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VAZIRLIGI

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Oliy matematika kafedrasi

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«CHIZIQLI FAZO VA CHIZIQLI OPERATORLAR »

O‘QUV USLUBIY QO‘LLANMA

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Ushbu qo‘llanma oily o‘quv yurtlari talabalari uchun mo‘ljallangan va o‘qitishning kredit sistemasiga moslangan bo‘lib, unda n o‘lchovli vektorlar sistemasi, n o‘lchovli vektorlar sistemasining rangi va bazisi, n o‘chovli arifmetik vector fazo, arifmetik vector fazoning kanonik bazisi, chiziqli fazo, chiziqli fazoning o‘lchovi va bazisi tushunchalari, hamda chiziqli operatorlar nazariyasi keltirilgan bo‘lib, bu mavzularga doir misol va masalalar hamda ularni yechishga doir uslubiy ko‘rsatmalar berilgan. Talabalarning mustaqil yechishlari uchun masalalar tavsiya qilingan bo‘lib, bu talabalarning o‘z bilimlarini mustahkamlashlariga imkon beradi.

Ushbu uslubiy qo‘llanma Muhammad al-Xorazmiy nomidagi Toshkent axborot texnologiyalari universiteti ilmiy uslubiy kengashi qaroriga asosan chop etildi.

O‘quv uslubiy qo‘llanma Muhammad al-Xorazmiy nomidagi Toshkent axborot texnologiyalari universiteti barcha yo‘nalishlarida ta’lim oluvchi talabalar uchun mo‘ljallangan.

O‘quv uslubiy qo‘llanma Muhammad al-Xorazmiy nomidagi Toshkent axborot texnologiyalari universiteti ilmiy uslubiy kengashining qarori bilan chop etishga tavsiya etildi (2019 yil “21” fevral “ 8(120)”- sonli bayonnoma).

Kirish

Chiziqli algebra, analitik geometriya va matematik analiz kurslarida turli tabiatli elementlar to‘plamlarini uchratish mumkin. Bular-haqiqiy va kompleks sonlar to‘plamlari; to‘gri chiziqdagi, tekislikdagi va fazodagi vektorlar to‘plamlari; oldingi mavzularda biz tanishgan matritsalar to‘plamlari; n o‘lchovli vektorlar to‘plamlari; darajali n dan oshmaydigan ko‘phadlar to‘plamlari; biror $[a, b]$ kesmada aniqlangan va uzluksiz funksiyalar to‘plamlari; chiziqli fazolarda aniqlangan operatorlar to‘plamlari va hakozo. Bu to‘plamlar turli tabiatli bo‘lsada, bu to‘plamlarning har birining elementlari orasida ularni qo‘shish va songa ko‘paytirish amallarini kiritish mumkin va bu to‘plamlar ustida kiritilgan qo‘shish va songa ko‘paytirish amallari juda ko‘p umumiy xossalarga ega bo‘ladi. Biz bu uslubiy qo‘llanmada to‘plam elementlarining tabiatini hisobga olmasdan bu to‘plamlar uchun umumiy bo‘lgan nazariya bilan tanishamiz. Bu ob’ektlardan biri chiziqli fazo bo‘lib, axborot-kommunikatsiya texnologiyalari sohasida juda muhim ahamiyatga ega.

1. Arifmetik vektor fazo

Bizga o‘rta maktab kursidan va oldingi mavzulardan ma’lumki, yo‘nalishga ega kesmalar vektorlar deyiladi va ular $\vec{a}, \vec{b}, \vec{c},...$ ko‘rinishda belgilanib, bu vektorlar ustida vektorlarni qo‘shish va songa ko‘paytirish amallari aniqlangan. Bunday aniqlangan vektor tushunchasidan tekislikda va R^3 fazoda foydalanish mumkin. Biz bu paragrafda umumiyroq vektor, ya’ni n o‘lchovli arifmetik vektor tushunchasini kiritib, bu vektorlar ustida bajariladigan chiziqli amallarni aniqlaymiz va bu amallar yordamida arifmetik vektor fazo tushunchasini kiritamiz.

1.1- ta‘rif. n ta sonning tartiblangan tizimiga n o‘lchovli vektor deyiladi.

Vektorlarni lotin alifbosining bosh harflari bilan $A, B, ..., X, Y,...$ ko‘rinishda belgilaymiz va quyidagi bir ustundan iborat matritsa ko‘rinishida yozamiz:

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}.$$

Izoh:

1. Amaliyotda $A = (a_1, a_2, ..., a_n)$ shakldagi satr matritsa vektorlardan ham foydalaniladi.

2. Ba’zida vektorlar matritsalardan farq qilishi uchun lotin alifbosining kichik harflari bilan ham belgilanishi mumkin.

3. Oldingi mavzularda ikki va uch o‘lchovli geometrik vektorlar o‘rganilgan. Bu mavzuda o‘rganiladigan vektorlar bu vektorlarning umumlashmasidan iboratdir.

n o‘lchovli vektorlar ustida qo‘shish va songa ko‘paytirish amallari xuddi matritsalardagi kabi aniqlanadi.

1) X va Y vektorlarning yig‘indisi, deb shunday bir $C = X + Y$ vektorga aytiladiki, bu vektor quyidagicha aniqlanadi:

$$C = X + Y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ \vdots \\ x_n + y_n \end{pmatrix};$$

2) X vektorning λ songa ko‘paytmasi quyidagicha aniqlanadi:

$$\lambda X = \lambda \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} \lambda x_1 \\ \lambda x_2 \\ \vdots \\ \lambda x_n \end{pmatrix}.$$

Aniqlanishiga ko‘ra ikkita n o‘lchovli vektorlar yig‘indisi, shuningdek, vektorni songa ko‘paytirish natijasida yana n o‘lchovli vektor hosil bo‘ladi, ya‘ni n o‘lchovli vektorlar to‘plami kiritilgan bu amallarga nisbatan yopiq to‘plam bo‘ladi.

1-misol. Quyidagi vektorlar uchun $5A+7B-2A$ ni toping:

$$A = \begin{pmatrix} 2 \\ 5 \\ 3 \\ -4 \end{pmatrix}; \quad B = \begin{pmatrix} -1 \\ 5 \\ 6 \\ 7 \end{pmatrix}.$$

Yechish.

$$5A+7B-2A = 5\begin{pmatrix} 2 \\ 5 \\ 3 \\ -4 \end{pmatrix} + 7\begin{pmatrix} -1 \\ 5 \\ 6 \\ 7 \end{pmatrix} - 2\begin{pmatrix} 2 \\ 5 \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} -1 \\ 50 \\ 51 \\ 37 \end{pmatrix}.$$

Vektorlar ustida kiritilgan bu chiziqli amallar quyidagi xossalarga ega:

1) $X + Y = Y + X$;

2) $X + (Y + Z) = (X + Y) + Z$;

3) $X + \Theta = X$, bunda $\Theta = (0, 0, ..., 0)^T$;

4) $X + (-X) = \Theta$;

5) $1 \cdot X = X$;

6) $(\alpha + \beta)X = \alpha X + \beta X$, bunda α va β ixtiyoriy sonlar;

7) $\alpha(X+Y)=\alpha X+\alpha Y$;

8) $\alpha(\beta X)=(\alpha\beta)X$. bu yerda, X, Y va Z n o‘lchovli vektorlar.

1.2- ta’rif. Barcha n o‘lchovli vektorlar to‘plami yuqorida kiritilgan vektorlarni qo‘shish va songa ko‘paytirish amallari bilan birgalikda *n o‘lchovli arifmetik vektor fazo* deyiladi.

Agar vektorlarning komponentlari haqiqiy sonlardan iborat bo‘lsa bu arifmetik vektor fazoga haqiqiy arifmetik vektor fazo deyiladi va R^n bilan belgilanadi.

Agar vektorlarning komponentlari kompleks sonlardan iborat bo‘lsa bu arifmetik vektor fazoga kompleks arifmetik vektor fazo deyiladi va C^n bilan belgilanadi.

Izoh. Vektor tushunchasining umumlashtirilishi vektor komponentlarini turlicha talqin qilishga imkon beradi.

2- misol. Korxona o‘zining ishlab chiqarish jarayonida n turdagi xom ashyodan foydalanib m xildagi mahsulot ishlab chiqarsin. Korxonaning bir sutkada xom ashyoga bo‘lgan ehtiyojini va bir sutkada ishlab chiqargan mahsulotlarini ifodalovchi vektorlarni yozing.

Yechish. Agar x_k kattalik $k-$xom ashyoga bo‘lgan korxonaning bir sutkalik ehtiyojini, y_i kattalik esa bir sutkada ishlab chiqarilgan $i-$mahsulot miqdorini bildirsa, u holda quyidagi $X=(x_1,x_2,...,x_n)^T$ va $Y=(y_1,y_2,...,y_m)^T$ vektorlar mos ravishda korxonaning barcha xom ashyoga bo‘lgan bir sutkalik ehtiyojini va bir kunda, ishlab chiqarilgan mahsulotning turlari miqdorini bildiradi.

3- misol. Ikkita korxona bir xil 4 turdagi mahsulot ishlab chiqaradi. Korxonalarning har bir mahsulotdan bir sutkada qanchadan ishlab chiqarishi quyidagi jadvalda berilgan:

Mahsulot turlari	1	2	3	4
1-korxona	24	36	50	80
2-korxona	30	25	20	10

Birinchi korxona bir oyda 22 kun, ikkinchi korxona esa 20 kun ishlaydi. Bir oyda ikkala korxona har bir turdagi mahsulotlardan birgalikda qancha miqdorda ishlab chiqaradi.

Yechish. Korxonalarning bir sutkada ishlab chiqargan mahsulotlari vektorlarini quyidagicha yozamiz:

$$A=\begin{pmatrix}24\\36\\50\\80\end{pmatrix} \quad va \quad B=\begin{pmatrix}30\\25\\20\\10\end{pmatrix}.$$

U holda ikkala korxonaning birgalikdagi bir oyda ishlab chiqarish vektori quyidagicha topiladi:

$$22A+20B=22\begin{pmatrix}24\\36\\50\\80\end{pmatrix}+20\begin{pmatrix}30\\25\\20\\10\end{pmatrix}=\begin{pmatrix}528\\792\\1100\\1760\end{pmatrix}+\begin{pmatrix}600\\500\\400\\200\end{pmatrix}=\begin{pmatrix}1128\\1292\\1500\\1960\end{pmatrix}.$$

1.3- ta’rif. Ikkita bir xil o‘lchovli

$$X=\begin{pmatrix}x_1\\x_2\\\vdots\\x_n\end{pmatrix} \quad va \quad Y=\begin{pmatrix}y_1\\y_2\\\vdots\\y_n\end{pmatrix}$$

vektorlarning *skalyar ko‘paytmasi* deb shu vektorlar mos koordinatalari ko‘paytmalarining yig‘indisiga teng songa aytiladi va

$$(X,Y)=x_1y_1+x_2y_2+\cdots+x_ny_n$$

shaklda yoziladi.

Skalyar ko‘paytmani matritsalar ko‘paytmasi shaklida quyidagicha ifodalashimiz mumkin:

$$(X,Y)=X^TY=Y^TX\ .$$

4- misol. Quyidagi vektorlarning skalyar ko‘paytmasini toping:

$$X = \begin{pmatrix} 2 \\ 5 \\ 3 \\ -4 \end{pmatrix}; \quad Y = \begin{pmatrix} -1 \\ 5 \\ 6 \\ 7 \end{pmatrix}.$$

Yechish. $$(X,Y) = X^T Y = \begin{pmatrix} 2 & 5 & 3 & -4 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 5 \\ 6 \\ 7 \end{pmatrix} =$$

$$= 2 \cdot (-1) + 5 \cdot 5 + 3 \cdot 6 + (-4) \cdot 7 = -2 + 25 + 18 - 28 = 13.$$

5- misol. Korxona 5 turdagi mahsulot ishlab chiqaradi. Korxonaning bir sutkada har bir turdagi mahsulotdan qanchadan ishlab chiqarganligi va har bir mahsulotning bir birligining narxi quyidagi jadvalda berilgan:

Mahsulot turlari	1	2	3	4	5
Korxonaning bir sutkada i/ch.mahsuloti miqdori	23	54	26	46	68
Bir birlik mahsulot narxi(sh.p.b)	32	56	36	65	35

Korxonaning bir sutkalik daromadi qancha bo‘ladi?

Yechish. Agar korxonaning ishlab chiqarish vektorini X va narx vektorini P bilan belgilasak, u holda

$$X = \begin{pmatrix} 23 \\ 54 \\ 26 \\ 46 \\ 68 \end{pmatrix}; \quad P = \begin{pmatrix} 32 \\ 56 \\ 36 \\ 65 \\ 35 \end{pmatrix}$$

bo‘ladi. Korxonaning bir sutkalik daromadini topish uchun bu vektorlarni skalyar ko‘paytiramiz:

$$(X,P) = X^T P = \begin{pmatrix} 23 & 54 & 26 & 46 & 68 \end{pmatrix} \begin{pmatrix} 32 \\ 56 \\ 36 \\ 65 \\ 35 \end{pmatrix} = 10066.$$

Skalyar ko‘paytma quyidagi xossalarga ega:

1) $(X,Y)=(Y,X)$;

2) $(X,Y+Z)=(X,Y)+(X,Z)$;

3) $(\lambda X,Y)=\lambda(X,Y)$.

4) $(X,X)\geq 0$; $(X,X)=0 \Leftrightarrow X=\theta$;

bu yerda X,Y, Z n o‘lchovli vektorlar va λ ixtiyoriy son.

1.4- ta’rif. Vektor komponentlari kvadratlari yig‘indisining kvadrat ildiziga teng bo‘lgan $|X|=\sqrt{(X,X)}=\sqrt{x_1^2+x_2^2+\cdots+x_n^2}$ songa n o‘lchovli X vektor *uzunligi (moduli, normasi)* deyiladi.

Vektor uzunligi quyidagi xossalarga ega:

1) $|X|\geq 0$;

2) $|\lambda X|=|\lambda|\cdot|X|$;

3) $|X+Y|\leq|X|+|Y|$ (uchburchak tengsizligi)

bu yerda, X,Y n o‘lchovli vektorlar va λ ixtiyoriy son.

6- misol. Quyidagi vektorlarning uzunliklarini toping:

$$1)\ A=\begin{pmatrix}3\\0\\4\end{pmatrix};\quad 2)\ B=\begin{pmatrix}2\\5\\-2\\3\end{pmatrix};\quad 3)\ C=\begin{pmatrix}1\\2\\3\\-4\\-3\end{pmatrix}.$$

Yechish. 1) $|A|=\sqrt{3^2+0^2+4^2}=\sqrt{9+16}=\sqrt{25}=5$

2) $|B|=\sqrt{2^2+5^2+(-2)^2+3^2}=\sqrt{4+25+4+9}=\sqrt{42}$

3) $|C|=\sqrt{1^2+2^2+3^2+(-4)^2+(-3)^2}=\sqrt{1+4+9+16+9}=\sqrt{39}$.

1.5- ta’rif. Agar ikkita noldan farqli vektorlarning skalyar ko‘paytmasi nolga teng bo‘lsa, u holda bunday vektorlar *ortogonal vektorlar* deyiladi.

7- misol. a parametrning qanday qiymatida quyidagi vektorlar ortogonal bo‘ladi:

$$X=\begin{pmatrix}3\\0\\a\\-1\end{pmatrix}\quad va\quad Y=\begin{pmatrix}-2\\5\\6\\0\end{pmatrix}.$$

Yechish. Bu vektorlarning skalyar ko‘paytmasini hisoblaymiz $(X,Y)=3\cdot(-2)+0\cdot 5+a\cdot 6+(-1)\cdot 0=6a-6.$ Masala shartiga ko‘ra, $6a-6=0 \Rightarrow a=1.$

Endi biz R^3 fazodagi vektorlarga tegishli ba‘zi ma‘lumotlarni keltiramiz. Geometrik masalalarini yechishda zarur bo‘ladigan “vektor ko‘paytma” va “aralash ko‘paytma” tushuchalarini kiritamiz. Bu ucun biz avval uchta nokomplanar vektor uchligining R^3 fazoda joylashishiga aloqador bo‘lgan quyidagi tushunchani kiritamiz.

1.6- ta’rif. Agar uchlari bir nuqtaga qo‘yilgan, o‘zaro nokomplanar $\{\vec{a};\vec{b};\vec{c}\}$ vektorlar uchligi uchun :

1) $\vec{c}$ vektorning uchidan qaraganda $\vec{a}$ vektordan $\vec{b}$ vektorga eng qisqa burilish soat strelkasiga teskari yo‘nalishda bo‘lsa u holda $\{\vec{a};\vec{b};\vec{c}\}$ vektorlar uchligi o‘ng vektorlar uchligini tashkil qiladi deyiladi;

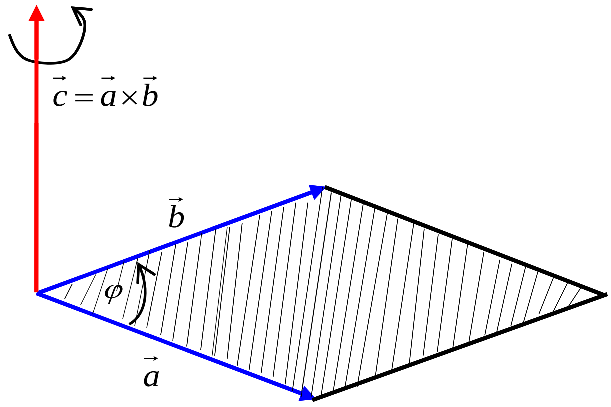
2) $\vec{c}$ vektorning uchidan qaraganda $\vec{a}$ vektordan $\vec{b}$ vektorga eng qisqa burilish soat strelkasi yo‘nalishida bo‘lsa u holda $\{\vec{a};\vec{b};\vec{c}\}$ vektorlar uchligi chap vektorlar uchligini tashkil qiladi deyiladi;

1.7- ta’rif. Agar uchta o‘zaro nokomplanar $\vec{a}$,$\vec{b}$ va $\vec{c}$ vektorlar uchun:

1) $\{\vec{a};\vec{b};\vec{c}\}$ vektorlar uchligi o‘ng vektorlar uchligini tashkil qilib $(\vec{a},\vec{c})=0\ \ va\ \ (\vec{b},\vec{c})=0$ bo‘lsa;

2) $\vec{c}$ vektor uzunligi $|\vec{c}|=|\vec{a}||\vec{b}|\sin\alpha$ (bu son jihatidan $\vec{a}$ va $\vec{b}$ vektorlarga qurilgan parallrlogrammning yuziga teng) tenglik bilan aniqlansa;

u holda $\vec{c}$ vektor ikki $\vec{a}$ va $\vec{b}$ vektorlarning *vektor ko'paytmasi* deyiladi va $\vec{c}=[\vec{a},\vec{b}]$ ko'rinishda belgilanadi.



1-chizma.

Bu yerda $\alpha-$ burchak $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak bo'lib $0<\alpha<\pi$ bo'ladi.

Ikkita $\vec{a}$ va $\vec{b}$ vektorlarning vektor ko'paytmasi quyidagi xossalarga ega:

1. $[\vec{a},\vec{b}]=-[\vec{b},\vec{a}]$;

2. $\vec{a}\perp\vec{b}\Rightarrow\left|[\vec{a},\vec{b}]\right|=\left|\vec{a}\right|\left|\vec{b}\right|$;

3. $\alpha[\vec{a},\vec{b}]=[\alpha\vec{a},\vec{b}]=[\vec{a},\alpha\vec{b}]$;

4. $[\vec{a},(\vec{b}+\vec{c})]=[\vec{a},\vec{b}]+[\vec{a},\vec{c}]$.

R^3 arifmetik vector fazoda berilgan $\vec{a}(x_1,y_1,z_1)$ *va* $\vec{b}(x_2,y_2,z_2)$ vektorlarning vector ko'paytmasi $\vec{c}=[\vec{a},\vec{b}]$ ni uchinchi tartibli determinant yordamida quyidagicha hisoblash mumkin $\vec{c}=\left[\vec{a},\vec{b}\right]=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ x_1 & x_2 & x_3\\ y_1 & y_2 & y_3\end{vmatrix}$.

R^3 arifmetik vector fazoda ABC uchburchak uchlarining koordinatalari bilan berilgan bo'lsin: $\overrightarrow{A}(x_1,y_1,z_1)$; $\overrightarrow{B}(x_2,y_2,z_2)$ *va* $\overrightarrow{C}(x_3,y_3,z_3)$.U holda vektor ko'paytmaning ta'rifiga asosan $S_{\Delta ABC}=\frac{1}{2}\left|\left[\overrightarrow{AB},\overrightarrow{AC}\right]\right|$ o'rinli bo'ladi.

8- misol. $\vec{i}$ *va* $\vec{j}$ birlik vektorlar bo‘lib ular orasidagi burchak 45° ga teng bolsin. $\vec{a} = 3\vec{i} - \vec{j}$ *va* $\vec{b} = 2\vec{i} + 3\vec{j}$ vektorlarga yasalgan parallelogramning yuzini hisoblang.

Yechish.

$$\left[\vec{a}, \vec{b}\right] = \left[3\vec{i} - \vec{j},\ 2\vec{i} + 3\vec{j}\right] = \left[3\vec{i},\ 2\vec{i}\right] + \left[3\vec{i},\ 3\vec{j}\right] + \left[-\vec{j},\ 2\vec{i}\right] + \left[-\vec{j},\ 3\vec{j}\right] =$$
$$9\left[\vec{i}, \vec{j}\right] - 2\left[\vec{j}, \vec{i}\right] = 11\left[\vec{i}, \vec{j}\right].$$

$$\left|\left[\vec{a}, \vec{b}\right]\right| = \left|11\left[\vec{i}, \vec{j}\right]\right| = 11|\vec{i}| \cdot |\vec{j}| \sin 45^\circ = 11 \cdot 1 \cdot 1 \frac{\sqrt{2}}{2} = 5{,}5\sqrt{2}$$

Demak $S = 5{,}5 \cdot \sqrt{2}$ kv. birlik.

9- misol. Uchlarining koordinatalari $A(5;4;3)$, $B(2;-1;0)$, $C(-3;2;1)$ bo‘lgan ABC uchburchakning yuzini toping.

Yechish.

$$\overrightarrow{AB} = \{-3;-5;-3\},\ \overrightarrow{AC} = \{-8;-2;-2\}^{\rightarrow}. \text{ Bundan}$$

$$\left[\overrightarrow{AB}, \overrightarrow{AC}\right] = \left\{\begin{vmatrix} -5 & -3 \\ -2 & -2 \end{vmatrix}; \begin{vmatrix} -3 & -3 \\ -2 & -8 \end{vmatrix}; \begin{vmatrix} -3 & -5 \\ -8 & -2 \end{vmatrix}\right\} = \{4;\ 18;\ -34\}$$

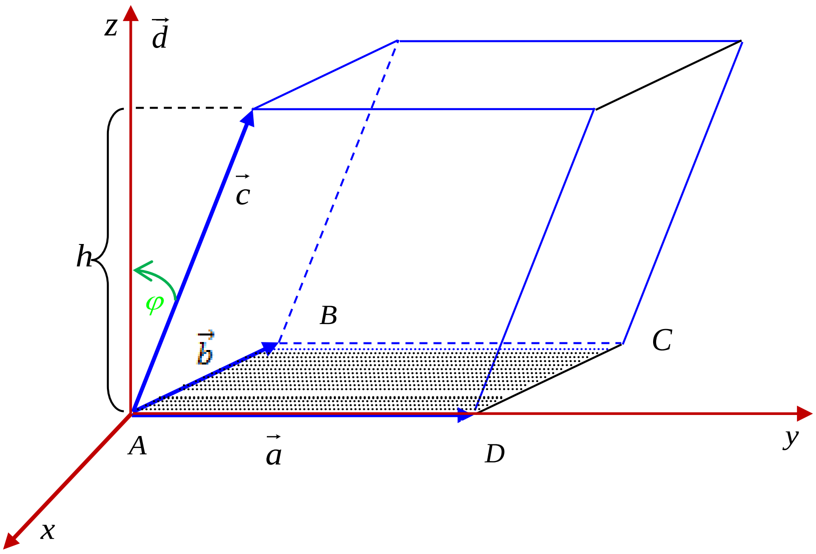
$$\left|\left[\overrightarrow{AB}; \overrightarrow{AC}\right]\right| = \sqrt{4^2 + 18^2 + (-34)^2} = \sqrt{1496} = 2\sqrt{374}.$$

Bu erdan yuqoridagi formulaga kora $S_{\Delta ABC} = \frac{1}{2} \cdot 2 \cdot \sqrt{374} \Rightarrow S_{\Delta ABC} = \sqrt{374}$ kv.birlik.

1.8- ta’rif. Agar ikkita $\vec{a}$ va $\vec{b}$ vektorlarning vektor ko‘paytmasi uchinchi $\vec{c}$ vektorga skalyar ko‘paytirilsa, u holda hosil bo‘lgan songa $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlarning aralash ko‘paytmasi deyiladi va $\left([\vec{a}, \vec{b}], \vec{c}\right)$ ko‘rinishda belgilanadi.

Izoh: Uchta vektorning aralash ko‘paytmasi absolyut qiymati jihatidan shu vektorlarga qurilgan parallelepipedning hajmiga teng boladi (2-chizma).

$\vec{d}=[\vec{a},\vec{b}]$ vektorning moduli $\vec{a}$ va $\vec{b}$ vektorlardan yasalgan parallelogram yuziga teng (2-chizma) bo‘lgani uchun skalyar ko‘paytma ta‘rifiga ko‘ra $(\vec{d},\vec{c})=|\vec{d}|\cdot \mathrm{Pr}_{\vec{d}}\,\vec{c}$.



2-chizma.

Ammo $Pr_{\vec{d}}\,\vec{c}=h$ miqdorning moduli, ya’ni $|h|$ son $\vec{a}$, $\vec{b}$, $\vec{c}$ vektorlarga yasalgan parallelepipedning balandligini anglatadi. Bundan

$$V_{parallelopiped}=\left|\left([\vec{a},\vec{b}],\vec{c}\right)\right|.$$

10- misol. 2) $\vec{a}=\vec{i}-2\vec{j}+\vec{k};\ \vec{b}=3\vec{i}+\vec{j}-2\vec{k}$ va $\vec{c}=7\vec{i}+14\vec{j}-13\vec{k}$ vektorlarni komplanarlikka tekshiring.

Yechish. $[\vec{a},\vec{b}]=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & -2 & 1\\ 3 & 1 & -2\end{vmatrix}=3\vec{i}+5\vec{j}+7\vec{k},\quad \left(\left[\vec{a},\vec{b}\right],\vec{c}\right)=21+70-91=0.$

Demak, bu vektorlar komplanar.

R^n arifmetik fazoda kiritilgan skalyar ko‘paytma xossalaridan foydalanib quyidagi teoremani isbotlaymiz.

Teorema *(Koshi – Bunyakovskiy tengsizligi).* R^n arifmetik fazodan olingan ixtiyoriy X va Y vektorlar uchun

$$\left|(X,Y)\right| \le |X| \cdot |Y| \quad yoki \quad \left|\sum_{i=1}^{n} x_i y_i\right| \le \sqrt{\sum_{i=1}^{n} x_i^2} \cdot \sqrt{\sum_{i=1}^{n} y_i^2}.$$

Isbot. Ixtiyoriy $\lambda \in R$ uchun

$$0 \le (X+\lambda Y, X+\lambda Y) = (X,X) + 2\lambda(X,Y) + \lambda^2(Y,Y)$$

hosil bo‘lgan kvadrat uchhad nomanfiy bo‘lganligi sababli bu kvadrat uchhadning diskriminanti musbat bo‘lmaydi. Bundan

$$4(X,Y)^2 - 4(X,X)(Y,Y) \le 0 \text{ yoki } \left|(X,Y)\right| \le |X| \cdot |Y|.$$

Bu teorema asosida R^n arifmetik fazo vektorlari orasidagi burchak tushunchasini kiritamiz.

1.6- ta’rif. Ikkita n o‘lchovli noldan farqli X va Y vektorlar orasidagi φ burchak

$$\cos\varphi = \frac{(X,Y)}{|X| \cdot |Y \cdot|} = \frac{\sum_{i=1}^{n} x_i y_i}{\sqrt{\sum_{i=1}^{n} x_i^2} \cdot \sqrt{\sum_{i=1}^{n} y_i^2}}, \quad \varphi \in [0;\pi]$$

formula bilan aniqlanadi.

Izoh: R^n arifmetik fazodagi n o‘lchovli vektorlar orasidagi burchak ta‘rifining korrektligi yuqorida isbotlangan Koshi – Bunyakovskiy tengsizligidan kelib chiqadi.

11- misol. $X(3;-4;2;5)$ va $Y(-1;3;-7;2)$ vektorlar berilgan:

$a)$ $3X+2Y$ vektorni toping;

$b)$ $\left(X,Y\right)$ skalyar ko‘paytmani toping;

$c)$ X va Y vektorlar orasidagi burchakni toping;

$d)$ Koshi – Bunyakovskiy tengsizligini tekshiring.

Yechish.

$$a)\ 3X+2Y = 3\begin{pmatrix} 3 \\ -4 \\ 2 \\ 5 \end{pmatrix} + 2\begin{pmatrix} -1 \\ 3 \\ -7 \\ 2 \end{pmatrix} = \begin{pmatrix} 7 \\ -6 \\ -8 \\ 19 \end{pmatrix}.$$

$b)\ (X,Y) = -3-12-14+10 = -19.$

$c)\ |X| = \sqrt{9+16+4+25} = \sqrt{54};\quad |Y| = \sqrt{1+9+49+4} = \sqrt{63}.$

$$\cos\varphi = \frac{-19}{\sqrt{54}\sqrt{63}};\quad \varphi = \arccos\left(\frac{-19}{\sqrt{54}\sqrt{63}}\right) = \pi - \arccos\left(\frac{19}{9\sqrt{42}}\right).$$

$d)\ |-19| < \sqrt{54}\cdot\sqrt{63}\qquad 19 < 9\sqrt{42}\qquad 9\sqrt{42} \approx 58{,}33.$

12- misol. $A_1(3;-4;1;7;-2)$ va $A_2(4;6;-3;3;6)$ nuqtalar berilgan. $\vec{a} = \overrightarrow{A_1A_2}$ vektorning koordinatalarini toping.

Yechish: ushbu holda $x_1 = 3,\quad x_2 = -4,\quad x_3 = 1\quad,\quad x_4 = 7\quad,\quad x_5 = -2,$ va $y_1 = 4,\ y_2 = 6,\ y_3 = -3,\ y_4 = 3,\ y_5 = 6.$ $\vec{a} = \overrightarrow{A_1A_2}$ vektorning koordinatalarini $\vec{a} = \overrightarrow{A_1A_2} = (y_1 - x_1;\ y_2 - x_2;;\ y_3 - x_3;\ y_4 - x_4;\ y_5 - x_5;)$ formula bo‘yicha hisoblab $\vec{a} = \overrightarrow{A_1A_2} = (1;\ 10;\ -4;-4;8)$ ga ega bo‘lamiz.

13- misol. Parallelogrammning uchta ketma-ket uchlari: $A(1;-2;3), B(3;2;1)$ $C(6;4;4)$ koordinatalari bilan berilgan. Parallelogrammning to‘rtinchi D uchini toping.

Yechish: parallelogrammning to‘rtinchi D uchining koordinatalarini x, y, z bilan belgilaymiz $D(x;y;z)$. $\overrightarrow{BC}$ va $\overrightarrow{AD}$ vektorlarning koordinatalarini topamiz. $\overrightarrow{BC} = (6-3;\ 4-2;\ 4-1),\qquad \overrightarrow{BC} = (3;\ 2;\ 3),\qquad \overrightarrow{AD} = (x-1; y+2; z-3).$

$ABCD$ parallelogrammda $\overrightarrow{BC}$ va $\overrightarrow{AD}$ vektorlar tengligidan, $x-1=3,$ $y+2=2,\ z-3=3.$ Demak, $x=4,\ y=0,\ z=6 \Rightarrow D(4;\ 0;\ 6).$

14- misol. Uch o‘lchovli fazoda $\vec{a}$ vektor berilgan bo‘lib, bu vector Ox va Oy o‘qlari bilan mos ravishda $\alpha = 60^0$ va $\beta = 120^0$ burchak tashkil qiladi. Agar $\left|\vec{a}\right| = 2$ bo‘lsa, $\vec{a}$ vektorning koordinatalarini toping.

Yechish. $\vec{a}$ vektorning koordinatalarini x, y, z deb olamiz, ya‘ni $\vec{a}=(x;y;z)$. U holda $\vec{a}$ vektorning yo‘naltiruvchi kosinuslarini $\cos\alpha=\frac{x}{\left|\vec{a}\right|}$, $\cos\beta=\frac{y}{\left|\vec{a}\right|}$, $\cos\gamma=\frac{z}{\left|\vec{a}\right|}$ munosabatlardan topamiz. Bizga ma‘lumki

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1 \Rightarrow \cos^2\gamma=1-\cos^2 60^0-\cos^2 120^0 \Rightarrow \cos^2\gamma=\frac{1}{2}.$$

Bu yerdan $\cos\gamma=\frac{\sqrt{2}}{2}$ yoki $\cos\gamma=-\frac{\sqrt{2}}{2}$. Masala shartini ikkita $\vec{a}_1$ va $\vec{a}_2$. vektolar qanoatlantiradi:

1) $\vec{a}_1$ vektorning yo‘naltiruvchi kosinuslari $\cos\alpha=\frac{1}{2}$, $\cos\beta=-\frac{1}{2}$, $\cos\gamma=\frac{\sqrt{2}}{2} \Rightarrow \frac{1}{2}=\frac{x_1}{2}$, $-\frac{1}{2}=\frac{y_1}{2}$, va $-\frac{1}{2}=\frac{y_1}{2} \Rightarrow x_1=1$, $y_1=-1$, $z_1=\sqrt{2} \Rightarrow$ $\vec{a}_1=\left(1;-1;\sqrt{2}\right)$

2) $\vec{a}_2$ vektorning yo‘naltiruvchi kosinuslari $\cos\alpha=\frac{1}{2}$, $\cos\beta=-\frac{1}{2}$, $\cos\gamma=-\frac{\sqrt{2}}{2}$ $\Rightarrow \frac{1}{2}=\frac{x_2}{2}, -\frac{1}{2}=\frac{y_2}{2}$, $-\frac{\sqrt{2}}{2}=\frac{z_2}{2} \Rightarrow x_2=1$, $y_2=-1$, $z_2=-\sqrt{2} \Rightarrow \vec{a}_2=\left(1;-1;-\sqrt{2}\right)$.

15- misol. $\vec{a}=m\vec{i}+3\vec{j}+4\vec{k}$ va $\vec{b}=4\vec{i}+m\vec{j}-7\vec{k}$ vektorlar berilgan. m ning qanday qiymatlarida vektorlar perpendikulyar bo‘ladi.

Yechish. Bu vektorlarning skalyar ko‘paytmasini topamiz: $(\vec{a}, \vec{b})=4m+3m-28$; $\vec{a}\perp\vec{b}$ bo‘lgani uchun $(\vec{a},\vec{b})=0$ bo‘ladi. Bundan $7m-28=0$, ya‘ni $m=4$.

16- misol. $\vec{a}=\vec{i}+2\vec{j}+3\vec{k}$ va $\vec{b}=6\vec{i}+4\vec{j}-2\vec{k}$ vektorlar orasidagi burchakni hisoblang.

Yechish. $\left(\vec{a},\vec{b}\right)=|\vec{a}||\vec{b}|Cos\varphi$ bo‘lgani uchun $Cos\varphi=\dfrac{\left(\bar{a},\bar{b}\right)}{|\vec{a}||\vec{b}|}$

$\left(\bar{a},\bar{b}\right)=1\cdot 6+2\cdot 4+3\cdot(-2)=8,\quad |\vec{a}|=\sqrt{1+4+9}=\sqrt{14},\quad |\vec{b}|=\sqrt{36+16+4}=2\sqrt{14}.$

Demak, $\cos\varphi=\dfrac{8}{\sqrt{14}\cdot 2\sqrt{14}}=\dfrac{2}{7}$ va $\varphi=\arccos\dfrac{2}{7}.$

17- misol. $\vec{a}=2\vec{i}+3\vec{j}+5\vec{k}$ va $\vec{b}=\vec{i}+2\vec{j}+\vec{k}$ vektorlarning vektor ko‘paytmasini toping.

Yechish. $\left[\vec{a}\times\vec{b}\right]=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 2 & 3 & 5\\ 1 & 2 & 1\end{vmatrix}=\vec{i}\begin{vmatrix}3 & 5\\ 2 & 1\end{vmatrix}-\vec{j}\begin{vmatrix}2 & 5\\ 1 & 1\end{vmatrix}+\vec{k}\begin{vmatrix}2 & 3\\ 1 & 2\end{vmatrix}$, ya‘ni

$$\left[\vec{a}\times\vec{b}\right]=-7\vec{i}+3\vec{j}+\vec{k}.$$

18- misol. $\vec{a}=2\vec{i}+5\vec{j}+7\vec{k}$, $\vec{b}=\vec{i}+\vec{j}-\vec{k},$ $\vec{c}=\vec{i}+2\vec{j}+2\vec{k}$ vektorlarning komplanarligini ko‘rsating.

Yechish. Uch vektorning aralash ko‘paytmasini topamiz:

$$\left(\left[\vec{a}\times\vec{b}\right]\cdot\vec{c}\right)=\begin{vmatrix}2 & 5 & 7\\ 1 & 1 & -1\\ 1 & 2 & 2\end{vmatrix}=2\cdot\begin{vmatrix}1 & -1\\ 2 & 2\end{vmatrix}-5\cdot\begin{vmatrix}1 & -1\\ 1 & 2\end{vmatrix}+7\cdot\begin{vmatrix}1 & 1\\ 1 & 2\end{vmatrix}=8-15+7=0,$$

$\left(\left[\vec{a}\times\vec{b}\right]\cdot\vec{c}\right)=0$ bo‘lgani uchun $\vec{a},\ \vec{b},\ \vec{c}$ lar komplanar.

Tayanch so‘z va iboralar: *arifmetik vekto fazo, vektor, nol vektor, vektorlar ustida chiziqli amallar, vektorlarning skalyar ko‘paytmasi vektor uzunligi, vektorlar orasidagi burchak, uchburchak tengsizligi, Koshi-Bunyakovskiy tengsizligi.*

O‘z-o‘zini tekshirish uchun savollar

1. N-o‘lchovli haqiqiy arifmetik vektor fazo deganda nimani tushunasiz?
2. Vektorlar ustida chiziqli amallar deganda qanday amallar tushuniladi?

3. Vektorlar ustida bajariladigan chiziqli amallar bo‘ysunadigan xossalarni sanab o‘ting?
4. Vektorlarning skalyar ko‘paytmasi deb nimaga aytiladi?
5. Arifmetik vektor uzunligi deb nimaga aytiladi?
6. Vektorlarning uzunligi bo‘ysunadigan qanday xossalarni bilasiz?
7. Vektorlarning skalyar ko‘paytmasi bo‘ysunadigan qanday xossalarni bilasiz?
8. Koshi-Bunyakovskiy tengsizligini yozing?

Mustaqil yechish uchun misollar

19. $A(1;3;2;-7;6;-1)$ *va* $B(5;8;-1;5;3;-1)$ nuqtalar berilgan bo‘lsa $\vec{a}=\left|\overrightarrow{AB}\right|$ vektorni toping.

20. $A(-2;3;5;6;9)$ *va* $B(3;-8;-1;-2;9)$ nuqtalar berilgan bo‘lsa $\vec{a}=\left|\overrightarrow{AB}\right|$ vektorni toping.

21. Parallelogramning ketma-ket uchta uchlari: $A(1;-2;3)$, $B(3;2;1)$ va $C(6;4;4)$ koordinatalari bilan berilgan. Uning to‘rtinchi D uchini toping.

22. $\vec{a}(2;-1;3;4;6)$ va $\vec{b}(5;2;-2;6;-5)$ vektorlar berilgan:

$a)$ $\left(\vec{a},\vec{b}\right)$; $\left(3\vec{a}+\vec{b},\vec{a}-2\vec{b}\right)$ skalyar ko‘paytmalarini toping;

$b)$ $\vec{a}$ *va* $\vec{b}$ vektorlar orasidagi burchak kosinusini toping.

23. $\vec{a}(1;\,-3;\,2;\,0)$, $\vec{b}(4;\,-2;\,1;\,3)$, $\vec{c}(5;\,-3;\,2;\,1)$, $\vec{d}(1;\,2;\,2;\,-3)$ vektorlar uchun quyidagilarni hisoblang:

a) ortogonal vektorlarni aniqlang;

b) $(\vec{a}\,\hat{}\,\vec{b}),(\vec{b}\,\hat{}\,\vec{c}),(\vec{b}\,\hat{}\,\vec{d})$ burchaklarni hisoblang.

24. Quyidagi vektorlarning vektor ko‘paytmasini toping:

1) $\vec{a}(4;3;-1)$ *va* $\vec{b}(5;-1;4)$; 2) $\vec{a}(0;5;6)$ *va* $\vec{b}(12;1;5)$;

3) $\vec{a}(7;2;-2)$ *va* $\vec{b}(4;-1;6)$; 4) $\vec{a}(2;3;6)$ *va* $\vec{b}(-1;3;5)$;

25. Quyidagi vektorlarni komplanarlikka tekshiring:

1) $\vec{a}(4;3;-1), \vec{b}(15;-1;4);$ *va* $\vec{c}(-1;0;-1);$

2) $\vec{a}(-4;3;7), \vec{b}(12;1;-5)$ *va* $\vec{c}(4;-1;-1)$

3) $\vec{a}(7;2;-2), \vec{b}(4;-11;5)$ *va* $\vec{c}(-4;-1;6);$

4) $\vec{a}(12;-3;6), \vec{b}(-2;4;6)$ *va* $\vec{c}(1;-2;-3);$

26. $\vec{r} = \overline{OM} = 2\vec{i} + 3\vec{j} + 6\vec{k}$ vektorning uzunligini hamda yo‘nalishini aniqlang. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ formula bo‘yicha tekshiring.

27. Uchlari $A(2;-1;3);$ $B(1;1;1)$ *va* $C(0;0;5)$ nuqtalarda bo‘lgan uchburchakning barcha burchaklari aniqlansin.

28. Uchlari $A(3;2;3);$ $B(-1;1;-1)$ *va* $C(5;-3;5)$ nuqtalarda bo‘lgan uchburchakning barcha burchaklari aniqlansin.

29. $\vec{a} = \vec{i} + 2\vec{j}$ va $\vec{b} = -\vec{j} + 2\vec{k}$ vektorlarga qurilgan parallelogramm dioganallari orasidagi burchak topilsin.

30. $\vec{a} = 2\vec{i} + \vec{j}$ va $\vec{b} = -2\vec{j} + \vec{k}$ vektorlarga qurilgan parallelogram dioganallari orasidagi burchak topilsin.

31. $\bar{a} = \bar{i} + \bar{j} + 2\bar{k}$ va $\bar{b} = \bar{i} - \bar{j} + 4\bar{k}$ vektorlar berilgan. $\text{Pr}_{\bar{b}}\left(\vec{a}\right)$ va $\text{Pr}_{\bar{a}}\left(\vec{b}\right)$ larni toping.

32. $\bar{a} = 2\bar{i} + \bar{j}$ va $\bar{b} = -\overline{2j} + \bar{k}$ vektorlar berilgan. $\text{Pr}_{\bar{b}}\left(\vec{a}\right)$ va $\text{Pr}_{\bar{a}}\left(\vec{b}\right)$ larni toping.

33. Agar $\vec{m}$ *va* $\vec{n}$ vektorlar o‘zaro 30° burchak tashkil etuvchi birlik vektorlar bo‘lsa, u holda $\left(\vec{m} + \vec{n}\right)^2$ ni hisoblang.

34. Agar $|\bar{a}| = 2\sqrt{2}$, $|\bar{b}| = 4$ hamda $\left(\vec{a} \,\widehat{}\, \vec{b}\right) = 135^\circ$ bo‘lsa, u holda $\left(\vec{a} - \vec{b}\right)^2$ ni hisoblang.

35. Teng yonli $OABC$ trapetsiyada M va N nuqtalar mos ravishda $BC=2,\ AB=2$ tomonlarning o‘rtalari. Trapetsiyaning o‘tkir burchagi $60°$ ga teng. $\overrightarrow{OM}$ *va* $\overrightarrow{ON}$ vektorlar orasidagi burchakni toping.

36. $A(2;\ 2;\ 0)$ *va* $B(0;\ -2;\ 5)$ nuqtalar berilgan. $\overline{AB}=\bar{u}$ vektorning uzunligi va yo‘nalishi aniqlansin.

37. Tetraedrning bir uchidan o‘tkazilgan ikki tekis burchagining bissektrisalari orasidagi burchak kosinusini aniqlang.

38. $\overrightarrow{OA}=\vec{a}$ *va* $\overrightarrow{OB}=\vec{b}$ vektorlar berilgan. $|\vec{a}|=4;\ |\vec{b}|=2$ va $\left(\vec{a}\,\hat{}\,\vec{b}\right)=30^{\circ}$ OAB uchburchakning OM medianasi bilan OA tomoni orasidagi burchakni aniqlang.

39. $\overrightarrow{OA}=\vec{a}$ *va* $\overrightarrow{OB}=\vec{b}$ vektorlar berilgan. $|\vec{a}|=2;\ |\vec{b}|=4$ va $\left(\vec{a}\,\hat{}\,\vec{b}\right)=60^{\circ}$ OAB Uchburchakning OM medianasi bilan OA tomoni orasidagi burchakni aniqlang.

40. O‘zaro komplanar $\vec{a},\vec{b}$ *va* $\vec{c}$ vektorlar berilgan bo‘lib $|\bar{a}|=4,|\bar{b}|=2,|\bar{c}|=3$ va $\left(\vec{a}\,\hat{}\,\vec{b}\right)=30°,\left(\vec{b}\,\hat{}\,\vec{c}\right)=30°$ bo‘lsa, u holda

a) $\vec{u}=\vec{a}+\vec{b}+\vec{c}$ vektor uchun $|\vec{u}|=\sqrt{\left(\vec{a}+\vec{b}+\vec{c}\right)^{2}}$ formula bo‘yicha uning modulini hisoblang.

b) $\vec{u}=\vec{a}+\vec{b}-\vec{c}$ vektor uchun $|\vec{u}|=\sqrt{(\vec{a}+\vec{b}-\vec{c})^{2}}$ formula bo‘yicha uning modulini hisoblang.

41. O‘zaro komplanar $\vec{a},\ \vec{b}$ *va* $\vec{c}$ vektorlar berilgan bo‘lib $|\bar{a}|=3,|\bar{b}|=2,|\bar{c}|=5$ va $\left(\vec{a}\,\hat{}\,\vec{b}\right)=60°,\left(\vec{b}\,\hat{}\,\vec{c}\right)=60°$ bo‘lsa, u holda

a) $\vec{u}=\vec{a}+\vec{b}+\vec{c}$ vektor uchun $\left|\vec{u}\right|=\sqrt{\left(\vec{a}+\vec{b}+\vec{c}\right)^2}$ formula bo‘yicha uning modulini hisoblang.

b) $\vec{u}=\vec{a}+\vec{b}-\vec{c}$ vektor uchun $\left|\vec{u}\right|=\sqrt{(\vec{a}+\vec{b}-\vec{c})^2}$ formula bo‘yicha uning modulini hisoblang.

42. Quyidagi vektorlar uchun Koshi–Bunyakovskiy tengsizligini tekshiring;

1. $\vec{a}(1;\,2;\,3;\,4;5)$ *va* $\vec{b}(3;\,2;\,4;\,1;5)$; **2**. $\vec{a}(2;\,3;\,5;\,1;\,0)$ *va* $\vec{b}(4;\,3;\,2;\,1;\,1)$;

3. $\vec{a}(4;\,0;\,1;\,3;\,2)$ *va* $\vec{b}(2;\,3;\,5;\,4;\,2)$; **4**. $\vec{a}(1;\,3;\,7;\,5;\,4)$ *va* $\vec{b}(4;\,2;\,0;\,3;\,5)$

2.Chiziqli fazo

To‘plam elementlari orasida ularni qo‘shish va songa ko‘paytirish amallarini kiritish mumkin va to‘plamlar turli tabiatli bo‘lishiga qaramasdan ular ustida kiritilgan qo‘shish va songa ko‘paytirish amallari juda ko‘p umumiy xossalarga ega bo‘ladi. Biz quyida to‘plam elementlarining tabiatini hisobga olmasdan bu to‘plamlar uchun umumiy bo‘lgan nazariya bilan tanishamiz.

2.1- ta’rif. Agar elementlari ixtiyoriy tabiatli bo‘lgan L to‘plam berilgan va bu toplam elementlari orasida qo‘shish va songa ko‘paytirish amallari kiritilgan, ya‘ni

1) ixtiyoriy $x\in L$ va $y\in L$ elementlar juftiga x va y elementlarning yig‘indisi, deb ataluvchi yagona $z=x+y\in L$ element mos qo‘yilgan;

2) $x\in L$ element va $\lambda\in K$ (K-haqiqiy yoki kompleks sonlar to‘plami) songa x vektorning λ songa ko‘paytmasi deb ataluvchi yagona $z=\lambda x\in L$ element mos qo‘yilgan bo‘lib, aniqlangan bu qo‘shish va songa ko‘paytirish amallari quyidagi 8 ta aksiomani bajarsa, u holda L to‘plam *chiziqli (yoki vektor) fazo* deyiladi:

1. Qo‘shish kommutativ, $x+y=y+x$;

2. Qo‘shish assotsiativ, $(x+y)+z=x+(y+x)$;

3. L to‘plamda barcha x elementlar uchun $x+\theta=x$ shartni qanoatlantiradigan nol element θ mavjud;

4. L to‘plamda har qanday x element uchun $x+(-x)=\theta$ shartni qanoatlantiradigan $-x$ qarama-qarshi element mavjud;

5. $\alpha\left(x+y\right)=\alpha x+\alpha y$;

6. $\left(\alpha+\beta\right)x=\alpha x+\beta x$;

7. $\alpha\left(\beta x\right)=\left(\alpha\beta\right)x$;

8. $1\cdot x=x$.

Bundan keyin biz chiziqli fazo elementlarini vektorlar deb aytamiz. Agar chiziqli fazodagi vektorlar uchun faqat haqiqiy songa ko‘paytirish amali aniqlangan bo‘lsa, u holda bunday fazo *haqiqiy chiziqli fazo* deyiladi. Agar chiziqli fazodagi vektorlar uchun kompleks songa ko‘paytirish amali aniqlangan bo‘lsa, u holda bunday fazoga *kompleks chiziqli fazo* deyiladi.

Chiziqli fazoni aniqlovchi aksiomalardan, quyidagi xossalarni ajratish mumkin:

1- **xossa.** Har qanday chiziqli fazo uchun yagona θ-nol vektor mavjud.

2- **xossa.** Har qanday chiziqli fazoda har bir x vektor uchun unga qarama-qarshi bo‘lgan yagona $\left(-x\right)$ vektor mavjud.

3- **xossa.** Har qanday chiziqli fazoda har bir x vektor uchun $0\cdot x=\theta$ tenglik o‘rinli.

4- **xossa.** Har qanday λ haqiqiy sonva $\vec{\theta}\in L$ element uchun $\lambda\cdot\vec{\theta}=\vec{\theta}$ munosabat hamma vaqt bajariladi.

5-xossa. $\lambda\cdot\vec{a}=\theta\Rightarrow$ *yoki* $\lambda=0$ *yoki* $\vec{a}=\theta$

Izoh. $y-x$ vektorlar ayirmasi deb, y va $-x$ vektorlar yig‘indisi tushuniladi.

Yuqoridagi aniqlashimizga ko‘ra chiziqli fazo elementlari turli tabiatli bo‘lishi mumkin. Quyida biz chiziqli fazolarni aniq misollarda ko‘rib chiqamiz.

1- misol. Barcha haqiqiy sonlar to‘plami - haqiqiy sonlarni qo‘shish va ko‘paytirish amallariga nisbatan chiziqli fazo tashkil qiladi.

2- misol. Barcha kompleks sonlar to‘plami kompleks sonlarni qo‘shish va ko‘paytirish amallariga nisbatan chiziqli fazo tashkil qiladi.

3- misol. Oldingi mavzularda ko‘rgan $R^n\ (n=1,2,3,...,k)$ fazolar n o‘lchovli vektorlarni qo‘shish va songa ko‘paytirish amallariga nisbatan chiziqli fazo tashkil qiladi.

4- misol. Elementlari $n\times m$- tartibli matritsalardan iborat bo‘lgan $M^{n\times m}$ matritsalar to‘plami matritsalarni qo‘shish va songa ko‘paytirish amallariga nisbatan chiziqli fazo tashkil qiladi.

5- misol. $C\left[a,b\right]-\left[a,b\right]$ kesmada aniqlangan va uzluksiz barcha haqiqiy $f\equiv f(t)$ funksiyalar to‘plami funksiyalarni qo‘shish $\left(f+g\right)t\equiv f(t)+g(t)$ va songa ko‘paytirish $\lambda f(t)$ amallariga nisbatan chiziqli fazo tashkil qiladi.

6- misol. Darajasi n dan yuqori bo‘lmagan barcha ko‘phadlar to‘plami ko‘phadlarni qo‘shish va songa ko‘paytirish amallariga nisbatan chiziqli fazo tashkil qiladi.

7- misol. Darajasi roppa-rosa n ga teng bo‘lgan barcha ko‘phadlar to‘plami ko‘phadlarni qo‘shish va songa ko‘paytirish amallariga nisbatan chiziqli fazo tashkil qilmaydi. Haqiqatan ham

$$P_n(t)=a_nt^n+a_{n-1}t^{n-1}+\ldots+a_1t+a_0 \text{ va } Q_n(t)=-a_nt^n+b_{n-1}t^{n-1}+\ldots+b_1t+b_0$$

$n-$ darajali ko‘phadlar, lekin $P_n(t)+Q_n(t)$ ko‘phadning darajasi n dan kichik.

8- misol. Quyidagi chiziqli bir jinsli tenglamalar sistemasini qaraymiz

$$\begin{cases} a_{11}x_1+a_{12}x_2+...+a_{1n}x_n=0 \\ a_{21}x_1+a_{22}x_2+...+a_{2n}x_n=0 \\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \\ a_{m1}x_1+a_{m2}x_2+...+a_{mn}x_n=0 \end{cases}$$

Bizga ma‘lumki, agar $X_1\ va\ X_2$ vektorlar chiziqli bir jinsli tenglamalar sistemasining echimlari bo‘lsa, u holda bu vektorlarning chiziqli kombinatsiyasi $\lambda_1X_1+\lambda_2X_2$ ham bu sistemaning echimi bo‘ladi. Demak chiziqli bir jinsli tenglamalar sistemasining echimlari to‘plami chiziqli fazo tashkil qiladi.

9- misol. Agar $a\ va\ b$ haqiqiy sonlar bo‘lsa, u holda

$$M=\left\{a\cdot e^z+b\cdot e^{-z};\ (-\infty<z<+\infty\right\}$$

funktsiyalar to‘plami chiziqli fazo tashkil qiladi.

2.2- ta’rif. L chiziqli fazodan olingan $x_1, x_2, ..., x_n$ elementlar va $\lambda_i \in R$, ($i = 1...n$) sonlar yordamida qurilgan $\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + ... + \lambda_n x_n$ ifodaga $x_1, x_2, ..., x_n$ - elementlarning *chiziqli kombinatsiyasi* deyiladi.

2.3- ta’rif. Agar $y = \lambda_1 x_1 + \lambda_2 x_2 + \cdots + \lambda_n x_n$ tenglik o‘rinli bo‘lsa, u holda y element $x_1, x_2, ..., x_n$ elementlarning *chiziqli kombinatsiyasidan iborat* deyiladi.

2.4- ta’rif. Agar $\lambda_1, \lambda_2, ..., \lambda_n$ koeffitsiyentlardan hech bo‘lmaganda bittasi noldan farqli bo‘lganda $\lambda_1 x_1 + \lambda_2 x_2 + \cdots + \lambda_n x_n = \theta$ tenglik o‘rinli bo‘lsa, u holda $x_1, x_2, ..., x_n$ elementlar *chiziqli bog‘liq* deyiladi.

Agar $\lambda_1 x_1 + \lambda_2 x_2 + \cdots + \lambda_n x_n = \theta$ tenglik $\lambda_1, \lambda_2, ..., \lambda_n$ koeffitsiyentlardan barchasi nolga teng bo‘lgandagina o‘rinli bo‘lsa, u holda $x_1, x_2, ..., x_n$ - elementlar chiziqli erkli , aks holda $x_1, x_2, ..., x_n$ - elementlar *chiziqli bogliqli* deyiladi . Bu yerda, θ -chiziqli fazoning nol elementi.

2.5- ta’rif. Agar L chiziqli fazoda n ta chiziqli erkli elementlar mavjud bo‘lib, har qanday $n+1$ ta element chiziqli bog‘liqli bo‘lsa, u holda L chiziqli fazoning *o‘lchovi n ga teng* deyiladi.

2.6- ta’rif. n o‘lchovli L chiziqli fazoda har qanday n ta chiziqli erkli vektorlar sistemasi bu fazoning *bazisi* deyiladi.

Odatda bazis vektorlar sistemasi $e_1, e_2, ..., e_n$ kabi belgilanadi.Masalan, darajasi n dan oshmaydigan barcha ko‘phadlar to‘plami chekli o‘lchovli, ya‘ni $(n+1)$ o‘lchovli chiziqli fazo tashkil qiladi. Bu fazoning bazisini $\{1, t, t^2, ..., t^n\}$ vektorlar sistemasi tashkil qiladi.

10- misol. Barcha ikkinchi tartibli matritsalarning chiziqli fazosi

$M^2 = \left\{ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} : \; a_{11}, a_{12}, a_{21}, a_{22} \in R \right\}$ berilgan bo‘lsin. Bu chiziqli fazoning bazisi va o‘lchamini toping.

Yechish. Bu fazoning bazislaridan biri sifatida quyidagi matritsalar sistemasini olish mumkin.

$$e_1=\begin{pmatrix}1&0\\0&0\end{pmatrix},\ e_2=\begin{pmatrix}0&1\\0&0\end{pmatrix},\ e_3=\begin{pmatrix}0&0\\1&0\end{pmatrix},\ e_4=\begin{pmatrix}0&0\\0&1\end{pmatrix}$$

Chunki ixtiyoriy 2-tartibli matritsani bu matritsalarning chiziqli kombinatsiyasi orqali quyidagicha yozish mumkin

$$\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}=a_{11}e_1+a_{12}e_2+a_{21}e_3+a_{22}e_4$$

$e_1=\begin{pmatrix}1&0\\0&0\end{pmatrix},\ e_2=\begin{pmatrix}0&1\\0&0\end{pmatrix},\ e_3=\begin{pmatrix}0&0\\1&0\end{pmatrix},\ e_4=\begin{pmatrix}0&0\\0&1\end{pmatrix}$ matritsalar sistemasining chiziqli erkliligini ko‘rsatamiz. Buning uchun quyidagi tenglikni qaraymiz:

$\lambda_1e_1+\lambda_2e_2+\lambda_3e_3+\lambda_4e_4=\begin{pmatrix}0&0\\0&0\end{pmatrix}$. Bu tenglik faqat va faqat $\lambda_1=0,\ \lambda_2=0,\ \lambda_3=0,$ $\lambda_4=0$ bajarilsagina o‘rinli bo‘lgani uchun $e_1=\begin{pmatrix}1&0\\0&0\end{pmatrix},\ e_2=\begin{pmatrix}0&1\\0&0\end{pmatrix},\ e_3=\begin{pmatrix}0&0\\1&0\end{pmatrix},$ $e_4=\begin{pmatrix}0&0\\0&1\end{pmatrix}$ matritsalar sistemasi M^2 fazoning bazisi hisoblanadi. Bundan M^2 fazoning o‘lchovi 4 ga tengligi ham kelib chiqadi.

Teorema. n o‘lchovli L chiziqli fazoning har bir elementi bazis vektorlarining chiziqli kombinatsiyasi ko‘rinishida bir qiymatli yoziladi.

Isbot. Faraz qilaylik $\{\ e_1,e_2,...,e_n\ \}$-elementlar sistemasi L fazoning bazisi va $x\in L$ ixtiyoriy element bo‘lsin. U holda $\{\ e_1,e_2,...,e_n,x\ \}$ elementlar sistemasi L fazoda chiziqli bog‘liq bo‘ladi. U holda barchasi bir vaqtda nolga teng bo‘lmagan $\{\ \lambda_1,\lambda_2,...,\lambda_n,\lambda\ \}$ sonlar ketma-ketligi mavjudki,

$$\lambda_1x_1+\lambda_2x_2+\cdots+\lambda_nx_n+\lambda x=\theta \tag{1}$$

tenglik o‘rinli bo‘ladi. Bu yerda $\lambda\neq0$ bo‘ladi, aks holda $\lambda_1x_1+\lambda_2x_2+\cdots+\lambda_nx_n=\theta$ tenglikda $\{\ \lambda_1,\lambda_2,...,\lambda_n\ \}$ sonlarning hech bo‘lmaganda bittasi noldan farqli bo‘lishi kerak, ammo bu $\{\ e_1,e_2,...,e_n\ \}$ elementlar sistemasining bazisligiga ziddir. Chunki

$$\lambda_1e_1+\lambda_2e_2+\cdots+\lambda_ne_n=\theta\Rightarrow\lambda_1=...=\lambda_n=0. \tag{1}$$

tenglikdan quyidagiga ega bo‘lamiz:

$x = -\frac{\lambda_1}{\lambda} e_1 + \frac{\lambda_2}{\lambda} e_2 + ... + \frac{\lambda_n}{\lambda} e_n$, yoki $\mu_i = -\frac{\lambda_i}{\lambda} (i = 1,2,...,n)$ belgilashdan,

$$x = \mu_1 e_1 + \mu_2 e_2 + ... + \mu_n e_n \tag{2}$$

ya‘ni L fazoning ixtiyoriy elementi bazis elementlarining kombinatsiyasi, ko‘rinishida ifodalanadi.

Endi (2) yoyilma bir qiymatli yo‘zilishini isbotlaymiz. Faraz qilaylik bu x elementni boshqa ko‘rinishda ham ifodalash mumkin bo‘lsin:

$$x = \gamma_1 e_1 + \gamma_2 e_2 + ... + \gamma_n e_n \tag{3}$$

(2) va (3) ifodalarni hadma-had ayirib quyidagini hosil qilamiz $(\mu_1 - \gamma_1)e_1 + (\mu_2 - \gamma_2)e_2 + ... + (\mu_n - \gamma_n)e_n = \theta$. Bu tenglikdan va $\{\ e_1, e_2, ..., e_n\ \}$ elementlar sistemasining bazisligidan $\mu_1 - \gamma_1 = \mu_2 - \gamma_2 = ... = \mu_n - \gamma_n = 0$ yani $\mu_1 = \gamma_1, \mu_2 = \gamma_2, ..., \mu_n = \gamma_n$. Demak (2) yo‘yilma yagona bo‘ladi.

2.7- ta’rif. (2) tenglik $x \in L$ elementning $\{\ e_1, e_2, ..., e_n\ \}$ *bazis vektorlari bo‘yicha yoyilmasi* deyiladi, $\lambda_1, \lambda_2, ..., \lambda_n$ sonlarga esa x elementning bu bazis vektorlar bo‘yicha koordinatalari deyiladi

Chiziqli fazo elementlari uchun chiziqli bog‘liqlik va erklilik tushunchalariga misollar ko‘ramiz.

11- misol. $C[a,b]$ fazoda $x_1 = e^t$ va $x_2 = 3e^t$ funksiyalar chiziqli bog‘liq bo‘ladimi?

Yechish. Bu vektorlarning quyidagicha chiziqli kombinatsiyasini tuzamiz va uni nolga tenglaymiz: $\lambda_1 x_1 + \lambda_2 x_2 = 0 \Rightarrow \ \lambda_1 e^t + 3\lambda_2 e^t = 0 \Rightarrow$, $3\lambda_1 - \lambda_2 = 0$.

Demak, bu funksiyalar chiziqli bog‘liq.

Xuddi shunga o‘xshab ko‘rsatish mumkinki $C[a,b]$ fazoda $y_1 = sin^2 t$, $y_2 = cos^2 t$, $y_3 = \frac{1}{2}$ funksiyalar ham chiziqli bog‘liq bo‘ladi. Chunki $y_1 + y_2 - 2y_3 \equiv 0$.

2.8- ta’rif. Agar chiziqli fazo cheksiz sondagi chiziqli erkli vektorlar sistemasiga ega bo‘lsa, u holda bunday chiziqli fazoga *cheksiz o‘lchovli* chiziqli fazo deyiladi.

Yuqorida ko‘rilgan $C[a,b]$ fazo cheksiz o‘lchovli chiziqli fazo bo‘ladi, chunki $\{1, t, t^2,...,t^n\}$ funksiyalar barcha $n \in N$ lar uchun chiziqli erkli bo‘ladi.

2.9- ta’rif. L chiziqli fazoning V qism to‘plamining o‘zi ham L da aniqlangan elementlarni qo‘shish va elementlarni songa ko‘paytirish amallariga nisbatan chiziqli fazo tashkil qilsa, u holda V fazo L fazoning chiziqli *qism fazosi* deyiladi.

12- misol. Barcha n-tartibli kvadrat matritsalar chiziqli fazosini qaraymiz. Bu fazo uchun barcha n-tartibli diagonal matritsalar fazosi chiziqli qism fazo bo‘ladimi?

Yechish. Ixtiyoriy

$$D_1 = \begin{pmatrix} a_{11} & 0 & ... & 0 \\ 0 & a_{22} & ... & 0 \\ ... & ,,, & ... & ... \\ 0 & 0 & ... & a_{nn} \end{pmatrix} \qquad D_2 = \begin{pmatrix} b_{11} & 0 & ... & 0 \\ 0 & b_{22} & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & b_{nn} \end{pmatrix}$$

matritsalarni qaraymiz. Ma‘lumki bunda

$$D_1 + D_2 = \begin{pmatrix} a_{11} + b_{11} & 0 & ... & 0 \\ 0 & a_{22} + b_{22} & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & a_{nn} + b_{nn} \end{pmatrix}$$

ya‘ni ikkita diagonal matritsaning yig‘indisi yana diagonal matritsa bo‘ladi.
Endi diagonal matritsaning λ songa ko‘paytmasini tekshiramiz:

$$\lambda D_1 = \lambda \cdot \begin{pmatrix} a_{11} & 0 & ... & 0 \\ 0 & a_{22} & ... & 0 \\ ... & ,,, & ... & ... \\ 0 & 0 & ... & a_{nn} \end{pmatrix} = \begin{pmatrix} \lambda a_{11} & 0 & ... & 0 \\ 0 & \lambda a_{22} & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \lambda a_{nn} \end{pmatrix}$$

ya‘ni diagonal matritsani λ songa ko‘paytirsak yana diagonal matritsa hosil bo‘ladi. Bundan tashqari bizga ma‘lumki, $n-$tartibli matritsalar uchun chiziqli fazo uchun o‘rinli bo‘lgan yuqoridagi 8 ta aksioma bajariladi. Demak, n-tartibli diagonal matritsalar to‘plami $n-$tartibli matritsalar fazosining chiziqli qism fazosini tashkil qiladi. Endi biz oldingi mavzuda R^n arifmetik fazo uchun kiritilgan ckalyar ko‘paytma tushunchasini chiziqli fazo uchun umumlashtiramiz.

2.10- ta’rif. L chiziqli fazoning har bir x va y vektorlar juftligiga biror qoida bilan haqiqiy son $(x,\, y)$ mos qo‘yilgan bo‘lib, bu moslik uchun quyidagi shartlar:

1) $(x,\, y)=(y, x)$;

2) $(x+y,\, z)=(x,\, z)+(y,\, z)$;

3) $(\alpha x,\, y)=\alpha(x,\, y)$.

4) $(x,\, x)\geq 0$, ixtiyoriy $x\in L$ uchun $(x,\, x)=0 \Leftrightarrow x=\theta$;

bajarilsa, u holda $(x,\, y)$ son x va y vektorlarning *skalyar ko‘paytmasi* deyiladi.

2.11- ta’rif. Agar chiziqli fazo elementlari orasida skalyar ko‘paytma aniqlangan bo‘lsa, bu fazo *Yevklid fazosi* deyiladi va E^n ko‘rinishda belgilanadi. Har qanday n o‘lchovli haqiqiy arifmetik fazoda skalyar ko‘paytmani aniqlash orqali uni Yevklid fazosiga aylantirish mumkin.

2.12- ta’rif. Yevklid fazosidan olingan $\vec{x}$ vektor uchun quyidagicha

$$\left|\vec{x}\right| = \sqrt{(\vec{x}, \vec{x})}$$

aniqlangan songa $\vec{x}$ vektorning *normasi* (*uzunligi)* deb aytiladi:

Vektorning uzunligi uchun quyidagi xossalar o‘rinlidir:

1. $\left|\vec{x}\right|\geq 0$ barcha $\vec{x}\in L$ elementlar uchun. $\left|\vec{x}\right|=0 \Leftrightarrow \vec{x}=\theta$

2. $\left|\lambda\vec{x}\right|=\left|\lambda\right|\cdot\left|\vec{x}\right|$, bunda $\lambda\in R$;

3. $\left|(\vec{x},\vec{y})\right|\leq\left|\vec{x}\right|\cdot\left|\vec{y}\right|$ (Koshi-Bunyakovskiy tengsizligi);

4. $\left|\vec{x}+\vec{y}\right|\leq\left|\vec{x}\right|+\left|\vec{y}\right|$ (uchburchak tengsizligi).

2.13- ta’rif. Agar $\vec{x},\vec{y}\in E^n$ elementlar uchun $(\vec{x},\vec{y})=0$ bo‘lsa u holda $\vec{x}$ *va* $\vec{y}$ elementlar ortogonal vektorlar deyiladi.

2.14- ta'rif. Noldan farqli $\overrightarrow{a_1}, \overrightarrow{a_2}, ..., \overrightarrow{a_n} \in E^n$ elementlardan tashkil topgan vektorlar sistemasidagi vektorlarning har qanday ikki jufti o'zaro ortogonal bo'lsa, u holda bu sistema *ortogonal vektorlar sistemasi* deb ataladi.

2.15- ta'rif. Agar $\left\{\overrightarrow{a_1}, \overrightarrow{a_2}, ..., \overrightarrow{a_n}\right\} \subset E^n$ ortogonal vektorlar sistemasi bo'lib $\left|\vec{a}_i\right| = 1 \left(i = 1, 2, ..., n\right)$ bo'lsa, u holda $\left\{\vec{a}_1, \vec{a}_2, \vec{a}_3, ..., \vec{a}_n\right\}$ vektorlar sistemasi *ortonormal vektorlar sistemasi* deyiladi.

2.16- ta'rif. Agar $\left\{\overrightarrow{e_1}, \overrightarrow{e_2}, \overrightarrow{e_3}, ..., \overrightarrow{e_n}\right\} \subset E^n$ vektorlar sistemasi E^n fazoning bazisi bo'lib, ortonormal vektorlar sistemasini tashkil qilsa, u holda bu bazisga *ortonormal bazis* deyiladi.

Ortonormallangan $\left\{\overrightarrow{e_1}, \overrightarrow{e_2}, \overrightarrow{e_3}, ..., \overrightarrow{e_n}\right\} \subset E^n$ bazis uchun quyidagi munosabat o'rinli:

$$\left(\vec{e}_i, \vec{e}_k\right) = \begin{cases} 1, agar\ i = k\ bo'lsa \\ 0, agar\ i \neq k\ bo'lsa \end{cases}$$

Teorema. (*Pifagor teoremasining umumlashmasi*) Agar $\left\{\overrightarrow{a_1}, \overrightarrow{a_2}, ..., \overrightarrow{a_n}\right\} \subset E^n$ vektorlar sistemasi juft-jufti bilan ortogonal bo'lsa, u holda quyidagi munosabat o'rinli

$$\left|\vec{a}_1 + \vec{a}_2 + ... + \vec{a}_n\right|^2 = \left|\vec{a}_1\right|^2 + \left|\vec{a}_2\right|^2 + ... + \left|\vec{a}_n\right|^2$$

Teorema. Agar $\left\{\overrightarrow{a_1}, \overrightarrow{a_2}, ..., \overrightarrow{a_n}\right\} \subset E^n$ vektorlar noldan farqli va juft-jufti bilan orthogonal bo'lsa u holda bu vektorlar chiziqli erkli bo'ladi.

Isbot. Bu vektorlarning chiziqli kombinatsiyasini tuzib uni nolga tenglaymiz

$$\lambda_1 \vec{a}_1 + \lambda_2 \vec{a}_2 + ... + \lambda_n \vec{a}_n = 0$$

Bu tenglikning ikkala tomonini a_1 ga skalyar ko'paytiramiz:

$$\lambda_1 (\vec{a}_1, \vec{a}_1) + \lambda_2 (\vec{a}_2, \vec{a}_1) + ... + \lambda_n (\vec{a}_n, \vec{a}_1) = 0$$

Teorema shartiga ko'ra $(\vec{a}_1, \vec{a}_1) \neq 0, (\vec{a}_1, \vec{a}_i) \neq 0 \left(i = 2, 3, ..., n\right)$ bo'lgani uchun oxirgi tenglikdan $\lambda_1 (\vec{a}_1, \vec{a}_1) = \lambda_1 \left\|\vec{a}_1\right\|^2 = 0,$ ga ega bo'lamiz. Bundan $\lambda_1 = 0$ ekani kelib chiqadi. Xuddi shunga o‘xshab $\lambda_2 = \lambda_3 = ... = \lambda_n = 0$ ekanligi isbotlanadi. Demak $\vec{a}_1, \vec{a}_2, ..., \vec{a}_k \in E^n$ chiziqli erkli vektorlar sistemasini tashkil qiladi. Teorema isbotlandi.

Teorema. Har qanday ***n*** o‘lchovli haqiqiy Evklid fazosida ortonormallangan bazis mavjud.

Isbot. Faraz qilaylik $\{\vec{e}_1, \vec{e}_2, \vec{e}_3, ..., \vec{e}_n\} \subset E^n$ vektorlar sistemasi E^n fazoning ortonormall bo‘lmagan bazislaridan biri bo‘lsin. Biz bu bazisdan ortonormallangan bazisni quramiz. Buning uchun Shmidt formulalaridan foydalanamiz:

1) $\vec{\boldsymbol{e}}' = \vec{\boldsymbol{e}}_1$, deb olib keyingi qadamda

$$2) \vec{\boldsymbol{e}}'_t = \vec{\boldsymbol{e}}_t - \sum_{i=1}^{t-1} \frac{\left(\vec{\boldsymbol{e}}'_i \cdot \vec{\boldsymbol{e}}_t\right)}{\left(\vec{\boldsymbol{e}}'_i \cdot \vec{\boldsymbol{e}}'_i\right)} \vec{\boldsymbol{e}}'_i, \quad t = 2, 3, ..., k$$

Teorema isbotlandi.

13- misol. R^3 fazoda berilgan $\vec{a}_1(1, 1, 1)$, $\vec{a}_2(0, 1, 1)$, $\vec{a}_3(0, 0, 1)$ vektorlar sistemasidan ortonormallangan bazis quring.

Yechish. Birinchi navbatda $\vec{a}_1(1, 1, 1)$, $\vec{a}_2(0, 1, 1)$, $\vec{a}_3(0, 0, 1)$ vektorlar sistemasining rangini aniqlab olamiz

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

$rang(\vec{a}_1, \ \vec{a}_2, \ \vec{a}_3) = 3$ bo‘lganligi sababli bu sistemadagi vektorlar chiziqli erkli. Sistemani ortogonal sistemaga aylantirish uchun Shmidt formulasidan foydalanamiz:

1) $\vec{b}_1 = \vec{a}_1(1, 1, 1)$;

$$2) \ \vec{b}_2 = \vec{a}_2 - \frac{\left(\vec{b}_1, \vec{a}_2\right)}{\left(\vec{b}_1, \vec{b}_1\right)} \vec{b}_1 = (0, 1, 1) - \frac{2}{3}(1, 1, 1) = \left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right);$$

$$3) \vec{b}_3 = \vec{a}_3 - \frac{\left(\vec{b}_1, \vec{a}_3\right)}{\left(\vec{b}_1, \vec{b}_1\right)} \vec{b}_1 - \frac{\left(\vec{b}_2 \, \vec{a}_3\right)}{\left(\vec{b}_2 \, \vec{b}_2\right)} \vec{b}_2 = \left(0; -\frac{1}{2}; \frac{1}{2}\right).$$

Berilgan vektorlar sistemasi ustida qurilgan ortogonal sistema vektorlarini butun koordinatali vektorlarga aylantirish uchun $\vec{c}_1 = \vec{b}_1(1,1,1)$; $\vec{b}_2 = \left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right)$ ni unga kollinear bo‘lgan $\vec{c}_2(-2,1,1) = 3\vec{b}_2$ bilan; $\vec{b}_3 = \left(0; -\frac{1}{2}; \frac{1}{2}\right)$ ni esa unga kollinear bo‘lgan $\vec{c}_3(0,\,-1,1) = 2\vec{b}_3$ bilan almashtirib va $\vec{c}_1 = \vec{b}_1(1,1,1)$ belgilash kiritib: $\vec{c}_1(1,1,1)$, $\vec{c}_2(-2,1,1)$, $\vec{c}_3(0,\,-1,1)$ ortogonal vektorlar sistemasini hosil qilamiz.

Nol bo‘lmagan $\vec{c}$ vektorning birlik vektori, deb $\frac{\vec{c}}{|\vec{c}|}$ vektorga aytiladi.

Yuqoridagi misolda topilgan ortogonal $\vec{c}_1(1,1,1)$, $\vec{c}_2(-2,1,1)$, $\vec{c}_3(0,\,-1,1)$ vektorlar sistemasini ortonormal vektorlar sistemasiga keltiramiz.

$$\frac{\vec{c}_1}{|\vec{c}_1|} = \frac{1}{\sqrt{1^2+1^2+1^2}}(1,1,1) = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$$\frac{\vec{c}_2}{|\vec{c}_2|} = \frac{1}{\sqrt{(-2)^2+1^2+1^2}}(-2,1,1) = \left(-\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$$

$$\frac{\vec{c}_3}{|\vec{c}_3|} = \frac{1}{\sqrt{0^2+(-1)^2+1^2}}(0,\,-1,\,1) = \left(0,\,-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

Tayanch so‘z va iboralar: *chiziqli fazo, elementlarning chiziqli kombinatsiyasi, chiziqli kombinatsiya koeffitsientlari, chiziqli bog‘liq va chiziqli erkli elementlar, chiziqli fazo bazisi, chiziqli fazo o‘lchami, qism fazo, Yevklid fazosi*

O‘z-o‘zini tekshirish uchun savollar

1. Chiziqli fazo deb nimaga aytiladi?
2. Chiziqli fazoning qism osti fazosi deb nimaga aytiladi? Misollar keltiring.
3. Chiziqli fazoda elementlarning chiziqli kombinatsiyasi.
4. *n*-o‘lchovli chiziqli fazo deb qanday chiziqli fazoga aytiladi?
5. Chiziqli fazo o‘lchovi deb nimaga aytiladi?
6. *n*-o‘lchovli chiziqli fazo bazisi deb nimaga aytiladi?
7. Chiziqli fazo elementlarining bazis vektorlari bo‘yicha yoyilmasi, elementlarining bazisdagi koordinatalari.

8. Har qanday x∊L^n vektorni fazoning bazisi orqali yoyish mumkinmi?

9. Vektorning biror-bir bazisdagi koordinatalari deb nimaga aytiladi?

10. Qanday chiziqli fazoga Yevklid fazo deyiladi?

Mustaqil yechish uchun misollar

14. Barcha n-tartibli kvadrat matritsalar fazosini va barcha n-tartibli simmetrik matritsalar to‘plamini qaraymiz. Agar barcha n-tartibli simmetrik matritsalar to‘plami barcha n-tartibli kvadrat matritsalar fazosining chiziqli qism fazosi bo‘lsa, chiziqli qism fazoning o‘lchovini toping.

15. Tekislikda boshi koordinatalar boshida uchi I chorakda bo‘lgan vektorlar to‘plami vektorlarni qo‘shish va songa ko‘paytirish amallariga nisbatan chiziqli fazo tashkil qiladimi?

16. Tekislikda birorta vektorga parallel bo‘lgan vektorlar to‘plami vektorlarni qo‘shish va songa ko‘paytirish amallariga nisbatan chiziqli fazo tashkil qiladimi?

17. R_+^1 $-$barcha musbat haqiqiy sonlar to‘plami bo‘lsin. Bu to‘plamda quyidagicha amal kiritamiz: ikki son yig‘ndisi sifatida ularning oddiy ko‘paytmasini, $r \in R_+^1$ ning λ songa ko‘paytmasi sifatida esa r^λ ni tushunamiz. Bu kiritilgan amallarga nisbatan R_+^1 chiziqli fazo tashkil qiladimi?

18. $P(t) = 5 - 2(t+1) + 3(t+1)^2 + (t+1)^3$ ko‘phadning quyidagi bazisga nisbatan koordinatalarini toping.

a) $e_1 = 1, e_1 = t, e_1 = t^2, \ e_1 = t^3$;

b) $e_1 = 1, e_2 = t+1, e_3 = (t+1)^2, \ e_4 = (t+1)^3$.

19. $\vec{x}(2;-1)$ vektor $\vec{e_1}, \vec{e_2}$ bazisda berilgan. Vektorning $\begin{cases} \vec{e_1'} = \vec{e_1} - 3\vec{e_2} \\ \vec{e_2'} = 2\vec{e_1} + \vec{e_2} \end{cases}$

bazisdagi koordinatalarini toping.

20. $\vec{x}(3;-2)$ vector $\vec{e_1}, \vec{e_2}$ bazisda berilgan. Vektorning $\begin{cases} \vec{e_1'} = 2\vec{e_1} - \vec{e_2} \\ \vec{e_2'} = \vec{e_1} + \vec{e_2} \end{cases}$

bazisdagi koordinatalarini toping.

21. $\vec{x}(1;2;-2)$ vektor $\vec{e_1},\vec{e_2},\vec{e_3}$ bazisda berilgan vektorning $\begin{cases} \vec{e_1'} = \vec{e_1} + \vec{e_2} - \vec{e_3} \\ \vec{e_2'} = 2\vec{e_1} - \vec{e_2} + \vec{e_3} \end{cases}$ bazisdagi koordinatalarini toping.

22. Quyida berilgan ikki vektorlar sistemalaridan har biri bazis bo‘la olishini isbotlang. Ushbu bazislarda berilgan aynan bir vektorning koordinatalari orasida munosabatlarni o‘rnating:

1) $\begin{cases} \vec{e_1}(1;2) \\ \vec{e_2}(1;1) \end{cases}$ va $\begin{cases} \vec{e_1'}(1;1) \\ \vec{e_2'}(3;4) \end{cases}$ 2) $\begin{cases} \vec{e_1}(1;3) \\ \vec{e_2}(2;3) \end{cases}$ va $\begin{cases} \vec{e_1'}(1;0) \\ \vec{e_2'}(0;-3) \end{cases}$

3) $\begin{cases} \vec{e_1}(2;3) \\ \vec{e_2}(2;4) \end{cases}$ va $\begin{cases} \vec{e_1'}(0;-1) \\ \vec{e_2'}(6;11) \end{cases}$ 4) $\begin{cases} \vec{e_1}(2;1;-1) \\ \vec{e_2}(3;1;2) \\ \vec{e_3}(1;0;4) \end{cases}$ va $\begin{cases} \vec{e_1'}(1;1;-1) \\ \vec{e_2'}(2;3;-2) \\ \vec{e_3'}(3;4;-4) \end{cases}$

23. R^4 fazoda quyidagi vektorlar bazis tashkil qiladimi?

a) $\vec{e_1} = \{1,1,1,0\}$, $\vec{e_3} = \{1,1,2,1\}$,

$\vec{e_2} = \{1,2,1,1\}$, $\vec{e_4} = \{1,3,2,5\}$.

b) $\vec{e_1} = \{2,3,4,-3\}$, $\vec{e_3} = \{1,0,0,0\}$,

$\vec{e_2} = \{5,4,9,-2\}$, $\vec{e_4} = \{3,5,5,3\}$.

3. Chiziqli operatorlar va ularning xossalari

Matritsalar algebrasining asosiy tushunchalaridan biri – chiziqli operatorlar tushunchasidir. Faraz qilaylik bizga $L,\ L_1$ chiziqli fazolar berilgan bo‘lsin.

3.1- ta’rif. Agar biror A qoida yoki qonun bo‘yicha har bir $x \in L$ elementga $y \in L_1$ element mos qo‘yilgan bo‘lsa, u holda L fazoni L_1 fazoga o‘tkazuvchi A *operator (almashtirish, akslantirish)* aniqlangan deyiladi va $y = A(x)$ ko‘rinishda belgilanadi.

3.2- ta’rif. Agar ixtiyoriy $x, y \in L,\ \lambda \in R$ uchun:

1) $\tilde{A}(x+y) = \tilde{A}(x) + \tilde{A}(y)$ (*operatorning additivligi*);

2) $\tilde{A}(\lambda x)=\lambda\tilde{A}(x)$ *(operatorning bir jinsliligi)* munosabatlar o‘rinli bo‘lsa, u holda bu operator chiziqli operator deyiladi.

1- misol. $\tilde{A}:R^2\to R^3$ operator $\tilde{A}(x,y)=(x,y,x+y)$ qoida bilan aniqlangan bo‘lsin, u holda bu operatorning chiziqli operator ekanligini ko‘rsating.

Yechish. Ma‘lumki, $\vec{a}_1=(x_1,y_1)$ va $\vec{a}_2=(x_2,y_2)$ vektor uchun $\vec{a}_1+\vec{a}_2=(x_1+x_2,y_1+y_2)$. U holda $\vec{a}_1+\vec{a}_2=(x_1+x_2,y_1+y_2)$ elementga A operatorni ta‘sir ettirsak, quyidagiga ega bo‘lamiz:

$$\tilde{A}(a_1+a_2)=\tilde{A}(x_1+x_2,y_1+y_2)=(x_1+x_2,y_1+y_2,x_1+x_2+y_1+y_2)=$$
$$=(x_1,y_1,x_1+y_1)+(x_2,y_2,x_2+y_2)=\tilde{A}(a_1)+\tilde{A}(a_2).$$

Bu esa $\tilde{A}$ operatorning additivligini ko‘rsatadi.

Endi operatorning bir jinsli ekanligini tekshiramiz. Ma‘lumki, $ka_1=(kx_1,ky_1)$. U holda

$$\tilde{A}(ka_1)=\tilde{A}(kx_1,ky_1)=(kx_1,ky_1,kx_1+ky_1)=k(x_1,y_1,x_1+y_1)=k\tilde{A}(a_1).$$

Demak, biz o‘rganayotgan operator chiziqli operatordir.

$y=A(x)\in L_1$ element $x\in L$ elementning *aksi*, $x\in L$ elementning o‘zi esa $y\in L_1$ elementning *proobrazi* deyiladi. Agar $L=L_1$ bo‘lsa, u holda $\tilde{A}$ operator L fazoni o‘zini o‘ziga akslantiruvchi operator bo‘ladi. Biz ko‘proq fazoni o‘zini o‘ziga akslantiruvchi operatorlarni o‘rganamiz.

Teorema. Har bir $\tilde{A}:L^n\to L^n$ chiziqli operatorga berilgan bazisda $n-$tartibli matritsa mos keladi va aksincha har bir $n-$tartibli matritsaga n o‘lchovli chiziqli fazoni, n o‘lchovli chiziqli fazoga akslantiruvchi $\tilde{A}$ chiziqli operator mos keladi.

Isbot. Faraz qilaylik $\tilde{A}:L^n\to L^n$ chiziqli operator bo‘lsin. Agar $\{\vec{e}_1,\vec{e}_2,...,\vec{e}_n\}\subseteq L^n$ vektorlar sistemasi L^n fazoning bazisi bo‘lsa, u holda ixtiyoriy $\vec{x}\in L^n$ elementni bu bazis elementlari orqali yozish mumkin:

$$\vec{x}=x_1\vec{e}_1+...+x_n\vec{e}_n. \tag{1}$$

Bu yerda biz $\tilde{A}$ operatorning chiziqliligidan foydalanib, $\tilde{A}(x)$ ni quyidagicha yoza olamiz:

$$\tilde{A}(x)=\tilde{A}\left(x_1\vec{e}_1+\ldots+x_n\vec{e}_n\right)=x_1\tilde{A}(\vec{e}_1)+\ldots+x_n\tilde{A}(\vec{e}_n)\,. \qquad (2)$$

Bu yerda har bir $\tilde{A}(e_i)$ $(i=\overline{1,n})$ elementlar o‘z navbatida L^n fazoning elementlari bo‘lganligi sababli, bu elementlarni ham $\left\{\vec{e}_1,\vec{e}_2,\ldots,\vec{e}_n\right\}$ bazis orqali yozish mumkin:

$$\tilde{A}(\vec{e}_i)=a_{1i}\vec{e}_1+\ldots+a_{ni}\vec{e}_n\,. \qquad (3)$$

U holda (3) dan foydalanib (2) ifodani quyidagicha yozish mumkin:

$$\begin{aligned}&\tilde{A}(x)=x_1(a_{11}\vec{e}_1+a_{21}\vec{e}_2+\ldots+a_{n1}\vec{e}_n)+x_2(a_{12}\vec{e}_1+a_{22}\vec{e}_2+\ldots+a_{n2}\vec{e}_n)+\ldots\\&+x_n(a_{1n}\vec{e}_1+a_{2n}\vec{e}_2+\ldots+a_{nn}\vec{e}_n)=(a_{11}x_1+a_{12}x_2+\ldots+a_{1n}x_n)\vec{e}_1+\\&+(a_{21}x_1+a_{22}x_2+\ldots+a_{2n}x_n)\vec{e}_2+\ldots+(a_{n1}x_1+a_{n2}x_2+\ldots+a_{nn}x_n)\vec{e}_n\end{aligned} \qquad (4)$$

Ikkinchi tomondan $y=\tilde{A}(x)$ element ham $\left\{\vec{e}_1,\vec{e}_2,\ldots,\vec{e}_n\right\}$ bazis elementlari bo‘yicha quyidagi yoyilmaga ega:

$$y=\tilde{A}(x)=y_1\vec{e}_1+y_2\vec{e}_2+\ldots+y_n\vec{e}_n\,. \qquad (5)$$

Vektorning bitta bazis bo‘yicha yoyilmasi yagonaligidan (4) va (5) tengliklarning o‘ng tomonlarini tenglashtirib, quyidagini olamiz.

$$\begin{cases}y_1=a_{11}x_1+a_{12}x_2+\ldots+a_{1n}x_n\\y_2=a_{21}x_1+a_{22}x_2+\ldots+a_{2n}x_n\\\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\y_n=a_{n1}x_1+a_{n2}x_2+\ldots+a_{nn}x_n\end{cases}$$

yoki matritsa ko‘rinishida $Y=AX$, bu yerda

$$A=\begin{pmatrix}a_{11}&a_{12}&\ldots&a_{1n}\\a_{21}&a_{22}&\ldots&a_{2n}\\\cdot&\cdot&\ldots&\cdot\\a_{n1}&a_{n2}&\ldots&a_{nn}\end{pmatrix},\ X=\begin{pmatrix}x_1\\x_2\\\cdot\\x_n\end{pmatrix},\ Y=\begin{pmatrix}y_1\\y_2\\\cdot\\y_n\end{pmatrix}$$

3.3- ta’rif. $A=(a_{ij})$ $(i,j=1,2,\ldots,n)$ matritsa $\tilde{A}$ operatorning $\left\{\vec{e}_1,\vec{e}_2,\ldots,\vec{e}_n\right\}$ bazisdagi matritsasi, $A=(a_{ij})$ $(i,j=1,2,\ldots,n)$ matritsaning rangi esa $\tilde{A}$ *operatorning rangi* deyiladi.

L fazoning barcha vektorlarini θ nol vektorga akslantiruvchi $\theta(x)=\theta$ operator *nol operator*, $E(x)=x$ tenglikni qanoatlantiruvchi operator *birlik operator* deb ataladi.

2- misol. R^3 fazoda $\{e_1, e_2, e_n\}$ bazisda chiziqli operator matritsasi

$$A=\begin{pmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{pmatrix}$$

berilgan bo‘lsin. $x=4\vec{e}_1-3\vec{e}_2+\vec{e}_3$ vektorning $y=A(x)$ aksini toping.

Yechish. Yuqorida qayd qilingan formulaga ko‘ra

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}=\begin{pmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{pmatrix}\begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}=\begin{pmatrix} 10 \\ -13 \\ -18 \end{pmatrix}$$

Demak, $y=10\vec{e}_1-13\vec{e}_2-18\vec{e}_3$.

3- misol. $T: R^3 \to R^4$; $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} x_1+x_2 \\ x_1-x_2 \\ x_3 \\ x_1 \end{pmatrix}$ operatorning matritsasini toping.

Yechish. $A=\begin{bmatrix} T(\vec{e}_1) & T(\vec{e}_2) & T(\vec{e}_3) \end{bmatrix}$ matritsaning har bir elementini topamiz:

$$T(\vec{e}_1)=T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}=\begin{pmatrix} 1+0 \\ 1-0 \\ 0 \\ 1 \end{pmatrix}=\begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix},$$

$$T(\vec{e}_2)=T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}=\begin{pmatrix} 0+1 \\ 0-1 \\ 0 \\ 0 \end{pmatrix}=\begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix},$$

$$T(\vec{e}_3) = T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0+0 \\ 0-0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$

U holda

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

Chiziqli operatorlar ustida bajariladigan chiziqli amallar bilan tanishib chiqamiz. L^n chiziqli fazoda $A,\ B$ operatorlar berilgan bo‘lsin.

3.4- ta’rif. $\left(A+B\right)(x) = A(x) + B(x)$ tenglik bilan aniqlanadigan operatorni $A,\ B$ *operatorlarning yig‘indisi* deb ataladi.

Teorema. Agar A va B operatorlar chiziqli operatorlar bo‘lsa, u holda $A+B$ operator ham chiziqli operator bo‘ladi .

Isbot. Ixtiyoriy $x, y \in R^n$ vektorlar va $\alpha \in R$ son uchun:

1) $\left(\tilde{A}+\tilde{B}\right)(x+y) = \tilde{A}(x+y) + \tilde{B}(x+y) = \tilde{A}(x) + \tilde{A}(y) + \tilde{B}(x) + \tilde{B}(y) =$

$$= \left(\tilde{A}+\tilde{B}\right)(x) + \left(\tilde{A}+\tilde{B}\right)(y);$$

2) $\left(\tilde{A}+\tilde{B}\right)(\alpha x) = \tilde{A}(\alpha x) + \tilde{B}(\alpha x) = \alpha(\tilde{A}(x)) + \alpha(\tilde{B}(x)) = \alpha(\tilde{A}(x) + \tilde{B}(x)) =$

$$= \alpha\left[(\tilde{A}+\tilde{B})(x)\right]$$

munosabatlar o‘rinli. Bu esa $A+B$ operator chiziqli ekanligini ko‘rsatadi.

3.5- ta’rif. $\left(\tilde{A}\tilde{B}\right)(x) = \tilde{B}\left(\tilde{A}(x)\right)$ tenglik bilan aniqlanadigan, ya‘ni $A,\ B$ operatorlarni ketma-ket bajarishdan hosil bo‘lgan $A \cdot B$ operator *A va B operatorlarning ko‘paytmasi* deyiladi.

Teorema. Agar A va B operatorlar chiziqli operatorlar bo‘lsa, u holda $A \cdot B$ operator ham chiziqli operator bo‘ladi .

Isbot. Ixtiyoriy $\vec{x},\vec{y}\in R^n$ vektorlar va $\alpha\in R$ son uchun:

1) $\left(\tilde{A}\tilde{B}\right)(\vec{x}+\vec{y})=\tilde{B}[\tilde{A}(\vec{x}+\vec{y})]=\tilde{B}(\tilde{A}(\vec{x})+\tilde{A}(\vec{y}))=\left(\tilde{A}\tilde{B}\right)(\vec{x})+\left(\tilde{A}\tilde{B}\right)(\vec{y});$

2) $\left(\tilde{A}\tilde{B}\right)(\alpha\vec{x})=\tilde{B}[\tilde{A}(\alpha\vec{x})]=\tilde{B}[\alpha(\tilde{A}(\vec{x}))]=\alpha[\tilde{B}(\tilde{A}(\vec{x}))]=\alpha[(\tilde{A}\tilde{B})(\vec{x})]$

munosabat o‘rinli. Bu esa $A\cdot B$ operator chiziqli ekanligini ko‘rsatadi.

3.6- ta’rif. $\left(\alpha\tilde{A}\right)(\vec{x})=\alpha\left(\tilde{A}(\vec{x})\right)$ tenglik bilan aniqlanadigan αA operator A operatorlarning α *songa ko‘paytmasi* deyiladi.

Teorema. Agar A operator chiziqli operator bo‘lsa, u holda αA operator ham chiziqli operator bo‘ladi .

Isbot. Ixtiyoriy,ixtiyoriy $\vec{x},\vec{y}\in R^n$ vektorlar va $\alpha,\beta\in R$ sonlar uchun:

1) $\left(\alpha\tilde{A}\right)(\vec{x}+\vec{y})=\alpha[\tilde{A}(\vec{x}+\vec{y})]=\alpha(\tilde{A}(\vec{x})+\tilde{A}(\vec{y}))=$

$=\alpha(\tilde{A}(\vec{x}))+\alpha(\tilde{A}(\vec{y}))=\left(\alpha\tilde{A}\right)(\vec{x})+\left(\alpha\tilde{A}\right)(\vec{y});$

2) $\left(\alpha\tilde{A}\right)(\beta\vec{x})=\alpha[\tilde{A}(\beta\vec{x})]=\alpha[\beta(\tilde{A}(\vec{x}))]=\beta[\alpha(\tilde{A}(\vec{x}))]=\beta[(\alpha\tilde{A})(\vec{x})]$

munosabat o‘rinli. Bu esa αA operator chiziqli ekanligini ko‘rsatadi.

Yuqoridagilardan quyidagi xulosalarni chiqarish mumkin.

I. Ixtiyoriy bazisda chiziqli operatorlar yig‘indisining matritsasi bu operatorlarning o‘sha bazisdagi matritsalari yig‘indisiga teng.

II. Ixtiyoriy bazisda chiziqli operatorlar ko‘paytmasining matritsasi bu operatorlarning o‘sha bazisdagi matritsalari ko‘paytmasiga teng.

III. Biror bir bazisda A chiziqli operatorning α songa ko‘paytmasini beruvchi matritsa bu operatorning shu bazisdagi matritsasini α songa ko‘paytirilganiga teng.

3.7- ta’rif. $\tilde{A}(x)$ operator uchun $\tilde{A}\tilde{A}^{-1}=\tilde{A}^{-1}\tilde{A}=\tilde{E}$ munosabat o‘rinli bo‘lsa, u holda A^{-1} operator A operatorga *teskari operator* deb ataladi.

Teorema. $\tilde{A}(x)$ operatorga teskari operator mavjud bo‘lishi uchun uning har qanday bazisdagi A matritsasi xosmas bo‘lishi zarur va etarlidir.

Ta'rif. Matritsasi xosmas bo'lgan operatorga *xosmas operator*, deb ataladi.

4- misol. Quyida

$$\tilde{A}(x_1, x_2, x_3) = \left(2x_2,\, -2x_1 + 3x_2 + 2x_3,\, 4x_1 - x_2 + 5x_3\right) \text{ va}$$

$$\tilde{B}(x_1, x_2, x_3) = \left(-3x_1 + x_2,\, 2x_2 + x_3,\, -x_2 + 3x_3\right)$$

operatorlar berilgan. $C = A \cdot B$ operator va uning matritsasi topilsin.

Yechish. Avval A va B matritsalarni topib olamiz:

$$\tilde{A}(\vec{e}_1) = \begin{pmatrix} 0 \\ -2 \\ 4 \end{pmatrix},\ \tilde{A}(\vec{e}_2) = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix},\ \tilde{A}(\vec{e}_3) = \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix},$$

$$\tilde{B}(\vec{e}_1) = \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix},\ B(\vec{e}_2) = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix},\ \tilde{B}(\vec{e}_3) = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}.$$

U holda

$$A = \begin{pmatrix} 0 & 2 & 0 \\ -2 & 3 & 2 \\ 4 & -1 & 5 \end{pmatrix},\quad B = \begin{pmatrix} -3 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 3 \end{pmatrix}.$$

$$C = AB = \begin{pmatrix} 0 & 4 & 2 \\ 6 & 2 & 9 \\ -12 & -3 & 14 \end{pmatrix}.$$

Bundan

$$\tilde{C}(\vec{e}_1) = \begin{pmatrix} 0 \\ 6 \\ -12 \end{pmatrix}, \tilde{C}(\vec{e}_2) = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix}, \tilde{C}(\vec{e}_3) = \begin{pmatrix} 2 \\ 9 \\ 14 \end{pmatrix}.$$

$$\tilde{C}(\vec{x}) = \left(4x_2 + 2x_3,\, 6x_1 + 2x_2 + 9x_3,\, -12x_1 - 3x_2 + 14x_3\right).$$

Bitta chiziqli operatorning turli bazislardagi matritsalari orasidagi bog'lanish haqidagi teoremani keltiramiz.

Teorema. Agar A chiziqli operatorning $\left\{\vec{e}_1, \vec{e}_2, ..., \vec{e}_n\right\}$ va $\left\{\overrightarrow{e_1^*}, \overrightarrow{e_2^*}, ..., \overrightarrow{e_n^*}\right\}$ bazislardagi matritsalari mos ravishda A va A^* matritsalardan iborat bo'lsa, u holda $A^* = C^{-1}AC$ munosabat o‘rinli bo‘ladi. Bu yerda C o‘tish matritsasi deb ataladi.

5- misol. $\{\vec{e}_1, \vec{e}_2\}$ bazisda chiziqli operator matritsasi $A = \begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix}$ berilgan bo‘lsin. Yangi $\begin{cases} \vec{e}_1^{\,*} = \vec{e}_1 - 2\vec{e}_2 \\ \vec{e}_2^{\,*} = 2\vec{e}_1 + \vec{e}_2 \end{cases}$ bazisdagi chiziqli operator matritsasini toping.

Yechish. O‘tish matritsasi $C = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$, unga teskari matritsa $C^{-1} = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$. Demak, yangi bazisda operatorning matritsasi quyidagi ko‘rinishda bo‘ladi:

$$A^* = C^{-1}AC = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 20 \end{pmatrix}.$$

3.8- ta’rif. Agar A chiziqli operator va λ son uchun $\tilde{A}(x) = \lambda x$ tenglik o‘rinli bo‘lsa, u holda λ son $\tilde{A}(x)$ operatorning *xos soni,* unga mos $\vec{x}$ vektorga esa operatorning *xos vektori* deb ataladi.

Yuqoridagi tenglikni operatorning matritsasidan foydalanib yozsak, u holda quyidagi tenglamalar sistemasini hosil qilamiz:

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = \lambda \cdot x_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = \lambda \cdot x_2 \\ ------------------ \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = \lambda \cdot x_n \end{array}\right\} \Rightarrow \left.\begin{array}{l} (a_{11} - \lambda)x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = 0 \\ a_{21}x_1 + (a_{22} - \lambda)x_2 + \cdots + a_{2n}x_n = 0 \\ ------------------ \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + (a_{nn} - \lambda)x_n = 0 \end{array}\right\}$$

Bundan $\left[A - \lambda E\right] \cdot X = 0$.

Bizga ma‘lumki bir jinsli chiziqli tenglamalar sistemasi har doim trivial yechimga ega. Chiziqli tenglamalar sistemasi trivial bo‘lmagan yechimga ega bo‘lishi uchun esa uning koeffitsiyentlaridan tuzilgan determinantning qiymati nolga teng bo‘lishi zarur va etarli, ya‘ni

$$|A-\lambda E|=\begin{vmatrix} a_{11}-\lambda & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22}-\lambda & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn}-\lambda \end{vmatrix}=0 \qquad (6)$$

$|A-\lambda E|$ determinant λ ga nisbatan n darajali ko‘phaddir. Bu ko‘phad $\tilde{A}(x)$ operatorning xarakteristik ko‘phadi deb ataladi. (6) tenglama $\tilde{A}(x)$ operatorning xarakteristik tenglamasi deyiladi. Chiziqli operatorning xarakteristik ko‘phadi bazisni tanlashga bog‘liq emas.

6- misol. $\tilde{A}(\vec{x})=\left(2x_1-x_2+2x_3,\,5x_1-3x_2+3x_3,\,-x_1-2x_3\right)$ operatorning xos soni va xos vektorlarini toping.

Yechish. Avval $\tilde{A}$ operatorning matritsasini tuzib olamiz:

$$A=\begin{pmatrix} 2 & -1 & 2 \\ 5 & -3 & 3 \\ -1 & 0 & -2 \end{pmatrix}.$$

Berilgan operatorga mos keluvchi bir jinsli tenglamalar sistemasi quyidagi ko‘rinishni oladi:

$$\begin{cases} (2-\lambda)x_1-x_2+2x_3=0 \\ 5x_1-(3+\lambda)x_2+3x_3=0 \\ -x_1-(2+\lambda)x_3=0. \end{cases}$$

Bundan xarakteristik ko‘phadni topamiz:

$$p(\lambda)\equiv\begin{vmatrix} 2-\lambda & -1 & 2 \\ 5 & -3-\lambda & 3 \\ -1 & 0 & -2-\lambda \end{vmatrix}=-(\lambda+1)^3.$$

Demak, xos son $\lambda=-1$ ekan. Bu sonni sistemaga qo‘ysak,

$$\begin{cases} 3x_1-x_2+2x_3=0, \\ 5x_1-2x_2+3x_3=0, \\ -x_1-x_3=0. \end{cases}$$

Bundan $x_1 = x_2$, $x_1 = -x_3$. Demak, $X = \alpha\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$.

7- misol. Ushbu

$$A = \begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}$$

matritsaning xos soni va xos vektorlarini toping.

Yechish. Xarakteristik tenglamani tuzib yechamiz:

$$\begin{Vmatrix} 7-\lambda & -2 & 0 \\ -2 & 6-\lambda & -2 \\ 0 & -2 & 5-\lambda \end{Vmatrix} = 0;$$

$$-\lambda^3 + 18\lambda^2 - 99\lambda + 162 = 0,$$

$$\lambda_1 = 3,\ \lambda_2 = 6,\ \lambda_3 = 9.$$

$\lambda_1 = 3$ xos son uchun xos vektor

$$\left.\begin{aligned} 4x_1 - 2x_2 &= 0 \\ -2x_1 + 3x_2 - 2x_3 &= 0 \\ -2x_2 + 2x_3 &= 0 \end{aligned}\right\}$$

tenglamalar sistemasidan aniqlanadi. $x_1 = m$, deb qabul qilib, $x_2 = 2m, x_3 = 2m$ ni hosil qilamiz. Xos vektor: $\vec{\tau}_1 = m\vec{i} + 2m\vec{j} + 2m\vec{k}$. Shunga o‘xshash $\vec{\tau}_2 = m\vec{i} + \frac{1}{2}m\vec{j} - m\vec{k}$; $\vec{\tau}_3 = -m\vec{i} + m\vec{j} - \frac{1}{2}m\vec{k}$ xos vektorlarni topamiz.

8- misol. Agar R^3 da chiziqli A operator $\vec{e_1}, \vec{e_2}, \vec{e_3}$ bazisda o‘zining $A = \begin{pmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{pmatrix}$ matritsasi bilan berilgan bo‘lsa, $\vec{x} = 4\vec{e_1} - 3\vec{e_2} + \vec{e_3}$ vektorning $\vec{y} = A(\vec{x})$ aksini toping.

Yechish. $Y = AX$ formulaga binoan, $\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ -13 \\ -18 \end{pmatrix}$

Demak, $\vec{y} = 10\vec{e_1} - 13\vec{e_2} - 18\vec{e_3}$

9- misol. $\vec{e_1}$, $\vec{e_2}$ bazisda A operator $A = \begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix}$ matritsaga ega. $\begin{cases} \vec{e_1'} = \vec{e_1} - 2\vec{e_2} \\ \vec{e_2'} = 2\vec{e_1} + \vec{e_2} \end{cases}$ bazisda A operatorning matritsasini toping.

Yechish. O'tish matritsasi $C = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$ ning teskari matrisasi $C^{-1} = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$ Demak,

$$B = C^{-1}AC = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 8 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 20 \end{pmatrix}$$

10- misol. $\vec{e_1}$, $\vec{e_2}$ bazisda chiziqli operatorning matritsasi $A = \begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix}$ ko'rinishga ega. Yangi $\begin{cases} \vec{e_1'} = 5\vec{e_1} - 3\vec{e_2} \\ \vec{e_2'} = \vec{e_1} + 2\vec{e_2} \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

11- misol. $\vec{e_1}, \vec{e_2}, \vec{e_3}$ bazisda chiziqli operatorning matritsasi $A = \begin{pmatrix} 0 & 1 & 2 \\ -4 & 3 & 0 \\ 2 & 1 & 2 \end{pmatrix}$ ko'rinishda. Yangi $\begin{cases} \vec{e_1'} = 2\vec{e_1} + 2\vec{e_2} + 2\vec{e_3} \\ \vec{e_2'} = \vec{e_1} - \vec{e_2} + \vec{e_3} \\ \vec{e_3'} = \vec{e_1} + \vec{e_2} - \vec{e_3} \end{cases}$ bazisda A operatorning matritsasini toping.

12- misol. $A\left(\vec{x}\right)=\left(2x_1+x_3;\,4x_2-2x_3;\,3x_1+x_2-x_3\right)$ operatorni chiziqlilikka tekshiring.

Yechish. Operatorni chiziqlilikka tekshirish uchun $A\left(\vec{x}+\vec{y}\right)=A\left(\vec{x}\right)+A\left(\vec{y}\right)$ hamda $A\left(\alpha\vec{x}\right)=\alpha A\left(\vec{x}\right)$ tengliklarni bajarilishini tekshirish kifoya.

$$A\left(\vec{x}+\vec{y}\right)=\begin{pmatrix} 2\left(x_1+y_1\right)+x_3+y_3 \\ 4\left(x_2+y_2\right)-2\left(x_3+y_3\right) \\ 3\left(x_1+y_1\right)+x_2+y_2-\left(x_3+y_3\right) \end{pmatrix}=\begin{pmatrix} 2x_1+x_3+2y_1+y_3 \\ 4x_2-2x_3+4y_2-2y_3 \\ 3x_1+x_2-x_3+3y_1+y_2-y_3 \end{pmatrix}=$$

$$=\begin{pmatrix} 2x_1+x_3 \\ 4x_2-2x_3 \\ 3x_1+x_2-x_3 \end{pmatrix}+\begin{pmatrix} 2y_1+y_3 \\ 4y_2-2y_3 \\ 3y_1+y_2-y_3 \end{pmatrix}=A\left(\vec{x}\right)+A\left(\vec{y}\right)$$

$$A\left(\alpha\vec{x}\right)=\begin{pmatrix} 2\alpha x_1+\alpha x_3 \\ 4\alpha x_2-2\alpha x_3 \\ 3\alpha x_1+\alpha x_2-\alpha x_3 \end{pmatrix}=\alpha\begin{pmatrix} 2x_1+x_3 \\ 4x_2-2x_3 \\ 3x_1+x_2-x_3 \end{pmatrix}=\alpha A\left(\vec{x}\right)$$

13- misol Chizqli A operator $A=\begin{pmatrix} 1 & 4 \\ 9 & 1 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operatorning hos qiymatlari va hos vektorlarini toping.

Yechish. Xarakteristik tenglama tuzamiz: $\left|A-\lambda E\right|=\begin{vmatrix} 1-\lambda & 4 \\ 9 & 1-\lambda \end{vmatrix}=0$

$\lambda^2-2\lambda-35=0,\ \lambda_1=-5,\ \lambda_2=7$

$\lambda_1=-5$ xos qiymatga mos $\vec{X}^{(1)}=\left(x_1;\,x_2\right)$ xos vektorni topamiz. Buning uchun quyidagi tenglamani yechamiz:

$$\lambda_1=-5,\ \left(A-\lambda E\right)\cdot\vec{x}=\theta,\ \begin{pmatrix} 6 & 4 \\ 9 & 6 \end{pmatrix}\cdot\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix},\ x_2=-1{,}5x_1$$

Agar $x_1=C$ deb olsak, $x_2=-1{,}5C$ bo‘ladi. Demak $\vec{x}^{(1)}=\left(c;\,-1.5c\right)$ vector A operatorning $\lambda_1=-5$ xos qiymatiga mos xos vector bo‘ladi. Xuddi shunga o‘xshab

$\lambda_2 = 7$ xos qiymatga mos xos vektorlarni $\vec{x}^{(2)} = \left(\frac{2}{3}C_1;\ C_1\right)$, $C_1 \neq 0$ aniqlash mumkin.

Tayanch so‘z va iboralar. *Operator, chiziqli operator, operatorning matritsasi, operatorning rangi, nol operator, birlik operator, matritsaning xos vektori, matritsaning xos qiymati, xarakteristik tenglama, chiziqli operator matritsasining diagonal shakli.*

O‘z-o‘zini tekshirish uchun savollar

1. L^n fazoda bir bazisdan ikkinchi bazisga o‘tish matritsasi qanday tuziladi?
2. Chiziqli fazoning chiziqli almashtirishi yoki operatori deb nimaga aytiladi?
3. Chiziqli operator ustida bajariladigan qanday amallarni bilasiz?
4. Chiziqli operatorning xos vektori va xos qiymati deb nimaga aytiladi?
5. Xos vektorlarning qanday xossalarini bilasiz?
6. Matritsaning xarakteristik ko`phadi bazis tanlanishiga bog‘liqmi?
7. Qanday bazisda matritsa diagonal ko`rinishga ega bo`ladi?

Mustaqil yechish uchun misollar

14 $T: R^2 \to R^2,\ T(x_1, x_2) = (x_1 + x_2, -3x_1 + 2x_2)$ operator berilgan. Bu operatorning chiziqli isbotlang.

15 $\tilde{A}(x_1, x_2, x_3) = \left(4x_2,\ x_1 - 2x_2 + x_3,\ 5x_1 - x_2 + 4x_3\right)$ operator berilgan. Bu operatorlarning chiziqli ekanligini isbotlang.

16 $\tilde{A}(x_1, x_2) = (x_1 + x_2, x_1 - x_2),\ \tilde{B}(x_1, x_2) = (x_1 - x_2, x_1 + x_2)$ operatorlar berilgan. Bu operatorlarning chiziqli ekanligini isbotlang.

17 R^4 fazoning $\{\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4\}$ bazisida A chiziqli operator matritsasi

$$A = \begin{pmatrix} 1 & 2 & 0 & 3 \\ 4 & 5 & 2 & 5 \\ 3 & 4 & 7 & 0 \\ 6 & 3 & 2 & 1 \end{pmatrix}$$

ko‘rinishda berilgan bo‘lsin. $\vec{x} = 3\vec{e}_1 - 2\vec{e}_2 + \vec{e}_3 - 2\vec{e}_4$ vektorning aksini toping.

18 R^4 fazoning $\{\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4\}$ bazisida $\mathring{A}$ chiziqli operator matritsasi

$$A = \begin{pmatrix} 2 & 2 & 3 & 3 \\ 0 & 5 & 2 & 5 \\ 1 & 4 & 7 & 0 \\ 1 & 3 & 2 & 1 \end{pmatrix}$$

ko‘rinishda berilgan bo‘lsin. $\vec{x} = 3\vec{e}_1 + 4\vec{e}_2 + 3\vec{e}_3 - \vec{e}_4$ vektorning aksini toping.

19 R^4 fazoda $\vec{e_1}, \vec{e_2}, \vec{e_3}, \vec{e_4}$ bazisda chiziqli operator matritsasi

$A = \begin{pmatrix} -5 & 2 & 0 & -3 \\ 2 & 5 & 2 & 5 \\ 3 & 4 & 6 & 0 \\ -6 & 0 & 2 & 1 \end{pmatrix}$ ko‘rinishda berilgan bo‘lsin. $\vec{e_1}, \vec{e_2}, \vec{e_3}, \vec{e_4}$

$\vec{x} = 2\vec{e_1} - 2\vec{e_2} + 4\vec{e_3} + \vec{e_4}$ vektorning $A(\vec{x})$ aksini toping.

20 R^3 fazoda $\vec{e_1}, \vec{e_2}, \vec{e_3}$ bazisda chiziqli operator matritsasi

$A = \begin{pmatrix} -5 & 3 & 4 \\ -2 & 6 & 8 \\ 1 & -7 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x} = 4\vec{e_1} - 3\vec{e_2} - \vec{e_3}$ vektorning aksi

$\vec{y} = A(\vec{x})$ ni toping.

21. R^3 fazoda $\vec{e_1}, \vec{e_2}, \vec{e_3}$ bazisda chiziqli operator matritsasi

$A = \begin{pmatrix} 5 & 3 & -6 \\ -1 & 3 & 6 \\ 1 & 4 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x} = -2\vec{e_1} - 3\vec{e_2} + 5\vec{e_3}$ vektorning

aksi $\vec{y} = A(\vec{x})$ ni toping.

22. $\tilde{A}(x_1, x_2, x_3) = (x_2 - x_3,\ 4x_1 + 3x_2 - x_3,\ -3x_1 - x_2 + 6x_3)$ va $\tilde{B}(x_1, x_2, x_3) =$ $= (x_1 - 3x_2,\ 4x_2 + x_3,\ -5x_2 + 3x_3)$ operatorlar berilgan. $C = A \cdot B$ operator va uning matritsasi topilsin.

23 $\{\vec{e}_1, \vec{e}_2\}$ bazisda chiziqli operator $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ ko‘rinishga ega.

Yangi $\begin{cases} \vec{e}_1^{\,*} = \vec{e}_1 - \vec{e}_2 \\ \vec{e}_2^{\,*} = \vec{e}_1 + \vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

24 $\{\vec{e}_1, \vec{e}_2\}$ bazisda chiziqli operatorning matritsasi $A = \begin{pmatrix} 3 & 3 \\ 2 & 4 \end{pmatrix}$ ko‘rinishga ega.Yangi $\begin{cases} \vec{e}_1^{\,*} = 2\vec{e}_1 - \vec{e}_2 \\ \vec{e}_2^{\,*} = \vec{e}_1 + \vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

25 $\{\vec{e}_1, \vec{e}_2\}$ bazisda chiziqli operator $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ ko‘rinishga ega.

Yangi $\begin{cases} \vec{e}_1^{\,*} = \vec{e}_1 - \vec{e}_2 \\ \vec{e}_2^{\,*} = \vec{e}_1 + \vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

26. $\{\vec{e}_1, \vec{e}_2\}$ bazisda chiziqli operatorning matritsasi $A = \begin{pmatrix} 3 & 3 \\ 2 & 4 \end{pmatrix}$ ko‘rinishga ega.Yangi $\begin{cases} \vec{e}_1^{\,*} = 2\vec{e}_1 - \vec{e}_2 \\ \vec{e}_2^{\,*} = \vec{e}_1 + \vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

27. $A = \begin{pmatrix} 6 & 24 \\ 54 & 6 \end{pmatrix}$ matritsa bilan berilgan chiziqli operatorning xos soni va xos vektorlarini toping.

29. $\tilde{A}(\vec{x}) = \left(4x_1 - 2x_2 + 4x_3, 10x_1 - 6x_2 + 6x_3, -2x_1 - 4x_3\right)$ operatorning xos soni va unga mos keluvchi xos vektorlarini toping.

30. $\overrightarrow{e_1}, \overrightarrow{e_2}$ bazisda chiziqli operatorning matritsasi $A = \begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix}$ ko‘rinishga ega. Yangi $\begin{cases} \overrightarrow{e_1'} = 5\overrightarrow{e_1} - 3\overrightarrow{e_2} \\ \overrightarrow{e_2'} = \overrightarrow{e_1} + 2\overrightarrow{e_2} \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

31. $\vec{e_1},\vec{e_2}$ bazisda chiziqli operatorning matritsasi $A=\begin{pmatrix}5 & 3\\ 2 & 6\end{pmatrix}$ ko‘rinishga ega. Yangi $\begin{cases}\vec{e_1'}=2\vec{e_1}-3\vec{e_2}\\ \vec{e_2'}=3\vec{e_1}+\vec{e_2}\end{cases}$ bazisda chiziqli operatorning matritsasini toping.

32. Berilgan $A\left(\vec{x}\right)=\left(8x_2;\,5x_1-3x_2+x_3;2x_1-x_2+2x_3\right)$ operatorlarning chiziqli operator ekanligini isbotlang.

33. Quyidagi operatorlarning chiziqli operator ekanligini isbotlang.

$$A\left(x_1;\,x_2\right)=\left(x_1+2x_2;\,3x_1-x_2\right),\quad B\left(x_1;\,x_2\right)=\left(4x_1-x_2;\,7x_1+x_2\right)$$

34. $A=\begin{pmatrix}6 & 24\\ 54 & 6\end{pmatrix}$ matritsa bilan berilgan chiziqli operatorning xos soni va xos vektorlarini toping.

35. $\tilde{A}(x)=\left(4x_1-2x_2+4x_3,\,10x_1-6x_2+6x_3,\,-2x_1-4x_3\right)$ operatorning xos soni va unga mos keluvchi xos vektorlarini toping.

Quyidagi matritsalari bilan berilgan operatorlarning xos qiymatlari va xos vektorlarini toping:

36. $A=\begin{pmatrix}5 & 4\\ 8 & 9\end{pmatrix}$	**37.** $A=\begin{pmatrix}2 & 4\\ -1 & -3\end{pmatrix}$	**38** $A=\begin{pmatrix}0 & 1 & 0\\ -4 & 4 & 0\\ -2 & 1 & 2\end{pmatrix}$
39 $A=\begin{pmatrix}4 & -5 & 2\\ 5 & -7 & 3\\ 6 & -9 & 4\end{pmatrix}$	**40** $A=\begin{pmatrix}1 & 2 & -2\\ 1 & 0 & 3\\ 1 & 3 & 0\end{pmatrix}$	**41** $A=\begin{pmatrix}1 & -3 & 3\\ -3 & 7 & 3\\ -1 & -4 & 8\end{pmatrix}$

42. Berilgan $A\left(\vec{x}\right)=\left(3x_2-x_3;\,2x_1+x_2-x_3;-2x_1-x_2+4x_3\right)$ va $B\left(\vec{x}\right)=\left(x_1-2x_2;\,x_2+x_3;-2x_2+3x_3\right)$ operatorlarga ko‘ra $C=A\cdot B$ operator hamda uning C matritsasi topilsin.

43. $A\left(x_1;\, x_2\right)=\left(x_1+4x_2;\, 2x_1-5x_2\right),\, B\left(x_1;\, x_2\right)=\left(x_1-2x_2;\, 3x_1-x_2\right)$ operatorlarga ko‘ra $C=A\cdot B$ operator hamda uning C matritsasini toping.

44. Quyidagi operatorning xos qiymat va unga mos keluvchi xos vektorlarini toping;

$$A\left(\vec{x}\right)=\left(4x_1-2x_2+4x_3;\, 10x_1-6x_2+6x_3;\, -2x_1-4x_3\right)$$

45. $A=\begin{pmatrix}1 & 4\\ 9 & 1\end{pmatrix}$ matritsa bilan berilgan chiziqli operatorning xos soni va xos vektorini toping.

4. Xos vektorlari bazis tashkil qiladigan chiziqli operatorlar

R^n fazodagi eng sodda chiziqli operatorlar shunday operatorlarki, ular n ta chiziqli erklli vektorga ega.

Haqiqatan, $T:R^n\rightarrow R^n$ operator chiziqli erkli $\left\{\vec{e_1},\, \vec{e_2},...,\vec{e_n}\right\}$ vektorlarga ega bo‘lgan operator bo‘lsin. Shu vektorlarni bazis uchun qabul qilamiz. U holda

$$\left.\begin{array}{l} T(\vec{e_1})=\lambda_1\vec{e_1},\\ T(\vec{e_2})=\lambda_2\vec{e_2},\\ \dots\dots\dots\dots\dots\dots\\ T(\vec{e_n})=\lambda_n\vec{e_n}, \end{array}\right\}$$

bunda $\lambda_1,\lambda_2,\ldots,\lambda_n$ sonlar T operatorning $\left\{\vec{e_1},\, \vec{e_2},...,\vec{e_n}\right\}$ xos vektorlariga mos kelgan xos qiymatlari.

Bundan $\left\{\vec{e_1},\, \vec{e_2},...,\vec{e_n}\right\}$ xos vektorlar tashkil qilgan bazisda T operatorning matritsasi ushbu eng sodda, diagonal ko‘rinishga ega bo‘ladi:

$$A=\begin{bmatrix}\lambda_1 & 0 & \dots & 0\\ 0 & \lambda_2 & \dots & 0\\ . & . & ... & .\\ 0 & 0 & \dots & \lambda_n\end{bmatrix} \qquad (4.1)$$

Aksincha, agar biror $\{\overrightarrow{e_1}, \overrightarrow{e_2},...,\overrightarrow{e_n}\}$ bazisda T operatorga bunday dioganal matritsa mos kelsa, u holda $\overrightarrow{e_1}, \overrightarrow{e_2},...,\overrightarrow{e_n}$ vektorlar T ning xos vektorlari, $\lambda_1,\lambda_2,\ldots,\lambda_n$ esa operatorning $\overrightarrow{e_1},\overrightarrow{e_2},\ldots,\overrightarrow{e_n}$ vektorlariga mos keladigan xos qiymatlaridir.

Haqiqatan, A matritsaning xossasidan uning ustunlari $T(\overrightarrow{e_1}),T(\overrightarrow{e_1}),\ldots,T(\overrightarrow{e_n})$ vektorning $\overrightarrow{e_1},\overrightarrow{e_2},\ldots,\overrightarrow{e_n}$ bazisdagi komponentlaridan iboratligi kelib chiqadi. Shu sababli

$$T(\overrightarrow{e_1})=\lambda_1\overrightarrow{e_1},T(\overrightarrow{e_2})=\lambda_2\overrightarrow{e_2},\ldots,T(\overrightarrow{e_n})=\lambda_n\overrightarrow{e_n}.$$

Shuning o‘zi aytilgan tasdiqni isbotlaydi.

Teorema. Agar R^n da T chiziqli operatorning xos qiymatlari $\lambda_1,\lambda_2,\ldots,\lambda_3$ $(s\le n)$ haqiqiy sonlar to‘plamiga tegishli juft-jufti bilan har xil sonlar bo‘lsa, bu xos qiymatlarga mos keluvchi $\overrightarrow{e_1},\overrightarrow{e_2},\ldots,\overrightarrow{e_s}$ xos vektorlar chiziqli erkli bo‘ladi. Xususan, agar $(s=n)$ bo‘lsa, $\overrightarrow{e_1},\overrightarrow{e_2},\ldots,\overrightarrow{e_n}$ xos vektorlar R^n da bazis tashkil qiladi.

Isbot. Isbotni induksiya metodi bilan olib boriladi. $s=1$ da tasdiqning to‘g‘riligi ravshan. Tasdiq $s-1$ ta vektor uchun o‘rinli deb faraz qilamiz va uni s ta vektor uchun isbotlaymiz. Agar s ta vektor uchun tasdiq to‘g‘rimas deb faraz qilinsa, u holda R da hammasi bir vaqtda nolga teng bo‘lmagan va

$$\gamma_1\overrightarrow{e_1}+\gamma_2\overrightarrow{e_2}+\ldots\gamma_s\overrightarrow{e_s}=0 \qquad (*)$$

munosabatni qanoatlantiruvchi

$$\gamma_1,\gamma_2,\ldots,\gamma_s$$

sonlar topiladi. Aniqlik uchun $\gamma_1\ne 0$ deb faraz qilaylik. oxirgi tenglikka T operatorni qo‘llanib, quyidagini topamiz:

$$T(\gamma_1\overrightarrow{e_1}+\gamma_2\overrightarrow{e_2}+\ldots\gamma_3\overrightarrow{e_3})=T(0)=0,$$

ammo

$$T(\gamma_1\overrightarrow{e_1}+\gamma_2\overrightarrow{e_2}+\ldots\gamma_s\overrightarrow{e_s})=\gamma_1\lambda_1\overrightarrow{e_1}+\gamma_2\lambda_2\overrightarrow{e_2}+\ldots\gamma_s\lambda_s\overrightarrow{e_s}$$

va shuning uchun

$$\gamma_1\lambda_1\overrightarrow{e_1}+\gamma_2\lambda_2\overrightarrow{e_2}+\ldots\gamma_s\lambda_s\overrightarrow{e_s}=0$$

Agar oxirgi tenglikdan (*) tenglikni λ_s ga ko‘paytirib ayirilsa, ushbuga ega bo‘lamiz:

$$\gamma_1(\lambda_1-\lambda_2)\vec{e_1}+\gamma_2(\lambda_2-\lambda_s)\vec{e_2}+\ldots\gamma_{s-1}(\lambda_{s-1}-\lambda_s)\vec{e}_{s-1}=0,$$

farazga ko‘ra $\gamma_1\neq 0$ va $\lambda_1-\lambda_s\neq 0$ bo‘lgani uchun $\gamma_1(\lambda_1-\lambda_s)\neq 0$, lekin $(s-1)$ ta $\vec{e_1},\vec{e_2},\ldots,\vec{e}_{s-1}$ vektorlar chiziqli erkli edi. Biz bunda zid natijaga keldik. Demak, induksiya s uchun ham to‘g‘ri ekanini isbot etdik. **Teorema to‘la isbot bo‘ldi.**

1- misol. Shunday $T:R^3\to R^3$ chiziqli operator berilganki, berilgan tayin $\{\vec{e_1},\vec{e_2},\vec{e_3}\}$ bazis uchun T ning matritsasi ushbu ko‘rinishga ega:

$$A=\begin{bmatrix}-1 & 3 & -1\\ -3 & 5 & -1\\ -3 & 3 & 1\end{bmatrix}$$

T operatorning xos sonlari, xos vektorlarini va (agar mumkin bo‘lsa) T operatorning matritsasi diagonal ko‘rinishni oladigan bazisni toping.

Yechish. T operatorning xarakteristik ko‘phadi ushbu ko‘rinishga ega:

$$\det[A-\lambda E]=\begin{vmatrix}-1-\lambda & 3 & -1\\ -3 & 5-\lambda & -1\\ -3 & 3 & 1-\lambda\end{vmatrix}=(\lambda-1)(\lambda-2)^2.$$

Bundan T operatorning xarakteristik sonlari $\lambda=1, \lambda_2=\lambda_3=2$ bo‘ladi. $\lambda_1=1$ songa to‘g‘ri keladigan $q_1=x_1\vec{e_1}+x_2\vec{e_2}+x_3\vec{e_3}$ xos vektor ushbu sistemaning yechimi sifatida topiladi:

$$\left.\begin{aligned}-x_1+3x_2-x_3&=x_1,\\ -3x_1+5x_2-x_3&=x_2,\\ -3x_1-3x_2-x_3&=x_3.\end{aligned}\right\}$$

$q_1=\vec{e_1}+\vec{e_2}+\vec{e_3}$ vektor $\lambda_1=1,$ songa to‘g‘ri keladigan xos vektor ekanini tekshirish oson. $\lambda_2=\lambda_3=2$ xarakteristik sonlarga to‘g‘ri keladigan $\vec{q_2}$ va $\vec{q_3}$ xos vektorlarni topish sistemasi ushbu ko‘rinishga ega:

$$\left.\begin{aligned} -x_1+3x_2-x_3&=2x_1,\\ -3x_1-5x_2-x_3&=2x_2,\\ -3x_1+3x_2+x_3&=2x_3.\end{aligned}\right\}$$

Bevosita tekshirish yo‘li bilan $\overrightarrow{q_2}=\overrightarrow{e_1},\ \overrightarrow{q_3}=\overrightarrow{e_1}-3\overrightarrow{e_2}$ vektorlar T operatorning $\lambda_2=\lambda_3=2$ sonlarga mos xos vektorlari ekaniga ishonch hosil qilamiz. $\overrightarrow{q_3}$ vektorlar chiziqli erkli ekanini ko‘rish oson:

$$\begin{vmatrix} 1 & 1 & 1\\ 1 & 0 & 0\\ 1 & 0 & -3\end{vmatrix}=-\begin{vmatrix} 1 & 1\\ 0 & -3\end{vmatrix}=3\neq 0$$

Shu sababli $\overrightarrow{q_1},\overrightarrow{q_2},\overrightarrow{q_3}$ vektorlar bazis tashkil qiladi. $\overrightarrow{q_1},\overrightarrow{q_2},\overrightarrow{q_3}$ lar φ operatorning xos vektorlari bo‘lgani uchun:

$$\left.\begin{aligned} T(\overrightarrow{q_1})&=\overrightarrow{q_1}+0\cdot\overrightarrow{q_2}+0\cdot\overrightarrow{q_3},\\ T(\overrightarrow{q_2})&=0\cdot\overrightarrow{q_1}+2\cdot\overrightarrow{q_2}+0\cdot\overrightarrow{q_3},\\ T(\overrightarrow{q_3})&=0\cdot\overrightarrow{q_1}+0\cdot\overrightarrow{q_2}+2\overrightarrow{q_3}.\end{aligned}\right\}$$

Shu sababli T operatorning $\overrightarrow{q_1},\overrightarrow{q_2},\overrightarrow{q_3}$ bazisdagi matritsasi bunday:

$$\mathrm{B}=\begin{bmatrix} 1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2\end{bmatrix}$$

2- misol. $T: R^2\to R^2$ operator $\overrightarrow{e_1},\overrightarrow{e_2}$ bazis vektorlarini $T(\overrightarrow{e_1})=\overrightarrow{e_1}+\overrightarrow{ie_2}$, $\varphi(\overrightarrow{e_2})=\overrightarrow{e_2}+i\overrightarrow{e_1}$ vektorga o‘tkazuvchi operator bo‘lsin. T operatorning matritsasi diagonal ko‘rinishida bo‘ladigan bazisini topish talab qilinadi.

Yechish. $\overrightarrow{e_1},\overrightarrow{e_2}$ bazisda T ning matritsasi ushbu ko‘rinishda bo‘ladi:

$$A=\begin{bmatrix} 1 & i\\ i & 1\end{bmatrix}.$$

Shunig uchun A operatorning xarakteristik polinomi bunday:

$$\det|(A-\lambda E)|=\begin{vmatrix} 1-\lambda & i\\ i & 1-\lambda\end{vmatrix}=(1-\lambda)^2-i^2=\lambda^2-2\lambda+2$$

Ushbu $\lambda_1 = 1 - i$ va $\lambda_2 = 1 + i$ sonlar T operatorning xarakteristik sonlari bo‘ladi. λ_1 va λ_2 xos sonlarga to‘g‘ri keladigan mos $\overrightarrow{q_1} = x_1\overrightarrow{e_1} + x_2\overrightarrow{e_2}$ va $\overrightarrow{q_2} = y_1\overrightarrow{e_1} + y_2\overrightarrow{e_2}$ xos vektorlar quyidagi tenglamalar sistemasidan topiladi:

$$\begin{cases} x_1 + ix_2 = (1-i)x_1 \\ x_1 + ix_2 = (1+i)x_1 \\ ix_1 + x_2 = (1-i)x_2 \\ ix_1 + x_2 = (1+i)x_2 \end{cases}$$

Bundan $\overrightarrow{q_1}$ va $\overrightarrow{q_2}$ vektorlar sifatida $\overrightarrow{q_1} = \overrightarrow{e_1} - \overrightarrow{e_2}$ va $\overrightarrow{q_2} = \overrightarrow{e_1} + \overrightarrow{e_2}$ chiziqli erkli vektorlarni olish mumkinligi kelib chiqadi, $\overrightarrow{q_1}, \overrightarrow{q_2}$ bazisda φ vektorning matritsasi ushbu ko‘rinishga ega:

$$B = \begin{vmatrix} 1-i & 0 \\ 0 & 1+\lambda \end{vmatrix}$$

Chunki

$$\varphi(\overrightarrow{q_1}) = (1-i)\overrightarrow{q_1},$$
$$\varphi(\overrightarrow{q_2}) = (1-i)\overrightarrow{q_2}.$$

Mustaqil yechish uchun misollar

3. $T : R^4 \to R^5$ akslantirish $\overrightarrow{e_1}, \overrightarrow{e_2}, \overrightarrow{e_3}, \overrightarrow{e_4}$ vektorlarni

$$T(\overrightarrow{e_1}) = \{1,1,0,0,0\},$$
$$T(\overrightarrow{e_2}) = \{0,1,1,0,0\},$$
$$T(\overrightarrow{e_3}) = \{0,0,1,1,0\},$$
$$T(\overrightarrow{e_4}) = \{0,0,0,1,1\}$$

vektorlarga o‘tkazuvchi chiziqli akslantirish bo‘lsin. Shu akslantirishning matritsasini va uning koordinatalar bo‘yicha tasvirini (ifodasini) yozing.

4. $T : R^4 \to R^4$ almashtirish $\overrightarrow{e_1} + \overrightarrow{e_2}$, $\overrightarrow{e_1} - \overrightarrow{e_2}$, $\overrightarrow{e_3}, \overrightarrow{e_4}$ vektorlarni

$$T(\overrightarrow{e_1} + \overrightarrow{e_2}) = \{0,0,1,-1\}, \quad T(\overrightarrow{e_1} - \overrightarrow{e_2}) = \{0,0,1,2\},$$
$$T(\overrightarrow{e_3}) = \{1,2,0,0\}, \qquad T(\overrightarrow{e_4}) = \{0,-3,2,0\}$$

vektorlarga o‘tkazuvchi chiziqli akslantirish bo‘lsin. Shu akslantirishning matritsasi va uning koordinatalar orqali tasvirini yozing.

5. $T: R^5 \to R^3$ akslantirish quyidagi

$$\alpha_1 = x_1 + 2x_2 + x_3 + x_4 + 5x_5,$$
$$\alpha_2 = 2x_1 + x_2 + 3x_3 - x_4 + 2x_5,$$
$$\alpha_3 = -x_1 + x_2 - x_3 + 4x_4 - 6x_5,$$

koordinatalar orqali tasvirlangan chiziqli akslantirish bo‘lsin. $T(\overrightarrow{e_1} - \overrightarrow{e_2} + \overrightarrow{e_3})$, $T(\overrightarrow{e_1} + \overrightarrow{e_4} - 2\overrightarrow{e_5})$ vektorlarni toping.

6. $T: R^3 \to R^4$ chiziqli akslantirishning bazisdagi matritsasi ushbu ko‘rinishga ega:

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 3 & 0 & 1 & -2 \\ 2 & 4 & 3 & 1 \\ 1 & 2 & -1 & 3 \end{bmatrix}$$

$T: R^4 \to R^4$ ning quyidagi bazislardagi matritsalarini toping:

a) $\overrightarrow{e_1}, \overrightarrow{2e_2}, \overrightarrow{3e_3}, \overrightarrow{e_3} + \overrightarrow{e_4}$;

b) $\overrightarrow{e_1},\ \overrightarrow{e_1} + \overrightarrow{e_2},\ \overrightarrow{e_1} + \overrightarrow{e_2} + \overrightarrow{e_3} + \overrightarrow{e_4}$.

7. Ushbu $T: R^2 \to R^2$ operator $\overrightarrow{e_1}, \overrightarrow{e_2}$ bazis vektorlarni

$$\varphi(\overrightarrow{e_1}) = \overrightarrow{e_1} + i\overrightarrow{e_2},$$
$$\varphi(\overrightarrow{e_2}) = \overrightarrow{e_2} + i\overrightarrow{e_1},$$

vektorlarga o‘tkazuvchi operator bo‘lsin. $T: R^2 \to R^2$ operatorning matritsasi diagonal ko‘rinishda bo‘ladigan bazisni toping.

Javob:

$$\overrightarrow{q_1} = \overrightarrow{e_1} + \overrightarrow{e_2},$$
$$\overrightarrow{q_2} = \overrightarrow{e_1} + \overrightarrow{e_2}.$$

$\overrightarrow{q_1}, \overrightarrow{q_2}$ bazisda φ operatorning matritsasi ushbu ko‘rinishga ega:

$$\mathrm{B} = \begin{vmatrix} 1-i & 0 \\ 0 & 1+i \end{vmatrix}.$$

8. $T: R^2 \to R^2$ operator $\overrightarrow{q_1} = \{1,2\}$, $\overrightarrow{q_2} = \{2,3\}$ bazisdagi chiziqli operator bo‘lib, uning matritsasi

$$A_T = \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}$$

dan iborat bo‘lsin, $\overrightarrow{e_1} = \{3,1\}, \overrightarrow{e_2} = \{4,2\}$ bazisdagi $T_1: R^2 \to R^2$ chiziqli operator esa

$$B_{T_1} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

matritsa bilan beriladi. $A_{T+T_1}, A_{T-T_1}, A_{T\cdot T_1}$ matritsalarni aniqlang.

9. Tayinlangan $\{\overrightarrow{e_1}, \overrightarrow{e_2}, \overrightarrow{e_3}\}$ bazisda quyidagi matritsalar yordamida berilgan chiziqli operatorlarning xos vektorlarni toping:

a) $A = \begin{bmatrix} 2 & -1 & 2 \\ 5 & -3 & 2 \\ -1 & 0 & 2 \end{bmatrix}$, c) $C = \begin{bmatrix} 4 & -3 & 2 \\ 5 & -7 & 3 \\ 6 & 9 & 4 \end{bmatrix}$,

b) $B = \begin{bmatrix} 0 & 1 & 0 \\ -4 & 4 & 0 \\ -2 & 1 & 2 \end{bmatrix}$, d) $D = \begin{bmatrix} 1 & -3 & 3 \\ -2 & -6 & 13 \\ -1 & -4 & 8 \end{bmatrix}$.

10. Agar tayinlangan $\{\overrightarrow{e_1}, \overrightarrow{e_2}, \overrightarrow{e_3}\}$ bazisda (yoki $\{\overrightarrow{e_1}, \overrightarrow{e_2}, \overrightarrow{e_3}, \overrightarrow{e_4}\}$ bazisda) chiziqli operatorlar

a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, b) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$,

matritsalar bilan berilgan bo‘lsa, shu chiziqli operatorlar R^3 va R^4 da diagonal ko‘rinishda bo‘ladigan bazislarni toping.

11. n-tartibli

$$A=\begin{bmatrix} 0 & 0 & 0 & . & . & . & 0 & 0 & 1 \\ 0 & 0 & 0 & . & . & . & 0 & 1 & 0 \\ 0 & 0 & 0 & . & . & . & 1 & 0 & 0 \\ . & . & . & . & . & . & . & . & . \\ 0 & 1 & 0 & . & . & . & 0 & 0 & 0 \\ 1 & 0 & 0 & . & . & . & 0 & 0 & 0 \end{bmatrix}$$

matritsa uchun shunday maxsusmas n-tartibli B matritsa topish kerakki,

$$C = B^{-1}AB$$

matritsa diagonal matritsa bo‘lsin.

12. Tayinlangan bazisda

$$A=\begin{bmatrix} 4 & -2 & 2 \\ 2 & 0 & 2 \\ -1 & 1 & 1 \end{bmatrix}$$

matritsa bilan berilgan $T : R^3 \to R^3$ chiziqli operatorning barcha invariant qism fazolarini toping.

13. R^3 dagi $\{\vec{e_1}, \vec{e_2}, \vec{e_3}\}$ bazisda

$$A=\begin{bmatrix} 5 & -1 & -1 \\ -1 & 5 & -1 \\ -1 & -1 & 5 \end{bmatrix} \qquad \text{va} \qquad B=\begin{bmatrix} -6 & 2 & 3 \\ 2 & -3 & 6 \\ 3 & 6 & 2 \end{bmatrix}$$

matritsalar bilan berilgan ikki chiziqli operatorning umumiy invariant qism fazolarini toping.

5. Mustaqil bajarish uchun nazorat variantlar

1-variant

1. R^2 fazoda $\vec{a}=2\vec{m}+4\vec{n}$ va $\vec{b}=\vec{m}-\vec{n}$ vektorlar berilgan. Bu yerda $\vec{m}$ va $\vec{n}$– birlik vektorlar bo‘lib, ular orasidagi burchak 120^0 ga teng bo‘lsa $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchakni toping.

2. T operatorning $\{\vec{e_1}, \vec{e_2}, \vec{e_3}\}$ bazisdan $\{\vec{e'_1}, \vec{e'_2}, \vec{e'_3}\}$ bazisga o`tish matritsasi $A=\begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 3 & 1 & -5 \end{pmatrix}$ ko‘rinishda berilgan bo‘lsin. $\vec{e'}_3$ vektorning $\{\vec{e_1}, \vec{e_2}, \vec{e_3}\}$ bazisdagi koordinatalarini toping.

3. $\tilde{A}:R^3 \to R^3$ chiziqli operator $\{\vec{e_1}, \vec{e_2}, \vec{e_3}\}$ bazisida $\begin{pmatrix} -1 & 0 & 2 \\ 2 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$ matritsa bilan berilgan bo‘lib, $\vec{x}=2\vec{e}_1+4\vec{e}_2-\vec{e}_3$ bo`lsa, $y=\tilde{A}(x)$ vektorning $\{\vec{e_1}, \vec{e_2}, \vec{e_3}\}$ bazisidagi koordinatalarini toping.

2-variant

1. R^4 fazodan olingan $\vec{a}=(3;2;-4;1)$ va $\vec{b}=(1;-7;2;0)$ vektorlar berilgan bo‘lsin. $2\left(3\vec{a}+2\vec{b}\right)-3\vec{a}+\vec{b}+7\left(\vec{a}-\vec{b}\right)$ vektorni toping.

2. $\{\vec{e}_1,\vec{e}_2,\vec{e}_3\}$ bazisda berilgan $\vec{x}=(4;0;-12)\in R^3$ vektorning $(\vec{e'}_1=\vec{e}_1+2\vec{e}_2+\vec{e}_3;$ $\vec{e'}_2=2\vec{e}_1+3\vec{e}_2+4\vec{e}_3; \vec{e'}_3=3\vec{e}_1+4\vec{e}_2+3\vec{e}_3)$ bazisdagi koordinatalarini toping.

3. $\{\vec{e_1}, \vec{e_2}, \vec{e_3}\}$ bazisda $\tilde{A}$ operator $A = \begin{pmatrix} 0 & -2 & 1 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{pmatrix}$ matritsa bilan berilgan bo‘lsin.

Agar $\vec{e'}_1 = 3\vec{e}_1 + \vec{e}_2 + 2\vec{e}_3,\ \vec{e'}_2 = 2\vec{e}_1 + \vec{e}_2 + 2\vec{e}_3,\ \overrightarrow{e'_3} = -\vec{e}_1 + 2\vec{e}_2 + 5\vec{e}_3$ bo‘lsa, $\tilde{A}$ operatorning $\{\vec{e'_1}, \vec{e'_2}, \vec{e'_3}\}$ bazisdagi matritsasini toping.

3-variant

1. $\vec{a} = -2\vec{j} + \vec{k}$, $\vec{b} = 2\vec{i} + \vec{j}$ vektorlarga qurilgan parallelogramm diagonallari orasidagi burchakni toping.

2. $\vec{b} = (1; m; 3)$ vektor $\vec{a}_1 = (2;3;7)$ $\vec{a}_2 = (3;-2;4)$ va $\vec{a}_3 = (-1;1;-1)$ vektorlar orqali chiziqli ifodalanadigan m ning barcha qiymatlarini toping.

3. $\{\vec{e_1}, \vec{e_2}\}$ bazisda $\tilde{A}$ chiziqli operator $A = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$ matritsa bilan berilgan, $\vec{x} = \vec{e}_1 + 3\vec{e}_2$ bo`lsa, $y = \tilde{A}(x)$ vektorning $\{\vec{e_1}, \vec{e_2}\}$ bazisdagi koordinatalarini toping.

4-variant

1. Vektorlar uzunliklari $|\vec{a}| = 11$; $|\vec{b}| = 23$; va $|\vec{a} - \vec{b}| = 30$ bo‘lsa, $|\vec{a} + \vec{b}|$ ni aniqlang.

2. $\{\vec{e_1}, \vec{e_2}, \vec{e_3}\}$ bazisda $\vec{a_1} = (1; 1; 1)$ $\vec{a_2} = (0; 2; 3)$ va $\vec{a_3} = (0; 1; 5)$ vektorlar berilgan. $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ bazisda $\vec{d} = 2\vec{a}_1 - \vec{a}_2 + 5\vec{a}_3$ vektorning koordinatalarini toping.

3. Biror bazisda $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$ matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

5-variant

1. α va β ning qanday qiymatlarida $\vec{a}=-2\vec{i}+3\vec{j}+\beta\vec{k}$ va $\vec{b}=\alpha\vec{i}-6\vec{j}+2\vec{k}$ vektorlar a) kolleniar b) ortogonal bo‘ladi.

2. $\{\vec{e_1}, \vec{e_2}, \vec{e_3}\}$ bazisdan $\{\vec{e_1}', \vec{e_2}', \vec{e_3}'\}$ bazisga o`tish matritsasi $A=\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 2 \\ -1 & 3 & 1 \end{pmatrix}$ berilgan. $\vec{e_1}, \vec{e_2}$ *va* $\vec{e_3}$ vektorlarning $\{\vec{e_1}', \vec{e_2}', \vec{e_3}'\}$ bazisdagi koordinatalarini toping.

3. Biror bazisda $A=\begin{pmatrix} 3 & 1 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & 4 \end{pmatrix}$ matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

6-variant

1. Oxy tekisligida $\vec{a}=2\vec{i}$, $\vec{b}=3\vec{i}+3\vec{j}$ va $\vec{c}=2\vec{i}+6\vec{j}$ vektorlarni yasang. $\vec{c}$ ni $\vec{a}$ va $\vec{b}$ vektorlar orqali analitik va geometrik ifodalang.

2. Ortonormallangan bazis tashkil qiluvchi $\{\vec{e_1}, \vec{e_2}, \vec{e_3}\}$ vektorlar sistemasi berilgan. $\vec{x}=5\vec{e_1}+\vec{e_3}$ va $\vec{y}=\vec{e_1}+\vec{e_2}+\vec{e_3}$ vektorlar orasidagi burchakni toping**.**

3. Chiziqli operatorning $\{\vec{e_1}, \vec{e_2}, \vec{e_3}\}$ bazisdagi matritsasi $A=\begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ ko‘rinishga ega. Agar $\vec{e}'_1=2\vec{e_1}+\vec{e_2}-\vec{e_3}$; $\vec{e}'_2=2\vec{e_1}-\vec{e_2}+2\vec{e_3}$; $\vec{e}'_3=3\vec{e_1}+\vec{e_3}$ bo‘lsa, chiziqli operatorning $\{\vec{e_1}', \vec{e_2}', \vec{e_3}'\}$ bazisdagi matritsasini toping.

7-variant

1. R^3 fazoda $\vec{a}=(2;1;0)$, $\vec{b}=(1;-1;2)$, $\vec{c}=(2;2;-1)$ va $\vec{d}=(3;7;-7)$ vektorlar berilgan. $\vec{a}$ ni $\vec{b}, \vec{c}$ *va* $\vec{d}$ vektorlar orqali ifodalang.

2. $\left\{\vec{e_1}, \vec{e_2}, \vec{e_3}\right\}$ bazisdan $(\vec{e'}_1 = \vec{e}_1 + \vec{e}_3; \quad \vec{e'}_2 = -\vec{e}_1 + 2\vec{e}_2; \quad \vec{e'}_3 = \vec{e}_1 + \vec{e}_2)$ bazisga o`tish matritsasini toping.

3. $\left\{\vec{e}_1, \vec{e}_2\right\}$ bazisda chiziqli $\tilde{A}$ operator $\begin{pmatrix} -1 & 1 \\ 2 & 1 \end{pmatrix}$ matritsa bilan berilgan $\vec{x} = (2;-1)$ bo`lsa, $y = \tilde{A}(x)$ vektorning koordinatalarini toping.

8-variant

1. $\vec{a} = 2\vec{i} + 3\vec{j} - 6\vec{k}$ vektor uzunligi va uning yo‘naltiruvchi kosinuslarini toping.

2. $\left\{\vec{e_1}, \vec{e_2}, \vec{e_3}\right\}$ bazisdan $(\vec{e'}_1 = \vec{e}_2 + \vec{e}_3; \ \vec{e'}_2 = -\vec{e}_1 + 2\vec{e}_3; \ \vec{e'}_3 = \vec{e}_1 + \vec{e}_2)$ bazisga o`tish matritsasini toping.

3. Biror bazisda $A = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 6 & 0 \\ 2 & 3 & 5 \end{pmatrix}$ matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

9-variant

1.Agar $\vec{a}$ vektor Oy va Oz o‘qlari bilan mos ravishda 60^0 va 120^0 burchak tashkil qiladi. Ox o‘qi bilan qanday burchak tashkil qiladi.

2. Agar $\left\{\vec{e_1}, \vec{e_2}, \vec{e_3}\right\}$ bazisdan $\left\{\vec{e}_1{}', \vec{e}_2{}', \vec{e}_3{}'\right\}$ bazisga o`tish matritsasi $A = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$ berilgan. $\vec{e_2'}$ vektorning $\left\{\vec{e_1}, \vec{e_2}, \vec{e_3}\right\}$ bazisdagi koordinatalarini toping.

3.Chiziqli operatorning $\left\{\vec{e}_1, \vec{e}_2\right\}$ bazisdagi matritsasi $A = \begin{pmatrix} -2 & 1 \\ 3 & -1 \end{pmatrix}$ ko‘rinishga ega. Agar $\vec{e}_1{}' = \vec{e}_2, \vec{e}_2{}' = \vec{e}_1 + \vec{e}_2$ bo‘lsa, chiziqli operatorning $\left\{\vec{e}_1{}', \vec{e}_2{}'\right\}$ bazisdagi matritsasini toping.

10-variant

1. $\vec{a}=6\vec{i}-8\vec{j}+5\sqrt{2}\vec{k}$ va $\vec{b}=2\vec{i}-4\vec{j}+\sqrt{2}\vec{k}$ vektorlar berilgan. $\vec{a}-\vec{b}$ vektorning Ox o‘qi bilan hosil qilgan burchakni toping.

2. Biror bazisda $\vec{a}_1=(2;1)$ va $\vec{a}_2=(-1;3)$ vektorlar berilgan. $\vec{b}=(1;m)$ vektor $\vec{a}_1$ *va* $\vec{a}_2$ vektorlar orqali chiziqli ifodalanadigan m ning barcha qiymatlarini toping.

3. Biror bazisda $A=\begin{pmatrix}0&2&0\\1&5&4\\0&3&0\end{pmatrix}$. matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

11-variant

1. m ning qanday qiymatlarida $\vec{a}=m\vec{i}-3\vec{j}+2\vec{k}$ va $\vec{b}=\vec{i}+2\vec{j}-m\vec{k}$ vektorlar perpendikulyar bo‘ladi.

2. $\{\vec{e}_1,\vec{e}_2\}$ bazisda $\vec{a}_1=2\vec{e}_1+\vec{e}_2$ *va* $\vec{a}_1=2\vec{e}_1+\vec{e}_2$ vektorlar berilgan. $\vec{a}_1$ *va* $\vec{a}_2$ vektorlar bazis tashkil qilishini isbotlang. $\vec{a}_3=3\vec{e}_1+2\vec{e}_2$ vektorning $\{\vec{a}_1,\vec{a}_2\}$ bazisdagi koordinatalarini toping.

3. Biror bazisda $A=\begin{pmatrix}0&1&0\\1&1&1\\0&1&0\end{pmatrix}$ matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

12-variant

1. $\vec{a}=\vec{i}+\vec{j}+2\vec{k}$ vektorning $\vec{b}=\vec{i}-\vec{j}+4\vec{k}$ vektordagi proeksiyasini toping.

2. Biror bazisda $\vec{a_1}=(1;2;1)$, $\vec{a_2}=(2;1;1)$ va $\vec{a_3}=(-1;-2;-1)$ vektorlar berilgan. $\vec{b}=(2;3;m)$ vektorni $\vec{a_1},\vec{a_2},\vec{a_3}$ vektorlar orqali chiziqli ifodalanadigan m ning barcha qiymatlarini toping.

3. Biror bazisda $A=\begin{pmatrix} 7 & -2 & -2 \\ 2 & 2 & -1 \\ 0 & 0 & 3 \end{pmatrix}$ matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

13-variant

1. $\vec{a}=3\vec{i}-6\vec{j}-\vec{k}$, $\vec{b}=\vec{i}+4\vec{j}-5\vec{k}$, $\vec{c}=3\vec{i}+4\vec{j}+2\vec{k}$ vektorlar berilgan. $\vec{a}+\vec{c}$ vektorning $\vec{b}+\vec{c}$ vektordagi proeksiyasini toping.

$\vec{b}=(5;9;m)$ vektor $\vec{a_1}=(4;4;3)$, $\vec{a_2}=(7;2;1)$ va $\vec{a_3}=(4;1;6)$ vektorlar orqali chiziqli ifodalanadigan m ning barcha qiymatlarini toping.

3.Biror bazisda $A=\begin{pmatrix} 1 & 2 & -2 \\ -1 & 0 & 3 \\ 1 & 3 & 0 \end{pmatrix}$. matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping

14-variant

1. $\vec{c}=\vec{i}+2\vec{j}+2\vec{k}$ vektordagi proeksiyasi 1 ga teng bo‘lgan, $\vec{a}=\vec{i}+\vec{k}$, va $\vec{b}=2\vec{j}-\vec{k}$, vektorlarga perpendikulyar $\vec{d}$ vektorni toping.

2. Quyidagi vektorlar sistemalari chiziqli bog‘liq yoki chiziqli erkliligini ko‘rsating: $\vec{a_1}=(-7;5;19)$, $\vec{a_2}=(-5;7;-7)$, $\vec{a_3}=(-8;7;14)$.

3.Biror bazisda $A=\begin{pmatrix} 1 & 0 & 3 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

15-variant

1. $\vec{a}=(1;-1;2)$ va $\vec{b}=(1;0;1)$ vektorlar uzunliklarini va ular orasidagi burchakni toping.

2. $\{\vec{e}_1,\vec{e}_2\}$ bazisdan $\{\vec{e}\,'_1,\vec{e}\,'_2\}$ bazisga o\`tish matritsasi $A=\begin{pmatrix}2 & -1\\ 3 & 5\end{pmatrix}$ ko‘rinishda berilgan. $\vec{e}_1,\vec{e}_2$ vektorlarning $\{\vec{e}\,'_1,\vec{e}\,'_2\}$ bazisdagi koordinatalarini toping.

3. Biror bazisda $A=\begin{pmatrix}1 & 1 & 8\\ 0 & 2 & 0\\ 1 & 0 & -1\end{pmatrix}$ matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

16-variant

1. $M_1=(1;2;3)$ va $M_2=(3;-4;6)$ nuqtalar berilgan. $\overrightarrow{M_1M_2}$ vektorning uzunligi va uning yo‘naltiruvchi kosinuslarini toping.

2. $\{\vec{e}_1,\vec{e}_2,\vec{e}_3\}$ vektorlar ortonormallangan bazisni tashkil qiladi. $\vec{x}=3\vec{e}_2-\vec{e}_3$ va $\vec{y}=4\vec{e}_1+\vec{e}_2-2\vec{e}_3$ vektorlar orasidagi burchakni toping.

3. Biror bazisda $A=\begin{pmatrix}2 & 0 & -6\\ 1 & 3 & 2\\ -1 & 0 & 1\end{pmatrix}$. matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

17-variant

1. M nuqtaning radius vektori Oy o‘qi bilan 60^0, Oz o‘qi bilan 45^0 li burchak tashkil qiladi, uning uzunligi $r=8$. M nuqtaning absissasi manfiy bo‘lsa uni toping.

2. Quyidagi vektorlar sistemalari chiziqli bog‘liq yoki chiziqli erkliligini ko‘rsating: $\vec{a}_1=(1;8;-1)$, $\vec{a}_2=(-2;3;3)$, $\vec{a}_3=(4;-11;9)$.

3. Biror bazisda $A=\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 1 & -1 & 1 \end{pmatrix}$. matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

18-variant

1. $\vec{a}=\vec{i}+2\vec{j}+3\vec{k}$ va $\vec{b}=6\vec{i}+4\vec{j}-2\vec{k}$ vektorlar orasidagi burchakni toping.

2. R^3 uch o‘lchovli fazoda $\vec{a}_1=(1;1;1)$, $\vec{a}_2=(1;0;1)$, $\vec{a}_3=(2;1;2)$ vektorlar bazis tashkil qiladimi.

3. Biror bazisda $A=\begin{pmatrix} 3 & -1 & 1 \\ 0 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$. matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

19-variant

1. m ning qanday qiymatlarida $\vec{a}=m\,\vec{i}+3\,\vec{j}+4\,\vec{k}$ va $\vec{b}=4\,\vec{i}+m\,\vec{j}-7\,\vec{k}$ vektorlar perpendikulyar.

2. $\vec{b}=(1;3;5)$ vektor $\vec{a}_1=(3;2;5)$, $\vec{a}_2=(2;4;7)$ va $\vec{a}_3=(5;6;m)$ vektorlar orqali chiziqli ifodalanadigan m ning barcha qiymatlarini toping.

3. Biror bazisda $A=\begin{pmatrix} 5 & -1 & -1 \\ 0 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix}$. matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

20-variant

1. $\vec{a}=(1;6;1)$, $\vec{b}=(0;1;-2)$, $\vec{c}=(1;-1;0)$ va $\vec{d}=(2;-1;3)$ vektorlar berilgan. $\vec{a}$ ni $\vec{b},\vec{c}$ *va* $\vec{d}$ vektorlarning chiziqli kombinatsiyasi orqali ifodalang.

2. R^4 to'rt o'lchovli fazoda $\vec{a}_1=(1;1;1;1)$, $\vec{a}_2=(1;0;1;0)$, $\vec{a}_3=(0;-1;0;1)$, $\vec{a}_4=(1;0;0;1)$ vektorlar bazis tashkil qiladimi.

3. Biror bazisda $A=\begin{pmatrix}1&0&0\\3&6&4\\2&3&5\end{pmatrix}$. matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

21-variant

1. α va β ning qanday qiymatlarida $\vec{a}=-2\vec{i}+3\vec{j}+\alpha\vec{k}$ va $\vec{b}=\beta\vec{i}-6\vec{j}+2\vec{k}$ vektorlar kollinear bo'ladi.

2.Biror bazisda $\vec{a}_1=(4;5;2)$, $\vec{a}_2=(3;0;1)$, $\vec{a}_3=(-1;4;2)$ va $\vec{b}=(5;7;8)$. vektorlar berilgan. $\vec{a}_1,\vec{a}_2,\vec{a}_3$ vektorlar bazis tashkil qilishini ko'rsating. $\vec{b}$ vektorning bu bazisdagi koordinatalarini toping.

3. Biror bazisda $A=\begin{pmatrix}0&0&0\\0&0&0\\1&0&0\end{pmatrix}$. matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

22-variant

1. $\vec{c}=(9;4)$, $\vec{a}=(1;2)$, $\vec{b}=(2;-3)$ vektorlar berilgan. $\vec{c}$ ni $\vec{a},\vec{b}$ vektorlar orqali ifodalang.

2. Biror bazisda $\vec{a}_1=(3;-5;2)$, $\vec{a}_2=(4;5;1)$, $\vec{a}_3=(-3;0;-4)$ va $\vec{b}=(-4;5;-16)$. vektorlar berilgan. $\vec{a}_1,\vec{a}_2,\vec{a}_3$ vektorlar bazis tashkil qilishini ko'rsating. $\vec{b}$ vektorning bu bazisdagi koordinatalarini toping.

3. $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ bazisda chiziqli $\tilde{A}$ operator $A = \begin{pmatrix} 0 & 0 & 3 \\ 1 & 1 & 4 \\ 3 & 1 & 5 \end{pmatrix}$ matritsa bilan berilgan $\vec{x} = -\overline{e}_1 + 2\overline{e}_2 + \overline{e}_3$ bo`lsa, $y = A(\vec{x})$vektorning koordinatalarini toping.

23-variant

1. $\vec{a} = (2;3)$, $\vec{b} = (1;-3)$, $\vec{c} = (-1;3)$ vektorlar berilgan. α ning qanday qiymatlarida $\vec{p} = \vec{a} + \alpha\vec{b}$ va $\vec{q} = \vec{a} + 2\vec{c}$ vektorlar kollinear.

2. Biror bazisda $\vec{a}_1 = (-2;3;5)$, $\vec{a}_2 = (1;-3;4)$, $\vec{a}_3 = (7;8;-1)$ va $\vec{b} = (1;20;1)$. vektorlar berilgan. $\vec{a}_1, \vec{a}_2, \vec{a}_3$ vektorlar bazis tashkil qilishini ko‘rsating. $\vec{b}$ vektorning bu bazisdagi koordinatalarini toping.

3. Chiziqli operatorning $\{\vec{e}_1, \vec{e}_2\}$ bazisdagi matritsasi $A = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$ ko‘rinishga ega.Agar $\vec{e}_1^{\,'} = 3\vec{e}_1 - \vec{e}_2$, $\vec{e}_2^{\,'} = 2\vec{e}_1 - 3\vec{e}_2$, bo‘lsa, chiziqli operatorning $\{\vec{e}_1', \vec{e}_2'\}$ bazisdagi matritsasini toping.

24-variant

1. $\vec{a} = (1;1;1)$ va $\vec{b} = (0;1;1)$ vektorlar uzunliklarini va ular orasidagi burchakni toping.

2. Quyidagi vektorlar sistemalari chiziqli bog‘liq yoki chiziqli erkliligini ko‘rsating: $\vec{a}_1 = (1;4;6)$, $\vec{a}_2 = (1;-1;1)$, $\vec{a}_3 = (1;1;3)$.

3. Biror bazisda $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$. matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

25-variant

1. Tekislikda uch vektor joylashgan $\vec{a}$, $\vec{b}$, $\vec{c}$. va $|\vec{a}| = 2$, $|\vec{b}| = 3$, $|\vec{c}| = 5$, $\left(\vec{a} \wedge \vec{b}\right) = 60^0$, $\left(\vec{b} \wedge \vec{c}\right) = 60^0$. $\vec{d} = -\vec{a} + \vec{b} - \vec{c}$ vektorning uzunligini toping.

2. $\left(\vec{e}_1, \vec{e}_2, \vec{e}_3\right)$vektorlar ortogonal bazisni tashkil qiladi. Agar $|\vec{e}_1| = 1$, $|\vec{e}_2| = 2$, $|\vec{e}_3| = 2$ bo`lsa, $\vec{x} = 2\vec{e}_1 - 3\vec{e}_2 + 4\vec{e}_3$ va $\vec{y} = \vec{e}_1 + \vec{e}_2 - 5\vec{e}_3$ vektorlar uzunliklarini va ularning skalyar ko`paytmasini toping.

3. Biror bazisda $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$. matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

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M U N D A R I J A

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