O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

ISLOM KARIMOV NOMIDAGI TOSHKENT DAVLAT TEXNIKA UNIVERSITETI

OLIY MATEMATIKA

fanining

TEKISLIKDA ANALITIK GEOMETRIYA

qismidan

USLUBIY QO‘LLANMA

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“Oliy matematikaning tekislikda analitik geometriya qismidan” uslubiy qo‘llanma ‟Analitik geometriya” kursi "Ikkinchi tartibli chiziqlar" bo‘limiga tegishli vazifa va masalalar to‘plamidan iborat bo‘lib, bu masalalarni yechish uchun ikkinchi tartibli chiziqlarning kanonik ko‘rinishi, bu egri chiziqlarning turli xarakteristikalarini bog‘lovchi asosiy formulalari, shuningdek tekislikdagi dekart koordinatalarini o‘zgartirish usullaridan foydalanishligini talab etadi.

Islom Karimov nomidagi ToshDTU ilmiy-uslubiy kengashi qaroriga asosan chop etildi.

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KIRISH

“Oliy matematikani tekislikda analitik geometriya qismidan” tekislikdagi dekart koordinatalarini o‘zgartirish usullaridan foydalanishligini talab etadi. Bunda yangi bazisga o‘tuvchi hamda berilgan burchakka asosan koordinata boshiga nisbatan burilish matritsasini qurish va koordinata boshini o‘zgartirish(siljitish) uslublarini bilish talab etiladi. Shuningdek ikkinchi tartibli egri chiziqlarni qutb koordinatalari sistemasidagi tenglamalari va ularning tekislikdagi to‘g‘riburchakli dekart koordinatalar sistemasidagi tenglamalari va ularning tekislikdagi to‘g‘riburchakli dekart koordinatalar sistemasidagi tenglamalari va ularning tekislikdagi to‘g‘ri burchakli dekart koordinatalar bilan bog‘lovchi formulalarini bilishlari lozim bo‘ladi. Egri chiziq tenglamalarini boshqa koordinatalar sistemasiga o‘tkazish birmuncha murakkabligidan bunday masalalarni yechishga alohida e’tibor berishimizni talab etadi.

Qo‘llanma ikkinchi tartibli egri chiziqlarni uni xarakterlovchi nuqtalari (fokuslari uchlari) va to‘g‘ri chiziqlari (direktrisalari, asimptotalari) bilan o‘zaro bog‘lovchi asosiy formulalar ro‘yxat berishi bilan boshlanadi. Shundan so‘ng yangi bazisga o‘tuvchi matritsalar ta‘rifi, koordinata boshiga nisbatan siljish formulalari berilib aytilgan qiyinchiliklarni tug‘diruvchi masalalarni yechish uslublari ko‘rsatilgan. Shundan so‘ng mustaqil yechish uchun masalalar to‘plami berilgan.

Asosiy formulalar

Aylananing asosiy formulalari va kanonik tenglamasi:

T a‘ r i f. Aylana deb shunday nuqtalarning geometrik o‘rniga aytiladiki, bu nuqtalardan markaz deb ataluvchi nuqtagacha bo‘lgan masofalar bir biriga teng bo‘ladi.

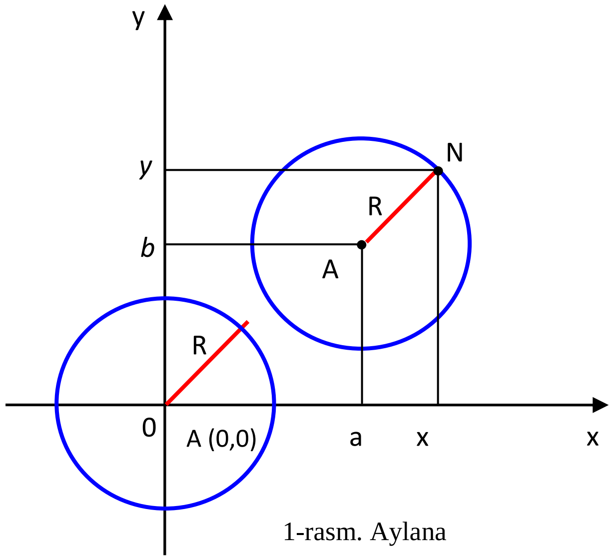
T a‘ r i f. Markaz deb ataluvchi nuqtadan teng uzoqlikda yotgan nuqtalarning geometrik o‘rniga aylana deyiladi.

To‘g‘ri burchakli koordinatalar sistemasida aylananing radiusi *R* va markazi A *(a ; b)* nuqtada bo‘lsin. *N (x ; y)* aylanadagi ixtiyoriy nuqta. Aylananing ta‘rifiga ko‘ra: *AN=R.*
Ikki nuqta orasidagi masofani topish formulasiga asosan:

$$AN=\sqrt{(x-a)^2+(y-b)^2}.$$

Tenglikning ikkita tomonini kvadratga ko‘tarib, AN=R ekanligini e‘tiborga olsak:

$$(x-a)^2+(y-b)^2=R^2$$



1-rasm. Aylana

Markazi A *(a ; b)* nuqtada bo‘lib, radiusi *R* ga teng bo‘lgan aylana tenglamasi hosil bo‘ladi.

Xususiy holda, agar aylananing markazi koordinatalar boshida bo‘lsa, uning tenglamasi: $x^2+y^2=R^2$

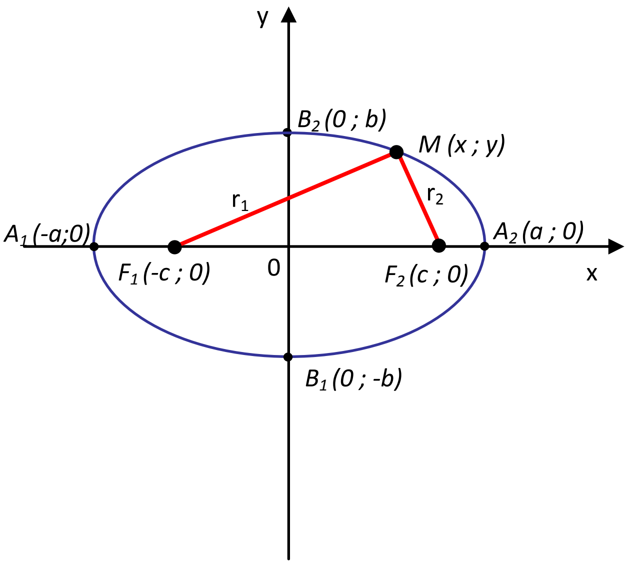
Ellipsning asosiy formulalari va kanonik tenglamasi:

T a’ r i f

Ellips deb shunday nuqtalarning geometrik o‘rniga aytiladiki, bu nuqtalardan fokuslar deb ataluvchi nuqtalargacha bo‘lgan masofalarning yig‘indisi o‘zgarmas songa teng.

T a‘ r i f

Fokuslar deb ataluvchi nuqtalargacha bo‘lgan masofalarning yig‘indisi o‘zgarmas songa teng bo‘lgan nuqtalarning geometrik o‘rinlariga ellips deyiladi.



2-rasm. Ellips

Ta’rifga ko‘ra:

$$|MF_1| + |MF_2| = const = 2a$$

Ikki nuqta orasidagi masofa formulasidan foydalanib,

$MF_1 = \sqrt{(x+c)^2 + y^2}$ va $MF_2 = \sqrt{(x-c)^2 + y^2}$ ni hosil qilamiz, demak,

$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a$ yoki $(a^2 - c^2)x^2 + a^2 y^2 = a^2(a^2 - c^2)$

Ellipsning ta‘rifiga ko‘ra $2a > 2c$ bo‘lgani uchun $a^2 - c^2$ son musbat:

$a^2 - c^2 = b^2$ belgilash kiritamiz. U holda tenglama $b^2x^2 + a^2y^2 = a^2b^2$ yoki

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ko‘rinishni oladi.

(2a-ellipsning katta o‘qi, 2b- ellipsning kichik o‘qi)

Fokuslari: $F_1(c, 0)$, $F_2(-c, 0)$, $c^2=a^2-b^2$ $(a>b)$.

Ekssenrisiteti:

$(e<1)$: $e = \sqrt{1 - \frac{b^2}{a^2}}$, $e = \frac{c}{a}$.

Direktrisalari:

δ_1: $x = \frac{a}{e}$, δ_2: $x = -\frac{a}{e}$.

Fokusdan mos direktrisalarigacha bo‘lgan masofa:

$$p = a\left(\frac{1}{e} - e\right), \quad p = \frac{b^2}{c}.$$

Fokal radiuslari (ellipsning ixtiyoriy M(x,y) nuqtasidan fokuslarigacha bo‘lgan masofa):

$$r_1 = |MF_1| = a - ex,$$

$$r_2 = |MF_1| = a + ex.$$

Ellipsning qutb koordinatalar sistemasidagi tenglamasi:

$$p = \frac{ep}{1 - e\cos\theta}.$$

<u>Giperbolaning asosiy formulalari va kanonik tenglamasi:</u>

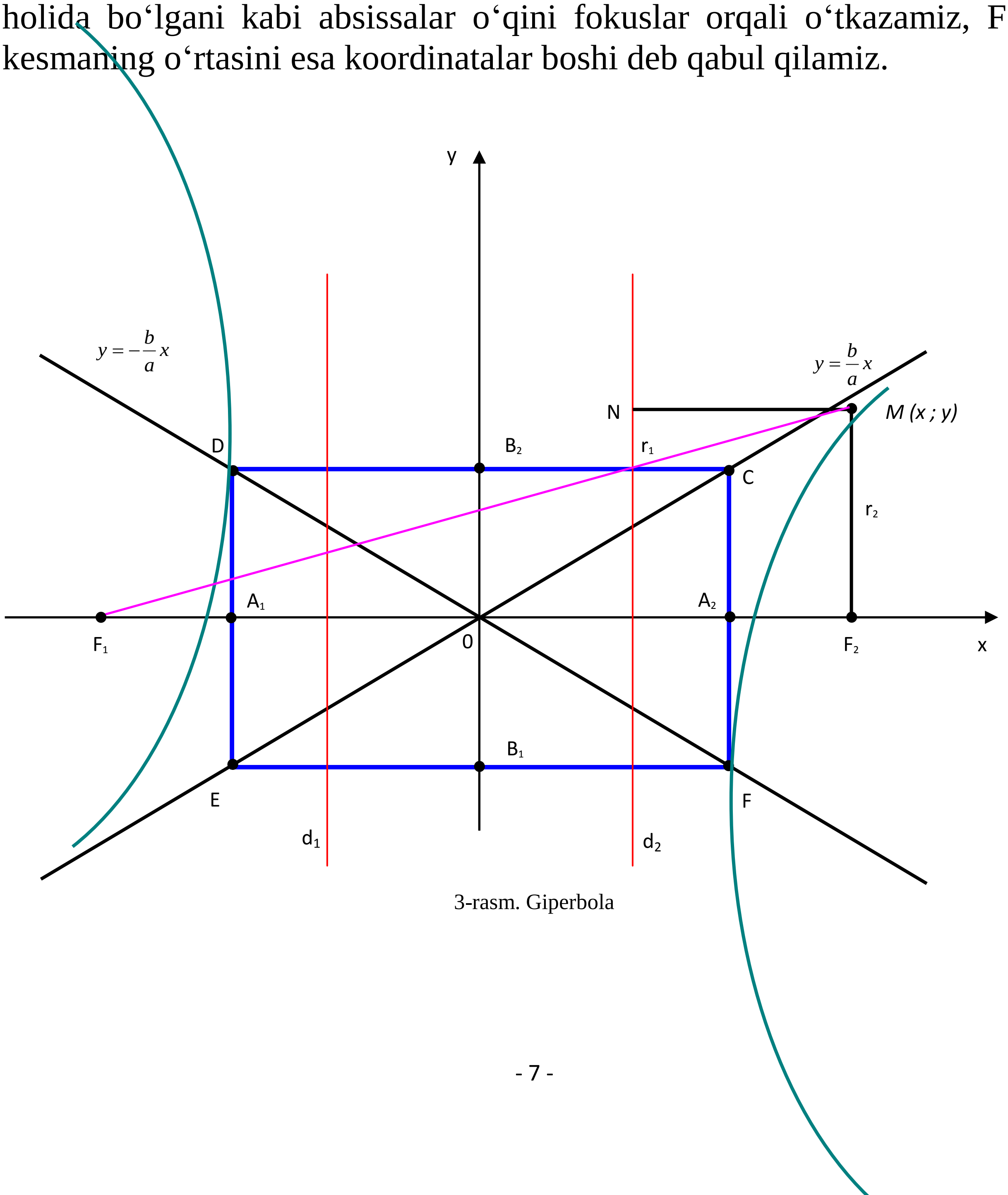
T a' r i f

Giperbola deb shunday nuqtalarning geometrik o'rniga aytiladiki, bu nuqtalardan fokuslar deb ataluvchi nuqtalargacha bo'lgan masofalarning ayirmasi o'zgarmas songa teng.

T a' r i f

Fokuslar deb ataluvchi nuqtalargacha bo'lgan masofalarning ayirmasi songa teng bo'lgan nuqtalarning geometrik o'rinlariga giperbola deyiladi.

F_1 va F_2 fokuslar orasidgi masofani $2c$ orqali, giperbolaning har bir nuqtasidan fokuslargacha bo'lgan masofalar ayirmasining moduliga teng bo'lgan o'zgarmas miqdorni 2a orqali (0<2a<2c) belgilaymiz. Ellips holida bo'lgani kabi absissalar o'qini fokuslar orqali o'tkazamiz, F_1 F_2 kesmaning o'rtasini esa koordinatalar boshi deb qabul qilamiz.



3-rasm. Giperbola

Fokuslari Ox o‘qida yotgan giperbola tenglamasini, uning ta‘rifiga asoslanib keltirib chiqaramiz. Ikki nuqt orasidagi masofa formulasiga ko‘ra:

$$\sqrt{(x+c)^2+y^2}-\sqrt{(x-c)^2+y^2}=\pm 2a$$

Soddalashtirishlarni bajargandan so‘ng, quyidagi tenglamani hosil qilamiz:

$$(a^2-c^2)x^2+a^2y^2=a^2(a^2-c^2)$$

tenglamada giperbola uchun 2a < 2c bo‘lgandan ayirma noldan kichik:

$a^2-c^2<0$. Shuning uchun $c^2-a^2=b^2$ deb olamiz. U holda tenglama quyidagi ko‘rinishga keladi:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

bu fokuslari Ox o‘qida yotgan giperbolaning kanonik (sodda) tenglamasidir (2a-giperbolaning haqiqiy o‘qi; 2b-giperbolaning mavhum o‘qi).

Fokuslari: $F_1(c,0)$, $F_2(-c,0)$, $c^2=a^2+b^2$

Ekssentrisiteti

(e >1): $e=\sqrt{1+\frac{b^2}{a^2}}$, $e=\frac{c}{a}$

Direktrisalari:

δ_1: $x=\frac{a}{e}$, δ_2: $x=-\frac{a}{e}$.

Asimptotalari: $y=\frac{b}{a}x, y=-\frac{b}{a}x$.

Fokusdan mos direktrisalarigacha bo‘lgan masofa:

$$p = a\left(e - \frac{1}{e}\right), \quad p = \frac{b^2}{c}.$$

Fokal radiuslari (giperbolaning ixtiyoriy M(x,y) nuqtasidan fokuslarigacha bo‘lgan masofa):

$$r_1 = |MF_1| = \begin{cases} ex - a, agar\ x \geq a, \\ a - ex, agar\ x \leq -a \end{cases}$$

$$r_2 = |MF_2| = \begin{cases} ex + a, agar\ x \geq a, \\ -(a + ex), agar\ x \leq -a \end{cases}$$

Qo‘shma(sopryajennaya) giperbola:

$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

(2a-giperbolaning mavhum o‘qi, 2b-giperbolaning haqiqiy o‘qi).
Teng yonli (ravnostoronnaya) giperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$$

Giperbolaning qutb koordinatalar sistemasidagi tenglamasi:
$p = \frac{ep}{1 - e\cos\theta}$ - o‘ng shox tenglamasi;
$p = \frac{-ep}{1 + e\cos\theta}$ - chap shox tenglamasi;

Parabolaning asosiy formulalari va kanonik tenglamasi:

T a' r i f

Direktrisa deb ataluvchi to'g'ri chiziq nuqtalardan fokus deb ataluvchi nuqtagacha bo'lgan masofalari teng bo'lgan nuqtalarning geometrik o'rinlariga parabola deyiladi.

Fokus $F\left(\frac{p}{2}, 0\right)$

Ekssentrisiteti: e=1

Direktrisasi: δ: $x = -\frac{p}{2}$ bo'lgan parabolaning kanonik tenglamasi:

$y^2 = 2px$ ko'rinishda bo'ladi.

Fokal radius(parabolaning ixtiyoriy M(x,y) nuqtasidan fokusigacha bo'lgan masofa):

$$r = |MF| = x + \frac{p}{2};$$

Berilgan tenglamasi $y^2 = -2px$ ko'rinishdagi parabola uchun

$F\left(-\frac{p}{2}, 0\right)$ δ: $x = -\frac{p}{2}$;

Tenglamasi $x^2 = 2py$ ko'rinishdagi parabola uchun

$$F\left(0, \frac{p}{2}\right),\ \delta:\ y = -\frac{p}{2};$$

tenglamasi $x^2 = -2py$ ko'rinishdagi parabola uchun $F\left(0, -\frac{p}{2}\right)$, δ: $y = \frac{p}{2}$;

Parabolaning qutb koordinatalar sistemasidagi tenglamasi:

$$\rho = \frac{p}{1 - cos\theta}$$

Tekislikda koordinatalarni o‘zgartirish(almashtirish)

Faraz qilaylik tekislikda e_1, e_2– eski va e_1', e_2'- yangi bazislar berigan bo‘lib, yangi basis eski bazis bilan quyidagi munosabatni qanoatlantiruvchi koordinatalari bilan berilgan bo‘lsin:

$$e_1' = c_{11}e_1 + c_{21}e_2$$

$$e_2' = c_{12}e_1 + c_{22}e_2.$$

U holda C matritsa:

$$C=\begin{pmatrix} c_{11} & c_{21} \\ c_{12} & c_{22} \end{pmatrix}$$

e_1, e_2-bazasidan e_1', e_2' bazasiga o‘tish matritsasi deyiladi.

Agarda almashtirish koordinata boshiga nisbatan α burchakka burish natijasida bajarilsa, yangi bazisga o‘tish matritsasi:

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

ko‘rinishda bo‘ladi.

Bu matritsa ortogonal bo‘ladi. Chunki bir burchakli dekart koordinatalar sistemasidan ikkinchi to‘g‘ri burchakli koordinatalar sistemasiga o‘tish matritsasi doimo ortogonal bo‘lishi bizga ma‘lum. Bunda tekislikda ortogonal almashtirish matritsalari ikki xil ko‘rinishda bo‘lishi mumkin bo‘lib, birinchisi ko‘rinishdagi burish matritsasi bo‘lib, ikkinchisi

$$\begin{pmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{pmatrix}$$

ko‘rinishdagi (birinchi koordinatalar o‘qiga nisbatan) burish matritsasidan iborat.

Ortogonal matritsalar uchun

$$C^T = C^{-1}$$

munosabat o‘rinligidan, ortogonal matritsaga teskari bo‘lgan matritsani uni transponirlash orqali osonlik bilan aniqlash mumkin.

Tekislikning ixtiyoriy eski (x, y) koordinatalar sistemasidan yangi koordinatalari (x', y') bo‘lgan yangi sistema bilan bog‘lash formulasi:

$$x = c_{11}x' + c_{12}y'$$

$$y = c_{21}x' + c_{22}y'$$

ko‘rinishda bo‘lib, o‘tish matritsasiga ega bo‘lgan holda koordinata boshiga nisbatan burish yangi bazisi;

$$x = x'cos\alpha - y'sin\alpha$$

$$y = x'sin\alpha y +' cos\alpha$$

ko‘rinishda, akslantirib burilgan holatda esa:

$$x = x'cos\alpha + y'sin\alpha$$

$$y = x'sin\alpha - y'cos\alpha$$

ko‘rinishda bo‘ladi.

Koordinata boshini $O' = (x_0, y_0)$ nuqtaga siljitish formulasi

$$x = x' + x_0$$

$$y = y' + y_0$$

ko‘rinishda bo‘lib, C o‘tish matritsaga ega bo‘lib, koordinata boshiga nisbatan siljitilgan holatda, o‘zgartirish (almashtirish) koordinatalarini quyidagi umumiy formuladan aniqlanadi:

$$x = c_{11}x' + c_{12}y' + x_0$$

$$y = c_{21}x' + c_{22}y' + y_0$$

Teskari o‘zgartirish koordinatalarni aniqlash uchun formulani

$$\begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix} = \begin{pmatrix} c_{11} & c_{21} \\ c_{12} & c_{22} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

yozib olib hosil bo‘lgan matritsali tenglikni har ikki tomoniga C^{-1} matritsani qo‘llash lozim. Natijada eski (x, y)koordinatalar orqali ifodalangan yangi(x', y') koordinatalarni aniqlash munosabatini hosil qilamiz. Yuqorida takidlanganimizdek matritsa ortogonal bo‘lganda teskari matritsani aniqlash qulay bo‘ladi

<u>1-misol</u>

Asimptotalari $y = \pm\frac{2}{5}x$ dan iborat bo‘lib, $A(45,12\sqrt{2})$ nuqtadan o‘tuvchi giperbola tenglamasi tuzilsin.

<u>Yechish.</u>

Asimptotalar koordinata boshida kesishganligidan, koordinata boshi simmetriya markazidan iborat giperbola kanonik tenglamasini

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

ko‘rinishda ishlaymiz.

Asimptotalar tenglamasidan $\frac{b}{a} = \frac{2}{5}$

bu yerdan, $a = \frac{5b}{2}$

u holda giperbola tenglamasi

$$\frac{x^2}{\frac{25b^2}{4}} - \frac{y^2}{b^2} = 1$$

ko‘rinishni oladi.

Bu tenglamaga A nuqtaning koordinatalarini qo‘ysak

$$\frac{45^2 \cdot 4}{25b^2} - \frac{12^2 \cdot 2}{b^2} = 1$$

bu yerdan, $b^2 = 36$

$a^2 = \frac{25}{4} \cdot 36 = 225$

giperbola tenglamasi: $\frac{x^2}{225} - \frac{y^2}{36} = 1$

<u>2-misol</u>

Cho‘qqilari $A_1(12,0)$ va $A_2(-4,0)$ nuqtalarda bo‘lib, $0y$ o‘qning uzunligi $6\sqrt{3}$ bo‘lgan xord (kesma) bo‘yicha kesuvchi ellips tenglamasi tuzilsin.

<u>Yechish.</u>

Ellips cho‘qqilari $0x$ o‘qida yotganligidan $0x$ o‘qi simmetriya o‘qi bo‘ladi. Simmetriya markazi O' cho‘qqilarning o‘rtasida yotganligidan $O'(4,0)$. Cho‘qqilari orasidagi masofa uning o‘qiga tengligidan, $2a = 16$ ya‘ni $a = 8$. Ellipsning ikkinchi simmetriya o‘qi $x = 4$ chiziqda yotganligidan ellipsning tenglamasi

$$\frac{(x-4)^2}{64} - \frac{y^2}{b^2} = 1$$

ko‘rinishni oladi.

Masalaning shartiga ko‘ra ellipsning $0y$ o‘qini ($0x$ o‘qiga nisbatan simmetrik bo‘lgan) kesuvchi nuqtalarini koordinatalari $B_1(0,3\sqrt{3})$ va $B_2(0,-3\sqrt{3})$.

Bularni ellips tenglamasiga qo‘yib

$$\frac{16}{64} + \frac{27}{b^2} = 1$$

ni hosil qilamiz. Bu yerdan $\frac{27}{b^2} = \frac{3}{4}$ va $b^2 = 36$ ni aniqlab, so‘ralgan ellipsning

$$\frac{(x-4)^2}{64} - \frac{y^2}{36} = 1$$

tenglamasini hosil qilamiz.

<u>3-misol</u>

Direktrisasi $y = 3$, ekssentrisiteti $e = 4$ ga teng bo‘lib, fokusi $F(5,33)$ nuqtada bo‘lgan ikkinchi tartibli chiziq tenglamasi tuzilsin.

<u>Yechish.</u>

Ekssentrisitet birdan katta demak bu giperbola. Giperbola direktrisasi $0x$ o‘qiga parallelligidan giperbolaning haqiqiy o‘qi $0y$ o‘qiga, mavhum o‘qi esa $0x$ o‘qiga parallel. U holda qo‘shma(сопряженный) giperbola

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1,$$

bu yerda $O' = (x_0, y_0)$ giperbolaning simmetriya markazi.

Fokusdan direktrisagacha bo‘lgan masofa $p = 30$, ikkinchi tomondan

$p = b\left(e - \frac{1}{e}\right)$ (b-giperbolamizning haqiqiy yarim o‘qi),

Demak, $30 = b\left(4 - \frac{1}{4}\right)$ bu yerda $b = 8$.

U holda $c = be = 8 \cdot 4 = 32$ va $a^2 = c^2 - b^2 = 32^2 - 8^2 = 960$

Endi giperbolaning simmetriya markazini topamiz. Fokus giperbolaning haqiqiy o‘qida yotganligidan tenglamasi $x = 5$ bo‘ladi, bu yerda $x_0 = 5$

C -simmetriya markazidan fokusigacha bo‘lgan masofa ekanligidan,
$33-y_0=c=32,$ bundan $y_0=1.$

Natijada giperbola tenglamasi

$$-\frac{(x-5)^2}{960}+\frac{(y-1)^2}{64}=1$$

Ikkinchi fokus $x=5$ to‘g‘ri chiziqda yotganligidan $,F_2(5,-31)$.

Ikkinchi direktrisa berilgan mavhum o‘q $y=1$ ga nisbatan simmetrik ekanligidan, $y=-1$ dan iborat bo‘ladi

<u>4-misol</u>

Fokusi F (3,0) nuqtada, direktrisasi $x=\frac{8}{3}$ dan iborat bo‘lib, A(4,-1) nuqtadan o‘tgan ikkinchi tartibli chiziq tenglamasi, ikkinchi fokus va ikkinchi direktrisalari aniqlansin.

<u>*Yechish:*</u>

A nuqtadan fokusgachan bo‘lgan masofani, A nuqtadan direktrisagacha bo‘lgan masofaga nisbati egri chiziq ekssentrisitetiga teng ya‘ni:

$$e=\frac{\sqrt{2}}{\frac{4}{3}}=\frac{3\sqrt{2}}{4}=\frac{3}{2\sqrt{2}}$$

Bu yerda $e>1$, demak so‘ralayotgan egri chiziq giperbola ekan. $\frac{3}{2\sqrt{2}}=e=\frac{c}{a}$ ga asosan $c=\frac{3}{2\sqrt{2}}a$ $b^2=c^2-a^2=\frac{9}{8}a^2-a^2=\frac{1}{8}a^2$.

Giperbola fokusi $0x$ o‘qida yotganligi va direktrisa bu o‘qqa perpendikulyarligidan, giperbola simmetriya markazi $O'(x_0,0)$ $,O'$ nuqtada bo‘ladi. U holda so‘ralayotgan giperboola tenglamasi:

$$\frac{(x-x_0)^2}{8b^2}-\frac{y^2}{b^2}=1$$

Fokusdan direktrisagacha bo‘lgan masofa $\frac{1}{3}$ ga teng. Ikkinchi tomondan, $p=a(e-\frac{1}{e})$

demak,

$$\frac{1}{3}=a\left(\frac{3}{2\sqrt{2}}-\frac{2\sqrt{2}}{3}\right)=\frac{a}{6\sqrt{2}}\ ,$$

bu munosabatdan $a=2\sqrt{2}$ ya‘ni , $a^2=8$ $b^2=1$ ya‘ni

$c^2 = a^2 + b^2 = 8 + 1 = 9 \quad c = 3$ c-fokusdan simmetriya markazigacha bo‘lgan masofa ekanligidan simmetriya markazi koordinata boshidaligi kelib chiqadi. Demak, so‘ralgan giperbola tenglamasi:

$\frac{x^2}{8} - \frac{y^2}{1} = 1$ ekan.

<u>5-misol</u>

Direktrisa va asimptotalarining to‘rtta kesishgan nuqtalari koordinatalari $(\pm 6, \pm 3)$ dan iborat bo‘lgan giperbolaning tenglamasi tuzilsin.

<u>*Yechish.*</u> Masala shartini qanoatlantiruvchi giperbola ikkita. Bittasini haqiqiy o‘qi-$0x$, ikkinchisiniki-$0y$. Dastlab haqiqiy o‘q-$0x$ bo‘lgan hol uchun ko‘raylik. Bu holda direktrisa tenglamalari $x = 6$ va $x = -6$, asimptota tenglamalari esa $y = \pm\frac{1}{2}x$ lardan iborat bo‘lib, giperbola kanonik tenglamasi $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ bo‘lgan holda asimptotalari tenglamasi $y = \pm\frac{b}{a}x$ ekanligidan $\frac{b}{a} = \frac{1}{2}$ yoki $a = 2b$,direktrisa tenglamalari esa $x = \pm\frac{a}{e}$.

Direktrisalari orasidagi masofa $\frac{2a}{e} = 12$. Bundan tashqari $e = \sqrt{1 + \frac{b^2}{a^2}}$ ekanligidan quyidagi sistemani hosil qilamiz

$$\begin{cases} a = 2b \\ 2a = 12e \\ e^2 = 1 + \frac{1}{4} = \frac{5}{4} \end{cases}$$

Sistemani yechib $e = \frac{\sqrt{5}}{2}, a = 3\sqrt{5}, b = \frac{3\sqrt{5}}{2}$ larni aniqlaymiz.

Demak, so‘ralgan giperbola tenglamasi

$$\frac{x^2}{45} - \frac{y^2}{\frac{45}{4}} = 1$$

Endi giperbola haqiqiy o‘qi- $0y$ bo‘lgan holni qaraylik. Bu holda direktrisalari $y = \pm 3$, asimptotalari esa $y = \pm\frac{1}{2}x$ tenglamalardan iborat bo‘lib, giperbola tenglamasi

$$\frac{-x^2}{a^2} + \frac{y^2}{b^2} = 1$$

ko‘rinishni oladi. Direktrisalar tenglamasi $y = \pm\frac{b}{e}$ ekanligidan quyidagi sistemani hosil qilamiz:

$$\begin{cases} a = 2b \\ 2b = 6e \\ e^2 = 1 + \dfrac{a^2}{b^2} = 5 \end{cases}$$

bu yerdan $e = \sqrt{5}$, $a = 6\sqrt{5}$ $b = 3\sqrt{5}$ giperbola tenglamasini topamiz

$$\frac{-x^2}{180} + \frac{y^2}{45} = 1$$

<u>6-misol.</u>

A(7,6) nuqtalardan o‘tib $x = 5$ va $y = 3$ asimptotalarga ega bo‘lgan giperbola tenglamasi tuzilsin.

<u>*Yechish.*</u>

Giperbola asimptotalari simmetriya markazida kesishadi. Demak $O'(5{,}3)$ nuqta giperbolaning simmetriya markazi bo‘ladi. $x = 5$ va $y = 3$ to‘g‘ri chiziqlar o‘zaro perpendikulyar, ya‘ni giperbola teng tomonli bo‘lib, asimptotalari simmetriya o‘qlarining bissektrisalaridan iborat. Maktab matematika kursidan ma’lumki simmetriya markazi koordinata boshida bo‘lgan giperbola tenglamasi: $y = \frac{k}{x}$ ekanligi. Simmetriya markazi $O'(5{,}3)$ nuqtaga su‘rilganligidan:

$$\begin{cases} x = x' + 5 \\ y = y' + 3, \end{cases}$$

bu yerdan

$$\begin{cases} x' = x - 5 \\ y' = y - 3. \end{cases}$$

$O'x'y'$ sistemada koordinata boshi O' nuqtada bo‘lib, $O'x'$ va $O'y'$ o‘qlar mos ravishda Ox va Oy o‘qlarga parallel bo‘lib, izlanayotgan giperbola tenglamasi $y' = \frac{k}{x'}$ ko‘rinishni oladi. U holda Oxy sistemasida tenglama :

$$y - 3 = \frac{k}{x-5} \quad (7)$$

k ni aniqlash uchun giperbolaning A(7,6) nuqtadan o‘tishidan foydalanamiz. Bu nuqta koordinatalarini (7) ga qo‘ysak: $6 - 3 = \frac{k}{7-5}$ bu yerdan k=6 ni topib (7)ga qo‘ysak:

$$y - 3 = \frac{6}{x - 5}$$

yoki

$$xy - 3x - 5y + 9 = 0$$

so‘ralgan giperbolaning tenglamasi hosil bo‘ladi.

7-misol.

$2x + 3y - 19 = 0$ va $3x - 2y - 9 = 0$ to‘g‘ri chiziqlar mos ravishda katta va kichik yarim o‘qlaridan bo‘lib, yarim o‘q uzunliklari *a*=2 va *b* =1 larga teng bo‘lgan ellips tenglamasi tuzilsin.

Yechish.

Ellips o‘qlarining kesishishi nuqtasi O' ellipsning simmetriya markaziga tengligidan:

$$\begin{cases} 2x + 3y - 19 = 0 \\ 3x - 2y - 9 = 0 \end{cases}$$

ellipsning simmetriya markazi $O'(5,3)$ ekanligi aniqlanadi.

Katta va kichik yarim o‘qlar yo‘naltiruvchi vektorlari esa $p = (3, -2)$ va $q = (2,3)$ bo‘ladi.

Yangi ortonormallashgan vektor bazislari sifatida normallashgan *p* va *q* vektrolarni olsak:

$$e'_1 = \frac{p}{|p|} = \left(\frac{3}{\sqrt{13}}, -\frac{2}{\sqrt{13}}\right)$$

$$e'_2 = \frac{q}{|q|} = \left(\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\right).$$

U holda koordinatalarini almashtirish formulalari:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{3}{\sqrt{13}} & \frac{2}{\sqrt{13}} \\ -\frac{2}{\sqrt{13}} & \frac{3}{\sqrt{13}} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} + \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

yoki

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \frac{3}{\sqrt{13}} & -\frac{2}{\sqrt{13}} \\ \frac{2}{\sqrt{13}} & \frac{3}{\sqrt{13}} \end{pmatrix} \begin{pmatrix} x - 5 \\ y - 3 \end{pmatrix},$$

ko‘rinishlarda bo‘lib, yangi koordinatalar sistemasidagi ellipsning kanonoik tenglamasi

$\frac{x'^2}{4}+\frac{y'^2}{1}=1$ ko‘rinishda bo‘ladi.

U holda bu tenglamaga x' va y' larni eski koordinatalar sistemasidagi ifodasini qo‘ysak:

$$\frac{(3x-2y-9)^2}{13\cdot 4}+\frac{(2x+3y-19)^2}{13}=1$$

$$(3x-2y-9)^2+4(2x+3y-19)^2=5$$

yoki

$$25x^2+36xy+40y^2-358x-420y+1473=0$$

so‘ralgan ellipsning tenglamasini hosil qilamiz.

8-misol.

Ellipsning katta o‘qi 12 ga teng, $x=\pm 9$ to‘g‘ri chiziqlar esa uning direktrisalari bo‘lsin. Ellipsning kanonik tenglamasini va ekssentrisitetini toping.

Yechish.

Ellipsning kanonik tenglamasini topish uchun a va b yarim o‘qlarni bilish kerak. Shart bo‘yicha $2a=12\Rightarrow a=6$

b yarim o‘qni ellipsni direktrisasini ifodalovchi formuladan foydalanib, quyidagicha topamiz:

$$x=\frac{a^2}{c}\Rightarrow c=\frac{a^2}{x}=\frac{36}{9}=4,\ b^2=a^2-c^2=36-16=20.$$

Ellips tenglamasi: $\frac{x^2}{36}+\frac{y^2}{20}=1$.

Ellips ekssentrisiteti: $\varepsilon = \frac{c}{a} = \frac{4}{6} = \frac{2}{3}$

9-misol.

Ellipsning kichik o‘qi 8 ga, ekssentrisiteti $\varepsilon = 0{,}6$ ga teng bo‘lsa ellipsning kanonik tenglamasini va direktrisa tenglamasini yozing.

Yechish.

Shartga ko‘ra $2b = 8 \Rightarrow b = 4$. Ekssentrisiteti $\varepsilon = \sqrt{\frac{a^2 - b^2}{a}} = \sqrt{1 - \left(\frac{b}{a}\right)^2}$

formulasiga asosan: $\varepsilon = \sqrt{1 - \frac{b^2}{a^2}} \Leftrightarrow \sqrt{1 - \frac{16}{a^2}} = 0{,}6$.

Bundan $a^2 = 25$.

Ellips tenglamasi, $\frac{x^2}{25} + \frac{y^2}{16} = 1$ ko‘rinishda bo‘ladi.

$c^2 = a^2 - b^2 \Leftrightarrow c = \sqrt{25 - 16} = 3$

direktrisasini ifodalovchi formuladan foydalanib direktrisa tenglamasini topamiz:

$$x = \pm \frac{a^2}{c} \Leftrightarrow x = \pm \frac{25}{3}$$

10-misol.

Asimptotalar orasidgi burchak 150^o va fokuslari absissalar o‘qida bo‘lib, ular orasidagi masofa $2c = 8\sqrt{3}$ bo‘lsa giperbola tenglamasini tuzing.

Yechish.

Agar giperbola asimptotalari o‘zaro 150^{o} li burchak tashkil etsa, ularda bittasi bilan Ox o‘qning musbat yo‘nalishi orasidagi burchak 30^{o} bo‘ladi.

Shuning uchun: $\frac{b}{a} = tg30^{0} \Rightarrow a = \sqrt{3}b$.

a va b larning qiymatlarini aniqlaymiz. Masala shartiga asosan: $c^2 = 48$.

Bundan: $\begin{cases} a = \sqrt{3}b \\ a^2 + b^2 = 48 \end{cases} \Rightarrow \begin{cases} 3b^2 + b^2 = 48 \\ a^2 = 3b^2 \end{cases} \Rightarrow \begin{cases} b^2 = 12 \\ a^2 = 36 \end{cases}$

Demak, izlanayotgan giperbola tenglamasi:

$$\frac{x^2}{36} - \frac{y^2}{12} = 1$$

11-Misol.

Quyidagi giperbolaning tenglamasini eng sodda shaklga keltiring:

$16x^2 - 4y^2 + 32x + 16y - 64 = 0$.

Yechish.

Bu berilgan tenglamani giperbolaning kanonik ko‘rinishdagi tenglamasiga keltiramiz.

$16x^2 + 32x = 16(x^2 + 2x + 1) - 16$ va $-4y^2 + 16y = -4(y^2 - 4y + 4) + 16$

ekanliklarini e‘tiborga olsak, berilgan tenglamaning ko‘rinishi:

$16(x+1)^2 - 4(y-2)^2 = 64$ yoki $\frac{(x+1)^2}{4} - \frac{(y-2)^2}{16} = 1$. Bu tenglama markazi $(-1;2)$ nuqtada, haqiqiy yarim o‘qi 2 ga mavhum yarim o‘qi esa 4 bo‘lgan giperbolaning kanonik tenglamasidir.

12-misol.

Uchi A (1 ; 3) nuqtada bolib, M (5 ; 7) nuqtadan o‘tuvchi, simmetriya o‘qi Ox o‘qqa parallel bo‘lgan parabola tenglamasini toping.

Yechish.

Shartga muvofiq, izlanayotgan parabola tenglamasi $(y-b)^2=2p(x-a)$ ko‘rinishda bo‘ladi, chunki M (5 ; 7) nuqta parabolaning uchidan o‘ngda joylashgan. Demak, parabolaning tarmoqlari o‘ngga yo‘nalgan. p parametrning qiymatini hisoblash uchun A va M nuqtalarning koordinatalarini $(y-b)^2=2p(x-a)$ tenglama qo‘yamiz:

$$(7-3)^2=2p(5-1)\Rightarrow 16=8p\Rightarrow p=2.$$

Topilgan $p=2$ qiymatni va A uchning koordinatalarini $(y-b)^2=2p(x-a)$ tenglamaga qo‘yib, izlanayotgan tenglamani hosil qilamiz:

$$(y-3)^2=4(x-1)$$

13-misol.

$y^2+4y-24x+76=0$ parabola uchi va fokusining koordinatalarini toping. Direktrisasining tenglamasini tuzing.

Yechish.

Parabola tenglamasini $(y-b)^2=2p(x-a)$ ko‘rinishga keltiramiz :

$$y^2+4y=24x-76\Rightarrow y^2+2\cdot 2y+2^2=24x-72\Rightarrow (y+2)^2=24(x-3).$$

Bundan parabola uchining koordinatalari: A (3;-2);

$$2p = 24 \Rightarrow p = 12.$$

Parabola uchidan fokusigacha bo‘lgan masofa $\frac{p}{2} = \frac{12}{2} = 6$ ga teng.

Fokusning abssissasi: $3 + \frac{p}{2} = 3 + 6 = 9.$

Fokus parabola uchidan o‘ngda joylashgan, chunki parabolaning tarmoqlari o‘ngga yo‘nalgan; fokusning ordinatasi parabola uchining ordinatasiga teng, chunki parabolaning o‘qi Ox o‘qqa parallel, u holda fokusning koordinatalari F (9 ; -2) bo‘ladi.

Parabolaning tarmoqlari o‘ngga yo‘nalgani uchun direktrisa parabola uchidan chaproqdan o‘tadi. U koordinatalar boshidan ham chapdan o‘tadi, chunki uchidan Oy o‘qqacha masofa 3 ga teng, uchidan direktrisagacha bo‘lgan masofa 6 ga teng. Direktrisaning abssissasi minus ishora bilan olingan ushbu ayirmaga teng: $\frac{p}{2} - 3 = 6 - 3 = 3$.

Shuning uchun direktrisaning tenglamasi: $x = -3$

<u>14-misol.</u>

Qutb koordinatalar sistemasida berilgan egri chiziq tenglamasi

$\rho = \frac{12}{\sqrt{10} - 4cos\theta}$ ni kanonik ko‘rinishiga keltiring.

<u>Yechish.</u>

Qutb koordinatalar sistemasida egri chiziq tenglamasi $\rho = \frac{ep}{1 - ecos\theta}$ ko‘rinishda bo‘lishligidan, berilgan tenglamani shu ko‘rinishga keltiramiz. Buning uchun surat va maxrajni $\sqrt{10}$ ga bo‘lib yuborsak:

$$\rho = \frac{\frac{12}{\sqrt{10}}}{1 - \frac{4}{\sqrt{10}} cos\theta}$$

Bu munosabatdan $e = \frac{4}{\sqrt{10}}$ va $ep = \frac{12}{\sqrt{10}}$, Demak $p = 3$, $e > 1$, ya'ni egri chizig'imiz giperbola. Giperbola uchun $p = a\left(e - \frac{1}{e}\right)$ ekanligidan

$3 = a\left(\frac{4}{\sqrt{10}} - \frac{\sqrt{10}}{4}\right)$, bu yerdan $a = 2\sqrt{10}$,

$c = ae = 8$, $b^2 = c^2 - a^2 = 64 - 40 = 24$

Demak so'ralgan giperbola kanonik tenglamasi:

$$\frac{x^2}{40} - \frac{y^2}{24} = 1$$

Tekislikda to'g'ri chiziqning umumiy tenglamasi

Tekislikda dekart koordinatalar sistemasi berilgan bo'lsin. A($x_1; y_1$), B($x_2; y_2$) nuqtalarni qaraymiz. Bu nuqtalardan bir xil masofada yotuvchi $C(x, y)$ nuqtalar toplami to'g'ri chiziq hosil qilib, AB o'rta perpendikulyari hisoblanadi. $|AC| = |CB|$ tenglikdan

$$\sqrt{(x - x_1)^2 + (y - y_1)^2} = \sqrt{(x - x_2)^2 + (y - y_2)^2}$$

ga ega bo'lamiz. Tomonlarini kvadratga oshirib, qavslarni ochamiz:

$$x^2 - 2xx_1 + x_1^2 + y^2 - 2yy_1 + y_1^2 = x^2 - 2xx_2 + x_2^2 + y^2 - 2yy_2 + y_2^2$$

o'xshash hadlarni ixchamlab, $2(x_2 - x_1)x + 2(y_2 - y_1)y + x_1^2 + y_1^2 - x_2^2 - y_2^2 = 0$ tenglamaga ega bo'lamiz.

Agar $A = 2(x_2 - x_1), B = 2(y_2 - y_1), C = x_1^2 + y_1^2 - x_2^2 - y_2^2$ belgilashlar kiritsak, tenglama

$$Ax + By + C = 0$$

ko'rinish oladi.

Bu tenglama to'g'ri chiziq umumiy tenglamasi deyiladi.

Masalan, P(4;1), Q(-1;2) nuqtalardan bir xil masofoda yotuvchi to'g'ri chiziq tenglamasini topamiz.

$$\sqrt{(x - 4)^2 + (y - 1)^2} = \sqrt{(x + 1)^2 + (y - 2)^2},$$

$$x^2 - 8x + 16 + y^2 - 2y + 1 = x^2 + 2x + 1 + y^2 - 4y + 4$$

o'xshash hadlarni ixchamlab, $10x - 2y - 12 = 0$ yoki

$5x - y - 6 = 0$ tenglamaga egamiz.

To‘g‘ri chiziq umumiy tenglamasidagi A, B, C-sonlari tenglama koeffitsiyentlari deyilib, quyidagicha xususiy hollar bo‘lishi mumkin:

1°. $A \neq 0, B \neq 0, C = 0$ bu holda tenglama $Ax + By = 0$ ko‘rinish olib, koordinata boshidan o‘tuvchi to‘g‘ri chiziq bo‘ladi, chunki, O(0;0) nuqta tenglamani qanoatlantiradi.

2°. $A \neq 0, B = 0, C \neq 0$. Bu holda tenglama $Ax + C = 0$ bo‘lib, uni

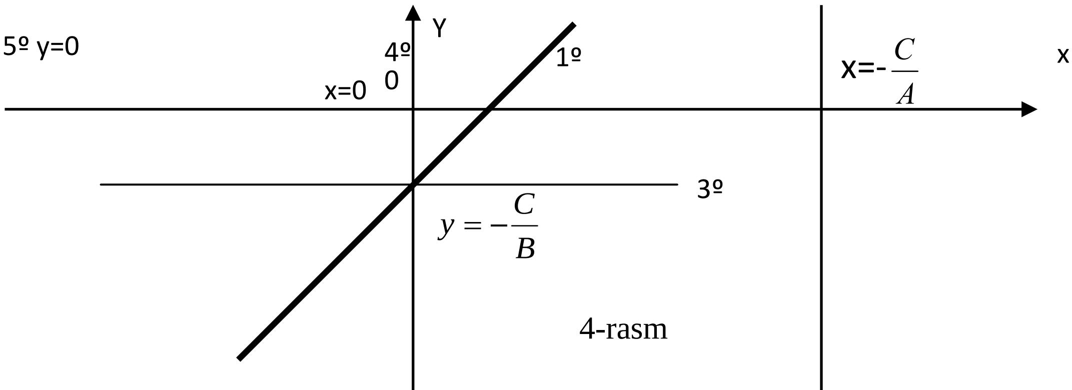
$x = \frac{C}{A}$ ko‘rinishda yozish mumkin. Demak, abssissa biror o‘zgarmas songa teng, ordinata ixtiyoriy qiymat qabul qiladi. Bu to‘g‘ri chiziqning Oy o‘qiga parallelligini bildiradi.

3°. $A = 0, B \neq 0, C \neq 0$. Bu holda $By + C = 0$ hosil bo‘lib, $y = -\frac{C}{B}$ tarzida yoziladi. To‘g‘ri chiziq Ox o‘qiga parallel.

4°. $A \neq 0, B = C = 0$. Tenglama $Ax = 0$ ko‘rinishida bo‘lib,

$x = 0$ tenglama kelib chiqadi va Oy o‘qini ifodalaydi.

5°. $, B \neq 0, A = C = 0$ Bu holda $y = 0$ kelib chiqadi va bu tenglama Ox o‘qini bildiradi



4-rasm

To‘g‘ri chiziqning burchak koeffitsiyentli tenglamasi.

Dekart koordinatalar sistemasida ordinatalar o‘qidan O(0;0) dan hisoblanganda uzunligi b ga teng kesma ajratadigan, abssissa o‘qi bilan α burchak hosil qiluvchi to‘g‘ri chiziqni qaraymiz. To‘g‘ri chiziq ixtiyoriy C(x;y) nuqtasini olamiz.

Hosil bo‘lgan to‘g‘ri burchakli uchburchakdan $\frac{y-b}{x} = tg\alpha$ ekanligini topamiz. Bu tenlamadagi $tg\alpha$ to‘g‘ri chiziqning burchak koeffitsiyenti deyiladi va k bilan belgilanadi:
$k = tg\alpha.$

To‘g‘ri chiziq tenglamasi $\frac{y-b}{x} = k$ ko‘rinish oladi. Undan to‘g‘ri chiziqning burchak koeffitsiyentli tenglamasi deb ataluvchi

$$y = kx + b$$

tenglamani olamiz .

To‘g‘ri chiziq holati k va b koeffitsiyentlari bilan to‘la aniqlanadi .To‘g‘ri chiziq umumiy $Ax + By + C = 0$ tenglamasidan burchak koeffitsiyentlisiga o‘tish uchun bu tenglamani y ga nisbatan yechish kifoya .

$$y = -\frac{A}{B}x - \frac{C}{B}$$

Bunda $k = -\frac{A}{B}$, $b = -\frac{C}{B}$ belgilashlar kiritilsa, tenglama

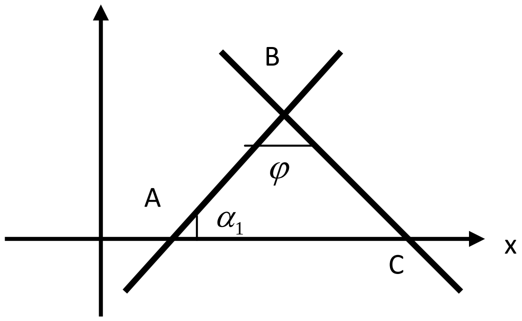
$$y = kx + b$$

ko‘rinishga keladi.

Ma‘lumki, $y = kx + b$ funksiya chiziqli deyilar edi. Demak, chiziqli funksiya grafigi to‘g‘ri chiziq bo‘lar ekan. $b = 0$ bo‘lsa $y = kx$ hosil bo‘lib, x va y o‘zaro proporsional, k-esa proporsionallik koeffitsiyenti deyiladi .

Ikki to‘g‘ri chiziq orasidagi burchak

Tekislikda ikki $y = k_1x + b_1, y = k_2x + b_2$ to‘g‘ri chiziqlar orasidagi φ burchakni topish masalasini ko‘ramiz, bunda $k_1 = \text{tg}\,\alpha_1$, $k_2 = \text{tg}\,\alpha_2$.



5-rasm

Uchburchak tashqi burchagi xossasidan: $\alpha_2 = \varphi + \alpha_1$.

Izlanayotgan burchak $\varphi = \alpha_2 - \alpha_1$ burchakni esa burchak koeffitsiyentlari orqali topish qulay: $\operatorname{tg}\varphi = \operatorname{tg}(\alpha_2 - \alpha_1) = \frac{\operatorname{tg}\alpha_2 - \operatorname{tg}\alpha_1}{1+\operatorname{tg}\alpha_1 \cdot \operatorname{tg}\alpha_2} = \frac{k_2 - k_1}{1+k_1 \cdot k_2}$

Berilgan to‘g‘ri chiziqlar orasidagi o‘tkir burchakni topish uchun $\operatorname{tg}\varphi = \left|\frac{k_2 - k_1}{1+k_1 \cdot k_2}\right|$ ko‘rinishida yozish kifoya.

Masalan, $y = -2x$ va $y = 3x - 4$ to‘g‘ri chiziqlar uchun $\operatorname{tg}\varphi = \frac{3-(-2)}{1+(-2)\cdot 3} = -1$, demak ular orasidagi o‘tmas burchak $\frac{3\pi}{4}$ ga, o‘tkir burchak esa $\frac{\pi}{4}$ ga teng.

1. Agar to‘g‘ri chiziqlar parallel bo‘lsa, $\varphi = 0$ yoki $\varphi = \pi$ bo‘lib $k_2 - k_1 = 0$ kelib chiqadi. Demak, to‘g‘ri chiziqlar parallellik sharti $k_2 = k_1$ dir .

2. To‘g‘ri chiziqlar o‘zaro perpendikulyar bo‘lsa, $\pi = \frac{\pi}{2}$, $\operatorname{tg}\frac{\pi}{2} = \infty$, $1 + k_1 k_2 = 0$ shart kelib chiqadi. Demak, to‘g‘ri chiziqlar perpendikulyarlik sharti $k_2 = -\frac{1}{k_1}$ dir.

Agar to‘g‘ri chiziqlar $A_1x + B_1y + C_1 = 0, A_2x + B_2y + C_2 = 0$ formulalar bilan berilsa, ularni y ga nisbatan yechib $k_1 = -\frac{A_1}{B_1}$,

$k_2 = -\frac{A_2}{B_2}$ bo‘lishini topamiz .

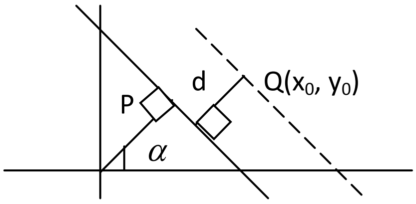
Demak, to‘g‘ri chiziqlar umumiy tenglamasi bilan berilsa,

$$tg\varphi = \frac{-\frac{A_2}{B_2} - \left(-\frac{A_1}{B_1}\right)}{1 + \left(-\frac{A_1}{B_1}\right) \cdot \left(-\frac{A_2}{B_2}\right)} = \frac{A_1B_2 - A_2B_1}{A_1A_2 + B_1B_1}$$

formulaga ega bo‘lamiz. Unda to‘g‘ri chiziqlar parallel bo‘lishi uchun $A_1B_2 - A_2B_1 = 0$, yani $\frac{A_1}{A_2} = \frac{B_1}{B_2}$ bo‘lishi, perpendikulyar bo‘lishi uchun esa $A_1A_2 + B_1B_1 = 0$ bo‘lishi kerak.

Nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa

Normal tenglamasi bilan berilgan $xcos\alpha + ysin\alpha - p = 0$ to‘g‘ri chiziq va unda yotmagan biror Q($x_0; y_0$) nuqta berilgan bo‘lsin. Q($x_0; y_0$) nuqtadan berilgan to‘g‘ri chiziqqacha bo‘lgan d masofani topish masalasini qaraymiz. Q($x_0; y_0$) dan o‘tib, $xcos\alpha + ysin\alpha - p = 0$ ga parallel to‘g‘ri chiziqni $xcos\alpha + ysin\alpha - q = 0$ tenglama bilan beriladi, bunda $q = p + d$, lekin $q = x_0cos\alpha + y_0sin\alpha$ ekanligidan



$$d = q - p = x_0cos\alpha + y_0sin\alpha - p$$

kelib chiqadi. Agar $q < p$ bo‘lsa $d = q - p$ bo‘lishini hisobga olsak,

$$d = |x_0cos\alpha + y_0sin\alpha - p|$$

formulaga ega bo‘lamiz.

Agar to‘g‘ri chiziq $Ax + By + C = 0$ umumiy tenglamasi bilan berilsa, masofa formulasi $d = \frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$ ko‘rinishida bo‘ladi. Masalan, A(4;2), B(1;1) dan $3x - 4y - 4 = 0$ gacha masofani hisoblaymiz. $d_A = \frac{|3\cdot4-4\cdot2-4|}{\sqrt{3^2+(-4)^2}} = 0$, $d = \frac{|3\cdot1-4\cdot1-4|}{\sqrt{3^2+(-4)^2}} = \frac{|-5|}{5} = 1$, demak, A to‘g‘ri chiziqqa tegishli, B nuqta esa to‘g‘ri chiziqdan 1 birlik uzoqlikda joylashgan.

Normal tenglamasi bilan berilgan $xcos\alpha + ysin\alpha - p = 0$ va $xcos\alpha + ysin\alpha - q = 0$ to‘g‘ri chiziqlar orasidagi masofa $d = |p - q|$ bo‘lishi tushunarli. Agar to‘g‘ri chiziqlar $xcos\alpha + ysin\alpha - p = 0$, $\lambda xcos\alpha + \lambda ysin\alpha - q = 0$ tenglamalar bilan berilsa, masofa $d = \left|p - \frac{q}{\lambda}\right|$ bo‘ladi. Demak ikki parallel $A_1x + B_1y + C_1 = 0$

$\lambda A_1x + \lambda B_1y + C_1 = 0$ to‘g‘ri chiziqlar orasidagi masofa

$d = \left|\frac{C_1-\frac{C_2}{\lambda}}{\sqrt{A^2+B^2}}\right|$ formula yordamida topiladi.

Masalan, $6x + 8y + 7 = 0, 3x - 4y - 7 = 0$ to‘g‘ri chiziqlar o‘zaro parallel, ularni

$3x + 4y + \frac{7}{2} = 0$, $3x - 4y - 7 = 0$ tarzida yozsak,

$d = \frac{\left|\frac{7}{2}+7\right|}{\sqrt{3^2+4^2}} = \frac{21}{10} = 2,1$ ekanligi kelib chiqadi .

Bazi hollarda birinchi tog‘ri chiziqdan biror nuqta tanlab ikkinchisigacha masofani hisoblasa ham bo‘ladi, masalan $C(0;\frac{7}{8})$ nuqta birinchi to‘g‘ri chiziqqa tegishli, undan ikkinchi to‘g‘ri chiziqqacha masofa esa

$$d = \frac{\left|3\cdot 0-4\cdot\frac{7}{8}-7\right|}{\sqrt{3^2+(-4)^2}} = \frac{\left|-\frac{7}{2}-7\right|}{5} = \frac{21}{10} = 2.1.$$

Masofa formulasi yordamida ikki kesishuvchi $A_1x + B_1y + C_1 = 0$ va $A_2x + B_2x + C_2 = 0$ to‘g‘ri chiziq bissektrisalari tenglamasini keltirib chiqaramiz.

Bissektrisadagi ixtiyoriy C(x;y) nuqtadan berilgan to‘g‘ri chiziqqacha masofalar tengligidan $\frac{|A_1x+B_1y+C_1|}{\sqrt{{A_1}^2+{B_1}^2}} = \frac{|A_2x+B_2x+C_2|}{\sqrt{{A_2}^2+{B_2}^2}}$ yoki $\frac{A_1x+B_1y+C_1}{\sqrt{{A_1}^2+{B_1}^2}} = \pm\frac{A_2x+B_2y+C_2}{\sqrt{{A_2}^2+{B_2}^2}}$ kelib chiqadi.

<u>Misol.</u>

Tekislikda ABC uchburchakning uchlarining koordinatalari berilgan, ya‘ni A(2, 5), B(-3, 1), C(0, 4)

a) AB tomonning tenglamasi tuzilsin:

Ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasini topish radiusi

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}$$

Shunga ko‘ra A nuqtaning koordinatalari x_1, y_1B nuqtasini x_2, y_2bo‘lsa, u holda

$$\frac{x-2}{-3-2} = \frac{y-5}{1-5}$$

$\frac{x-2}{-5} = \frac{y-5}{-4}$ yoki AB tenglamaning umumiy tenglamasi

$-4x + 8 = -5y + 25$

$4x - 5y + 17 = 0$

b) CH balandlik tenglamasini topish uchun, balandlik AB tomonga tushirilgan, shuning uchun $AB \perp CH$ Perpendikulyar shartidan foydalanib, ya‘ni $x_2 = -\frac{1}{k_1}$

$AB: 4x - 5y + 17 = 0$

$k_1 = \frac{4}{5}$, $k_2 = -\frac{5}{4}$

CH balandlik tenglamasi H nuqatadan o‘tib, burchak koeffitsiyenti

$k_2 = -\frac{5}{4}$

$$y - y_0 = k(x - x_0)$$

x_0, y_0 - C nuqtaning koordinatalari

$y - 4 = -\frac{5}{4}\ (x - 0)$

$4y - 16 = -5x$

$5x + 4y - 16 = 0$ CH tenglamasi

c) AM medianasining tenglamasi tuzilgan, M nuqta BC tomonning o‘rtasi.

Yechish:

BC ni o‘rtasi, M nuqta koordinatalari

$$\frac{x_1+x_2}{2} = X_M \qquad \frac{y_2+y_2}{2} = Y_M$$

Formula orqali topamiz.

$$X_M = \frac{-3+0}{2}, \quad Y_M = \frac{1+4}{2}$$

M $(-\frac{3}{2};\ \frac{5}{2})$

A nuqta berilgan, M nuqtani topdik, ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasini topish uchun

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

formuladan foydalanamiz.

A (2;5) nuqta

$$\frac{x-2}{-\frac{3}{2}-2} = \frac{y-5}{\frac{5}{2}-5} \Rightarrow \frac{x-2}{-\frac{7}{2}} = \frac{y-5}{\frac{5}{-2}} \Rightarrow$$

$5x - 10 = 7y - 35$

$5x - 7y + 25 = 0$ AM to‘g‘ri chiziq tenglamasi

d) N nuqta tenglamasi
AM va CH to‘g‘ri chiziqlarning kesishish nuqtasidir.

$$\begin{cases} 5x-7y+25=0 & (AM) \\ 5x+4y-16=0 & (CH) \end{cases}$$

$$-11y=41$$

$$y=-\frac{41}{11}$$

$$5x=16-4y$$

$$x=\frac{16}{5}-\frac{4y}{5}=\frac{16}{5}+4\cdot\frac{41}{11}=\frac{176+820}{55}=\frac{996}{55}=18\frac{6}{55}$$

$$N(18\frac{6}{55};-3\frac{8}{11})$$

e) C nuqtadan o‘tuvchi va AB tomonga parallel bo‘lgan to‘gri chiziq tenglamasini tuzilsin.
Yechish:
C(0;4) $AB: 4x-5y+17=0$
Izlanyotgan to‘g‘ri chiziq AB tomonga parallel bo‘lgani uchun ularning burchak koeffitsiyentlari teng bo‘ladi, ya‘ni $k_1=k_2$.
AB to‘g‘ri chiziqning burchak koeffitsiyenti
$5y=4x+17$
$y=\frac{4x}{5}+\frac{17}{5}$; $k_1=\frac{4}{5}$.
$y-y_0=\frac{4}{5}(x-x_0)$ $y-4=\frac{4}{5}(x-0)$ $5\cdot(y-4)=4x$ $4x-5y+20=0$ C nuqtadan o‘tuvchi va AB to‘g‘ri chiziqqa parallel bo‘lgan to‘g‘ri chiziq tenglamasi.

T O P SH I R I Q L A R

1-topshiriq

Ellipsning tenglamasi tuzilsin, agar u koordinata o‘qlarining birini $A(x_1, y_1)$ va $B(x_2, y_2)$ nuqtalarda kesib o‘tsa, hamda ikkinchi koordinata o‘qini esa $C(x_3, y_3)$da urinib o‘tsa. Ellipsning o‘qlari koordinata o‘qlariga parallel va simmetrikdir.

1.(3, 0), (9, 0), (0, 5);

2. (0, -15), (0, -3), (15, 0);

3. (2, 0), (12, 0), (0, -4);

4. (0, 6), (0, 2), (9, 0);

5. (3, 0), (15, 0), (0, 5);

16. (0, 2), (0, 8), (-16, 0);

17. (5, 0), (11, 0), (10, 0);

18. (0 ,10), (0, 18), (-15, 0);

19. (1, 0), (3, 0), (0, 6);

20. (0, -1), (0, -5), (10, 0);

6. (0, 6), (0, 14), (7, 0);
7. (-2, 0), (-8, 0), (0,16);
8. (0, 6), (0, 16), (-8, 0);
9. (-9, 0), (-1, 0), (0, 3);
10. (0, -3), (0, -1), (-6, 0);
11. (-12, 0), (-4, 0), (0, 9);
12. (0, 3), (0, 15), (10, 0);
13. (2, 0), (8, 0), (0, -4);
14. (0, 4), (0, 12), (-6, 0);
15. (-16, 0), (-8, 0), (0, 8);
21. (4, 0), (10, 0), (0,-8);
22. (0, -9), (0, -3), (-5, 0);
23. (6, 0), (14, 0), (0, -7);
24. (0, -1), (0, -9), (6, 0);
25. (-6, 0), (-2, 0), (0, -9);
26. (0, -20), (0, -4), (15, 0);
27. (-12, 0), (-2, 0), (0, 12);
28. (0, -8), (0, -2), (4, 0);
29. (-7, 0), (-3, 0), (0, -7);
30. (0, -6), (0, -14), (-21, 0);

2-topshiriq

Ellipsning tenglamasi tuzilsin, agar uning o‘qlari koordinata o‘qlariga parallel bo‘lsa, hamda Ox va Oy o‘qiga mos ravishda $A(x_1, y_1)$ va $B(x_2, y_2)$nuqtalarda urinib o‘tsa

1. (2, 0), (0, 9);
2. (4, 0), (0, 6);
3. (-5, 0), (0, 11);
4. (7, 0), (0, -8);
5. (1, 0), (0, -2);
6. (-6, 0), (0, 12);
7. (4, 0), (0, -5);
8. (3, 0), (0, 7);
16. (-1, 0), (0, 4);
17. (9, 0), (0, 3);
18. (2, 0), (0, 8);
19. (-3, 0), (0, 15);
20. (11, 0), (0, 5);
21. (10, 0), (0, -6);
22. (-6, 0), (0, -11);
23. (6, 0), (0, 8);

9. (-8, 0), (0, 3);

10. (10, 0), (0, 6);

11. (5, 0), (0, 2);

12. (-9, 0), (0, -7);

13. (11, 0), (0, 5);

14. (3, 0), (0, 8);

15. (-12, 0), (0, -9);

24. (12, 0), (0, -5);

25. (4, 0), (0, 16);

26. (7, 0), (0, 4);

27. (-9, 0), (0, -13);

28. (7, 0), (0, -14);

29. (10, 0), (0,12);

30. (-8, 0), (0, 17);

3-topshiriq

Ellipsning tenglamasi tuzilgan, agar uning uchlari $A(x_1, y_1)$ va $B(x_2, y_2)$ nuqtalarda bo‘lsa, hamda bu ellips $Ox(Oy)$ o‘qida p-uzunlikka teng bo‘lgan vatarni kesib o‘tsa.

1. (0, 6), (0, -2), $p = 6$;

2. (-5, 0), (11, 0), $p =$55/4;

3 .(0, 8), (0, -4), $p =$4;

4. (-7, 0), (5, 0), $p =$7;

5. (0, 9), (0, -3), $p =$6;

6. (-14, 0), (2, 0), $p =$14;

7. (0, 8), (0, -14), $p =$8;

8. (-10, 0), (2, 0), $p =$10;

9. (0, 4), (0, -8), $p =$8;

16. (1, 0), (-7, 0), $p =$14;

17. (0, -8), (0, 14), $p =$56;

18. (-2, 0), (8, 0), $p =$8;

19. (0, -16), (0, 4), $p =$8;

20. (10, 0), (-2, 0), $p =$20;

21. (0, -2), (0, 8), $p =$32;

22. (-16, 0), (4, 0), $p =$24;

23. (0, 6), (0, -2), $p =$30;

24. (-4, 0), (8, 0), $p =$4;

10. (16, 0), (-4, 0), p =16;

11. (0, -6), (0, 2), p =12;

12. (-4, 0), (12, 0), p =6;

13. (0, -9), (0, 3), p =12;

14. (5, 0), (-11, 0), p =55;

15. (0, 2), (0, -8), p =8;

25. (0, 9), (0, -3), p =30;

26. (-1, 0), (7, 0), p =14;

27. (0, -4), (0, 12), p =6;

28. (-8, 0), (4, 0), p =8;

29. (0, -14), (0, 2), p =14;

30. (-2, 0), (14, 0), p =14;

4-topshiriq

Asimptotalari $y = \pm\frac{a}{b}x$ dan iborat bo‘lgan va $A(x_0, y_0)$ nuqtadan o‘tuvchi giperbolaning tenglamasi tuzilsin.

1. $A(4,\ 3),\ y = \pm\frac{3}{2}x$;
2. $A(3,\ 2),\ y = \pm 2x$;
3. $A(4,\ 1),\ y = \pm\frac{1}{2}x$;
4. $A(3,\ 1),\ y = \pm\frac{2}{3}x$;
5. $A(6,\ 1),\ y = \pm\frac{1}{3}x$;
6. $A(\ 8,\ 9),\ y = \pm\frac{3}{2}x$;
7. $A(4,\ 2),\ y = \pm 2x$;
8. $A(6,\ 2),\ y = \pm\frac{1}{2}x$;
9. $A(6,\ 2),\ y = \pm\frac{2}{3}x$;
10. $A(9,\ 2),\ y = \pm\frac{1}{3}x$;
11. $A(10,\ 9),\ y = \pm\frac{3}{2}x$;
12. $A(5,\ 2),\ y = \pm 2x$;
16. $A(6,\ 3),\ y = \pm\frac{3}{2}x$;
17. $A(5,\ 6),\ y = \pm 2x$;
18. $A(8,\ 3),\ y = \pm\frac{1}{2}x$;
19. $A(9,\ 4),\ y = \pm\frac{2}{3}x$;
20. $A(15,\ 3),\ y = \pm\frac{1}{3}x$;
21. $A(8,\ 3),\ y = \pm\frac{3}{2}x$;
22. $A(4,\ 4),\ y = \pm 2x$;
23. $A(8,\ 2),\ y = \pm\frac{1}{2}x$;
24. $A(12,\ 2),\ y = \pm\frac{2}{3}x$;
25. $A(15,\ 4),\ y = \pm\frac{1}{3}x$;
26. $A(4,\ 6),\ y = \pm 2x$;
27. $A(12,\ 4),\ y = \pm\frac{2}{3}x$;

13. $A(6,\ 1),\ y=\pm\frac{1}{2}x$;

14. $A(9,\ 2),\ y=\pm\frac{2}{3}x$;

15. $A(12,\ 2),\ y=\pm\frac{1}{3}x$;

28. $A(18,\ 5), y=\pm\frac{1}{3}x$;

29. $A(6,\ 4),\ y=\pm 2x$;

30. $A(12,\ 6),\ y=\pm\frac{2}{3}x$;

5-topshiriq

Asimptotalari $y=\pm\frac{b}{a}x$ dan iborat bo‘lgan va $A(x_0, y_0)$nuqtadan o‘tuvchi giperbolaning tenglamasi tuzilsin.

1. $A(5,\ 12),\ y=\pm 2x$;
2. $A(2,\ 3),\ y=\pm\frac{1}{2}x$;
3. $A(3,\ 4), y=\pm\frac{2}{3}x$;
4. $A(3,\ 2),\ y=\pm\frac{1}{3}x$;
5. $A(2,\ 6),\ y=\pm\frac{3}{2}x$;
6. $A(4,\ 12),\ y=\pm 2x$;
7. $A(2,\ 4),\ y=\pm\frac{1}{2}x$;
8. $A(3,\ 6),\ y=\pm\frac{2}{3}x$;
9. $A(3,\ 3),\ y=\pm\frac{1}{3}x$;
10. $A(2,\ 9),\ y=\pm\frac{3}{2}x$;
11. $A(3,\ 12),\ y=\pm 2x$;
12. $A(2,\ 5),\ y=\pm\frac{1}{2}x$;
13. $A(3,\ 8),\ y=\pm\frac{2}{3}x$;
16. $A(2, 12),\ y=\pm 2x$;
17. $A(4, 3),\ y=\pm\frac{1}{2}x$;
18. $A(6,\ 3),\ y=\pm\frac{1}{3}x$;
19. $A(2,\ 12),\ y=\pm\frac{3}{2}x$;
20. $A(4,\ 10),\ y=\pm 2x$;
21. $A(4,\ 4),\ y=\pm\frac{1}{2}x$;
22. $A(6,\ 4),\ y=\pm\frac{1}{3}x$;
23. $A(4,\ 12),\ y=\pm\frac{3}{2}x$;
24. $A(3,\ 10),\ y=\pm 2x$;
25. $A(4,\ 6),\ y=\pm\frac{1}{2}x$;
26. $A(9,\ 4),\ y=\pm\frac{1}{3}x$;
27. $A(6,\ 12),\ y=\pm\frac{3}{2}x$;
28. $A(2,\ 8),\ y=\pm 2x$;

14. $A(3,\ 5),\ y = \pm\frac{1}{3}x$;

15. $A(4,\ 9),\ y = \pm\frac{3}{2}x$;

29. $A(9,\ 5),\ y = \pm\frac{1}{3}x$;

30. $A(3,\ 8),\ y = \pm 2x$;

6-topshiriq

Asimptotalari l_1 va l_2 to‘g‘ri chiziqdan iborat bo‘lgan va $A(x_0, y_0)$ nuqtadan o‘tuvchi giperbolaning tenglamasi tuzilsin.

1. A(0, 0), x = 1, y = 2;
2. A(1, 3), x = -3, y = 2;
3. A(2, 3), x = 5, y = -2;
4. A(1, 5), x = -1, y = 4;
5. A(4, -1), x = 2, y = 0;
6. A(4, 1), x = 2, y = 3;
7. A(3, 2), x = -2, y = 4;
8. A(4, 2), x = 3, y = 0;
9. A(1, 0), x = -1, y = -2;
10. A(-1, -2), x = 0, y = -3;
11. A(-2, 3), x = -4, y = 2;
12. A(3, -2), x = 6, y = -1;
13. A(1, 1), x = 0, y = -3;
14. A(-2, 1), x = -6, y = 3;
15. A(1, 3), x = -5, y = 1;
16. A(6, 1), x = =4, y = -2;
17. A(2, 1), x = 4, y = -2;
18. A(0, 2), x = -1, y = -1;
19. A(1, 3), x = 2, y = 0;
20. A(8, 5), x = -2, y = 2;
21. A(4, 5), x = -3, y = 4;
22. A(5, -1), x = 7, y = -2;
23. A(1, 8), x = -7, y = 3;
24. A(-1, 5), x = -2, y = 7;
25. A(1, -1), x = -1, y = 2;
26. A(1, -2), x = 2, y = -1;
27. A(4, 5), x = 3, y = 3;
28. A(0, 2), x = 1, y = 1;
29. A(3, 8), x = 2, y = -2;
30. A(-2, 1), x = -5, y = 3;

7-topshiriq.

$A(x, y)$ va $B(x, y)$ nuqtalardan o‘tib, Ox (yokiOy) o‘q simmetriya o‘qidan iborat bo‘lgan, ikkinchi koordinata o‘qi esa uchiga urinma bo‘lib o‘tadigan ikkinchi tartibli chiziq tenglamasi tuzilsin.

1. Ox, A(5, 6), B(9, -6);
2. Ox, A(3, 4), B(9, -4);
3. Ox, A(8, 2), B(-8, 8);
4. Ox, A(6, 4), B(-6, 12);
5. Ox, A(4, 4), B(10, -4);
6. Ox, A(-1, 2), B(-11, -2);
7. Oy, A(-5, 2), B(5, 10);
8. Ox, A(2, 4), B(8, -4);
9. Oy, A(7, -2), B(-7, -14);
10. Oy, A(1, 1), B(-1, 5);
11. Ox, A(2, -3), B(6, 3);
12. Oy, A(-3, -2), B(3, -18);
13. Ox, A(-10, 5), B(-2, -5);
14. Oy, A(2, 3), B(-2, 5);
15. Ox, A(-8, 2), B(-4, -2);
16. Oy, A(6, -5), B(-6, -15);
17. Ox, A(-11, 5), B(-5, -5);
18. Ox, A(1, -3), B(9, 3);
19. Oy, A(5, -3), B(-5, -15);
20. Ox, A(-12, 3), B(-4, -3);
21. Oy, A(-6, 6), B(6, 18);
22. Oy, A(-12, -9), B(12, -3);
23. Ox, A(-15, -9), B(-5, 9);
24. Oy, A(3, 4), B(-3, 12);
25. Ox, A(-18, -6), B(-8, 6);
26. Ox, A(2, -9), B(6, 9);
27. Oy, A(6, 2), B(-6, 18);
28. Ox, A(-12, 6), B(-2, -6);
29. Oy, A(-8, 6), B(8, 12);
30. Ox, A(-1, 5), B(-5, -5);

8-topshiriq

Berilgan nuqtadan o‘tib, simmetriya o‘qi $Ox(Oy)$ o‘qqa parallel bo‘lgan, ikkinchi koordinata o‘qi esa vatarni q masofada kesib o‘tadigan parabola tenglamasi tuzilsin.

1. Oy, (-5, 12), q=$12\sqrt{2}$;
2. Ox, (-4, 3), q=8;
3. Oy, (3, 5), q=10;
4. Ox, (3, -1), q=12;
5. Oy, (-3, -8), q=8;
16. Oy, (9, -3), q=12;
17. Ox, (4, -5), q=16;
18. Oy, (9, 10), q=$4\sqrt{10}$;
19. Oy, (-2, 4), q=8;
20. Ox, (5, 2), q=$4\sqrt{5}$;

6. Ox, (-10, 1), q=20;

7. Ox, (4, 8), q=12;

8. Oy, (3, 2), q=4;

9. Ox, (-5, -1), $q=10\sqrt{2}$;

10. Oy, (1, -12), q=12;

11. Oy, (1, -3), q=6;

12. Ox, (7, -3), q=28;

13. Oy, (-8, 10), $q=8\sqrt{5}$;

14. Ox, (-12, 3), q=24;

15. Ox, (-1, -3), q=4;

21. Oy, (-11, -6), $q=6\sqrt{2}$;

22. Ox, (-6, 2), q=12;

23. Ox, (3, 2), q=12;

24. Oy, (-6, 4), q=16;

25. Ox, (8, 5), q=16;

26. Oy, (6, -8), q=8;

27. Ox, (-4, -3), q=8;

28. Oy, (7, 8), q=8;

29. Ox, (6, 4), q=12;

30. Oy, (-7, -4), q=8;

9-topshiriq

Bir uchi $A(x,y)$ nuqtada bo‘lib, haqiqiy o‘qi $Ox(Oy)$ o‘qqa parallel bo‘lgan, ikkinchi o‘qi esa uzunligiga teng bo‘lgan vatarni kesib o‘tuvchi teng yonli giperbola tenglamasi tuzilsin:

1. Oy, A(-7, 8), q=24;

2. Ox, A(-2, 3), q=8;

3. Oy, A(3, 4), q=16;

4. Ox, A(-2, 2), q=8;

5. Oy, A(1, 1), q=14;

6. Ox, A(-1, -2), q=6;

7. Oy, A(1, -1), q=6;

8. Ox, A(4, 2), q=16;

16. Oy, A(4, 2), q=16;

17. Ox, A(2, -7), q=12;

18. Oy, A(10, -10), q=40;

19. Ox, A(1, -3), q=10;

20. Oy, A(-1, 2), q=12;

21. Ox, A(-8, -8), q=32;

22. Ox, A(10, -17), q=40;

23. Oy, A(11, -6), q=48;

9. Oy, A(-8, 4), q=24;

10. Oy, A(-2, -1), q=10;

11. Ox, A(-1, -6), q=10;

12. Oy, A(1, 1), q=6;

13. Oy, A(-4, 9), q=6;

14. Ox, A(-4, 4), q=16;

15. Oy, A(4, -2), q=12;

24. Ox, A(1, 2), q=10;

25. Ox, A(-5, 11), q=30;

26. Oy, A(-9, 10), q=40;

27. Oy, A(3, 2), q=8;

28. Ox, A(-8, 9), q=32;

29. Ox, A(4, -13), q=24;

30. Oy, A(-1, -1), q=6;

10-topshiriq.

Fokusi $F(x, y)$, unga mos bo‘lgan direktrisa va ekssentrisitetlari ma’lum bo‘lgan ikkinchi tartibli chiziq tenglamasi tuzilsin. Ikkinchi fokus va ikkinchi direktrisalari ham aniqlansin:

1. $F(4,0)$, $\delta\colon\ x = 5,\ e = \frac{\sqrt{3}}{2}$;

2. $F(1,4)$, $\delta\colon\ x = \frac{44}{3},\ e = \frac{3}{5}$;

3. $F(4,0)$, $\delta\colon\ x = 8,\ e = \frac{1}{\sqrt{3}}$;

4. $F(-1,10)$, $\delta\colon\ x = \frac{85}{7},\ e = \frac{7}{8}$;

5. $F(2,-1)$, $\delta\colon\ x = 5,\ e = \frac{2}{\sqrt{10}}$;

6. $F(1,6)$, $\delta\colon\ x = \frac{54}{5},\ e = \frac{5}{7}$;

7. $F(3,-1)$, $\delta\colon\ x = 5,\ e = \frac{1}{\sqrt{2}}$;

8. $F(-1,3)$, $\delta\colon\ x = \frac{45}{4},\ e = \frac{5}{7}$;

9. $F(1,0)$, $\delta\colon\ x = 2,\ e = \frac{2}{\sqrt{6}}$;

10. $F(4,0)$, $\delta\colon\ x = 5,\ e = \frac{\sqrt{3}}{2}$;

11. $F(1,4)$, $\delta\colon\ x = \frac{44}{3},\ e = \frac{3}{5}$;

12. $F(4,0)$, $\delta\colon\ x = 8,\ e = \frac{1}{\sqrt{3}}$;

13. $F(-1,10)$, $\delta\colon\ x = \frac{85}{7},\ e = \frac{7}{8}$;

14. $F(2,-1)$, $\delta\colon\ x = 5,\ e = \frac{2}{\sqrt{10}}$;

15. $F(1,6)$, $\delta\colon\ x = \frac{54}{5},\ e = \frac{5}{7}$;

21. $F(3,-1)$, $\delta\colon\ x = 5,\ e = \frac{1}{\sqrt{2}}$;

22. $F(-1,3)$, $\delta\colon\ x = \frac{45}{4},\ e = \frac{5}{7}$;

23. $F(1,0)$, $\delta\colon\ x = 2,\ e = \frac{2}{\sqrt{6}}$;

24. $F(-1,7)$, $\delta\colon\ x = \frac{46}{5},\ e = \frac{5}{6}$;

25. $F(1,1)$, $\delta\colon\ x = \frac{3}{2},\ e = \frac{2}{\sqrt{5}}$;

26. $F(2,2)$, $\delta\colon\ x = 7,\ e = \frac{2}{3}$;

27. $F(3,1)$, $\delta\colon\ x = \frac{13}{3},\ e = \frac{3}{\sqrt{13}}$;

28. $F(0,5)$, $\delta\colon\ x = \frac{29}{4},\ e = \frac{4}{5}$;

29. $F(6,0)$, $\delta\colon\ x = \frac{23}{3},\ e = \frac{3}{\sqrt{14}}$;

30. $F(1,3)$, $\delta\colon\ x = 12,\ e = \frac{1}{2}$;

31. $F(4,2)$, $\delta\colon\ x = \frac{13}{3},\ e = \frac{3}{\sqrt{10}}$;

32. $F(1,1)$, $\delta\colon\ x = 17,\ e = \frac{1}{3}$;

33. $F(1,-1)$, $\delta\colon\ x = \frac{9}{2},\ e = \frac{2}{\sqrt{11}}$;

34. $F(0,4)$, δ: $x = \frac{13}{2}$, $e = \frac{2}{3}$;

35.$F(1,1)$, δ: $x = 53$ $e = \frac{3}{\sqrt{15}}$;

16. $F(3,3)$, δ: $x = \frac{16}{3}$, $e = \frac{3}{4}$;

17. $F(2,2)$, δ: $x = \frac{14}{3}$, $e = \frac{3}{\sqrt{17}}$;

18. $F(3,4)$, δ: $x = \frac{28}{3}$, $e = \frac{3}{5}$;

19. $F(1,-2)$, δ: $x = \frac{5}{2}e = \frac{2}{\sqrt{7}}$;

20.$F(-1,8)$, δ: $x = \frac{88}{7}$, $e = \frac{7}{9}$;

36. $F(3,0)$, δ: $x = 27$, $e = \frac{1}{2}$;

37. $F(1,11)$, δ: $x = \frac{37}{2}$, $e = \frac{2}{3}$;

38. $F(6,0)$, δ: $x = \frac{27}{2}$, $e = \frac{2}{3}$;

39. $F(-3,7)$, δ: $x = \frac{91}{5}$, $e = \frac{5}{9}$;

11-topshiriq.

Tekislikda ABC uchburchakning uchlarining koordinatalari berilgan, ya‘ni A, B, C.

a) ABC uchburchakning AB tomoni tenglamasi tuzilsin.
b) C uchidan AB tomoniga tushirilgan CH balandlik tenglamasi tuzilsin.
c) AM mediananing tenglamasi tuzilsin, bu yerda M nuqta BC tomonni o‘rtasi
d) AM mediana va CH balandlikning kesishish nuqtasi N topilsin.
e) Uchburchakning C uchidan o‘tuvchi va AB tomoniga parallel bo‘lgan to‘g‘ri chiziq tenglamasi tuzulsin.

1. A(2, 5), B(-3, 1), C(0, 4);

16. A(4, -4), B(6, 2), C(-1, 8);

2. A(-2, -6), B(-3, 5),C (4, 0);

3. A(7, 0), B(1, 4), C(-8, -4);

4. A(0, 6), B(0, 2), C(9, 0);

5. A(-4, 2), B(6, -4), C(4, 10);

6. A(-3, 8), B(-6, 2), C(0, -5);

7. A(-4, 2), B(8, -6), C(2,6);

8. A(4, -4), B(8, 2), C(3, 8);

9. A(-2, -3), B(1, 6), C(6, 1);

10. A(1, 7), B(-3, -1), C(11, -3);

11. A(-5, 1), B(8, -2), (1, 4);

12. A(0, 2), B(-7, -4), C(3, 2);

13. A(-3, -1), B(-4, -5), C(8, 1);

14. A(-7, -2), B(-7, 4), C(5, -5);

15. A(4, 1), B(-3, -1), C(7, -3);

17. A(1, -6), B(3, 4), C(-3, 3);

18. A(4 , -3), B(7, 3), C(1, 10);

19. A(1, -2), B(7, 1), C(3, 7);

20. A(-3, -2), B(14, 4), C(6, 8);

21. A(1, -3), B(0, 7), C(-2,4);

22. A(7, 0), B(1, 4), C(-8, -4);

23. A(-7, -2), B(3, -8), C(-4, 6);

24. A(10, -2), B(4, -5), C(-3, 1);

25. A(3, -1), B(11, 3), C(-6, 2);

26. A(2, -3), B(0, 5), C(-3, 4);

27. A(3, 0), B(-1, 4), C(8, -4);

28. A(-1, -3), B(3, -9), C(-4, 2);

29. A(3, -2), B(4, -3), C(3, 1);

30. A(2, -3), B(6, 3), C(0, 2);

F O Y D A L A N I L G A N A D A B I Y O T L A R

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M U N D A R I J A

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