) $M'(f) = T^{-1} \cdot M(f) \cdot T \wedge M(f) = T \cdot M'(f) \cdot T^{-1}$.

11.1. $\vec{x}=(1, 2, 3)$; $\vec{e}_1'(2, 1, 0)$, $\vec{e}_2'(0, 3, 1)$, $\vec{e}_3'(-1,0,2)$.

11.2. $\vec{x}=(6,-5,13)$; $\vec{e}_1'(3, 1,-2)$, $\vec{e}_2'(1, 3, 1)$, $\vec{e}_3'(1,5,0)$.

11.3. $\vec{x}=(1, 2, 3)$; $\vec{e}_1'(1, 0, 3)$, $\vec{e}_2'(1, 1, -2)$, $\vec{e}_3'(2,-1,2)$.

11.4. $\vec{x}=(-8, 5, 2)$; $\vec{e}_1'(1, 1, 1)$, $\vec{e}_2'(1, 2, 3)$, $\vec{e}_3'(1,3,3)$.

11.5. $\vec{x}=(7,-2,-4,3)$; $\vec{e}_1'(1, 2, 3,-1)$, $\vec{e}_2'(2, 3, 4,1)$, $\vec{e}_3'(1,4,3,3)$.

11.6. $\vec{x}=(3,-5,7)$; $\vec{e}_1'(-1, 1, 1)$, $\vec{e}_2'(-1, 2, 3)$, $\vec{e}_3'(-1,3,3)$.

11.7. $\vec{x}=(3,1,9)$; $\vec{e}_1'(1, 2, 3)$, $\vec{e}_2'(2, 3, 4)$, $\vec{e}_3'(1,4,3)$.

11.8. $\vec{x}=(6,-4,5)$; $\vec{e}_1'(2, 2, 3)$, $\vec{e}_2'(1, -1, 0)$, $\vec{e}_3'(-1,2,1)$.

11.9. $\vec{x}=(5,0,1)$; $\vec{e}_1'(1, 2, 3)$, $\vec{e}_2'(-1, 2, 0)$, $\vec{e}_3'(-1,2,1)$.

11.10. $\vec{x}=(-4,5,-2)$; $\vec{e}_1'(2,-1, 0)$, $\vec{e}_2'(0,-3,-1)$, $\vec{e}_3'(1,0,-2)$.

11.11. $\vec{x}=(3,3,-2)$; $\vec{e}_1'(1, 1, 0)$, $\vec{e}_2'(0, 1, 1)$, $\vec{e}_3'(1,0,1)$.

11.12. $\vec{x}=(5,6,7)$; $\vec{e}_1'(2, 3, 0)$, $\vec{e}_2'(0, 2, 3)$, $\vec{e}_3'(2,0,3)$.

11.13. $\vec{x}=(-1,-2,-3)$; $\vec{e}_1'(4, 0, 4)$, $\vec{e}_2'(4, 4, 0)$, $\vec{e}_3'(0,4,4)$.

Takrorlash uchun savollar

1. Chiziqli akslantirish, chiziqli operator deb nimaga aytiladi?
2. Additiv, bir jinsli operatorlarga misol keltiring.
3. Nol, birlik operatorlar deb nimaga aytiladi?
4. O‘xshashlik, proyeksiyalovchi operatorlar ta’rifini ayting.
5. Chiziqli operatorning yadrosi haqida tushuncha bering.
6. Chiziqli operatorning obrazini misol yordamida tushuntiring.
7. Chiziqli operator matritsasi qanday topiladi?
8. Chiziqli operator rangi deb nimaga aytiladi?
9. Chiziqli operatorning turli bazislarga nisbatan matritsalari orasidagi bog‘lanish formulasini tushuntiring.
10. O‘xshash matritsalar deb nimaga aytiladi?
11. $\overline{x}$ va $\varphi(\overline{x})$ vektorlar ustun koordinatalari orasidagi bog‘lanish qanday o‘rnatiladi?

22-§. Chiziqli operatorlar ustida amallar. Chiziqli algebralar. Teskari operator. Xos vektorlar va xos qiymatlar

✔ **Asosiy tushunchalar**: chiziqli algebra, rangi, chiziqli algebralar ustida amallar, operatorlar chiziqli algebrasi, chiziqli operator xos qiymati, xos vektorlari.

$\mathcal{F}$ maydon ustidagi V chiziqli fazo elementlari uchun quyidagi

1) $\overline{x}\overline{y} \in V\ (\forall \overline{x}, \overline{y} \in V)$;

2) $\overline{x}(\overline{y}\overline{z}) = (\overline{x}\overline{y})\overline{z}\ (\forall \overline{x}, \overline{y}, \overline{z} \in V)$;

3) $\overline{x}(\overline{y} + \overline{z}) = \overline{x}\overline{y} + \overline{x}\overline{z}$ va $(\overline{y} + \overline{z})\overline{x} = \overline{y}\overline{x} + \overline{z}\overline{x}\ (\forall \overline{x}, \overline{y}, \overline{z} \in V)$;

4) $\lambda(\overline{x}\overline{y}) = (\lambda\overline{x})\overline{y} = \overline{x}(\lambda\overline{y})\ (\lambda \in F,\ \forall \overline{x}, \overline{y} \in V)$

shartlar bajarilsa, u holda V fazoni $\mathcal{F}$ maydon ustidagi *chiziqli algebra* deyiladi.

Agar V chiziqli algebrada $\overline{x} \cdot \overline{y} = \overline{y} \cdot \overline{x}\ (\forall \overline{x}, \overline{y} \in V)$ shart bajarilsa, V *kommutativ chiziqli algebra* deyiladi.

V chiziqli algebraning rangi deb V fazoning o'lchoviga aytiladi.

V fazo $\mathfrak{I}$ maydon ustidagi vektor fazo bo'lib, φ, ψ lar shu vektor fazoning chiziqli operatorlari bo'lsin, u holda:

1) $(\varphi + \psi)(\overline{x}) = \varphi(\overline{x}) + \psi(\overline{x})$;

2) $(\lambda\varphi)(\overline{x}) = \lambda\varphi(\overline{x})$;

3) $(\varphi\psi)(\overline{x}) = \varphi(\psi(\overline{x}))$.

Hom(V, V) to'plam $\mathcal{F}$ maydon ustida vektor fazo tashkil qiladi.

$<$Hom (V, V), $+$, $\{\omega_\lambda | \lambda \in F\}$, $\cdot>$ algebra V vektor fazoning *chiziqli operatorlar algebrasi* deyiladi va quyidagicha belgilanadi:

$$\text{End}\,V = <\text{Hom}\ (V, V),\ +,\ \{\omega_\lambda | \lambda \in F\},\ \cdot> .$$

U va U' algebralar $\mathcal{F}$ maydon ustidagi chiziqli algebralar va $\varphi: U \to U'$ akslantirish biyektiv akslantirish bo'lib, quyidagi

$\varphi(\overline{a} + \overline{b}) = \varphi(\overline{a}) + \varphi(\overline{b})$;

$\varphi(\lambda\overline{a}) = \lambda\varphi(\overline{a})$;

$\varphi(\overline{a} \cdot \overline{b}) = \varphi(\overline{a}) \cdot \varphi(\overline{b}),\ \forall \overline{a}, \overline{b} \in V \wedge \forall \lambda \in F$

shartlar bajarilsa, u holda φ akslantirish U va U' *chiziqli algebralar izomorfizmi* deyiladi.

Kompleks sonlar maydoni ustida qurilgan V_n vektor fazo va $\varphi: V_n \to V_n$ chiziqli operator berilgan bo'lsin. Ushbu $\varphi(\bar{x}) = \lambda(\bar{x})$ $(\forall \bar{x} \in V_n, x \neq \bar{0}, \lambda \in F)$ tenglikni qanoatlantiruvchi α son φ chiziqli operatorning *xos qiymati*, $\bar{x}$ vektor esa λ xos qiymatga mos keluvchi *xos vektori* deyiladi.

V_n vektor fazoning $\bar{e}_1, \bar{e}_2, \ldots, \bar{e}_n$ bazisida φ chiziqli operator

$$A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ . & . & . & . \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{pmatrix}$$ matritsa yordamida berilgan bo'lsa,

$|A - \lambda E| = 0$ tenglama φ chiziqli operatorning *xarakteristik tenglamasi* deyiladi.

1-misol. Berilgan $A = \begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix}$ operatorga teskari operatorni toping.

Yechish. Berilgan A operatorga teskari operatorni topish uchun A matritsaga teskari matritsa topiladi:

$$\left(\begin{array}{cccc|cccc} 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cccc|cccc} 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -4 & -1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & -1 & 0 & 0 & 1 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -4 & -1 & 0 & -2 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & -1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cccc|cccc} 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 4 & -2 & 1 & 4 & 0 \\ 0 & 0 & -1 & 1 & -1 & 0 & 1 & 1 \end{array}\right) \sim$$

$$\sim\left(\begin{array}{cccc|cccc}1 & 2 & 1 & 0 & 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0\\ 0 & 0 & 1 & -4 & 2 & -1 & -4 & 0\\ 0 & 0 & 0 & -3 & 1 & -1 & -3 & 1\end{array}\right)\sim\left(\begin{array}{cccc|cccc}1 & 2 & 1 & 0 & 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0\\ 0 & 0 & 1 & -4 & 2 & -1 & -4 & 0\\ 0 & 0 & 0 & 1 & -\frac{1}{3} & \frac{1}{3} & 1 & -\frac{1}{3}\end{array}\right)\sim$$

$$\sim\left(\begin{array}{cccc|cccc}1 & 2 & 1 & 0 & 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0\\ 0 & 0 & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & -\frac{4}{3}\\ 0 & 0 & 0 & 1 & -\frac{1}{3} & \frac{1}{3} & 1 & -\frac{1}{3}\end{array}\right)\sim\left(\begin{array}{cccc|cccc}1 & 2 & 1 & 0 & 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & \frac{1}{3} & -\frac{1}{3} & 0 & \frac{1}{3}\\ 0 & 0 & 1 & 0 & \frac{2}{3} & 3 & 0 & -\frac{4}{3}\\ 0 & 0 & 0 & 1 & -\frac{1}{3} & \frac{1}{3} & 1 & -\frac{1}{3}\end{array}\right)\sim$$

$$\sim\left(\begin{array}{cccc|cccc}1 & 2 & 0 & 0 & \frac{1}{3} & -\frac{1}{3} & 0 & \frac{4}{3}\\ 0 & 1 & 0 & 0 & \frac{1}{3} & -\frac{1}{3} & 0 & \frac{1}{3}\\ 0 & 0 & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & -\frac{4}{3}\\ 0 & 0 & 0 & 1 & -\frac{1}{3} & \frac{1}{3} & 1 & -\frac{1}{3}\end{array}\right)\sim\left(\begin{array}{cccc|cccc}1 & 0 & 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & 0 & \frac{2}{3}\\ 0 & 1 & 0 & 0 & \frac{1}{3} & -\frac{1}{3} & 0 & \frac{1}{3}\\ 0 & 0 & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & -\frac{4}{3}\\ 0 & 0 & 0 & 1 & -\frac{1}{3} & \frac{1}{3} & 1 & -\frac{1}{3}\end{array}\right),$$

$$A^{-1}=\frac{1}{3}\begin{pmatrix}-1 & 1 & 0 & 2\\ 1 & -1 & 0 & 1\\ 2 & 1 & 0 & -4\\ -1 & 1 & 3 & -1\end{pmatrix}.$$

Tekshirish: $\frac{1}{3}\begin{pmatrix}1 & 2 & 1 & 0\\ 2 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\\ 1 & 1 & 0 & 0\end{pmatrix}\begin{pmatrix}-1 & 1 & 0 & 2\\ 1 & -1 & 0 & 1\\ 2 & 1 & 0 & -4\\ -1 & 1 & 3 & -1\end{pmatrix}=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}.$

2-misol. f chiziqli operator (a) bazisda A matritsa orqali, φ chiziqli operator (b) bazisda B matritsa orqali berilgan bo'lsa, $4f+2\varphi$ operatorning bazisdagi matritsasini toping:

$(a): \bar{a}_1 = (1,1),\ \bar{a}_2 = (2,1);\ A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix};$

$(b): \bar{b}_1 = (2,3),\ \bar{b}_2 = (3,2);\ B = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}.$

Yechish. (a) bazisdan (e) bazisga o'tish matritsasini topamiz:

$$\bar{a}_1 = (1,1) = 1\cdot\bar{e}_1 + 1\cdot\bar{e}_2;\ \bar{a}_2 = (2,1) = 2\cdot\bar{e}_1 + 1\cdot\bar{e}_2 \text{ va } T = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}.$$

O'tish matritsasiga teskari matritsani topamiz:

$$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -1 & -1 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -1 \end{array}\right), \text{ bundan}$$

$$T^{-1} = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}.$$

3-misoldagi $M'(f) = T^{-1}M(f)T$ bog'lanishdan foydalanamiz:

$$M'(f) = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} =$$
$$= \begin{pmatrix} 5 & 2 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 10 \\ -1 & -3 \end{pmatrix}.$$

Demak, f operatorning (e) bazisdagi matritsasi $\begin{pmatrix} 5 & 10 \\ -1 & -3 \end{pmatrix}.$

Endi (b) bazisdan (e) bazisga o'tish matritsasini topamiz:

$$\bar{b}_1 = (2,3) = 2\cdot\bar{e}_1 + 3\cdot\bar{e}_2;\ \bar{b}_2 = (3,2) = 3\cdot\bar{e}_1 + 2\cdot\bar{e}_2.$$

O'tish matritsasi $T = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}$. Uning teskarisini topamiz:

$$\left(\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & -\frac{5}{2} & -\frac{3}{2} & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & -\frac{2}{5} & \frac{3}{5} \\ 0 & 1 & \frac{3}{5} & -\frac{2}{5} \end{array}\right).$$

Bundan $T^{-1} = \frac{1}{5}\begin{pmatrix} -2 & 3 \\ 3 & -2 \end{pmatrix}$.

Endi φ operatorning (e) bazisdagi matritsasini topamiz:

$$M'(\varphi) = \frac{1}{5}\begin{pmatrix} -2 & 3 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 6 & 7 \\ -2 & -3 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} =$$

$$= \frac{1}{5}\begin{pmatrix} 33 & 32 \\ -13 & -12 \end{pmatrix}.$$

Demak, (e) bazisda f operator matritsasi $\begin{pmatrix} 5 & 10 \\ -1 & -3 \end{pmatrix}$; φ operator matritsasi $\frac{1}{5}\begin{pmatrix} 33 & 32 \\ -13 & -12 \end{pmatrix}$ ga teng. U holda

$$4f + 2\varphi = 4\begin{pmatrix} 5 & 10 \\ -1 & -3 \end{pmatrix} + \frac{2}{5}\begin{pmatrix} 33 & 32 \\ -13 & -12 \end{pmatrix} = \begin{pmatrix} 20 & 40 \\ -4 & -12 \end{pmatrix} + \begin{pmatrix} \frac{66}{5} & \frac{64}{5} \\ -\frac{26}{5} & -\frac{24}{5} \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{166}{5} & \frac{264}{5} \\ -\frac{46}{5} & -\frac{84}{5} \end{pmatrix}.$$

3-misol. $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 3 & 1 & 0 \end{pmatrix}$ operatorning xos vektorlari va xos qiymatlarini toping.

Yechish. Berilgan operatorning xos qiymatlarini topish uchun $|\lambda E - A| = 0$ tenglikdan foydalanamiz:

$$\left|\lambda\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 3 & 1 & 0 \end{pmatrix}\right| = \begin{vmatrix} \lambda-1 & 0 & -1 \\ 0 & \lambda-1 & -1 \\ -3 & -1 & \lambda \end{vmatrix} = \lambda(\lambda-1)^2 - 3(\lambda-1) =$$

$$= (\lambda-1)\ (\lambda(\lambda-1)-4) = (\lambda-1)(\lambda^2-\lambda-4) = 0.$$

Bundan $\lambda_1 = 1, \lambda_{\frac{2}{3}} = \frac{1\pm\sqrt{17}}{2}$, xos qiymatlarni topamiz.

Endi, berilgan operatorning xos vektorlarini topish uchun $(A-\lambda E)X=0$ tenglamadan foydalanamiz. Bu tenglamalarning noldan farqli yechimlari berilgan operatorning xos vektorlari bo'ladi.

$\lambda_1=1$ uchun xos vektorlarni aniqlaymiz:

$$(A-E)X=0 \Leftrightarrow \left(\begin{pmatrix}1&0&1\\0&1&1\\3&1&0\end{pmatrix}-\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\right)\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{pmatrix}0&0&1\\0&0&1\\3&1&-1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix} \Leftrightarrow \begin{cases}x_3=0,\\x_3=0,\\3x_1+x_2-x_3=0\end{cases} \Leftrightarrow \begin{cases}x_1\in R,\\x_2=-3x_1,\\x_3=0.\end{cases}$$

Demak, $\lambda_1=1$ xos qiymat uchun berilgan operatorning xos vektorlari $\{(x_1,-3x_1,0)\,|\,x_1\in R\wedge x_1\neq 0\}$ to'plamdan iborat.

$\lambda_2=\frac{1-\sqrt{17}}{2}$ uchun xos vektorlarni topamiz:

$$(A-\lambda E)X=0 \Leftrightarrow \left(A-\frac{1-\sqrt{17}}{2}E\right)X=0 \Leftrightarrow$$

$$\Leftrightarrow \left(\begin{pmatrix}1&0&1\\0&1&1\\3&1&0\end{pmatrix}-\begin{pmatrix}\frac{1-\sqrt{17}}{2}&0&0\\0&\frac{1-\sqrt{17}}{2}&0\\0&0&\frac{1-\sqrt{17}}{2}\end{pmatrix}\right)\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{pmatrix}1-\frac{1-\sqrt{17}}{2}&0&1\\0&1-\frac{1-\sqrt{17}}{2}&1\\3&1&-\frac{1-\sqrt{17}}{2}\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \left(1-\frac{1-\sqrt{17}}{2}\right)x_1 + x_3 = 0, \\ \left(1-\frac{1-\sqrt{17}}{2}\right)x_2 + x_3 = 0, \\ 3x_1 + x_2 - 1 - \frac{1-\sqrt{17}}{2}x_3 = 0 \end{cases} \Leftrightarrow \begin{cases} \left(1-\frac{1-\sqrt{17}}{2}\right)x_1 + x_3 = 0, \\ \left(1-\frac{1-\sqrt{17}}{2}\right)x_2 + x_3 = 0, \\ \left(\frac{1-\sqrt{17}}{2}\right)x_2 + x_3 = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x_1 = -\frac{2}{1+\sqrt{17}}x_3 = 0, \\ x_2 = -\frac{2}{1+\sqrt{17}}x_3 = 0, \\ x_3 \in R. \end{cases}$$

Demak, $\lambda_2 = \frac{1-\sqrt{17}}{2}$ xos qiymat uchun operatorning xos vektorlari $\left\{\left(-\frac{2}{1+\sqrt{17}}x_3;\ -\frac{2}{1+\sqrt{17}}x_3;\ x_3\right) \middle| x_3 \in R \wedge x_3 \neq 0\right\}$ to'plamdan iborat.

$\lambda_3 = \frac{1+\sqrt{17}}{2}$ uchun xos vektorlarni topamiz:

$$\left(A - \frac{1+\sqrt{17}}{2}E\right)X = 0 \Leftrightarrow \left(\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 3 & 1 & 0 \end{pmatrix} - \begin{pmatrix} \frac{1+\sqrt{17}}{2} & 0 & 0 \\ 0 & \frac{1+\sqrt{17}}{2} & 0 \\ 0 & 0 & \frac{1+\sqrt{17}}{2} \end{pmatrix}\right)\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{pmatrix} 1-\frac{1+\sqrt{17}}{2} & 0 & 1 \\ 0 & 1-\frac{1+\sqrt{17}}{2} & 1 \\ 3 & 1 & -\frac{1+\sqrt{17}}{2} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \frac{1-\sqrt{17}}{2}x_1 + x_3 = 0, \\ \frac{1-\sqrt{17}}{2}x_2 + x_3 = 0, \\ 3x_1 + x_2 - \frac{1+\sqrt{17}}{2}x_3 = 0 \end{cases} \Leftrightarrow \begin{cases} \frac{1-\sqrt{17}}{2}x_1 + x_3 = 0, \\ \frac{1-\sqrt{17}}{2}x_2 + x_3 = 0, \\ \frac{1-\sqrt{17}}{2}x_2 - x_3 = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \frac{1-\sqrt{17}}{2}x_1 + x_3 = 0, \\ \frac{1-\sqrt{17}}{2}x_2 + x_3 = 0, \\ -2x_3 = 0 \end{cases} \Leftrightarrow \begin{cases} x_1 = 0, \\ x_2 = 0, \\ x_3 = 0. \end{cases}$$

Demak, $\lambda = \frac{1-\sqrt{17}}{2}$ xos qiymat uchun xos vektorlar mavjud emas.

Misol va mashqlar

1. f chiziqli operator $\vec{a}_1$, $\vec{a}_2$ bazisda A matritsa, φ chiziqli operator $\vec{b}_1$, $\vec{b}_2$ bazisda V matritsa yordamida berilgan bo'lsa, $f+\varphi$ operatorning matritsasini toping:

1.1. $\vec{a}_1=(1, -2)$, $\vec{a}_2=(3, -5)$, $A = \begin{pmatrix} 37 & -13 \\ 108 & -38 \end{pmatrix}$,

$\vec{b}_1=(1, 2)$, $\vec{b}_2=(2, 5)$, $B = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$.

1.2. $\vec{a}_1=(7, 3)$, $\vec{a}_2=(2, 1)$, $A = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$.

$\vec{b}_1=(6, 1)$, $\vec{b}_2=(5, 1)$, $B = \begin{pmatrix} 3 & -2 \\ 6 & -6 \end{pmatrix}$.

2. f chiziqli operator $\vec{a}_1=(-3, 1)$, $\vec{a}_2=(7, 2)$ bazisda $A = \begin{pmatrix} 2 & -1 \\ -2 & 2 \end{pmatrix}$ matritsa, φ chiziqli operator $\vec{b}_1=(3, 2)$, $\vec{b}_2=(4, 3)$

bazisda $B=\begin{pmatrix}-14 & 10\\ -21 & 15\end{pmatrix}$ matritsa yordamida berilgan bo'lsa, $\varphi\varphi$ operatorning matritsasini toping.

3. f chiziqli operator (a) bazisda A matritsa orqali, φ chiziqli operator (b) bazisda B matritsa orqali berilgan bo'lsa, $\lambda f+\mu\varphi$ operatorning (e) bazisdagi matritsasini toping:

3.1. (a): $\vec{a}_1(1,2)$, $\vec{a}_2(0,1)$; $A=\begin{pmatrix}2 & 1\\ 3 & -1\end{pmatrix}$;

(b): $\vec{b}_1(3,1)$, $\vec{b}_2(2,1)$; $B=\begin{pmatrix}-1 & 1\\ 4 & 2\end{pmatrix}$; $3f+\varphi$.

3.2. (a): $\vec{a}_1(-1,3)$, $\vec{a}_2(1,1)$; $A=\begin{pmatrix}2 & 1\\ 3 & -1\end{pmatrix}$;

(b): $\vec{b}_1(-2,1)$, $\vec{b}_2(2,4)$; $B=\begin{pmatrix}-1 & 1\\ 4 & 2\end{pmatrix}$; $-2f+3\varphi$.

3.3. (a): $\vec{a}_1(8,9)$, $\vec{a}_2(3,1)$; $A=\begin{pmatrix}7 & -2\\ 1 & 5\end{pmatrix}$;

(b): $\vec{b}_1(4,5)$, $\vec{b}_2(1,-4)$; $B=\begin{pmatrix}6 & -5\\ 0 & -2\end{pmatrix}$; $f-4\varphi$.

3.4. (a): $\vec{a}_1(1,2)$, $\vec{a}_2(3,4)$; $A=\begin{pmatrix}-4 & 5\\ -7 & 6\end{pmatrix}$;

(b): $\vec{b}_1(-1,-2)$, $\vec{b}_2(-3,-4)$; $B=\begin{pmatrix}1 & -8\\ -9 & 10\end{pmatrix}$; $5f-2\varphi$.

3.5. (a): $\vec{a}_1(-7,6)$, $\vec{a}_2(5,-4)$; $A=\begin{pmatrix}0 & 2\\ -3 & 4\end{pmatrix}$;

(b): $\vec{b}_1(2,-1)$, $\vec{b}_2(-4,3)$; $B=\begin{pmatrix}3 & -5\\ -4 & 0\end{pmatrix}$; $-3f+\varphi$.

3.6. (a): $\vec{a}_1(1,-1)$, $\vec{a}_2(-2,1)$; $A=\begin{pmatrix}3 & 7\\5 & 4\end{pmatrix}$;

(b): $\vec{b}_1(2,1)$, $\vec{b}_2(1,3)$; $B=\begin{pmatrix}1 & 5\\3 & 2\end{pmatrix}$; $2f+5\varphi$.

4. $C=\{a+bi\,|\,\forall a,b\in R,\ i^2=-1\}$ to'plam R maydon ustida rangi ikkiga teng bo'lgan chiziqli algebra tashkil etishini isbotlang.

5. Barcha n-tartibli kvadrat matritsalar to'plami $F^{n\times n}$ ning $\mathcal{F}$ maydon ustida rangi n^2 bo'lgan chiziqli algebra tashkil etishini isbotlang.

6. Berilgan operatorlarga teskari operatorni toping:

6.1. $A=\begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}$. 6.2. $A=\begin{pmatrix}0 & -1 & -1\\1 & 2 & 3\\-2 & 1 & -1\end{pmatrix}$.

6.3. $A=\begin{pmatrix}1 & 0 & 1 & 2\\0 & 1 & -1 & -1\\3 & 1 & 2 & 4\\4 & 2 & 3 & 1\end{pmatrix}$. 6.4. $A=\begin{pmatrix}1 & 1 & 1 & 1\\0 & 1 & 1 & 1\\0 & 0 & 1 & 1\\0 & 0 & 0 & 1\end{pmatrix}$.

6.5. $A=\begin{pmatrix}5 & 4 & 62 & -79\\0 & 0 & 2 & 3\\6 & 5 & 183 & 201\\0 & 0 & 3 & 4\end{pmatrix}$. 6.6. $A=\begin{pmatrix}3 & 2 & 1 & 2\\7 & 5 & 2 & 5\\0 & 0 & 9 & 4\\0 & 0 & 11 & 5\end{pmatrix}$.

6.7. $A=\begin{pmatrix}2 & 5 & 4 & 1\\1 & 3 & 2 & 1\\2 & 10 & 9 & 7\\3 & 8 & 9 & 2\end{pmatrix}$. 6.8. $A=\begin{pmatrix}3 & 5 & -3 & 2\\4 & -2 & 5 & 3\\7 & 8 & -1 & 5\\6 & 4 & 5 & 3\end{pmatrix}$.

6.9. $A=\begin{pmatrix}1 & 2 & 3 & 4\\2 & 3 & 1 & 2\\1 & 1 & 1 & -1\\1 & 0 & -2 & -6\end{pmatrix}$. 6.10. $A=\begin{pmatrix}0 & 0 & 0 & 1 & -1\\0 & 0 & 1 & 0 & 2\\0 & 1 & 0 & 0 & 2\\1 & 0 & 0 & 0 & 1\\1 & -2 & 4 & 1 & 5\end{pmatrix}$.

7. Quyidagi chiziqli operatorlarning xos qiymatlari va xos vektorlarini toping:

7.1. $A = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$. 7.2. $A = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$.

7.3. $A = \begin{pmatrix} 1 & -3 & 0 \\ -1 & -2 & 3 \\ -1 & -4 & 4 \end{pmatrix}$. 7.4. $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 2 \\ 0 & -2 & 3 \end{pmatrix}$.

7.5. $A = \begin{pmatrix} 6 & 1 & -5 \\ 3 & 12 & -3 \\ 7 & 1 & -6 \end{pmatrix}$. 7.6. $A = \begin{pmatrix} 0 & 1 & 0 \\ -4 & 4 & 0 \\ -2 & 1 & 2 \end{pmatrix}$.

7.7. $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. 7.8. $A = \begin{pmatrix} 3 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 3 & 0 & 5 & -3 \\ 4 & -1 & 3 & -1 \end{pmatrix}$.

7.9. $A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 4 \\ 0 & 1 & 1 & 2 \end{pmatrix}$. 7.10. $A = \begin{pmatrix} 3 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 3 & 0 & 5 & -3 \\ 4 & -1 & 3 & -1 \end{pmatrix}$.

Takrorlash uchun savollar

1. Chiziqli operatorlar ustida qanday amallar bajariladi?
2. Chiziqli algebra deb nimaga aytiladi?
3. Chiziqli operatorlar algebrasi deb nimaga aytiladi?
4. Matritsalar chiziqli algebrasini tushuntiring.
5. Chiziqli operatorning teskarisi qanday topiladi?
6. Chiziqli operatorning xos qiymatlari, xos vektorlari deb nimaga aytiladi?

X MODUL. CHIZIQLI TENGSIZLIKLAR SISTEMASI

23-§. Chiziqli tengsizliklar sistemasi. Qavariq konus

✓ **Asosiy tushunchalar:** chiziqli tengsizliklar sistemasi, yechim, chiziqli kombinatsiya, qavariq konus, yo'ldosh sistema.

Ushbu

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n + b \geq 0 \qquad (1)$$

tengsizlik R haqiqiy sonlar maydoni ustidagi *n ta noma'lumli chiziqli tengsizlik* deyiladi. (1) da x_1, x_2, ..., x_n – noma'lumlar, a_i, $b \in R$ ($i = \overline{1,n}$) esa koeffitsiyentlar deyiladi.

Agar (1) da b=0 bo'lsa, (1) bir jinsli, $b \neq 0$ bo'lsa, (1) *bir jinsli bo'lmagan chiziqli tengsizlik* deyiladi.

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n + b_i \geq 0, \; i = \overline{1,m} \qquad (2)$$

sistemaning barcha tengsizliklarini qanoatlantiruvchi $x_1=\alpha_1$, $x_2=\alpha_2$, ..., $x_n=\alpha_n$ sonlar (2) *sistemaning yechimi* deyiladi.

Agar (2) tengsizlik bitta ham yechimga ega bo'lmasa, ya'ni $0 \cdot x_1 + 0 \cdot x_2 + \ldots + 0 \cdot x_n + b \geq 0$ ($b < 0$) bo'lsa, u *ziddiyatli tengsizlik* deyiladi.

(2) sistemaning tengsizliklarini, mos ravishda, $k_1 \geq 0$, $k_2 \geq 0$, ..., $k_m \geq 0$ sonlarga ko'paytirib, ularni hadlab qo'shsak, hosil bo'lgan ushbu

$$\sum_{j=1}^{m} k_j a_{j1} x_1 + \sum_{j=1}^{m} k_j a_{j2} x_2 + \ldots + \sum_{j=1}^{m} k_j a_{jm} x_m + \sum_{j=1}^{m} k_j b_j \geq 0$$

tengsizlik (2) *sistemaning manfiymas chiziqli kombinatsiyasi* deyiladi.

Bir xil x_1, x_2, ..., x_n noma'lumli ikkita hamjoyli tengsizliklar sistemasidan birining istalgan yechimi ikkinchisi uchun ham yechim bo'lsa yoki ikkala sistema ham hamjoysiz sistema bo'lsa, ular *teng kuchli sistemalar* deyiladi.

Vektorlarni qo'shish va manfiymas haqiqiy songa ko'paytirish amallariga nisbatan yopiq bo'lgan V vektor fazoning vektorlaridan tuzilgan bo'sh bo'lmagan to'plam *V vektor fazoning qavariq konusi* deyiladi.

Chiziqli tengsizliklar sistemasidan noma'lumlar sonini bittaga kamaytirib tuzilgan yangi sistema *berilgan sistemaga yo'ldosh sistema* deyiladi.

(2) sistemadan

$$\begin{cases} P_1 \geq x_n, \\ P_2 \geq x_n, \\ \cdots\cdots \\ P_p \geq x_n, \end{cases} \quad \begin{cases} x_n \geq Q_1, \\ x_n \geq Q_2, \\ \cdots\cdots \\ x_n \geq Q_n, \end{cases} \quad \begin{cases} R_1 \geq 0, \\ R_2 \geq 0, \\ \cdots\cdots \\ R_r \geq 0 \end{cases} \tag{3}$$

sistemani hosil qilamiz.

Bundan

$$\begin{cases} P_\alpha \geq Q_\beta \ (\alpha = \overline{1,p}, \ \beta = \overline{1,q}), \\ R_\gamma \geq 0 \ (\gamma = \overline{1,r}) \end{cases}$$

sistemani hosil qilamiz.

Agar (3) sistemada birinchi yoki ikkinchi blok tengsizliklari bo'lmasa, u holda yo'ldosh sistema faqat $R_\gamma \geq 0$ tengsizliklardan iborat bo'ladi. Agar (3) sistemada birinchi va uchinchi yoki ikkinchi va uchinchi bloklar bo'lmasa, u holda yo'ldosh sistema mavjud emas. Ya'ni bu sistemani ayniy ($0 \geq 0$) deb qarash va uning yechimlari sifatida ixtiyoriy n o'lchovli arifmetik vektorni olish mumkin.

1-misol. Chiziqli tengsizliklar sistemasini algebraik va geometrik usullarda yeching:

$$\begin{cases} 2x_1 + x_2 \leq 1, \\ 3x_1 - x_2 \leq 2, \\ x_1 + 2x_2 \leq 4, \\ -x_1 - 3x_2 \leq 3. \end{cases}$$

Yechish. **1) Algebraik usul:**

$$\begin{cases} 2x_1 + x_2 \le 1, \\ 3x_1 - x_2 \le 2, \\ x_1 + 2x_2 \le 4, \\ -x_1 - 3x_2 \le 3 \end{cases} \Leftrightarrow \begin{cases} 2x_1 \le 1 - x_2, \\ 3x_1 \le 2 + x_2, \\ x_1 \le 4 - 2x_2, \\ -3x_2 \le 3 + x_1, \end{cases} \Leftrightarrow \begin{cases} x_1 \le \frac{1}{2} - \frac{1}{2}x_2, \\ x_1 \le \frac{2}{3} + \frac{1}{3}x_2, \\ x_1 \le 4 - 2x_2, \\ -3x_2 - 3 \le x_1 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} -3x_2 - 3 \le \frac{1}{2} - \frac{1}{2}x_2, \\ -3x_2 - 3 \le \frac{2}{3} + \frac{1}{3}x_2, \\ -3x_2 - 3 \le 4 - 2x_2 \end{cases} \Leftrightarrow \begin{cases} -\frac{7}{2} \le \frac{5}{2}x_2, \\ -\frac{11}{3} \le \frac{10}{3}x_2, \\ -7 \le x_2 \end{cases} \Leftrightarrow \begin{cases} x_2 \ge -\frac{7}{5}, \\ x_2 \ge -\frac{11}{10}, \\ x_2 \ge -7 \end{cases} \Leftrightarrow x_2 \ge -\frac{11}{10}.$$

$x_2 = -1$ deb olamiz. U holda

$$\begin{cases} x_1 \le \frac{1}{2} + \frac{1}{2}, \\ x_1 \le \frac{2}{3} - \frac{1}{3}, \\ x_1 \le 4 + 2, \\ 3 - 3 \le x_1 \end{cases} \Leftrightarrow \begin{cases} x_1 \le 1, \\ x_1 \le \frac{1}{3}, \\ x_1 \le 6, \\ x_1 \ge 0 \end{cases} \Leftrightarrow 0 \le x_1 \le \frac{1}{3}.$$

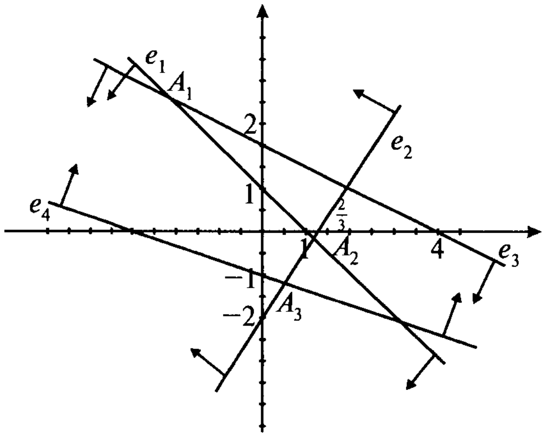
$x_1 = 0$ deb olsak, u holda berilgan chiziqli tengsizliklar sistemasining xususiy yechimi sifatida $(0; -1)$ vektorni olish mumkin.

2) Geometrik usul:

$$\begin{cases} 2x_1 + x_2 \le 1, \\ 3x_1 - x_2 \le 2, \\ x_1 + 2x_2 \le 4, \\ -x_1 - 3x_2 \le 3 \end{cases}$$

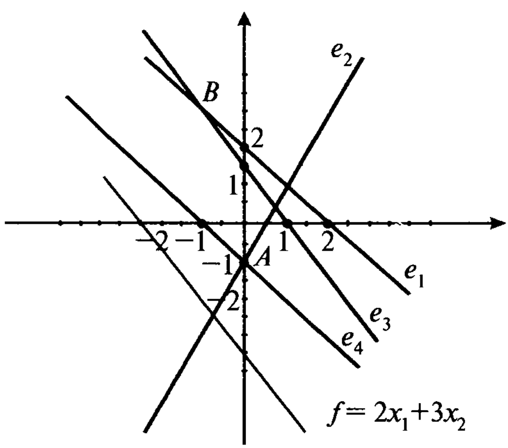
chiziqli tengsizliklar sistemasini tashkil etgan 4 ta tengsizlikning har biri tekislikda yarimtekislikni bildiradi. Ularning umumiy qismi berilgan sistemaning yechimi bo‘ladi.

Chiziqli tengsizliklar sistemasining yechimi Dekart koordinatalar sistemasidagi $A_1A_2A_3$ siniq chiziq bilan chegaralangan sohadan iborat.



2-misol. $\begin{cases} x_1 + x_2 \le 2, \\ 2x_1 - x_2 \le 1, \\ 3x_1 + 2x_2 \le 3, \\ -x_1 - x_2 \le 1 \end{cases}$ chiziqli tengsizliklar sistemasini geometrik usulda yechib, uning manfiymas yechimlari orasidan berilgan $f = 2x_1 + 3x_2$ chiziqli formani minimallashtiruvchi va maksimallashtiruvchi nuqtalarini aniqlang.

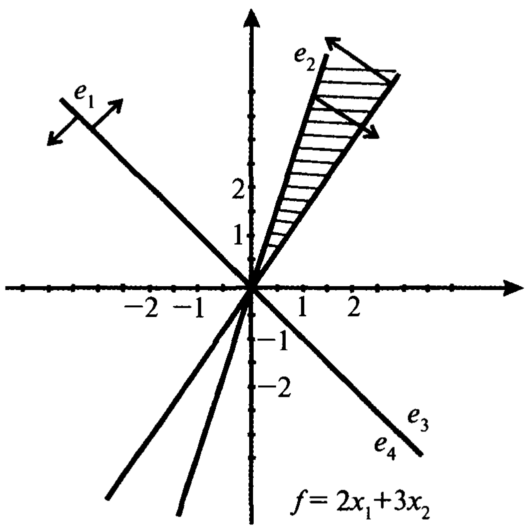
Yechish.



f chiziqli formani minimallashtiruvchi nuqtalar $A(-1; 0)$, maksimallashtiruvchi nuqta $B(-1; 3)$.

3-misol. $\begin{cases} 2x_1 + x_2 \geq 0, \\ 3x_1 - x_2 \geq 0, \\ -x_1 + 2x_2 \geq 0 \end{cases}$ chiziqli tengsizliklar sistemasi yechimlarining qavariq konusini tekislikda tasvirlang.

Yechish. Bir jinsli chiziqli tengsizliklar sistemasi yechimlarining qavariq konusi uning nolmas yechimlaridan iborat. Shuning uchun tekislikda berilgan bir jinsli chiziqli tengsizliklar sistemasining nolmas yechimlarini topamiz.



Demak, berilgan chiziqli tengsizliklar sistemasi yechimlar to'plami tashkil etgan qavariq konus chizmadagi shtrixlangan sohadan iborat.

4-misol. $\begin{cases} 2x_1 + 3x_2 - x_3 \geq 1, \\ -x_1 + 2x_2 + 3x_3 \leq 2, \\ -3x_1 - x_2 - 2x_3 \geq -2 \end{cases}$ chiziqli tengsizliklar sistemasini yeching.

Yechish. Berilgan chiziqli tengsizliklar sistemasidan x_2 ni yo'qotamiz. Buning uchun berilgan tengsizliklar sistemasiga yo'ldosh sistemani hosil qilamiz:

$$\begin{cases} 2x_1+3x_2-x_3\ge 1, \\ -x_1+2x_2+3x_3\le 2, \\ -3x_1-x_2-2x_3\ge -2 \end{cases} \Leftrightarrow \begin{cases} 3x_2\ge -2x_1+x_3+1, \\ 2x_2\le x_1-3x_3+2, \\ x_2\le -3x_1-2x_3+2 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x_2\ge -\frac{2}{3}x_1+\frac{1}{3}x_3+\frac{1}{3}, \\ x_2\ge \frac{1}{2}x_1-\frac{3}{2}x_3+1, \\ x_2\le -3x_1-2x_3+2 \end{cases} \Leftrightarrow \begin{cases} -\frac{2}{3}x_1+\frac{1}{3}x_3+\frac{1}{3}\le \frac{1}{2}x_1-\frac{3}{2}x_3+1, \\ -\frac{2}{3}x_1+\frac{1}{3}x_3+\frac{1}{3}\le -3x_1-2x_3+2 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \left(-\frac{2}{3}-\frac{1}{2}\right)x_1+\left(\frac{1}{3}+\frac{3}{2}\right)x_3+\left(\frac{1}{3}-1\right)\le 0, \\ \left(-\frac{2}{3}+3\right)x_1+\left(\frac{1}{3}+2\right)x_3+\left(\frac{1}{3}-2\right)\le 0 \end{cases} \Leftrightarrow \begin{cases} -\frac{7}{6}x_1+\frac{11}{6}x_3-\frac{2}{3}\ge 0, \\ \frac{7}{3}x_1+\frac{7}{3}x_3-\frac{5}{3}\le 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x_1\le -x_3+\frac{5}{7}, \\ x_1\ge \frac{11}{7}x_3-\frac{4}{7} \end{cases} \Leftrightarrow \frac{11}{7}x_3-\frac{4}{7}\le -x_3+\frac{5}{7} \Leftrightarrow$$

$$\Leftrightarrow \left(\frac{11}{7}+1\right)x_3\le \frac{5}{7}+\frac{4}{7} \Leftrightarrow \frac{18}{7}x_3\le \frac{9}{7} \Leftrightarrow 2x_3\le 1 \Leftrightarrow x_3\le \frac{1}{2}.$$

x_3 ning topilgan sohasidan $x_3=0$ qiymatni olsak, $\begin{cases} x_1\le \frac{5}{7}, \\ x_1\ge -\frac{4}{7}, \end{cases}$

ya'ni $-\frac{4}{7}\le x_1\le \frac{5}{7}$ hosil bo'ladi. Agar $x_1=0$ deb olsak, u holda,

$\begin{cases} x_2\ge \frac{1}{3}, \\ x_2\le 1, \\ x_2\le 2 \end{cases}$ ya'ni $\frac{1}{3}\le x_2\le 1$ hosil bo'ladi.

Demak, berilgan chiziqli tengsizliklar sistemasining xususiy yechimlaridan biri (0, 1, 0) bo'ladi.

Misol va mashqlar

1. Quyidagi tengsizliklar sistemalarining yechimlar sohasini aniqlang:

1.1. $\begin{cases} x_1 + x_2 \geq -2, \\ 2x_1 - x_2 \geq -1. \end{cases}$ 1.2. $\begin{cases} x_1 - x_2 \geq -1, \\ 2x_1 - x_2 \geq 3. \end{cases}$

1.3. $\begin{cases} 3x_1 - 2x_2 \geq 6, \\ x_1 \leq 3. \end{cases}$ 1.4. $\begin{cases} 4x_1 - 3x_2 \leq 1, \\ x_1 + x_2 \geq 0. \end{cases}$

1.5. $\begin{cases} 2x_1 + x_2 \leq -3, \\ x_1 - 3x_2 \leq 1, \\ 4x_1 + 3x_2 \leq 2. \end{cases}$ 1.6. $\begin{cases} -x_1 + x_2 \geq 4, \\ x_1 - 3x_2 \leq -1, \\ 3x_1 - 3x_2 \leq -2. \end{cases}$

1.7. $\begin{cases} 4x_1 + 5x_2 - 20 \leq 0, \\ -7x_1 + 3x_2 - 12 \leq 0, \\ -3x_1 + 8x_2 + 15 \geq 0. \end{cases}$ 1.8. $\begin{cases} 2x_1 - x_2 - 3 \leq 0, \\ x_1 + x_2 - 6 \leq 0, \\ x_1 - 2x_2 \leq 0. \end{cases}$

1.9. $\begin{cases} 8x_1 + 8x_2 - 64 \leq 0, \\ 4x_1 - 4x_2 - 16 \leq 0, \\ -4x_1 + 4x_2 - 16 \leq 0, \\ 8x_1 + 8x_2 + 64 \geq 0. \end{cases}$ 1.10. $\begin{cases} x_1 - x_2 \leq 0, \\ -x_1 + x_2 - 4 \leq 0, \\ x_1 - x_2 + 6 \geq 0, \\ 2x_1 - 2x_2 + 8 \leq 0. \end{cases}$

2. Chiziqli tengsizliklar sistemasining har bir manfiymas chiziqli kombinatsiyasi shu sistemaning natijasi bo‘lishini isbotlang.

3. Quyidagi tengsizliklar sistemalarining biri ikkinchisiga natija bo‘ladimi? Teng kuchli sistemalarni aniqlang:

3.1. 1) $x_1 - x_2 \leq 0,$ 2) $x_1 - x_2 + 6 \geq 0.$

3.2. 1) $-x_1 + x_2 - 4 \leq 0,$ 2) $x_1 - x_2 + 6 \geq 0.$

3.3. 1) $\begin{cases} 4x_1 + 5x_2 - 20 \leq 0, \\ -7x_1 + 3x_2 - 12 \leq 0, \\ -3x_1 + 8x_2 + 15 \geq 0, \end{cases}$ 2) $x_2 - 5 \leq 0.$

3.4. 1) $\begin{cases} x_1 - 2x_2 \leq 0, \\ -2x_1 + x_2 + 3 \leq 0, \\ x_1 + x_2 - 10 \leq 0; \end{cases}$ 2) $\begin{cases} 2x_1 - x_2 - 3 \geq 0, \\ x_1 + x_2 - 6 \leq 0, \\ x_1 - 2x_2 \leq 0. \end{cases}$.

3.5. 1) $\begin{cases} x_1 - x_2 \geq 0, \\ x_2 - 3 \geq 0, \\ 4x_1 + 4x_2 + 16 \leq 0; \end{cases}$ 2) $\begin{cases} x_1 + x_2 - 3 \leq 0, \\ -x_1 - x_2 + 5 \leq 0. \end{cases}$

3.6. 1) $\begin{cases} 8x_1 + 8x_2 - 64 \leq 0, \\ 4x_1 - 4x_2 - 16 \leq 0, \\ -4x_1 + 4x_2 - 16 \leq 0, \\ 8x_1 + 8x_2 + 64 \geq 0. \end{cases}$ 2) $5x_1 - 3x_2 - 30 \leq 0.$

4. $\bar{a} \in R^n$ va $\bar{a} \neq \bar{0}$ lar uchun $\{\lambda\bar{a} \mid \lambda \geq 0, \lambda \in R\}$ to‘plam R^n fazoning qavariq konusi bo‘lishini isbotlang. Bu qavariq konus $\bar{a}$ vektor yaratgan to‘g‘ri chiziq deyiladi.

5. $\bar{a}_1, \ldots, \bar{a}_m \in R^n$ vektorlar sistemasining barcha manfiymas chiziqli kombinatsiyalar to‘plami R^n fazoning qavariq konusi bo‘lishini isbotlang.

6. Bir jinsli chiziqli tengsizliklar sistemasining barcha yechimlar to‘plami $V = R^n$ fazoning qavariq konusi bo‘lishini isbotlang.

7. Chiziqli tengsizliklar sistemasi hamjoysiz bo‘lishi uchun uning biror chiziqli kombinatsiyasi ziddiyatli tengsizlik bo‘lishi zarur va yetarli ekanligini isbotlang.

8. Bir jinsli chiziqli tengsizliklar sistemasining har bir natijasi bu sistemaning manfiymas koeffitsiyentli chiziqli kombinatsiyasidan iboratligini isbotlang.

9. Yo‘ldosh sistemaning har bir tengsizligi berilgan tengsizliklar sistemasining chiziqli kombinatsiyasi bo‘lishini isbotlang.

10. Chiziqli tengsizliklar sistemasini algebraik va geometrik usullarda yeching:

10.1. $\begin{cases} -2x_1 + 5x_2 \leq -1, \\ x_1 + 3x_2 \leq 13, \\ -3x_1 + 2x_2 \leq 5, \\ -x_1 + 3x_2 \leq 2. \end{cases}$ 10.2. $\begin{cases} 5x_1 + 3x_2 \geq 3, \\ x_1 - x_2 \geq 6, \\ 2x_1 - x_2 \geq 1, \\ -x_1 + 8x_2 \geq 2. \end{cases}$

10.3. $\begin{cases} 8x_1 + 4x_2 \le 1, \\ -8x_1 - 4x_2 \le -1, \\ 2x_1 - 3x_2 \le 5, \\ -2x_1 + 3x_2 \le -5. \end{cases}$ 10.4. $\begin{cases} 11x_1 - 3x_2 \le 30, \\ 2x_1 + 5x_2 \le -6, \\ 12x_1 - 6x_2 \le 4, \\ -5x_1 + 7x_2 \le 12. \end{cases}$

10.5. $\begin{cases} 5x_1 - 9x_2 \le 4, \\ -5x_1 + 9x_2 \le -4, \\ -2x_1 + 3x_2 \le 5, \\ 2x_1 - 3x_2 \ge -5, \\ x_1 + x_2 \le -1. \end{cases}$ 10.6. $\begin{cases} 2x_1 + 5x_2 \le -10, \\ x_1 + 3x_2 \le 2, \\ 6x_1 + 7x_2 \ge 5, \\ 4x_1 + 3x_2 \le 12. \end{cases}$

10.7. $\begin{cases} -2x_1 + x_2 \ge -1, \\ 4x_1 + 3x_2 \le 3, \\ -x_1 + 7x_2 \ge 5, \\ -4x_1 + 3x_2 \le -2. \end{cases}$ 10.8. $\begin{cases} 5x_1 - x_2 \le 4, \\ 3x_1 + 11x_2 \le 2, \\ 6x_1 + 7x_2 \ge -5, \\ -x_1 + 3x_2 \le 0. \end{cases}$

11. Chiziqli tengsizliklar sistemasi yechimlarining qavariq konusini tekislikda tasvirlang:

11.1. $\begin{cases} x_1 + x_2 \ge 0, \\ 2x_1 - x_2 \le 0. \end{cases}$ 11.2. $\begin{cases} x_1 - x_2 \ge 0, \\ 2x_1 - x_2 \ge 0. \end{cases}$

11.3. $\begin{cases} -x_1 + x_2 \ge 0, \\ 2x_1 - 3x_2 \ge 0. \end{cases}$ 11.4. $\begin{cases} 4x_1 - 3x_2 \le 0, \\ x_1 + x_2 \ge 0. \end{cases}$

11.5. $\begin{cases} 9x_1 + 8x_2 \ge 0, \\ 2x_1 + 3x_2 \ge 0, \\ -4x_1 + 2x_2 \ge 0, \\ -7x_1 + 3x_2 \ge 0. \end{cases}$ 11.6. $\begin{cases} 3x_1 - 4x_2 \ge 0, \\ -2x_1 + 13x_2 \ge 0, \\ -4x_1 + 3x_2 \ge 0, \\ 7x_1 + 3x_2 \ge 0. \end{cases}$

11.7. $\begin{cases} 2x_1 + 8x_2 \ge 0, \\ x_1 + 3x_2 \ge 0, \\ -x_1 + 2x_2 \ge 0, \\ -3x_1 + 3x_2 \ge 0. \end{cases}$ 11.8. $\begin{cases} 4x_1 + 8x_2 \ge 0, \\ 6x_1 - 3x_2 \ge 0, \\ 7x_1 + 2x_2 \ge 0, \\ -x_1 - 3x_2 \ge 0. \end{cases}$

11.9. $\begin{cases} x_1 + x_2 - 8 \le 0, \\ x_1 - x_2 - 4 \le 0, \\ -x_1 + x_2 - 3 \le 0, \\ 2x_1 + 2x_2 + 16 \ge 0. \end{cases}$ 11.10. $\begin{cases} -8x_1 + 8x_2 + 64 \le 0, \\ 4x_1 - 4x_2 - 16 \le 0, \\ -4x_1 + 4x_2 - 6 \le 0, \\ -8x_1 + 8x_2 - 64 \ge 0. \end{cases}$

12. Chiziqli tengsizliklar sistemasini yeching:

12.1. $\begin{cases} 6x_1 - 5x_2 + 2x_4 \le 11, \\ 2x_1 + 4x_2 - x_3 + 3x_4 \le -1, \\ x_1 + 2x_2 + 2x_3 + 13x_4 \ge 10, \\ 2x_1 - 4x_2 + 7x_3 - 2x_4 \ge 0. \end{cases}$

12.2. $\begin{cases} 6x_1 - 5x_2 + x_3 - 2x_4 \le 11, \\ x_1 + 4x_2 - x_3 + 3x_4 \le 1, \\ 2x_2 + 2x_3 + 13x_4 \le 17, \\ 2x_1 - 7x_3 - 2x_4 \ge 0. \end{cases}$

12.3. $\begin{cases} -2x_1 + 3x_2 - 6x_3 - x_4 \le 11, \\ 3x_1 + 6x_1 + 5x_2 - 12x_4 \le 2, \\ x_1 - 7x_2 + x_3 + 4x_4 \ge 23, \\ x_2 + 23x_4 \le 2, \\ -2x_1 + 7x_2 + 2x_3 + 2x_4 \le 14. \end{cases}$

12.4. $\begin{cases} -2x_1 + 3x_2 + 3x_3 + 2x_4 \le 4, \\ 2x_1 + 4x_2 - 3x_3 + 3x_4 \le -1, \\ 13x_1 - x_2 + 2x_3 + 11x_4 \ge 0, \\ 12x_1 + 4x_2 - 6x_3 + 2x_4 \le 10. \end{cases}$

12.5. $\begin{cases} -4x_1 + 5x_2 + 3x_3 + 4x_4 \le 1, \\ -2x_1 + 3x_2 - 9x_3 - x_4 \ge 2, \\ -9x_1 + 10x_2 - x_3 \ge 3. \end{cases}$

12.6. $\begin{cases} -9x_1 + 5x_2 + 3x_3 - 5x_4 \le 1, \\ 3x_1 + 3x_2 - 9x_3 + x_4 \le 2, \\ 7x_1 + 10x_2 - x_3 + x_4 \le 3. \end{cases}$

12.7. $\begin{cases} 2x_1 + 5x_2 + 3x_3 - 3x_4 \geq 1, \\ x_1 + 3x_2 - 9x_3 + x_4 \leq 2, \\ 2x_1 + 10x_2 - x_3 + 3x_4 \leq 3. \end{cases}$

12.8. $\begin{cases} 2x_1 - 3x_2 - 9x_3 + 5x_4 \leq 11, \\ 3x_1 + x_2 + 8x_3 - 3x_4 \leq 0, \\ -2x_1 - 4x_2 - 2x_3 \geq 2. \end{cases}$

12.9. $\begin{cases} x_1 + 2x_2 + x_3 - x_4 + x_5 \geq 21, \\ 2x_1 - 3x_2 - 9x_3 - 3x_4 \geq 3, \\ 3x_1 + x_2 + 8x_3 + 2x_4 + x_5 \leq 3. \end{cases}$

12.10. $\begin{cases} x_1 - x_3 + x_3 + 5x_4 - 4x_5 \geq 7, \\ x_1 - x_2 + x_3 + 3x_4 - 5x_5 \leq 6, \\ x_1 - 3x_2 - x_3 + x_4 \geq 9. \end{cases}$

12.11. $\begin{cases} -2x_1 - x_2 + 3x_3 + 4x_4 \leq -11, \\ -3x_1 + 3x_2 - 9x_3 - x_4 \geq 2, \\ x_1 + 10x_2 - x_3 \geq 3. \end{cases}$

12.12. $\begin{cases} 2x_1 - x_2 + 5x_3 - 5x_4 \leq 3, \\ x_1 + 3x_2 - 4x_3 + x_4 \geq 2, \\ -x_1 + 10x_2 - x_3 + x_4 \leq 3. \end{cases}$

12.13. $\begin{cases} 5x_1 - 2x_2 + 3x_3 - 4x_4 \leq 1, \\ x_1 + 3x_2 - 9x_3 - x_4 \geq 2, \\ 5x_1 + 3x_2 - x_3 \geq 3. \end{cases}$

12.14. $\begin{cases} 3x_1 + x_2 - 2x_3 - x_4 \leq 4, \\ -2x_1 + 3x_2 - 4x_3 + x_4 \leq 2, \\ -3x_1 - x_2 - x_3 + x_4 \leq 3. \end{cases}$

13. f chiziqli formaning minimum qiymatini va uni minimum qiymatga keltiruvchi nuqtani toping:

13.1. $f = -x_2 + x_1,\quad \begin{cases} x_1 + 3x_2 \leq 12, \\ 3x_1 - x_2 \geq 6, \\ 3x_1 + 4x_2 \geq 0, \\ x_1 \geq 0, \\ 3x_2 \geq 0. \end{cases}$

13.2. $f = x_1 - 4x_2,$ $\begin{cases} x_1 + 2x_2 \le 4, \\ x_1 \le 3, \\ x_1 - 2x_2 \ge -1, \\ x_1 \ge 0, \\ 3x_2 \ge 0. \end{cases}$

13.3. $f = 2x_1 - x_2,$ $\begin{cases} 2x_1 - x_2 \le 12, \\ x_1 + x_2 \le 6, \\ x_1 + 3x_2 \ge 1, \\ x_1 \ge 0, \\ 3x_2 \ge 0. \end{cases}$

13.4. $f = x_1 + 2x_2 + 3,$ $\begin{cases} 2x_1 + 4x_2 \le 8, \\ 3x_1 \le 6, \\ 5x_2 \le 5, \\ x_1 \ge 0, \\ 3x_2 \ge 0. \end{cases}$

14. f chiziqli formaning maksimum qiymatini va uni maksimum qiymatga keltiruvchi nuqtani toping:

14.1. $f = 2x_1 + 4x_2,$ $\begin{cases} 4x_1 + 3x_2 \le 40, \\ 12x_1 + 3x_2 \le 24, \\ 2x_1 \le 6, \\ x_2 \le 3, \\ x_1 \ge 0, \\ x_2 \ge 0. \end{cases}$

14.2. $f = -x_1 + 4x_2,$ $\begin{cases} 3x_1 + 2x_2 \le 12, \\ 2x_1 - x_2 \le 0, \\ -3x_1 + 2x_2 \le 3, \\ x_1 + 2x_2 \le 3, \\ x_1 \ge 0, \\ x_2 \ge 0. \end{cases}$

14.3. $f = 2x_1 + x_2, \quad \begin{cases} 4x_1 - x_2 \ge -4, \\ 2x_1 + 3x_2 \le 12, \\ 5x_1 - 3x_2 \le 15, \\ x_2 \le 7, \\ x_1 \ge 0, \\ x_2 \ge 0. \end{cases}$

14.4. $f = 3x_1 + 2x_2, \quad \begin{cases} x_1 + x_2 \ge 1, \\ -5x_1 + x_2 \le 0, \\ 5x_1 - x_2 \ge 0, \\ x_1 - x_2 \ge -1, \\ x_1 + x_2 \le 6, \\ x_1 \ge 0, \\ x_2 \ge 0. \end{cases}$

Takrorlash uchun savollar

1. Chiziqli tengsizliklar sistemasining umumiy ko‘rinishini yozing.
2. CHTS ning yechimi deb nimaga aytiladi?
3. Hamjoyli va hamjoysiz CHTS ta’riflarini ayting.
4. CHTSning natijasi deb nimaga aytiladi?
5. CHTSning manfiymas chiziqli kombinatsiyasini tuzing.
6. Bir jinsli CHTS deb nimaga aytiladi?
7. Qavariq konus ta’rifini ayting.
8. Ziddiyatli tengsizlik deb nimaga aytiladi?

XI MODUL. BURUN SONLAR HALQASIDA BO'LINISH MUNOSABATI

24-§. Tub va murakkab sonlar. EKUB. EKUK

Asosiy tushunchalar: tub son, murakkab son, natural son, natural bo'luvchilar soni va yig'indisi, EKUB, EKUK, Eyler funksiyasi.

Faqat ikkita turli natural bo'luvchilarga ega bo'lgan natural son *tub son* deyiladi.

Natural bo'luvchilarining soni ikkitadan ortiq bo'lgan natural son *murakkab son* deyiladi.

$a > 1$ natural son bo'lsin. $a = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_n^{\alpha_n}$ tenglik a sonning *kanonik yoyilmasi* deyiladi.

Agar a va $b \neq 0$ butun sonlar uchun $a = bq$ munosabatni qanoatlantiruvchi q butun son mavjud bo'lsa, u holda *a son b songa bo'linadi* yoki *b son a sonni bo'ladi* deyiladi.

$a = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_n^{\alpha_n}$ son uchun $\tau(a)=(\alpha_1+1)(\alpha_2+1)\dots(\alpha_k+1)$ va

$$\sigma(n) = \frac{p_1^{\alpha_1+1}-1}{p_1-1} + \frac{p_2^{\alpha_2+1}-1}{p_2-1} + \dots + \frac{p_n^{\alpha_n+1}-1}{p_n-1} \text{ bo'ladi.}$$

a va b butun sonlarning ikkisini ham bo'ladigan son shu sonlarning *umumiy bo'luvchisi* deyiladi.

a va b natural sonlar umumiy bo'luvchilarining eng kattasi shu sonlarning *eng katta umumiy bo'luvchisi* (EKUB) deyiladi va u $(a; b)$ ko'rinishda belgilanadi.

Agar $(a; b)=1$ bo'lsa, u holda a va b natural sonlar *o'zaro tub sonlar* deyiladi.

$a_1, a_2, \dots, a_n$ butun sonlarning barchasini bo'ladigan son shu sonlarning *umumiy bo'luvchisi* deyiladi.

Agar $(a_1, a_2, \dots, a_n)=1$ bo'lsa, u holda $a_1, a_2, \dots, a_n$ natural sonlar *o'zaro tub sonlar* deyiladi.

Agar quyidagi ikkita shart bajarilsa, u holda $\varphi(m)$ sonli funksiya *Eyler funksiyasi* deyiladi:

1. $\varphi(1)=1$.

2. $\varphi(m)$ funksiya m dan kichik va m bilan o'zaro tub bo'lgan natural sonlar soni.

Natural sonlar to'plamida aniqlangan f funksiya uchun $(m; n)=1$ bo'lganda $f(m \cdot n)=f(m) \cdot f(n)$ tenglik bajarilsa, u holda f funksiya *multiplikativ funksiya* deyiladi.

Eyler funksiyasi $\varphi(m)$ ni hisoblash formulalari quyidagicha:

$m=p$ tub son bo'lsa, u holda $\varphi(p)=r-1$.

$m=r^{\alpha}$ (r – tub son, α – natural son) bo'lsa, u holda $\varphi(p^{\alpha})=p^{\alpha-1}\cdot(p-1)$.

$m=p_1^{\alpha_1}p_2^{\alpha_2}\ldots p_k^{\alpha_k}$ bo'lsa, u holda

$$\varphi(m)=\varphi(p_1^{\alpha_1}p_2^{\alpha_2}\ldots p_k^{\alpha_k})=m\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\ldots\left(1-\frac{1}{p_k}\right).$$

1-misol. $\forall n\in N$ uchun $n(n+1)(2n+1)$ ning 6 ga bo'linishini isbotlang.

Yechish. **1-usul**. Matematik induksiya metodi. $n=1$ bo'lsa, u holda $n(n+1)(2n+1)$ ifoda 6 ga bo'linadi. Faraz qilamiz, $n=k$ uchun $k(k+1)(2k+1)$ ifoda 6 ga bo'linsin, u holda $n=k+1$ da $(k+1)(k+2)(2k+3)$ ifoda 6 ga bo'linadi. Haqiqatan ham $(k+1)(k+2)(2k+3)=k(k+1)(2k+1)+6(k+1)^2$ bo'lganligi va qo'shiluvchilarning har biri 6 ga bo'linganligi uchun $(k+1)(k+2)(2k+3)$ ifoda 6 ga bo'linadi.

2-usul. Natural sonlar qatorida 2 ta ketma-ket kelgan sonlar ko'paytmasi $n(n+1)$ 2 ga bo'linishidan $n(n+1)(2n+1)$ ifoda ham 2 ga bo'linadi. $n=1$ bo'lganda $1\cdot2\cdot3=6$ va $(2,3)=1$ ekanligidan $(k+1)(k+2)(2k+3)$ ifodaning 6 ga bo'linishi uchun $n(n+1)(2n+1)$ ifodaning 3 ga bo'linishini ko'rsatish kifoya. Qoldiqli bo'lish haqidagi teoremaga ko'ra har qanday natural sonni $n=3k$ yoki $n=3k+1$, yoki $n=3k+2$ ko'rinishida ifodalash mumkin. Bundan

1) agar $n=3k$ bo'lsa, u holda $n(n+1)(2n+1)$ ifoda 3 ga bo'linadi;

2) agar $n=3k+1$ ko'rinishda bo'lsa, u holda $2n+1=6k+3$ va $n(n+1)(2n+1)$ ifoda 3 ga bo'linadi;

3) agar $n=3k+2$ ko'rinishida bo'lsa, u holda $n+1=3k+3$ va $n(n+1)(2n+1)$ ifoda 3 ga bo'linadi.

Demak, $n(n+1)(2n+1)$ ifoda 6 ga bo'linadi.

3-usul. Agar $n(n+1)(2n+1)=n(n+1)[(n-1)+(n+2)]=$ $=(n+1)^n(n+1)+n(n+1)(n+2)$ shakl almashtirishdan foydalansak, u holda $n(n+1)(2n+1)$ ifodani 2 ta ketma-ket keluvchi 3 ta son ko'paytmasidan iborat 2 ta qo'shiluvchili yig'indi ko'rinishiga keladi. Ketma-ket kelgan 3 ta natural son ko'paytmasining 6 ga bo'linishidan $n(n+1)(2n+1)$ ifodaning 6 ga bo'linishi kelib chiqadi.

2-misol. Berilgan 150 va 200 sonlar orasidagi barcha tub sonlarni aniqlang.

Yechish. 150 va 200 sonlar orasidagi barcha natural sonlarni tartib bilan yozib olamiz:

150	151	152	153	154	155	156	157	158	159
160	161	162	163	164	165	166	167	168	169
170	171	172	173	174	175	176	177	178	179
180	181	182	183	184	185	186	187	188	189
190	191	192	193	194	195	196	197	198	199
200									

Tuzilgan qatorning birinchi soni 150 – juft son. Demak, 2 ga bo'linadi. 150 dan boshlab qatorning har 2-sonini o'chirib chiqamiz:

~~150~~ 151 ~~152~~ 153 ~~154~~ 155 .... ~~200~~

Berilgan qatordan 2 ga bo'linuvchi sonlarni o'chirib chiqdik. Endi sonlar qatoridan raqamlari yig'indisi 3 ga bo'linadigan birinchi sonni topamiz. Bu sonni va undan keyin keluvchi har 3-sonni qatordan o'chiramiz. Bunda o'chirilgan sonlar o'rni ham hisobga olinadi. Bu jarayonni $\sqrt{200}\approx 14$ dan katta bo'lmagan tub songa bo'linadigan sonlarni o'chirguncha davom ettiramiz. Berilgan qatorning o'chirilmay qolgan sonlari 150 dan 200 gacha bo'lgan tub sonlardir.

~~150~~	151	~~152~~	~~153~~	~~154~~	~~155~~	~~156~~	157	~~158~~	~~159~~
~~160~~	~~161~~	~~162~~	163	~~164~~	~~165~~	~~166~~	167	~~168~~	~~169~~
~~170~~	~~171~~	~~172~~	173	~~174~~	~~175~~	~~176~~	~~177~~	~~178~~	179
~~180~~	181	~~182~~	~~183~~	~~184~~	~~185~~	~~186~~	~~187~~	~~188~~	~~189~~
~~190~~	191	~~192~~	193	~~194~~	~~195~~	~~196~~	197	~~198~~	199
200									

Demak, 150 bilan 200 orasidagi tub sonlarni topish uchun 2, 3, 5, 7, 11, 13 ga bo‘linadigan sonlar qatordan o‘chirildi va berilgan oraliqdagi tub sonlar Eratosfen g‘alviri yordamida aniqlandi. Ular 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199.

3-misol. Berilgan 1321 sonning tub yoki murakkab ekanligini aniqlang.

Yechish. Berilgan natural sonning tub yoki murakkab ekanligini aniqlash uchun berilgan sonning $\sqrt{a}$ songacha bo‘lgan tub sonlarga bo‘linishi yoki bo‘linmasligi aniqlanadi. Agar berilgan a son $\sqrt{a}$ gacha bo‘lgan birorta ham tub songa bo‘linmasa, u holda u tub son bo‘ladi.

Demak, $\sqrt{1321} \approx 36$ ni topamiz. Berilgan 1321 sonning 36 gacha bo‘lgan tub sonlar 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 ga bo‘linish-bo‘linmasligini tekshiramiz.

2 ga bo‘linmaydi, chunki 1321 toq son;

3 ga bo‘linmaydi, chunki $1+3+2+1=7/3$;

5 ga bo‘linmaydi, chunki 1321 ning oxirgi raqami 1;

$1321 : 7 \approx 188$;

$1321 : 11 \approx 120$;

$1321 : 13 \approx 101$;

$1321 : 17 \approx 77$;

$1321 : 19 \approx 69$;

$1321 : 23 \approx 54$;

$1321 : 29 \approx 45$;

$1321 : 31 \approx 42$.

Demak, 1321 soni 36 gacha bo‘lgan tub sonlarga bo‘linmaydi. U tub son.

4-misol. Berilgan 123 va 321 sonlarning EKUB va EKUKlarini ikki usulda toping. EKUBni berilgan sonlar orqali chiziqli ifodalang.

Yechish. Berilgan natural sonlarning EKUB va EKUKlarini topish uchun ularni tub ko'paytiruvchilarga yoyilmasidan yoki Evklid algoritmidan foydalanish mumkin.

1-usul. Berilgan sonlarning tub ko'paytiruvchilarga kanonik yoyilmasini topamiz:

$$\begin{array}{r|l} 123 & 3 \\ 41 & 41 \\ 1 & \end{array} \qquad \begin{array}{r|l} 321 & 3 \\ 107 & 107 \\ 1 & \end{array}$$

$123 = 3 \cdot 41 = 3^1 \cdot 41^1 \cdot 107^0$;

$321 = 3 \cdot 107 = 3^1 \cdot 41^0 \cdot 107^1$.

$n = P_1^{\alpha_1} ... P_n^{\alpha_n}$ va $m = P_1^{\beta_1} ... P_n^{\beta_n}$ sonlar uchun

EKUB $(n;m) = P_1^{\min(\alpha_1,\beta_1)} \cdot P_2^{\min(\alpha_2,\beta_2)} \cdot ... \cdot P_n^{\min(\alpha_n,\beta_n)}$;

EKUK $[n;m] = P_1^{\max(\alpha_1,\beta_1)} \cdot P_2^{\max(\alpha_2,\beta_2)} ... P_n^{\max(\alpha_n,\beta_n)}$.

Demak, $(123; 321) = 3$ va $[123; 321] = 3 \cdot 41 \cdot 107 = 13161$.

2-usul. Berilgan sonlar uchun qoldiqli bo'lish teoremasi yordamida Evklid algoritmini tuzamiz:

$$\begin{array}{ll} 321 = 123 \cdot 2 + 75; & 75 = 321 - 123 \cdot 2; \\ 123 = 75 \cdot 1 + 48; & 48 = 123 - 75 \cdot 1; \\ 75 = 48 \cdot 1 + 27; & 27 = 75 - 48 \cdot 1; \\ 48 = 27 \cdot 1 + 21; & 21 = 48 - 27 \cdot 1; \\ 27 = 21 \cdot 1 + 6; & 6 = 27 - 21 \cdot 1; \\ 21 = 6 \cdot 3 + 3; & 3 = 21 - 6 \cdot 3. \\ 6 = 3 \cdot 2 + 0. & \end{array}$$

Evklid algoritmidagi oxirgi noldan farqli qoldiq EKUB ni beradi.

Demak, $(321; 123) = 3$. Bundan $[321;123] = \dfrac{321 \cdot 123}{(321,123)} = 13161$.

Topilgan EKUB $(321; 123) = 3$ ning 123 va 321 lar yordamidagi chiziqli ifodasini topamiz. Tuzilgan Evklid algoritmidagi qoldiqlarni bo'linuvchi va bo'luvchilar yordamidagi ifodalarini topamiz:

$3 = 21 - 6 \cdot 3 = (48 - 27 \cdot 1) - (27 - 21 \cdot 1) \cdot 3 = 48 - 27 \cdot 4 + 21 \cdot 3 =$
$= 123 - 75 \cdot 1 - (75 - 48 \cdot 1) \cdot 4 + (48 - 27 \cdot 1) \cdot 3 = 123 - 75 \cdot 5 + 48 \cdot 7 -$
$-27 \cdot 3 = 123 - (321 - 123 \cdot 2) \cdot 5 + (123 - 75 \cdot 1) \cdot 7 - (75 - 48 \cdot 1) \cdot 3 =$
$= 123 \cdot 18 - 321 \cdot 5 - 75 \cdot 10 + 48 \cdot 3 = 123 \cdot 18 - 321 \cdot 5 -$
$-(321 - 123 \cdot 2) \cdot 10 + (123 - 75 \cdot 1) \cdot 3 = 123 \cdot 41 - 321 \cdot 15 - 75 \cdot 3 =$
$= 123 \cdot 41 - 321 \cdot 15 - (321 - 123 \cdot 2) \cdot 3 = 123 \cdot 47 - 321 \cdot 18 =$
$= 123 \cdot 47 + 321 \cdot (-18).$

Bundan, $3 = 123 \cdot 47 + 321 \cdot (-18)$ kelib chiqadi.

5-misol. Berilgan n=126 sonning natural bo'linuvchilari soni va yig'indisini, undan katta bo'lmagan va u bilan o'zaro tub sonlar sonini toping.

Yechish. Berilgan n sonining natural bo'luvchilari soni $\tau(n)$ va natural bo'luvchilari yig'indisini $\sigma(n)$, n dan katta bo'lmagan u bilan o'zaro tub sonlar soni $\varphi(n)$ larni aniqlash uchun n sonining tub ko'paytuvchilarga kanonik yoyilmasini topamiz. Agar $n = p_1^{\alpha_1} \dots p_n^{\alpha_n}$ bo'lsa, u holda

$$\tau(n) = (\alpha_1 + 1)(\alpha_2 + 1)\dots(\alpha_n + 1);$$

$$\sigma(n) = \frac{p_1^{\alpha_1+1} - 1}{p_1 - 1} \cdot \frac{p_2^{\alpha_2+1} - 1}{p_2 - 1} \cdot \dots \cdot \frac{p_n^{\alpha_n+1} - 1}{p_n - 1};$$

$$\varphi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\dots\left(1 - \frac{1}{p_n}\right)$$

bo'ladi.

n=126 ning tub bo'luvchilarga kanonik yoyilmasini topamiz:

126	2
63	3
21	3
7	7
1	

Bundan, $126 = 2^1 \cdot 3^2 \cdot 7^1$ ekan. U holda

a) $\tau(126)=(1+1)(2+1)(1+1)=2\cdot3\cdot2=12$. Demak, 126 ning natural bo‘luvchilari 12 ta. Haqiqatan ham ular: 1, 2, 3, 6, 7, 9, 14, 18, 21, 42, 63, 126;

b) $\sigma(126)=\frac{2^2-1}{2-1}\cdot\frac{3^3-1}{3-1}\cdot\frac{7^2-1}{7-1}=\frac{3}{1}\cdot\frac{26}{2}\cdot\frac{48}{6}=26\cdot12=312$.

Haqiqatan ham,

$$1+2+3+6+7+9+14+18+21+42+63+126=312.$$

d) $\varphi(126)=126\cdot\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{7}\right)=126\cdot\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{6}{7}=36$.

Demak, 126 dan katta bo‘lmagan, u bilan o‘zaro tub sonlar soni 36 ta.

6-misol. 23! ning tub ko‘paytiruvchilarga kanonik yozilmasini toping.

Yechish. Berilgan *n*! sonning tub ko‘paytuvchilarga yoyilmasini topish uchun, *n* dan katta bo‘lmagan tub sonlar qanday daraja bilan kanonik yoyilmada qatnashishini topamiz.

23 dan katta bo‘lmagan tub sonlar 2, 3, 5, 7, 11, 13, 17, 19, 23.

2 ning 23! ning kononik yoyilmasidagi darajasini topamiz. Buning uchun 23 ni 2 ga bo‘lamiz. Bu jarayonni bo‘linma 2 dan kichik son bo‘lguncha davom ettiramiz:

$23=2\cdot11+1,$

$11=2\cdot5+1,$

$5=2\cdot2+1,$

$2=2\cdot1+0.$

Demak, 2 ning kanonik yoyilmadagi darajasi 11+5+2+1=19.

3 ning darajasini topamiz:

$23=3\cdot7+2,$

$7=3\cdot2+1.$

3 ning darajasi 7+2=9.

5 ning darajasini topamiz: $23=5\cdot4+3$.

5 ning darajasi 4.

$23=7\cdot3+2$.

7 ning darajasi 3.

$23 = 11 \cdot 2 + 1$.

11 ning darajasi 2.

13 ning darajasi 1, chunki $23 = 13 \cdot 1 + 10$.

Xuddi shunday, 17, 19, 23 larning ham yoyilmadagi darajalari 1 ga teng.

Demak, $23! = 2^{19} \cdot 3^9 \cdot 5^4 \cdot 7^3 \cdot 11^2 \cdot 13 \cdot 17 \cdot 19 \cdot 23$.

7-misol. $\begin{cases} a \cdot b = 768, \\ (a,b) = 8 \end{cases}$ sistemani qanoatlantiruvchi a va b sonlarni toping.

Yechish. Berilgan a va b sonlarning eng katta umumiy bo'luvchisi 8 ekanligidan, bu sonlarni $a=8k$ va $b=8l$, $k,l \in Z$ ko'rinishda yozib olamiz. Bu yerda $(k,l)=1$. Bundan $a \cdot b = 8k \cdot 8l = 64 \cdot k \cdot l = 768$ va $k \cdot l = 12$ larni hosil qilamiz. Demak, 12 o'zaro tub k va l sonlarning ko'paytmasi ko'rinishida ifodalanadi. Quyidagi holatlar bo'lishi mumkin:

k	l	$k \cdot l$
1	12	12
3	4	12
4	3	12
12	1	12

Bundan,

a	b	$a \cdot b$
8	96	768
24	32	768
32	24	768
96	8	768

Demak, $(a,b): (8;96), (24;32), (32;24), (96;8)$

Misol va mashqlar

1. Tub va murakkab sonlarning quyidagi xossalarini isbotlang:

1.1. $a > 1$ murakkab sonning 1 dan boshqa eng kichik natural bo'luvchisi r bo'lsa, u holda r son tub son bo'ladi.

1.2. Har qanday natural a va r tub sonlar yoki o'zaro tub, yoki a son r ga bo'linadi.

1.3. Agar ab ko'paytma biror r tub songa bo'linsa, u holda ko'paytuvchilardan kamida bittasi r ga bo'linadi.

1.4. Agar ko'paytma r tub songa bo'linib, uning barcha ko'paytuvchilari tub sonlardan iborat bo'lsa, u holda bu ko'paytuvchilardan biri r ga teng bo'ladi.

1.5. 1 dan boshqa ixtiyoriy natural son yoki tub son, yoki tub sonlar ko'paytmasi shaklida yoziladi, agar bu ko'paytmada ko'paytuvchilarning o'rni e'tiborga olinmasa, u holda bu ko'paytma yagona bo'ladi.

2. Eratosfen g'alviri yordamida berilgan sonlar orasidagi barcha tub sonlarni aniqlang:

2.1. 1050 va 1150. 2.2. 2100 va 2200.
2.3. 1100 va 1200. 2.4. 2550 va 2650.
2.5. 1880 va 2000. 2.6. 4550 va 4670.
2.7. 5555 va 5750. 2.8. 4660 va 4770.
2.9. 4422 va 4525. 2.10 1122 va 1222.

3. Berilgan natural sonning tub yoki murakkab ekanligini aniqlang:

3.1. $n = 1559$. 3.2. $n = 1627$.
3.3. $n = 1783$. 3.4. $n = 3061$.
3.5. $n = 3709$. 3.6. $n = 4057$.
3.7. $n = 1987$. 3.8. $n = 2339$.
3.9. $n = 2671$. 3.10. $n = 3343$.

4. Butun sonlar halqasida bo'linish munosabatining quyidagi xossalarini isbotlang:

4.1. $(\forall\ a \in Z,\ a \neq 0)\ 0 \vdots a$.

4.2. $(\forall\ a \in Z,\ a \neq 0)\ a \vdots a$.

4.3. $(\forall\ a \in Z)\ a \vdots 1$.

4.4. $(\forall\ a, b, c \in Z,\ b \neq 0,\ c \neq 0)\ ((a \vdots b) \wedge (b \vdots c)) \Rightarrow (a \vdots c)$.

4.5. $(\forall\ a, b \in Z,\ a \neq 0,\ b \neq 0)\ ((a \vdots b) \wedge (b \vdots a)) \Rightarrow (b = \pm a)$.

4.6. $(\forall\ a, b, c \in Z,\ c \neq 0)\ a \vdots c \Rightarrow ab \vdots c$.

4.7. $(\forall\ a, b \in Z,\ c \neq 0)\ ((a \vdots c) \wedge (b \vdots c)) => (a \pm b) \vdots c$.

4.8. $(\forall a, b_i \in Z, a \neq 0, i=\overline{1,n})\ ((b_1 \vdots a) \wedge (b_2 \vdots a) \wedge \ldots \wedge (b_n \vdots a)) \Rightarrow$
$\Rightarrow (b_1c_1 \pm b_2c_2 \pm \ldots \pm b_nc_n) \vdots a\ (c_i \in Z,\ i = \overline{1,n})$.

5. Ixtiyoriy a butun, b natural sonlar uchun shunday yagona q butun son va yagona manfiymas r butun son topiladiki, natijada ushbu $a=bq+r,\ 0 \leq r < b$ munosabatlar o'rinli bo'ladi. Isbotlang.

6. $n = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_k^{\alpha_k}$ sonning bo'luvchisi d bo'lishi uchun d sonning kanonik yoyilmasi $d = p_1^{\beta_1} p_2^{\beta_2} \ldots p_k^{\beta_k}$ bo'lib, bunda $\beta_i \leq \alpha_i$ $(i=\overline{1,k})$ bo'lishi zarur va yetarli ekanligini isbotlang.

7. Berilgan n natural sonning natural bo'luvchilari soni va yig'indisini:

7.1. $n = 60$.	7.9. $n = 1000$.
7.2. $n = 100$.	7.10. $n = 1200$.
7.3. $n = 360$.	7.11. $n = 1542$.
7.4. $n = 375$.	7.12. $n = 3500$.
7.5. $n = 720$.	7.13. $n = 680$.
7.6. $n = 957$.	7.14. $n = 865$.
7.7. $n = 988$.	7.15. $n = 779$.
7.8. $n = 990$.	7.16. $n = 410$.

8. $n!$ ning tub ko'paytuvchilarga kanonik yoyilmasini toping:

8.1. $n = 55$.	8.7 $n = 53$.
8.2. $n = 92$.	8.8. $n = 45$.
8.3. $n = 87$.	8.9. $n = 50$.
8.4. $n = 63$.	8.10. $n = 38$.
8.5. $n = 34$.	8.11. $n = 90$.
8.6. $n = 99$.	8.12. $n = 100$.

9. Quyidagi xossalarni isbotlang:

9.1. $(a,b)=(a+b,a+2b)$.

9.2. $(a;\ b) = d$ bo'lsa, u holda shunday u va v butun sonlar topiladiki, ular uchun $au+bv=d$ tenglik bajariladi.

9.3. $(a;\ c)=1 \wedge (b;\ c)=1 \Rightarrow ((ab;\ c)=1)$.

9.4. $((ab : c) \wedge (a,c)=1) \Rightarrow (b : c)\ (c \neq 0)$.

9.5. $((a;b)=1) \Rightarrow ((a^n;b^n)=1)\ (\forall n \in N)$.

9.6. $((a;b)=d) \Rightarrow ((\frac{a}{d};\frac{b}{d})=1)$.

9.7. $((a\vdots b)\wedge(a\vdots c)\wedge((b;c)=1) \Rightarrow (a\vdots bc)$ $(b \neq 0,\ c \neq 0)$.

9.8. $a=bq+r \Rightarrow (a;b)=(b;r)$.

9.9. d son a va b sonlarning EKUBi bo‘lishi uchun d umumiy bo‘luvchi a va b sonlarning har qanday umumiy bo‘luvchisiga bo‘linishi zarur va yetarli.

9.10. Agar $(a_1,a_2,...,a_n)=d$ bo‘lib, $(a_1,a_2)=d_2$, $(d_2,a_3)=d_3$, ..., $(d_{n-1};a_n)=d_n$ bo‘lsa, u holda $d_n=d$ bo‘ladi.

10. Berilgan sonlarning EKUBini ikki usulda toping:

10.1. 1232, 1672.

10.2. 135, 8211.

10.3. 589, 343.

10.4. 29719, 76501.

10.5. 469459, 519203.

10.6. 179370199, 4345121.

10.7. 12606, 6494.

10.8. 162891, 32176.

10.9. 7650, 25245.

10.10. 35574, 192423.

10.11. 10140, 92274.

10.12. 46550, 37730.

11. x va y natural sonlarni toping:

11.1. $\begin{cases} x+y=150, \\ (x,y)=30. \end{cases}$

11.2. $\begin{cases} x\cdot y=8400, \\ (x,y)=20. \end{cases}$

11.3. $\begin{cases} x+y=667, \\ [x,y]=120\cdot(a,b). \end{cases}$

11.4. $\begin{cases} \frac{x}{y}=\frac{11}{7}, \\ (x,y)=45. \end{cases}$

11.5. $\begin{cases} \frac{x}{(x,y)}+\frac{y}{(x,y)}=18, \\ [x,y]=975. \end{cases}$

11.6. $\begin{cases} x\cdot y=20, \\ [x,y]=10. \end{cases}$

11.7. $\begin{cases} (x,y)=24, \\ [x,y]=2496. \end{cases}$

11.8. $\begin{cases} x\cdot y=168, \\ (x,y)=14. \end{cases}$

11.9. $\begin{cases} \frac{x}{y} = \frac{5}{9}, \\ (x, y) = 28. \end{cases}$ 11.10. $\begin{cases} x + y = 100, \\ [x, y] = 495. \end{cases}$

Takrorlash uchun savollar

1. Arifmetikaning asosiy teoremasini bayon eting.
2. Tub va murakkab sonlarning qanday xossalarini bilasiz?
3. Bo'linish munosabati xossalarini bayon eting.
4. Qoldiqli bo'lish haqidagi teoremani bayon eting.
5. Sonli funksiya deb nimaga aytiladi?
6. $\tau(n)$ va $\sigma(n)$ sonli funksiyalar qanday hisoblanadi?
7. Ikkita sonning EKUBi deb nimaga aytiladi?
8. n ta sonning EKUBi qanday topiladi?
9. Ikkita sonning EKUKi deb nimaga aytiladi?
10. n ta sonning EKUKi qanday topiladi?
11. O'zaro tub sonlar deb nimaga aytiladi?
12. Evklid algaritimini tushuntiring.

25-§. Chekli zanjir kasrlar. Munosib kasrlar

Asosiy tushunchalar: uzluksiz zanjir kasr, chekli zanjir kasr, munosib kasr.

Ushbu

$$a_0 + \cfrac{b_1}{a_1 + \cfrac{b_2}{a_2 + \dots \; \dots + \cfrac{b_k}{a_k}}}$$

(a_i ($i=\overline{0,k}$), b_j ($j=\overline{1,k}$) butun sonlar) ko'rinishdagi ifoda *uzluksiz zanjir kasr* deyiladi. Agar (1) da $b_1=b_2=\dots=b_k=1$, a_0 – butun son, a_1, a_2, ..., a_k – natural sonlar bo'lib, $a_k > 1$ bo'lsa, u holda ushbu

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \dots \qquad \atop \dots + \cfrac{1}{a_k}}}$$

ifoda *chekli zanjir kasr* deyiladi.

$$T = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \dots \qquad \atop \dots + \cfrac{1}{a_n}}}$$ bo‘lsin.

$A_0=a_0$ deb olaylik. U holda bu *nolinchi tartibli munosib kasr* deyiladi.

$A_1 = a_0 + \frac{1}{a_1} = \frac{a_0 a_1 + 1}{a_1}$ – *birinchi tartibli munosib kasr* deyiladi.

$A_2 = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2}}$ – *ikkinchi tartibli munosib kasr* deyiladi.

. . . .

$A_n=T$ esa *n-tartibli munosib kasr* deyiladi.

$A_1 = \frac{a_0}{1} = \frac{P_0}{Q_0}$ deb belgilaylik. U holda $R_0=a_0$, $Q_0=1$ hosil bo‘ladi.

$A_1 = a_0 + \frac{1}{a_1} = \frac{a_0 a_1 + 1}{a_1} = \frac{P_1}{Q_1}$ desak, u holda $R_1=a_0a_1+1$, $Q=a_1$ hosil bo‘ladi.

$A_2 = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2}} = \frac{P_2}{Q_2}$ – ikkinchi tartibli munosib kasr;

. . . .

$A_n = T = \frac{P_n}{Q_n}$ – n- tartibli munosib kasr.

Shu yo‘l bilan R_0, R_1, R_2, ..., Q_0, Q_1, Q_2, ... ketma-ketliklarni hosil qilamiz. Bu ketma-ketliklardan quyidagi rekurrent formulalarni hosil qilamiz:

$$R_k = P_{k-1}a_k + R_{k-2}, \quad Q_k = Q_{k-1}a_k + Q_{k-2}.$$

$\frac{P_k}{Q_k}$ – *k- tartibli munosib kasr* deyiladi.

$R_{-2}=0$, $R_{-1}=1$, $Q_{-2}=1$, $Q_{-1}=0$ deb belgilaylik. Lekin ularning o‘zi ma’noga ega emas. Yuqoridagi tushunchalardan quyidagi jadvalni tuzamiz:

k	-2	-1	0	1	2	...	$n-1$	n
A_k	–	–	a_0	a_1	a_2	...	a_{n-1}	a_n
P_k	0	1	P_0	P_1	P_2	...	P_{n-1}	P_n
Q_k	1	0	Q_0	Q_1	Q_2	...	Q_{n-1}	Q_n

1-misol. Berilgan $\frac{104}{23}$ kasrni chekli zanjir kasr ko‘rinishida ifodalang va uning munosib kasrlarini toping.

Yechish. $\frac{104}{23}$ kasrni chekli zanjir kasr ko‘rinishida ifodalash maqsadida 104 va 23 sonlari uchun Evklid algoritmini tuzamiz:

$$104 = 23 \cdot 4 + 12;$$
$$23 = 12 \cdot 1 + 11;$$
$$12 = 11 \cdot 1 + 1;$$
$$11 = 1 \cdot 11 + 0.$$

Evklid algoritmidagi tengliklarning har ikkala tomonini bo‘luvchilarga bo‘lamiz:

$$\frac{104}{23} = 4 + \frac{12}{23};$$
$$\frac{23}{12} = 1 + \frac{11}{12};$$
$$\frac{12}{11} = 1 + \frac{1}{11};$$
$$\frac{11}{1} = 11.$$

Hosil bo'lgan tengliklarning o'ng tomonidagi kasr sonni uning teskarisi bilan almashtirish natijasida

$$\frac{104}{23}=4+\frac{12}{23}=4+\frac{1}{\frac{23}{12}}=4+\frac{1}{1+\frac{11}{12}}=4+\frac{1}{1+\frac{1}{\frac{12}{11}}}=4+\frac{1}{1+\frac{1}{1+\frac{1}{11}}}$$

chekli zanjirni hosil qilamiz. Uni qisqacha $\frac{104}{23}=[4;\ 1,\ 1,\ 11]$ ko'rinishda ifodalaymiz. Agar berilgan kasr manfiy bo'lsa, birinchi qoldiqni musbat qilib olamiz. Masalan, $-\frac{23}{13}=-2+\frac{3}{13}$ va kasr qismi chekli zanjir ko'rinishida ifodalanadi:

$$-\frac{23}{13}=-2+\frac{3}{13}=-2+\frac{1}{\frac{13}{3}}=-2+\frac{1}{4+\frac{1}{3}}=[-2;\ 4,\ 3].$$

Berilgan $\frac{104}{23}=[4;\ 1,\ 1,\ 11]$ ning munosib kasrlarini topish uchun quyidagi jadvalni tuzamiz:

k	-1	0	1	2	3
q_k	–	4	1	1	11
P_k	1	4	5	9	104
Q_k	0	1	1	2	23

Demak, $\frac{P_0}{Q_0}=4;\quad \frac{P_1}{Q_1}=5;\quad \frac{P_2}{Q_2}=\frac{9}{2};\quad \frac{P_3}{Q_3}=\frac{104}{23}.$

2-misol. Berilgan $\sqrt{14}$ sonni zanjir kasr ko'rinishida ifodalang.

Yechish. $\sqrt{14}=3+\frac{1}{\alpha_1};$

$$\alpha_1=\frac{1}{\sqrt{14}-3}=\frac{\sqrt{14}+3}{5}=1+\frac{1}{\alpha_2};$$

$$\alpha_2=\frac{1}{\frac{\sqrt{14}+3}{5}-1}=\frac{5}{\sqrt{14}-1}=\frac{\sqrt{14}+2}{2}=2+\frac{1}{\alpha_3};$$

$$\alpha_3=\frac{1}{\frac{\sqrt{14}+2}{2}-2}=\frac{2}{\sqrt{14}-2}=\frac{\sqrt{14}+2}{5}=1+\frac{1}{\alpha_4};$$

$$\alpha_4=\frac{1}{\frac{\sqrt{14}+2}{5}-1}=\frac{5}{\sqrt{14}-3}=\sqrt{14}+3=6+\frac{1}{\alpha_5};$$

$$\alpha_5=\frac{1}{\sqrt{14}+3-6}=\frac{1}{\sqrt{14}-3}.$$

$\alpha_5=\alpha_1$ bo'lganligi uchun, yana yuqoridagi jarayon hosil bo'ladi.

Demak, $\sqrt{14}=[3;\ (1,\ 2,\ 1,\ 6)]$.

3-misol. $-117x+343y=119$ tenglamani butun sonlar to'plamida yeching.

Yechish. Tenglamani $117(-x)+343y=119$ ko'rinishida yozib olamiz va $ax+by+c$ tenglama agar $(a,b)=1$ bo'lsa,

$$x=(-1)^{n-1}\cdot c\cdot Q_{n-1}+bt,$$
$$y=(-1)^{n}\cdot c\cdot P_{n-1}-at,\quad t\in z$$

formulalar orqali topiladigan butun yechimlarga ega. Buning uchun $\frac{a}{b}$ kasrning munosib kasrlari topiladi.

$\frac{a}{b}=\frac{117}{343}$ uchun chekli zanjir kasrni topamiz:

$117=0\cdot343+117;$
$343=117\cdot2+109;$
$117=109\cdot1+8;$
$109=8\cdot13+5;$
$8=5\cdot1+3;$
$5=3\cdot1+2;$
$3=2\cdot1+1;$
$2=1\cdot2+0.$

Demak, $\frac{117}{343}=[0;2,\ 1,\ 13,\ 1,\ 1,\ 1,\ 2]$. Munosib kasrlar jadvalini tuzamiz:

k	-1	0	1	2	3	4	5	6	7
q_k	–	0	2	1	13	1	1	1	2
P_k	1	0	1	1	14	15	29	44	117
Q_k	0	1	2	3	41	44	85	129	343

$P_6 = 44$, $Q_6 = 129$ lardan foydalanamiz.

Xususiy yechim: $\begin{cases} -x_0 = (-1)^6 \cdot 119 \cdot 129 = 15351; \\ y_0 = (-1) \cdot 119 \cdot 44 = -5236. \end{cases}$

Umumiy yechim:

$\begin{cases} -x = 15351 + 343t, \\ y = -5236 - 117t, \ t \in z \end{cases}$ yoki $\begin{array}{l} x = -15351 - 343t, \\ y = -5236 - 117t, \quad t \in z. \end{array}$

Berilgan misolni yechishda $-\frac{117}{343}$ uchun zanjir kasrni tuzish ham mumkin.

U holda $-\frac{117}{343} = [-1; 1, \ 1, \ 1, \ 13, \ 1, \ 1, \ 1, \ 2]$ bo‘lib, k=8, $a = -117$, $b = 343$, $c = 119$, $P_{n-1} = P_7 = -44$, $Q_{n-1} = Q_7 = 129$ bo‘ladi.

Undan $\begin{cases} x = -15351 + 343t, \\ y = 5236 + 117t, \ t \in z \end{cases}$ yechimlar hosil bo‘ladi.

Misol va mashqlar

1. Quyidagi tasdiqlarni isbotlang:

1.1. Har qanday ratsional con chekli zanjir kasrga yoyiladi va bu yoyilma yagona bo‘ladi.

1.2. $A_k = \frac{P_k}{Q_k} \ (k = \overline{0, n})$.

1.3. $R_k Q_{k-1} - P_{k-1} Q_k = (-1)^{k-1}$ tenglik k ning har qanday qiymatida to‘g‘ri bo‘ladi.

1.4. $A_k = \frac{P_k}{Q_k}$ munosib kasrning surati bilan maxraji o‘zaro tub, ya’ni $(R_k; \ Q_k)=1$ bo‘ladi.

2. Berilgan kasrni chekli zanjir kasr ko'rinishida ifodalang:

2.1. $\frac{323}{17}$. 2.2. $\frac{135}{279}$. 2.3. $-\frac{187}{63}$. 2.4. $\frac{96}{67}$.

2.5. $\frac{30}{337}$. 2.6. $-\frac{12}{15}$. 2.7. $\frac{127}{52}$. 2.8. $\frac{24}{35}$.

2.9. 1,23. 2.10. $\frac{71}{41}$. 2.11. $\frac{157}{225}$. 2.12. $\frac{507}{1001}$.

3. Berilgan irratsional sonlarni chekli zanjir kasr orqali ifodalang:

3.1. $\sqrt{11}$. 3.2. $\sqrt{12}$. 3.3. $\sqrt{13}$.

3.4. $\sqrt{28}$. 3.5. $\sqrt{30}$. 3.6. $\sqrt{59}$.

3.7. $1+\sqrt{2}$. 3.8. $\frac{1+\sqrt{3}}{2}$. 3.9. $\frac{2+\sqrt{5}}{3}$.

3.10. $\frac{3+\sqrt{5}}{2}$. 3.11. $\frac{2+\sqrt{7}}{2}$. 3.12. $\frac{3+\sqrt{10}}{3}$.

4. Quyidagi zanjir kasrlar orqali ifodalanuvchi qisqarmas kasrlarni toping:

4.1. [2;1,3,4,2]. 4.7. [4;(3,2,1)].
4.2. [2;1,19,1,3]. 4.8. [(2,1)].
4.3. [2;1,1,3,1,2]. 4.9. [3;(3,6)].
4.4. [1;1,2,3,4]. 4.10. [1;(1,2)].
4.5. [0;4,1,2,5,6]. 4.11. [1;7,(1,6)].
4.6. [−2;1,3,1,1,5]. 4.12. [3;(5,2,1,2)].

5. Berilgan tenglamalarni butun sonlar to'plamida yeching:

5.1. $38x + 117y = 209$. 5.2. $23x - 42y = 72$.

5.3. $119x - 68y = 34$. 5.4. $15x + 28y = 185$.

5.5. $41x + 114y = 5$. 5.6. $90x - 5y = 5$.

5.7. $49x + 9y = 400$. 5.8. $10x - 11y = 15$.

5.9. $12x + 31y = 170$. 5.10. $31x - 47y = 23$.

5.11. $37x + 23y = 15$. 5.12. $101x + 39y = 89$.

5.13. $53x + 17y = 25$. 5.14. $-26x + 174y = 2$.

5.15. $64x - 39y = 15$. 5.16. $-6x + 11y = 29$.

5.17. $3827x+3293y=1869$. 5.18. $-10x+23y=17$.

5.19. $571x+359y=-10$. 5.20. $903x+5y=43$

Takrorlash uchun savollar

1. Uzluksiz kasr deb nimaga aytiladi?
2. Chekli zanjir kasr deb nimaga aytiladi?
3. Ratsional sonni chekli zanjir kasrga yagona yo'l bilan yoyishni bayon eting.
4. Munosib kasrlar haqida tushuncha bering.
5. Munosib kasrlar haqidagi teoremalarni bayon eting.
6. Chekli zanjir kasrlar tatbiqiga misollar keltiring.

26-§. Sistematik sonlar va ular ustida amallar

✓ **Asosiy tushunchalar:** sanoq sistemasi, asosi g ga teng bo'lgan sistematik son, sistematik sonlarni qo'shish, ayirish, ko'paytirish, bo'lish.

O'nlik sanoq sistemasidan boshqa 2, 5, 7, 12, 60, ... sanoq sistemalari ham mavjud. Bu sanoq sistemalarining barchasi bitta umumiy yo'nalish asosida quriladi.

$m>1$ natural son bo'lib, $M=\{0, 1, 2, ..., m-1\}$ to'plam berilganda har qanday a natural son uchun ushbu $a=a_0+a_1m+a_2m^2+...+a_nm^n=a_0m^0+a_1m^1+...+a_nm^n$ ($a_i\in M$, $i=\overline{1,n}$, $a_n\neq 0$) yoyilma mavjud va yagonadir. a natural sonning bu ko'rinishi *a ni m ning darajalari bo'yicha yoyish* deyiladi.

Ixtiyoriy $g\geq 2$ natural son va har qanday m natural son uchun

$$m=a_ng^n+a_{n-1}g^{n-1}+...+a_1g+a_0 \quad (0\leq a_i\leq g-1,\ i=\overline{0,n-1},\ 1\leq a_n\leq g-1)$$

tenglikni yoza olamiz. Undagi a_0, a_1, ..., a_n lar m sonning *raqamlari* deyiladi, uni $m=\left(\overline{a_na_{n-1},...,a_1a_0}\right)$ ko'rinishda qisqacha yozish mumkin. Bu ko'rinishdagi son *asosi g ga teng bo'lgan sistematik son* deyiladi.

g asosli ixtiyoriy a va b sonlarni qo'shish, ayirish, ko'paytirish va bo'lish ko'phadni ko'phadga ko'paytirish kabi bajariladi.

1-misol. Hisoblang:

$$(202332_4 + 22201_4) + (220111_4 - 32303_4) - 23230301_4 : 113_4 .$$

Yechish. 4 lik sanoq sistemasida berilgan amallarni bajarish uchun qo'shish va ko'paytirish amallari jadvallarini tuzib olamiz:

+	0	1	2	3
0	0	1	2	3
1	1	2	3	10
2	2	3	10	11
3	3	10	11	12

×	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	10	12
3	0	3	12	21

Berilgan misoldagi amallarni bajaramiz:

1) $$+\begin{array}{r} 202332_4 \\ 22201_4 \\ \hline 231133_4 \end{array}$$ *Tekshirish:* $$-\begin{array}{r} 231133_4 \\ 202332_4 \\ \hline 22201_4 \end{array}$$

2) $$-\begin{array}{r} 220111_4 \\ 32303_4 \\ \hline 121202_4 \end{array}$$ *Tekshirish:* $$+\begin{array}{r} 121202_4 \\ 32303_4 \\ \hline 220111_4 \end{array}$$

3) $$+\begin{array}{r} 231133_4 \\ 121202_4 \\ \hline 1013001_4 \end{array}$$ *Tekshirish:* $$-\begin{array}{r} 1013001_4 \\ 231133_4 \\ \hline 121202_4 \end{array}$$

4) $$\begin{array}{l|l} -23230301_4 & 113_4 \\ \;\;232 & \overline{200203_4} \\ \hline \;\;\;-303 & \\ \;\;\;\;\;232 & \\ \hline \;\;\;\;\;-1101 & \\ \;\;\;\;\;\;\;1011 & \\ \hline \;\;\;\;\;\;\;\;\;30_4 & \end{array}$$

Tekshirish: $$\times\begin{array}{r} 200203_4 \\ 113_4 \\ \hline 1201221 \\ +\;200203 \\ 200203 \\ \hline 23230211_4 \end{array}$$

$23230211_4 + 30_4 = 2323301_4 .$

5) $$-\begin{array}{r} 1013001_4 \\ 200203_4 \\ \hline 1213210_4 \end{array}$$

Demak, javob: 1213210_4.

2-misol. n asosda berilgan sonni m va k asoslarga o‘tkazing:

$$a = 211, \quad n = 3, \quad m = 2, \quad k = 4.$$

Yechish. Berilgan a sonni 3 lik sanoq sistemasidan 2 lik sanoq sistemasiga o‘tkazish uchun berilgan sonni va hosil bo‘ladigan bo‘linmalarni 2 ga bo‘lamiz:

$$
\begin{array}{r|l} -211_3 & 2_3 \\ \underline{2} & 102_3 \\ -11 & \\ \underline{11} & \\ 0 & \end{array}
\qquad
\begin{array}{r|l} -102_3 & 2_3 \\ \underline{2} & 12_3 \\ -12 & \\ \underline{11} & \\ 1 & \end{array}
\qquad
\begin{array}{r|l} -12 & 2_3 \\ \underline{11} & 2_3 \\ 1 & \end{array}
\qquad
\begin{array}{r|l} -2_3 & 2_3 \\ \underline{2_3} & 1_3 \\ 0 & \end{array}
\qquad
\begin{array}{r|l} -1_3 & 2_3 \\ \underline{0} & 0_3 \\ 1 & \end{array}
$$

Bu jarayonni bo‘linmada 0 hosil bo‘lguncha davom ettiramiz. Oxirgi qoldiqdan boshlab barcha qoldiqlar yordamida berilgan sonning 2 lik sanoq sistemasidagi ifodasini topamiz: $211_3 = 10110_2$.

Tekshirish ikki usulda bajariladi:

1-usul. 211_3 va 10110_2 sonlar o‘nlik asosga o‘tkazilib solishtiriladi.

2-usul. 10110_2 uchlik asosga o‘tkaziladi.

$$211_3 = 2 \cdot 3^2 + 1 \cdot 3^1 + 1 \cdot 3^0 = 18 + 3 + 1 = 22_{10},$$

$$10110_2 = 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 16 + 4 + 2 = 22_{10}.$$

Demak, 211_3 son ikkilik asosda to‘g‘ri ifodalangan. 211_3 ning to‘rtlik asosdagi ifodasini topamiz. Buning uchun 211_3 ning o‘nlik asosdagi ifodasini topib, hosil bo‘lgan sonni to‘rtlik asosga o‘tkazamiz: $211_3 = 22_{10}$.

$$
\begin{array}{r|l} -22_{10} & 4_{10} \\ \underline{20} & 5_{10} \\ 2 & \end{array}
\qquad
\begin{array}{r|l} -5_{10} & 4_{10} \\ \underline{4} & 1_{10} \\ 1 & \end{array}
\qquad
\begin{array}{r|l} -1_{10} & 4_{10} \\ \underline{0} & 0_{10} \\ 1 & \end{array}
$$

Demak, $22_{10} = 112_4$, bundan $211_3 = 112_4$. Tekshirish yuqoridagi usullarda bajariladi.

Misol va mashqlar

1. Hisoblang:

1.1. $1101_2 + 1011_2$. 1.2. $1011_2 \cdot 1101_2$.

1.3. $1000110_2 - 11011_2$. 1.4. $100011_2 : 101_2$.

1.5. $3604_7 \cdot 423_7$. 1.6. $7(10)1_{12} \cdot 5(11)73_{12}$.

1.7. $23054_7 + 4326_7$.

1.8. $(10)(11)792_{12} + 9534(10)_{12} + 70(10)0_{12}$.

1.9. $26153_7 : 326_7$. 1.10. $8(1005)(11)_{12} : 9(10)_{12}$.

1.11. $101_8 : 32_8$.

2. Hisoblang:

2.1. $11011{,}101_2 + 101{,}011_2$. 2.2. $11{,}001_2 \cdot 1{,}01_2$.

2.3. $111{,}01_2 \cdot 101{,}101_2$. 2.4. $0{,}25_8 \cdot 0{,}43_8$.

2.5. $2{,}5_8 \cdot 3{,}4_8$.

3. Amallarni bajaring:

3.1. $7306_8 + 25645_8 - 6774_8 - 26156_8$.

3.2. $(425_6 \cdot 54_6 - 531_6 \cdot 43_6). : 245_6$.

3.3. $20671_8 : 131_8 - 140_8$.

3.4. $23213_5 : 32_5 + 113_5 \cdot 3_5 - 1242_5$.

3.5. $232011_5 : 104 + 1234_5 \cdot 322_5 - 1022131_5$.

3.6. $(563_8 + 217_8) \cdot 15_8 + (2365_8 - 636_8) : 17_8 - 15122_8$.

3.7. $120111_3 : 102_3 + 201_3 \cdot 12_3 - 11220_3$.

3.8. $6325_7 - 456_7 - 150335_7 : 23_7 - 551_7$.

3.9. $3215_7 \cdot 24_7 - 11461_7 : 25_7 + 1532_7 - 115044_7$.

3.10. $(4123_8 - 4221_8) \cdot 11_8 + (1222_8 + 773_8) : 3_8$.

3.11. $(3333_4 + 2222_4) \cdot 12_4 - (231020_4 + 3333333_4) : 23_4$.

3.12. $[(215_8 + 532_8) \cdot 16_8 - (11031_8 - 527_8)32_8] : 14775_8$.

3.13. $[(351_6 \cdot 14_6 - 1153_6 : 31_6 - 150_6) : 205_6] : 25_6$.

4. Berilgan sonlarni o‘nlik sanoq sistemasida ifodalang:

4.1. 100111_2. 4.2. 11001101_2.

4.3. 345_8. 4.4. 5071_8.

4.5. 1300_8. 4.6. 33311_7.

4.7. 4602_7. 4.8. $(10)6(11)_{12}$.

4.9. 26014_7. 4.10. 42125_6.

4.11. 530415_6.

5. Berilgan sonlarni o‘nlik sanoq sistemasida ifodalang:

5.1. $0{,}111_2$. 5.2. $0{,}110_2$.

5.3. $11001{,}1111_2$. 5.4. $437{,}321_8$.

5.5. $0{,}027_8$.

6. Bir sanoq sistemasidan ikkinchisiga o‘ting:

6.1. $33311_7 \to x_{12}$. 6.2. $2100012212 2_3 \to x_9$.

6.3. $4672510_9 \to x_3$. 6.4. $1111011101110000 1_2 \to x_8$.

6.5. $21066754_8 \to x_2$. 6.6. $206315_7 \to x_5$.

6.7. $32014 \to x_8$.

7. O‘nlik sanoq sistemasidan berilgan sanoq sistemalariga o‘ting:

7.1. $2042 \to x_2$, y_3, z_5. 7.2. $2786 \to x_2$, y_3, z_5.

7.3. $729 \to x_7$. 7.4. $231632 \to x_7$.

7.5. $23163 \to x_8$. 7.6. $17527 \to x_8$.

8. x ni toping:

8.1. $201_x=41_8$. 8.2. $203_x=53_{10}$. 8.3. $106_x=153_7$.

8.4. $236_x=1240_5$. 8.5. $324_x=10022_3$. 8.6. $541_x = 2014_6$.

8.7. $364_x=3001_4$. 8.8. $401_x=265_7$. 8.9. $100_x=34_7$.

9. Quyidagi tengliklar o‘rinli bo‘lgan sanoq sistemasini toping:

9.1. $12 + 13 = 30$. 9.2. $15 + 16 = 33$;

9.3. $35 + 40 = 115$. 9.4. $236 - 145 = 61$.

9.5. $263 - 214 = 46$. 9.6. $216 \cdot 3 = 654$.

9.7. $656 : 5 = 124$. 9.8. $736 : 6 = 121$.

9.9. $1520 : 12 = 123$. 9.10. $10 \cdot 10 = 100$.

Takrorlash uchun savollar

1. Sanoq sistemalari haqida tushuncha bering.
2. Sistematik son deb nimaga aytiladi?
3. Sistematik sonlar ustida amallar qanday bajariladi?
4. Bir sanoq sistemasidan boshqa sanoq sistemasiga o‘tishni tushuntiring.

XII MODUL. TAQQOSLAMALAR

27-§. Butun sonlar halqasida taqqoslamalar. Eyler va Ferma teoremalari

✓ **Asosiy tushunchalar:** «taqqoslanadi» munosabati, chegirmalar sinfi, chegirmalarning to'la sistemasi, chegirmalarning keltirilgan sistemasi, Eyler teoremasi, Ferma teoremasi.

Z – butun sonlar halqasi bo'lib, $m \geq 1$ natural son bo'lsin.

Agar Z halqaga tegishli a va b sonlarni m natural songa bo'lganda hosil bo'lgan qoldiqlar teng bo'lsa, yoki $a - b$ ayirma m ga bo'linsa, ya'ni $a=b+mq$ tenglik o'rinli bo'lsa, u holda a va b sonlar m modul bo'yicha *taqqoslanadi* deyiladi va u $a \equiv b(\mathrm{mod}\, m)$ ko'rinishda belgilanadi.

m ga bo'linganda r ga teng bir xil qoldiq beradigan butun sonlar to'plami m modul bo'yicha *chegirmalar sinfi* deyiladi va $\bar{r}$ kabi belgilanadi.

m modul bo'yicha tuzilgan har bir chegirmalar sinfidan ixtiyoriy bittadan element olib tuzilgan to'plam m modul bo'yicha *chegirmalarning to'la sistemasi* deyiladi.

m modul bilan o'zaro tub bo'lgan barcha chegirmalar sinfidan ixtiyoriy bittadan chegirma olib tuzilgan to'plam chegirmalarning *m modul bo'yicha keltirilgan sistemasi* deyiladi.

Eyler teoremasi. *Agar* $(a;m)=1$ *bo'lsa, u holda* $a^{\varphi(m)} \equiv 1\ (\mathrm{mod}\, m)$ *taqqoslama o'rinli bo'ladi.*

Ferma teoremasi. *Agar* $(a;r)=1$ *bo'lsa, u holda* $a^{p-1} \equiv 1$ $(\mathrm{mod}\, m)$ *taqqoslama o'rinli bo'ladi.*

1-misol. $a=2511$ sonini $b=123$ ga bo'lgandagi qoldiqni toping.

Yechish. Qoldiqli bo'lishi haqidagi teoremadan foydalanib $a=bq+r$, $0 \leq r \leq b$ ifodani topamiz: $2511=123 \cdot 20+51$.

Demak, $a=2511$ ni $b=123$ ga bo'lganda $r = 51$ qoldiq qoladi.

2-misol. $a=25^{112}$ ni $b=16$ ga bo'lgandagi qoldiqni toping.

Yechish. $a=25^{112}$ sonini 16 ga bo'lish uchun taqqoslamaning xossalaridan foydalanamiz. $25=16 \cdot 1+9$ ekanligidan $25 \equiv 9(\mathrm{mod}\, 16)$ kelib chiqadi. Bundan

$25^{112} \equiv 9^{112} \equiv (9^2)^{56} \equiv 81^{56}$. $81=16\cdot 5+1$ ekanligini e'tiborga olsak, u holda $25^{112} \equiv 81^{56} \equiv 1^{56} \equiv 1 (\mathrm{mod} 16)$.

Demak, 25^{112} ni 16 ga bo'lganda 1 qoldiq qoladi.

3-misol. Agar $100a+10\varnothing b+c \equiv 0 \pmod{21}$ bo'lsa, u holda $a - 2b + 4c \equiv 0 \pmod{21}$ ekanligini isbotlang.

Isbot. Taqqoslamaning ikkala tomonini modul bilan o'zaro tub 4 songa ko'paytiramiz: $400a+40b+4c \equiv 0 \pmod{21}$.

$400=21\cdot 19+1$, $40=21\cdot 2+(-2)$, $4=21\cdot 0+4$ lardan foydalanib, quyidagi taqqoslamalarni yozamiz :

$400a \equiv a (\mathrm{mod} 21)$, chunki $400a - a = 399a \vdots 21$;

$40b \equiv -2b \pmod{21}$, chunki $40b - (-2b) = 42b \vdots 21$;

$4c \equiv 4c \pmod{21}$, chunki $4c - 4c = 0 \vdots 21$.

Berilgan taqqoslamadan yuqoridagi taqqoslamalarni e'tiborga olib, $400a+40b+4c \equiv a - 2b + 4c \pmod{21}$ taqqoslamani hosil qilamiz.

Demak, $400a+40b+4c \equiv 0 \pmod{21}$ shartdan $a-2b+4c \equiv$ $\equiv 0 (\mathrm{mod} 21)$ kelib chiqadi.

Misol va mashqlar

1. Butun sonlar halqasida aniqlangan taqqoslama munosabatining quyidagi xossalarini isbotlang:

1.1. Taqqoslama ekvivalent binar munosabat.

1.2. Bir xil modulli taqqoslamalarni hadma-had qo'shish (ayirish) mumkin.

1.3. Taqqoslamaning bir qismidagi sonni uning ikkinchi qismiga qarama-qarshi ishora bilan o'tkazish mumkin.

1.4. Taqqoslamaning ixtiyoriy qismiga modulga karrali sonni qo'shish mumkin.

1.5. Bir xil modulli taqqoslamalarni hadma-had ko'paytirish mumkin.

1.6. Taqqoslamaning ikkala qismini (modulni o'zgartirmay) bir xil natural darajaga ko'tarish mumkin.

1.7. Modulni o'zgartirmagan holda taqqoslamaning ikkala qismini bir xil butun songa ko'paytirish mumkin.

1.8. Agar $x \equiv y (\mathrm{mod} m)$ bo'lsa, u holda ixtiyoriy butun koeffitsiyentli $f(x)=a_0x^n+a_1x^{n-1}+\ldots+a_{n-1}x+a_n$, $f(y)=a_0y^n+a_1y^{n-1}+\ldots+$

$+a_{n-1}y+a_n$ ko‘phadlar uchun $f(x) \equiv f(y) \pmod m$ taqqoslama o‘rinli bo‘ladi.

1.9. Agar bir vaqtda $a_i \equiv b_i \pmod m (i=\overline{1,n})$ va $x \equiv y \pmod m$ taqqoslamalar o‘rinli bo‘lsa, u holda

$a_0x^n+a_1x^{n-1}+...+a_{n-1}x+a_n \equiv b_0y^n+b_1y^{n-1}+...+b_{n-1}y+b_n \pmod m$

taqqoslama o‘rinli bo‘ladi.

1.10. Taqqoslamada qatnashuvchi qo‘shiluvchini o‘zi bilan teng qoldiqli bo‘lgan ikkinchi songa almashtirish mumkin.

1.11. Taqqoslamaning ikkala qismini modul bilan o‘zaro tub bo‘lgan ko‘paytuvchiga qisqartirish mumkin.

1.12. Taqqoslamaning ikkala qismi va modulini bir xil musbat songa ko‘paytirish mumkin.

1.13. Taqqoslamaning ikkala qismi va moduli umumiy ko‘paytuvchiga ega bo‘lsa, u holda bu taqqoslamaning ikkala qismi va modulini bu umumiy ko‘paytuvchiga bo‘lish mumkin.

1.14. Agar taqqoslama bir nechta modul bo‘yicha o‘rinli bo‘lsa, u holda bu taqqoslama shu modullarning eng kichik umumiy bo‘linuvchisi bo‘yicha ham o‘rinli bo‘ladi.

1.15. Agar taqqoslama biror m modul bo‘yicha o‘rinli bo‘lsa, u holda bu taqqoslama modulning ixtiyoriy bo‘luvchisi bo‘yicha ham o‘rinli bo‘ladi.

1.16. Taqqoslamaning bir qismi va modulining EKUBi bilan uning ikkinchi qismi va modulining EKUBi o‘zaro teng bo‘ladi.

1.17. Sinfning bitta chegirmasi m modul bilan o‘zaro tub bo‘lsa, u holda bu sinfning barcha elementlari ham m modul bilan o‘zaro tub bo‘ladi.

2. Bo‘lish natijasida hosil bo‘lgan qoldiqni toping:

2.1. 15^{231} ni 14 ga.

2.2. $15^{231}+2$ ni 16 ga.

2.3. 1532^5-1 ni 9 ga.

2.4. $12^{1231}+14^{4324}$ ni 13 ga.

2.5. 208^{208} ni 23 ga.

2.6. $2^{15783}-7$ ni 25 ga.

2.7. $3^{79821}+5$ ni 17 ga.

2.8. $10^{2732}+10$ ni 22 ga.

2.9. $18^{2815}-3$ ni 14 ga.

2.10. $2^{100}+5^{200}$ ni 29 ga.

2.11. $13^{1054}-23\cdot16^{285}+22^{17}$ ni 15 ga.

2.12. $29^{2929}-34^{3434}+29\cdot41\cdot6^{231}-24\cdot17^{120}$ ni 31 ga.

3. Har qanday a, b lar uchun quyidagilarni isbotlang:

3.1. $(11a+5)^{2n+1}+(11b+6)^{2n+1}\equiv0 \pmod{11}$.

3.2. $(13a+3)^{3n+2}+(13b-4)^{3n+2}+1\equiv0 \pmod{13}$.

3.3. $9^{3n+1}+3^{3n+1}+1\equiv0 \pmod{13}$.

4. Berilgan sonlarning oxirgi ikkita raqamini toping:

4.1. 2^{999}. 4.2. 3^{999}. 4.3. 2^{341}.

4.4. 289^{289}. 4.5. 203^{203203}. 4.6. $14^{14^{14}}$.

4.7. 9^{9^9}. 4.8. $7^{9^{9^9}}$.

5. Isbotlang:

5.1. Agar $(a+b-c)\,\vdots\,2$ bo'lsa, u holda $(a-b-c)\,\vdots\,2$.

5.2. Agar $(11a+2b)\,\vdots\,19$ bo'lsa, u holda $(18a+5b)\,\vdots\,19$.

5.3. Agar $(a-5b)\,\vdots\,17$ bo'lsa, u holda $(2a+7b)\,\vdots\,17$.

5.4. Agar $(12a-7b)\,\vdots\,16$ bo'lsa, u holda $(4a+23b)\,\vdots\,16$.

5.5. Agar $(a-5b)\,\vdots\,19$ bo'lsa, u holda $(10a+7b)\,\vdots\,19$.

5.6. Agar $(16a-11b+c)\,\vdots\,21$ bo'lsa, u holda $(11a-b+2c)\,\vdots\,21$.

5.7. Agar $(6a-11b)\,\vdots\,31$ bo'lsa, u holda $(a-7b)\,\vdots\,31$.

5.8. Agar $(50a+8b+c)\,\vdots\,21$ bo'lsa, u holda $(a+b+8c)\,\vdots\,21$.

5.9. Agar $(15a+3b)\,\vdots\,17$ bo'lsa, u holda $(5a+b)\,\vdots\,17$.

5.10. Agar $(50a-b+60c)\,\vdots\,388$ bo'lsa, u holda $(a-4b+41c)\,\vdots\,194$.

6. Quyidagilarning qaysilari uchun Eyler teoremasi o'rinli ekanligini aniqlang:

6.1. $a=2$, $m=9$. 6.2. $a=2$, $m=15$.

6.3. $a=3$, $m=4$. 6.4. $a=3$, $m=9$.

6.5. $a=3$, $m=16$. 6.6. $a=4$, $m=9$.

6.7. $a=5$, $m=24$. 6.8. $a=2$, $m=33$.

6.9. $a=3$, $m=24$.

7. Quyidagilarning qaysilari uchun Ferma teoremasi o'rinli ekanligini aniqlang:

7.1. $a=2$, $p=3$. 7.2. $a=2$, $p=5$.

7.3. $a=3$, $p=2$. 7.4. $a=10$, $p=5$.

7.5. $a=5$, $p=2$. 7.6. $a=5$, $p=3$.

7.7. $a=5$, $p=7$. 7.8. $a=4$, $p=3$.

7.9. $a=4$, $p=5$. 7.10. $a=14$, $p=7$.

8. Eyler teoremasi yordamida bo'lishdan hosil bo'lgan qoldiqni toping:

8.1. 7^{67} ni 12 ga. 8.2. 109^{345} ni 14 ga.
8.3. 197^{157} ni 35 ga. 8.4. 356^{273} ni 39 ga.
8.5. 383^{175} ni 45 ga. 8.6. 293^{275} ni 48 ga.
8.7. 439^{291} ni 60 ga. 8.8. 527^{144} ni 65 ga.
8.9. 353^{160} ni 75 ga. 8.10. 485^{84} ni 129 ga.

9. Ferma teoremasi yordamida bo'lishdan hosil bo'lgan qoldiqni toping:

9.1. 93^{253} ni 7 ga. 9.2. 5008^{10000} ni 5, 7, 11, 13 ga.
9.3. 42^{50} ni 17 ga. 9.4. 20^{59} ni 17 ga.
9.5. 2598^{33} ni 17 ga. 9.6. 230^{347} ni 37 ga.
9.7. 71^{50} ni 67 ga. 9.8. 512^{402} ni 101 ga.

10. Bo'lish natijasida hosil bo'lgan qoldiqni toping:

10.1. 45^{83} ni 24 ga. 10.2. 6^{76} ni 26 ga.
10.3. 96^{113} ni 92 ga. 10.4. 204^{41} ni 111 ga.
10.5. 460^{150} ni 425 ga. 10.6. 763^{17} ni 29 ga.
10.7. 342^{256} ni 29 ga. 10.8. 581^{3792} ni 37 ga.
10.9. 10^{10} ni 67 ga. 10.10. 244^{408} ni 73 ga.
10.11. 749^{193} ni 79 ga. 10.12. 341^{245} ni 89 ga.
10.13. 175^{411} ni 629 ga. 10.14. 272^{1141} ni 135 ga.
10.15. 35^{100} ni 1242 ga. 10.16. 20^{6n+5} ni 9 ga, $n \in N$.

11. Bo'lish natijasida hosil bo'lgan qoldiqni toping:

11.1. $7^{100} + 8^{100}$ ni 5 ga. 11.2. $10^{100} + 40^{100}$ ni 7 ga.
11.3. $3^{100} + 4^{100}$ ni 7 ga. 11.4. $5^{50} + 25^{70}$ ni 9 ga.
11.5. $25^{80} + 40^{80}$ ni 11 ga. 11.6. $15^{60} + 20^{30}$ ni 13 ga.
11.7. $5^{70} + 7^{50}$ ni 12 ga. 11.8. $3^{500} + 7^{500}$ ni 101 ga.
11.9. $(12371^{56} + 145)^{28}$ ni 111 ga.
11.10. $3 \cdot 5^{75} + 4 \cdot 7^{100}$ ni 132 ga. 11.11. $53^{29} \cdot 43^{17}$ ni 37 ga.
11.12. $378^{561} \cdot 427^{921}$ ni 41 ga. 11.13. $37^{20} \cdot 23^{12}$ ni 61 ga;
11.14. $3^{19 \cdot 37 - 1}$ ni $19 \cdot 37$ ga;
11.15. $(5622 + 179 - 346) \cdot 923$ ni 23 ga.
11.16. $(631^{57} + 250^{58}) \cdot 926$ ni 23 ga.
11.17. $7^{161} - 3^{80}$ ni 100 ga.
11.18. $(12371^{56} + 34)^{28}$ ni 111 ga.

12. Quyidagi sonlarning oxirgi ikkita raqamini toping:

12.1. 3^{100}. 12.2. 3^{219}. 12.3. 11^{243}.

12.4. 13^{219}. 12.5. 17^{900}. 12.6. 19^{882}.

12.7. 903^{1294}. 12.8. 573^{1931}. 12.9. 2^{100}.

12.10. 2^{153}. 12.11. 102^{54}.

13. Isbotlang:

13.1. $2^{11\ 31} \equiv 2 \pmod{11 \cdot 31}$.

13.2. $2^{19(73-1)} \equiv 1 \pmod{19 \cdot 73}$.

13.3. $2^{17 \cdot 19} \equiv 23 \pmod{17 \cdot 19}$.

13.4. $2^{1093 \cdot 1092} \equiv 1 \pmod{1093^2}$.

13.5. $2^{73 \cdot 37 - 1} \equiv 1 \pmod{73 \cdot 37}$.

14. Isbotlang:

14.1. $a^7 - a \vdots 42$. 14.2. $a^{11} - a \vdots 66$.

14.3. $a^{21} - a^3 \vdots 27$. 14.4. $a^{42} - a^2 \vdots 100$.

14.5. $a^{103} - a^3 \vdots 125$.

14.6. $a^{12} - b^{12} \vdots 65$, $(a,65) = (b,65) = 1$.

14.7. $a^{13} - a \vdots 2730$.

14.8. $a^{560} - 1 \vdots 561$, $(a, 561) = 1$.

14.9. $a^{561} - a \vdots 11$.

14.10. $a^{10} - a^6 - a^4 + 1 \vdots 35$, $(a, 35) = 1$.

14.11. $14^{120} - 1 \vdots 45$.

14.12. $13^{176} - 1 \vdots 89$.

14.13. $372654^{500} + 72 \cdot 10^7 \vdots 18$.

14.14. $2^{1093} - 2 \vdots 1093^2$.

14.15. $43^{23} + 23^{43} \vdots 66$.

14.16. $222^{555} + 555^{222} \vdots 7$.

14.17. $220^{119^9} + 69^{220^{119}} + 119^{69^{220}} \vdots 102$.

15. Ixtiyoriy m,n natural sonlar uchun quyidagilarni isbotlang:

15.1. $n^7 + 6n \vdots 7$. 15.2. $10^n(9n - 1) + 1 \vdots 9$.

15.3. $3 \cdot 5^{2n+1} + 2^{3n+1} \vdots 17$. 15.4. $6^{2n+1} + 5^{n+2} \vdots 31$.

15.5. Agar $m = 2n$ bo'lsa, $20m + 16m - 3m - 1 \vdots 323$.

15.6. $mn(m^{60} - n^{60}) \vdots 56786730$.

15.7. Agar $(m,12) = (n,12) = 1$ bo'lsa, $m^{96} - b^{96} \vdots 144$.

16. Isbotlang:

16.1. Agar $a_1 + a_2 + ... + a_n \equiv 0 \pmod{30}$, $a_1, a_2, ... a_n \in Z$ bo'lsa, u holda $a_1^5 + a_2^5 + ... + a_n^5 \equiv 0 \pmod{30}$.

16.2. Agar $n \in N$, $(a, 10) = 1$ bo'lsa, u holda $a^{100n+1} \equiv a \pmod{1000}$.
16.3. Agar $(n,6) = 1$ bo'lsa, u holda $n^2 \equiv 1 \pmod{24}$.
16.4. $a^{6m} + a^{6n} \equiv 0 \pmod 7 \Rightarrow a \,\vdots\, 7$, $m, n \in N$.
16.5. Agar a butun sonning kubi bo'lsa, u holda

$$(a-1)a(a+2) \equiv 0 \pmod{504}.$$

Takrorlash uchun savollar

1. Taqqoslama deb nimaga aytiladi?
2. Taqqoslamaning sodda xossalarini bayon eting.
3. Modul bo'yicha chegirmalarning to'la sistemasi deb nimaga aytiladi?
4. Modul bo'yicha chegirmalarning keltirilgan sistemasi deb nimaga aytiladi?
5. Eyler va Ferma teoremalarini bayon eting.

28-§. Birinchi darajali va tub modul bo'yicha yuqori darajali taqqoslamalar

✔ **Asosiy tushunchalar:** bir noma'lumli n- darajali taqqoslama, taqqoslamaning yechimi, teng kuchli taqqoslamalar, bir noma'lumli birinchi darajali taqqoslama, tub modulli taqqoslama.

Koeffitsiyentlari butun sonlardan iborat

$$f(x) = a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n$$

ko'phad berilgan bo'lsin.

Ushbu $f(x) \equiv 0 \pmod m$ (a_0 son m ga bo'linmaydi, $a_i \in Z$, $m \geq 1$) ko'rinishdagi taqqoslama *bir noma'lumli n- darajali taqqoslama* deyiladi.

Agar $x=c$ bo'lganda $f(c) \equiv 0 \pmod m$ taqqoslama to'g'ri bo'lsa, u holda c son $f(x)$ taqqoslamani *qanoatlantiradi* deyiladi.

Agar c son $f(x)$ taqqoslamani qanoatlantirsa, u holda $\bar{c}$ chegirmalar sinfi $f(x)$ *taqqoslamaning yechimi* deyiladi.

Yechimlari to'plami ustma-ust tushgan taqqoslamalar *teng kuchli taqqoslamalar* deyiladi.

Ushbu $ax \equiv b \pmod m$ ($a,b \in Z$, $\forall m \in N$) ko'rinishdagi taqqoslama *bir noma'lumli birinchi darajali taqqoslama* deyiladi.

Agar $f(x)=a_0x^p+a_1x^{n-1}+...+a_{n-1}x+a_n$, $a_i \in Z$, p – tub son, $(a_0, p)=1$ bo‘lsa, u holda $f(x) \equiv 0 \pmod p$ taqqoslama *tub modulli n-darajali bir noma'lumli taqqoslama* deyiladi.

1-misol. $7 \cdot x \equiv 10 \pmod 4$ taqqoslamaning yechimlarini taqqoslama xossalaridan foydalanib toping.

Yechish. $(7,4)=1$ ekanligidan taqqoslama yagona yechimga ega ekanligi kelib chiqadi. 7 va 11 sonlari 4 dan katta bo‘lganligi uchun $7 \cdot x \equiv 3x \pmod 4$ va $10 \equiv 2 \pmod 4$ lardan foydalanib, $3x \equiv 2 \pmod 4$ ni hosil qilamiz. Bundan $3x \equiv -x \pmod 4$ ni e'tiborga olib, $-x \equiv 2 \pmod 4$ ni, va nihoyat $x \equiv -2 \pmod 4$ ni hosil qilamiz.

Agar $-2 \equiv 2 \pmod 4$ ni qo‘llasak, u holda $x \equiv 2 \pmod 4$ kelib chiqadi.

Tekshirish: $7 \cdot 2 \equiv 10 \pmod 4$,

$14 \equiv 10 \pmod 4 \Rightarrow (14 - 10) = 4 \vdots 4$

kelib chiqadi

2-misol. Taqqoslama xossalaridan foydalanib, $27x \equiv 47 \pmod{38}$ taqqoslamaning yechimlarini toping.

Yechish. $47 \equiv 9 \pmod{38}$ dan $27x \equiv 9 \pmod{38}$ hosil bo‘ladi. $(27,38)=1$ bo‘lgani uchun taqqoslama yagona yechimga ega. $(9,38)=1$ bo‘lgani uchun taqqoslamaning ikkala tomonini 9 ga bo‘lamiz: $3x \equiv 1 \pmod{38}$.

Taqqoslamaning o‘ng tomoniga 38 ni qo‘shamiz: $3x \equiv 39 \pmod{38}$. Hosil bo‘lgan taqqoslamaning ikkala tomonini $(3,38)=1$ bo‘lgani uchun 3 ga bo‘lamiz: $x \equiv 13 \pmod{38}$.

Tekshirish: $27 \cdot 13 - 47 = 304 = (38 \cdot 8) \vdots 38$.

3-misol. Berilgan $7x \equiv 10 \pmod 4$ taqqoslamani tanlash usuli bilan yeching.

Yechish. $ax \equiv b \pmod m$ taqqoslamaning yechimlarini tanlash usuli bilan topish uchun avval yechimlar sonini aniqlaymiz. So‘ngra m modul bo‘yicha chegirmalar to‘la sistemasidagi har bir sinfning yechim bo‘lish-bo‘lmasligini tekshiramiz.

$7x \equiv 10 \pmod 4$ taqqoslamada $(7,4) = 1$.

Demak, yagona yechim mavjud. 4 modul bo‘yicha chegirmalar to‘la sistemasidagi 0, 1, 2, 3 sonlar x noma'lum o‘rniga birmabir qo‘yib tekshiriladi. Qaysidir chegirmalar sinfi yechim bo‘lishi ma'lum bo‘lsa, tekshirish jarayonini to‘xtatamiz:

$x=0$ da $7 \cdot 0 \equiv 10 \pmod 4$ o'rinli emas, chunki $(0-10) \not\vdots\, 4$;

$x=1$ da $7 \cdot 1 \equiv 10 \pmod 4$ o'rinli emas, chunki $7-10=3 \not\vdots\, 4$;

$x=2$ da $7 \cdot 2 \equiv 10 \pmod 4$ o'rinli, chunki $14-10=4 \,\vdots\, 4$.

$x \equiv 2 \pmod 4$ yechim bo'ladi. Qolgan sinflar berilgan taqqoslamaning birgina yechimi mavjud bo'lganligi sababli, tekshirilmaydi.

Tekshirish. $7 \cdot 2 - 10 = 14 - 10 = 4 \,\vdots\, 4$.

4-misol. $2x \equiv 5 \pmod 9$ taqqoslamaning yechimlarini tanlash usuli yordamida toping.

Yechish. 9 modul bo'yicha 0, ±1, ±2, ±3, ±4 chegirmalar sinflaridan $(2;9)=1$ bo'lganligi uchun berilgan taqqoslamaning yagona yechimini topamiz:

$$2 \cdot 0 = 0 \not\equiv 5 \pmod 9;$$
$$2 \cdot 1 = 2 \not\equiv 5 \pmod 9;$$
$$2 \cdot (-1) = -2 \not\equiv 5 \pmod 9;$$
$$2 \cdot 2 = 4 \not\equiv 5 \pmod 9;$$
$$2 \cdot (-2) = -4 \equiv 5 \pmod 9.$$

Demak, $x \equiv -2 \pmod 9$, ya'ni $x \equiv 7 \pmod 9$ berilgan taqqoslamaning yechimi.

Tekshirish: $2 \cdot 7 - 5 = 14 - 5 = 9 \,\vdots\, 9$.

5-misol. $7x \equiv 10 \pmod 4$ taqqoslamani Eyler teoremasi yordamida yeching.

Yechish. Agar $a \cdot x \equiv b \pmod m$ taqqoslama $(a,m)=1$ bo'lsa, u holda uning yechimi $x = b \cdot a^{\varphi(m)-1} \pmod m$ formula yordamida topiladi. Haqiqatan ham, Eyler teoremasiga ko'ra, $a^{\varphi(m)} \equiv$ $\equiv 1 \pmod m$. Bundan $a^{\varphi(m)} b \equiv b \pmod m$ va $a \cdot a^{\varphi(m)-1} b \equiv b \pmod m$ larni hosil qilsak, $x \equiv b a^{\varphi(m)-1} \pmod m$ kelib chiqadi.

$7x \equiv 10 \pmod 4$ dan $a = 7$, $b = 10$, $m = 4$ yechim $x \equiv 10 \cdot 7^{\varphi(4)-1} \pmod 4$ ni topish uchun $\varphi(4)$ ni aniqlaymiz. $4=2^2$ ekanligidan $\varphi(4) = 4 \cdot \left(1 - \frac{1}{2}\right) = 2$ kelib chiqadi.

Demak, $x = 10 \cdot 7^{2-1} \pmod 4$. Agar $10 \equiv 2 \pmod 4$, $7 \equiv 3 \pmod 4$ va $6 \equiv 2 \pmod 4$ taqqoslamalardan foydalansak, $x \equiv 10 \cdot 7^{2-1} \equiv$ $\equiv 2 \cdot 3 \equiv 6 \equiv 2 \pmod 4$, ya'ni $x \equiv 2 \pmod 4$ yechimni hosil qilamiz.

Tekshirish: $2 \cdot 10 - 10 = 14 - 10 = 4 \vdots 4$.

6-misol. $27x \equiv 24 \pmod{102}$ taqqoslamaning yechimlarini Eyler metodidan foydalanib toping.

Yechish. (27,102)=3 va 24=3·8. Demak, taqqoslama 3 ta yechimga ega. Berilgan taqqoslamaning ikkala qismi va modulni 3 ga bo'lamiz: $9x \equiv 8 \pmod{34}$.

Bunda $a = 9$, $m = 34$, $b = 8$ bo'lgani uchun $x \equiv b \cdot a^{\varphi(m)-1} \pmod m$ dan $x \equiv 8 \cdot 9^{\varphi(34)-1} \pmod{34}$ ga ega bo'lamiz.

$\varphi(34) = 2 \cdot 17\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{17}\right) = 16$ ekanligini e'tiborga olamiz:

$$x \equiv 8 \cdot 9^{15} \equiv 8 \cdot 9 \cdot 9^{14} \equiv 4 \cdot (9^2)^7 \equiv 4 \cdot 13^7 \equiv 4 \cdot 13^7 \equiv 4 \cdot 13 \cdot (13^2)^3 \equiv$$
$$\equiv 18 \cdot 33^3 \equiv 18 \cdot 33 \cdot (33)^2 \equiv 16 \cdot 1^2 \equiv 16 \pmod{34}.$$

Bundan $x \equiv 16 \pmod{34}$ ga ega bo'lamiz.

Tekshirish: $9 \cdot 16 - 8 = 136 \vdots 34$. U holda $27x \equiv 24 \pmod{102}$ taqqoslama

$x \equiv 16 \pmod{102}$,

$x \equiv 16 + 34 \pmod{102}$,

$x \equiv 16 + 34 \cdot 2 \pmod{102}$ yechimlarga, ya'ni

$x \equiv 16 \pmod{102}$,

$x \equiv 50 \pmod{102}$,

$x \equiv 84 \pmod{102}$ yechimlarga ega.

Tekshirish: $27 \cdot 16 - 24 = 408 \vdots 102$;

$27 \cdot 50 - 24 = 3126 \vdots 102$;

$27 \cdot 84 - 24 = 2244 \vdots 102$.

7-misol. $7x \equiv 10 \pmod 4$ taqqoslamani munosib kasrlar yordamida yeching.

Yechish. Agar $ax \equiv b \pmod m$ taqqoslamada $(a,m)=1$ va P_{n-1} son $\frac{m}{a}$ ning oxiridan oldingi munosib kasr surati bo'lsa, u holda $x \equiv b\cdot(-1)^{n-1}P_{n-1} \pmod m$ berilgan taqqoslamaning yechimi bo'ladi.

Berilgan taqqoslamada m=4, a=7 bo'lganidan, $\frac{4}{7}$ ning munosib kasrlarini topamiz:

$$4 = 7\cdot 0 + 4;$$
$$7 = 4\cdot 1 + 3;$$
$$4 = 3\cdot 1 + 1;$$
$$3 = 1\cdot 3 + 0.$$

Bundan $\frac{4}{7} = [0;\ 1,\ 1,\ 3]$ ko'rinishda bo'ladi.

Munosib kasrlar jadvalini tuzamiz:

k	-1	0	1	2	3
q_k	–	0	1	1	3
P_k	1	0	1	1	4
Q_k	0	1	1	2	7

Demak, $P_{n-1} = P_2 = 1$ va $x \equiv b\cdot(-1)^{n-1}P_{n-1} \equiv 10\cdot(-1)^{3-1}\cdot 1 \equiv$ $\equiv 10 \equiv 2 \pmod 4$. Berilgan taqqoslamaning yechimi mavjud ekan.

Tekshirish: $7\cdot 2 - 10 = 14 - 10 = 4 \,\vdots\, 4$.

8-misol. $220x \equiv 28 \pmod{348}$ taqqoslamaning yechimlarini munosib kasrlar yordamida toping.

Yechish. $(220,348)=4$ va $28 \,\vdots\, 4$ dan berilgan taqqoslama 4 ta yechimga ega ekanligi kelib chiqadi. Taqqoslamaning ikkala tomoni va modulni 4 ga bo'lamiz: $55x \equiv 7 \pmod{87}$. $\frac{87}{55}$ kasrni chekli zanjir kasr ko'rinishiga keltirib, munosib kasrlar jadvalini tuzamiz: $\frac{87}{55}=[1; 1,\ 1,\ 1,\ 2,\ 1,\ 1,\ 4]$. Bundan,

k	1	0	1	2	3	4	5	6
q_k	–	1	1	1	2	1	1	4
P_k	1	1	2	3	8	11	19	87

va n=6, $P_{n-1}=P_5=19$, b=7, m=87 larni $x \equiv (-1)^n P_{n-1} b \pmod{m}$ formulaga qo'ysak, $x \equiv (-1)^6 \cdot 19 \cdot 7 \equiv 133 \equiv 46 \pmod{87}$ kelib chiqadi.

Demak, $55x \equiv 7 \pmod{87}$ ning yechimi $x \equiv 46 \pmod{87}$ va $220x \equiv 28 \pmod{348}$ ning yechimlari $x \equiv 46;\ 133;\ 220;\ 307 \pmod{348}$.

Tekshirish:
$$220 \cdot 46 - 28 = 10092 \vdots 348;$$
$$220 \cdot 133 - 28 = 29232 \vdots 348;$$
$$220 \cdot 220 - 28 = 48372 \vdots 348;$$
$$220 \cdot 307 - 28 = 67512 \vdots 348.$$

9-misol. $7x \equiv 10 \pmod 4$ taqqoslamani 7 ga 4 modul bo'yicha teskari sinfi orqali yeching.

Yechish. $ax \equiv b \pmod{m}$ taqqoslamada $(a,m)=1$ bo'lsa, u holda 1 ning a va m sonlarga chiziqli yoyilmasini topamiz: $1=au+mv$ yoyilmadagi u soni a soniga m modul bo'yicha teskari son bo'ladi.

Evklid algoritmi yordamida berilgan $\frac{7}{4}$ sonlarning eng katta umumiy bo'luvchisining chiziqli ifodasini topamiz:

$7 = 4 \cdot 1 + 3;$ $\qquad 3 = 7 - 4 \cdot 1;$

$4 = 3 \cdot 1 + 1;$ $\qquad 1 = 4 - 3 \cdot 1.$

$3 = 1 \cdot 3 + 0.$

Bundan $1 = 4 - 3 \cdot 1 = 4 - (7 - 4 \cdot 1) = 4 \cdot 2 - 7 = 4 \cdot 2 + 7(-1)$. Demak, $1 = 4 \cdot 2 + 7(-1)$. 7 soniga 4 modul bo'yicha teskari son -1 yoki $-1 \equiv 3 \pmod 4$ ekanligidan 3 soni bo'ladi.

$7x \equiv 10 \pmod 4$ taqqoslamaning ikkala tomonini 7 ga 4 modul bo'yicha teskari son 3 ga ko'paytiramiz: ((3,4)=1).

$$7 \cdot 3x \equiv 10 \cdot 3 \pmod 4;$$
$$21x \equiv 30 \pmod 4;$$
$$21x \equiv x \pmod 4;$$
$$30x \equiv 2 \pmod 4$$

lardan $x \equiv 2$ yechimni topamiz.

Tekshirish: $7 \cdot 2 - 10 = 14 - 10 = 4 \vdots 4$.

10-misol. $37x \equiv 25 \pmod{107}$ taqqoslamani teskari sinf yordamida yeching.

Yechish. (37,107)=1 dan berilgan taqqoslamaning yagona yechimi mavjudligi kelib chiqadi. 107 modulda 37 ga teskari sonni topamiz:

$$107 = 37 \cdot 2 + 33;$$
$$37 = 33 \cdot 1 + 4;$$
$$33 = 4 \cdot 8 + 1;$$
$$4 = 1 \cdot 4 + 0.$$

$$1 = 33 - 4 \cdot 8 = 33 - (37 - 33 \cdot 1) \cdot 8 = 33 \cdot 9 + 37(-8) =$$
$$= (107 - 37 \cdot 2) \cdot 9 + 37(-8) = 107 \cdot 9 + 37(-26).$$

Bundan $1 = 107 \cdot 9 + 37(-26)$, ya'ni 107 modulda 37 ga teskari sinf −26 ni musbat son bilan almashtiramiz: −26+107=81. Hosil bo'lgan 81 ga berilgan taqqoslamaning ikkala qismini ko'paytiramiz va $37 \cdot 81x \equiv 28 \cdot 81 \pmod{107}$ dan $x \equiv 2025 \pmod{107}$, ya'ni $x \equiv 99 \pmod{107}$ yechimni topamiz.

Tekshirish: $37 \cdot 99 - 25 = 3638 \vdots 107$.

11-misol. $27x + 38y = 47$ tenglamani taqqoslamalar yordamida yeching.

Yechish. Tenglamaning butun yechimlarini taqqoslamalardan foydalanib topish uchun $27x \equiv 47 \pmod{38}$ bir o'zgaruvchili taqqoslamani tuzib olamiz. (27,38)=1 ekanligidan taqqoslamaning bitta yechimi mavjud. $47 \equiv 9 \pmod{38}$ dan $27x \equiv 9 \pmod{38}$ ni hosil qilamiz. Bundan $3x \equiv 1 \pmod{38}$ va $x \equiv 13 \pmod{38}$ kelib chiqadi.

$x \equiv 13 \pmod{38}$ berilgan $27x \equiv 9 \pmod{38}$ taqqoslamaning yechimi. U holda $\left\{13, \frac{47-27\cdot 13}{38}\right\} = \{13, -8\}$ berilgan tenglamaning yechimlaridan biri bo'ladi.

$ax + by = c$ tenglamaning barcha yechimlari $x' = x_0 + \frac{m}{d}t$, $y' = y_0 - \frac{a}{d}t$ ko'rinishda bo'lib, bu yerda $x_0=13$, $y_0=-8$, $m=38$, $a=27$, $d=1$. Demak, $\begin{cases} x' = 13 + 38t, \\ y' = -8 - 27t, \quad t \in Z. \end{cases}$

Tekshirish:

$$27(13 + 38t) + 38(-8 - 27t) = 47;$$
$$351 + 1026t - 304 - 1026t = 47;$$
$$47 = 47.$$

12-misol. $\begin{cases} 3x \equiv 11 \pmod{17}, \\ 15x \equiv 35 \pmod{13}, \\ 21x \equiv 33 \pmod{30} \end{cases}$ taqqoslamalar sistemasini yeching.

Yechish. Berilgan taqqoslamalar sistemasidagi har bir taqqoslama yechimlari yuqoridagi misollarda keltirilgan usullardan biri yordamida topiladi:

$$\begin{cases} x \equiv 15 \pmod{17}, \\ x \equiv 11 \pmod{13}, \\ x \equiv 3 \pmod{10}. \end{cases}$$

Hosil qilingan taqqoslamalar sistemasidagi taqqoslamalar modullari o'zaro tub bo'lganligi uchun ularning eng kichik umumiy karralisi M bo'yicha quyidagi qiymatlarni topamiz:

$M = 17 \cdot 13 \cdot 10 = 2210;$

$$M_1=\frac{2210}{17}=130;$$
$$M_2=\frac{2210}{13}=170;$$
$$M_3=\frac{2210}{10}=221.$$

Quyidagi taqqoslamalarni tuzib, yechimini topamiz:

1) $130y_1\equiv 1 \pmod{17}$,
$y_1=14$;

2) $170y_2\equiv 1 \pmod{13}$,
$y_2=1$;

3) $221y_3\equiv 1 \pmod{10}$,
$y_3=1$.

Bundan berilgan taqqoslamalar sistemasining yechimi

$$x=x_0=130\cdot14\cdot15+170\cdot1\cdot11+211\cdot1\cdot3=29833\equiv1103 \pmod{2210},$$

ya'ni $x\equiv 1103 \pmod{2210}$ kelib chiqadi.

Agar berilgan taqqoslamalar sistemasidagi uchinchi taqqoslamaning 3 ta yechimi borligini e'tiborga olsak, u holda taqqoslamalar sistemasining 3 ta yechimini topish mumkin:

$$\begin{cases}x\equiv15 \pmod{17},\\ x\equiv11 \pmod{13},\\ x\equiv3 \pmod{30},\end{cases}\quad \begin{cases}x\equiv15 \pmod{17},\\ x\equiv11 \pmod{13},\\ x\equiv13 \pmod{30},\end{cases}\quad \begin{cases}x\equiv15 \pmod{17},\\ x\equiv11 \pmod{13},\\ x\equiv23 \pmod{30}.\end{cases}$$

$$x\equiv5523 \pmod{6630};\quad x\equiv3313 \pmod{6630};\quad x\equiv1103 \pmod{6630}$$

yechimlar hosil qilinadi.

13-misol. $\begin{cases}x\equiv2 \pmod{15},\\ x\equiv7 \pmod{20},\\ x\equiv12 \pmod{35}\end{cases}$ taqqoslamalar sistemasini yeching.

Yechish. Taqqoslama ta'rifiga ko'ra birinchi taqqoslamadan $x=2+15t,\ t\in Z$ ifodani hosil qilamiz. Bu qiymatni ikkinchi taqqoslamaga qo'yamiz: $2+15t\equiv7 \pmod{20}$. Bundan, $15t\equiv5 \pmod{20}$ yoki $t\equiv3 \pmod{4}$ ni olamiz. Yana taqqoslama ta'rifini qo'llab,

$z \equiv 3+4k,\ k \in Z$ ifodani olamiz. Bu ifodadan $x = 2+15t =$ $= 2+15(3+4k) = 47+60k$ kelib chiqadi. Hosil qilingan x ning ifodasini uchinchi taqqoslamaga qo'yamiz: $47+60k \equiv 12 \pmod{35}$ taqqoslamani yechib, $k \equiv 0 \pmod 7$ yechimni topamiz.

Bundan $k = 7l,\ l \in Z$ kelib chiqadi. Hosil bo'lgan ifodani x ning ifodasiga qo'llaymiz: $x = 47+60k = 47+60\cdot 7l = 47+420l$.

Demak, $x \equiv 47 \pmod{420}$ berilgan taqqoslamalar sistemasining yechimi.

Tekshirish: $\begin{cases} 47-2 = 45 \vdots 15, \\ 47-7 = 40 \vdots 20, \\ 47-12 = 35 \vdots 35. \end{cases}$

14-misol. $251x^{54}+63x^{25}-7x^{11}+4x^3+2 \equiv 0 \pmod 5$ taqqoslamani soddalashtiring.

Yechish. Berilgan taqqoslamani soddalashtirish uchun taqqoslamalar xossalari va Eyler teoremasidan foydalanamiz:

$251 \equiv 1 \pmod 5$;
$63 \equiv 3 \pmod 5$;
$7 \equiv 2 \pmod 5$;
$4 \equiv 4 \pmod 5$;
$2 \equiv 2 \pmod 5$.

$\varphi(5)=4$ dan

$x^{54} \equiv (x^4)^{13}\cdot x^2 \equiv x^2 \pmod 5$;
$x^{25} \equiv (x^4)^6\cdot x \equiv x \pmod 5$;
$x^{11} \equiv (x^4)^2\cdot x^3 \equiv x^3 \pmod 5$.

Keltirilgan taqqoslamalar yordamida berilgan taqqoslamani soddalashtiramiz:

$$251x^{54}+63x^{25}-7x^{11}+4x^3+2 = x^2+3x-2x^3+4x^3+2 =$$
$$= 2x^3+x^2+3x+2 = 0 \pmod 5.$$

Misol va mashqlar

1. Quyidagi xossalarni isbotlang:

1.1. Agar c son $f(x)\equiv 0$ (mod$m$) taqqoslamani qanoatlantirsa, u holda $\overline{c}$ chegirmalar sinfiga tegishli ixtiyoriy son ham shu taqqoslamani qanoatlantiradi.

1.2. Agar $(a;m)=1$ bo'lsa, u holda $ax\equiv b$ (mod m) taqqoslama yagona yechimga ega bo'ladi.

1.3. Agar $(a;\ m)=d$ bo'lib, b son d ga bo'linmasa, u holda $ax\equiv b$ (mod m) taqqoslama yechimga ega emas.

1.4. Agar $ax\equiv b$ (mod m) taqqoslamada $(a;\ m)=d$ bo'lib, b son d ga bo'linsa, u holda taqqoslama soni d ga teng bo'lgan ushbu

$\overline{\alpha}, \overline{\alpha+\frac{m}{d}}, ..., \overline{\alpha+\frac{(d-1)m}{d}}$ yechimlarga ega bo'lib, bundagi α yechim

$\frac{a}{d}x\equiv\frac{b}{a}\left(\mathrm{mod}\frac{m}{d}\right)$ taqqoslamaning yagona yechimi bo'ladi.

1.5. Agar $f(x)$ va $g(x)$ koeffitsiyentlari butun sonlardan iborat ko'phadlar bo'lsa, u holda $f(x)\equiv 0$(mod p) va $(x)-(x^p-x)g(x)\equiv$ $\equiv 0$ (mod$p$) taqqoslamalar teng kuchli bo'ladi.

1.6. Darajasi n $(n>r)$ bo'lgan r tub modulli taqqoslama darajasi $r-1$ dan katta bo'lmagan taqqoslamaga teng kuchli bo'ladi.

1.7. Tub modulli n-darajali taqqoslama yechimlari soni n tadan ortiq emas.

2. Quyidagi taqqoslamalarni tanlash usulida yeching:

2.1. $2x\equiv 1$ (mod 3). 2.2. $8x\equiv 3$ (mod 4).
2.3. $6x\equiv 7$ (mod 5). 2.4. $3x\equiv 22$ (mod 7).
2.5. $4x\equiv 6$ (mod 10). 2.6. $12x\equiv 1$ (mod 7).
2.7. $5x\equiv 7$ (mod 11). 2.8. $8x\equiv 1$ (mod 16).

3. Quyidagi taqqoslamalarni taqqoslama xossalari yordamida yeching:

3.1. $7x\equiv 8$ (mod 13). 3.2. $6x\equiv 11$ (mod 14).
3.3. $8x\equiv 10$ (mod 14). 3.4. $11x\equiv -32$ (mod 27).
3.5. $16x\equiv 50$ (mod 23). 3.6. $25x\equiv 1$ (mod 37).
3.7. $17x\equiv 23$ (mod 41). 3.8. $32x\equiv 43$ (mod 51).

4. Berilgan taqqoslamalarni Eyler teoremasi yordamida yeching:

4.1. $5x \equiv 7 \pmod{13}$. 4.2. $29x \equiv 3 \pmod{12}$.

4.3. $5x \equiv 26 \pmod{12}$. 4.4. $8x \equiv 17 \pmod{19}$.

4.5. $27x \equiv 11 \pmod{34}$. 4.6. $24x \equiv 1 \pmod{15}$.

4.7. $15x \equiv 23 \pmod{22}$. 4.8. $12x \equiv 51 \pmod{39}$.

5. Berilgan taqqoslamalarni chekli zanjir kasrlar yordamida yeching:

5.1. $15x \equiv 37 \pmod{98}$. 5.2. $32x \equiv 182 \pmod{119}$.

5.3. $105x \equiv 72 \pmod{147}$. 5.4. $97x \equiv 53 \pmod{169}$.

5.5. $-50x \equiv 67 \pmod{177}$. 5.6. $69x \equiv 393 \pmod{201}$.

5.7. $192x \equiv 9 \pmod{327}$. 5.8. $365x \equiv 50 \pmod{395}$.

5.9. $-639x \equiv 177 \pmod{924}$. 5.10. $1296x \equiv 1105 \pmod{2413}$.

5.11. $1215x \equiv 550 \pmod{2755}$. 5.12. $1919x \equiv 1717 \pmod{4009}$.

6. Berilgan $ax \equiv b \pmod{m}$ taqqoslamalarni a ga teskari sinf orqali yeching:

6.1. $21x \equiv 17 \pmod{23}$. 6.2. $5x \equiv 7 \pmod{24}$.

6.3. $17x \equiv 19 \pmod{24}$. 6.4. $13x \equiv -1 \pmod{30}$.

6.5. $28x \equiv 33 \pmod{35}$. 6.6. $12x \equiv 24 \pmod{30}$.

6.7. $9x \equiv 18 \pmod{41}$. 6.8. $11x \equiv 31 \pmod{50}$.

7. Quyidagi taqqoslamalarni yeching:

7.1. $(a+b)x \equiv a^2+b^2 \pmod{ab}$, $(a,b) = 1$.

7.2. $(a^2+b^2)x \equiv a - b \pmod{ab}$, $(a,b) = 1$.

7.3. $(a+b)^2x \equiv a^2 - b^2 \pmod{ab}$, $(a,b) = 1$.

7.4. $(a-b)x \equiv a^2+b^2 \pmod{ab}$, $(a,b) = 1$.

7.5. $2x \equiv 1 + p \pmod{p}$, bu yerda p — tub toq son.

7.6. $(m-1)x \equiv 1 \pmod{m}$.

7.7. $(m+1)^2x \equiv a \pmod{m}$.

7.8. $ax \equiv 1 \pmod{p}$, bu yerda p tub son va $(a, p) = 1$.

8. Berilgan tenglamalarni taqqoslamalar yordamida yeching:

8.1. $2x + 3y = 4$. 8.2. $4x - 3y = 2$.

8.3. $3x + 4y = 13$. 8.4. $5x + 4y = 3$.

8.5. $3x + 8y = 5$. 8.6. $17x + 13y = 1$.

8.7. $23x + 15y = 19$. 8.8. $17x - 16y = 31$.

8.9. $91x - 28y = 35$. 8.10. $17x - 39y = 26$.

8.11. $50x - 42y = 34$. 8.12. $47x - 105y = 4$.

8.13. $47x - 111y = 89$.

9. Taqqoslamalar sistemasini yeching:

9.1. $\begin{cases} 3x \equiv 5 \pmod{7}, \\ 2x \equiv 1 \pmod{5}. \end{cases}$ 9.2. $\begin{cases} 3x \equiv 1 \pmod{20}, \\ 2x \equiv 3 \pmod{15}. \end{cases}$

9.3. $\begin{cases} 3x \equiv 1 \pmod{5}, \\ 5x \equiv 4 \pmod{7}. \end{cases}$ 9.4. $\begin{cases} 14x \equiv 12 \pmod{18}, \\ x \equiv 5 \pmod{25}. \end{cases}$

9.5. $\begin{cases} x \equiv b_1 \pmod{13}, \\ x \equiv b_2 \pmod{17}. \end{cases}$ 9.6. $\begin{cases} 3x + 4y \equiv 29 \pmod{143}, \\ 2x - 9y \equiv 59 \pmod{143}. \end{cases}$

9.7. $\begin{cases} x + 2y \equiv 0 \pmod{5}, \\ 3x + 2y \equiv 2 \pmod{5}. \end{cases}$ 9.8. $\begin{cases} 5x - y \equiv 3 \pmod{6}, \\ 2x + 2y \equiv 5 \pmod{6}. \end{cases}$

10. Taqqoslamalar sistemasini yeching:

10.1. $\begin{cases} x \equiv 3 \pmod{8}, \\ x \equiv 11 \pmod{20}, \\ x \equiv 1 \pmod{15}. \end{cases}$

10.2. $\begin{cases} x \equiv 2 \pmod{3}, \\ x \equiv 3 \pmod{4}, \\ x \equiv 4 \pmod{5}. \end{cases}$

10.3. $\begin{cases} x \equiv 1 \pmod{2}, \\ x \equiv 3 \pmod{5}, \\ x \equiv 6 \pmod{9}. \end{cases}$

10.4. $\begin{cases} x \equiv 2 \pmod{7}, \\ x \equiv 5 \pmod{9}, \\ x \equiv 11 \pmod{15}. \end{cases}$

10.5. $\begin{cases} x \equiv 4 \pmod{7}, \\ x \equiv 9 \pmod{13}, \\ x \equiv 1 \pmod{17}. \end{cases}$

10.6. $\begin{cases} x \equiv 5 \pmod{12}, \\ x \equiv 2 \pmod{8}, \\ x \equiv 2 \pmod{11}. \end{cases}$

10.7. $\begin{cases} x \equiv 2 \pmod{15}, \\ x \equiv 7 \pmod{20}, \\ x \equiv 12 \pmod{35}. \end{cases}$

10.8. $\begin{cases} x \equiv 4 \pmod{5}, \\ x \equiv 1 \pmod{12}, \\ x \equiv 7 \pmod{14}. \end{cases}$

10.9. $\begin{cases} x \equiv 5 \pmod{8}, \\ x \equiv 4 \pmod{11}, \\ x \equiv 6 \pmod{17}. \end{cases}$

10.10. $\begin{cases} x \equiv b_1 \pmod{25}, \\ x \equiv b_2 \pmod{27}, \\ x \equiv b_3 \pmod{59}. \end{cases}$

10.11. $\begin{cases} x \equiv 1 \pmod{3}, \\ x \equiv 4 \pmod{5}, \\ x \equiv 2 \pmod{7}, \\ x \equiv 9 \pmod{11}, \\ x \equiv 3 \pmod{13}. \end{cases}$

11. Taqqoslamalar sistemasini yeching:

11.1. $\begin{cases} 3x \equiv 1 \pmod{10}, \\ 4x \equiv 3 \pmod{5}, \\ 2x \equiv 7 \pmod{9}. \end{cases}$

11.2. $\begin{cases} 2x \equiv 3 \pmod{5}, \\ 3x \equiv 5 \pmod{7}, \\ 3x \equiv 3 \pmod{9}. \end{cases}$

11.3. $\begin{cases} 4x \equiv 1 \pmod{9}, \\ 5x \equiv 3 \pmod{7}, \\ 4x \equiv 5 \pmod{12}. \end{cases}$

11.4. $\begin{cases} 7x \equiv 3 \pmod{11}, \\ 3x \equiv 2 \pmod{5}, \\ 15x \equiv 5 \pmod{35}. \end{cases}$

11.5. $\begin{cases} 3x \equiv 7 \pmod{10}, \\ 2x \equiv 5 \pmod{15}, \\ 7x \equiv 5 \pmod{12}. \end{cases}$

11.6. $\begin{cases} 5x \equiv 3 \pmod{9}, \\ 4x \equiv 7 \pmod{13}, \\ 8x \equiv 4 \pmod{14}, \\ x \equiv 2 \pmod{17}. \end{cases}$

11.7. $\begin{cases} 2x \equiv 7 \pmod{13}, \\ 5x \equiv 8 \pmod{17}, \\ 14x \equiv 35 \pmod{19}, \\ 3x \equiv 7 \pmod{31}. \end{cases}$

12. *a* ning qanday qiymatlarida taqqoslamalar sistemasi yechimga ega?

12.1. $\begin{cases} x \equiv a \pmod{6}, \\ x \equiv 1 \pmod{10}, \\ x \equiv 2 \pmod{21}, \\ x \equiv 3 \pmod{11}. \end{cases}$

12.2. $\begin{cases} 2x \equiv a \pmod{4}, \\ 3x \equiv 4 \pmod{10}. \end{cases}$

12.3. $\begin{cases} x \equiv 5 \pmod{18}, \\ x \equiv 8 \pmod{21}, \\ x \equiv a \pmod{35}. \end{cases}$

12.4. $\begin{cases} x \equiv a \pmod{6}, \\ x \equiv 1 \pmod{10}, \\ x \equiv 2 \pmod{21}, \\ x \equiv 3 \pmod{11}. \end{cases}$

13. Darajasi berilgan taqqoslama darajasiga, bosh koeffitsiyenti 1 ga teng bo‘lgan teng kuchli taqqoslamani toping:

13.1. $3x^3 - 5x^2 - 2 \equiv 0 \pmod{11}$.

13.2. $27x^3 + 14x^2 - 10x + 13 \equiv 0 \pmod{59}$.

13.3. $70x^6 + 78x^5 + 25x^4 + 68x^3 + 52x^2 + 4x + 3 \equiv 0 \pmod{101}$.

13.4. $a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n \equiv 0 \pmod{m}$, $(a_0, m) \equiv 1$.

14. Darajasi moduldan kichik, berilgan taqqoslamaga teng kuchli taqqoslamani toping:

14.1. $x^8 + 2x^7 + x^5 - x^4 - x + 3 \equiv 0 \pmod{5}$.

14.2. $3x^{14} + 4x^{13} + 3x^{12} + 2x^{11} + x^9 + 2x^8 + 4x^7 +$
$+ x^6 + 3x^4 + x^3 + 4x^2 + 2x \equiv 0 \pmod{5}$.

14.3. $x^{16} + 3x^8 - 5x^7 - x^4 + 6x - 2 \equiv 0 \pmod{7}$.

14.4. $2x^{17} + 6x^{16} + x^{14} + 5x^{12} + 3x^{11} + 2x^{10} + x^9 + 5x^8 + 2x^7 +$
$+ 3x^5 + 4x^4 + 6x^3 + 4x^2 + x + 4 \equiv 0 \pmod{7}$.

14.5. $6x^{18} + 18x^{15} + 3x^4 - 8x^3 + x^2 + 3 \equiv 0 \pmod{11}$.

15. Berilgan taqqoslamani soddalashtiring (darajasini pasaytiring, koeffitsiyentlarni moduldan kichik sonlar bilan almashtiring, bosh koeffitsiyenti 1ga teng bo‘lsin) va tanlash usulida yeching:

15.1. $x^5 + x^3 + x^2 + 4 \equiv 0 \pmod{3}$.

15.2. $6x^4 + 17x^2 - 16 \equiv 0 \pmod{3}$.

15.3. $28x^9 + 29x^8 - 26x^7 + 20x^4 - 17x + 23 \equiv 0 \pmod{3}$.

15.4. $x^5 + 2x^4 - 2x^3 - 2x^2 + 2x - 1 \equiv 0 \pmod{3}$.

15.5. $x^5 + x^4 - x^2 - 5x + 1 \equiv 0 \pmod{3}$.

15.6. $x^7 + 2x^6 + x^5 + 4x^3 - 2x^2 - 4x + 2 \equiv 0 \pmod{5}$.

15.7. $x^7 + 3x^6 + x^5 - x^3 - 3x^2 - 4x + 4 \equiv 0 \pmod{5}$.

15.8. $x^7 + 5x^5 - x^3 - 9x + 3 \equiv 0 \pmod{5}$.

15.9. $34x^{10} - 29x^7 + 43x^4 - 19x + 37 \equiv 0 \pmod{5}$.

15.10. $6x^{10} - 12x + 1 \equiv 0 \pmod{5}$.

15.11. $x^7 - 3x^6 + x^5 - 15x^4 - x^3 + 4x^2 - 4x + 2 \equiv 0 \pmod{5}$.

16. Berilgan taqqoslamalarni soddalashtiring va tanlash usulida yeching:

16.1. $5x^{24} + 4x^{23} + 4x^{22} + 2x^{21} + x^{20} + 6x^{19} + 4x^{18} + 3x^{17} +$
$+ 4x^{16} + 6x^{15} + 5x^{14} + 2x^{13} + x^{12} + 2x^{11} + x^{10} + 3x^9 +$
$+ 4x^8 + 2x^7 + 5x^6 + 6x^5 + 5x^4 + 3x^3 + 4x^2 +$
$+ 4x + 2 \equiv 0 \pmod{7}$.

3-usul. Berilgan $\frac{219}{383}$ kasrning maxraji suratidan katta bo'lgani uchun 9-xossani qo'llash mumkin:

$$\left(\frac{219}{383}\right)=\left(\frac{383}{219}\right)\cdot(-1)^{\frac{383-1}{2}\cdot\frac{219-1}{2}}=-\left(\frac{383}{219}\right)=\left(\frac{164}{219}\right)=\left(\frac{41\cdot 2^2}{219}\right)=$$

$$=-\left(\frac{41}{219}\right)=-\left(\frac{219}{41}\right)\cdot(-1)^{\frac{219-1}{2}\cdot\frac{41-1}{2}}=-\left(\frac{219}{41}\right)=\left(\frac{14}{41}\right)=-\left(\frac{2}{41}\right)\left(\frac{7}{41}\right)=$$

$$=(-1)^{\frac{41^2-1}{8}}\left(\frac{7}{41}\right)=-\left(\frac{7}{41}\right)=-\left(\frac{41}{7}\right)\cdot(-1)^{\frac{41-1}{2}\cdot\frac{7-1}{2}}=-\left(\frac{41}{7}\right)=-\left(\frac{-1}{7}\right)=$$

$$=(-1)^{\frac{7-1}{2}}=1.$$

Demak, $\left(\frac{219}{383}\right)=1$.

2-misol. $\frac{383}{219}$ ning Yakobi simvolini aniqlang.

Yechish. Yakobi simvolining Lejandr simvolidan farqi Yakobi simvoli o'zaro tub bo'lgan a va m (m>1) sonlardan tuzilgan $\frac{a}{m}$ uchun aniqlanadi. $\left(\frac{a}{m}\right)$ belgilash «a ning m modul bo'yicha Yakobi simvoli» deb o'qiladi. Yuqoridagi misoldagi Lejandr simvolining xossalari va $\left(\frac{a}{m}\right)=\left(\frac{a}{p_1\ldots p_n}\right)=\left(\frac{a}{p_1}\right)\ldots\left(\frac{a}{p_n}\right)$ xossadan:

$$\left(\frac{383}{219}\right)=\left(\frac{383}{3\cdot 73}\right)=\left(\frac{383}{3}\right)\cdot\left(\frac{383}{73}\right)=\left(\frac{2}{3}\right)\cdot\left(\frac{18}{73}\right)=\left(\frac{2}{3}\right)\cdot\left(\frac{23^2}{73}\right)=$$

$$=\left(\frac{2}{3}\right)\cdot\left(\frac{2}{73}\right)=(-1)^{\frac{3^2-1}{8}}(-1)^{\frac{73^2-1}{8}}=(-1)\cdot 1=-1.$$

Demak, $\left(\frac{383}{219}\right)=-1$, ya'ni $x^2\equiv 383 \pmod{219}$ taqqoslama uchun 383 kvadrat chegirma emas.

3-misol. $p=17$ modul bo'yicha $g=6$ boshlang'ich ildizning indekslar jadvalini tuzing.

Yechish. p tub modul bo‘yicha boshlang‘ich ildiz bu shunday g chegirmalar sinfiki, uning uchun $g^{p-1} \equiv (\text{mod } p)$ bo‘lib, $p-1$ dan kichik natural darajalarda modulda 1 bilan taqqoslanmaydi.

$g = 6$ ning mod 17 da boshlang‘ich ildiz bo‘lishini tekshiramiz. Buning uchun $p-1$ ning n bo‘luvchilarida $6^n \equiv 1 \pmod p$ shartni tekshiramiz:

$p = 17$, $p - 1 = 16$, 16 ning natural bo‘luvchilari n=1, 2, 4, 8, 16. Bundan:

$6^1 \equiv 6 \pmod{17}$;
$6^2 \equiv 2 \pmod{17}$;
$6^4 \equiv 4 \pmod{17}$;
$6^8 \equiv 16 \pmod{17}$;
$6^{16} \equiv 1 \pmod{17}$.

Demak, 17 modulda 6 boshlang‘ich ildiz bo‘ladi. 6^0, 6^1, 6^2, ..., 6^{15} lardan 17 modul bo‘yicha taqqoslamalar tuzamiz:

$6^0 \equiv 1 \pmod{17}$; $6^5 \equiv 7 \pmod{17}$; $6^{10} \equiv 15 \pmod{17}$;
$6^1 \equiv 6 \pmod{17}$; $6^6 \equiv 8 \pmod{17}$; $6^{11} \equiv 5 \pmod{17}$;
$6^2 \equiv 2 \pmod{17}$; $6^7 \equiv 14 \pmod{17}$; $6^{12} \equiv 13 \pmod{17}$;
$6^3 \equiv 12 \pmod{17}$; $6^8 \equiv 16 \pmod{17}$; $6^{13} \equiv 10 \pmod{17}$;
$6^4 \equiv 4 \pmod{17}$; $6^9 \equiv 11 \pmod{17}$; $6^{14} \equiv 9 \pmod{17}$;
$6^{15} \equiv 3 \pmod{17}$.

Tuzilgan taqqoslamalar yordamida quyidagi jadvallarni tuzamiz:

1-jadval

N	0	1	2	3	4	5	6	7	8	9
0		0	2	15	4	11	1	5	6	14
1	13	9	3	12	7	10	8			

1-jadval uchun taqqoslamalarning ikkinchi tomonidagi songa mos daraja topiladi.

2-jadval uchun taqqoslamalarning birinchi tomonidagi darajaga mos qoldiq topiladi.

2-jadval

I	0	1	2	3	4	5	6	7	8	9
0	1	6	2	12	4	7	8	14	16	11
1	15	5	13	10	9	3				

4-misol. $15x^{19} \equiv 28 \pmod{17}$ taqqoslamani yeching.

Yechish. $15x^{19} \equiv 28 \pmod{17}$ taqqoslamani taqqoslama xossalari yordamida soddalashtiramiz: $15x^3 \equiv 11 \pmod{17}$. Hosil bo'lgan taqqoslamaning indekslar xossalariga ko'ra, $\mathrm{ind}15 + 3\mathrm{ind}x \equiv \mathrm{ind}11 \pmod{16}$ taqqoslamani hosil qilamiz.

Yuqorida tuzilgan jadvaldan $\mathrm{ind}15 = 10$, $\mathrm{ind}11 = 9$ larni topamiz:

$10 + 3\mathrm{ind}x \equiv 9 \pmod{16}$,

$3\mathrm{ind}x = -1 \pmod{16}$.

(3,16)=1 ekanligidan taqqoslama yagona yechimga ega. Taqqoslama xossalaridan $\begin{matrix} 3\mathrm{ind}x \equiv 15 \pmod{16}; \\ \mathrm{ind}x \equiv 5 \pmod{16} \end{matrix}$ larni va 2-jadval yordamida $x \equiv 7 \pmod{17}$ yechimni hosil qilamiz.

Tekshirish:

$$15 \cdot 7^{19} - 28 = -2(7^2)^9 7 - 11 \equiv -2(49)^9 \cdot 7 - 11 \equiv -2(-2)^9 \cdot 7 - 11 \equiv$$
$$-2(-2)^5 \cdot (-2)^4 7 - 11 \equiv -2(-32) \cdot 16 \cdot 7 - 11 \equiv -2 \cdot 2 \cdot (-1) \cdot 7 - 11 \equiv$$
$$\equiv 28 - 11 \equiv 17 \equiv 0 \pmod{17}.$$

Demak, $15 \cdot 7^{19} - 28 \equiv 0$.

Misol va mashqlar

1. Lejandr simvolini toping:

1.1. $\left(\frac{13}{7}\right)$. 1.2. $\left(\frac{22}{13}\right)$. 1.3. $\left(\frac{19}{67}\right)$.

1.4. $\left(\frac{37}{67}\right)$. 1.5. $\left(\frac{56}{73}\right)$. 1.6. $\left(\frac{47}{73}\right)$.

1.7. $\left(\frac{54}{83}\right)$. 1.8. $\left(\frac{68}{113}\right)$. 1.9. $\left(\frac{63}{131}\right)$.

2. Yakobi simvolini toping:

2.1. $\left(\frac{283}{563}\right)$. 2.2. $\left(\frac{251}{577}\right)$. 2.3. $\left(\frac{241}{593}\right)$.

2.4. $\left(\frac{323}{607}\right)$. 2.5. $\left(\frac{346}{643}\right)$. 2.6. $\left(\frac{3153}{1201}\right)$.

2.7. $\left(\frac{20470}{1847}\right)$. 2.8. $\left(\frac{2108}{2003}\right)$. 2.9. $\left(\frac{3149}{5987}\right)$.

3. Quyidagi taqqoslamalarning yechimlar sonini aniqlang:

3.1. $x^2 \equiv 3 \pmod{31}$. 3.2. $x^2 \equiv 2 \pmod{31}$.

3.3. $x^2 \equiv 5 \pmod{73}$. 3.4. $x^2 \equiv 3 \pmod{101}$.

3.5. $x^2 \equiv 226 \pmod{563}$. 3.6. $x^2 \equiv 429 \pmod{563}$.

3.7. $x^2 \equiv 579 \pmod{821}$. 3.8. $x^2 \equiv 728 \pmod{919}$.

3.9. $x^2 \equiv 847 \pmod{1087}$. 3.10. $x^2 \equiv 3766 \pmod{5987}$.

4. Quyidagi tasdiqlarni isbotlang:

4.1. Biror m modul bo'yicha tuzilgan bitta sinfning chegirmalari shu modul bo'yicha bir xil ko'rsatkichga tegishli bo'ladi.

4.2. Agar $(a;\ m)=1$ bo'lganda $a\delta\equiv 1(\text{mod} m)$ bo'lsa, u holda $a^0, a^1, ..., a^{\delta-1}$ sonlar sistemasi m modul bo'yicha o'zaro taqqoslanmaydi.

4.3. Agar $\delta=\varphi(m)$ bo'lsa, u holda $a^0, a^1, ..., a^{\delta-1}$ sistema m modul bo'yicha chegirmalarning keltirilgan sistemasini tashkil qiladi.

4.4. a son m modul bo'yicha δ ko'rsatkichga tegishli bo'lsa, u holda $\alpha^{\gamma}\equiv\alpha^{\gamma_1}(\text{mod} m)$ taqqoslama o'rinli bo'lishi uchun $\gamma\equiv\gamma_1(\text{mod}\delta)$ taqqoslamaning o'rinli bo'lishi zarur va yetarli.

4.5. $\gamma = 0(\text{mod}\ \delta)$ bo'lganda va faqat shu holdagina $a\gamma = 1(\text{mod}\ m)$ taqqoslama o'rinli bo'ladi.

4.6. a sonning m modul bo‘yicha δ ko‘rsatkichi $\varphi(m)$ning bo‘luvchisi bo‘ladi.

4.7. Agar a son m modul bo‘yicha δ ko‘rsatkichga tegishli bo‘lsa, u holda a^k soni shu modul bo‘yicha $\frac{\delta}{(\delta; k)}$ ko‘rsatkichga tegishli bo‘ladi.

4.8. Agar $(\delta;k)=1$ bo‘lsa, u holda a son δ ko‘rsatkichga tegishli bo‘ladi.

4.9. r tub modul bo‘yicha tuzilgan $r - 1$ sonning har bir δ bo‘luvchisi $\varphi(\delta)$ ta sinfning ko‘rsatkichi bo‘ladi. Xususiy holda $\varphi(r - 1)$ ta boshlang‘ich ildizlar sinfi mavjud.

5. a sonining m modul bo‘yicha tartibini aniqlang:

5.1. $a = 2, \ m = 5$.

5.2. $a = 4, \ m = 5$.

5.3. $a = 5, \ m = 8$.

5.4. $a = 10, m = 13$.

5.5. $a = 4, \ m = 15$.

5.6. $a = 2, \ m = 15$.

5.7. $a = 2, \ m = 17$.

5.8. $a = 7, \ m = 20$.

5.9. $a = 7, \ m = 22$.

5.10. $a = 7, \ m = 43$.

5.11. $a = 5, \ m = 108$.

5.12. $a = 2, \ m = 133$.

6. a, b, c, d sonlarning m modul bo‘yicha tartibini aniqlang:

6.1. $a = 7, b = 9, c =12; m =13$.

6.2. $a = 5, b = 8, c = 13; m =17$.

6.3. $a = 5, b = 8, c =10; d =16; m =13$.

6.4. $a =10, b = 25, c = 50; m = 39$.

6.5. $a = 5, b =15, c = 21; d = 35; m = 44$.

7. Berilgan modul bo‘yicha barcha boshlang‘ich ildizlarni toping:

7.1. 11.	7.2. 13.
7.3. 15.	7.4. 19.
7.5. 49.	7.6. 81.

8. Berilgan modul bo‘yicha boshlang‘ich ildizlar sonini va ularning eng kichigini toping:

8.1. 10. 8.2. 18. 8.3. 19.

8.4. 31. 8.5. 37.

9. Berilgan modul bo‘yicha boshlang‘ich ildizlarning eng kichigini toping:

9.1. 7. 9.2. 17. 9.3. 23.

9.4. 41. 9.5. 53. 9.6. 50.

9.7. 54. 9.8. 71. 9.9. 242.

9.10. 289. 9.11. 578. 9.12. 625.

10. r modul bo‘yicha g asosga ko‘ra indekslar jadvalini tuzing:

10.1. $p = 3$, $g = 2$.

10.2. $p = 5$, $g = 2$.

10.3. $p = 5$, $g = 3$.

10.4. $p = 7$, $g = 3$.

10.5. $p = 7$, $g = 5$.

10.6. $p = 11$, $g = 2$.

10.7. $p = 13$, $g = 2$.

10.8. $p = 29$, $g = 2$.

11. Indekslarning quyidagi xossalarini isbotlang:

11.1. $a \equiv b \pmod{r} \Leftrightarrow \text{ind}a = \text{ind}b$.

11.2. Agar $(a;r)=1$, $(b;r)=1$ bo‘lsa, u holda $\text{ind}(ab)=\text{ind}a+$ $+\text{ind}b(\text{mod}\, p-1)$ bo‘ladi.

11.3. Agar $(a;r)=1$ va $\forall n \in N$ bo‘lsa, u holda $\text{ind}(a^n)\equiv$ $\equiv n \cdot \text{ind}a \pmod{p-1}$ taqqoslama o‘rinli bo‘ladi.

11.4. $\text{ind}\left(\frac{a}{b}\right) \equiv \text{ind}a - \text{ind}b \pmod{p-1}$ taqqoslama o‘rinli.

11.5. $\text{ind}\, 1 = 0$, $\text{ind}_g g = 1$.

12. Quyidagi taqqoslamalarni yeching:

12.1. $7x \equiv 23 \pmod{17}$.

12.2. $5x \equiv 13 \pmod{27}$.

12.3. $8x \equiv -11 \pmod{37}$.

12.4. $47x \equiv 23 \pmod{73}$.

12.5. $53x \equiv 37 \pmod{79}$.

12.6. $125x \equiv 7 \pmod{79}$.
12.7. $65x \equiv 38 \pmod{83}$.
12.8. $23x \equiv 9 \pmod{97}$.
12.9. $37x \equiv 5 \pmod{221}$.

13. Quyidagi ikkinchi darajali taqqoslamalarni yeching:

13.1. $x^2 \equiv 15 \pmod{17}$.
13.2. $x^2 \equiv 10 \pmod{27}$.
13.3. $x^2 \equiv 47 \pmod{53}$.
13.4. $x^2 \equiv 58 \pmod{61}$.
13.5. $x^2 \equiv 59 \pmod{67}$.
13.6. $x^2 \equiv -28 \pmod{67}$.
13.7. $x^2 \equiv 54 \pmod{71}$.
13.8. $x^2 \equiv 40 \pmod{83}$.
13.9. $3x^2 - 5x - 2 \equiv 0 \pmod{11}$.
13.10. $2x^2 - 7x + 28 \equiv 0 \pmod{43}$.
13.11. $3x^2 - 8x + 44 \equiv 0 \pmod{47}$.
13.12. $x^2 \equiv 29 \pmod{59^2}$.
13.13. $x^2 \equiv 61 \pmod{73^2}$.

14. Quyidagi taqqoslamalarning yechimlar sonini aniqlang:

14.1. $x^{15} \equiv 6 \pmod{37}$.
14.2. $x^{16} \equiv 10 \pmod{37}$.
14.3. $3x^3 \equiv 2 \pmod{37}$.
14.4. $7x^7 \equiv 11 \pmod{41}$.
14.5. $3x^{12} \equiv 31 \pmod{41}$.
14.6. $5x^{30} \equiv 37 \pmod{41}$.
14.7. $x^5 \equiv 3 \pmod{71}$.
14.8. $x^{21} \equiv 5 \pmod{71}$.
14.9. $x^{15} \equiv 46 \pmod{97}$.
14.10. $x^{55} \equiv 17 \pmod{97}$.
14.11. $x^{60} \equiv 79 \pmod{97}$.

15. Quyidagi ikkihadli taqqoslamalarni yeching:

15.1. $x^{10} \equiv 33 \pmod{37}$.
15.2. $x^3 \equiv 34 \pmod{41}$.
15.3. $x^8 \equiv 31 \pmod{41}$.

15.4. $x^{12} \equiv 37 \pmod{41}$.

15.5. $x^{5} \equiv 37 \pmod{43}$.

15.6. $x^{27} \equiv 39 \pmod{43}$.

15.7. $x^{35} \equiv 17 \pmod{67}$.

15.8. $x^{30} \equiv 14 \pmod{67}$.

15.9. $x^{12} \equiv 27 \pmod{83}$.

15.10. $x^{48} \equiv 2 \pmod{97}$.

16. Quyidagi ikkihadli taqqoslamalarni yeching:

16.1. $3x^{3} \equiv 4 \pmod{7}$.

16.2. $2x^{8} \equiv 5 \pmod{13}$.

16.3. $15x^{4} \equiv 17 \pmod{23}$.

16.4. $27x^{5} \equiv 25 \pmod{31}$.

16.5. $13x^{3} \equiv 24 \pmod{37}$.

16.6. $37x^{8} \equiv 59 \pmod{61}$.

16.7. $23x^{6} \equiv 15 \pmod{73}$.

16.8. $37x^{6} \equiv 69 \pmod{73}$.

16.9. $37x^{15} \equiv 62 \pmod{73}$.

16.10. $44x^{21} \equiv 53 \pmod{73}$.

16.11. $27x^{30} \equiv 41 \pmod{79}$.

Takrorlash uchun savollar

1. Sonning modulga ko‘ra ko‘rsatkichi deb nimaga aytiladi?
2. Sonning modul bo‘yicha boshlang‘ich ildizi deb nimaga aytiladi?
3. Sonning modul bo‘yicha indeksini tushuntiring.
4. Indeksning qanday xossalarini bilasiz?
5. n-darajali ikki hadli taqqoslama deb nimaga aytiladi?
6. n-darajali ikki hadli taqqoslama yechimlari soni nechta bo‘ladi?
7. Tub modulli ikki hadli kvadratik taqqoslamaning yechimlari nechta?

XIII MODUL. KO‘PHADLAR

30-§. Bir o‘zgaruvchili ko‘phadlar

✓ **Asosiy tushunchalar:** n-darajali ko‘phad, ko‘phadning ildizi, Bezu teoremasi, algebraik teng ko‘phadlar, funksional teng ko‘phadlar.

Agar $a_n \neq 0$ bo‘lsa, u holda ushbu

$$a_nx^n+a_{n-1}x^{n-1}+...+a_1x+a_0=\sum_{i=0}^{n} a_ix^i \quad (a_i \in K,\ i=\overline{0,n},\ \forall n \in N)$$

ifoda *K maydon ustidagi n-darajali ko‘phad* deyiladi.

Agar K butunlik sohasining biror c elementi uchun $f(c)=0$ tenglik o‘rinli bo‘lsa, u holda c element $f(x)$ ko‘phadning yoki $f(x)=0$ *tenglamaning ildizi* deyiladi.

Bezu teoremasi. *$f(x)$ ko‘phadni $x - c$ ikkihadga bo‘lishdan hosil bo‘lgan qoldiq $f(c)$ ga teng.*

$x=c$ element $f(x)$ ko‘phadning ildizi bo‘lishi uchun $f(x)$ ning $x-c$ ikkihadga bo‘linishi zarur va yetarli.

Agar c_1, c_2, ..., c_k lar $f(x)$ ko‘phadning turli ildizlari bo‘lsa, u holda $f(x)$ ko‘phad $(x - c_1)(x - c_2)...(x - c_k)$ ko‘paytmaga bo‘linadi.

Noldan farqli n-darajali ko‘phad $(n \geq 1)$ K butunlik sohasida n tadan ortiq ildizga ega emas.

Agar $f(x) \in K[x]$ va $0 \neq \varphi(x) \in K[x]$ ko‘phadlar berilgan bo‘lib, shunday $g(x) \in K[x]$ ko‘phad topilsaki, natijada $f(x)=\varphi(x)g(x)$ tenglik o‘rinli bo‘lsa, u holda $f(x)$ ko‘phad $\varphi(x)$ ko‘phadga bo‘linadi deyiladi va u $f(x) \vdots \in \varphi(x)$ yoki $f(x)/\varphi(x)$ ko‘rinishlarda belgilanadi.

O‘zgaruvchining bir xil darajalari oldidagi koeffitsiyentlari teng bo‘lgan ko‘phadlar *o‘zaro algebraik ma’nodagi teng ko‘phadlar* deyiladi.

Agar o‘zgaruvchining biror cheksiz sohadan olingan har qanday qiymatlariga mos keluvchi ko‘phadlarning qiymatlari ustma-ust tushsa, u holda bunday ko‘phadlar *o‘zaro funksional ma’nodagi teng ko‘phadlar* deyiladi.

Berilgan $f(x) = a_n x^n + \ldots + a_1 x + a_0$ ko‘phadni

$$g(x) = b_m x^m + \ldots + b_1 x + b_0$$

ko‘phadga bo‘lishni quyidagi jadval asosida bajarish mumkin:

	a_n	a_{n-1}	a_{n-2}	...	a_m	a_{m-1}	...	a_0
b_m	a_n	$b_{m-1}\frac{a_n}{b_m}$	$b_{m-2}\frac{a_n}{b_m}$	...				
b_{m-1}		$a_{n-1}-\sigma_1$	$b_{m-1}\frac{a_{n-1}-\sigma_1}{b_m}$	...				
. . .								
b_1								
b_0								
					$a_m-\sigma_{n-m}$	$b_{m-1}\frac{a_m-\sigma_{n-m}}{b_m}$	...	$b_0\frac{a_m-\sigma_{m-n}}{b_m}$
	$\underbrace{\frac{a_n}{b_m}}_{c_{n-m}}$	$\underbrace{\frac{a_{n-1}-\sigma_1}{b_m}}_{c_{n-m-1}}$	$\underbrace{\frac{a_{n-2}-\sigma_2}{b_m}}_{c_{n-m-2}}$	...	$\underbrace{\frac{a_m-\sigma_{n-m}}{b_m}}_{c_0}$	$\underbrace{a_{m-1}-\delta_{m-1}}_{d_{m-1}}$	...	$\underbrace{a_0-\delta_0}_{d_0}$

1-misol. $g(x) \in Z[x]$, $f(x) = x^4 + ax^3 + bx^2 - 8x + 4$ uchun $f(x) = (g(x))^2$ shartni qanoatlantiruvchi barcha a va b butun sonlarni toping.

Yechish. $f(x)$ ko‘phadning darajasi 4 ga teng. Demak, $g(x)$ ning darajasi 2 ga teng. $g(x) = mx^2 + nx + P$, $m \neq 0$ bo‘lsin. Bundan

$$(g(x))^2 = (mx^2 + nx + p)^2 =$$

$$= m^2x^4 + 2mnx^3 + (2mp + n^2)x^2 + 2npx + p^2.$$

$f(x) = (g(x))^2$ dan quyidagi sistemani hosil qilamiz:

$$\begin{cases} m^2 = 1, \\ 2mn = a, \\ 2mp + n^2 = b, \\ 2np = -8, \\ p^2 = 4. \end{cases}$$

Sistemadan $m = \pm 1$ va $p = \pm 2$ larni hosil qilsak, u quyidagi 4 ta sistemaga ajraladi:

$$1)\ \begin{cases} m = 1, \\ p = 2, \\ n = -2, \\ a = -4, \\ b = 8; \end{cases} \quad 2)\ \begin{cases} m = 1, \\ p = -2, \\ n = 2, \\ a = 4, \\ b = 0; \end{cases} \quad 3)\ \begin{cases} m = -1, \\ p = 2, \\ n = -2, \\ a = 4, \\ b = 0; \end{cases} \quad 4)\ \begin{cases} m = -1, \\ p = -2, \\ n = 2, \\ a = -4, \\ b = 8. \end{cases}$$

Demak, agar $a = -4$ va $b = 8$ bo‘lsa, $g_1(x) = -x^2 + 2x - 2$ va $g_2(x) = x^2 - 2x + 2$;

agar $a = 4$ va $b = 0$ bo‘lsa, $g_1(x) = x^2 + 2x - 2$ va $g_2(x) = -x^2 - 2x + 2$ bo‘ladi.

2-misol. Ozod hadi 7 ga bo‘linadigan barcha $f(x) \in Z[x]$ lar to‘plami K halqa tashkil etishini tekshiring.

Yechish. $f(x) = a_n x^n + ... + a_1 x + 7a_0$, $g(x) = b_m x^n + ... + b_1 x + 7b_0$ va $m \geq n$ bo‘lsin. U holda

$$f(x) + g(x) = b_m x^m + ... + (a_n + b_n)x^n + ... + (a_1 + b_1)x + (7a_0 + 7b_0) =$$
$$= b_m x^m + ... + (a_n + b_n)x^n + ... + (a_1 + b_1)\,x + 7(a_0 + b_0);$$

$$f(x) - g(x) = (-b_m)x^m + ... + (a_n - b_n)x^n + ... + (a_1 - b_1)x + 7(a_0 - b_0);$$
$$f(x) \cdot g(x) = a_n b_m\, x^{n+m} + \ ...\ + 7(a_1 b_0 + a_0 b_1)x + 7 \cdot 7a_0 b_0.$$

Bundan $f(x) + g(x)$, $f(x) - g(x)$ va $f(x) \cdot g(x)$ lar K to‘plamning elementlari ekanligi kelib chiqadi.

Demak, K $Z[x]$ ning qism halqasi.

3-misol. $f(x) \in Z[x]$, $\deg f(x) \leq 4$ uchun barcha $\overline{a} \in Z_5$ da $f(\overline{a}) = \overline{0}$ bo‘lsa, $f(x)$ – nol ko‘phad ekanligini isbotlang.

Yechish. $f(x) = \overline{a}x^4 + \overline{b}x^3 + \overline{c}x^2 + \overline{d}x + \overline{e}$ bo‘lsin. U holda

	2	0	4	−1	−6	−1	3	−2
1			2	0	2	−1	−8	0
	2	0	2	−1	−8	0	11	−2
1			2	0	0	−1		
	2	0	0	−1	−8	1		
1			2	0				
	2	0	−2	−1				
1								
	2	0						

Demak, $f(x) = 2x(x^2 + 1)^3 + (-2x - 1)(x^2 + 1)^2 +$
$+ (-8x + 1)(x^2 + 1) + 11x - 2.$

9-misol. $C[x]$ da berilgan $f(x) = (x + a + b)^{2005} - x^{2005} - a^{2005} - b^{2005}$ ning $g_1(x)=x+a$ va $g_2(x)=x+b$ ikkihadlarga bo'linishini isbotlang.

Isbot. $f(x)$ ko'phadning $x = -a$ va $x = -b$ dagi qiymatlarini topamiz:

$f(-a) = b^{2005} + a^{2005} - a^{2005} - b^{2005} = 0,$

$f(-b) = a^{2005} + b^{2005} - a^{2005} - b^{2005} = 0.$

Bezu teoremasiga ko'ra $f(x)$ ko'phad $g_1(x)$ va $g_2(x)$ larga bo'linadi.

10-misol. $Q[x]$ halqada $f(x) = 3x^2 - 6x^2 + 5x - 10$ va $g(x) = 2x^3 - 4x^2 + 3x - 6$ ko'phadning EKUB, EKUKlarini toping.

Yechish. $2f(x) - 3g(x) = x - 2$ va $f(2) = g(2) = 0$ dan $(f, g) = x - 2$ kelib chiqadi.

$[f,g] = \frac{f(x)g(x)}{(f,g)}$ ni e'tiborga olsak, $[f, g]=(3x^3 - 6x^2 + 5x + 10)\times$
$\times(2x^2 + 3)$ hosil bo'ladi.

11-misol. $Z_7[x]$ halqada $f(x) = x^4 + \bar{4}x^3 + \bar{4}x^2 + \bar{6}x + \bar{6}$ va $g(x) = x^3 - \bar{1}$ ko'phadlarning EKUBini chiziqli ifodasini toping.

Yechish. $f(x)$ va $g(x)$ larning EKUBini topamiz:

$f(x) = g(x)h_1(x) + r_1(x)$ dagi $h_1(x) = x + \overline{4}$, $r_1(x) = \overline{4}x^2 + \overline{3}$;

$g(x) = r_1(x)h_2(x) + r_2(x)$ dagi $h_2(x) = \overline{2}x$; $r_2(x) = -\overline{6}x - \overline{1} = x + \overline{6}$;

$r_1(x) = r_2(x)h_3(x)$ da $h_3(x) = \overline{4}x + \overline{3}$.

Demak, $(f, g) = x + \overline{6} = r_2(x)$ ekan.

Evklid algoritmi yordamida

$r_2(x) = g(x) - r_1(x)h_2(x) = g(x) - (f(x) - g(x)h_1(x))h_2(x) =$
$= f(x)(-h(x)) + g(x)(\overline{1} + h_1(x)h_2(x))$

ifodani hosil qilamiz. Unda $u(x) = -h_2(x) = -\overline{2}x = \overline{5}x$ va $v(x) = \overline{1} + h_1(x)h_2(x) = \overline{1} + (x + \overline{4})\overline{2}x = \overline{2}x^2 + x + \overline{1}$ belgilashlarni kiritsak, $(f, g) = f(x)u(x) + g(x)v(x)$ hosil bo'ladi.

12-misol. $Q[x]$ halqaning $u(x) = (x^2 - 1) + v(x)(x^2 + 2x + 1) = x^3 + 1$ tenglamani qanoatlantiruvchi $u(x)$ va $v(x)$ ko'phadlarini toping.

Yechish. Berilgan tenglamada shakl almashtirish bajaramiz:

$u(x)(x - 1)(x + 1) + v(x)(x + 1)^2 = (x + 1)(x^2 - x + 1)$ va

$$u(x)(x - 1) + v(x)(x + 1) = x^2 - x + 1$$

ga ega bo'lamiz.

$g_1(x) = x - 1$ va $g_2(x) = x + 1$ ko'phadlar uchun $(g_1, g_2) = 1$ bo'lganligi uchun EKUBni $-\frac{1}{2}g_1(x) + \frac{1}{2}g_2(x) = 1$ ko'rinishda ifodalash mumkin.

Hosil bo'lgan ifodaning ikkala tomonini $(x^2 - x + 1)$ ga ko'paytirsak, $-\frac{1}{2}(x^2 - x + 1)g_2(x) + \frac{1}{2}(x^2 - x + 1)g_2(x) = x^2 - x + 1$ ga ega bo'lamiz. Bundan berilgan tenglamani qanoatlantiruvchi $u(x)$ va $v(x)$ larning xususiy qiymatlari

$$u_0(x) = \frac{1}{2}(x^2 - x + 1),$$

$$v_0(x) = \frac{1}{2}(x^2 - x + 1)$$

kelib chiqadi.

Demak, berilgan tenglamani qanoatlantiruvchi $u(x)$ va $v(x)$ lar quyidagi ko'rinishda bo'ladi:

12. $R[x]$ da $f(x)$ ni $g(x)$ ga bo'lganda 3, $(f(x))^2$ ni $(g(x))^2$ ga bo'lganda 9 qoldiq qoldi. $f(x)$ ni $(g(x))^2$ ga bo'lgandagi qoldiqni toping.

13. Berilgan halqada $f(x)$ ni $g(x)$ ga bo'ling:

13.1. $f(x) = 10x^4-23x^3+26x^2-9x-2$; $g(x) = 2x-3$; $Z[x]$.

13.2. $f(x) = (2 + 2i)x^4-6x^3+(2-4i)x^2+(1 + 11i)x + 2-5i$;
$g(x) = (1-i)x + 3i$; $C[x]$.

13.3. $f(x) = 2x^5 + 12{,}5x^3 - 4x^2 + 5{,}5x - 2{,}5$;
$g(x) = 4x^2 +1$; $Q[x]$.

13.4. $f(x) = -\bar{5}x^6+\bar{5}x^5-\bar{2}x^3+\bar{3}x^2-\bar{2}x+\bar{5}$;
$g(x) = \bar{6}x^3 + \bar{4}$; $Z_7[x]$.

14. $f(x)$ ko'phadning x_0 nuqtadagi qiymatini toping:

14.1. $f(x) = x^4-3x^3+6x^2-10x + 16$; $x_0=4$, $Z[x]$.

14.2. $f(x) = x^5 + (1 + 2i)x^4 - (1 + 3i)x^2 + 7$, $x_0 = -2- i$, $C(x)$.

14.3. $f(x) = x^5+x^4+\bar{3}x^2+\bar{1}$, $x_0=\bar{3}$, $Z_5[x]$.

14.4. $f(x) = x^3-(1 + \sqrt{2})x^2+(1 +\sqrt{2})$, $x_0=1-\sqrt{2}$,
$Q[(\sqrt{2})][x]$.

15. Har qanday $a\in K$ uchun $f(x)\in K[x]$ko'phad $g(x) = x - a$ ikkihadga bo'linishini isbotlang.

15.1. $f(x) = x^7 - x$; $K= Z_7$.

15.2. $f(x) = x^{10} - x^5$; $K= Z_5$.

15.3. $f(x) = x^p - x$; $K= Z_p$.

16. $R[x]$ halqada n – toq natural sonlar uchun $f(x) = (x + a+ + b)^4 - x^4 - a^4 - b^4$ ko'phad $g_1(x)=x+a$ va $g_2(x)=x+b$ ikkihadlarga bo'linishini isbotlang.

17. $f(x)$ ko'phadni $x - a$ darajalariga yoying:

17.1. $f(x) = x^4 - 2x^3 + 3x^2 - 5x + 1$, $a = 1$; $Q[x]$.

17.2. $f(x) = \bar{2}x^4+ x^3 +x^2 +\bar{2}$, $a = \bar{1}$; $Z_3[x]$.

17.3. $f(x) = x^5 - 3ix^3 - 4x^2 + 5ix -1$, $a = -i$; $C[x]$.

18. Berilgan ko'phadlarning EKUBini toping:

18.1. $f(x) = x^3 + 3x^2 - 2$, $g(x) = x^3 + 3x^2 - x - 3$.

18.2. $f(x) = x^4 + x^3 - 3x^2 - 2x - 2$; $g(x) = -x^3 + 3x^2 + 2x + 2$.

18.3. $f(x) = 2x^3 + x^2 + 4x + 2$, $g(x) = 2x^3 + x^2 + 6x + 3$.

19. Evklid algoritmi yordamida berilgan ko'phadlar EKUB ini toping.

19.1. $f(x) = x^3 + x^2 - x - 6$; $g(x) = x^3 + x^2 - 10x - 6$, $Q[x]$.

19.2. $f(x) = x^4 + x^3 + x^2 + x + 1$,
$g(x) = 4x^3 + 3x^2 + 2x + 1$, $Q[x]$.

19.3. $f(x) = x^3 + \bar{3}x^2 + \bar{2}x + \bar{1}$, $g(x) = x^3 + \bar{2}x^2 + x + \bar{2}$, $Z_5[x]$.

19.4. $f(x) = x^4 + 2ix^3 - 2x^2 - 2ix + 1$;
$g(x) = x^3 + (i + 1)x^2 + ix$, $C[x]$.

20. Berilgan ko'phadlar EKUKini toping.

20.1. $f(x) = 2x^3 + 7x^2 + 4x - 3$, $g(x) = x^3 + x^2 - 3x + 1$, $Q[x]$.

20.2. $f(x) = x^3 + \bar{6}x^2 + \bar{4}x + 1$, $g(x) = x^3 + x^2 + \bar{3}x - 4$, $Z_7[x]$.

20.3. $f(x) = x^3 - x^2 + 3x - 3$, $g(x) = x^4 + 2x^3 + 2x - 1$, $R[x]$.

20.4. $f(x) = x^4 + 2ix^3 - 2x^2 - 2ix + 1$,
$g(x) = x^3 + (i + 1)x^2 + ix$, $C[x]$.

21. Berilgan ko'phadlar EKUBining chiziqli ifodasidagi $u(x)$ va $v(x)$ larni toping.

21.1 $f(x) = x^3 + 5x^2 + 6x + 2$, $g(x) = x^2 + 6x + 5$; $Q[x]$;

21.2. $f(x) = x^3 + \bar{3}x^2 + \bar{2}x + \bar{1}$, $g(x) = x^3 + \bar{2}x^2 + x + \bar{2}$, $Z_5[x]$.

21.3. $f(x) = 4x^4 - 2x^3 - 16x^2 + 5x + 9$,
$g(x) = 2x^3 - x^2 - 5x + 4$, $Q[x]$.

22. $K[x]$ halqada $f(x)u(x) + g(x)v(x) = \varphi(x)$ tenglamaning barcha yechimlarini toping.

22.1. $f(x) = x^2 - 1$, $g(x) = x^2 - 2x + 1$; $\varphi(x) = x^3 - 1$, $K = Q$.

22.2. $f(x) = x^3 - \bar{1}$, $g(x) = x^2 + \bar{3}$, $\varphi(x) = x^3 - \bar{2}x^2 + x$, $K = Z_5$.

23. $C[x]$ halqada $f(x)u(x) + g(x)v(x) + h(x)\varphi(x) = S(x)$ tenglama yechimga ega ekanligini tekshiring. Bu yerda $f(x) = x^4 + 2x^2 + 1$; $g(x) = x^3 - x^2 + x - 1$; $h(x) = x^2 + (i + 1)x + i$; $S(x) = x^2 + (i - 1)x - i$.

24. K maydon ustida berilgan ko'phadlar uchun quyidagi xossalarni isbotlang:

24.1. $((f(x) \vdots \varphi(x)) \wedge (\varphi(x) \vdots \psi(x)) \Rightarrow (f(x) \vdots \psi(x))$,
$(\varphi(x) \neq 0, \psi(x) \neq 0)$.

24.2. $(f_i(x) \vdots \varphi(x)) \Rightarrow (f_1(x) \pm f_2(x) \pm \ldots \pm f_m(x)) \vdots \varphi(x)$
$(i = \overline{1, m})$, $(\varphi(x) \neq 0)$.

24.3. $f_i(x)$ $(i=\overline{1,m})$ ko'phadlarning kamida bittasi $\varphi(x) \neq 0$ ga bo'linsa, u holda ularning ko'paytmasi ham $\varphi(x)$ ga bo'linadi.

24.4. Agar $f_i(x)$ $(i=\overline{1,m})$ ko'phadlarning har biri $\varphi(x) \neq 0$ ga bo'linib, $g_i(x)$ lar ixtiyoriy ko'phadlar bo'lsa, u holda $(f_1(x)g_1(x) \pm \pm f_2(x)g_2(x) \pm \ldots \pm f_m(x)g_m(x)) \vdots \varphi(x)$ bo'ladi.

24.5. Har qanday $f(x)$ ko'phad ixtiyoriy nolinchi darajali ko'phadga bo'linadi.

24.6. $f(x) \vdots \varphi(x) \Rightarrow f(x) \vdots \alpha\varphi(x)$ $(0 \neq a \in K$ maydon, $\varphi(x) \neq 0)$.

24.7. $f(x) \neq 0$ va $\varphi(x) \neq 0$ ko'phadlar bir-biriga bo'linsa, u holda ular bir-biridan o'zgarmas $a \neq 0$ ko'paytuvchi bilan farq qiladi.

Takrorlash uchun savollar

1. Ko'phad darajasining xossalarini ayting.
2. Ko'phadni ikkihadga bo'lishni tushuntiring.
3. Ko'phad ildizi va uning xossalarini ayting.
4. Ko'phadlarning tengligi (algebraik, funksional).
5. Qoldiqli bo'lish haqidagi teoremani tushuntiring.
6. Evklid algoritmi qanday tuziladi?

31-§. Ko'p o'zgaruvchili ko'phadlar

✓ **Asosiy tushunchalar:** ko'p o'zgaruvchili ko'phad, ko'phadning darajasi, bir jinsli ko'phad, leksikografik yozilgan ko'phad, simmetrik ko'phad, asosiy (elementar) simmetrik ko'phadlar, ko'phadlarning rezultanti.

Kamida ikkita o'zgaruvchiga bog'liq bo'lgan ko'phad *ko'p o'zgaruvchili ko'phad* deyiladi.

n ta noma'lumli ko'phad $x_1^{\alpha_i} x_2^{\beta_i} \ldots x_n^{\gamma_i}$ ko'rinishdagi chekli sondagi hadlarning algebraik yig'indisidan iborat.

n ta $x_1, x_2, \ldots, x_n$ o'zgaruvchili ko'phad $f(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} a_i x_1^{\alpha_i} x_2^{\beta_i} \ldots x_n^{\gamma_i}$ ko'rinishda bo'ladi. Bunda $a_i \in K$.

n ta noma'lumli ko'phadning *darajasi* deb, bu ko'phaddagi qo'shiluvchilar darajalarining kattasiga aytiladi.

Barcha qo'shiluvchilarining darajalari bir xil bo'lgan ko'phad *bir jinsli ko'phad* yoki *forma* deyiladi.

$f(x_1,x_2,...,x_n)$ ko'phad berilgan bo'lib, uning ikkita hadidan qaysi birida x_1 ning darajasi katta bo'lsa, o'sha had yuqori had deb yuritiladi. Agar bu hadlardagi x_1 ning darajasi teng bo'lib, qaysi birida x_2 ning darajasi katta bo'lsa o'sha had yuqori deb hisoblanadi va h.k.

$f(x_1,x_2,...,x_n)$ ko'phadni birinchi o'rinda eng yuqori hadni, ikkinchi o'rinda qolgan hadlar orasida eng yuqori bo'lgan hadni va shu jarayon oxirgi had uchun yozilgan bo'lsa, u holda $f(x_1,x_2,...,x_n)$ ko'phad *leksikografik yozilgan* deyiladi.

Agar ko'p noma'lumli ko'phaddagi ixtiyoriy ikkita noma'lumning o'rinlarini almashtirganda ko'phad o'zgarmasa, u holda bunday ko'phad *simmetrik ko'phad* deyiladi.

$x_1,x_2,...,x_n$ o'zgaruvchilardan tuzilgan

$$\begin{cases} \tau_1 = x_1 + x_2 + ... + x_n, \\ \tau_2 = x_1x_2 + x_1x_3 + ... + x_{n-1}x_n, \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ \tau_n = x_1x_2...x_n \end{cases} \qquad (1)$$

sistemadagi simmetrik ko'phadlar *asosiy (elementar) simmetrik ko'phadlar* deyiladi.

Simmetrik ko'phadlar haqidagi asosiy teorema. *F maydon ustidagi har qanday simmetrik ko'phad shu F maydon ustidagi elementar simmetrik ko'phadlar orqali yagona usulda ifodalanadi.*

1-misol. $f(x, y, z) = (x + 2y)(z^2 - 1) + (y - z)^2 - (x + z)(y - 2)$ ko'phadni bir jinsli ko'phadlar yig'indisiga keltiring.

Yechish. Berilgan ko'phaddagi qavslarni ochib, guruhlasak, $f(x, y, z) = (xz^2 + 2yz^2) + (-xy + y^2 - 3yz + z^2) + (x - 2y + 2z)$ hosil bo'ladi.

2-misol. $f(x, y, z) = 3x^3 + 3y^3 + 3z^3 + 5xyz + 2x^2 + 2y^2 + 2z^2$ simmetrik ko'phadni elementar simmetrik ko'phadlarga yoying.

Yechish. Berilgan ko‘phaddan

$$2x^2 + 2y^2 + 2z^2 = 2((x + y + z)^2 - 2(xy + yz + xz)) = 2\sigma_1{}^2 - 4\sigma_2$$

ko‘phadning $3x^3 + 3y^3 + 3z^3 + 5xyz$ qismini noma’lum koeffitsiyentlar usulida elementar simmetrik ko‘phadlarga yoyamiz. Buning uchun quyidagi jadvalni tuzamiz:

Yuqori had darajalari			Yuqori had	Elementar simmetrik ko’phadlar yoyilmasi
x	y	z		
3	0	0	$3x^3$	$3\sigma_1^{3-0}\sigma_2^{0-0}\sigma_3^0 = 3\sigma_1^3$
2	1	0	Ax^2y	$a\sigma_1^{2-1}\sigma_2^{1-0}\sigma_3^0 = a\sigma_1\sigma_2$
1	1	1	$bxyz$	$b\sigma_1^{1-1}\sigma_2^{1-1}\sigma_3^1 = b\sigma_3$

Bu jadvaldan $3x^3 + 3y^3 + 3z^3 + 5xyz = 3\sigma_1{}^3 + a\sigma_1\sigma_2 + b\sigma_3$ ga ega bo‘lamiz. O‘zgaruvchilarga qiymat berish yordamida a, b parametrlarni topamiz:

x	y	z	σ_1	σ_2	σ_3	$3x^3+3y^3+3z^3+5xyz = = 3\sigma_1^3 + a\sigma_1\sigma_2 + b\sigma_3$
1	1	0	2	1	0	$6 = 24 + 2a$
1	1	−2	0	−3	−2	$-28 = -2b$

Demak, $a = -9$, $b = 14$, ya’ni $f(x, y, z) = 3\sigma_1{}^3 - 9\sigma_1\sigma_2 + 14\sigma_3 + 2\sigma_1{}^2 - 4\sigma_2$.

3-misol. $x_1^2x_2^2$ hadning orbitasini elementar simmetrik ko‘phadlar yordamida ifodalang.

Yechish. $x_1^2x_2^2$ hadning orbitasi yoki $x_1^2x_2^2$ had yordamida hosil qilingan monogen ko‘phad $o(x_1, x_2, \dots, x_n) = x_1^2x_2^2 + \dots + x_{n-1}^2x_n^2$ ko‘rinishda bo‘lib, uni topish uchun quyidagi jadvalni tuzamiz:

Yuqori had darajalari						Yuqori had	Hosil bo'lgan elementar ko'phadlar
x_1	x_2	x_3	x_4	...	x_n		
2	2	0	0	...	0	$x_1^2x_2^2$	$\sigma_1^{2-2}\sigma_2^{2-0}=\sigma_2^2$
2	1	1	0	...	0	$ax_1^2x_2x_3$	$a\sigma_1^{2-1}\sigma_2^{1-1}\sigma_3^1=a\sigma_1\sigma_3$
1	1	1	1	...	0	$bx_1x_2x_3x_4$	$b\sigma_1^{1-1}\sigma_2^{1-1}\sigma_3^{1-1}\sigma_4^1=b\sigma_4$

Demak, $o(x_1,x_2,...,x_n)=\sigma_2^2+a\sigma_1\sigma_3+b\sigma_4$.

O'zgaruvchilarga qiymatlar beramiz:

x_1	x_2	x_3	x_4	x_5	...	x_n	σ_1	σ_2	σ_3	σ_4	σ_5	...	σ_n	$o(x_1,x_2,...,x_n)=$ $=\sigma_2^2+a\sigma_1\sigma_3+b\sigma_4$
1	1	1	0	0	...	0	3	3	1	0	0	...	0	$3=9+3a$
1	1	0	1	0	...	0	4	6	3	1	0	...	0	$6=36+16a+b$

Bundan $a=-2$, $b=2$ qiymatlarni hosil qilamiz.

Demak, $o(x_1,x_2,...,x_n)=\sigma_2^2-a\sigma_1\sigma_3+2\sigma_4$.

4-misol. $x\frac{19-x}{x+1}\left(x+\frac{19-x}{x+1}\right)=84$ tenglamani yeching.

Yechish. $u=x\frac{19-x}{x+1}$, $v=x+\frac{19-x}{x+1}$ belgilashlarni kiritib,

$\begin{cases} u+v=19, \\ u\cdot v=84 \end{cases}$ sistemani tuzamiz. Bu sistemaning $u_1=7$, $v_1=12$; $u_2=12$, $v_2=7$ yechimlari mavjud. Ular yordamida

$$\begin{cases} x\frac{19-x}{x+1}=7, \\ x+\frac{19-x}{x+1}=12, \end{cases} \qquad \begin{cases} x\frac{19-x}{x+1}=12, \\ x+\frac{19-x}{x+1}=7 \end{cases}$$

tenglamalar sistemasini hosil qilamiz. Ularning yechimlari $x_{1,2}=6\pm\sqrt{29}$ va $x_3=4$, $x_4=3$.

P maydonda $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$,
$g(x) = b_m x^m + b_{m-1}x^{m-1} + ... + b_1 x + b_0$
ko'phadlar berilgan va $a_n b_m \neq 0 \wedge \alpha_1, \alpha_2, ..., \alpha_n$ lar $f(x)$ ning ildizlari bo'lsin.

$f(x)$, $g(x)$ ko'phadlarning rezultanti $R(f, g) = a_n^{\ m} g(\alpha_1) g(\alpha_2) ... (g(a_n)$ dan iborat.

Agar $\beta_1, \beta_2, ..., \beta_n - g(x)$ ning ildizlari bo'lsa, u holda

$$R(f,g) = a_n^m b_m^n \prod_{\substack{1 \le i \le n, \\ 1 \le j \le m}} (\alpha_i - \beta_j),$$

$R(g, f) = (-1)^{mn} R(f, g)$ lar o'rinli.

Silvestr formulasi yordamida rezultant quyidagicha topiladi:

$$R(f,g) = \begin{vmatrix} a_n & a_{n-1} & \dots & a_1 & a_0 & 0 & \dots & 0 \\ 0 & a_n & \dots & a_2 & a_1 & a_0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a_n & a_{n-1} & a_{n-2} & \dots & 0 \\ b_m & b_{m-1} & \dots & b_1 & b_0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & b_m & b_{m-1} & b_{m-2} & \dots & 0 \end{vmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} m \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} n \end{matrix}$$

$f(x)$ ko'phadning diskriminanti:

$$D(f) = (-1)^{\frac{n(n-1)}{2}} a_n^{-1} R(f, f') \text{ yoki } D(f) = a_n^{2n-2} \prod_{1 \le i \le j \le n} (\alpha_i - \alpha_j)^2.$$

5-misol. $f(x) = x^2 - 3x + 6$ va $g(x) = x^3 + x^2 - x - 1$ uchun rezultantni toping.

Yechish. **1-usul.** Berilgan $f(x)$ ning ildizlari kompleks sonlar. $g(x)$ ning ildizlari $x = \pm 1$.

$R(f, g) = (-1)^{3 \cdot 2} R(g, f) = R(g, f)$ bo'lganligi uchun

$R(f, g) = f(-1) f(-1) f(1) = 10 \cdot 10 \cdot 4 = 400$.

2-usul. α_1 va α_2 $f(x)$ ning ildizlari bo'lsin. U holda

$R(f,g) = (\alpha_1^3 + \alpha_1^2 - \alpha_1 - 1)(\alpha_2^3 + \alpha_2^2 - \alpha_2 - 1) =$
$= (\alpha_1\alpha_2)^3 + (\alpha_1\alpha_2)^2 (\alpha_1\alpha_2) - \alpha_1\alpha_2(\alpha_1^2 + \alpha_2^2) - (\alpha_1^3 + \alpha_2^3) +$
$+ (\alpha_1\alpha_2)^2 - \alpha_1\alpha_2(\alpha_1 + \alpha_2) - (\alpha_1^2 + \alpha_2^2) + (\alpha_1 + \alpha_2) + 1.$

Viyet formulalariga ko'ra, $\alpha_1 + \alpha_2 = 3$ va $\alpha_1\alpha_2 = 6$ bo'lib,

$\alpha_1^2 + \alpha_2^2 = (\alpha_1 + \alpha_2)^2 - 2\alpha_1\alpha_2 = 3$ va

$\alpha_1^3 + \alpha_2^3 = (\alpha_1 + \alpha_2)((\alpha_1 + \alpha_2)^2 - 3\alpha_1\alpha_2) = -27.$

Demak, $R(f, g) = 400$.

3-usul.

$$R(f,g)=\begin{vmatrix} 1 & -3 & 6 & 0 & 0 \\ 0 & 1 & -3 & 6 & 0 \\ 0 & 0 & 1 & -3 & 6 \\ 1 & 1 & -1 & -1 & 0 \\ 0 & 1 & 1 & -1 & -1 \end{vmatrix} = \ldots = 400.$$

$f(x)$ ko'phad karrali ildizga ega bo'lishi uchun $R(f, f') = 0$ bo'lishi zarur va yetarli.

Ikki o'zgaruvchili yuqori darajali tenglamalar sistemasini rezultant yordamida yechish mumkin.

6-misol. $\begin{cases} y^2 - y + x^2 - 3x = 0, \\ y^2 + (11 - 6x)y - x^2 + 7x - 12 = 0 \end{cases}$ sistemaning yechimlarini toping.

Yechish. Sistemani y o'zgaruvchiga nisbatan qaraymiz va uning rezultantini tuzamiz:

$$R(x)=\begin{vmatrix} 1 & -1 & x^2-3x & 0 \\ 0 & 1 & -1 & x^2-3x \\ 1 & 11-6x & x^2+7x-12 & 0 \\ 0 & 1 & 11-6x & -x^2+7x-12 \end{vmatrix} = \ldots =$$

$$= 2x(x - 2)(20x^2 - 100x + 120) = 40x(x - 2)^2(x - 3).$$

Bundan $x_1=0$, $x_2=2$, $x_3=0$ yechimlarni topamiz. x o'zgaruvchining qiymatlarini sistemaga qo'yib, y ning qiymatlarini topamiz.

1) $x = 0$ da $y = 1$ bo'lib, $(0; 1)$ yechim;

2) $x = 2$ da $y = -1 \wedge y = 2$ bo'lib, $(2; -1)\wedge(2; 2)$ yechimlar ;

3) $x = 3$ da $y = 0$ bo'lib, $(3; 0)$ yechim.

Misol va mashqlar

1. Berilgan ko‘phadlarni kanonik shaklga keltiring:

1.1. $f(x,y) = (x-y)^2(x^2+xy+y^2)(x+2y)+x^2-1.$

1.2. $f(x,y) = (x-y)(xy-z)(x-z)xyz.$

2. Berilgan ko‘phadlarni leksiografik tartibda yozing va uning yuqori hadini toping:

2.1. $f(x,y,z) = (\overline{2}x+\overline{3}y)^2z - x(y+z-\overline{3}xz)$, $Z_5[x,y,z]$.

2.2. $f(x,y,z,t) = (x+y)(z+y)+\overline{2}x(y+t+\overline{1})+(y+z)^3$, $Z_3[x,y,z,t]$.

3. Quyidagi ko‘phadlarni simmetrik ko‘phadga to‘ldiring:

3.1. $f(x,y) = x^2+2y.$

3.2. $f(x,y) = x^3+x^2y+xy.$

3.3. $f(x,y,z) = x^3+2xy+2yz+5.$

3.4. $f(x,y,z) = (x+y)^2+2xz+xyz.$

4. Quyidagi ko‘phadlarning yuqori hadini toping:

4.1. $f(x_1,y_2) = 5\varsigma_1^2\varsigma_2\varsigma_3.$

4.2. $f(x_1,y_2) = 5\varsigma_1^2+2\varsigma_2\varsigma_3-3\varsigma_3^2.$

5. Quyidagi ko‘phadlarni elementar ko‘phadlar yordamida ifodalang:

5.1. $f(x,y) = x^3y+y^3x+2x^2+2y^2.$

5.2. $f(x,y) = 2x^4y-5x^2y+2xy^4-5xy^2.$

5.3. $f(x,y,z) = x^2y+xy^2+x^2z+x^2+y^2z+yz^2.$

5.4. $f(x,y,z) = x^4+y^4+z^4-2x^2y^2-2x^2z^2-2y^2z^2.$

5.5. $f(x,y,z,t) = (xy+zt)(xz+yt)(xt+yz).$

5.6. $f(x,y,z) = (xy+z)(xz+y)(yz+x).$

6. Agar $n \in N$ uchun $S_n(x,y) = x^n+y^n$ bo‘lsa, barcha $k > 2$ uchun $S_k = \varsigma_1 S_{k-1} - \varsigma_2 S_{k-2}$ ekanligini isbotlang.

7. 6-misoldagi formula yordamida quyidagilarning o‘rinli ekanligini tekshiring:

7.1. $S_2 = \varsigma_1^2 - 2\varsigma_2.$

7.2. $S_3 = \varsigma_1^3 - 3\varsigma_1\varsigma_2.$

7.3. $S_4 = \varsigma_1^4 - 4\varsigma_1^2\varsigma_2 + 2\varsigma_2^2.$

7.4. $S_7 = \varsigma_1^7 - 7\varsigma_1^5\varsigma_2 + 14\varsigma_1^3\varsigma_2^2 - 7\varsigma_1\varsigma_2^3.$

8. Agar $n \in N$ uchun $S_n(x,y,z) = x^n + y^n + z^n$ bo'lsa, barcha $k > 3$ uchun $S_k = \sigma_1 S_{k-1} - \sigma_2 S_{k-2} + \sigma_3 S_{k-3}$ uchun ekanligini isbotlang.

9. 8-misoldan foydalanib, quyidagilar o'rinli ekanligini tekshiring:

9.1. $S_3 = \sigma_1^2 - 3\sigma_1\sigma_2 + 3\sigma_3$.

9.2. $S_4 = \sigma_1^4 - 4\sigma_1\sigma_2 + 2\sigma_2^2 + 4\sigma_1\sigma_3$.

9.3. $S_5 = \sigma_1^5 - 5\sigma_1^3\sigma_2 + 5\sigma_1\sigma_2^2 + 5\sigma_1^2\sigma_3 - 5\sigma_2\sigma_3$.

9.4. $S_6 = \sigma_1^6 - 6\sigma_1^4\sigma_2 + 9\sigma_1^2\sigma_2^2 - 2\sigma_2^3 + 6\sigma_1^3\sigma_3 - 12\sigma_1\sigma_2\sigma_3 + 3\sigma_3^2$.

10. Berilgan ratsional kasrlar surat va maxrajini elementar simmetrik ko'phadlar orqali ifodalab, qiymatini toping:

10.1. $\dfrac{f(x,y)}{g(x,y)} = \dfrac{(x-y)^4}{x+y}$ va $\varsigma_1 = 2$, $\varsigma_2 = 1$.

10.2. $\dfrac{f(x,y,z)}{g(x,y,z)} = \dfrac{1}{y} + \dfrac{1}{z} + \dfrac{1}{x} + \dfrac{2}{xy} + \dfrac{1}{xz} + \dfrac{2}{yz}$ va $\varsigma_1 = 0$, $\varsigma_2 = 1$, $\varsigma_3 = 2$.

11. Quyidagi hadlar orbitasini elementar simmetrik ko'phadlar yordamida ifodalang:

11.1. $x_1^2 x_3$, $P[x_1, x_2, x_3]$.

11.2. $x_1 x_2 x_3$, $P[x_1, x_2, x_3, x_4]$.

11.3. x_1^3, $P[x_1, x_2, \ldots, x_n]$.

12. Isbotlang:

12.1. $x^4 + y^4 + (x - y^4) = 2(x^2 + xy + y^2)^2$.

12.2. $(x + y)^3 + 3xy(1 - x - y) - 1 =$
$= (x + y - 1)(x^2 + y^2 - xy + x + y + 1)$.

12.3. $x(y + z)^2 + y(x + z)^2 + z(x + y)^2 =$
$= (y + z)(x + z)(x + y) + 4xyz$.

12.4. $(xy + xz + yz)^3 + (x^2 - yz)^2 + (y^2 - zx)^2 + (z^2 - xy)^2 =$
$= (x^2 + y^2 + z^2)^2$.

13. Agar $x + y + z = 0$ bo'lsa, quyidagi tengliklarni isbotlang:

13.1. $x^4 + y^4 + z^4 = 2(xy + xz + yz)^2$.

13.2. $2(x^4 + y^4 + z^4) = (x^2 + y^2 + z^2)^2$.

13.3. $2(x^5 + y^5 + z^5) = 5xyz(x^2 + y^2 + z^2)$.

14. Berilgan ko'phadlar rezultantini hisoblang:

14.1. $f(x) = 6x^2 + x - 2$, $g(x) = 3x^2 - 4x + 2$.

14.2. $f(x) = x^4 - 2x^2 + 3$, $g(x) = x^2 - x + 1$.

14.3. $f(x) = x^2 - 2x + 2$, $g(x) = 2x^2 + x - 5$.

14.4. $f(x) = x^3 + 2x^2 + 4x + 1$, $g(x) = 3x^2 + 4x + 4$.

15. a ning qanday qiymatlarida berilgan ko'phadlar umumiy ildizga ega:

15.1. $f(x) = 2x^2 + ax - 3$, $g(x) = ax^2 + x - 2$.

15.2. $f(x) = x^3 - 5x^2 + 4ax - 4$, $g(x) = 3x^2 - 5ax + 8$.

15.3. $f(x) = x^3 - ax + 2$, $g(x) = x^2 + ax + 2$.

15.4. $f(x) = x^3 + ax^2 - 9$, $g(x) = x^3 + ax - 3$.

16. Berilgan ko'phadlar diskriminantini hisoblang:

16.1. $f(x) = x^3 + 6x + 2$.

16.2. $f(x) = x^3 - 9x^2 + 21x - 5$.

16.3. $f(x) = x^5 + 2$.

16.4. $f(x) = ax^2 + bx + c$.

16.5. $f(x) = x^3 + px + q$.

16.6. $f(x) = x^3 + ax^2 + bx + c$.

17. Isbotlang:

17.1. $R(f, g_1 \pm g_2) = R(f_1 g_1) \pm R(f_1 g_2)$, $\deg f = 1$.

17.2. $R(f, g_1 \cdot g_2) = R(f_1 g_1) \cdot R(f_1 g_2)$.

17.3. $R(f_1 \cdot f_2, g_1 \cdot g_2) = R(f_1 g_1) \cdot R(f_1 g_2) \cdot R(f_2 g_1) \cdot R(f_2 g_2)$.

17.4. $D((x - a) \cdot f(x)) = D(f(x)) \cdot (f(a)^2$.

17.5. $D(f \cdot g) = D(f) \cdot D(g) \cdot (R(f, g))^2$.

18. a ning qanday qiymatlarida ko'phad karrali ildizga ega?

18.1. $f(x) = x^3 - 3x + a$.

18.2. $f(x) = x^4 - 4x + a$.

18.3. $f(x) = 4x^3 - ax + 1$.

18.4. $f(x) = x^3 + (2 - 3i)x^2 - ax - 2$.

19. Tenglamalar sistemasini yeching:

19.1. $\begin{cases} x^2 + 2y^2 = 17, \\ 6x^2 - xy - 12y^2 = 0. \end{cases}$

19.2. $\begin{cases} y^2 - 5y + 4x - 4 = 0, \\ 2y^2 + y - x^2 + 1 = 0. \end{cases}$

19.3. $\begin{cases} 5x^2 - 5y^2 - 3x + 9y = 0, \\ 5x^3 + 5y^3 - 15x^2 - 13xy - y^2 = 0. \end{cases}$

19.4. $\begin{cases} (y-1)x^2 + xy - 3 = 0, \\ (y-1)x^2 - 2x + y - 1 = 0. \end{cases}$

20. *R* maydonda quyidagi sistemalarni yeching:

20.1. $\begin{cases} x^3 + y^3 = 35, \\ x + y = 5. \end{cases}$

20.2. $\begin{cases} x^2 + xy + y^2 = 49, \\ x^4 + x^{2y} + y = 931. \end{cases}$

20.3. $\begin{cases} x + y + z = 1, \\ x^2 + y^2 + z^2 = 9, \\ x^3 + y^3 + z^3 = 1. \end{cases}$

20.4. $\begin{cases} x - y + z = 6, \\ x^2 + y^2 + z^2 = 14, \\ x^3 - y^3 + z^3 = 36. \end{cases}$

20.5. $\begin{cases} \sqrt{x} + \sqrt{y} = 9, \\ \sqrt[3]{x} + \sqrt[3]{y} = 5. \end{cases}$

20.6. $\begin{cases} \sqrt{\dfrac{x}{y}} + \sqrt{\dfrac{y}{x}} = \dfrac{61}{\sqrt{xy}} + 1, \\ \sqrt[4]{x^3 y} + \sqrt[4]{xy^3} = 78. \end{cases}$

21. Quyidagi tenglamalarni yeching.

21.1. $x + \sqrt{17 - x^2} + x\sqrt{17 - x^2} = 9$.

21.2. $\sqrt[3]{10 - x} - \sqrt[3]{3 - x} = 1$.

21.3. $\sqrt[4]{8 - x} + \sqrt[4]{89 + x} = 5$.

21.4. $\sqrt[4]{78 + \sqrt[3]{24 + \sqrt{x}}} - \sqrt[4]{84 - \sqrt[3]{30 - \sqrt{x}}} = 0$.

Takrorlash uchun savollar

1. Ko'p o'zgaruvchili ko'phadlar halqasi deb nimaga aytiladi?
2. Ko'phad darajasining qanday xossalarini bilasiz?
3. Ko'phad hadlarining leksikografik tartiblashni tushuntiring.
4. Simmetrik ko'phadlarga ta'rif bering.
5. Ikki ko'phad rezultanti qanday topiladi?
6. Ikki o'zgaruvchili yuqori darajali tenglamalarni yechish usullarini bayon eting.

32-§. Maydon ustida ko'phadlar

✓ **Asosiy tushunchalar:** keltiriladigan ko'phad, keltirilmaydigan ko'phad.

Agar F maydon ustida berilgan va darajasi nolga teng bo'lmagan $f(x)$ ko'phadni shu maydon ustidagi va darajalari $f(x)$ ning darajasidan kichik ikkita $g(x)$, $h(x)$ ko'phadlar ko'paytmasi shaklida ifodalash mumkin bo'lsa, u holda $f(x)$ ko'phadni F maydon ustida *keltiriladigan ko'phad*, aksincha, agar bunday ko'paytma shaklida ifodalash mumkin bo'lmasa, u holda $f(x)$ ni F maydon ustida *keltirilmaydigan ko'phad* deyiladi.

Algebrannig asosiy teoremasi. *Darajasi 1 dan kichik bo'lmagan kompleks koeffitsiyentli har qanday ko'phad kamida bitta kompleks ildizga ega.*

Agar $d(x)$ ko'phad $f(x)$ va $\varphi(x)$ ko'phadlarning umumiy bo'luvchisi bo'lib, $d(x)$ ko'phad $f(x)$ va $\varphi(x)$ larning ixtiyoriy umumiy bo'luvchisiga bo'linsa, u holda $d(x)$ bo'luvchini $f(x)$ va $\varphi(x)$ *ko'phadlarning eng katta umumiy bo'luvchisi* (EKUB) deyiladi va u $(f(x); \varphi(x))$ ko'rinishda belgilanadi.

$f(x)=a_0 + a_1x + ... + a_nx^n, f(y) = a_0 + a_1y + ... + a_ny^n$ bo'lsin.

$$f(x) - f(y) = \sum_{k=1}^{n} a_k(x^k - y^k) =$$

$$= (x-y)\sum_{k=1}^{n} a_k(x^{k-1}+x^{k-2}y + ... + y^{k-1}) = (x-y)F(x; y),$$

bu yerda $F(x;y)=\sum_{k=1}^{n} a_k(x^{k-1}+x^{k-2}y+...+y^{k-1})$. Aytaylik $x=y$ bo'lsin. U holda $F(x;x)=\sum_{k=1}^{n} ka_kx^{k-1}=a_1+2a_2x+...+na_nx^{n-1}$ bo'lib, $F(x;x)$ ni $f(x)$ *ko'phadning formal hosilasi* deyiladi va u $f'(x)$ yoki f' orqali belgilanadi.

$f(x)=a_0x^n+a_1x^{n-1}+...+a_{n-1}x+a_n$ ko'phad $x-c$ ning darajalari bo'yicha

$$f(x)=f(c)+f'(c)(x-c)+\frac{f''(c)}{2!}(x-c)^2+...+\frac{f^{(n)}(c)}{n!}(x-c)$$

ko'rinishda yoziladi.

Kompleks sonlar maydoni ustida $f(z)=z^n+c_1z^{n-1}+...+c_{n-1}z+c_n$ ko'phad berilgan bo'lib, $\alpha_1,\alpha_2,...,\alpha_n$ lar $f(z)$ ko'phadning ildizi bo'lsa, u holda ushbu

$$\begin{cases} c_1=-(\alpha_1+\alpha_2+...+\alpha_n); \\ c_2=\alpha_1\alpha_2+\alpha_2\alpha_3+...+\alpha_{n-1}\alpha_n; \\ c_3=-(\alpha_1\alpha_2\alpha_3+\alpha_2\alpha_3\alpha_4+...+\alpha_{n-2}\alpha_{n-1}\alpha_n); \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ c_n=(-1)^n\alpha_1\alpha_2\cdots\alpha_n \end{cases}$$

munosabatlar o'rinli bo'ladi.

Kompleks sonlar maydoni C ustidagi ushbu $ax^3+bx^2+cx+d=0$ $(a\neq0)$ ko'rinishdagi tenglama 3-darajali bir noma'lumli tenglama deyiladi. Uning har ikki qismini a ga bo'lib,

$$x^3+\frac{b}{a}x^2+\frac{c}{a}x+\frac{d}{a}=0$$

tenglamani hosil qilamiz. Unda $x=y-3\frac{b}{a}$ almashtirish bajarib, soddalashtirgandan so'ng $y^3+py+q=0$ tenglamani hosil qilamiz. Bunda $y=u+v$ almashtirishdan so'ng u va v larni shunday tanlab olamizki, natijada $3uv+p=0$ shart bajarilsin. U holda

$$\begin{cases} u^3+v^3=-q, \\ u^3v^3=-\frac{p^3}{27} \end{cases}$$

sistemaga ega bo'lamiz. Sistemadan ko'rinadiki u^3 va v^3 lar Viyet teoremasiga ko'ra qandaydir $z^2+qz-\frac{p^3}{27}=0$ tenglamaning ildizi bo'ladi. Bu kvadrat tenglamani yechib, $z_1=u^3$ dan $u=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$, $v=\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$ larni hosil qilamiz. u va v ning har biriga uchta qiymat, u o'zgaruvchi uchun esa to'qqizta qiymat topiladi.

Agar u, εu, $\varepsilon^2 u$ (bunda ε son 1 dan chiqarilgan 3-darajali ildiz) z_1 ning uchinchi darajali ildizlarining qiymatlari bo'lsa, unga mos z_2 ning uchinchi darajali ildizlari qiymatlari v, εv, $\varepsilon^2 v$ bo'ladi. Natijada keltirilgan tenglama $y_1=u+v$, $y_2=\varepsilon u+\varepsilon^2 v$, $y_3=\varepsilon^2 u+\varepsilon v$ ildizlarga ega bo'lib, unda $\varepsilon=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$ bo'lgani uchun $y_1=u+v$, $y_2=-\frac{1}{2}(u+v)+i\frac{\sqrt{3}}{2}(u-v)$, $y_3=-\frac{1}{2}(u+v)-i\frac{\sqrt{3}}{2}(u-v)$ bo'ladi.

Bu yerda $x=y-\frac{3b}{a}$ ni e'tiborga olib berilgan tenglamaning $x_1=y_1-\frac{3b}{a}$, $x_2=y_2-\frac{3b}{a}$, $x_3=y_3-\frac{3b}{a}$ ildizlari topiladi.

Kub tenglamani bu usulda yechish uni *Kardano usuli bilan yechish* deyiladi.

Agar $x^3+px+q=0$ tenglamada p, q lar haqiqiy sonlar bo'lib, $\Delta=\frac{q^2}{4}+\frac{p^3}{27}$ bo'lsa, u holda quyidagi mulohazalar o'rinli:

1) agar $\Delta>0$ bo'lsa, tenglama bitta haqiqiy va ikkita o'zaro qo'shma mavhum ildizlarga ega bo'ladi;

2) agar $\Delta=0$ bo'lsa, tenglamaning barcha ildizlari haqiqiy va kamida bitta ildizi karrali bo'ladi;

3) agar $\Delta<0$ bo'lsa, tenglamaning barcha ildizlari haqiqiy va turlicha.

Agar a butun son koeffitsiyentlari butun bo'lgan

$$a_0x^n + a_1x^{n-1} + ... + a_{n-1}x + a_n = 0$$

tenglamaning ildizi bo'lsa, u holda $\frac{f(1)}{a-1}$ va $\frac{f(-1)}{a+1}$ sonlar ham butun sonlar bo'ladi.

Agar p/q $(q > 0)$ qisqarmas kasr koeffitsiyentlari butun bo'lgan $a_0x^n + a_1x^{n-1} + ... + a_{n-1}x + a_n = 0$ tenglamaning ildizi bo'lsa, u holda p son a_n ozod hadning q son esa a_0 bosh koeffitsiyentning bo'luvchisi bo'ladi.

Eyzenshteyn kriteriyasi. Butun koeffitsiyentli

$$f(x) = c_nx^n + c_{n-1}x^{n-1} + ... + c_1x + c_0$$

ko'phadning bosh koeffitsiyenti c_n dan boshqa barcha koeffitsiyentlari p tub songa bo'linib, ozod had c_0 esa p^2 ga bo'linmasa, u holda $f(x)$ ko'phad ratsional sonlar maydoni ustida keltirilmaydigan ko'phad bo'ladi.

Kasrning maxrajidagi irratsionallikni yo'qotish mumkin, ya'ni F_1 sonlar maydoni ustida keltirilmaydigan n-darajali $p(x)=x^n + a_1x^{n-1} + .. + a_{n-1}x + a_n$ $(n \geq 2)$ ko'phad berilgan bo'lib, $x=\alpha$ uning ildizi bo'lsa, u holda $\frac{f(\alpha)}{g(\alpha)}(g(\alpha) \neq 0)$ kasr-ratsional ifodani shunday o'zgartirish mumkinki, natijada uning maxraji butun ratsional ifodaga aylanadi.

1-misol. $f(x) = x^4 + 2x^3 - 3x^2 - 5x + 2$ ko'phad maydonda keltiriluvchimi?

Yechish. $f(x)$ ko'phad darajasi 4 ga teng bo'lganligi uchun, agar u Q maydonda keltiriluvchi bo'lsa, u holda $f(x)$ ko'phadni ikkita 2-darajali yoki 1- va 3-darajali ko'phadlar ko'paytmasiga yoyish mumkin.

Agar $f(x) = (ax^2 + bx + c)(dx^2 + mx + n)$ deb faraz qilsak, bu tenglamaning yechimlari na butun sonlarda va na kasr sonlarda mavjud emasligiga ishonch hosil qilish mumkin.

Agar $f(x) = (ax + b)(cx^3 + dx^2 + mx + n)$ deb faraz qilsak, u holda tenglamadan

$$\begin{cases} ac = 1, \\ ad + bc = 2, \\ am + bd = -3, \\ an + bm = -5, \\ bn = 2 \end{cases}$$

sistemani hosil qilamiz. Bu sistemaning yechimlaridan biri $a=c=$ $=n=1$, $b=2$, $d=0$, $m=-3$. Demak, $f(x) = (x+2)(x^3-3x+1)$ bo'lib, berilgan $f(x)$ ko'phad Q maydonda keltiriluvchi ekan.

2-misol. $Q[x]$ da berilgan $f(x) = (x-2)(x-3)^2(x+1)$ va $g(x) = x^3 - 3x^2 - 2x + 6$ ko'phadlarning EKUB va EKUKini toping.

Yechish. $f(x)$ ko'phad kanonik yoyilma ko'rinishida berilganligi uchun $g(x)$ ko'phadni keltirilmaydigan ko'phadlar kanonik yoyilmasiga keltiramiz:

$$f(x) = (x^3 - 3x^2) - (2x - 6) = x^2(x-3) - 2(x-3) = (x-3)(x^2-2).$$

Demak, $(f, g) = x - 3$; $[f, g] = (x-2)(x^2-2)(x-3)^2(x+1)$.

3-misol. $Q[x]$ halqada berilgan $f(x) = x^4 - 2x^3 + 3x^2 - 5x + 1$ ko'phad hosilalarining $x_0=1$ nuqtadagi hosilalarini toping va berilgan ko'phadni $x-1$ ikkihad darajalariga yoying.

Yechish. **1-usul.**

$$f'(x) = 4x^3 - 6x^2 + 6x - 5;$$
$$f''(x) = 12x^2 - 12x + 6;$$
$$f'''(x) = 24x - 12;$$
$$f^{IV}(x) = 24.$$

U holda
$$f'(1) = 4 \cdot 1^3 - 6 \cdot 1 - 5 = -1;$$
$$f''(1) = 12 \cdot 1 - 12 \cdot 1 + 6 = 6;$$
$$f'''(1) = 24 \cdot 1 - 12 = 12;$$
$$f^{IV}(x) = 24.$$

Berilgan ko'phadning $(x-1)$ darajalariga yoyilmasini Teylor formulasidan foydalanib topamiz:

$$f(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 +$$
$$+ \frac{f'''(1)}{3!}(x-1)^3 + \frac{f^{IV}(1)}{4!}(x-1)^4.$$

Bu yerda $f(1) = -2$ bo‘lganligi uchun

$f(x) = -2 - (x - 1) + 3(x - 1)^2 + 2(x - 1)^3 + (x - 1)^4.$

2-usul. Gorner sxemasi yordamida yoyilmani topamiz:

	1	−2	3	− 5	1
1	1	−1	2	−3	−2
1	1	0	2	−1	
1	1	1	3		
1	1	2			
1	1				
1					

Jadvaldan $f(1) = -2$; $f'(1) = -1$; $\frac{f''(1)}{2!} = 3$; $\frac{f'''(1)}{3!} = 2$; $\frac{f^{IV}(1)}{4!} = 1$ larni aniqlaymiz.

Bundan, $f(x) = (x-1)^4 + 2(x - 1)^3 + 3(x - 1)^2 - (x - 1) - 2$ va $f'(1) = -1$; $f''(1) = 6$; $f'''(1) = 12$ larni topamiz.

4-misol. *a* ning qanday qiymatlarida $f(x) = x^3 + x^2ax + 3$ ko‘phad karrali ildizga ega bo‘ladi?

Yechish. Berilgan ko‘phadning karrali ildizi α bo‘lsin. U holda $f(x) = (x - \alpha)^2 \cdot h(x)$. Bu ko‘phadning hosilasi $f(x) = 2(x - \alpha)h(x) + h'(x)(x - \alpha)^2$ bo‘lib, $f'(\alpha) = 0$ bo‘ladi.

$f(\alpha) = 0$, $f'(\alpha) = 0$ lardan quyidagi tenglamalar sistemasini tuzamiz:

$$\begin{cases} \alpha^3 + \alpha^2 + a\alpha + 3 = 0, \\ 3\alpha^2 + 2\alpha + \alpha = 0. \end{cases}$$

Bundan $a = -3\alpha^2 - 2\alpha$ yordamida $\alpha^3 + \alpha^2 - 3\alpha^3 - 2\alpha^2 + 3 = 0$, ya’ni $-2\alpha^3 - \alpha^2 + 3 = 0$ tenglamaga ega bo‘lamiz. Uning $\alpha_1 = -\frac{3}{2}$; $\alpha_2 = 1$ yechimlari mavjud bo‘lib, $a_1 = -\frac{15}{4}$; $a_2 = -5$ ga ega bo‘lamiz.

Demak, $a = -5$ da berilgan ko‘phad karrali ildizga ega.

5-misol. Agar $Q[x]$ halqada 2 son $f(x)$ ko'phadning 3 karrali ildizi bo'lsa, u holda $g(x) = f'(x)(x^2 + 3) + (x + 3)f''(x)$ ko'phadning necha karrali ildizi bo'ladi?

Yechish. 2 soni $f(x)$ko'phadning 3 karrali ildizi bo'lganligi uchun $f(2) = f'(2) = f''(2) = 0$ va $f'''(2) \neq 0$. 2 sonni $g(x)$ uchun tekshiramiz: $g(2) = 0$ va $g'(2) \neq 0$.

Demak, 2 soni $g(x)$ uchun bir karrali ildiz.

6-misol. $f(x)$ ko'phadni karrali ko'paytuvchilarga yoying.

Yechish. $f(x)$ ko'phadni keltirilmaydigan ko'phadlarga kanonik yoyilmasini quyidagi jadvaldan foydalanib topamiz:

C maydonda $f(x) = \varphi_1\varphi_2{}^2...\varphi_m{}^n$, $1 \leq i \leq m$, bo'lsin, φ_i ko'phadlarni quyidagicha aniqlaymiz:

$f = \varphi_1\varphi_2^2...\varphi_m^m$	$q_1 = \frac{1}{d_1} = \varphi_1\varphi_2...\varphi_m$	$\varphi_1 = \frac{q_1}{q_2}$
$d_1 = (f, f') = \varphi_2\varphi_3^2...\varphi_m^{m-1}$	$q_2 = \frac{d_1}{d_2} = \varphi_2\varphi_3...\varphi_m$	$\varphi_2 = \frac{q_2}{q_3}$
$d_2 = (d_1 d_1') = \varphi_3......\varphi_m^{m-2}$	$q_3 = \frac{d_2}{d_3} = \varphi_3...\varphi_m$	$\varphi_3 = \frac{q_3}{q_4}$
...	...	...
$d_{m-1} = (d_{m-2}d'_{m-2}) = \varphi_m$	$q_m = \frac{d_{m-1}}{d_m} = \varphi_m$	$\varphi_m = q_m$
$d_m = 1$		

Berilgan $f(x)$ ko'phad uchun Evklid algoritmi yordamida $d_1, d_2, ..., d_m$ larni topamiz: $f'(x) = 4x^3 - 6ix^2 - 2i$.

$d_1 = (f, f') = (x - i)^2;$

$d_1' = 2x - 2i = 2(x - i);$

$d_2 = x - i;$

$d_2' = 1;$

$d_3 = (d_2, d_2') = 1;$

$$q_1 = \frac{f}{d_1} = x^2 + 1;$$

$$q_2 = \frac{d_1}{d_2} = x - i;$$

$$q_3 = \frac{d_2}{d_3} = x - i.$$

Bulardan, $\varphi_1 = \frac{q_1}{q_2} = x + i$; $\varphi_2 = \frac{q_2}{q_3} = 1$; $\varphi_3 = q_3 = x - i$ lar kelib chiqadi.

Demak, $f(x) = (x + i)(x - i)^3$.

7-misol. Q maydonda $\frac{f(x)}{g(x)} = \frac{x^2+4x+3}{(x^2-4x+4)(x+3)^2}$ kasrni elementar kasrlarga yoying.

Yechish. $f(x)$ va $g(x)$ ko'phadlarning o'zaro tub yoki tubmasligini tekshiramiz. Buning uchun ularning EKUBini topamiz. $(f, g) = x + 3$. $f(x)$ va $g(x)$ larni o'zaro tub holga keltiramiz va qisqarmas kasr $\frac{f(x)}{g(x)} = \frac{x+1}{(x-2)^2(x+3)}$ ni hosil qilamiz. Bundan, A, B, C parametrlar yordamida $\frac{x+1}{(x-2)^2(x+3)} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{C}{x+3}$ yoyilmani tuzamiz. Natijada $x + 1 = A(x - 2)(x + 3) + B(x + 3) + C(x - 2)^2$ tenglama kelib chiqadi va uning yechimlari

a) $x = 2$ da $B = \frac{3}{5}$;

b) $x = -3$ da $C = -\frac{2}{25}$;

d) $A + C = 0$ va $A = \frac{2}{25}$.

Demak, $\frac{f(x)}{g(x)} = \frac{x+1}{(x-2)^2(x+3)} = \frac{2}{25(x-2)} + \frac{2}{5(x-2)^2} + \frac{2}{25(x+3)}$.

Misol va mashqlar

1. $f(x)$ ko'phad berilgan maydonda keltirilmasligini isbotlang:

1.1. $f(x) = x^3 - 2$, Q.

1.2. $f(x) = x^2 + x + 1$, Q.

1.3. $f(x) = x^2 + x + 1$, Z_5.

1.4. $f(x) = x^6 + x^3 + 1$, Q.

2. Q maydonda berilgan quyidagi ko'phadlarni keltirilmaydigan ko'phadlar ko'paytmasiga yoying.

2.1. $f(x) = 2x^5 - x^4 - 6x^3 + 3x^2 + 4x - 2$.

2.2. $f(x) = 3x^5 + x^4 - 15x^3 - 5x^2 + 12x + 4$.

3. $f(x) = 2x^5 - x^4 - 2x^3 + x^2 - 4x + 2$ ko'phadning 2 juft bir-biriga qarama-qarshi ildizlari mavjudligi ma'lum bo'lsa, uni Q, R, C maydonlardagi keltirilmaydigan ko'phadlarga yoyilmasini toping.

4. Q maydonda berilgan 3-darajali ko'phad keltiriluvchi bo'lishi uchun uning bitta ildizi ratsional son bo'lishi zarur va yetarli ekanligini isbotlang.

5. $Z[x]$ halqada quyidagi ko'phadlar keltirilmasligini isbotlang:

5.1. $f(x) = x^5 - x^2 + 1$.

5.2. $f(x) = x^5 + x^4 + x^3 + x^2 + 1$.

5.3. $f(x) = x^3 - x^2 + x + 1$.

6. $f(x) = x^4 + 4$ ko'phad Z_5, Q, R, C maydonlarning qaysi birida keltiriluvchi?

7. Quyidagi ko'phadlarni keltirilmaydigan ko'phadlarga yoying:

7.1. $f(x) = x^4 - 6x^3 + 11x^2 - 6x + 1$, $R[x]$.

7.2. $f(x) = x^4 + 4$, $C[x]$.

7.3. $f(x) = (x^2 + x - 1)^2 + 3x(x^2 + x - 1) + 2x^2$.

7.4. $f(x) = x^4 + 4$; $C[x]$.

7.5. $f(x) = x^2(x - 3)^2 + 4x^2 - 12x + 4$.

7.6. $f(x) = x^6 + 27$; $C[x]$.

7.7. $f(x) = (x + 2)(x + 3)(x + 4)(x + 5) + 1$.

7.8. $f(x) = x^{2n} + x^4 + 1$; $C[x]$.

8. Quyidagi ko'phadlarning EKUB va EKUK larini toping:

8.1. $f(x) = (x-1)^2(x^2-5x+6)$, $g(x) = x^2 - x - 2$, $Z[x]$.

8.2. $f(x) = (x^2-2x+3)^2(x^2+5x-6)^2$,
$g(x) = (x^2-8x+12)^2(x^3-1)$, $Q[x]$.

8.3. $f(x) = x^4 + 2x^3 - 2x - 1$, $g(x) = (x+1)(x^2-x-2)$, $Q[x]$.

8.4. $f(x) = x^5 - x$, $g(x) = (x^2 + x + \overline{1})^2(\overline{2}x + \overline{4})$, $Z_5[x]$.

8.5. $f(x) = x^m - 1$, $g(x) = x^n - 1$.

8.6. $f(x) = x^m + 1$, $g(x) = x^n + 1$.

9. Quyidagi ko'phadlarning hosilasini toping:

9.1. $f(x) = (x^2+x-1)^3(x^3-2)$, $Q[x]$.

9.2. $f(x) = \overline{4}x^{10} + \overline{3}x^2(x+3)$, $Z_5[x]$.

10. Agar $Z_3[x]$ halqada $f'(x) = \overline{2}x + 1$ va $f(1) = 1$ bo'lsa, 6 darajali $f(x)$ ko'phadni toping.

11. Agar $Q[x]$ halqada $f''(x) = 24x + 2$, $f(0) = 1$ va $f(1) = 5$ va bo'lsa, $f(x)$ ko'phadni toping.

12. $Z_2[x]$ halqada darajasi 3 dan katta bo'lmagan va o'z hosilasiga bo'linuvchi barcha $f(x)$ ko'phadlarni toping.

13. $f(x)$ ko'phadni $x - a$ darajalariga yoying va hosilalarining a nuqtadagi qiymatini toping.

13.1. $f(x) = ix^4 + (1-i)x^3 - (2+i)x^2 + 3x - 3 - 4i$, $a = 2i$, $C[x]$.

13.2. $f(x) = x^5 - 3ix^3 - 4x^2 + 5ix - 1$, $a = -i$, $G[x]$.

13.3. $f(x) = (x-3)(x-2)(x+1)(x+4) + 1$, $a = -1$, $Q[x]$.

13.4. $f(x) = \overline{2}x^4 + x^3 + x^2$, $a = \overline{1}$, $Z_3[x]$.

13.5. $f(x) = x^4 - 8x^3 + 24x^2 - 50x + 90$, $a = 2$, $R[x]$.

13.6. $f(x) = x^5 - 4x^3 + 6x^2 - 8x + 10$, $a = 2$, $R[x]$.

14. Berilgan ildizlar necha karrali ekanligini aniqlang:

14.1. $\alpha = 3$; $f(x) = x^4 - 6x^3 + 10x^2 - 6x + 9$, $Q[x]$.

14.2. $\alpha = 2$; $f(x) = x^5 - 4x^4 + 7x^3 - 11x^2 + 4$, $Q[x]$.

14.3. $\alpha = 1 + i$; $f(x) = x^4 - (3+4i)x^3 + (3+3i)x^2 +$
$+ (8-2i)x - 2 - 2i$, $C[x]$.

14.4. $\alpha = 2$; $f(x) = x^5 - 5x^4 + 7x^3 - 2x^2 + 4x - 8$.

14.5. $\alpha = 3$; $f(x) = x^5 - 6x^4 + 2x^3 + 36x^2 - 27x - 54$.

15. $R[x]$ da b ning qanday qiymatlarida berilgan ko'phad karrali ildizga ega:

15.1. $f(x) = x^5 - 5x^3 + b$.

15.2. $f(x) = x^3 - 4x^2 - 3x + b$.

15.3. $f(x) = x^3 + 3x^2 + 3bx - 4$.

15.4. $f(x) = x^3 + 5x^2 + 8x + b$.

16. Berilgan ko'phadlarning karrali ildizga ega bo'lishining zarur va yetarli shartlarini aniqlang:

16.1. $f(x) = x^4 + ax + b$.

16.2. $f(x) = x^5 + ax^3 + b$.

17. Berilgan ko'phadlarning keltirilmaydigan ko'phadlar kanonik yoyilmasini toping:

17.1. $f(x) = x^5 + 4x^4 + 7x^3 + 8x^2 + 5x + 2$.

17.2. $f(x) = x^5 - ix^4 + 5x^3 - ix^2 + 8x + 4i$.

17.3. $f(x) = x^5 + 5x^4 + (6 - i)x^3 - (4 + 6i)x^2 - (8 + 12i)x - 8i$.

17.4. $f(x) = x^6 - 6x^4 - 4x^3 + 9x^2 + 12x + 4$.

18. Quyidagi shartlar asosida kompleks koeffitsiyentli eng kichik darajali ko'phadni aniqlang:

18.1. 1 – ikki karrali, 2, 3, $1+i$ – bir karrali ildizlar.

18.2. i – ikki karrali, $-1-i$ – bir karrali ildizlar.

19. $R[x]$ halqada berilgan kasrlarni qisqarmas kasrga keltiring:

19.1. $\frac{x^2-4x+3}{x^2-5x+6}$.

19.2. $\frac{x^8+x^4+1}{x^2+x+1}$.

20. Q maydonda berilgan kasrni elementar kasrlarga yoying.

20.1. $\frac{f(x)}{g(x)} = \frac{x+3}{(x^3-2)(x+1)}$.

20.2. $\frac{f(x)}{g(x)} = \frac{1}{x^4-2x}$.

20.3. $\frac{f(x)}{g(x)}=\frac{x^2}{x^4-4}$.

20.4. $\frac{f(x)}{g(x)}=\frac{1}{x^3+x}$.

21. *R* maydonda berilgan kasrni elementar kasrlarga yoying:

21.1. $\frac{f(x)}{g(x)}=\frac{x^3-1}{(x^2+x+1)^2(x^2+1)}$.

21.2. $\frac{f(x)}{g(x)}=\frac{x^4+2x^3-18x^2+54}{x^5+6x^4+9x^3}$.

21.3. $\frac{f(x)}{g(x)}=\frac{x^2+3x+2}{(x^4+4)(x+2)}$.

21.4. $\frac{f(x)}{g(x)}=\frac{x^2}{x^4-4}$.

21.5. $\frac{f(x)}{g(x)}=\frac{x+3}{(x^3-2)(x+1)}$.

21.6. $\frac{f(x)}{g(x)}=\frac{x^2}{(x^2+x+2)^2}$.

21.7. $\frac{f(x)}{g(x)}=\frac{x^2}{x^4-16}$.

21.8. $\frac{f(x)}{g(x)}=\frac{1}{x^4+4}$.

22. *C* maydonda berilgan kasrni elementar kasrlarga yoying:

22.1. $\frac{f(x)}{g(x)}=\frac{x^2}{(x-1)(x+2)(x+3)}$.

22.2. $\frac{f(x)}{g(x)}=\frac{1}{x^4+4}$.

22.3. $\frac{f(x)}{g(x)}=\frac{5x^2+6x-23}{(x-1)^3(x+1)^2(x-2)}$.

22.4. $\frac{f(x)}{g(x)}=\frac{i}{(x-i)(x+2i)}$.

22.5. $\frac{f(x)}{g(x)}=\frac{2x}{(x-1)(x^2+1)}$.

22.6. $\frac{f(x)}{g(x)}=\frac{x^2}{x^4-4}$.

23. Z_5 maydonda $\frac{f(x)}{g(x)}=\frac{\overline{1}}{x^p-x}$ (p — tub son) kasrni elementar kasrlarga yoying.

24. Maydon ustida keltirilmaydigan ko‘phadlarning quyidagi xossalarini isbotlang:

1°. Agar $p(x)$ va $g(x)$ keltirilmaydigan ko‘phadlar bo‘lib, $p(x) \vdots g(x)$ bo‘lsa, u holda $p(x) = ag(x)$ $(a \neq 0)$ bo‘ladi.

2°. Ixtiyoriy $f(x)$ ko‘phad keltirilmaydigan ixtiyoriy $p(x)$ ko‘phadga bo‘linadi yoki $(f(x); p(x))=1$ bo‘ladi.

3°. Agar $f_i(x)$ $(i=\overline{1,m})$ ko‘phadlarning hech biri keltirilmaydigan $p(x)$ ko‘phadga bo‘linmasa, u holda $f_1(x)\cdot f_2(x) \ldots f_m(x) \not\vdots\, p(x)$ bo‘ladi.

4°. Agar $f_1(x) f_2(x) \ldots f_m(x) \vdots\, p(x)$ ($p(x)$ — keltirilmaydigan ko‘phad), u holda $f_i(x)$ $(i=\overline{1,m})$ ko‘phadlarning aqalli bittasi $p(x)$ ga bo‘linadi.

5°. $p(x)$ keltirilmaydigan ko‘phad bo‘lsa, u holda $ap(x)$ $(0 \neq a \in F)$ ham keltirilmaydigan ko‘phad bo‘ladi.

25. Agar $x_1 = a + bi$ berilgan $f(x)$ ko‘phadning ildizi bo‘lsa, uning qolgan yechimlarini toping:

25.1. $f(x) = x^3 - 4x^2 + 3x + 30$; $x_1 = 3 + i\sqrt{6}$.

25.2. $f(x) = x^3 - 4x^2 + 3x + 30$; $x_1 = 3 - i\sqrt{6}$.

25.3. $f(x) = 4x^4 - 24x^3 + 53x^2 + 18x - 42$; $x_1 = 3 - i\sqrt{5}$.

25.4. $f(x) = x^4 + 2x^3 + 2x^2 + 6x - 3;\ x_1 = -1 - i\sqrt{2}$.

26. Kardano formulalari yordamida quyidagi tenglamalarni yeching:

26.1. $-2x^3 - 2x^2 + 12x - 24$.

26.2. $2x^3 + 4x^2 + 4x + 4$.

26.3. $2x^3 + 8x^2 - 2x + 5$.

26.4. $-5x^3 + 8x^2 - 3x - 3$.

26.5. $6x^3 - 8x^2 + 5x - 3$.

26.6. $-5x^3 + 8x^2 - 3x - 24$.

26.7. $2x^3 + 8x^2 - 12x + 12$.

26.8. $2x^3 + 4x^2 + 4x + 4$.

26.9. $5x^3 + x^2 - 3x - 2$.

27. Ferrari usuli bilan quyidagi tenglamalarni yeching:

27.1. $x^4 - 2x^3 - 2x^2 + 12x - 24$.

27.2. $x^4 + 2x^3 + 4x^2 + 4x + 4$.

27.3. $x^4 - 2x^3 + 8x^2 - 12x + 12$.

27.4. $x^4 - 5x^3 + 8x^2 - 3x - 24$.

27.5. $x^4 - 2x^3 + 8x^2 - 12x + 12$.

27.6. $x^4 - 5x^3 + x^2 - 3x - 2$.

28. Ko'phadning butun ildizlarini toping:

28.1. $f(x) = x^4 - 3x^2 - 14$.

28.2. $f(x) = 4x^4 + 3x^2 - 4$.

28.3. $f(x) = x^4 + 4x^3 + 27$.

28.4. $f(x) = x^4 - 3x^2 - 24$.

28.5. $f(x) = x^5 + 3x - 9$.

28.6. $f(x) = x^5 + 3x - 8$.

28.7. $f(x) = x^5 + 3x - 12$.

28.8. $f(x) = x^3 + 2x^2 - 3x + 2$.

29. Ko'phadning ratsional ildizlarini toping:

29.1. $f(x) = 4x^5 + 4x^4 + 7x^3 + 8x^2 + 5x + 2$.

29.2. $f(x) = 5x^5 - x^4 - 2x^3 - 27x^2 - 44x + 7$.

29.3. $f(x) = -4x^5 - x^4 - 6x^3 + 11x^2 - 6x + 1$.

29.4. $f(x) = 5x^5 + 4x^4 + 4x^3 + 13x^2 + 6x + 9$.

29.5. $f(x) = -7x^5 + x^4 - 5x^3 - 8x^2 + 19x - 3$.

30. Kasr maxrajini irratsionallikdan qutqaring:

30.1. $\frac{7}{1-\sqrt[4]{2}+\sqrt{2}}$.

30.2. $\frac{2}{\sqrt[4]{27}-2\sqrt[4]{9}+\sqrt[4]{3}-1}$.

30.3. $\frac{2\sqrt[3]{5}}{\sqrt[3]{9}-\sqrt[3]{3}+7}$.

30.4. $\frac{2}{\sqrt[3]{49}-\sqrt[3]{7}+3}$.

30.5. $\frac{2\sqrt{3}}{\sqrt[3]{25}-\sqrt[3]{5}+6}$.

30.6. $\frac{9\sqrt[3]{2}}{\sqrt[4]{49}-\sqrt[4]{7}+3}$.

30.7. $\frac{\sqrt[3]{2}}{\sqrt[3]{4}+2\sqrt[3]{2}}$.

30.8. $\frac{9}{\sqrt[2]{4}+\sqrt[3]{2}+3}$.

30.9. $\frac{5}{\sqrt[3]{9}-\sqrt[3]{7}}$.

Takrorlash uchun savollar

1. Ko'phadning butun va ratsional ildizlari.
2. Eyzenshteynning keltirilmaslik alomati.
3. Algebraik elementning minimal ko'phadi.
4. Maydonning oddiy kengaytmasi va uni qurish.
5. Kasr maxrajini algebraik irratsionallikdan qutqarish.
6. Maydonning chekli kengaytmasi. Maydonning murakkab kengaytmasi.
7. Algebraik sonlar maydoni.
8. Tenglamalarning radikallarda yechilishi.
9. Uchinchi darajali tenglamalarning kvadrat radikallarda yechilish sharti.
10. Kvadrat radikallarda yechilmaydigan masalalar.

JAVOBLAR

I MODUL. MATEMATIK MANTIQ ELEMENTLARI

1-§. Mulohaza. Mulohazalar ustida mantiq amallari

1. 1.2, 1.5, 1.8, 1.9, 1.10, 1.14, 1.18, 1.19, 1.20, 1.21, 1.22 – mulohaza. **2**. 2.1, 2.5, 2.7 – yolg'on. **3**. 3.3. $3 \le 2$, 3.8. barcha haqiqiy sonlar toq. 3.10. Shunday natural son mavjudki, u birdan katta emas. **4**. 4.2, 4.8. **5**. 5.1. $(x \ne 0) \wedge (y \ne 0)$. 5.2. $(x = 0) \vee (y = 0)$. 5.3. $(x = 0) \wedge (y = 0)$. 5.4. $(x = 0) \wedge (y \ne 0)$. 5.5. $(x > -6) \wedge (x < 6)$. 5.6. $(x = -2) \vee (y = 2)$. **6**. 6.1, 6.5, 6.6, 6.7, 6.9, 6.10 – yolg'on. **7**. 7.2, 7.4, 7.7, 7.8, 7.9 – rost. **8.** 8.1. $\rceil A \wedge \rceil B \Rightarrow A \vee B$. 8.2. $A \wedge \rceil B \Rightarrow \rceil C$. 8.3. $A \Rightarrow B \vee C \vee D$. 8.4. $A \Rightarrow B \vee C \vee D$. **9**. Mavjud emas. **10.** 10.2 – yolg'on, qolganlari rost.

2-§. Formula. Teng kuchli formulalar. Mantiq qonunlari.

2.2.3. $(((\rceil A) \Leftrightarrow (\rceil B)) \vee (C \wedge B))$, $(((\rceil A) \Leftrightarrow ((\rceil B) \vee C)) \wedge B)$, $(((\rceil A) \Leftrightarrow (\rceil (B \vee C))) \wedge B)$, $((\rceil (A \Leftrightarrow ((\rceil B) \vee C))) \wedge B)$, $(\rceil ((A \Leftrightarrow (\rceil B)) \vee C) \wedge B)$, $(\rceil ((A \Leftrightarrow ((\rceil B) \vee C)) \wedge B))$, $(\rceil (A \Leftrightarrow (((\rceil B) \vee C) \wedge B)))$, $(\rceil (A \Leftrightarrow ((\rceil B) \vee (C \wedge B))))$, $(\rceil (A \Leftrightarrow (\rceil (B \vee (C \wedge B)))))$, $(\rceil (A \Leftrightarrow (\rceil ((B \vee C) \wedge B))))$, $(\rceil ((A \Leftrightarrow (\rceil B)) \vee (C \wedge B)))$, $(\rceil ((A \Leftrightarrow (\rceil (B \vee C))) \wedge B))$, $(\rceil (A \Leftrightarrow ((\rceil (B \vee C)) \wedge B)))$, $((\rceil (A \Leftrightarrow (\rceil B))) \vee (C \wedge B))$, $((\rceil A) \Leftrightarrow (((\rceil B) \vee C) \wedge B))$, $((\rceil A) \Leftrightarrow ((\rceil (B \vee C)) \wedge B))$, $((\rceil (A \Leftrightarrow (\rceil (B \vee C)))) \wedge B)$. **3.** 3.1. $A, B, C, (\rceil C), (A \vee B), (A \Leftrightarrow B)$, $((A \vee B) \Rightarrow A), ((A \Leftrightarrow B) \wedge (\rceil C))$, $(((A \vee B) \Rightarrow A) \Rightarrow (\rceil C))$,

$(((A \Leftrightarrow B) \wedge (\rceil C)) \Rightarrow (((A \vee B) \Rightarrow A) \Rightarrow (\rceil C)))$. **11.** 11.1. 1. 11.2. $A \vee B$. 11.3. $A \wedge B$. 11.4. $\rceil A \wedge \rceil C$. 11.5. $A \vee (B \wedge C)$. 11.6. $B \Rightarrow \rceil A$. **12.** 12.1. $\rceil(\rceil A \wedge B \wedge \rceil C)$. 12.2. $\rceil(\rceil A \wedge \rceil B)$. 12.3. $\rceil(\rceil A \wedge B \wedge \rceil C)$. 12.4. $\rceil(A \wedge \rceil B) \wedge \rceil(A \wedge C)$. 12.5. $\rceil(\rceil A \wedge \rceil B) \wedge \rceil(\rceil A \wedge C)$. **13.** 13.1. $\rceil(\rceil A \vee B) \vee \rceil(\rceil B \vee \rceil C)$. 13.2. $A \vee B \vee \rceil(\rceil A \vee \rceil B)$.

13.3. $\rceil(\rceil(A \vee B) \vee C) \vee \rceil(B \vee \rceil C)$. 13.4. $\rceil(\rceil(\rceil A \vee \rceil(\rceil B \vee \rceil C)) \vee$ $\vee (\rceil A \vee B)) \vee \rceil B$. 13.5. $\rceil(\rceil(\rceil A \vee B) \vee \rceil(\rceil B \vee B \vee C)) \vee (\rceil A \vee C)$.

14. 14.1. $(\rceil A \vee (\rceil B \wedge C)) \wedge (A \vee \rceil B)$. 14.2. $((A \vee B \vee C) \wedge \rceil D) \vee Q \vee R \vee P$.

14.3. $(((A \vee (B \wedge \rceil C)) \wedge \rceil D) \vee Q) \wedge (R \vee (\rceil P \wedge F))$.

14.4. $((\rceil A \vee (B \wedge (C \vee \rceil D))) \wedge Q) \vee \rceil R$.

3-§. Predikatlar. Kvantorlar.

1. 1.2,1.4,1.5,1.7,1.8,1.10,1.11,1.14. **5**. 5.1. M\{3}. 5.2. {1,2,3,4,5,6}. 5.3. {2,4,5,6,8,10,12,14,15,16,18,20}. 5.4. M\{3}. 5.5. M\{5}. 5.6. M\{5}. 5.7. {1,5,7,11,13,17,19}. 5.10. {6,12,18}.

4-§. Matematik mantiqning tatbiqlari

4. 4.1. $\forall x \in M, \forall y \in M((x = y) \vee (x < y) \vee (x > y))$;
$\exists x \in M, \exists y \in M(\rceil(x = y) \vee \rceil(x < y) \vee \rceil(x > y))$.
4.2. $\exists L \in R_+, \forall y \in M(|f(x)| \le L)$; $\forall L \in R_+, \exists y \in M(|f(x)| > L)$.
4.3. $\forall x_1 \in M, \forall x_2 \in M((x_1 < x_2) \Rightarrow f(x_1) < f(x_2))$;
$\exists x_1 \in M, \exists x_2 \in M((x_1 < x_2) \wedge f(x_1) \ge f(x_2)))$.
4.4. $\exists T \in R \setminus \{0\}, \forall x \in M((x \pm T \in M) \wedge (f(x \pm T) = f(x))$;
$\forall T \in R \setminus \{0\}, \exists x \in M((x \pm T \notin M) \vee (f(x \pm T) \neq f(x))$.

II MODUL. TO‘PLAMLAR VA MUNOSABATLAR

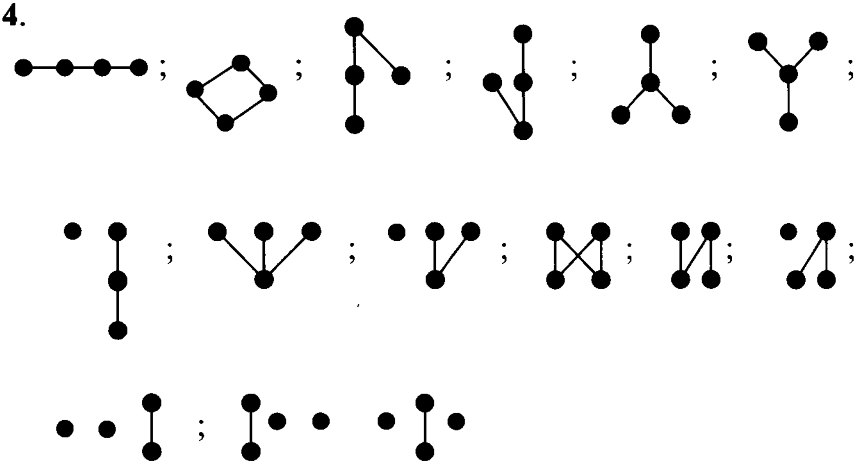
5-§. To‘plam. To‘plamlar ustida amallar. Eyler–Venn diagrammalari

2. A={1}, B={a,{1}}, C={{a,{1}}}. **4**. |R(M_1)|=2, |R(M_2)|=4, |R(M_3)|=16, |R(M_n)|=2^n. **6**. 10 nafardan kam emas.

7-§. Akslantirish (funksiya). Tartib munosabati. Graflar

3. $R = \{(1,1),(2,2),(2,1),(3,2),(3,3),(3,1),(4,4),(1,5),(2,5),(3,5)\}$.

4.



II MODUL. ALGEBRA VA ALGEBRAIK SISTEMALAR

8-§. Algebra. Faktor-algebra

1. 1.1. Amal emas. 1.2. Unar amal. 1.3. Amal emas. 1.4. Amal emas. 1.5. Amal emas, agar natural sonlar to‘plamida qaralsa, ternar amal. **2.** Neytral element mavjud emas. **3**. 3.1. Neytral element mavjud emas. 3.2. Neytral element 1. **4**. 4.1. Butun sonlar to‘plamida qo‘shish, ko‘paytirish va qarama-qarshi elementni topish. 4.2. Ratsional sonlar to‘plamida qo‘shish, ko‘paytirish, qarama-qarshi va teskari elementlarni topish. 4.3. Mulohazalar to‘plamida dizyunksiya, konyunksiya, implikatsiya, inkor amallari.

9-§. Gruppa. Halqa. Maydon

10. $4z \begin{cases} 0, \text{ agar } z = 4k, \\ e, \text{ agar } z = 4k+1, \\ a, \text{ agar } z = 4k+2, \\ b, \text{ agar } z = 4k+3 \end{cases}$ akslantirish gomomorfizmdir.

IV MODUL. ASOSIY SONLI SISTEMALAR

12-§. Kompleks sonlar maydoni

1. 1.1. $\operatorname{Re} z = 0$, $\operatorname{Im} z = \frac{14}{5}$. 1.2. $\operatorname{Re} z = 2$, $\operatorname{Im} z = \frac{3}{2}$. 1.3. $\operatorname{Re} z = 0$, $\operatorname{Im} z = -1$. 1.4. $\operatorname{Re} z = \frac{1}{2}$, $\operatorname{Im} z = 0$.

4. 4.1. $z = 2{,}5 + yi, y \in R$. 4.2. $z = -\frac{1}{4} + i$. 4.3. $z_{1,2} = \frac{-1\pm\sqrt{5}}{2}$. 4.4. $\varnothing$. 4.5. $z_1 = -1 + i$, $z_2 = -4 - i$. 4.6. $z_1 = \frac{3-7i}{8}$, $z_2 = \frac{1+3i}{8}$. 4.7. $z_1 = 1 - 2i$, $z_2 = -3 + i$. 4.8. $z_1 = -2i$, $z_2 = \frac{-1+3i}{2}$. **5.** 5.1. $-1-i$. 5.2. i. 5.3. $\varnothing$. **6.** 6.1. $1+i$. 6.2. $6+8i, 6+17i$. 6.3. $z_1 = 1 - i$, $z_2 = i$. 6.4. $z_1 = 1 + i, z_2 = -2i$. 6.5. $z = \frac{7}{6} + \frac{5}{6}i$. 6.6. $z_1 = -1{,}5 - 2i$; $z_2 = -1{,}5 - 4{,}25i$. **8.** 8.1. $2^n \cos\frac{n\pi}{3}$. 8.2. $\frac{2^n}{\sqrt{3}}\sin\frac{2n\pi}{3}$. 8.3. $2^n \cos\frac{5n\pi}{3}$. 8.4. $\frac{2^n}{3^{\frac{n-1}{2}}}\sin\frac{n\pi}{6}$. **9.** 9.1. $5(\cos 0 + i\sin 0)$. 9.2. $2(\cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi)$. 9.3. $\frac{1}{2\sqrt{2}}(\cos\frac{5}{4}\pi + i\sin\frac{5}{4}\pi)$. 9.4. $16(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$. 9.5. $\cos(\frac{\pi}{2} - \alpha) + i\sin(\frac{\pi}{2} - \alpha)$. **11.** 11.1. $-\frac{\cos(\alpha+\beta)}{\sin\beta}(\cos(2\beta - \theta) + i\sin(2\beta - \theta))$. 11.2. 4^{2008}. 11.3. $2^n \sin^n\frac{\alpha}{2}(\cos n(\frac{\pi}{2} - \alpha) + i\sin n(\frac{\pi}{2} - \alpha))$. 11.4. $-\frac{3^8}{\sqrt{3}\cdot 2^{971}}(-1 + i\sqrt{3})$.

12. 12.1. $\sqrt[6]{5}(\cos\frac{\varphi+2k\pi}{3} + i\sin\frac{\varphi+2k\pi}{3})$, $k = 0,1,2; \varphi = \arcsin(-\frac{1}{\sqrt{5}})$.

12.2. $2(\cos\frac{k\pi}{6} + i\sin\frac{k\pi}{6}), k = 0,1,2,3,4,5$.

12.3. $\sqrt[8]{13}(\cos\frac{\varphi+2k\pi}{4} + i\sin\frac{\varphi+2k\pi}{4}), k = 0,1,2,3; \varphi = \arcsin\frac{3}{\sqrt{13}}$.

12.4. $\sqrt{\frac{1}{2}}(\cos\frac{\frac{\pi}{3}+2k\pi}{4} + i\sin\frac{\frac{\pi}{3}+2k\pi}{4}), k = 0,1,2,3$.

12.5. $\cos\frac{\frac{11\pi}{6}+2k\pi}{5}+i\sin\frac{\frac{11\pi}{6}+2k\pi}{5}),\ k=0,1,2,3,4.$

12.6. $\sqrt[8]{2}(\cos\frac{\varphi+2k\pi}{4}+i\sin\frac{\varphi+2k\pi}{4}),\ k=0,1,2,3;\ \varphi=\arccos\frac{\sqrt{3}+1}{2\sqrt{2}}.$

12.7. $\frac{1}{\sqrt[12]{2}}(\cos\frac{\frac{11\pi}{12}+2k\pi}{6}+i\sin\frac{\frac{11\pi}{12}+2k\pi}{6}),\ k=0,1,2,3,4,5.$

12.8. $\sqrt[16]{74^3}(\cos\frac{3\varphi+2k\pi}{8}+i\sin\frac{3\varphi+2k\pi}{8}),k=0,1,2,3,4,5,6,7;\ \varphi=\arccos\frac{7}{\sqrt{74}}.$

V MODUL. ARIFMETIK VEKTOR FAZO. CHIZIQLI TENGLAMALAR SISTEMASI

13-§. Arifmetik vektor fazo

2. 2.1. $(8+\sqrt{2},0,-9,4)$. 2.2. $(-2,12,-17)$. 2.3. $(\frac{1}{2}\sin\alpha,\frac{1}{2},\frac{1}{2}\cos 3\alpha)$. 2.4. $(-\frac{31}{6},\frac{11}{2},-3)$.

14-§. Matritsa va uning rangi

3. Agar $\lambda=1\wedge\lambda=\frac{1}{4}$ bo‘lsa, rang 2 ga; agar $\lambda=1\vee\lambda=\frac{1}{4}$ bo‘lsa, rang 3ga teng. **4.** $\lambda=0$ da rang 2 ga, $\lambda\neq 0$ da rang 3 ga teng.

15-§. Chiziqli tenglamalar sistemasi

2. 2.1. (1,−1,0). 2.2. (1,2,−1,−2). 2.3. $(\frac{2}{3},-1,\frac{3}{2},0)$. 2.4. Hamjoysiz. **3.** 3.1. har qanday λ uchun CHTS hamjoyli. 3.2. $\lambda\neq -2$ da CHTS hamjoyli. 3.3. $\lambda=-3$ da CHTS hamjoysiz. 3.4. $\lambda=2$ da CHTS hamjoysiz. **4.** 4.1. $x=\frac{1}{4}(-a+b+c+d)$, $y=\frac{1}{4}(a-b+c+d)$, $z=\frac{1}{4}(a+b-c+d)$, $t=\frac{1}{4}(a+b+c-d)$.

4.2. $x=\frac{1}{A}(ap-bq-cr-ds)$, $y=\frac{1}{A}(aq+bp+cs-dr)$, $z=\frac{1}{A}(ar-bs+cp+dp)$, $t=\frac{1}{A}(as+br-cq+dp), A=a^2+b^2+c^2+d^2$.

4.3. $x_k=(-1)^{n+k}\sum_{i=1}^{n}\frac{b_i f_{ik}}{(a_i-a_1)\dots(a_i-a_{i-1})(a_i-a_{i+1})\dots(a_i-a_n)}$, bu yerda

f_{ik}, $a_1,...,a_{k-1},a_{k+1},...,a_n$ elementlarning $n-i$ tadan ko'paytmalari yig'indisi. 4.4. $x_k = \dfrac{\prod\limits_{i\neq k}(b-a_i)}{\prod\limits_{i\neq k}(a_k-a_i)} = \dfrac{f(b)}{(b-a_k)f'(a_k)}$, $f(x)=(x-a_1)(x-a_2)...(x-a_n)$. **5.** 5.1. $x_1 = \frac{x_3-9x_4-2}{11}$, $x_2 = \frac{-5x_3+x_4+10}{11}$. 5.2. $x_3 = 22x_1-33x_2-11, x_4 = -16x_1+24x_2+8$. 5.3. (3,2,1). 5.4. $x_1 = \frac{-6+8x_4}{7}, x_2 = \frac{1-13x_4}{7}, x_3 = \frac{15-6x_4}{7}$. 5.5. $x_3 = 13, x_4 = 19-3x_1-2x_2, x_5 = -34$. 5.6. $x_3 = \frac{4}{3}x_1+\frac{2}{3}x_2$, $x_4 = -\frac{14}{3}x_1-\frac{7}{3}x_2-1$, $x_5 = \frac{4}{3}x_1+\frac{2}{3}x_2+2$. **8.** 8.2. $x_1 = 2x_3+8x_4$, $x_2 = -x_3-2x_4, x_3, x_4 \in R, x_5 = 0$. $\vec{a}_1 = (2,-1,1,0,0)$, $\vec{a}_2 = (8,-2,0,1,0)$. 8.4. $x_1 = x_2 = x_3 = x_4 = 0$; fundamental sistema mavjud emas. 8.6. $x_1 = x_3+x_4+5x_5, x_2 = -2x_3-2x_4-6x_5$, $x_3,x_4,x_5 \in R$; $\vec{a}_1 = (1,-2,1,0,0), \vec{a}_2 = (1,-2,0,1,0), \vec{a}_3 = (5,-6,0,0,1)$. 8.8. Agar $\lambda = 0$ bo'lsa, $x_1,x_2,x_3,x_4 \in R$ va ortonormal sistema $\bar{e}_1,...,\bar{e}_4 \in R^4$ fundamental sistema bo'ladi; $\lambda \neq 0$ bo'lsa, $x_1 = -3x_3-4x_4$, $x_2 = -2x_3-3x_4$, $x_3,x_4 \in R$; $\vec{a}_1 = (-3,-2,1,0)$, $\vec{a}_2 = (-4,-3,0,1)$. 8.10. $\lambda = -1$, $x_1 = x_3 = x_4 = 0$, $x_2 \in R$, $\vec{a} = (0,1,0,0)$; $\lambda = -2$, $x_2 = -2x_1-2x_3-2x_4, x_1, x_3, x_4 \in R$; $\vec{a}_1 = (1,-2,0,0)$, $\vec{a}_2 = (0,-2,1,0)$, $\vec{a}_3 = (0,-2,0,1)$; $\lambda \notin \{-1,-2\}$, $x_1 = x_2 = x_3 = x_4 = 0$, fundamental sistema mavjud emas.

VI MODUL. MATRITSALAR

16-§. Matritsalar va ular ustida amallar

5. 5.1. $\begin{pmatrix}0 & 1 & 2 & -1\end{pmatrix}$. 5.2. $\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$. 5.3. $\begin{pmatrix}\cos(\alpha+\beta) & -\sin(\alpha+\beta)\\ \sin(\alpha+\beta) & \cos(\alpha+\beta)\end{pmatrix}$, va $\begin{pmatrix}\cos(\alpha+\beta) & -\sin(\alpha+\beta)\\ \sin(\alpha+\beta) & \cos(\alpha+\beta)\end{pmatrix}$. 5.4. $\begin{pmatrix}0 & 0\\0 & 0\end{pmatrix}$, $\begin{pmatrix}-4 & 6 & 0 & 2\\ -3 & 2 & -2 & 2\\ 4 & -1 & 4 & -3\\ 1 & 6 & 6 & -2\end{pmatrix}$.

5.5. Ko'paytma mavjud emas. 5.6. $\begin{pmatrix} 2 & -3 & -1 \\ 12 & -18 & -6 \\ -4 & 6 & 2 \end{pmatrix}$. **6.** 6.1. n – juft bo'lsa, $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, n – toq bo'lsa, $\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$. 6.2. $\begin{pmatrix} 2^n & 2^{n-1} \\ 0 & 2^n \end{pmatrix}$.

6.3. $\begin{pmatrix} \cos n\alpha & -\sin n\alpha \\ \sin n\alpha & \cos n\alpha \end{pmatrix}$ 6.4. $\begin{pmatrix} \lambda^n & n\lambda^{n-1} \\ 0 & \lambda^n \end{pmatrix}$. 6.5. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2^n & 0 & 0 \\ 0 & 0 & 3^n & 0 \\ 0 & 0 & 0 & 4^n \end{pmatrix}^n$.

6.6. $(\frac{1}{2}\sin 2\alpha)^n \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. **7.** 7.1. $\begin{pmatrix} a & 0 \\ b & a \end{pmatrix}$. 7.2. $\begin{pmatrix} a & 2b \\ 3b & a+3b \end{pmatrix}$.

7.3. $\begin{pmatrix} a & 3b \\ -5b & a+9b \end{pmatrix}$. 7.4. $\begin{pmatrix} a & b & c \\ 0 & a & b \\ 0 & 0 & a \end{pmatrix}$. 7.5. $\begin{pmatrix} a & b & c \\ 0 & a & b \\ 0 & 0 & a \end{pmatrix}$.

7.6. $\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$. 7.7. $\begin{pmatrix} a & 0 & c \\ 0 & b & 0 \\ c & 0 & a \end{pmatrix}$. 7.8. $\begin{pmatrix} a & b & c & d \\ 0 & a & b & c \\ 0 & 0 & a & b \\ 0 & 0 & 0 & a \end{pmatrix}$.

12. 12.1. $\begin{pmatrix} -2 & 3 \\ 3 & -4 \end{pmatrix}$. 12.2. $\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$. 12.3. $\begin{pmatrix} \frac{3}{2} & \frac{1}{2} & \frac{3}{2} \\ -\frac{3}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 1 & 0 & 1 \end{pmatrix}$.

12.4. $\begin{pmatrix} -8 & 29 & -11 \\ -5 & 18 & -7 \\ 1 & -3 & 1 \end{pmatrix}$. 12.5. $\frac{1}{4}\begin{pmatrix} 2 & 0 & 0 & 0 \\ -3 & 2 & 0 & 0 \\ 25 & -14 & 12 & -16 \\ -19 & 10 & -8 & 12 \end{pmatrix}$. 12.6. $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$.

12.7. $\begin{pmatrix} 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & -1 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}$. 12.8. $\begin{pmatrix} 1 & -a & 0 & 0 & \dots & 0 \\ 0 & 1 & -a & 0 & \dots & 0 \\ 0 & 0 & 1 & -a & \dots & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & \dots & 1 \end{pmatrix}$.

12.9. $\frac{1}{n+1}\begin{pmatrix} n & n-1 & n-2 & n-3 & \dots & 1 \\ n-1 & 2(n-1) & 2(n-2) & 2(n-3) & \dots & 2 \\ n-2 & 2(n-2) & 3(n-2) & 3(n-3) & \dots & 3 \\ n-3 & 2(n-3) & 3(n-3) & 4(n-3) & \dots & 4 \\ . & . & . & . & . & . \\ 1 & 2 & 3 & 4 & \dots & n \end{pmatrix}$.

12.10. $-\frac{1}{s}\begin{pmatrix} \frac{1+a_1 s}{a_1^2} & \frac{1}{a_1 a_2} & \frac{1}{a_1 a_3} & \dots & \frac{1}{a_1 a_n} \\ \frac{1}{a_2 a_1} & \frac{1+a_2 s}{a_2^2} & \frac{1}{a_2 a_3} & \dots & \frac{1}{a_2 a_n} \\ \frac{1}{a_3 a_1} & \frac{1}{a_3 a_2} & \frac{1+a_3 s}{a_3^2} & \dots & \frac{1}{a_3 a_n} \\ . & . & . & . & . \\ \frac{1}{a_n a_1} & \frac{1}{a_n a_2} & \frac{1}{a_n a_3} & \dots & \frac{1+a_n s}{a_n^2} \end{pmatrix}$, $s = \frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n}$.

14. 14.1. $\begin{pmatrix} -3 & -6 \\ 2 & 4 \end{pmatrix}$. 14.2. $\begin{pmatrix} -1 & -1 \\ 2 & 3 \end{pmatrix}$. 14.3. $\begin{pmatrix} \frac{3}{2} & \frac{1}{2} & \frac{3}{2} \\ -\frac{3}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 1 & 0 & 1 \end{pmatrix}$.

14.4. $\begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix}$. 14.5. $\frac{1}{17}\begin{pmatrix} 2 & 5 \\ 9 & 14 \end{pmatrix}$. 14.6. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. 14.7. $\begin{pmatrix} 6 & 4 & 5 \\ 2 & 1 & 2 \\ 3 & 3 & 3 \end{pmatrix}$.

14.8. $\begin{pmatrix} 35 & 32 & 49 \\ 15 & 14 & 22 \\ 31 & 29 & 48 \end{pmatrix}$. 14.9. $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$. 14.10. $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 2 \end{pmatrix}$.

14.11. $\begin{pmatrix}1&1&1\\1&2&3\\2&3&1\end{pmatrix}$. 14.12. $\frac{1}{3}\begin{pmatrix}-101&55&-176&217\\22&-11&40&-47\\8&-4&14&-19\\-3&3&-6&6\end{pmatrix}$. **15.** 15.1. $(1,-2,3)$. 15.2. $(15,\frac{59}{2},-\frac{22}{5})$. 15.3. $(1,2,2,0)$. 15.4. $(1,2,3,4)$. 15.5. $(2,0,2,2)$. 15.6. $(2,-\frac{37}{2},2,-\frac{9}{2})$. 15.7. $(-1,0,2,-1)$. 15.8. $(0,0,0,0)$.

16. 16.1. $X=\begin{pmatrix}2&3\\0&2\end{pmatrix}, Y=\begin{pmatrix}-1&-2\\0&-1\end{pmatrix}$. 16.2. $X\in R^{2\times2}, Y=2X+\begin{pmatrix}0&-1\\1&0\end{pmatrix}$.

VII MODUL. DETERMINANTLAR

17-§. O‘rniga qo‘yishlar

1. 1.1. $\begin{pmatrix}1&2&3&4&5\\2&1&3&4&5\end{pmatrix}$. 1.2. $\begin{pmatrix}1&2&3&4&5&6\\6&5&3&2&1&4\end{pmatrix}$.

2. 2.1. $(1\ \ 5\ \ 3)(2\ \ 4\ \ 7)$. 2.5. $(1\ \ 2)(3\ \ 4)\ldots(2n-1\ \ 2n)$. 2.6. $(1\ \ n+1)(2\ \ n+2)\ldots(n\ \ 2n)$. **4.** 4.1. $\alpha=2, \beta=4$. 4.2. $\alpha=10, \beta=6$. **5.** $(10\ 9\ 8\ 7\ 6\ 5\ 4\ 3\ 2\ 1)$, $C_{10}^2=45$ ta inversiya. **8.** 8.1. $\frac{3n(n-1)}{2}$. 8.2. $\frac{3n(n+1)}{2}$. 8.3. $\frac{n(3n+1)}{2}$. 8.4. $\frac{n(3n-1)}{2}$.

18-§. Determinantlar

1. 1.1. 6. 1.2. −1. 1.3. 2. 1.4. 0. 1.5. $-\sin\alpha$. 1.6. 1. 1.7. 1. 1.8. 0. 1.9. $3abc-a^3-b^3-c^3$. 1.10. $(ab+bc+ca)x+abc$. 1.11. 1. 1.12. $-\frac{3}{2}-\frac{3i\sqrt{3}}{2}$. 1.13. $1+\alpha^2+\beta^2+\gamma^2$. 1.14. Birinchi qatorga 2- va 3-qatorlarni qo‘shib Viyet formulalaridan foydalaning. **4.** 4.1. $n!$. 4.2. $(-1)^{n-1}n!$. 4.3. $x_1x_2\ldots x_n(1+\frac{1}{x_1}+\ldots+\frac{1}{x_n})$. 4.4. $x_1(x_2-a_{12})(x_3-a_{23})\ldots(x_n-a_{n-1,n})$. 4.5. $(x-1)(x-2)\ldots(x-n+1)$. 4.6. $(-1)^n(a-1)(a-2)\ldots(a-n)$. 4.7. $n+1$. 4.8. $2^{n+1}-1$. 4.9. $x^n+(a_1+a_2+\ldots+a_n)x^{n-1}$. 4.10. $\prod_{k=1}^{n}(1-a_{kk}x)$. **5.** 5.1. $(1,3,2)$. 5.2. $(2,1,3)$. 5.3. $(\frac{5}{3},0,-\frac{2}{3})$. 5.4. $(\frac{1}{3},0,-\frac{1}{6})$. 5.5. $(-1,-1,0,1)$.

5.6. $(-\frac{3}{5},\frac{6}{5},3,2)$. 5.7. $(-3,0,-\frac{1}{2},\frac{2}{3})$. 5.8. $(-1,2,0,1)$. 5.9. $(1,2,3,4)$. 5.10. $(1,0,-1,0)$. **8.** 8.1. 2. 8.2. 2. 8.3. 3. 8.4. 3. 8.5.3. 8.6. 2. 8.7. 5. 8.8. n, agar n – toq bo‘lsa, n – 1, agar n – juft bo‘lsa. **9.** 9.1. $\begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$. 9.2. $\begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix}$. 9.3. $\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$. 9.4. $\frac{1}{18-5i}\begin{pmatrix} 5+4i & -3-i \\ -1+i & 2-3i \end{pmatrix}$. 9.5. $\begin{pmatrix} 1 & -1 & 1 \\ -38 & 41 & -34 \\ 27 & -29 & 24 \end{pmatrix}$. 9.6. $-\frac{1}{34}\begin{pmatrix} -42 & -15 & 58 \\ 24 & 11 & -38 \\ 22 & 3 & -32 \end{pmatrix}$.

9.7. $\frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$. 9.8. $-\frac{1}{4}\begin{pmatrix} -1 & -1 & -1 & -1 \\ -1 & -i & 1 & -i \\ -1 & 1 & -1 & 1 \\ -1 & -i & 1 & i \end{pmatrix}$.

VIII MODUL. VEKTOR FAZOLAR

19-§. Vektor fazo. Fazoostilar kesishmasi, yigindisi

1. 1.1. $\dim V = 1$, bazislardan biri $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$. 1.2. $\dim V = 4$, bazislardan biri $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$. 1.3. $\dim V = 9$, bazislardan biri $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$,

$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$,

$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. 1.7. $\dim V = 6$, bazislardan biri $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$,

$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$.

3. 3.1. $\dim V = n-1$, bazis $\vec{a}_1 = (1,0,0,\ldots,0)$, ..., $\vec{a}_{n-1} = (0,0,0,\ldots,1,0)$. 3.2. $\dim V = n-1$, bazislardan biri $\vec{a}_1 = (1,-1,0,\ldots,0)$, $\vec{a}_2 = (0,1,-1,\ldots,0)$, ..., $\vec{a}_{n-1} = (0,0,0,\ldots,1,-1)$. 3.3. n – juft son bo'lsa, $\dim V = \frac{n}{2}$, bazislardan biri $\vec{a}_1 = (1,0,0,\ldots,0)$, $\vec{a}_2 = (0,0,1,\ldots,0)$, ..., $\vec{a}_{\frac{n}{2}} = (0,0,0,\ldots,1,0)$; n – toq son bo'lsa, $\dim V = \frac{n+1}{2}$, bazislardan biri $\vec{a}_1 = (1,0,0,\ldots,0)$, $\vec{a}_2 = (0,0,1,\ldots,0)$ ..., $\vec{a}_{\frac{n+1}{2}} = (0,0,0,\ldots,0,1)$.

4. 4.1. Tashkil etmaydi. 4.2. Tashkil etmaydi. 4.3. To'g'ri chiziq koordinatalar boshidan o'tgan bo'lsa tashkil etadi, aks holda tashkil etmaydi. 4.4. Tashkil etadi. 4.5. Tashkil etmaydi. 4.6. Tashkil etadi. **6.** 6.1. O'lchovi 3, bazislardan biri $\vec{a}_1, \vec{a}_2, \vec{a}_4$. 6.2. O'lchovi 3, bazislardan biri $\vec{a}_1, \vec{a}_2, \vec{a}_4$. 6.3. O'lchovi 3, bazislardan biri $\vec{a}_1, \vec{a}_2, \vec{a}_5$. 6.4. O'lchovi 3, sistema o'ziga bazis.

20-§. Skalar ko'paytmali vektor fazolar. Evklid vektor fazolar. Vektor fazolar izomorfizmi

5. 5.2. Vektorlar sistemasi chiziqli bog'liq bo'lganligi uchun uni ortogonallab bo'lmaydi. 5.3. $\vec{b}_1 = \vec{a}_1, \vec{b}_2 = \vec{a}_3, \vec{b}_3 = (1,-3,3,4)$. 5.4. $\vec{b}_1 = \vec{a}_1, \vec{b}_2 = (-2,1,2), \vec{b}_3 = (8,32,-8)$. 5.5. $\vec{b}_1 = \vec{a}_1, \vec{b}_2 = (0,1,0,0)$, $\vec{b}_3 = (-1,0,0,1), \vec{b}_4 = (-1,0,2,-1)$. 5.6. $\vec{b}_1 = \vec{a}_1, \vec{b}_2 = (2,1,-2,-2), \vec{b}_3 = (2,6,4,1)$. **11.** 11.1. $\vec{b}_1 = (2,1,0), \vec{b}_2 = (-3,0,1)$. 11.2. $\varnothing$. 11.3. $\vec{b}_1 = (-1,1,-1,0)$, $\vec{b}_2 = (2,0,5,1)$. 11.4. $\vec{b}_1 = (-1,1,1,0), \vec{b}_2 = (1,2,0,1)$. 11.5. $\vec{b}_1 = (1,-3,-2,0)$, $\vec{b}_2 = (0,-5,-3,1)$. **12.** 12.1. $\vec{b}_1 = (1,0,1,0), \vec{b}_2 = (0,1,0,1)$. 12.2. $\vec{b}_1 = (-1,1,0), \vec{b}_2 = (-1,0,1)$.

IX MODUL. CHIZIQLI AKSLANTIRISHLAR

21-§. Chiziqli akslantirish. Chiziqli operator yadrosi va obrazi. Chiziqli operator matritsasi

5. 5.1. $\begin{pmatrix} 2 & -11 & 6 \\ 1 & -7 & 4 \\ 2 & -1 & 0 \end{pmatrix}$. 5.2. $\frac{1}{3}\begin{pmatrix} -6 & 11 & 5 \\ -12 & 13 & 10 \\ 6 & -5 & -5 \end{pmatrix}$. **9.** 9.1. $r = 1$,

$d = 1$. 9.2. $r=2, d=0$. 9.3. $r=1, d=2$. 9.4. $r=2, d=1$. 9.5. $r=3, d=0$. 9.6. $r=1, d=3$. 9.7. $r=2, d=2$. 9.8. $r=3, d=1$.

22-§. Chiziqli operatorlar ustida amallar. Chiziqli algebralar. Teskari operator. Xos vektorlar va xos qiymatlar

1. 1.1. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. 1.2. $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$. **2.** $\begin{pmatrix} 2 & -2 \\ -10 & 10 \end{pmatrix}$. **6.** 6.1. $\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$.

6.2. $\begin{pmatrix} 2 & 0 & 2 \\ -8 & 1 & -5 \\ -1 & 0 & -1 \end{pmatrix}$. 6.3. $\begin{pmatrix} -16 & -5 & 7 & -1 \\ 14 & 5 & -6 & 1 \\ 11 & 3 & -5 & 1 \\ 3 & 1 & -1 & 0 \end{pmatrix}$.

6.4. $\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. **7.** 7.1. $\lambda_1 = 0, \vec{a} = c_1(1,-2); \lambda_2 = 4, \vec{a} = c_2(1,2)$, $c_1, c_2 \neq 0$. 7.2. $\lambda_1 = \lambda_2 = 2, \vec{a} = c_1(1,0), c_1 \neq 0$. 7.3. $\lambda_1 = \lambda_2 = \lambda_3 = 1$, $\vec{a} = c(1,3,-3), c \neq 0$. 7.4. $\lambda = 2, \vec{a} = c(1,0,0), c \neq 0$.

X MODUL. CHIZIQLI TENGSIZLIKLAR SISTEMASI

23-§. Chiziqli tengsizliklar sistemasi. Qavariq konus

3. 3.1. Ikkalasi ham natija emas. 3.2. 2) sistema 1) ning natijasi. 3.3. 2) sistema 1) ning natijasi. 3.4. 1) sistema 2) ning natijasi. 3.5. Teng kuchli. 3.6. 2) sistema 1) ning natijasi. **13.** 13.1. $f_{\min} = 0, x_1 = 3, x_2 = 3$. 13.2. $f_{\min} = -3\frac{1}{2}, x_1 = \frac{3}{2}, x_2 = \frac{5}{4}$. 13.3. $f_{\min} = -6, x_1 = 0, x_2 = 6$. 13.4. $f_{\min} = 3, x_1 = 0, x_2 = 0$. **14.** 14.1. $f_{\max} = 14\frac{1}{2}, x_1 = 1\frac{1}{4}, x_2 = 3$. 14.2. $f_{\max} = 17, x_1 = 5, x_2 = 1$. 14.3. $f_{\max} = 6, x_1 = 0, x_2 = \frac{3}{2}$. 14.4. $f_{\max} = 9\frac{1}{7}, x_1 = 3\frac{6}{7}, x_2 = 1\frac{3}{7}$.

XI MODUL. BUTUN SONLAR HALQASIDA BO'LINISH MUNOSABATI

24-§. Tub va murakkab sonlar. EKUB. EKUK

7. 7.1. 12, 168. 7.2. 9, 217. 7.3. 24, 1170. 7.4. 8, 624. 7.5. 30, 2418. 7.6. 8, 1440. 7.7. 12, 1960. 7.8. 24, 2808. 7.9. 16, 2340.

7.10. 30, 3844. 7.11. 8, 3096. 7.12. 24, 8736. **8.** 8.1. 88. 8.2. 357. 8.3. 1. 8.4. 113. 8.5. 3109. 8.6. 3911. 8.7. 382. 8.8. 2011. 8.12. 490. **9.** 9.1. 30,120; 60, 90; 90, 60; 120, 30. 9.2. 20,420; 60,140; 420,20; 140,60. 9.3. 552, 115; 435, 232; 232, 435; 115, 552. 9.4. 495, 315; 315, 495.

25-§. Chekli zanjir kasrlar. Munosib kasrlar

2. 2.1. [1,9]. 2.2. [0;2,15]. 2.3. [−2;1,30,2]. 2.4. [1;2,3,4]. 2.5. [0;1.4,3,2]. 2.6. [−3;1,1,2]. 2.7. [2;2,3,1,5]. 2.8. [0;1,2,5,2]. 2.9. [1;4.2.1.7]. 2.10. [1;1,2,1,2,1,2]. 2.11. [0;1,2,3,4,5]. 2.12. [0;1.1.38]. **3.** 3.1. [3;(3,6)]. 3.2. [3;(2,6)]. 3.3. [3;(1,1,1,1,6)]. 3.4. [5;(3,2,3,10)]. 3.5. [5;(2,10)]. 3.6. [7;(1,2,7,2,1,14)]. 3.7. [(2)]. 3.8. [(1,2)]. 3.9. [(2,2,2,1,12,1)]. 3.10. [2;(1)]. 3.11. [1;(1,1,4,1)]. 3.12. [2;(18,2)]. **4.** 4.1. $\frac{20}{31}$. 4.2. $\frac{131}{583}$. 4.3. $\frac{7}{23}$. 4.4. $\frac{97}{113}$. 4.5. $\frac{17}{83}$. 4.6. $\frac{359}{113}$. 4.7. $\frac{9+2\sqrt{39}}{5}$. 4.8. $1+\sqrt{3}$. 4.9. $\sqrt{11}$. 4.10. $\sqrt{3}$. 4.11. $5-\sqrt{15}$. 4.12. $\frac{245-\sqrt{85}}{74}$. **5.** 5.1. $x=-8360-117t$, $y=2717+38t, t\in Z$. 5.3. $x=-2+4t, y=-4+7t, t\in Z$. 5.5. $x=-125-114t, y=45+41t, t\in Z$. 5.7. $x=1-9t, y=39+49t$, $t\in Z$. 5.9. $x=9+31t, y=2-12t, t\in Z$. 5.11. $x=75+23t$, $y=-120-37t, t\in Z$. 5.13. $x=4+17t, y=-11-53t, t\in Z$. 5.15. $x=-15-39t, y=-25-64t, t\in Z$. 5.17. $x=-15+37t$, $y=18-43t, t\in Z$. 5.19. $x=1270-559t, y=-2020-571t, t\in Z$.

26-§. Sistematik sonlar va ular ustida amallar

1. 1.1. 11000_2. 1.2. 10001111_2. 1.3. 101011_2. 1.4. 111_2. 1.5. 2255025_7. 1.6. $3(10)94913_{12}$. 1.7. 30413_7. 1.8. 190000_{12}. 1.9. 56_7 va qoldiq 202_7. 1.10. $(10)94_{12}$ va qoldiq 87_{12}. 1.11. $2,4_8$. **2**. 2.1. 100001_2. 2.2. 11, 11101_2. 2.3. 100000, 11001_2. 2.4. $0,1337_8$. 2.5. 11, 14_8. **3**. 3.1. 1_8. 3.2. 1_6. 3.3. 1_8. 3.4. 1_5. 3.5. 1_5. 3.6. 1_8. 3.7. 1_3. 3.8. 1_7. 3.9. 1_7. 3.10. 1_8. 3.11. 1_4. 3.12. 1_8. 3.13. 1_6. **4**. 4.1. 39. 4.2. 205. 4.3. 229. 4.4. 2617. 4.5. 704. 4.6. 8387. 4.7. 1668. 4.8. 1523. 4.9. 6871. 4.10. 5669. 4.11. 42923. **5**. 5.1. 0,875. 5.2. 0,75. 5.3. 25,9365. 5.4. 287,388671875. 5.5. 0,044921875. **6**. 6.1. $4(10)2(11)_{12}$. 6.2. 230578_9. 6.3. 11202102120100_3. 6.4. 367341_8. 6.5. $100010001101101111101100_2$. 6.6. 2121311_5.

6.7. 4126_8. **7.** 7.1. $1111111010 = 2210122_3 = 31132_5$. 7.2. $1010111000010_2 = 1021101_3 = 42121_5$. 7.3. 2061_7. 7.4. 1653212_7. 7.5. 55173_8. 7.6. 42167_8. **8.** 8.1. 4. 8.2. 5. 8.3. 9. 8.4. 9. 8.5. 5. 8.6. 9. 8.7. 7. 8.8. 6. 8.9. 5. **9.** 9.1. 5. 9.2. 8. 9.3. 6. 9.4. 7. 9.5. 7. 9.6. 7. 9.7. 7. 9.8. 9. 9.9. 6. 9.10. $g, g \geq 2$.

XII MODUL. TAQQOSLAMALAR

27-§. Butun sonlar halqasida taqqoslamalar. Eyler va Ferma teoremalari

2. 2.1. 1. 2.2. 1. 2.3. 4. 2.4. 0. 2.5. 1. 2.6. 1. 2.7. 0. 2.8. 0. 2.9. 1. 2.10. 12. 2.11. 3. 2.12. 11. **4**. 4.1. 88. 4.2. 67. 4.3. 24. 4.4. 9. 4.5. 27. 4.6. 36. 4.7. $9^{10} \equiv 1 \pmod{100}$, $9^{10q+r} \equiv 9^r \pmod{100}$,

$9^9 \equiv 9 \pmod{10}$ $9^{9^9} \equiv 9^9 \equiv \varepsilon 9 \pmod{100}$.

4.8. $7^4 = 2401 \equiv 1 \pmod{100}$, $7^{100} \equiv 1 \pmod{100}$,

$7^{9^{99}} \equiv 7^{100q+89} \equiv 7^{89} \pmod{100}$, $7^{88} \equiv 1 \pmod{100}$,

$7^{89} \equiv 7 \pmod{100}$. **8**. 8.1. 7. 8.2. 1. 8.3. 22. 8.4. 5. 8.5. 32. 8.6. 29. 8.7. 19. 8.8. 1 . 8.9. 1. 8.10. 1. **9**. 9.1. 2. 9.2. 1, 4, 1, 4. 9.3. 13. 9.4. 7. 9.5. 14. 9.6. 14. 9.7. 65. 9.8. 49. **10.** 10.1. 21. 10.2. 22. 10.3. 64. 10.4. 21. 10.5. 375. 10.6. 4. 10.7. 24. 10.8. 1. 10.9. 23. 10.10. 8. 10.11. 8. 10.12. 60. 10.13. 147. 10.14. 48. 10.15. 127. 10.16. 5. **11.** 11.1. 2. 11.2. 6. 11.3. 1. 11.4. 5. 11.5. 2. 11.6. 0. 11.7. 2. 11.8. 2. 11.9. 70. 11.10. 7. 11.11. 19. 11.12. 30. 11.13. 20. 11.14. 1. 11.15. 12. 11.16. 10. 11.17. 6. 11.18. 70. **12**. 12.1. 01. 12.2. 67. 12.3. 31. 12.4. 97. 12.5. 01. 12.6. 61. 12.7. 61. 12.8. 97. 12.9. 76. 12.10. 92. 12.11. 84.

28-§. Birinchi darajali va tub modul bo'yicha yuqori darajali taqqoslamalar

2. 2.1. $x \equiv 2 \pmod 3$. 2.2. $\varnothing$. 2.3. $x \equiv 2 \pmod 5$. 2.4. $x \equiv 5 \pmod 7$. 2.5. $x \equiv 4{,}9 \pmod{10}$. 2.6. $x \equiv 3 \pmod 7$. 2.7. $x \equiv 8 \pmod{11}$. 2.8. $x \equiv 2{,}5{,}8{,}11 \pmod{12}$. **3.** 3.1. $x \equiv 3 \pmod{13}$. 3.2. $\varnothing$. 3.3. $x \equiv 3{,}10 \pmod{14}$. 3.4. $x \equiv 2 \pmod{27}$. 3.5. $x \equiv 6 \pmod{23}$. 3.6. $x \equiv 3 \pmod{37}$. 3.7. $x \equiv 11 \pmod{41}$. 3.8. $x \equiv 38 \pmod{51}$. **4**. 4.1. $x \equiv 4 \pmod{13}$. 4.2. $x \equiv 3 \pmod{12}$. 4.3. $x \equiv 10 \pmod{12}$. 4.4. $x \equiv 14 \pmod{19}$. 4.5. $x \equiv 13 \pmod{34}$.

4.6. $\varnothing$. 4.7. $x \equiv 3 \pmod{22}$. 4.8. $x \equiv 1,14,27 \pmod{39}$. **5**. 5.1. $x \equiv 9 \pmod{98}$. 5.2. $x \equiv 28 \pmod{119}$. 5.3. $\varnothing$. 5.4. $x \equiv 11 \pmod{169}$. 5.5. $x \equiv 73 \pmod{117}$. 5.6. $x \equiv 29 \pmod{201}$. 5.7. $x \equiv 29, 138, 247 \pmod{327}$. 5.8. $x \equiv 17, 96, 175, 254, 333 \pmod{395}$. 5.9. $x \equiv 153,461,769 \pmod{924}$. 5.10. $x \equiv 1630 \pmod{2413}$. 5.11. $x \equiv 200,751,1302,1853, 2404 \pmod{2755}$. 5.12. $\varnothing$. **6.** 6.1. $x \equiv 3 \pmod{23}$. 6.2. $x \equiv 11 \pmod{24}$. 6.3. $x \equiv 11 \pmod{24}$. 6.4. $x \equiv 23 \pmod{30}$. 6.5. $\varnothing$. 6.6. $x \equiv 2,7,12,17,22,27 \pmod{30}$. 6.7. $x \equiv 2 \pmod{41}$. 6.8. $x \equiv 21 \pmod{50}$. **7.** 7.1. $x \equiv a+b \pmod{ab}$. 7.2. $x \equiv (a-b)^{\varphi(ab)-1} \pmod{ab}$. 7.3. $x \equiv (a-b)(a+b)^{\varphi(ab)-1} \pmod{ab}$. 7.4. $x \equiv (a-b) \pmod{ab}$. 7.5. $x \equiv \frac{1+p}{2} \pmod{p}$. 7.6. $x \equiv m-1 \pmod{m}$. 7.7. $x \equiv a \pmod{m}$. 7.8. $x \equiv a^{p-2} \pmod{p}$. **8.** 8.1. $x = 2+3t,\ y = -2t,\ t \in Z$. 8.2. $x = 2+3t,\ y = 2+4t, t \in Z$. 8.3. $x = 3+4t,\ y = 1-3t,\ t \in Z$. 8.4. $x = 3+4t,\ y = -3-5t,\ t \in Z$. 8.5. $x = 7+8t,\ y = -2-3t, t \in Z$. 8.6. $x = -3+13t,\ y = 4-17t,\ t \in Z$. 8.7. $x = -7+15t,\ y = 12-23t,\ t \in Z$. 8.8. $x = -1+16t,\ y = -8+17t,\ t \in Z$. 8.9. $x = 1+4t,\ y = 2+13t,\ t \in Z$. 8.10. $\varnothing$. 8.11. $x = 20+21t,\ y = 23+25t,\ t \in Z$. 8.12. $x = 47+105t,\ y = 21+47t,\ t \in Z$. 8.13. $x = 94+111t, y = 39+47t, t \in Z$. **9.** 9.1. $x \equiv 18 \pmod{35}$. 9.2. $\varnothing$. 9.3. $x \equiv 12 \pmod{35}$. 9.4. $x \equiv 105 \pmod{225}$. 9.5. $x \equiv 170b_1 + 52b_2 \pmod{221}$. 9.6. $x \equiv 100 \pmod{143}$, $y = 111 \pmod{143}$. 9.7. $x \equiv 1 \pmod{5}$, $y \equiv 2 \pmod{5}$. 9.8. $\varnothing$. **10.** 10.1. $x \equiv 91 \pmod{120}$. 10.2. $x \equiv 59 \pmod{160}$. 10.3. $x \equiv 33 \pmod{90}$. 10.4. $x \equiv 86 \pmod{315}$. 10.5. $x \equiv 256 \pmod{1547}$. 10.6. $\varnothing$. 10.7. $x \equiv 47 \pmod{420}$. 10.8. $x \equiv 49 \pmod{420}$. 10.9. $x \equiv 125 \pmod{1496}$. 10.10. $x \equiv 11151b_1 + 11800b_2 + 16875b_3 \pmod{39825}$. 10.11. $x \equiv 8479 \pmod{15015}$. **11.** 11.1. $x \equiv 17 \pmod{90}$. 11.2. $x \equiv 4 \pmod{105}$. 11.3. $\varnothing$. 11.4. $x \equiv 299 \pmod{385}$.

11.5. $\varnothing$. 11.6. $x \equiv 9573 \pmod{13923}$.
11.7. $x \equiv 85056 \pmod{130169}$. **12.** 12.1. $a \equiv 5 \pmod 6$. 12.2. $a \equiv 0 \pmod 4$. 12.3. $a \equiv 1 \pmod 7$. 12.4. $a \equiv 1 \pmod 6$. **13.** 13.1. $x^3 + 2x^2 + 3 \equiv 0 \pmod{11}$. 13.2. $x^3 + 18x^2 + 4x - 17 \equiv 0 \pmod{59}$.
13.3. $x^6 + 4x^5 + 22x^4 + 76x^3 + 70x^2 + 52x + 39 \equiv 0 \pmod{101}$.
13.4. $x^n + a_1 x^{n-1} h + ... + a_n h \equiv 0 \pmod m$, bu yerda $a_0 h = 1 \pmod m$. **14.** 14.1. $2x^3 + 3 \equiv 0 \pmod 5$. 14.2. $3x^4 + 2x^3 + 3x^2 + 2x \equiv 0 \pmod 5$. 14.3. $3x^2 + x - 2 \equiv 0 \pmod 7$.
14.4. $5x^6 + x^5 + 5x^4 + 3x^2 + 3x + 4 \equiv 0 \pmod 7$.
14.5. $6x^8 + 7x^5 + 3x^4 + 3x^3 + x^2 + 3 \equiv 0 \pmod{11}$. **15.** 15.1. $x \equiv 2 \pmod 3$. 15.2. $\varnothing$. 15.3. $x \equiv 12 \pmod 3$.
15.4. $x \equiv 1 \pmod 3$. 15.5. $x \equiv 1 \pmod 3$. 15.6. $x \equiv 4 \pmod 5$, 15.7. $x \equiv 3 \pmod 5$. 15.8. $x \equiv 2 \pmod 5$. 15.9. $\varnothing$. 15.10. $x \equiv 1 \pmod 5$. 15.11. $x \equiv 1, 2 \pmod 5$. **16.** 16.1. $x \equiv 2 \pmod 7$. 16.2. $x \equiv 4 \pmod 7$. 16.3. $x \equiv 1, 2, 3, 4, 5, 6 \pmod 7$. 16.4. $x \equiv 4, 5 \pmod 7$. 16.5. $x \equiv 4 \pmod{11}$. 16.6. $\varnothing$. 16.7. $x \equiv 7, 9 \pmod{11}$. 16.8. $x \equiv 12 \pmod{13}$. 16.9. $x \equiv 7, 13 \pmod{23}$.
17. 17.1. $(x-3)(x-4)^2 \equiv 0 \pmod 5$. 17.2. $(x-1)(x-2)^2 \equiv 0 \pmod 5$.
17.3. $(x-1)(x-2)(x-3)(x-4) \equiv 0 \pmod 5$.
17.4. $3(x-1)(x-2)(x-3) \equiv 0 \pmod 5$.
17.5. $(x-1)(x-2)(x-3)(x-6) \equiv 0 \pmod 7$.
17.6. $5(x-1)(x-3)(x-5) \equiv 0 \pmod 7$.
17.7. $6(x-1)(x-2)(x-9) \equiv 0 \pmod{11}$.
17.8. $(x-2)(x-3)(x-9) \equiv 0 \pmod{17}$.
17.9. $(x-1)(x-13)(x-21) \equiv 0 \pmod{23}$.
17.10. $(x-2)^2(x-11)(x-28) \equiv 0 \pmod{29}$.
17.11. $(x-17)(x-28)(x-30) \equiv 0 \pmod{31}$.

29-§. Tub modul bo'yicha boshlang'ich ildizlar va indekslar

1. 1.1. −1. 1.2. 1. 1.3. 1. 1.4. 1. 1.5. −1. 1.6. −1. 1.7. −1. 1.8. −1. 1.9. 1. **2.** 2.1. −1. 2.2. 1. 2.3. −1. 2.4. −1. 2.5. −1. 2.6. 1. 2.7. −1. 2.8. 1. 2.9. 1. **3.** 3.1. 0. 3.2. 2. 3.3. 0. 3.4. 0. 3.5. 0.

3.6. 2. 3.7. 0. 3.8. 0. 3.9. 0. 3.10. 0. **5**. 5.1. 4.5.2. 2. 5.3. 2. 5.4. 6. 5.5. 2. 5.6. 4. 5.7. 8. 5.8. 4. 5.9. 10. 5.10. 6. 5.11. 18. 5.12. 18. **6**. 6.1. 12, 3 va 2. 6.2. 8, 8 va 4. 6.3. 10, 10, 2 va 5. 6.4. 6, 2 va 12. 6.5. 5, 10, 2 va 10. **7**. 7.1. 2, 6, 7, 8. 7.2. 2, 6, 7, 11. 7.3. $\varnothing$. 7.4. 2, 3, 10, 13, 14, 15. 7.5. 3, 5, 10, 12, 17, 19, 24, 26, 38, 40, 45, 47. 7.6. 2, 5, 11, 14, 20, 23, 29, 32, 38, 41, 47, 50, 56, 59, 65, 68, 74, 77. **8**. 8.1. 2, 3. 8.2. 2, 5. 8.3. 6, 2. 8.4. 8, 3. 8.5. 12,2. **9**. 9.1. 3.9.2. 3. 9.3. 5. 9.4. 6. 9.5. 2. 9.6. 27. 9.7. 5. 9.8. 7. 9.9. 7. 9.10. 3. 9.11. 3. 9.12. 2.**12.** 12.1. $x \equiv 13 \pmod{17}$. 12.2. $x \equiv 8 \pmod{27}$. 12.3. $x \equiv 31 \pmod{37}$. 12.4. $x \equiv 30 \pmod{73}$. 12.5. $x \equiv 32 \pmod{79}$. 12.6. $x \equiv 74 \pmod{79}$. 12.7. $x \equiv 44 \pmod{83}$. 12.8. $x \equiv 51 \pmod{97}$. 12.9. $x \equiv 30 \pmod{221}$. **13.** 13.1. $x \equiv 7, 10 \pmod{17}$. 13.2. $x \equiv 8, 19 \pmod{27}$. 13.3. $x \equiv 10, 43 \pmod{53}$. 13.4. $x \equiv 27, 34 \pmod{61}$. 13.5. $x \equiv 27, 40 \pmod{67}$. 13.6. $x \equiv 21, 46 \pmod{67}$. 13.7. $x \equiv 14, 57 \pmod{71}$. 13.8. $x \equiv 17, 66 \pmod{83}$. 13.9. $x \equiv 2, 7 \pmod{11}$. 13.10. $x \equiv 5, 20 \pmod{43}$. 13.11. $x \equiv 3, 31 \pmod{47}$. 13.12. $x \equiv 1634, 1847 \pmod{59^2}$. 13.13. $x \equiv 253, 4076 \pmod{73^2}$. **14**. 14.1. 3. 14.2. 4 14.3. 0. 14.4. 1. 14.5. 0. 14.6. 10. 14.7. 0. 14.8. 7. 14.9. 3. 14.10. 1. 14.11. 0. **15**. 15.1. $x \equiv 4, 33 \pmod{37}$. 15.2. $x \equiv 17 \pmod{41}$. 15.3. $\varnothing$. 15.4. $x \equiv 2, 18, 23, 39 \pmod{41}$. 15.5. $x \equiv 7 \pmod{43}$. 15.6. $\varnothing$. 15.7. $x \equiv 17 \pmod{67}$. 15.8. $x \equiv 8, 28, 31, 36, 39, 59 \pmod{67}$. 15.9. $x \equiv 30, 53 \pmod{83}$. 15.10. $\varnothing$. **16**. 16.1. $x \equiv 3, 5, 6 \pmod{7}$. 16.2. $x \equiv 2, 3, 10, 11 \pmod{13}$. 16.3. $x \equiv 10, 13 \pmod{23}$. 16.4. $\varnothing$. 16.5. $x \equiv 11, 27, 36 \pmod{37}$. 16.6. $x \equiv 25, 30, 31, 36 \pmod{61}$. 16.7. $x \equiv 17 \pmod{73}$. 16.8. $x \equiv 12, 23, 35, 38, 50, 61 \pmod{73}$. 16.9. $x \equiv 17, 63, 66 \pmod{73}$. 16.10. $x \equiv 3, 24, 46 \pmod{73}$. 16.11. $x \equiv 6, 14, 20, 59, 65, 73 \pmod{79}$.

XIII MODUL. KO‘PHADLAR

30-§. Bir o‘zgaruvchili ko‘phadlar

1. $f_1(x) = f_3(x)$; $f_2(x) = f_4(x)$. **2**. 2.1. $a = -5, b = -1, c = 6$.

2.2. $a = 2, b = 5, c = 7$. **3.** 3.1. a) $a = 6$, $g_1(x) = x^2 + 3x + 1$, $g_2(x) = -x^2 - 3x - 1$. 3.2. $a = 3, g_1(x) = \bar{2}x + \bar{2}, g_2(x) = \bar{3}x + \bar{3}$; $a = 2, g_1(x) = \bar{2}x + \bar{3}, g_2(x) = \bar{3}x + \bar{2}$; 3.3. $a = 4, g_1(x) = 3x^2 - 2x - 2$, $g_2(x) = -3x^2 + 2x - 2$. **4.** $a = -8, b = 18, g_1(x) = x^2 - 4x + 1$, $g_2(x) = -x^2 + 4x - 1$; $a = 8, b = 14, g_1(x) = x^2 + 4x - 1$, $g_2(x) = -x^2 - 4x + 1$. **5.** $a = 3, b = -7, c = 4$. **7**. 7.1. $Z[x]$ da $f(x) \vdots g(x), Q[x]$ da $f(x) \vdots g(x)$. 7.2. Bo'linadi. 7.3. Bo'linmaydi. **8.** 8.1. $b = -1 - a$, $a = c$. 8.2. $b = 1$, $c = 0$. 8.3. Agar $a = 0$ bo'lsa, u holda $b = c + 1$ va $c \in z$; agar $a \in z \setminus \{0\}$ bo'lsa, u holda $b = 2 - a^2$ va $c = 1$. **9**. 9.1. $r = 1 - i$. 9.2. $r = 7$. 9.3. $r(x) = x + 2$. 9.4. $r(x) = (2 + i)x + (1 - i)$. 9.5. $r(x) = -7x + 11$. **10**. 10.1. $f(x) = g(x)(x^3 - 3x + 5) + 2x - 3$.
10.2. $f(x) = g(x)(3 + 2i)x + (2 - 7i)x + (-2 + i)$.
10.3. $f(x) = g(x)(\bar{2}x^2 + \bar{3}x) + \bar{1}$. 10.4. $f(x) = g(x)(5x^5 - 8x^4 - 2x^3 + 3x^2 + 6) + 4x + 5$. **11.** $r(x) = (3x^2 - 4x + 1)^2$. **12.** $r = 3$. **13.** 13.1. $h(x) = 5x^3 - 4x^2 + 7x + 6$; $r(x) = 16$. 13.2. $h(x) = 2ix^3 + (3 - i)x - 2$; $r(x) = 2 + i$. 13.3. $h(x) = 0{,}5x^3 + 3x - 1$; $r(x) = 2{,}5x - 1{,}5$. 13.4. $h(x) = \bar{5}x^3 + \bar{2}x^2 + \bar{1}$; $r(x) = \bar{2}x^2 - \bar{2}x + 1$. **14.** 14.1. 136. 14.2. $-1-46i$. 14.3. $\bar{2}$. 14.4. $9 - 5\sqrt{2}$. **17.** 17.1. $f(x) = (x-1)^4 + 2(x-1)^3 + 3(x - 1)^2 - (x - 1) - 2$. 17.2. $f(x) = \bar{2}(x - \bar{1})^4 + (x - \bar{1})^2 + (x - \bar{1})$.
17.3. $f(x) = (x + i)^5 - 5i(x + i)^4 - (3i + 10)(x + i)^3 + (-13 + 10i)(x + i) + (22i + 5)(x + i) + 11 - i$. **18.** 18.1. $(f, g) = x + 1$. 18.2. $(f, g) = 1$.
18.3. $(f, g) = 2x + 1$. **19**. 19.1. $(f, g) = x - 3$. 19.2. $(f, g) = 1$.
19.3. $(f, g) = x + \bar{3}$. 19.4. $(f, g) = x^2(1 + i)x + i$. **20.** 20.1. $[f, g] = (2x^3 + 7x^2 + 4x - 3)(x - 1)$. 20.2. $[f, g] = (x^3 + \bar{6}x^2 + \bar{4}x + 1) \times (x^3 + x^2 + \bar{3}x - \bar{4}) : (x + \bar{2})$. 20.3. $[f, g] = (x^3 - x^2 + 3x - 3) \times (x^4 + 2x^3 + 2x - 1)$. 20.4. $[f, g] = x^5 + 2ix^4 - 2x^3 - 2ix^2 + x$.
21. 21.1. $u(x) = 1$, $v(x) = -x + 1$. 21.2. $u(x) = -x - \bar{1}$, $v(x) = x + \bar{2}$.

21.3. $u(x)=-\frac{x-1}{3}$, $v(x)=x^2-x-\frac{3}{2}$. **22.** 22.1. $u(x)=\frac{1}{2}(x^2+x+1)+$ $+(x-1)h(x)$, $v(x)=\frac{1}{2}(x^2+x+1)-(x+1)h(x)$, $h(x)\in Q[x]$.

22.2. $u(x)=\frac{x^2+\bar{4}x}{\bar{3}}+(x+\bar{2})h(x)$, $v(x)=\frac{\bar{4}+\bar{1}}{\bar{3}}(x^2+\bar{4}x)-$ $-(x^2x+\bar{1})h(x)$, $h(x)\in Z_5[x]$. **23.** $S(x)\vdots(f,g,h)$ bo'lganligi uchun tenglama yechimga ega.

31-§. Ko'p o'zgaruvchili ko'phadlar

1. 1.1. $f(x,y)=x^5+x^4y-2x^3y^2-xy^4+2y^5+x^2-1$.
1.2. $f(x,y,z)=x^3y^2z+y^3z^2x+zx^2y-xy^2z^3-yz^2x^3-zx^2y^3$.
2. 2.1. Yuqori hadi $\bar{2}x^2z$. 2.2. Yuqori hadi xz. **4.** 4.1. $5x^4y^2z$.
4.2. $-3^xy^2z^2$. **5.** 5.1. $f(x,y)=\sigma_1^2\sigma_2+2\sigma_1^2-2\sigma_2^2-4\sigma_2$.
5.2. $f(x,y)=2\sigma_1^2\sigma_2-6\sigma_1\sigma_2^2-5\sigma_1\sigma_2$. 5.3. $f(x,y,z)=\sigma_1\sigma_2-\sigma_3$.
5.4. $f(x,y,z)=\sigma_1^4-4\sigma_1^3\sigma_2+8\sigma_1\sigma_3$. 5.5. $f(x,y,z,,t)=\sigma_1^2\sigma_4+\sigma_3^2-4\sigma_2\sigma_4$.
10. 10.1. 0. 10.2. $\frac{1}{2}$. **11.** 11.1. $o(x_1,x_2,x_3)=\sigma_1^2 2\sigma_2-\frac{7}{3}\sigma_2^2$. 11.2. $o(x_1,x_2,x_3,x_4)=\sigma_2$. 11.3. $o(x_1,x_2,\ldots,x_n)=\sigma_1^3-3\sigma_1\sigma_2+3\sigma_3$.
14. 14.1. 162. 14.2. 10. 14.3. 41. 14.4. 59. **15.** 15.1. $a=1$.
15.2. $a=2$. 15.3. $a=3\wedge a=-1$. 15.4. $a=\pm i\sqrt{2}$ va $a=\pm 2i\sqrt{3}$.
16. 16.1. -108. 16.2. -27036. 16.3. 50000. 16.4. $a(b^2-4ac)$.
16.5. $-27q^2-4p^3$. 16.6. $-2c^2+18abc-4a^3c-4b^3+a^2b^2$.
18. 18.1. $a=\pm 2$. 18.2. $a\in\left\{3,3\left(-\frac{1}{2}\pm i\frac{\sqrt{3}}{2}\right)\right\}$. 18.3. $a=3$. 18.4. $a=2+4i$. **19.** 19.1. $\left\{(3,2),(-3,-2),\left(2\sqrt{2},-\frac{3\sqrt{2}}{2}\right),\left(-2\sqrt{2},\frac{3\sqrt{2}}{2}\right)\right\}$.
19.2. $\left\{(1,0),(2,1),\left(\frac{-19-\sqrt{177}}{2},\frac{9+\sqrt{177}}{2}\right)\right\}$. 19.3. $\{(0,0),(2,-1),(1,2),$ $(1,68),(2,52)\}$. 19.4. $(1,2)$. **20.** 20.1. $(2,3);(3,2)$. 20.2. $(1,2);(2,1)$.
20.3. $(1,2,-2),(1,-2,2),(2,1,-2),(2,-2,1),(-2,2,1),(-2,1,2)$.
20.4. $(1,-2,3),(1,-3,2),(2,-1,3),(2,-3,1),(3,-1,2),(3,-2,1)$.

20.5. $(1,64),(64,1)$. 20.6. $(16,81),(81,16),(-16,-81),(-81,-16)$.
21. 21.1. $\{1,4\}$. 21.2. $\{2,11\}$. 21.3. $\{-8,-73\}$. 21.4. $x=0$.

32-§. Maydon ustida ko'phadlar

2. 2.1. $f(x)=(x-1)(x+1)(x^2-2)(2x-1)$. 2.2. $f(x)=(x^2-1)(x^2-4)(3x+1)$.

3. $f(x)=(x^2+1)(x^2-2)(2x-1)=(x^2+1)(x-\sqrt{2})(x+\sqrt{2})(2x-1)=$
$=(x-i)(x+i)(x-\sqrt{2})(x+\sqrt{2})(2x-1)$.

6. $f(x)=(x+\bar{1})(x+\bar{2})(x+\bar{3})(x+\bar{4})$, Z_5; Q da keltirilmaydi, $f(x)=(x^2-\sqrt{2}x+2)(x^2+\sqrt{2}x+2)$, R; C da keltiriladi.

7. 7.1. $f(x)=\left(x-\frac{3-\sqrt{5}}{2}\right)^2\left(x-\frac{3+\sqrt{5}}{2}\right)^2$. 7.2. $f(x)=(x-1)(x-2)(x-3)$.

7.3. $f(x)=\left(x-\frac{-1-\sqrt{2}}{2}\right)\cdot\left(x-\frac{-1-\sqrt{2}}{2}\right)\cdot\left(x-\frac{-3+\sqrt{3}}{2}\right)\cdot\left(x-\frac{-3-\sqrt{3}}{2}\right)$.

7.4. $f(x)=(x-1-i)(x-1+i)(x+1-i)(x+1+i)$.

7.5. $f(x)=(x-1)^2(x-2)^2$.

7.6. $f(x)=(x-i\sqrt{3})(x+i\sqrt{3})\left(x-\frac{3}{2}-\frac{\sqrt{3}}{2}i\right)\left(x+\frac{3}{2}-\frac{\sqrt{3}}{2}i\right)\left(x+\frac{3}{2}+\frac{\sqrt{3}}{2}i\right)$.

7.7. $f(x)=\left(x-\frac{-7-\sqrt{5}}{2}\right)^2\left(x-\frac{-7+\sqrt{5}}{2}\right)^2$.

7.8. $\prod_{k=1}^{3n-1}\left(x-\cos\frac{2\pi k}{3n}-i\sin\frac{2\pi k}{3n}\right)$, $(k,3)=1$.

8. 8.1. $(f,g)=x^2-x-2$; $[f,g]=(x-1)^2(x^2+1)(x^2-5x+6)$.

8.2. $(f,g)=x-1$; $[f,g]=(x^2-2x+3)^2(x+6)^2(x-1)^2(x-2)^2(x-6)^2(x^2+x+1)$

8.3. $(f,g)=(x+1)^2$; $[f,g]=(x+1)^3(x-1)(x-2)$.

8.4. $(f,g)=x+\bar{2}$; $[f,g]=x(x+\bar{1})(x+\bar{3})(x+\bar{4})(x^2+x+\bar{1})^2$.

8.5. $(f,g)=x^{(m,n)}-1$. 8.6. Agar $\frac{m}{(m,n)},\frac{n}{(m,n)}$ lar toq son bo'lsa, $(f,g)=x^{(m,n)}+1$, qolgan hollarda $(f,g)=1$.

9. 9.1. $f'(x)=3(x^2+x-1)^2(2x+1)(x^3-2)+3x^2(x^2+x-1)$.

9.2. $f'(x)=x(x+\bar{3})+\bar{3}x^2$. **10.** $f(x)=\bar{2}x^6+x^3+x^2+x+\bar{2}$.

11. $f(x)=4x^3+x^2-x+1$. **12.** $f_1(x)=\bar{1}$, $f_2(x)=x^2$, $f_3(x)=x^2+\bar{1}$, $f_4(x)=x^3+x^2+x$ lardan tashqari barcha ko'phadlar.

13. 13.1. $f(x)=i(x-2i)^4+(-7-i)(x-2i)^3+(4-19i)(x-2i)^2+$ $+(27+4i)(x-2i)-3+14i$ va $f'(2i)=27+4i$; $f''(2i)=8-38i$; $f'''(2i)=-42-6i$; $f^{IV}(2i)=24i$.

13.2. $f(x)=(x-i)^5+5i(x+i)^4-(3i+10)(x+i)^3+(10i-13)(x+i)^2+$ $+(5+22i)(x+i)+11-i$ va $f'(-i)=22i+5$; $f''(-i)=20;-26$; $f'''(-i)=-60-18i$; $f^{IV}(-i)=-120i$; $f^{V}(-i)=120$.

13.3. $f(x)=(x+1)^4-4(x+1)^3-9(x+1)^2+36(x+1)+1$, $f'(-1)=36$; $f''(-1)=-18$; $f'''(-1)=-24$; $f^{IV}(-1)=24$.

13.4. $f(x)=\bar{2}(x-\bar{1})^4+(x-\bar{1})^2+(x-\bar{1})$, $f'(\bar{1})=\bar{1}$; $f''(\bar{1})=\bar{2}$; $f'''(\bar{1})=\bar{0}$; $f^{IV}(\bar{1})=\bar{0}$. 13.5. $f(x)=(x-2)^4-18(x-2)+38$; $f'(2)=-18$, $f''(2)=f'''(2)=0$, $f^{IV}(2)=24$.

13.6. $f(x)=(x-2)^5+10(x-2)^4+36(x-2)^3+62(x-2)^2+48(x-2)+18$; $f'(2)=48$, $f''(2)=124$, $f'''(2)=216$, $f^{IV}(2)=240$, $f^{V}(2)=120$

14. 14.1. 2. 14.2. 1. 14.3. 0. 14.4. 3. 14.5. 3. **15.** 15.1. $b=0$ da $\alpha=0$; $b=-6\sqrt{3}$ da $\alpha=-\sqrt{3}$; $b=6\sqrt{3}$ da $\alpha=\sqrt{3}$. 15.2. $b\in\left\{-\frac{14}{27}, 18\right\}$. 15.3. $b=0$. 15.4. $b=4$ da $\alpha=-2$; $b=\frac{102}{27}$ da $\alpha=-\frac{4}{3}$. **16.** 16.1. $27a^4=256b^3$. 16.2. $3125b^2+108a^5=0$.

17. 17.1. $f(x)=(x^2+x+1)^2(x+2)$. 17.2. $f(x)=(x+i)^3(x-2i)^2$. 17.3. $f(x)=(x+2)^3(x^2-x-i)$.

18. 18.1. $f(x)=(x-1)^2(x-2)(x-3)(x-1-i)$.

18.2. $f(x)=(x-i)^2(x+1+i)$. **19.** 19.1. $\frac{x-1}{x-2}$.

19.2. $(x^2-x+1)(x^4-x^2+1)$. **20.** 20.1. $\frac{f(x)}{g(x)}=-\frac{2}{3(x+1)}+\frac{2x^2-2x+5}{3(x^3-2)}$.

20.2. $\frac{f(x)}{g(x)}=-\frac{1}{2x}+\frac{x^2}{2(x^3-2)}$. 20.3. $\frac{f(x)}{g(x)}=\frac{1}{2(x^2-2)}+\frac{1}{2(x^3+2)}$.

20.4. $\frac{f(x)}{g(x)}=\frac{1}{x}-\frac{x}{x^2+1}$. **21.** 21.1. $\frac{f(x)}{g(x)}=\frac{x+1}{x^2+1}-\frac{x+2}{x^2+x+1}$.

21.2. $\frac{f(x)}{g(x)}=\frac{6}{x^3}-\frac{4}{x^2}+\frac{1}{x+3}+\frac{3}{(x+3)^2}$. 21.3. $\frac{f(x)}{g(x)}=\frac{x}{8(x^2+2x+2)}-\frac{x-4}{8(x^2-2x+2)}$.

21.4. $\frac{f(x)}{g(x)}=\frac{\sqrt{2}}{8(x-\sqrt{2})}-\frac{\sqrt{2}}{8(x+\sqrt{2})}+\frac{1}{2(x^2+2)}$.

21.5. $\frac{f(x)}{g(x)}=-\frac{2}{3(x+1)}+\frac{\frac{3}{2}+\frac{5}{12}\sqrt[3]{2}-\frac{1}{3}\sqrt[3]{4}}{x-\sqrt[3]{2}}+\frac{\left(\frac{4}{3}-\frac{5}{12}\sqrt[3]{2}+\frac{1}{3}\sqrt[3]{4}\right)x-\frac{3}{2}+\frac{2}{3}\sqrt[3]{2}-\frac{5}{3}\sqrt[3]{4}}{x^2+\sqrt[3]{2}x+\sqrt[3]{4}}$.

21.6. $\frac{f(x)}{g(x)}=\frac{1}{x^2+x+2}-\frac{x-2}{(x^2+x+2)^2}$. 21.7. $\frac{f(x)}{g(x)}=\frac{1}{8(x-2)}-\frac{1}{8(x+2)}+\frac{1}{2(x^2+4)}$.

21.8. $\frac{f(x)}{g(x)}=\frac{1}{8}\left(\frac{x+2}{x^2+2x+2}-\frac{x-2}{x^2-2x+2}\right)$.

22. 22.1. $\frac{f(x)}{g(x)}=\frac{1}{12(x-1)}-\frac{4}{3(x+2)}+\frac{9}{4(x+3)}$.

22.2. $\frac{f(x)}{g(x)}=-\frac{1}{16}\left(\frac{1+i}{x-1-i}+\frac{1-i}{x-1+i}+\frac{-1+i}{x+1-i}+\frac{-1+i}{x+1+i}\right)$.

22.3. $\frac{f(x)}{g(x)}=\frac{3}{(x-1)^3}-\frac{4}{(x-1)^2}-\frac{1}{x-1}-\frac{1}{(x+1)^2}-\frac{2}{x+1}+\frac{1}{x-2}$.

22.4. $\frac{f(x)}{g(x)}=\frac{1}{3(x-i)}-\frac{1}{3(x+2i)}$. 22.5. $\frac{f(x)}{g(x)}=\frac{1}{x-1}-\frac{1+i}{2(x-i)}+\frac{i-1}{2(x+i)}$.

22.6. $\frac{f(x)}{g(x)}=\frac{\sqrt{2}}{8(x-\sqrt{2})}-\frac{\sqrt{2}}{8(x+\sqrt{2})}+\frac{\sqrt{2}}{8(x-\sqrt{2}i)}-\frac{\sqrt{2}}{8(x+\sqrt{2}i)}$.

23. $\frac{f(x)}{g(x)}=\sum_{\alpha=\bar{0}}^{\overline{p-1}}\frac{\bar{1}}{(\overline{p-1})(x+\bar{\alpha})}$.

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MUNDARIJA

VII MODUL. DETERMINANTLAR

VIII MODUL. VEKTOR FAZOLAR

IX MODUL. CHIZIQLI AKSLANTIRISHLAR

X MODUL. CHIZIQLI TENGSIZLIKLAR SISTEMASI

XI MODUL. BUTUN SONLAR HALQASIDA BO'LINISH MUNOSABATI

XII MODUL. TAQQOSLAMALAR

XIII MODUL. KO'PHADLAR

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Oliy o'quv yurtlari talabalari uchun o'quv qo'llanma

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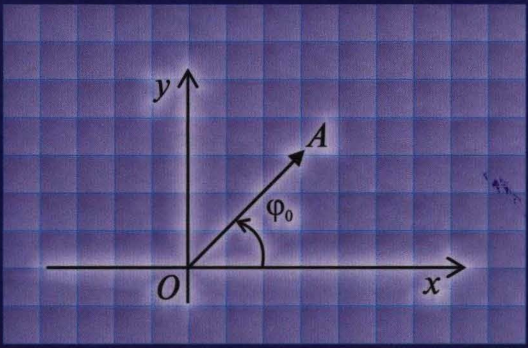

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