O‘ZBEKISTON RESPUBLIKASI OLIY TA’LIM, FAN VA INNOVATSIYALAR VAZIRLIGI

RENESSANS TA’LIM UNIVERSITETI

«MATEMATIKA VA AXBOROT TEXNOLOGIYALARI» KAFEDRASI

Sirtqi ta’lim yo’nalishidagi bakalavrlar uchun “Matematika” fanidan mustaqil ishlarni tashkil etish uchun uslubiy ishlanmasi

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Uslubiy ishlanma “Matematika va axborot texnologiyalari” kafedrasining 2023 yil ________________ dagi №____majlisida va institut kengashining 2023 yil ________________ dagi №____yigʻilishida muhokama etilgan va chop etishga tavsiya qilihgan.

M U A L L I F L A R : “Matematika va axborot texnologiyalari” kafedrasi
dotsenti G’.G’.Yunusov
katta o’qituvchi Z.Q.Hamroyeva
assistent M.A.Axmedov

T A Q R I Z CH I L A R: RTU dotsenti, f.-m.f.n.,
D.E.Davlatov
TKTI Professori, f.-m.f.d.
I.I.Safarov

M U H A R R I R : “Matematika va axborot texnologiyalari” kafedrasi
dotsenti I.X.Turopov

Uslubiy ishlanma o’quv rejasida “Amaliy matematika”, “Matematika” va “Oliy matematika” fanini o’qitish rejalashtirilgan bakalavriat yo’nalishlari uchun mo’ljallangan. Unda I bosqich bakalavrlari uchun fandan kuzgi oʻquv mavsumida mustaqil o’rganilishi ko’zda tutilgan mavzular ro’yxati, mustaqil ish topshiriqlari va ulardagi masalalarning namunaviy yechimlari keltirilgan, integrallar va Laplas tasvirlari jadvallari ilovalangan va oʻquv-uslubiy adabiyotlar roʻyxati ko’rsatilgan.

KIRISH

Hozirgi kunda respublikamizda oliy ta'limda kredit-modul tizimiga asoslangan o'qitish tizimiga o'tildi. Ushbu tizimda talabaning mustaqil ish uchun ajratilgan soatlar hajmi katta ahamiyatga ega. Shu munosabat bilan talabalarda mustaqil ta'limni shakllantirish va mustaqil o'rganishga bag'ishlangan uslubiy ko'rsatmalarga talab ortmoqda. Fan bo'yicha namunaviy fan dasturidagi talablar to'liq bajarilishi uchun talaba tomonidan mustaqil ravishda mavzular o'rganilishi zarurati tug'iladi.

Talabalar mustaqil ishi ularning auditoriya mashg'ulotlarida olgan bilimlarini mustahkamlash, chuqurlashtirish, kengaytirish va to'ldirishga xizmat qilishi kerak. Bundan tashqari fanning sillabusda rejalashtirilgan bir qator mavzularni talabalar o'quv adabiyotlari yordamida mustaqil o'rganishiga to'g'ri keladi.

Talabalarning fan bo'yicha mustaqil ishini tashkil etish va uni shaklini belgilash tegishli kafedra tomonidan amalga oshiriladi. Bu masala "Matematika va axborot texnologiyalari" kafedrasining 2023 yil ______________ dagi ___-majlisida muhokama etildi. Bu majlis qaroriga asosan kafedra fanlari bo'yicha talabalarning auditoriyadan tashqari mustaqil ishi ma'lum bir mavzu bo'yicha amaliy mazmunli hisob-kitob ko'rinishdagi topshiriqlarni bajarishdan iborat deb tasdiqlandi.

Ushbu uslubiy ishlanmada ishchi o'quv rejasida kuzgi mavsumdagi mustaqil ishni tashkil etish masalalari qaralgan. Unda fandan sillabusda nazarda tutilgan o'rganilishi rejalashtirilgan "Chiziqli algebra asoslari", "Vektorlar algebrasi", "Analitik geometriya" va "Matematik analiz" boʻlimlari boʻyicha muammoli misol–masalalardan iborat boʻlgan yozma ish topshiriqlari keltirilgan.

Talabalarga mustaqil ishni bajarish uchun uslubiy yordam sifatida topshiriqlardagi misol–masalalarning namunaviy yechimlari, ilova sifatida integrallar va Laplas tasvirlari jadvallari va adabiyotlar ro'yxati keltirilgan. Oʻquv guruhidagi talabalar soni 30 tagacha boʻlishini hisobga olib har bir topshiriq 30 variantdan iborat koʻrinishda tuzildi. Odatda talabaning varianti uning oʻquv guruhi jurnalidagi tartib raqami bilan aniqlanadi yoki oʻqituvchi tomonidan tayinlanadi. Yozma ish topshiriq variantlaridagi misol–masalalar tipik koʻrinishda boʻlib, bir–biridan asosan unga kiruvchi parametrlarning qiymatlari bilan farq qiladi. Shu sababli barcha variantlar boʻyicha topshiriqlar murakkabligi bir xil darajadadir.

Mustaqil ish topshiriqlari va referat mavzulari talabalarga kuzgi mavsum boshida tarqatiladi. Talabalar mustaqil ish topshiriqlari va referatlarni tegishli ma'ruzalar va amaliy mashgʻulotlar oʻtilayotgan davrda bajarib borishlari kerak.

Mustaqil ish topshirigʻi boʻyicha tegishli ma'ruza va amaliy mashgʻulotlar oʻtib boʻlingandan keyin bir hafta ichida talaba tegishli topshiriqni bajarishi va uni yozma koʻrinishda oʻqituvchiga topshirishi shart. Oʻz muddatida topshirilmagan mustaqil ish topshiriqlari bajarilmagan deb hisoblanadi va koʻrsatilgan vaqtdan keyin qabul qilinmaydi va ularga ball qo’yilmaydi.

Sirtqi ta’lim shaklida o’qiyotgan talabalar mustaqil ish topshiriqlarini 1 haftalik boshlang’ich mavsumda oladilar va mashg’ulotlarga qayta kelishlaridan oldin bajarib topshiradilar.

Uslubiy ishlanmada fandan talabalar mustaqil ishi topshiriqlari bajarilishini nazorat qilish tartibi va ularni baholash mezonlari ham keltirilgan. Oʻquv mavsumi davomida talabaning fandan mustaqil ish topshiriqlarini bajarish boʻyicha olgan baholari yoki to’plagan ballari joriy nazorat bahosiga yoki ballariga qoʻshib boriladi.

MUSTAQIL ISH TOPSHIRIQLARIDAGI MASALALARNING NAMUNAVIY YECHIMLARI.

I topshiriq

Berilgan uch noma'lumli chiziqli tenglamalar sistemasini Kramer, Gauss va matritsalar usullarida yeching:

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ 5x_1 - 3x_2 + 2x_3 = -3 \\ 2x_1 - x_2 + 3x_3 = 1 \end{cases}$$

Yechish: Berilgan sistemani Kramer usulida yechish uchun dastlab uning asosiy Δ va yordamchi Δ_1, Δ_2, Δ_3 aniqlovchilarini hisoblaymiz. Asosiy Δ aniqlovchi sistemaning koeffitsientlaridan tuziladi:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 5 & -3 & 2 \\ 2 & -1 & 3 \end{vmatrix} = 1\cdot(-3)\cdot 3 + (-1)\cdot 1\cdot 5 + 1\cdot 2\cdot 2 - 2\cdot 1\cdot(-3) - 2\cdot 1\cdot(-1) - 5\cdot 1\cdot 3 =$$

$$= -9 - 5 + 4 + 6 + 2 - 15 = -17,$$

Yordamchi Δ_1, Δ_2, Δ_3 aniqlovchilar asosiy Δ aniqlovchining mos ravishda birinchi, ikkinchi, uchinchi ustunlarini ozod hadlar bilan almashtirishdan hosil qilinadi:

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 1 \\ -3 & -3 & 2 \\ 1 & -1 & 3 \end{vmatrix} = 0 + 3 + 2 + 3 - 0 + 9 = 17,$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 & 1 \\ 5 & -3 & 2 \\ 2 & 1 & 3 \end{vmatrix} = -9 + 5 + 0 - (-6) - 2 - 0 = 0,$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 0 \\ 5 & -3 & -3 \\ 2 & -1 & 1 \end{vmatrix} = -3 + 0 - 6 - 0 - 3 - 5 = -17.$$

Bu aniqlovchilar yordamida berilgan chiziqli tenglamalar sistemasining ildizlarini Kramer formulalari orqali quyidagicha topamiz:

$$x_1 = \frac{\Delta_1}{\Delta} = \frac{17}{-17} = -1 \ , \qquad x_2 = \frac{\Delta_2}{\Delta} = \frac{0}{-17} = 0 \ , \qquad x_3 = \frac{\Delta_3}{\Delta} = \frac{-17}{-17} = 1.$$

Demak, berilgan sistemaning ildizlari $x_1 = -1$, $x_2=0$, $x_3=1$ boʻladi.

Yechim toʻgʻriligini tekshirish uchun bu ildizlar qiymatlarini berilgan sistemaga qoʻyamiz:

$$\begin{cases} x_1 + x_2 + x_3 = -1 + 0 + 1 \equiv 0 \\ 5x_1 - 3x_2 + 2x_3 = 5 \cdot (-1) - 3 \cdot 0 + 2 \cdot 1 \equiv -3 \\ 2x_1 - x_2 + 3x_3 = 2 \cdot (-1) - 0 + 3 \cdot 1 \equiv 1 \end{cases}.$$

Bu yerdan koʻrinadiki $x_1= -1$, $x_2=0$, $x_3=1$ boʻlganda berilgan sistemaning uchala tenglamasi ham ayniyat boʻldi. Demak, sistema toʻgʻri yechilgan va $x_1=-1$, $x_2=0$, $x_3=1$ berilgan sistema ildizlari boʻladi.

Bu sistemani Gauss usulida yechish uchun dastlab uni «toʻrtburchakli» shakldan «uchburchakli» shaklga keltiramiz. Buning uchun dastlab sistemaning ikkinchi va uchinchi tenglamalaridan x_1 noma'lumni yoʻqotamiz. Bunga erishish uchun sistemaning birinchi tenglamasini 5ga (yoki 2ga) koʻpaytirib, uning ikkinchi (yoki uchinchi) tenglamasidan ayiramiz. Natijada quyidagi sistemaga kelamiz:

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ -8x_2 - 3x_3 = -3 \\ -3x_2 + x_3 = 1 \end{cases}$$

Endi bu sistemaning uchinchi tenglamasidan x_2 noma'lumni yoʻqotamiz. Buning uchun oxirgi sistemaning ikkinchi tenglamasini 3 ga, uchinchi tenglamasini esa 8 ga koʻpaytirib, hosil boʻlgan uchinchi tenglamadan ikkinchi tenglamani ayiramiz. Natijada ushbu «uchburchak» shaklidagi sistemaga kelamiz:

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ -8x_2 - 3x_3 = -3 \\ -17x_3 = -17 \end{cases}.$$

Oxirgi uchburchakli sistemaning uchinchi tenglamasidan x_3 noma'lumni topamiz:

$$-17x_3 = -17 \Rightarrow x_3 = \frac{-17}{-17} = 1.$$

$x_3=1$ natijani uchburchakli sistemaning ikkinchi tenglamasiga qoʻyib, x_2 noma'lumni topamiz:

$$-8x_2 - 3 \cdot 1 = -3 \Rightarrow -8x_2 - 3 = -3 \Rightarrow -8x_2 = 0 \Rightarrow x_2 = 0.$$

Topilgan $x_3=1$ va $x_2=0$ natijalarni uchburchakli sistemaning birinchi tenglamasiga qoʻyib, x_1 noma'lum qiymatini topamiz:

$$x_1 + 0 + 1 = 0 \Rightarrow x_1 + 1 = 0 \Rightarrow x_1 = -1.$$

Demak, berilgan sistemaning ildizlari $x_1= -1$, $x_2=0$, $x_3=1$ boʻladi va Kramer usulida topilgan natijalar bilan ustma–ust tushadi.

Endi bu sistemani matritsalar usulida yechamiz. Buning uchun berilgan sistema boʻyicha quyidagi matritsalarni kiritamiz:

$$A=\begin{pmatrix}1 & 1 & 1\\ 5 & -3 & 2\\ 2 & -1 & 3\end{pmatrix},\qquad B=\begin{pmatrix}0\\ -3\\ 1\end{pmatrix},\qquad X=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}.$$

Bu holda berilgan chiziqli tenglamalar sistemasi $AX=B$ koʻrinishga keladi va uning ildizlaridan iborat X matritsa $X=A^{-1}\cdot B$ formula bilan topiladi. Bu yerda A^{-1} yuqoridagi A matritsaga teskari matritsa boʻlib, u

$$A^{-1}=\frac{1}{\det A}\begin{pmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{pmatrix}$$

formula orqali topiladi. Shu sababli dastlab Δ=detA aniqlovchini va A_{ij} algebraik toʻldiruvchilarni hisoblaymiz. Kramer usuli koʻrilayotganda

$$\Delta=\begin{vmatrix}1 & 1 & 1\\ 5 & -3 & 2\\ 2 & -1 & 3\end{vmatrix}=-17$$

ekanligi topilgan edi. Algebraik toʻldiruvchi ta’rifiga asosan

$$A_{11}=\begin{vmatrix}-3 & 2\\ -1 & 3\end{vmatrix}=-7\ ,\quad A_{12}=-\begin{vmatrix}5 & 2\\ 2 & 3\end{vmatrix}=-11\ ,\quad A_{13}=\begin{vmatrix}5 & -3\\ 2 & -1\end{vmatrix}=1$$

$$A_{21}=-\begin{vmatrix}1 & 1\\ -1 & 3\end{vmatrix}=-4\ ,\quad A_{22}=\begin{vmatrix}1 & 1\\ 2 & 3\end{vmatrix}=1\ ,\quad A_{23}=-\begin{vmatrix}1 & 1\\ 2 & -1\end{vmatrix}=3$$

$$A_{31}=\begin{vmatrix}1 & 1\\ -3 & 2\end{vmatrix}=5\ ,\quad A_{32}=-\begin{vmatrix}1 & 1\\ 5 & 2\end{vmatrix}=3\ ,\quad A_{33}=\begin{vmatrix}1 & 1\\ 5 & -3\end{vmatrix}=-8$$

ekanligini topamiz.

Demak,

$$A^{-1}=\frac{1}{-17}\begin{pmatrix}-7 & -4 & 5\\ -11 & 1 & 3\\ 1 & 3 & -8\end{pmatrix}=\begin{pmatrix}\frac{7}{17} & \frac{4}{17} & -\frac{5}{17}\\ \frac{11}{17} & -\frac{1}{17} & -\frac{3}{17}\\ -\frac{1}{17} & -\frac{3}{17} & \frac{8}{17}\end{pmatrix}$$

va matritsalarni koʻpaytirish ta’rifiga asosan

$$X=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=A^{-1}B=\begin{pmatrix}\frac{7}{17} & \frac{4}{17} & -\frac{5}{17}\\ \frac{11}{17} & -\frac{1}{17} & -\frac{3}{17}\\ -\frac{1}{17} & -\frac{3}{17} & \frac{8}{17}\end{pmatrix}\cdot\begin{pmatrix}0\\ -3\\ 1\end{pmatrix}=\frac{1}{17}\begin{pmatrix}-17\\ 0\\ 17\end{pmatrix}=\begin{pmatrix}-1\\ 0\\ 1\end{pmatrix}.$$

Bu yerdan yana bir marta berilgan sistemaning yechimi $x_1=-1$, $x_2=0$ va $x_3=1$ ekanligini koʻramiz.

<u>II topshiriq</u>

Fazoda uchlari $A(8,6,4)$, $B(10,5,5)$, $C(5,6,8)$ va $D(9,10,7)$ nuqtalarda joylashgan piramida berilgan. Bu piramida bo'yicha quyidagilarni bajaring:

1. $\overrightarrow{AB}$ vektor koordinatalarini toping va undan foydalanib AB qirra uzunligini hisoblang;

2. $\overrightarrow{AB}$ va $\overrightarrow{AD}$ vektorlardan foydalanib AB va AD qirralar orasidagi φ burchak kosinusini toping;

3. $\overrightarrow{AB}$ va $\overrightarrow{AD}$ vektorlardan foydalanib piramidaning ABD tomoni yuzasini toping ;

4. $\overrightarrow{AB}$, $\overrightarrow{AC}$ va $\overrightarrow{AD}$ vektorlar yordamida $ABCD$ piramidaning hajmini aniqlang;

5. AD qirra yotgan toʻgʻri chiziqning kanonik va parametrik tenglamalarini yozing;

6. ABC yoq yotgan tekislikning umumiy, kesmalardagi va normal tenglamalarni yozing;

7. Piramidaning ABC va ABD yoqlari orasigi ikki yoqli α burchak kosinusini toping;

8. Piramidaning D uchidan tushirilgan DH balandligi yotuvchi L to'g'ri chiziqning kanonik tenglamasini aniqlang;

9. Piramidaning D uchidan tushirilgan DH balandligining uzunligini toping.

<u>Yechish:</u> **1.** $\overrightarrow{AB}=(x,\ y,\ z)$ vektorning x, y va z koordinatalari uning B(10,5,5) uchi va A(8,6,4) boshi mos koordinatalarining ayirmasiga teng , ya'ni

$$\overrightarrow{AB}=(x, y, z)=(x_2-x_1, y_2-y_1, z_2-z_1)=(10-8,\ 5-6,\ 5-4)=(2,\ -1,\ 1)\ .$$

AB qirraning |AB| uzunligi topilgan $\overrightarrow{AB}$ vektor moduliga teng bo'ladi va $|\overrightarrow{AB}|$ modul formulasiga asosan

$$|AB|=|\overrightarrow{AB}|=\sqrt{x^2+y^2+z^2}=\sqrt{2^2+(-1)^2+1^2}=\sqrt{6}\ .$$

2. Dastlab yuqoridagi singari A(8,6,4) va D(9,10,7) nuqtalar bo'yicha $\overrightarrow{AD}$ vektor koordinatalarini topamiz:

$$\overrightarrow{AD}=(9-8,\ 10-6,\ 7-4)=(1,\ 4,\ 3)\ .$$

AB va AD qirralar orasidagi φ burchak kosinusini $\overrightarrow{AB}=(2,\ -1,\ 1)$ va $\overrightarrow{AD}=(1,\ 4,\ 3)$ vektorlar orasidagi burchak formulasi, vektorlar skalyar ko'paytmasi va modullarini koordinatalar orqali ifodasidan foydalanib topamiz:

$$\cos\varphi=\frac{\overrightarrow{AB}\cdot\overrightarrow{AD}}{|\overrightarrow{AB}|\cdot|\overrightarrow{AD}|}=\frac{2\cdot1+(-1)\cdot4+1\cdot3}{\sqrt{2^2+(-1)^2+1^2}\cdot\sqrt{1^2+4^2+3^2}}=\frac{1}{\sqrt{6}\cdot\sqrt{26}}=\frac{1}{\sqrt{156}}\ .$$

3. Piramidaning ABD yog'ining S yuzasini topish uchun $\overrightarrow{AB}=(2,\ -1,\ 1)$ va $\overrightarrow{AD}=(1,\ 4,\ 3)$ vektorlarning vektorial ko'paytmasidan foydalanamiz.

Vektorial ko’paytmaning koordinatalardagi ifodasi va III tartibli aniqlovchini hisoblash formulasiga asosan

$$\overrightarrow{AB}\times\overrightarrow{AD}=\begin{vmatrix} i & j & k\\ 2 & -1 & 1\\ 1 & 4 & 3\end{vmatrix}=-3i+8k+j+k-4i-6j=-7i-5j+9k=(-7,-5,9).$$

Bu yerdan, vektorial ko’paytma modulining geometrik ma’nosiga asosan,

$$S=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AD}\right|=\frac{1}{2}\sqrt{(-7)^2+(-5)^2+9^2}=\frac{\sqrt{155}}{2}\text{ kv.birlik}$$

javobga ega bo’lamiz.

4. Dastlab A(8,6,4) va C(5,6,8) nuqtalar bo’yicha $\overrightarrow{AC}$ vektor koordinatalarini topamiz:

$$\overrightarrow{AC}=(5–8,\ 6–6,\ 8–4)=(–3,\ 0,\ 4).$$

ABCD piramidaning V hajmini

$$\overrightarrow{AB}=(2,\ –1,\ 1),\quad \overrightarrow{AC}=(–3,\ 0,\ 4),\quad \overrightarrow{AD}=(1,\ 4,\ 3)$$

vektorlarning aralash ko’paytmasi yordamida topamiz. Aralash ko’paytmaning koordinatalar orqali ifodasi formulasidan foydalanib

$$V=\pm\frac{1}{6}\overrightarrow{AB}\cdot\overrightarrow{AC}\cdot\overrightarrow{AD}=\pm\frac{1}{6}\begin{vmatrix} 2 & -1 & 1\\ -3 & 0 & 4\\ 1 & 4 & 3\end{vmatrix}=$$

$$=\pm\frac{1}{6}(0+(-4)+(-12)-0-32-9)=\pm\frac{1}{6}\cdot(-57)=\frac{57}{6}=9\frac{1}{2}\text{ kub birlik}$$

natijani olamiz.

5. *AD* qirra yotgan toʻgʻri chiziqning kanonik tenglamasini ikkita A(8,6,4) va D(9,10,7) nuqtalardan oʻtuvchi toʻgʻri chiziq tenglamasi formulasidan foydalanib topamiz:

$$AD:\quad \frac{x-8}{9-8}=\frac{y-6}{10-6}=\frac{z-4}{7-4}\Rightarrow\frac{x-8}{1}=\frac{y-6}{4}=\frac{z-4}{3}.$$

Endi AD qirraning kanonik tenglamasidagi kasrlarni t parametrga tenglashtirib, uning parametrik tenglamasini hosil qilamiz :

$$\frac{x-8}{1}=\frac{y-6}{4}=\frac{z-4}{3}=t\Rightarrow x-8=t,\quad y-6=4t,\quad z-4=3t\Rightarrow$$

$$x=t+8,\quad y=4t+6,\quad z=3t+4.$$

6. *ABC* yoq yotgan tekislikning $Ax+By+Cz+D=0$ koʻrinishdagi umumiy tenglamasini uchta $A(8,6,4)$, $B(10,5,5)$ va $C(5,6,8)$ nuqtalardan oʻtuvchi tekislik tenglamasining ifodasi yordamida topamiz:

$$\begin{vmatrix} x-8 & y-6 & z-4 \\ 10-8 & 5-6 & 5-4 \\ 5-8 & 6-6 & 8-4 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x-8 & y-6 & z-4 \\ 2 & -1 & 1 \\ -3 & 0 & 4 \end{vmatrix} = 0$$

$$\Rightarrow -4(x-8) - 3(y-6) - 3(z-4) - 8(y-6) = 0 \Rightarrow$$

$$-4x + 32 - 3y + 18 - 3z + 12 - 8y + 48 = 0$$

$$-4x - 11y - 3z + 110 = 0 \Rightarrow 4x + 11y + 3z - 110 = 0$$

Endi *ABC* yoqning kesmalarga nisbatan $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ tenglamasini topish uchun uning umumiy tenglamasini $-D = 110$ ga boʻlamiz:

$$\frac{4x}{110} + \frac{11y}{110} + \frac{3z}{110} - \frac{110}{110} = 0 \Rightarrow \frac{x}{5/2} + \frac{y}{10} + \frac{z}{110/3} = 1.$$

Bu yerdan izlangan kesmalardagi tenglamada a=5/2 , b=10 va c=110/3 ekanligini koʻramiz.

ABC yoqning normal $x\cos\alpha + y\cos\beta + z\cos\gamma - p = 0$ tenglamasini topish uchun normallashtiruvchi M koʻpaytuvchini topib, *ABC* yoqning umumiy tenglamasining ikkala tomonini M ga koʻpaytiramiz. Umumiy tenglamada ozod had $D = -110<0$ boʻlgani uchun

$$M = \frac{1}{\sqrt{A^2 + B^2 + C^2}} = \frac{1}{\sqrt{4^2 + 11^2 + 3^2}} = \frac{1}{\sqrt{16 + 121 + 9}} = \frac{1}{\sqrt{146}} \quad \Rightarrow$$

$$\frac{4}{\sqrt{146}}x + \frac{11}{\sqrt{146}}y + \frac{3}{\sqrt{146}}z - \frac{110}{\sqrt{146}} = 0.$$

Demak, $\cos\alpha = 4/\sqrt{146}$, $\cos\beta = 11/\sqrt{146}$, $\cos\gamma = 3/\sqrt{146}$ va $p = 110/\sqrt{146}$.

7. Dastlab *ABD* yoq yotgan tekislikning umumiy tenglamasini topamiz:

$$\begin{vmatrix} x-8 & y-6 & z-4 \\ 10-8 & 5-6 & 5-4 \\ 9-8 & 10-6 & 7-4 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x-8 & y-6 & z-4 \\ 2 & -1 & 1 \\ 1 & 4 & 3 \end{vmatrix} = 0$$

$$\Rightarrow -3(x-8) + (y-6) + 8(z-4) + (z-4) - 6(y-6) + 4(x-8) = 0 \Rightarrow$$

$$\Rightarrow x - 5y + 9z - 14 = 0.$$

Umumiy tenglamalari $4x+11y+3z-110=0$ va $x-5y+9z-14=0$ bo’lgan *ABC* va *ABD* tekisliklar orasidagi burchak formulasiga asosan cosα qiymatini topamiz:

$$\cos\alpha = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2} \cdot \sqrt{A_2^2 + B_2^2 + C_2^2}} =$$

$$= \frac{4\cdot 1 + 11\cdot(-5) + 3\cdot 9}{\sqrt{4^2 + 11^2 + 3^2} \cdot \sqrt{1^2 + (-5)^2 + 9^2}} = -\frac{24}{\sqrt{146}\cdot\sqrt{107}} = -\frac{24}{\sqrt{15622}} .$$

8. Piramidaning D(9,10,7) uchidan tushirilgan DH balandlik yotgan L to'g'ri chiziq tenglamasini topish uchun dastlab bu nuqtadan o'tuvchi to'g'ri chiziqlar dastasi tenglamasidan foydalanamiz:

$$L: \quad \frac{x-9}{m}=\frac{y-10}{n}=\frac{z-7}{p}.$$

Bu to'g'ri chiziq ABC yoq yotgan va $4x+11y+3z-110=0$ umumiy tenglama bilan aniqlangan tekislikka perpendikulyar joylshgan. Shu sababli, fazodagi to'g'ri chiziq va tekislikning perpendikulyarlik shartiga asosan, $m=4$, $n=11$ va $p=3$ deb olish mumkin. Demak, DH balandlik yotgan L to'g'ri chiziqning kanonik tenglamasi quyidagicha bo'ladi:

$$L: \quad \frac{x-9}{4}=\frac{y-10}{11}=\frac{z-7}{3}.$$

9. Piramidaning D(9,10,7) uchidan tushirilgan DH balandlikning h uzunligini bu nuqtadan umumiy tenglamasi $4x+11y+3z-110=0$ boʻlgan ABC yoq yotgan tekislikkacha boʻlgan d masofa formulasidan foydalanib topamiz:

$$h=d=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}=\frac{|4\cdot 9+11\cdot 10+3\cdot 7-110|}{\sqrt{4^2+11^2+3^2}}=$$

$$=\frac{|36+110+21-110|}{\sqrt{146}}=\frac{57}{\sqrt{146}}.$$

III topshiriq

III.1-masala

Quyidagi berilgan funksiyalarning hosilalarini toping:

$$a)\ y=\frac{x}{\sqrt{a^2-x^2}}, \qquad \text{b)}\ y=(3x^2+5x-4)\sin x,$$

$$\text{c)}\ y=\ln(3tgx+e^x) \qquad \text{d)}\ x=t(\cos t-\sin t),\ y=t(\cos t+\sin t).$$

Yechish: *a*) Bo'linmaning hosilasi

$$\left(\frac{u}{v}\right)'=\frac{u'v-uv'}{v^2}$$

formulasida $u=x,\quad v=\sqrt{a^2-x^2}$ deb olib va hosilalar jadvalidan foydalanib, ushbu natijani olamiz:

$$y'=\left(\frac{x}{\sqrt{a^2-x^2}}\right)'=\frac{x'\sqrt{a^2-x^2}-x\left(\sqrt{a^2-x^2}\right)'}{(\sqrt{a^2-x^2})^2}=\frac{\sqrt{a^2-x^2}-\dfrac{1}{2\sqrt{a^2-x^2}}(a^2-x^2)'}{a^2-x^2}=$$

$$=\frac{\sqrt{a^2-x^2}+\dfrac{x^2}{\sqrt{a^2-x^2}}}{a^2-x^2}=\frac{(\sqrt{a^2-x^2})^2+x^2}{\sqrt{\left(a^2-x^2\right)^3}}=\frac{a^2-x^2+x^2}{\sqrt{\left(a^2-x^2\right)^3}}=\frac{a^2}{\sqrt{\left(a^2-x^2\right)^3}}\ .$$

b) Ko'paytmaning hosilasi $(uv)'=u'v+uv'$ formulasida

$$u=3x^2+5x-4,\quad v=\sin x$$

deb olib va hosilalar jadvalidan foydalanib, ushbu javobga kelamiz:

$$y'=\left((3x^2+5x-4)\sin x\right)'=(3x^2+5x-4)'\sin x+(3x^2+5x-4)(\sin x)'=$$
$$=(6x+5)\sin x+(3x^2+5x-4)\cos x.$$

c) Murakkab funksiyaning hosilasi $\left[f(u)\right]'=f'(u)\cdot u'$ formulasida $f(u)=\ln u,\quad u=3tgx+e^x$ deb olib va hosilalar jadvaliga asosan

$$y'=[\ln(3tgx+e^x)]'=(u=3tgx+e^x)=(\ln u)'=\frac{1}{u}u'=$$
$$=\frac{1}{3tgx+e^x}(3tgx+e^x)'=\frac{1}{3tgx+e^x}(3\cdot\frac{1}{\cos^2 x}+e^x)=\frac{3+e^x\cos^2 x}{(3tgx+e^x)\cos^2 x}$$

natijaga erishamiz.

d) Parametrik $x=\varphi(t)$, $y=\psi(t)$ ko'rinishda berilgan funksiyaning hosilasini topish

$$y'=\frac{\psi'(t)}{\varphi'(t)}$$

formulasida $x=\varphi(t)=t(\cos t-\sin t)$, $y=\psi(t)=t(\cos t+\sin t)$ deb, izlanayotgan y' hosilaning parametrik ko'rinishdagi ifodasini topamiz:

$$y'=\frac{[t(\cos t+\sin t)]'}{[t(\cos t-\sin t)]'}=\frac{t'(\cos t+\sin t)+t(\cos t+\sin t)'}{t'(\cos t-\sin t)+t(\cos t-\sin t)'}=$$
$$=\frac{(\cos t+\sin t)+t(-\sin t+\cos t)}{(\cos t-\sin t)+t(-\sin t-\cos t)}=\frac{\cos t+\sin t-t(\sin t-\cos t)}{\cos t-\sin t-t(\sin t+\cos t)}\ .$$

III.2-masala

Ushbu $y=\sqrt[3]{x^2}-2x-2$ funksiya grafigiga absissasi $x_0=1$ bo'lgan nuqtada o'tkazilgan urinma va normal tenglamasini tuzing.

Yechish: Ma'lumki, differentsiallanuvchi $y=f(x)$ funksiya grafigining $(x_0, y_0)=(x_0, f(x_0))$ nuqtasiga o'tkazilgan urinma tenglamasi

$$y-y_0=f'(x_0)(x-x_0)\,,$$

normal tenglamasi esa

$$y - y_0 = -\frac{1}{f'(x_0)}(x - x_0)$$

formulalar bilan topiladi. Bizning masalada $f(x) = \sqrt[3]{x^2} - 2x - 2$, x_0=1 va

$$y_0 = f(x_0) = f(1) = \sqrt[3]{1^2} - 2\cdot1 - 2 = 1 - 4 = -3,$$

$$y'(x) = f'(x) = \frac{2}{3\sqrt[3]{x}} - 2 \Rightarrow f'(x_0) = \frac{2}{3\sqrt[3]{x_0}} - 2 = \frac{2}{3\sqrt[3]{1}} - 2 = \frac{2}{3} - 2 = -\frac{4}{3}$$

bo’ladi. Bu yerdan urinma tenglamasi

$$y+3 = -\frac{4}{3}(x-1) \Rightarrow 3y+9 = -4x+4 \Rightarrow 4x+3y+5=0,$$

normal tenglamasi esa

$$y+3 = \frac{3}{4}(x-1) \Rightarrow 4y+12 = 3x-3 \Rightarrow 3x-4y-15=0$$

ko’rinishda ekanligi kelib chiqadi.

III.3-masala

Moddiy nuqta $s=t\sin^2 t$ tenglama bo’yicha harakatlanmoqda. Bu moddiy nuqtaning berilgan $t=\pi/4$ vaqtdagi $v(\pi/4)$ tezligini va $a(\pi/4)$ tezlanishini aniqlang.

Yechish: Harakat tenglamasi $s=s(t)$ bo’lgan moddiy nuqtaning $t=t_0$ vaqtdagi tezligi $v(t_0)=s'(t_0)$ va tezlanishi $a(t_0)=s''(t_0)$ hosilalar orqali topiladi. Shu sababli dastlab I tartibli $s'(t)$ va II tartibli $s''(t)$ hosilalarni hisoblaymiz:

$$s'(t) = (t\sin^2 t)' = t'\sin^2 t + t(\sin^2 t)' = \sin^2 t + t\cdot 2\sin t\cos t = \sin^2 t + t\sin 2t,$$

$$s''(t) = [s'(t)]' = [\sin^2 t + t\sin 2t]' = \sin 2t + \sin 2t + 2t\cos 2t = 2(\sin 2t + t\cos 2t)$$

Bu yerdan, yuqoridagi formulalarga asosan,

$$v(\frac{\pi}{4}) = s'(\frac{\pi}{4}) = \sin^2\frac{\pi}{4} + \frac{\pi}{4}\sin\frac{\pi}{2} = (\frac{\sqrt{2}}{2})^2 + \frac{\pi}{4}\cdot 1 = \frac{2+\pi}{4},$$

$$a(\frac{\pi}{4}) = s''(\frac{\pi}{4}) = 2(\sin\frac{\pi}{2} + \frac{\pi}{4}\cos\frac{\pi}{2}) = 2(1 + \frac{\pi}{4}\cdot 0) = 2\ .$$

III.4-masala

Berilgan $f(x)=x^3+4,5x^2-12x+1$ funksiyani ekstremumga tekshiring va uning monotonlik oraliqlarini toping.

Yechish: Berilgan $f(x)=x^3+4,5x^2-12x+1$ funksiyani ekstremumga tekshirish uchun dastlab $f'(x)=0$ tenglamadan uning kritik nuqtalarini topamiz:

$$f'(x)=(x^3+4,5x^2-12x+1)'=3x^2+9x-12=0 \Rightarrow 3x^2+9x-12=0 \Rightarrow x^2+3x-4=0,$$

$$x_{1,2}=\frac{-3\pm\sqrt{9+16}}{2}=\frac{-3\pm 5}{2} \Rightarrow x_1=-4,\ x_2=1.$$

Dastlab funksiyaning $x_1= -4$ kritik nuqtadagi xarakterini aniqlaymiz. Bunda $x<-4$ holda $f'(x)>0$ va $x>-4$ holda $f'(x)<0$ bo'ladi. Demak, $x_1= -4$ kritik nuqtada funksiya lokal maksimumga ega bo'ladi va

$$f_{max}=f(-4)=(-4)^3+4.5\cdot(-4)^2-12\cdot(-4)+1=57.$$

Endi funksiyaning $x_2= 1$ kritik nuqtadagi xarakterini aniqlaymiz. Bunda $x<1$ holda $f'(x)<0$ va $x>1$ holda $f'(x)>0$ bo'ladi. Demak, $x_2=1$ kritik nuqtada funksiya lokal minimumga ega bo'ladi va

$$f_{min}=f(1)=1^3+4.5\cdot1^2-12\cdot1+1= -5.5\ .$$

Funksiyaning monotonlk oraliqlari, ya'ni o'sish va kamayish sohalari, $f'(x)>0$ va $f'(x)<0$ tengsizliklarning yechimlari kabi topiladi. Bunda

$$f'(x)>0 \Rightarrow 3x^2+9x-12>0 \Rightarrow x<-4,\ x>1$$

bo'lgani uchun funksiyaning o'sish sohasi $(-\infty,-4)\cup(1,\infty)$ ekanligi kelib chiqadi.

Xuddi shunday tarzda

$$f'(x)<0 \Rightarrow 3x^2+9x-12<0 \Rightarrow -4<x<1$$

bo'lgani uchun funksiyaning kamayish sohasi $(-4,1)$ oraliqdan iborat ekanligi kelib chiqadi.

MUSTAQIL ISH TOPSHIRIQLARI

I topshiriq.

Ushbu chiziqli tenglamalar sistemasini Kramer, Gauss hamda matritsalar usulida yeching:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}$$

Izoh: Sistemadagi a_{ij} koeffitsient va b_i ozod hadlardan iborat parametrlar variant boʻyicha jadvaldan olinadi.

Variant №	Sistema tenglamalarining parametrlari											
	a_{11}	a_{12}	a_{13}	b_1	a_{21}	a_{22}	a_{23}	b_2	a_{31}	a_{32}	a_{33}	b_3
1	1	−3	3	–2	2	1	−3	1	1	−1	4	3
2	2	1	6	2	3	−1	−3	10	−2	4	−1	3
3	−2	3	2	0	4	−4	−4	4	1	6	1	13
4	5	−3	2	−3	−5	2	6	2	0	−1	−4	−1
5	1	1	1	0	1	0	2	−2	0	2	3	−1
6	3	−2	3	4	0	2	1	–4	2	−4	0	2
7	0	2	−1	3	1	3	0	9	5	−2	1	12
8	1	1	2	−3	2	1	1	−4	1	2	3	−7
9	2	−5	7	−1	1	1	−1	1	−3	2	−3	0
10	1	1	−1	1	1	−1	1	5	1	1	1	9
11	3	2	−1	5	0	2	−2	6	–3	7	−3	2
12	10	3	4	7	2	3	−4	–1	7	−5	−4	−9
13	3	2	−3	5	0	1	−1	–1	4	−2	8	4
14	8	1	−4	1	3	−3	1	–4	4	9	−1	1
15	9	−3	7	–7	−8	−2	1	−15	1	−1	1	–3
16	8	6	−1	−6	6	1	−2	0	2	4	2	−2
17	1	−6	−6	4	2	–1	2	5	1	3	6	1
18	1	−2	3	−1	2	1	−2	2	4	3	−3	10
19	5	3	4	−1	4	4	1	9	4	2	3	–1
20	1	0	−1	3	5	−1	7	–10	4	9	5	3
21	2	−3	6	–7	3	4	−1	–6	1	−5	2	10
22	1	4	−2	8	1	−5	2	−3	5	6	1	−1
23	2	−2	1	−6	4	3	−1	1	1	−4	2	−9
24	1	3	1	−2	1	4	2	−4	1	−5	−3	10
25	3	0	5	−1	0	2	1	−1	1	−3	1	2
26	3	2	1	9	2	3	1	5	2	1	3	11

27	4	−3	2	12	2	5	−3	−3	5	6	−2	0
28	1	1	−3	6	2	−1	1	−1	3	1	2	3
29	7	2	4	1	1	−3	−2	6	1	−4	−1	6
30	2	−3	−2	3	3	−2	1	1	3	−4	−1	5

II topshiriq

Fazoda uchlari $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ va $D(x_4, y_4, z_4)$ nuqtalarda joylashgan piramida berilgan. Bu piramida bo'yicha quyidagilarni bajaring:

1. $\overrightarrow{AB}$ vektor koordinatalarini toping va undan foydalanib AB qirra uzunligini hisoblang;

2. $\overrightarrow{AB}$ va $\overrightarrow{AD}$ vektorlardan foydalanib AB va AD qirralar orasidagi φ burchak kosinusini toping;

3. $\overrightarrow{AB}$ va $\overrightarrow{AD}$ vektorlardan foydalanib piramidaning ABD tomoni yuzasini toping ;

4. $\overrightarrow{AB}$, $\overrightarrow{AC}$ va $\overrightarrow{AD}$ vektorlar yordamida *ABCD* piramidaning hajmini aniqlang;

5. *AD* qirra yotgan toʻgʻri chiziqning kanonik va parametrik tenglamalarini yozing;

6. *ABC* yoq yotgan tekislikning umumiy, kesmalardagi va normal tenglamalarni yozing;

7. Piramidaning *ABC* va *ABD* yoqlari orasigi ikki yoqli α burchak kosinusini toping;

8. Piramidaning *D* uchidan tushirilgan *DH* balandligi yotuvchi *L* to'g'ri chiziqning kanonik tenglamasini aniqlang;

9. Piramidaning *D* uchidan tushirilgan *DH* balandligining uzunligini toping.

Izoh: $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ va $D(x_4, y_4, z_4)$ nuqtalarning koordinatalari variantga asosan jadvaldan olinadi.

Variant №	x_1	y_1	z_1	x_2	y_2	z_2	x_3	y_3	z_3	x_4	y_4	z_4
1	2	4	8	–3	5	1	6	–4	3	5	8	–1
2	–1	–3	–7	2	–4	0	–5	3	–2	–4	–7	2
3	3	5	9	0	6	–2	7	1	4	6	9	0
4	0	–2	7	–3	–3	5	–4	4	–1	–3	–6	3
5	–4	6	–3	7	7	–1	8	0	5	7	–3	1
6	1	–2	3	–4	5	–6	7	–8	9	–3	6	–5
7	2	–3	4	–5	6	–7	8	–9	3	–4	7	1
8	3	–4	–5	6	7	–8	9	–3	7	–4	–1	–2
9	4	5	–6	–7	8	–9	3	–5	7	1	2	–3

10	–5	6	7	–8	9	–2	–1	3	–4	–2	3	4
11	–6	7	–8	–9	0	3	2	1	–3	–3	–4	–5
12	7	–8	4	0	–1	2	1	–2	3	4	5	6
13	8	–9	1	–1	2	–3	–4	–5	–6	–7	0	4
14	9	–1	1	–2	1	–2	3	4	5	6	7	8
15	0	–1	2	1	–2	–3	–4	5	–6	7	–8	–9
16	1	–2	–1	–2	3	4	5	6	7	–5	0	8
17	2	1	2	3	–4	–5	–6	7	8	–9	0	–3
18	–3	4	–5	1	–8	7	–4	–2	1	2	–1	0
19	2	–5	3	–2	7	–8	3	–1	2	–3	1	5
20	–4	3	–5	0	–9	6	5	–3	0	1	–3	2
21	2	−3	6	17	3	4	−1	3	1	−5	2	10
22	1	4	−2	8	1	−5	−3	1	−4	6	1	4
23	2	−2	1	−6	4	3	−1	3	1	−4	2	−9
24	1	3	1	−2	1	4	2	−36	1	−5	−3	10
25	3	0	5	−1	0	2	1	−1	1	−3	1	2
26	3	2	1	5	2	3	1	1	2	1	3	11
27	4	−3	2	9	2	5	−3	4	5	6	−2	18
28	1	1	−3	6	2	−1	1	5	3	1	2	7
29	7	2	4	1	1	−3	−2	2	1	−4	−1	8
30	2	−3	−2	4	3	−2	1	11	3	−4	−1	7

III topshiriq

III.1-masala

Berilgan a), b), c) va d) hollardagi $y=f(x)$ funksiyalarning hosilalarini toping.

№	a) b)	$y = f(x)$	c) d)	$y = f(x)$
1	a)	$y = \dfrac{x-1}{x+1}$	c)	$y = \operatorname{arctg}(1+\ln x);$
	b)	$y = (x+1)\ln(x+1);$	d)	$x = \ln t\,,\ y = t^2$
2	a)	$y = \dfrac{x-2x^2}{1-\sin x};$	c)	$y = \arcsin(1-\ln x);$
	b)	$y = (x^2+1)\sin x;$	d)	$x = \sin t\,,\ y = t^2 - t$
3	a)	$y = \dfrac{10^x + x^{10}}{\sin x}$	c)	$y = \arccos\sqrt{1-\ln x};$
	b)	$y = (x^2+1)\operatorname{arcctg} x$	d)	$x = \cos t\,,\ y = t + t^2$

4	a)	$y=\dfrac{\mathrm{tg}x+\sin x}{x^2}$	c)	$y=e^{1-\cos 5x}$
	b)	$y=(1-x^2)\arcsin x$;	d)	$x=2t+1$, $y=\cos t^2$
5	a)	$y=\dfrac{\cos x+\sin x}{1+x}$	c)	$y=\arcsin(1-x^3)$
	b)	$y=x^2\ln(1+x^2)$;	d)	$x=\ln(t^2+1)$, $y=t^3$
6	a)	$y=\dfrac{\ln x}{1+x^2}$	c)	$y=\ln(1-\sqrt{x-1})$;
	b)	$y=x\mathrm{tg}(1+x^2)$;	d)	$x=e^{2t}$, $y=t^2$
7	a)	$y=\dfrac{x}{x^2-1}$;	c)	$y=\mathrm{arctg}\sqrt{1+x^2}$;
	b)	$y=(x+\sin x)(x-\cos x)$;	d)	$x=t^2$, $y=t^3+t^2+1$
8	a)	$y=\dfrac{1+\sin x}{1-\cos x}$;	c)	$y=\sqrt{1-\sin(x^2+1)}$;
	b)	$y=(x-\mathrm{tg}x)(x-\mathrm{ctg}x)$	d)	$x=t^2+t$, $y=t^3+1$
9	a)	$y=\dfrac{1-\mathrm{tg}x}{1+\mathrm{ctg}x}$	c)	$y=\sin(e^x+\cos x)$
	b)	$y=(x-1)\mathrm{arctg}\sqrt{x-2}$	d)	$x=t^2-4t$, $y=t^3+t$
10	a)	$y=\dfrac{\sqrt{x}}{1-\sqrt{x}}$;	c)	$y=\ln(x+\ln x)$
	b)	$y=(x-1)\arcsin\sqrt{2-x}$;	d)	$x=t^2-4t$, $y=(t+1)^3$
11	a)	$y=\dfrac{x-1}{5x-2}$	c)	$\left(\sqrt{x}+1\right)\left(\dfrac{1}{\sqrt{x}}-1\right)$
	b)	$y=\ln x\cdot\sin\sqrt{\ln x}$	d)	$x=(t-2)^2$, $y=t^3+t$
12	a)	$y=\dfrac{2x+3}{3x+7}$	c)	$y=5\mathrm{arctg}\,e^{\sqrt{5x}}$
	b)	$y=\left(x^2-3x+3\right)\left(x^2+2x-1\right)$	d)	$x=\sin(t-4)$, $y=\cos(t+3)$
13	a)	$y=\dfrac{5x^2}{x-3}$	c)	$(\dfrac{2}{\sqrt{x}}-\sqrt{3})(4\sqrt[3]{x}+\dfrac{\sqrt[3]{x^2}}{3x})$
	b)	$y=\ln\left(e^{5x}+1\right)$	d)	$x=\sin(2t-1)$, $y=\cos 2(2t-1)$
14	a)	$y=\dfrac{x^2+2x}{3-4x}$	c)	$y=\mathrm{tg}(2^x+x+1)$
	b)	$y=\left(1-x^2\right)\left(1-2x^3\right)$	d)	$x=2^t$, $y=t^2$
15	a)	$y=\dfrac{1+\sqrt{x}}{1-\sqrt{x}}$	c)	$y=\sin 3^x\cdot\cos^2 3^x$

	b)	$y=\left(x^2+x+1\right)\left(x^2-x+1\right)$	d)	$x=\sin(2t+1)$, $y=\cos 2(2t+1)$
16	a)	$y=\dfrac{x^2}{x+1}$	c)	$y=(x-1)(x-2)(x-3)$
	b)	$y=2\ln \mathrm{tg}(x/8)$	d)	$x=(2t-1)^2$, $y=\ln(2t+1)$
17	a)	$y=\dfrac{\sqrt{x}-2}{\sqrt{x}+2}$	c)	$y=\dfrac{1}{2}\cdot\left(\mathrm{tg}2x+\ln\cos^2 2x\right)$
	b)	$y=\left(\sqrt[3]{x}+1\right)(x-1)$	d)	$x=\ln(2t-1)$, $y=\ln(2t+1)$
18	a)	$y=\dfrac{\sqrt{x^3}-x}{x+\sqrt[3]{x^2}}$	c)	$y=\mathrm{ctg}^2\mathrm{ctg}x$
	b)	$y=\left(x^2-4\right)\left(x^2-9\right)$	d)	$x=\mathrm{tg}(2t-1)$, $y=(2t+1)^2$
19	a)	$y=\dfrac{x^2+7x+5}{x^2-3x}$	c)	$y=\arcsin\sqrt{1-e^x}$
	b)	$y=\left(1+\sqrt{2x}\right)\left(1+\sqrt{3x}\right)$	d)	$x=(2t-1)^2$, $y=(2t+1)^3$
20	a)	$y=\dfrac{-x^2+2x+3}{x^3-2}$	c)	$y=\ln\dfrac{1-\sin 3x}{1+\sin 3x}$
	b)	$y=\left(x^2+x-1\right)\left(x^3+1\right)$	d)	$x=10^{2t-1}$, $y=\lg(2t-1)$
21	a)	$y=\dfrac{x^2-1}{x^2+1}$;	c)	$y=\mathrm{tg}(1+\ln x)$
	b)	$y=(x+2)^2\ln(x+2)$;	d)	$y=(x^2-1)\mathrm{tg}x$;
22	a)	$y=\dfrac{2x-x^2}{1-\cos x}$;	c)	$y=\arccos(1+\ln x)$
	b)	$y=(x^2-1)\mathrm{tg}x$;	d)	$x=10^{\sin t}$, $y=t^2-2t$
23	a)	$y=\dfrac{1+e^{3x}}{\ln x}$	c)	$y=\cos\sqrt{1-\ln x}$
	b)	$y=(x^2-1)\mathrm{arcctg}x$	d)	$x=\arccos t$, $y=\arcsin t$
24	a)	$y=\dfrac{x+\ln x}{x^3}$	c)	$y=e^{\sin x}+e^{-\cos x}$
	b)	$y=(1+x^2)\mathrm{arctg}x$	d)	$x=(2t-3)^2$, $y=\sin t^2$
25	a)	$y=\dfrac{\sin x}{1+\cos x}$	c)	$y=\arccos(1+x^2)$
	b)	$y=e^x\ln(1+x^2)$	d)	$x=(t^2+1)^3$, $y=t^3$
26	a)	$y=\dfrac{x-1}{1+x^2}$	c)	$y=\ln(1+\sqrt{x+1})$
	b)	$y=x^3\sin(1+x^2)$	d)	$x=e^{-4t}$, $y=t^2+2t$

27	a)	$y=\dfrac{1+\sqrt{x}}{1-\sqrt{x}}$	c)	$y=\ln(x-\ln x)$
	b)	$y=(x-1)\arccos\sqrt{2-x}$	d)	$x=t^2+2t$, $y=(t+2)^3$
28	a)	$y=\dfrac{2x-1}{4x+3}$	c)	$y=\ln x\cdot\cos\sqrt{\ln x}$
	b)	$y=\left(\sqrt{x-1}\right)\left(1-\sqrt{x}\right)$	d)	$x=(t+2)^3$, $y=t^3+3t$
29	a)	$y=\dfrac{2x^2+1}{3x^2+5}$	c)	$y=\mathrm{ctg}(2^x-x^2+3)$
	b)	$y=\left(x^2+5x-3\right)\left(x^2-4x+5\right)$	d)	$x=\ln(t^2-4)$, $y=\lg(t+2)$
30	a)	$y=\dfrac{5x^2+3}{x^2-1}$	c)	$y=\ln\left(e^{5x}+1\right)$
	b)	$y=\left(1-x^2\right)\left(1-2x^3\right)$	d)	$x=\sin(2t+1)$, $y=\cos 2(2t+1)$

III.2-masala

$y=f(x)$ tenglama bilan berilgan egri chiziqning absissasi $x=x_0$ bo'lgan nuqtasiga o'tkazilgan urinma va normal tenglamalarini yozing.

№	$y=f(x)$	x_0	№	$y=f(x)$	x_0
1	$y=x^2+2x$	2	16	$y=3tg\,2x+1$	π/2
2	$y=80x-x^2$	–1	17	$y=1-4x+e^{3x}$	0
3	$y=1+2\cos x$	π/2	18	$y=6tg\,5x$	π/20
4	$y=\frac{1}{4}x^4+\frac{1}{3}x^3$	1	19	$y=4\sin 6x$	π/18
5	$y=\frac{1}{3}x^3+4x+3$	4	20	$y=\frac{x^2}{3}-\frac{x^2}{2}-7x+9$	1
6	$y=x+\sin 2x$	π/4	21	$y=x^2-3x+1$	–1
7	$y=xe^x$	0	22	$y=8x^3-x^2+1$	3
8	$y=13+tgx$	π/3	23	$y=1-2\cos x$	–π/2
9	$y=1-x^2$	1	24	$y=4tg\,3x$	π/9
10	$y=1+3x+e^{2x}$	0	25	$y=x^3-3x+5$	–2
11	$y=x^3-5x^2+7x-2$	–1	26	$y=x-\cos 2x$	π/4
12	$y=x^2-6x+2$	2	27	$y=e^x\cos x$	0

13	$y = \frac{x^2}{4} - x + 5$	4	28	$y = \text{ctg}x + \text{tg}x$	π/4
14	$y = \frac{x^4}{4} - 27x + 60$	–2	29	$y = \sin(1 - x^2)$	−1
15	$y = -\frac{x^2}{2} + 7x - \frac{15}{2}$	3	30	$y = 1 - 5x + e^{3x}$	0

III.3-masala

Moddiy nuqta s=s(t) tenglama bo'yicha harakatlanmoqda. Bu moddiy nuqtaning berilgan $t=t_0$ vaqtdagi $v(t_0)$ tezligini va $a(t_0)$ tezlanishini aniqlang.

№	$s = s(t)$	t_0	№	$s = s(t)$	t_0
1	$s = e^{\sin 2t}$	π/2	16	$s = 2^{\ln t}$	E
2	$s = te^t$	0	17	$s = e^t \cos t$	0
3	$s = \ln(t^2 - 9)$	5	18	$s = \ln^2(t - 1)$	2
4	$s = t^2 \ln t$	1	19	$s = \frac{\ln t}{t}$	E
5	$s = \frac{t^2}{t + 2}$	4	20	$s = \frac{1}{1 - e^t}$	ln2
6	$s = \frac{4t}{4 - t^2}$	$\sqrt{2}$	21	$s = \ln(t^2 + 1)$	0
7	$s = \frac{t^2 + 1}{t^2 - 1}$	3	22	$s = \frac{4t}{4 + \sin t}$	π/2
8	$s = \frac{t^2}{t - 2}$	5	23	$s = t\sqrt{5 + t}$	4
9	$s = \ln(4 - t^2)$	1	24	$s = \sqrt{t^2 - t}$	2
10	$s = \frac{t^2 + 1}{t - 1}$	0	25	$s = \frac{t^2}{t - 1}$	3
11	$s = e^{2\cos t}$	π/2	26	$s = \sqrt{t}e^t$	1
12	$s = t\sin t$	π/4	27	$s = t^3 \ln t$	1
13	$s = \ln(t^2 - 1)$	3	28	$s = \frac{t}{t^2 + 1}$	2
14	$s = (2 + t^2)\ln t$	e	29	$s = \ln^2(t + 1)$	0
15	$s = e^t \ln(t + 1)$	0	30	$s = \frac{t^2}{t + 4}$	0

III.4-masala

Berilgan $y = f(x)$ funksiyani ekstremumga tekshiring va uning monotonlik oraliqlarini toping.

№	$y = f(x)$	№	$y = f(x)$	№	$y = f(x)$
1	$y = e^{2x - x^2}$	11	$y = 2^{1/x}$	21	$y = e^{2x + x^2}$
2	$y = xe^{x^2}$	12	$y = x \cdot e^{-x}$	22	$y = xe^{x}$
3	$y = \ln(x^2 - 1)$	13	$y = e^{x} - x$	23	$y = \ln(x^2 - 9)$
4	$y = (2 + x^2)e^{-x^2}$	14	$y = \dfrac{\ln x}{x}$	24	$y = x^2 + 2\ln x$
5	$y = x^2 - 2\ln x$	15	$y = \ln(x^2 - 1)$	25	$y = \dfrac{x^2}{x + 2}$
6	$y = \dfrac{x^2}{x - 1}$	16	$y = \dfrac{1}{1 - e^{x}}$	26	$y = \dfrac{4x}{4 - x^2}$
7	$y = \dfrac{4x}{4 + x^2}$	17	$y = x - \ln x$	27	$y = \dfrac{x^2 + 1}{x^2 - 1}$
8	$y = \dfrac{x^2 - 1}{x^2 + 1}$	18	$y = x\sqrt{x + 5}$	28	$y = \dfrac{x^2}{x - 2}$
9	$y = \ln(9 - x^2)$	19	$y = \sqrt{x^2 - x}$	29	$y = \ln(4 - x^2)$
10	$y = \dfrac{x^2}{x + 4}$	20	$y = \sqrt{x - x^2}$	30	$y = \dfrac{x^2 + 1}{x - 1}$

ILOVA. HOSILALAR JADVALI

I. DARAJALI FUNKSIYALAR		
1	$(x^n)' = nx^{n-1}, \quad n \in (-\infty, \infty)$	2 $(u^n)' = nu^{n-1}u', \ u = u(x)$
3	$(C)' = 0, C - const. \quad (x)' = 1$ $(x^2)' = 2x \quad (x^3)' = 3x^2$	4 $(\frac{1}{x})' = -\frac{1}{x^2} \qquad (\sqrt{x})' = \frac{1}{2\sqrt{x}}$
II. KO'RSATGICHLI FUNKSIYALAR		
5	$(a^x)' = a^x \ln a, a > 0, a \neq 1$	6 $(a^u)' = a^u u' \ln a, u = u(x)$
7	$(e^x)' = e^x \qquad (10^x)' = 10^x \ln 10$	8 $(e^u)' = e^u \cdot u', \quad u = u(x)$
III. LOGARIFMIK FUNKSIYALAR		
9	$(\log_a x)' = \frac{1}{x \ln a} = \frac{\log_a e}{x}, a > 0, a \neq 1$	10 $(\log_a u)' = \frac{u'}{u \ln a} = \frac{u' \log_a e}{u}, u = u(x)$
11	$(\ln x)' = \frac{1}{x} \qquad (\lg x)' = \frac{1}{x \ln 10} = \frac{\lg e}{x}$	12 $(\ln u)' = \frac{1}{u} u', \quad u = u(x)$
IV. TRIGONOMETRIK FUNKSIYALAR		
13	$(\sin x)' = \cos x \qquad (tgx)' = \frac{1}{\cos^2 x}$	14 $(\sin u)' = \cos u \cdot u' \qquad (tgu)' = \frac{u'}{\cos^2 u}$
15	$(\cos x)' = -\sin x \quad (ctgx)' = -\frac{1}{\sin^2 x}$	16 $(\cos u)' = -\sin u \cdot u' \quad (ctgu)' = -\frac{u'}{\sin^2 u}$
17	$(\sec x)' = (\frac{1}{\cos x})' = \frac{\sin x}{\cos^2 x} = \sec x \cdot tgx$	18 $(\cos ecx)' = (\frac{1}{\sin x})' = -\frac{\cos x}{\sin^2 x} = -\cos ecx \cdot ctgx$
V. TESKARI TRIGONOMETRIK FUNKSIYALAR		
19	$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}, (\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$	20 $(\arcsin u)' = \frac{u'}{\sqrt{1-u^2}}, (\arccos u)' = -\frac{u'}{\sqrt{1-u^2}}$
21	$(arctgx)' = \frac{1}{1+x^2} \qquad (arcctgx)' = -\frac{1}{1+x^2}$	22 $(arctgu)' = \frac{u'}{1+u^2} \qquad (arcctgu)' = -\frac{u'}{1+u^2}$
VI. GIPERBOLIK FUNKSIYALAR		
23	$(shx)' = \left(\frac{e^x - e^{-x}}{2}\right)' = \frac{e^x + e^{-x}}{2} = chx$	24 $(chx)' = \left(\frac{e^x + e^{-x}}{2}\right)' = \frac{e^x - e^{-x}}{2} = shx$
25	$(thx)' = \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)' = \frac{e^x + e^{-x}}{e^x - e^{-x}} = cthx$	26 $(cthx)' = \left(\frac{e^x + e^{-x}}{e^x - e^{-x}}\right)' = \frac{e^x - e^{-x}}{e^x + e^{-x}} = thx$
VII. DIFFERENSIALLASH QOIDARLARI		
27	$(C \cdot u)' = C \cdot u' \qquad (u \pm v)' = u' \pm v'$	28 $(u \cdot v)' = u' \cdot v + u \cdot v'$
29	$(\frac{u}{v})' = \frac{u' \cdot v - u \cdot v'}{v^2}$	30 $[f(u)]' = f'(u) \cdot u', \quad u = u(x)$

31	$(u \cdot v)^{(n)} = \sum_{k=0}^{n} C_n^k u^{(n-k)} v^{(k)}, \quad C_n^k = \frac{k!}{n!(n-k)!}$	32	$(u^v)' = u^v v' \ln u + v u^{v-1} u'$

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